Computer algebra independent integration tests

4-Trig-functions/4.1-Sine/4.1.7-d-trig-^m-a+b-c-sin-^n-^p

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3.215 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 892

3.216 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 897

3.217 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 904

3.218 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 914

3.219 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 923

3.220 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 931

3.221 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 937

3.222 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 945

3.223 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$ 952

3.224 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 961

3.225 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 970

3.226 $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 979

3.227 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 988

3.228 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 996

3.229 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1004

3.230 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1017

3.231 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1031

3.232 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1046

3.233 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1060

3.234 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1075

3.235 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$ 1090

3.236 $\int \frac{1}{1-\sin^4(x)}\,dx$ 1102

3.237 $\int \frac{1}{a+b\sin^4(x)}\,dx$ 1105

3.238 $\int \frac{1}{1+\sin^4(x)}\,dx$ 1110

3.239 $\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$ 1116

3.240 $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$ 1120

3.241 $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1124

3.242 $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1128

3.243 $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1132

3.244 $\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1135

3.245 $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1138

3.246 $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1142

3.247 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1146

3.248 $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 1149

3.249 $\int \frac{1}{a+b\sin^5(x)}\,dx$ 1153

3.250 $\int \frac{1}{a+b\sin^6(x)}\,dx$ 1157

3.251 $\int \frac{1}{a+b\sin^8(x)}\,dx$ 1160

3.252 $\int \frac{1}{a-b\sin^5(x)}\,dx$ 1163

3.253 $\int \frac{1}{a-b\sin^6(x)}\,dx$ 1167

3.254 $\int \frac{1}{a-b\sin^8(x)}\,dx$ 1170

3.255 $\int \frac{1}{1+\sin^5(x)}\,dx$ 1173

3.256 $\int \frac{1}{1+\sin^6(x)}\,dx$ 1180

3.257 $\int \frac{1}{1+\sin^8(x)}\,dx$ 1183

3.258 $\int \frac{1}{1-\sin^5(x)}\,dx$ 1187

3.259 $\int \frac{1}{1-\sin^6(x)}\,dx$ 1194

3.260 $\int \frac{1}{1-\sin^8(x)}\,dx$ 1198

3.261 $\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx$ 1204

3.262 $\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx$ 1207

3.263 $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$ 1210

3.264 $\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx$ 1212

3.265 $\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$ 1214

3.266 $\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx$ 1216

3.267 $\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx$ 1219

3.268 $\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx$ 1222

3.269 $\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx$ 1224

3.270 $\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$ 1226

3.271 $\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx$ 1229
3.272 $\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx$ 1231
3.273 $\int \frac{\cos^9(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1233
3.274 $\int \frac{\cos^7(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1236
3.275 $\int \frac{\cos^5(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1238
3.276 $\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1240
3.277 $\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1242
3.278 $\int \frac{\sec(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1245
3.279 $\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1248
3.280 $\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1251
3.281 $\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1254
3.282 $\int \frac{\cos^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1256
3.283 $\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1258
3.284 $\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$ 1260
3.285 $\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1262
3.286 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1266
3.287 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1269
3.288 $\int \left(a+b\sin^2(e+fx)\right)\,dx$ 1272
3.289 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1274
3.290 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1277
3.291 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1279
3.292 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$ 1282
3.293 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1285
3.294 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1289
3.295 $\int \left(a+b\sin^2(e+fx)\right)^2\,dx$ 1293
3.296 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1296
3.297 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1299
3.298 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1302
3.299 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1305
3.300 $\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$ 1308
3.301 $\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$ 1311
3.302 $\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$ 1314
3.303 $\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$ 1318
3.304 $\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$ 1321
3.305 $\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$ 1324

3.306 $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$ 1327
3.307 $\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx$ 1330
3.308 $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$ 1333
3.309 $\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$ 1336
3.310 $\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx$ 1339
3.311 $\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$ 1343
3.312 $\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$ 1346
3.313 $\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$ 1350
3.314 $\int \frac{\cos^6(x)}{(a+b\sin^2(x))^2}\,dx$ 1353
3.315 $\int \frac{\cos^5(x)}{(a+b\sin^2(x))^2}\,dx$ 1357
3.316 $\int \frac{\cos^4(x)}{(a+b\sin^2(x))^2}\,dx$ 1360
3.317 $\int \frac{\cos^3(x)}{(a+b\sin^2(x))^2}\,dx$ 1364
3.318 $\int \frac{\cos^2(x)}{(a+b\sin^2(x))^2}\,dx$ 1367
3.319 $\int \frac{\cos(x)}{(a+b\sin^2(x))^2}\,dx$ 1370
3.320 $\int \frac{\sec(x)}{(a+b\sin^2(x))^2}\,dx$ 1373
3.321 $\int \frac{\sec^2(x)}{(a+b\sin^2(x))^2}\,dx$ 1377
3.322 $\int \frac{\sec^3(x)}{(a+b\sin^2(x))^2}\,dx$ 1380
3.323 $\int \frac{\sec^4(x)}{(a+b\sin^2(x))^2}\,dx$ 1385
3.324 $\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1389
3.325 $\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1393
3.326 $\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1396
3.327 $\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1400
3.328 $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1403
3.329 $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1407
3.330 $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1411
3.331 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$ 1415
3.332 $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1418
3.333 $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 1422
3.334 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1426
3.335 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1430
3.336 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1434
3.337 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1438
3.338 $\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1442

3.339 $\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1445
3.340 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1449
3.341 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1454
3.342 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1458
3.343 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1461
3.344 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 1465
3.345 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1469
3.346 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1472
3.347 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1475
3.348 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1478
3.349 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1482
3.350 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1486
3.351 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1489
3.352 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1492
3.353 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 1496
3.354 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1500
3.355 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1503
3.356 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1506
3.357 $\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1510
3.358 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1515
3.359 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1520
3.360 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1524
3.361 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1528
3.362 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 1531
3.363 $\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1536
3.364 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1540
3.365 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1543
3.366 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1546
3.367 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1551

3.368 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1556

3.369 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1561

3.370 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1566

3.371 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 1570

3.372 $\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx$ 1575

3.373 $\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1578

3.374 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1581

3.375 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1584

3.376 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1587

3.377 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1590

3.378 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1593

3.379 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1596

3.380 $\int \left(a+b\sin^2(e+fx)\right)^p\,dx$ 1599

3.381 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1602

3.382 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$ 1605

3.383 $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1608

3.384 $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1614

3.385 $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1619

3.386 $\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1624

3.387 $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1631

3.388 $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1640

3.389 $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1645

3.390 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$ 1649

3.391 $\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1652

3.392 $\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$ 1662

3.393 $\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1798

3.394 $\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1806

3.395 $\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1812

3.396 $\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1817

3.397 $\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1822

3.398 $\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1832

3.399 $\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1838

3.400 $\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1842

3.401 $\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$ 1845

3.402 $\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2} \, dx$ 1848

3.403 $\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2} \, dx$ 1853

3.404 $\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1859

3.405 $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1864

3.406 $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1868

3.407 $\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1872

3.408 $\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1876

3.409 $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1882

3.410 $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1890

3.411 $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1900

3.412 $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1910

3.413 $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1919

3.414 $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1925

3.415 $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1931

3.416 $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1935

3.417 $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1941

3.418 $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)} \, dx$ 1949

3.419 $\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1960

3.420 $\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1962

3.421 $\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1965

3.422 $\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1968

3.423 $\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1971

3.424 $\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1974

3.425 $\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1977

3.426 $\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1979

3.427 $\int \left(a+b\sin^4(e+fx)\right)^p \, dx$ 1981

3.428 $\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1983

3.429 $\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p \, dx$ 1985

3.430 $\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 1987

3.431 $\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 1989

3.432 $\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 1992

3.433 $\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 1995

3.434 $\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 1998

3.435 $\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 2000

3.436 $\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 2002

3.437 $\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 2004

3.438 $\int \left(a+b\sin^n(e+fx)\right)^p \, dx$ 2006

3.439 $\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$ 2008

3.440 $\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$ . . . 2010

3.441 $\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2012

3.442 $\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2015

3.443 $\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2018

3.444 $\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2021

3.445 $\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2024

3.446 $\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2027

3.447 $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2030

3.448 $\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2033

3.449 $\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2036

3.450 $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2040

3.451 $\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2043

3.452 $\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2046

3.453 $\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2049

3.454 $\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2052

3.455 $\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2055

3.456 $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$ . . . 2059

3.457 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^5(e+fx)\,dx$ . . . 2063

3.458 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^3(e+fx)\,dx$ . . . 2066

3.459 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan(e+fx)\,dx$ . . . 2069

3.460 $\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$ . . . 2072

3.461 $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$ . . . 2075

3.462 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^6(e+fx)\,dx$ . . . 2079

3.463 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$ . . . 2083

3.464 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^2(e+fx)\,dx$ . . . 2087

3.465 $\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$ . . . 2090

3.466 $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$ . . . 2093

3.467 $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$ . . . 2096

3.468 $\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ . . . 2099

3.469 $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ . . . 2102

3.470 $\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ . . . 2105

3.471 $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ . . . 2108

3.472 $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ . . . 2111

3.473 $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ 2115

3.474 $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ 2119

3.475 $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ 2122

3.476 $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ 2125

3.477 $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$ 2128

3.478 $\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2132

3.479 $\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2136

3.480 $\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2139

3.481 $\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2142

3.482 $\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2145

3.483 $\int \frac{\tan^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2149

3.484 $\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2153

3.485 $\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2156

3.486 $\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2159

3.487 $\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$ 2163

3.488 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^5(e+fx)\,dx$ 2167

3.489 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)\,dx$ 2172

3.490 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)\,dx$ 2176

3.491 $\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 2179

3.492 $\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 2182

3.493 $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 2186

3.494 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$ 2190

3.495 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^2(e+fx)\,dx$ 2194

3.496 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$ 2198

3.497 $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 2201

3.498 $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$ 2205

3.499 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$ 2209

3.500 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$ 2215

3.501 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$ 2220

3.502 $\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2224

3.503 $\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2227

3.504 $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2232
3.505 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$ 2237
3.506 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$ 2241
3.507 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2245
3.508 $\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2248
3.509 $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$ 2252
3.510 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2256
3.511 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2261
3.512 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2265
3.513 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2268
3.514 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2271
3.515 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2275
3.516 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2279
3.517 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2283
3.518 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2286
3.519 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2289
3.520 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$ 2292
3.521 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2296
3.522 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2303
3.523 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2308
3.524 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2312
3.525 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2316
3.526 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2320
3.527 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2325
3.528 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2330
3.529 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2335
3.530 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2338
3.531 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$ 2342
3.532 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2347

3.533 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2354
3.534 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2359
3.535 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2363
3.536 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2367
3.537 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2372
3.538 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2377
3.539 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2382
3.540 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2387
3.541 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2391
3.542 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$ 2396
3.543 $\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$ 2401
3.544 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx$ 2404
3.545 $\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx$ 2407
3.546 $\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$ 2409
3.547 $\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$ 2411
3.548 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx$ 2414
3.549 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx$ 2417
3.550 $\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$ 2420
3.551 $\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$ 2423
3.552 $\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$ 2426
3.553 $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$ 2432
3.554 $\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$ 2435
3.555 $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$ 2438
3.556 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2441
3.557 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2444
3.558 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2447
3.559 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2450
3.560 $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2453
3.561 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2457
3.562 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2461
3.563 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$ 2464
3.564 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$ 2468

3.565 $\int \left(a+b\sin^4(c+dx)\right)^p\tan^3(c+dx)\,dx$ 2470
3.566 $\int \left(a+b\sin^4(c+dx)\right)^p\tan(c+dx)\,dx$ 2474
3.567 $\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$ 2477
3.568 $\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$ 2480
3.569 $\int \left(a+b\sin^4(c+dx)\right)^p\tan^4(c+dx)\,dx$ 2483
3.570 $\int \left(a+b\sin^4(c+dx)\right)^p\tan^2(c+dx)\,dx$ 2485
3.571 $\int \left(a+b\sin^4(c+dx)\right)^p\,dx$ 2487
3.572 $\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$ 2489
3.573 $\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$ 2491
3.574 $\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx$ 2493
3.575 $\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx$ 2497
3.576 $\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$ 2501
3.577 $\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$ 2504
3.578 $\int \frac{\tan^m(c+dx)}{\left(a+b\sin^n(c+dx)\right)^2}\,dx$ 2506
3.579 $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$ 2508
3.580 $\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$ 2511
3.581 $\int (a+b\sin^n(c+dx))^p\tan^m(c+dx)\,dx$ 2514
3.582 $\int (a+b\sin^n(c+dx))^p\tan^3(c+dx)\,dx$ 2516
3.583 $\int (a+b\sin^n(c+dx))^p\tan(c+dx)\,dx$ 2518
3.584 $\int \cot(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$ 2520
3.585 $\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$ 2523
3.586 $\int (a+b\sin^n(c+dx))^p\tan^4(c+dx)\,dx$ 2526
3.587 $\int (a+b\sin^n(c+dx))^p\tan^2(c+dx)\,dx$ 2528
3.588 $\int (a+b\sin^n(c+dx))^p\,dx$ 2530
3.589 $\int \cot^2(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$ 2532
3.590 $\int \cot^4(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$ 2534
3.591 $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$ 2536
3.592 $\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx$ 2540
3.593 $\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$ 2543
3.594 $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$ 2546

4 Listing of Grading functions **2549**
4.0.1 Mathematica and Rubi grading function 2549
4.0.2 Maple grading function 2551
4.0.3 Sympy grading function 2554
4.0.4 SageMath grading function 2556

Chapter 1

Introduction

This report gives the result of running the computer algebra independent integration problems. The listing of the problems are maintained by and can be downloaded from `https://rulebasedintegration.org`

The number of integrals in this report is [594]. This is test number [79].

1.1 Listing of CAS systems tested

The following systems were tested at this time.

1. Mathematica 12.3 (64 bit) on windows 10.
2. Rubi 4.16.1 in Mathematica 12.1 on windows 10.
3. Maple 2021.1 (64 bit) on windows 10.
4. Maxima 5.44 on Linux. (via sagemath 9.3)
5. Fricas 1.3.7 on Linux (via sagemath 9.3)
6. Giac/Xcas 1.7 on Linux. (via sagemath 9.3)
7. Sympy 1.8 under Python 3.8.8 using Anaconda distribution on Ubuntu.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 under windows 10 (64 bit)

Maxima, Fricas and Giac/Xcas were called from inside SageMath. This was done using SageMath integrate command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly using Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	solved	Failed
Rubi	% 99.33 (590)	% 0.67 (4)
Mathematica	% 98.15 (583)	% 1.85 (11)
Maple	% 87.71 (521)	% 12.29 (73)
Maxima	% 55.89 (332)	% 44.11 (262)
Fricas	% 67.34 (400)	% 32.66 (194)
Sympy	% 10.77 (64)	% 89.23 (530)
Giac	% 54.04 (321)	% 45.96 (273)
Mupad	% 56.23 (334)	% 43.77 (260)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

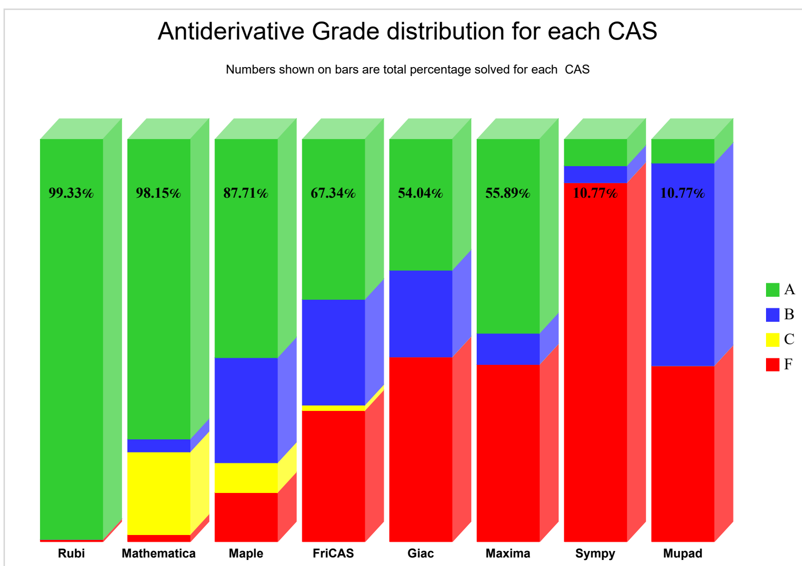
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

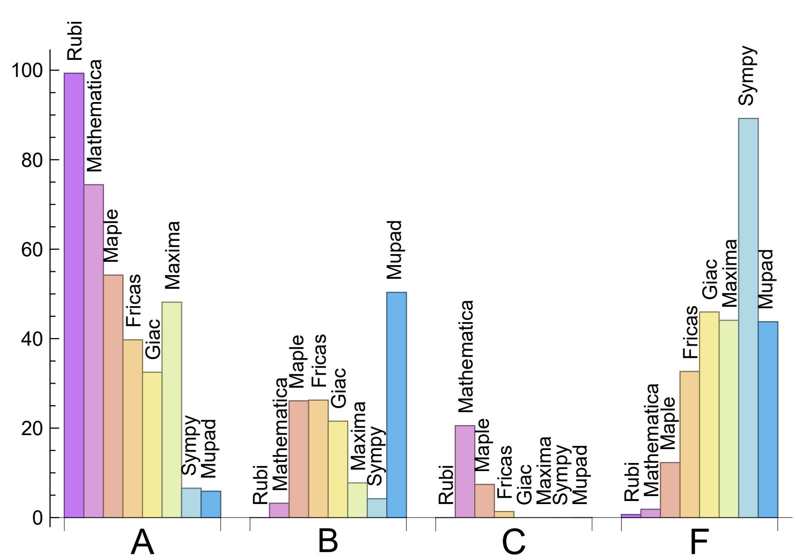
System	% A grade	% B grade	% C grade	% F grade
Rubi	99.33	0.00	0.00	0.67
Mathematica	74.41	3.20	20.54	1.85
Maple	54.21	26.09	7.41	12.29
Maxima	48.15	7.74	0.00	44.11
Fricas	39.73	26.26	1.35	32.66
Sympy	6.57	4.21	0.00	89.23
Giac	32.49	21.55	0.00	45.96
Mupad	5.89	50.34	0.00	43.77

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage time-out failure	Percentage exception failure
Rubi	4	100.00 %	0.00 %	0.00 %
Mathematica	11	100.00 %	0.00 %	0.00 %
Maple	73	94.52 %	5.48 %	0.00 %
Maxima	262	85.11 %	14.89 %	0.00 %
Fricas	194	77.84 %	17.53 %	4.64 %
Sympy	530	46.60 %	53.40 %	0.00 %
Giac	273	73.26 %	1.10 %	25.64 %
Mupad	260	98.85 %	1.15 %	0.00 %

Table 1.4: Time and leaf size performance for each CAS

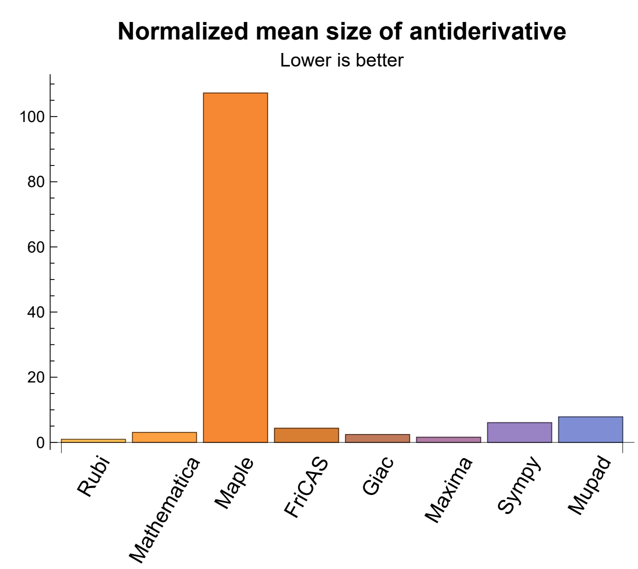
1.3 Performance

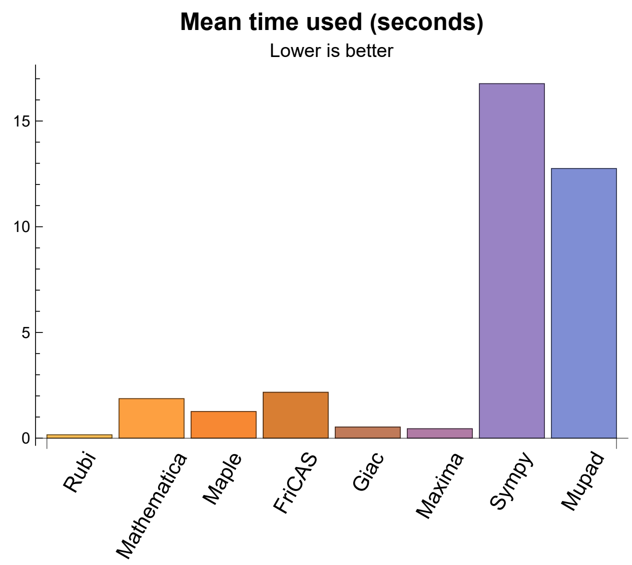
The table below summarizes the performance of each CAS system in terms of CPU time and leaf size of results.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.16	127.29	0.95	93.50	1.00
Mathematica	1.87	1004.00	3.07	98.00	0.97
Maple	1.26	8569.86	107.24	120.00	1.32
Maxima	0.45	130.71	1.58	69.00	1.07
Fricas	2.17	695.04	4.34	182.50	2.80
Sympy	16.77	248.06	6.04	156.00	3.21
Giac	0.53	315.14	2.41	86.00	1.38
Mupad	12.75	2012.82	7.84	99.50	1.14

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used columns from the above table.





1.4 list of integrals that has no closed form antiderivative

{399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590}

1.5 list of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {399, 400, 401, 402, 403}

Maple {399, 400, 401, 402, 403}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {399, 400, 401, 402, 403}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {}

Mathematica {132, 162, 171, 172, 173, 179, 188, 196, 198, 200, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 239, 240, 241, 242, 243, 244, 245, 249, 251, 252, 254, 328, 339, 348, 356, 357, 366, 372, 373, 378, 379, 380, 403, 565, 566, 574, 575, 576, 594}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AboluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of _int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call has completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out is not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for Rubi and Mathematica. Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative produced was correct.

Verification phase has 3 minutes time out. An integral whose result was not verified could still be correct. Further investigation is needed on those integrals which failed verifications. Such integrals are marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since these integrals are run in a batch mode, using an automated script, and by using sagemath (SageMath uses Maxima), then any integral where Maxima needs an interactive response from the user to answer a question during evaluation of the integral in order to complete the integration, will fail and is counted as failed.

The exception raised is `ValueError`. Therefore Maxima result below is lower than what could result if Maxima was run directly and each question Maxima asks was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the Timofeev test file, there were about 14 such integrals out of total 705, or about 2 percent. This pecrentage can be higher or lower depending on the specific input test file.

Such integrals can be indentified by looking at the output of the integration in each section for Maxima. The exception message will indicate of the error is due to the interactive question being asked or not.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath loading of Maxima `abs_integrate` was found to cause some problem. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS and Giac/XCAS results

There are Few integrals which failed due to SageMath not able to translate the result back to SageMath syntax and not because these CAS system were not able to do the integrations.

These will fail With error `Exception raised: NotImplementedError`

The number of such cases seems to be very small. About 1 or 2 percent of all integrals.

Hopefully the next version of SageMath will have complete translation of FriCAS and XCAS syntax and I will re-run all the tests again when this happens.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi and Maple, the buildin system function `LeafSize` is used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special buildin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy and Giac antiderivatives is determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which is called directly from Python, the following code is used to obtain the leafsize of its result

```
try:
```

```
# 1.7 is a fudge factor since it is low side from actual leaf count
leafCount = round(1.7*count_ops(anti))

except Exception as ee:
       leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative, Maple was used to determine the leaf size of Mupad output by post processing.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

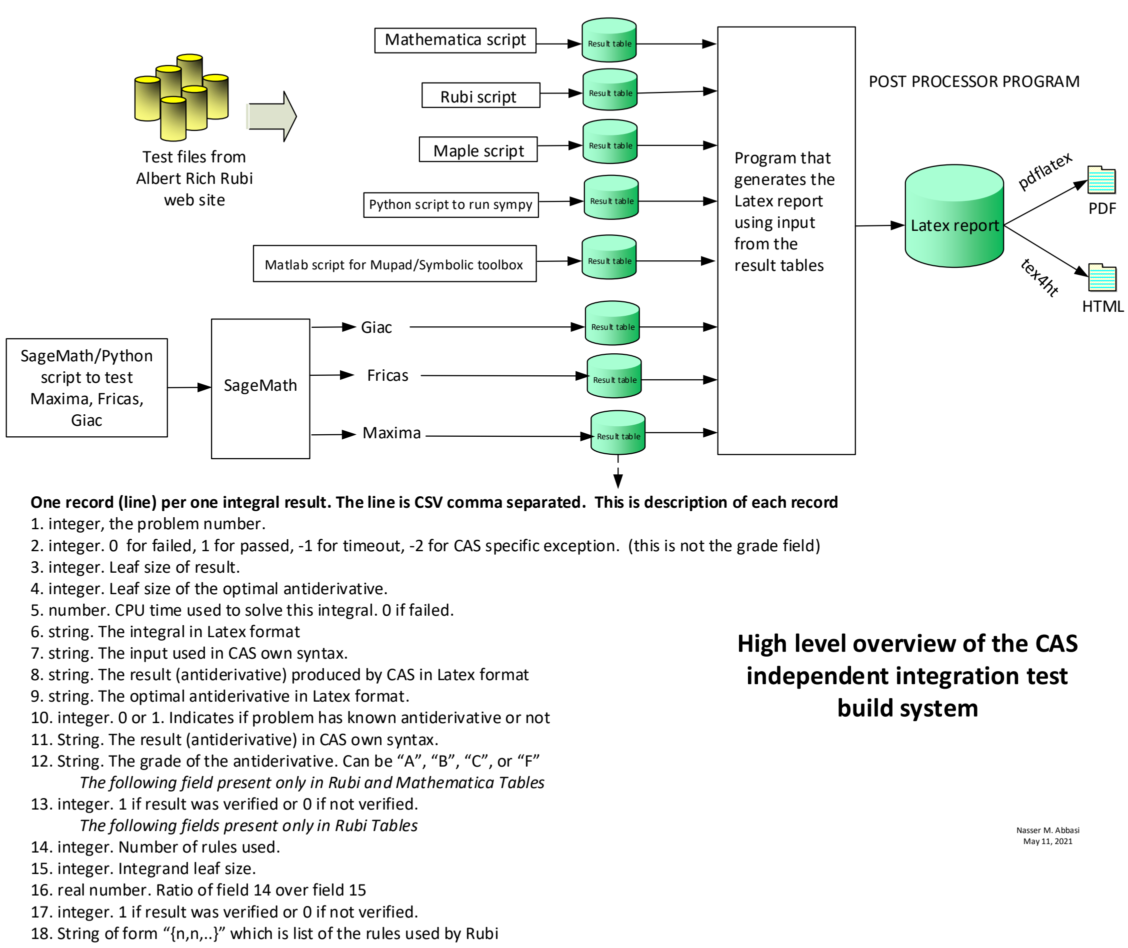
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



Chapter 2

detailed summary tables of results

2.1 List of integrals sorted by grade for each CAS

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592 }

B grade: { }

C grade: { }

F grade: { 391, 392, 593, 594 }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 71, 72, 73, 74, 75, 77, 85, 86, 87, 88, 89, 90, 91, 92, 93, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 116, 117, 118, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 174, 178, 203, 204, 205, 206, 207, 208, 209, 210, 211, 218, 219, 220, 221, 222, 223, 230, 231, 232, 233, 234, 235, 236, 256, 260, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 329, 330, 331, 332, 333, 334, 335, 336, 337, 340, 341, 342, 343, 344, 345, 346, 347, 349, 350, 351, 352, 353, 354, 355, 358, 359, 360, 361, 362, 363, 364, 365, 367, 368, 369, 370, 371, 372, 374, 375, 380, 384, 385, 395, 396, 399, 400, 401, 402, 403, 407, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 564, 567, 568, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 592 }

B grade: { 40, 54, 66, 67, 119, 120, 179, 265, 270, 276, 277, 308, 310, 312, 378, 379, 565, 566, 593 }

C grade: { 76, 78, 79, 80, 81, 82, 83, 84, 94, 95, 96, 97, 98, 99, 113, 114, 115, 172, 173, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 212, 213, 214, 215, 216, 217, 224, 225, 226, 227, 228, 229, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 294, 328, 338, 339, 348, 356, 357, 366, 373, 383, 386, 387, 388, 389, 390, 391, 392, 393, 394, 397, 398, 404, 405, 406, 408, 409, 410, 484, 521, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 561, 562, 563, 574, 575, 576, 591, 594 }

F grade: { 175, 176, 177, 180, 181, 376, 377, 381, 382, 423, 424 }

2.1.3 Maple

A grade: { 1, 2, 3, 6, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 107, 114, 116, 117, 118, 121, 127, 128, 129, 130, 131, 139, 140, 141, 142, 143, 148, 149, 151, 152, 154, 157, 158, 159, 160, 161, 162, 163, 164, 168, 170, 195, 196, 197, 198, 199, 200, 201, 202, 215, 217, 236, 238, 246, 256, 261, 262, 263, 264, 265, 267, 268, 271, 272, 273, 274, 275, 276, 279, 280, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 303, 305, 306, 307, 308, 309, 311, 313, 315, 317, 318, 319, 320, 321, 322, 324, 325, 329, 330, 331, 332, 333, 335, 340, 341, 342, 344, 345, 346, 349, 350, 352, 353, 354, 355, 358, 359, 360, 361, 364, 365, 368, 383, 384, 385, 386, 394, 395, 396, 397, 399, 400, 401, 402, 403, 407, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 450, 451, 452, 453, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 491, 492, 493, 494, 495, 496, 497, 498, 502, 503, 504, 505, 507, 508, 509, 513, 514, 515, 516, 519, 520, 524, 525, 526, 527, 528, 529, 530, 531, 541, 542, 552, 553, 554, 557, 564, 569, 570, 571, 572, 573, 577, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 5, 34, 84, 85, 106, 108, 109, 110, 111, 112, 113, 115, 120, 122, 123, 124, 125, 126, 132, 133, 134, 135, 136, 137, 138, 144, 145, 146, 147, 153, 155, 156, 165, 166, 167, 169, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 216, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 247, 259, 260, 266, 270, 277, 278, 298, 301, 302, 304, 310, 312, 314, 316, 323, 326, 327, 328, 334, 336, 337, 338, 339, 343, 347, 348, 356, 357, 362, 363, 366, 367, 369, 370, 371, 387, 393, 398, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 446, 447, 448, 449, 454, 455, 456, 488, 489, 490, 499, 500, 501, 506, 510, 511, 512, 517, 521, 522, 523, 532, 533, 534, 535, 536, 537, 538, 539, 540, 562, 591, 593 }

C grade: { 7, 8, 9, 10, 11, 12, 119, 150, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 239, 241, 242, 243, 249, 250, 251, 252, 253, 254, 255, 257, 258, 269, 281, 351, 388, 389, 390, 391, 392, 518, 594 }

F grade: { 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 240, 244, 245, 248, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 420, 421, 422, 423, 424, 431, 432, 433, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 558, 559, 560, 561, 563, 565, 566, 567, 568, 574, 575, 576, 578, 584, 585, 592 }

2.1.4 Maxima

A grade: { 3, 4, 13, 14, 15, 16, 17, 18, 31, 32, 33, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 116, 117, 118, 122, 123, 124, 132, 133, 134, 144, 145, 153, 154, 163, 164, 236, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 311, 313, 314, 315, 316, 317, 318, 319, 320, 321, 324, 325, 326, 334, 335, 336, 345, 346, 354, 355, 363, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 464, 465, 466, 467, 468, 469, 472, 478, 479, 481, 482, 488, 489, 491, 492, 493, 499, 500, 502, 503, 504, 511, 513, 514, 515, 522, 524, 525, 526, 533, 535, 536, 537, 552, 553, 554, 555, 558, 559, 560, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 5, 6, 34, 112, 119, 120, 121, 146, 155, 162, 165, 265, 270, 276, 277, 310, 312, 322, 323, 347, 356, 366, 441, 462, 463, 470, 471, 473, 474, 475, 476, 477, 480, 483, 484, 485, 486, 487, 490, 501, 510, 512, 521, 523, 532, 534 }

C grade: { }

F grade: { 1, 2, 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 125, 126, 127, 128, 129, 130, 131, 135, 136, 137, 138, 139, 140, 141, 142, 143, 147, 148, 149, 150, 151, 152, 156, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 260, 327, 328, 329, 330, 331, 332, 333, 337, 338, 339, 340, 341, 342, 343, 344, 348, 349, 350, 351, 352, 353, 357, 358, 359, 360, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 399, 400, 401, 402, 403, 411, 412, 413, 414, 415, 416, 417, 418, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 556, 557, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.5 FriCAS

A grade: { 1, 2, 3, 5, 6, 13, 14, 15, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 97, 100, 116, 117, 118, 120, 121, 122, 125, 126, 132, 133, 136, 137, 147, 154, 163, 256, 261, 262, 263, 264, 266, 267, 268, 269, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 310, 312, 317, 319, 324, 328, 334, 338, 339, 355, 364, 365, 385, 395, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 451, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 499, 500, 501, 502, 503, 504, 510, 511, 512, 513, 514, 515, 521, 555, 558, 559, 560, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 4, 16, 67, 84, 89, 90, 91, 92, 93, 94, 95, 96, 98, 99, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 119, 123, 124, 134, 135, 144, 145, 146, 153, 155, 156, 162, 164, 165, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 260, 265, 270, 276, 277, 309, 311, 313, 314, 315, 316, 318, 320, 321, 322, 323, 325, 326, 327, 335, 336, 337, 345, 346, 347, 348, 354, 356, 357, 363, 366, 396, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 447, 448, 449, 450, 452, 453, 454, 455, 456, 522, 523, 524, 525, 526, 532, 533, 534, 535, 536, 537, 556, 557 }

C grade: { 383, 384, 386, 387, 393, 394, 397, 552 }

F grade: { 7, 8, 9, 10, 11, 12, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 127, 128, 129, 130, 131, 138, 139, 140, 141, 142, 143, 148, 149, 150, 151, 152, 157, 158, 159, 160, 161, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 259, 329, 330, 331, 332, 333, 340, 341, 342, 343, 344, 349, 350, 351, 352, 353, 358, 359, 360, 361, 362, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 398, 399, 400, 401, 402, 403, 420, 421, 422, 423, 424, 431, 432, 433, 494, 495, 496, 497, 498, 505, 506, 507, 508, 509, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.6 Sympy

A grade: { 3, 29, 31, 35, 36, 37, 38, 42, 43, 44, 45, 49, 50, 51, 52, 55, 56, 57, 58, 64, 65, 68, 69, 70, 74, 269, 281, 285, 286, 287, 288, 293, 294, 295, 307, 319, 385, 407, 577 }

B grade: { 32, 33, 34, 61, 62, 63, 75, 76, 77, 236, 261, 262, 263, 264, 265, 267, 268, 273, 274, 275, 276, 277, 279, 280, 282 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 39, 40, 41, 46, 47, 48, 53, 54, 59, 60, 66, 67, 71, 72, 73, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 266, 270, 271, 272, 278, 283, 284, 289, 290, 291, 292, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594 }

2.1.7 Giac

A grade: { 1, 2, 3, 4, 5, 6, 13, 14, 15, 16, 31, 32, 33, 34, 37, 38, 42, 43, 44, 45, 46, 47, 48, 49, 51, 52, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 68, 69, 70, 72, 73, 74, 75, 76, 77, 80, 81, 85, 86, 87, 90, 91, 92, 93, 96, 97, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 113, 114, 115, 120, 154, 237, 238, 261, 262, 263, 264, 266, 267, 268, 271, 272, 273, 274, 275, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 290, 291, 292, 293, 294, 295, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 314, 315, 316, 317, 318, 319, 320, 321, 355, 364, 365, 383, 384, 385, 386, 387, 393, 394, 395, 396, 397, 398, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 445, 446, 452, 453, 454, 460, 471, 481, 491, 502, 513, 524, 535, 552, 553, 554, 558, 564, 569, 570, 571, 572, 573, 577, 578, 579, 580, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 29, 35, 36, 39, 40, 41, 50, 53, 54, 66, 67, 71, 78, 79, 82, 83, 84, 88, 89, 94, 95, 98, 99, 112, 116, 117, 118, 121, 126, 136, 137, 156, 162, 163, 164, 198, 199, 203, 204, 205, 206, 207, 208, 209, 210, 211, 215, 216, 218, 219, 220, 221, 222, 223, 227, 228, 230, 231, 232, 233, 234, 235, 236, 256, 259, 260, 265, 269, 270, 276, 277, 289, 296, 310, 311, 312, 313, 322, 323, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 441, 442, 443, 444, 447, 448, 449, 450, 451, 455, 456, 478, 479, 480, 488, 489, 490, 499, 500, 501, 510, 511, 512, 515, 521, 522, 523, 525, 526, 532, 533, 534, 536, 537 }

C grade: { }

F grade: { 7, 8, 9, 10, 11, 12, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 119, 122, 123, 124, 125, 127, 128, 129, 130, 131, 132, 133, 134, 135, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 157, 158, 159, 160, 161, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 200, 201, 202, 212, 213, 214, 217, 224, 225, 226, 229, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 257, 258, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 388, 389, 390, 391, 392, 399, 420, 421, 422, 423, 424, 431, 432, 433, 457, 458, 459, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 472, 473, 474, 475, 476, 477, 482, 483, 484, 485, 486, 487, 492, 493, 494, 495, 496, 497, 498, 503, 504, 505, 506, 507, 508, 509, 514, 516, 517, 518, 519, 520, 527, 528, 529, 530, 531, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 555, 556, 557, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 584, 585, 591, 592, 593, 594 }

2.1.8 Mupad

A grade: { 399, 400, 401, 402, 403, 419, 425, 426, 427, 428, 429, 430, 434, 435, 436, 437, 438, 439, 440, 564, 569, 570, 571, 572, 573, 577, 578, 581, 582, 583, 586, 587, 588, 589, 590 }

B grade: { 3, 16, 17, 18, 29, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 118, 154, 163, 164, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 325, 335, 346, 355, 364, 365, 375, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 422, 433, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 465, 466, 467, 468, 469, 470, 475, 476, 477, 478, 479, 480, 485, 486, 487, 552 }

C grade: { }

F grade: { 1, 2, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 30, 114, 115, 116, 117, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 155, 156, 157, 158, 159, 160, 161, 162, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 324, 326, 327, 328, 329, 330, 331, 332, 333, 334, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 347, 348, 349, 350, 351, 352, 353, 354, 356, 357, 358, 359, 360, 361, 362, 363, 366, 367, 368, 369, 370, 371, 372, 373, 374, 376, 377, 378, 379, 380, 381, 382, 420, 421, 423, 424, 431, 432, 460, 461, 462, 463, 464, 471, 472, 473, 474, 481, 482, 483, 484, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 565, 566, 567, 568, 574, 575, 576, 579, 580, 584, 585, 591, 592, 593, 594 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$

Problem 1	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	0	43	0	45	-1
normalized size	1	1.00	0.68	0.60	0.00	0.81	0.00	0.85	-0.02
time (sec)	N/A	0.029	0.033	0.709	0.000	0.433	0.000	3.741	0.000

Problem 2	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	0	29	0	24	-1
normalized size	1	1.00	0.76	0.71	0.00	0.85	0.00	0.71	-0.03
time (sec)	N/A	0.019	0.039	0.819	0.000	0.424	0.000	0.130	0.000

Problem 3	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	14	14	14	16	13	19	20	17	40
normalized size	1	1.00	1.00	1.14	0.93	1.36	1.43	1.21	2.86
time (sec)	N/A	0.010	0.004	0.614	0.705	0.437	0.579	0.134	13.638

Problem 4	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	17	30	49	26	70	0	15	-1
normalized size	1	1.00	1.76	2.88	1.53	4.12	0.00	0.88	-0.06
time (sec)	N/A	0.011	0.015	0.905	0.634	0.448	0.000	0.159	0.000

Problem 5	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	55	70	314	58	0	61	-1
normalized size	1	1.00	1.31	1.67	7.48	1.38	0.00	1.45	-0.02
time (sec)	N/A	0.024	0.062	1.558	0.690	0.446	0.000	0.227	0.000

Problem 6	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	77	89	931	78	0	80	-1
normalized size	1	1.00	1.26	1.46	15.26	1.28	0.00	1.31	-0.02
time (sec)	N/A	0.030	0.207	1.454	0.868	0.450	0.000	0.247	0.000

Problem 7	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	65	155	0	0	0	0	-1
normalized size	1	1.00	0.53	1.26	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.175	0.427	0.000	0.469	0.000	0.000	0.000

Problem 8	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	54	337	0	0	0	0	-1
normalized size	1	1.00	0.74	4.62	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.101	0.507	0.000	0.476	0.000	0.000	0.000

Problem 9	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	41	124	0	0	0	0	-1
normalized size	1	1.00	0.82	2.48	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.018	0.031	0.482	0.000	0.440	0.000	0.000	0.000

Problem 10	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	37	330	0	0	0	0	-1
normalized size	1	1.00	0.77	6.88	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.017	0.024	0.392	0.000	0.444	0.000	0.000	0.000

Problem 11	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	48	360	0	0	0	0	-1
normalized size	1	1.00	0.62	4.68	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.025	0.068	0.458	0.000	0.435	0.000	0.000	0.000

Problem 12	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	60	1301	0	0	0	0	-1
normalized size	1	1.00	0.49	10.58	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.216	0.400	0.000	0.440	0.000	0.000	0.000

Problem 13	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	53	63	85	82	0	57	-1
normalized size	1	1.00	0.40	0.48	0.64	0.62	0.00	0.43	-0.01
time (sec)	N/A	0.044	0.175	0.472	0.544	0.453	0.000	0.146	0.000

Problem 14	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	38	47	55	56	0	27	-1
normalized size	1	1.00	0.49	0.60	0.71	0.72	0.00	0.35	-0.01
time (sec)	N/A	0.028	0.099	0.280	0.610	0.443	0.000	0.128	0.000

Problem 15	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	25	33	22	36	0	15	-1
normalized size	1	1.00	0.69	0.92	0.61	1.00	0.00	0.42	-0.03
time (sec)	N/A	0.013	0.017	0.278	0.477	0.438	0.000	0.143	0.000

Problem 16	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	15	9	36	0	9	7
normalized size	1	1.00	1.00	0.94	0.56	2.25	0.00	0.56	0.44
time (sec)	N/A	0.014	0.006	0.261	0.517	0.422	0.000	0.130	13.712

Problem 17	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	34	29	23	74	0	0	44
normalized size	1	1.00	0.50	0.43	0.34	1.09	0.00	0.00	0.65
time (sec)	N/A	0.020	0.033	0.227	0.721	0.421	0.000	0.000	14.279

Problem 18	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	47	41	35	104	0	0	117
normalized size	1	1.00	0.40	0.35	0.30	0.88	0.00	0.00	0.99
time (sec)	N/A	0.028	0.052	0.303	0.589	0.433	0.000	0.000	16.558

Problem 19	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	74	0	0	0	0	0	-1
normalized size	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.039	0.170	6.446	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	72	0	0	0	0	0	-1
normalized size	1	1.00	0.87	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.038	0.114	0.550	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	68	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.036	0.068	0.665	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	72	0	0	0	0	0	-1
normalized size	1	1.00	0.90	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.042	0.076	0.542	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	71	0	0	0	0	0	-1
normalized size	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.108	0.551	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F(-2)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	73	0	0	0	0	0	-1
normalized size	1	1.00	0.82	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.041	0.107	0.535	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	71	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.058	0.865	0.000	0.448	0.000	0.000	0.000

Problem 26	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	61	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.072	1.844	0.000	0.446	0.000	0.000	0.000

Problem 27	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.035	0.074	1.714	0.000	0.437	0.000	0.000	0.000

Problem 28	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	0	0	0	0	0	-1
normalized size	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.033	0.084	2.094	0.000	0.431	0.000	0.000	0.000

Problem 29	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	0	0	16	61	384	36
normalized size	1	1.00	1.00	0.00	0.00	0.64	2.44	15.36	1.44
time (sec)	N/A	0.019	0.033	0.851	0.000	0.443	2.107	3.426	13.748

Problem 30	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	73	0	0	0	0	0	-1
normalized size	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.037	0.051	1.021	0.000	0.460	0.000	0.000	0.000

Problem 31	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	18	17	12	15	17	11
normalized size	1	1.00	1.00	1.12	1.06	0.75	0.94	1.06	0.69
time (sec)	N/A	0.009	0.003	0.072	0.323	0.419	0.099	0.121	13.575

Problem 32	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	43	40	28	110	25	33
normalized size	1	1.00	0.79	1.30	1.21	0.85	3.33	0.76	1.00
time (sec)	N/A	0.026	0.003	0.337	0.349	0.424	1.122	0.139	13.762

Problem 33	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	34	72	69	37	233	34	42
normalized size	1	1.00	0.74	1.57	1.50	0.80	5.07	0.74	0.91
time (sec)	N/A	0.032	0.003	0.436	0.319	0.449	2.663	0.128	13.685

Problem 34	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	42	105	104	46	376	43	51
normalized size	1	1.00	0.71	1.78	1.76	0.78	6.37	0.73	0.86
time (sec)	N/A	0.042	0.003	0.544	0.323	0.454	7.317	0.122	13.694

Problem 35	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	58	45	50	46	314	149	54
normalized size	1	1.00	0.94	0.73	0.81	0.74	5.06	2.40	0.87
time (sec)	N/A	0.087	0.058	0.326	0.362	0.439	35.438	0.149	13.648

Problem 36	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	43	35	40	36	143	105	38
normalized size	1	1.00	0.93	0.76	0.87	0.78	3.11	2.28	0.83
time (sec)	N/A	0.080	0.037	0.306	0.362	0.443	14.348	0.142	0.057

Problem 37	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	27	27	25	23	27	25	36	29	25
normalized size	1	1.00	0.93	0.85	1.00	0.93	1.33	1.07	0.93
time (sec)	N/A	0.065	0.032	0.296	0.344	0.438	6.935	0.134	0.045

Problem 38	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	16	15	15	34	15	15
normalized size	1	1.00	1.00	1.23	1.15	1.15	2.62	1.15	1.15
time (sec)	N/A	0.034	0.012	0.192	0.361	0.392	2.485	0.126	13.601

Problem 39	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	46	51	46	55	0	62	31
normalized size	1	1.00	1.59	1.76	1.59	1.90	0.00	2.14	1.07
time (sec)	N/A	0.058	0.039	0.442	0.349	0.441	0.000	0.146	0.084

Problem 40	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	146	87	70	98	0	149	55
normalized size	1	1.00	2.52	1.50	1.21	1.69	0.00	2.57	0.95
time (sec)	N/A	0.096	0.262	0.507	0.363	0.432	0.000	0.213	0.088

Problem 41	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	132	123	90	135	0	181	74
normalized size	1	1.00	1.61	1.50	1.10	1.65	0.00	2.21	0.90
time (sec)	N/A	0.095	4.262	0.490	0.344	0.429	0.000	0.174	0.096

Problem 42	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	44	84	72	56	1161	63	68
normalized size	1	1.00	0.60	1.15	0.99	0.77	15.90	0.86	0.93
time (sec)	N/A	0.090	0.186	0.323	0.426	0.431	28.493	0.148	13.718

Problem 43	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	49	49	34	56	49	45	502	50	45
normalized size	1	1.00	0.69	1.14	1.00	0.92	10.24	1.02	0.92
time (sec)	N/A	0.082	0.126	0.317	0.442	0.424	12.748	0.137	13.532

Problem 44	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	27	30	26	34	100	26	20
normalized size	1	1.00	1.35	1.50	1.30	1.70	5.00	1.30	1.00
time (sec)	N/A	0.063	0.012	0.232	0.452	0.432	4.572	0.154	13.737

Problem 45	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	14	13	21	41	13	13
normalized size	1	1.00	1.00	1.08	1.00	1.62	3.15	1.00	1.00
time (sec)	N/A	0.022	0.006	0.319	0.359	0.402	1.888	0.142	13.595

Problem 46	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	16	25	28	36	0	19	17
normalized size	1	1.00	0.57	0.89	1.00	1.29	0.00	0.68	0.61
time (sec)	N/A	0.075	0.027	0.456	0.348	0.401	0.000	0.159	13.504

Problem 47	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	49	35	42	56	0	42	38
normalized size	1	1.00	1.07	0.76	0.91	1.22	0.00	0.91	0.83
time (sec)	N/A	0.080	0.044	0.470	0.326	0.421	0.000	0.153	13.727

Problem 48	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	70	45	52	77	0	52	50
normalized size	1	1.00	1.13	0.73	0.84	1.24	0.00	0.84	0.81
time (sec)	N/A	0.084	0.037	0.509	0.326	0.402	0.000	0.168	13.955

Problem 49	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	59	47	52	46	156	57	48
normalized size	1	1.00	0.91	0.72	0.80	0.71	2.40	0.88	0.74
time (sec)	N/A	0.084	0.050	0.304	0.345	0.431	106.837	0.175	13.643

Problem 50	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	42	37	41	38	156	106	36
normalized size	1	1.00	0.89	0.79	0.87	0.81	3.32	2.26	0.77
time (sec)	N/A	0.069	0.033	0.276	0.343	0.428	49.027	0.162	0.054

Problem 51	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	31	29	28	28	156	28	26
normalized size	1	1.00	0.94	0.88	0.85	0.85	4.73	0.85	0.79
time (sec)	N/A	0.062	0.031	0.244	0.334	0.422	20.999	0.144	13.557

Problem 52	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	16	156	16	16
normalized size	1	1.00	1.00	0.94	0.89	0.89	8.67	0.89	0.89
time (sec)	N/A	0.042	0.013	0.147	0.345	0.423	9.724	0.139	13.588

Problem 53	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	61	67	59	70	0	107	41
normalized size	1	1.00	1.30	1.43	1.26	1.49	0.00	2.28	0.87
time (sec)	N/A	0.062	0.039	0.427	0.327	0.440	0.000	0.177	0.088

Problem 54	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	208	104	86	118	0	175	70
normalized size	1	1.00	2.67	1.33	1.10	1.51	0.00	2.24	0.90
time (sec)	N/A	0.085	0.441	0.520	0.335	0.445	0.000	0.167	13.761

Problem 55	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	46	73	64	59	1275	68	66
normalized size	1	1.00	0.67	1.06	0.93	0.86	18.48	0.99	0.96
time (sec)	N/A	0.082	0.216	0.354	0.450	0.430	72.425	0.169	13.808

Problem 56	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	42	46	37	49	551	44	31
normalized size	1	1.00	1.11	1.21	0.97	1.29	14.50	1.16	0.82
time (sec)	N/A	0.056	0.015	0.287	0.427	0.424	29.678	0.150	13.479

Problem 57	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	17	16	32	94	16	16
normalized size	1	1.00	1.00	0.94	0.89	1.78	5.22	0.89	0.89
time (sec)	N/A	0.067	0.016	0.205	0.327	0.397	12.971	0.146	13.376

Problem 58	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	26	25	25	34	238	25	24
normalized size	1	1.00	0.81	0.78	0.78	1.06	7.44	0.78	0.75
time (sec)	N/A	0.025	0.039	0.298	0.353	0.411	5.172	0.127	13.455

Problem 59	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	50	37	40	46	0	48	36
normalized size	1	1.00	1.06	0.79	0.85	0.98	0.00	1.02	0.77
time (sec)	N/A	0.078	0.042	0.448	0.337	0.398	0.000	0.147	13.575

Problem 60	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	46	47	52	72	0	34	48
normalized size	1	1.00	0.71	0.72	0.80	1.11	0.00	0.52	0.74
time (sec)	N/A	0.081	0.027	0.506	0.340	0.406	0.000	0.152	13.682

Problem 61	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	362	22	21
normalized size	1	1.00	1.07	0.69	0.76	0.86	12.48	0.76	0.72
time (sec)	N/A	0.021	0.005	0.161	0.367	0.399	7.347	0.131	13.410

Problem 62	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	28	31	675	28	33
normalized size	1	1.00	1.11	0.65	0.76	0.84	18.24	0.76	0.89
time (sec)	N/A	0.022	0.006	0.161	0.341	0.400	22.196	0.122	13.408

Problem 63	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	51	32	34	37	1083	34	43
normalized size	1	1.00	1.00	0.63	0.67	0.73	21.24	0.67	0.84
time (sec)	N/A	0.026	0.006	0.157	0.340	0.431	69.507	0.125	13.345

Problem 64	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	77	54	43	43	107	67	44
normalized size	1	1.00	1.51	1.06	0.84	0.84	2.10	1.31	0.86
time (sec)	N/A	0.045	0.030	0.442	0.339	0.412	3.315	0.131	13.366

Problem 65	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	54	34	34	27	58	40	27
normalized size	1	1.00	1.74	1.10	1.10	0.87	1.87	1.29	0.87
time (sec)	N/A	0.022	0.021	0.329	0.334	0.417	0.999	0.128	13.313

Problem 66	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	26	63	35	38	42	0	58	23
normalized size	1	1.00	2.42	1.35	1.46	1.62	0.00	2.23	0.88
time (sec)	N/A	0.025	0.024	0.309	0.335	0.456	0.000	0.164	13.366

Problem 67	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	118	63	58	95	0	121	42
normalized size	1	1.00	2.95	1.58	1.45	2.38	0.00	3.02	1.05
time (sec)	N/A	0.031	0.042	0.518	0.332	0.455	0.000	0.164	13.388

Problem 68	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	70	86	104	69	258	68	92
normalized size	1	1.00	0.79	0.97	1.17	0.78	2.90	0.76	1.03
time (sec)	N/A	0.053	0.108	0.459	0.439	0.451	5.373	0.134	13.944

Problem 69	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	45	65	74	50	158	43	68
normalized size	1	1.00	0.74	1.07	1.21	0.82	2.59	0.70	1.11
time (sec)	N/A	0.040	0.094	0.345	0.455	0.445	1.934	0.132	13.545

Problem 70	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	29	29	51	25	27
normalized size	1	1.00	1.10	1.07	0.97	0.97	1.70	0.83	0.90
time (sec)	N/A	0.015	0.031	0.070	0.326	0.426	0.349	0.122	13.401

Problem 71	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	16	22	23	32	0	39	16
normalized size	1	1.00	1.00	1.38	1.44	2.00	0.00	2.44	1.00
time (sec)	N/A	0.023	0.019	0.431	0.457	0.429	0.000	0.133	13.357

Problem 72	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	49	35	28	54	0	37	29
normalized size	1	1.00	1.14	0.81	0.65	1.26	0.00	0.86	0.67
time (sec)	N/A	0.036	0.028	0.588	0.331	0.395	0.000	0.146	13.368

Problem 73	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	95	56	45	81	0	61	49
normalized size	1	1.00	1.46	0.86	0.69	1.25	0.00	0.94	0.75
time (sec)	N/A	0.042	0.029	0.576	0.382	0.410	0.000	0.154	13.383

Problem 74	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	17	17	16	15	17	15
normalized size	1	1.00	1.00	0.89	0.89	0.84	0.79	0.89	0.79
time (sec)	N/A	0.009	0.004	0.062	0.337	0.428	0.063	0.133	13.310

Problem 75	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	43	42	39	47	110	42	44
normalized size	1	1.00	0.86	0.84	0.78	0.94	2.20	0.84	0.88
time (sec)	N/A	0.016	0.061	0.344	0.332	0.409	0.758	0.131	13.521

Problem 76	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	80	73	71	81	246	76	118
normalized size	1	1.00	0.92	0.84	0.82	0.93	2.83	0.87	1.36
time (sec)	N/A	0.083	0.102	0.467	0.347	0.427	2.757	0.147	14.129

Problem 77	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	113	110	108	123	410	118	147
normalized size	1	1.00	0.81	0.79	0.77	0.88	2.93	0.84	1.05
time (sec)	N/A	0.167	0.155	0.563	0.341	0.442	7.521	0.136	13.632

Problem 78	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	180	110	116	272	0	332	112
normalized size	1	1.00	1.70	1.04	1.09	2.57	0.00	3.13	1.06
time (sec)	N/A	0.113	1.441	0.324	0.452	0.462	0.000	0.177	0.163

Problem 79	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	150	70	88	218	0	173	72
normalized size	1	1.00	1.95	0.91	1.14	2.83	0.00	2.25	0.94
time (sec)	N/A	0.092	0.506	0.298	0.442	0.475	0.000	0.154	0.110

Problem 80	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	125	45	67	165	0	57	44
normalized size	1	1.00	2.40	0.87	1.29	3.17	0.00	1.10	0.85
time (sec)	N/A	0.070	0.243	0.269	0.425	0.468	0.000	0.169	0.097

Problem 81	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	97	29	50	117	0	37	29
normalized size	1	1.00	2.62	0.78	1.35	3.16	0.00	1.00	0.78
time (sec)	N/A	0.040	0.137	0.195	0.432	0.443	0.000	0.171	0.087

Problem 82	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	143	67	83	161	0	100	457
normalized size	1	1.00	2.60	1.22	1.51	2.93	0.00	1.82	8.31
time (sec)	N/A	0.064	0.291	0.459	0.433	0.460	0.000	0.163	13.727

Problem 83	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	85	85	224	142	120	327	0	196	592
normalized size	1	1.00	2.64	1.67	1.41	3.85	0.00	2.31	6.96
time (sec)	N/A	0.116	2.210	0.554	0.429	0.486	0.000	0.207	13.914

Problem 84	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	657	255	181	612	0	334	1105
normalized size	1	1.00	5.26	2.04	1.45	4.90	0.00	2.67	8.84
time (sec)	N/A	0.188	6.304	0.523	0.433	0.509	0.000	0.246	13.928

Problem 85	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	133	361	192	453	0	233	2244
normalized size	1	1.00	0.82	2.21	1.18	2.78	0.00	1.43	13.77
time (sec)	N/A	0.365	1.184	0.306	0.445	0.486	0.000	0.179	15.319

Problem 86	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	95	196	128	372	0	157	1892
normalized size	1	1.00	0.81	1.68	1.09	3.18	0.00	1.34	16.17
time (sec)	N/A	0.224	0.460	0.325	0.449	0.496	0.000	0.157	14.820

Problem 87	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	69	94	78	305	0	114	481
normalized size	1	1.00	0.90	1.22	1.01	3.96	0.00	1.48	6.25
time (sec)	N/A	0.114	0.321	0.248	0.436	0.480	0.000	0.152	13.974

Problem 88	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	46	46	46	50	46	260	0	81	104
normalized size	1	1.00	1.00	1.09	1.00	5.65	0.00	1.76	2.26
time (sec)	N/A	0.075	0.150	0.242	0.451	0.470	0.000	0.149	13.481

Problem 89	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	30	29	236	0	64	33
normalized size	1	1.00	1.00	0.83	0.81	6.56	0.00	1.78	0.92
time (sec)	N/A	0.024	0.081	0.381	0.429	0.450	0.000	0.134	13.527

Problem 90	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	52	48	313	0	83	45
normalized size	1	1.00	1.00	0.98	0.91	5.91	0.00	1.57	0.85
time (sec)	N/A	0.073	0.308	0.539	1.337	0.458	0.000	0.190	13.475

Problem 91	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	119	85	69	451	0	111	68
normalized size	1	1.00	1.55	1.10	0.90	5.86	0.00	1.44	0.88
time (sec)	N/A	0.106	0.669	0.573	0.428	0.476	0.000	0.189	13.431

Problem 92	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	147	138	98	595	0	155	95
normalized size	1	1.00	1.35	1.27	0.90	5.46	0.00	1.42	0.87
time (sec)	N/A	0.125	1.516	0.582	0.457	0.460	0.000	0.168	13.798

Problem 93	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	137	207	137	789	0	215	130
normalized size	1	1.00	0.98	1.48	0.98	5.64	0.00	1.54	0.93
time (sec)	N/A	0.156	1.708	0.605	0.562	0.483	0.000	0.185	15.074

Problem 94	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	194	118	154	529	0	322	123
normalized size	1	1.00	1.52	0.92	1.20	4.13	0.00	2.52	0.96
time (sec)	N/A	0.186	1.569	0.326	0.719	0.505	0.000	0.206	0.217

Problem 95	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	172	94	131	427	0	342	95
normalized size	1	1.00	1.69	0.92	1.28	4.19	0.00	3.35	0.93
time (sec)	N/A	0.146	0.915	0.325	0.900	0.473	0.000	0.210	0.164

Problem 96	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	160	80	111	327	0	93	71
normalized size	1	1.00	1.93	0.96	1.34	3.94	0.00	1.12	0.86
time (sec)	N/A	0.089	0.485	0.293	0.423	0.458	0.000	0.176	13.490

Problem 97	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	149	68	98	282	0	79	62
normalized size	1	1.00	2.01	0.92	1.32	3.81	0.00	1.07	0.84
time (sec)	N/A	0.054	0.295	0.219	0.461	0.452	0.000	0.157	0.105

Problem 98	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	194	150	149	455	0	246	2039
normalized size	1	1.00	1.88	1.46	1.45	4.42	0.00	2.39	19.80
time (sec)	N/A	0.129	0.804	0.509	0.571	0.536	0.000	0.194	14.631

Problem 99	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	390	226	223	838	0	512	2338
normalized size	1	1.00	2.55	1.48	1.46	5.48	0.00	3.35	15.28
time (sec)	N/A	0.243	1.571	0.618	0.477	0.573	0.000	0.201	14.946

Problem 100	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	106	187	181	623	0	223	2295
normalized size	1	1.00	0.72	1.26	1.22	4.21	0.00	1.51	15.51
time (sec)	N/A	0.292	1.532	0.342	0.454	0.493	0.000	0.183	16.064

Problem 101	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	93	140	109	492	0	140	1959
normalized size	1	1.00	1.00	1.51	1.17	5.29	0.00	1.51	21.06
time (sec)	N/A	0.138	0.856	0.274	0.483	0.484	0.000	0.192	15.227

Problem 102	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	74	77	74	419	0	109	72
normalized size	1	1.00	0.95	0.99	0.95	5.37	0.00	1.40	0.92
time (sec)	N/A	0.088	0.534	0.247	0.468	0.457	0.000	0.182	13.433

Problem 103	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	84	119	89	463	0	113	79
normalized size	1	1.00	0.97	1.37	1.02	5.32	0.00	1.30	0.91
time (sec)	N/A	0.063	0.413	0.358	0.483	0.469	0.000	0.155	13.426

Problem 104	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	130	155	144	133	588	0	179	132
normalized size	1	1.02	1.22	1.13	1.05	4.63	0.00	1.41	1.04
time (sec)	N/A	0.146	1.185	0.540	0.468	0.485	0.000	0.178	13.599

Problem 105	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	202	179	172	843	0	174	164
normalized size	1	1.00	1.25	1.10	1.06	5.20	0.00	1.07	1.01
time (sec)	N/A	0.203	1.300	0.556	0.562	0.500	0.000	0.194	14.398

Problem 106	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	134	363	234	950	0	224	3189
normalized size	1	1.00	0.91	2.45	1.58	6.42	0.00	1.51	21.55
time (sec)	N/A	0.279	2.698	0.327	0.517	0.549	0.000	0.243	17.957

Problem 107	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	97	136	158	683	0	152	149
normalized size	1	1.00	0.88	1.24	1.44	6.21	0.00	1.38	1.35
time (sec)	N/A	0.096	1.317	0.265	0.514	0.490	0.000	0.204	13.689

Problem 108	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	112	278	191	771	0	191	159
normalized size	1	1.00	0.85	2.12	1.46	5.89	0.00	1.46	1.21
time (sec)	N/A	0.149	1.389	0.260	0.948	0.500	0.000	0.198	13.854

Problem 109	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	125	334	211	843	0	211	176
normalized size	1	1.00	0.87	2.32	1.47	5.85	0.00	1.47	1.22
time (sec)	N/A	0.146	1.270	0.398	0.551	0.537	0.000	0.138	13.817

Problem 110	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	214	367	270	1003	0	232	251
normalized size	1	1.00	1.09	1.87	1.38	5.12	0.00	1.18	1.28
time (sec)	N/A	0.255	1.716	0.582	0.513	0.528	0.000	0.209	15.241

Problem 111	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	201	705	378	1361	0	344	339
normalized size	1	1.00	0.98	3.42	1.83	6.61	0.00	1.67	1.65
time (sec)	N/A	0.296	1.473	0.396	0.670	0.544	0.000	0.171	15.436

Problem 112	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	312	1249	588	2017	0	524	450
normalized size	1	1.00	1.12	4.48	2.11	7.23	0.00	1.88	1.61
time (sec)	N/A	0.531	1.927	0.407	0.533	0.605	0.000	0.168	16.104

Problem 113	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	11	11	29	33	10	57	0	10	18
normalized size	1	1.00	2.64	3.00	0.91	5.18	0.00	0.91	1.64
time (sec)	N/A	0.026	0.064	0.755	0.513	0.429	0.000	0.197	13.377

Problem 114	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	53	51	25	71	0	25	-1
normalized size	1	1.00	1.77	1.70	0.83	2.37	0.00	0.83	-0.03
time (sec)	N/A	0.030	0.053	1.110	0.544	0.464	0.000	0.149	0.000

Problem 115	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	15	15	39	57	11	94	0	11	-1
normalized size	1	1.00	2.60	3.80	0.73	6.27	0.00	0.73	-0.07
time (sec)	N/A	0.030	0.093	1.158	0.440	0.449	0.000	0.379	0.000

Problem 116	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	36	32	31	40	0	84	-1
normalized size	1	1.00	0.68	0.60	0.58	0.75	0.00	1.58	-0.02
time (sec)	N/A	0.054	0.022	0.877	0.522	0.446	0.000	0.206	0.000

Problem 117	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	26	24	17	26	0	57	-1
normalized size	1	1.00	0.76	0.71	0.50	0.76	0.00	1.68	-0.03
time (sec)	N/A	0.036	0.010	0.851	0.494	0.426	0.000	0.182	0.000

Problem 118	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	13	13	13	15	6	15	0	27	46
normalized size	1	1.00	1.00	1.15	0.46	1.15	0.00	2.08	3.54
time (sec)	N/A	0.026	0.005	0.525	0.477	0.423	0.000	0.146	0.216

Problem 119	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	C	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	16	16	46	20	38	65	0	0	-1
normalized size	1	1.00	2.88	1.25	2.38	4.06	0.00	0.00	-0.06
time (sec)	N/A	0.031	0.021	0.120	0.509	0.433	0.000	0.000	0.000

Problem 120	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	91	70	304	40	0	44	-1
normalized size	1	1.00	2.17	1.67	7.24	0.95	0.00	1.05	-0.02
time (sec)	N/A	0.037	0.067	1.339	0.535	0.434	0.000	0.233	0.000

Problem 121	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	72	89	933	49	0	129	-1
normalized size	1	1.00	1.18	1.46	15.30	0.80	0.00	2.11	-0.02
time (sec)	N/A	0.050	0.159	1.391	0.715	0.419	0.000	0.272	0.000

Problem 122	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	119	311	176	501	0	0	-1
normalized size	1	1.00	0.95	2.49	1.41	4.01	0.00	0.00	-0.01
time (sec)	N/A	0.128	0.430	1.904	0.464	0.813	0.000	0.000	0.000

Problem 123	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	93	182	70	433	0	0	-1
normalized size	1	1.00	1.19	2.33	0.90	5.55	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.265	1.327	0.443	0.551	0.000	0.000	0.000

Problem 124	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	99	174	132	1158	0	0	-1
normalized size	1	1.00	1.19	2.10	1.59	13.95	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.116	2.128	0.451	0.648	0.000	0.000	0.000

Problem 125	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	100	227	0	338	0	0	-1
normalized size	1	1.00	1.19	2.70	0.00	4.02	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.289	1.794	0.000	0.537	0.000	0.000	0.000

Problem 126	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	127	379	0	520	0	962	-1
normalized size	1	1.00	0.89	2.65	0.00	3.64	0.00	6.73	-0.01
time (sec)	N/A	0.132	0.508	2.205	0.000	0.731	0.000	0.548	0.000

Problem 127	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	199	413	0	0	0	0	-1
normalized size	1	1.00	0.77	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.316	1.428	1.550	0.000	0.480	0.000	0.000	0.000

Problem 128	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	159	266	0	0	0	0	-1
normalized size	1	1.00	1.00	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.191	0.828	1.453	0.000	0.475	0.000	0.000	0.000

Problem 129	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	-1
normalized size	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.087	0.872	0.000	0.439	0.000	0.000	0.000

Problem 130	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	137	156	0	0	0	0	-1
normalized size	1	1.00	0.79	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.163	0.606	1.318	0.000	0.436	0.000	0.000	0.000

Problem 131	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	188	342	0	0	0	0	-1
normalized size	1	1.00	0.80	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.260	3.194	1.487	0.000	0.451	0.000	0.000	0.000

Problem 132	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	152	446	248	579	0	0	-1
normalized size	1	1.00	0.90	2.64	1.47	3.43	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.781	1.822	0.452	1.948	0.000	0.000	0.000

Problem 133	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	113	309	106	495	0	0	-1
normalized size	1	1.00	0.99	2.71	0.93	4.34	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.415	1.684	0.432	0.821	0.000	0.000	0.000

Problem 134	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	141	255	179	1282	0	0	-1
normalized size	1	1.00	1.16	2.09	1.47	10.51	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.920	2.080	0.481	0.882	0.000	0.000	0.000

Problem 135	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	147	287	0	1449	0	0	-1
normalized size	1	1.00	1.15	2.24	0.00	11.32	0.00	0.00	-0.01
time (sec)	N/A	0.151	1.221	2.365	0.000	0.944	0.000	0.000	0.000

Problem 136	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	114	376	0	484	0	960	-1
normalized size	1	1.00	0.89	2.94	0.00	3.78	0.00	7.50	-0.01
time (sec)	N/A	0.125	0.719	1.949	0.000	0.743	0.000	0.695	0.000

Problem 137	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	161	565	0	752	0	1708	-1
normalized size	1	1.00	0.82	2.87	0.00	3.82	0.00	8.67	-0.01
time (sec)	N/A	0.175	1.177	2.516	0.000	1.780	0.000	1.139	0.000

Problem 138	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	249	602	0	0	0	0	-1
normalized size	1	1.00	0.77	1.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.482	2.654	1.608	0.000	0.517	0.000	0.000	0.000

Problem 139	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	201	429	0	0	0	0	-1
normalized size	1	1.00	0.92	1.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.304	1.382	1.408	0.000	0.482	0.000	0.000	0.000

Problem 140	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0	-1
normalized size	1	1.00	1.01	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.164	0.804	1.435	0.000	0.462	0.000	0.000	0.000

Problem 141	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	141	174	0	0	0	0	-1
normalized size	1	1.00	0.78	0.96	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	1.366	1.404	0.000	0.496	0.000	0.000	0.000

Problem 142	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	201	408	0	0	0	0	-1
normalized size	1	1.00	0.85	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.293	4.511	1.728	0.000	0.437	0.000	0.000	0.000

Problem 143	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	194	437	0	0	0	0	-1
normalized size	1	1.00	0.92	2.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.282	1.419	1.619	0.000	0.468	0.000	0.000	0.000

Problem 144	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	105	186	75	438	0	0	-1
normalized size	1	1.00	1.27	2.24	0.90	5.28	0.00	0.00	-0.01
time (sec)	N/A	0.096	0.279	1.476	0.431	0.567	0.000	0.000	0.000

Problem 145	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	53	99	24	370	0	0	-1
normalized size	1	1.00	1.29	2.41	0.59	9.02	0.00	0.00	-0.02
time (sec)	N/A	0.047	0.110	1.183	0.442	0.520	0.000	0.000	0.000

Problem 146	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	41	41	48	112	109	219	0	0	-1
normalized size	1	1.00	1.17	2.73	2.66	5.34	0.00	0.00	-0.02
time (sec)	N/A	0.076	0.165	1.389	0.438	0.509	0.000	0.000	0.000

Problem 147	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	102	231	0	347	0	0	-1
normalized size	1	1.00	1.15	2.60	0.00	3.90	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.314	2.098	0.000	0.559	0.000	0.000	0.000

Problem 148	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	163	268	0	0	0	0	-1
normalized size	1	1.00	0.79	1.30	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.199	0.901	1.436	0.000	0.451	0.000	0.000	0.000

Problem 149	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	78	93	0	0	0	0	-1
normalized size	1	1.00	0.70	0.84	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.126	0.224	1.237	0.000	0.445	0.000	0.000	0.000

Problem 150	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0	-1
normalized size	1	1.00	1.18	1.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.088	0.322	0.000	0.447	0.000	0.000	0.000

Problem 151	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	138	140	0	0	0	0	-1
normalized size	1	1.00	0.78	0.79	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	0.636	1.526	0.000	0.455	0.000	0.000	0.000

Problem 152	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	195	354	0	0	0	0	-1
normalized size	1	1.00	0.80	1.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.267	3.980	1.483	0.000	0.477	0.000	0.000	0.000

Problem 153	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	96	156	81	564	0	0	-1
normalized size	1	1.00	1.22	1.97	1.03	7.14	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.423	2.377	0.585	0.639	0.000	0.000	0.000

Problem 154	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	34	34	41	31	32	57	0	53	119
normalized size	1	1.00	1.21	0.91	0.94	1.68	0.00	1.56	3.50
time (sec)	N/A	0.045	0.117	0.885	0.388	0.443	0.000	0.580	15.179

Problem 155	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	93	165	165	422	0	0	-1
normalized size	1	1.00	1.18	2.09	2.09	5.34	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.353	2.383	0.487	0.574	0.000	0.000	0.000

Problem 156	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	134	274	0	634	0	520	-1
normalized size	1	1.00	1.00	2.04	0.00	4.73	0.00	3.88	-0.01
time (sec)	N/A	0.164	0.703	2.803	0.000	0.852	0.000	0.865	0.000

Problem 157	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	197	405	0	0	0	0	-1
normalized size	1	1.00	0.72	1.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.314	1.313	1.934	0.000	0.506	0.000	0.000	0.000

Problem 158	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	136	241	0	0	0	0	-1
normalized size	1	1.00	0.67	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.200	0.694	1.671	0.000	0.477	0.000	0.000	0.000

Problem 159	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	138	191	0	0	0	0	-1
normalized size	1	1.00	0.90	1.25	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.192	0.452	1.517	0.000	0.454	0.000	0.000	0.000

Problem 160	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0	-1
normalized size	1	1.00	0.89	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.057	0.150	1.740	0.000	0.450	0.000	0.000	0.000

Problem 161	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	170	199	0	0	0	0	-1
normalized size	1	1.00	0.72	0.85	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	1.284	1.795	0.000	0.485	0.000	0.000	0.000

Problem 162	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	133	243	283	885	0	338	-1
normalized size	1	1.00	0.97	1.77	2.07	6.46	0.00	2.47	-0.01
time (sec)	N/A	0.137	0.780	3.285	0.460	1.205	0.000	0.951	0.000

Problem 163	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	64	64	121	137	0	149	176
normalized size	1	1.00	0.79	0.79	1.49	1.69	0.00	1.84	2.17
time (sec)	N/A	0.095	0.324	1.314	0.345	0.658	0.000	0.790	20.866

Problem 164	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	60	55	63	134	0	137	159
normalized size	1	1.00	0.82	0.75	0.86	1.84	0.00	1.88	2.18
time (sec)	N/A	0.058	0.179	1.311	0.366	0.601	0.000	0.705	20.795

Problem 165	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	129	129	127	249	306	752	0	0	-1
normalized size	1	1.00	0.98	1.93	2.37	5.83	0.00	0.00	-0.01
time (sec)	N/A	0.153	0.630	3.709	0.530	0.905	0.000	0.000	0.000

Problem 166	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	285	285	192	698	0	0	0	0	-1
normalized size	1	1.00	0.67	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.332	2.141	1.883	0.000	0.557	0.000	0.000	0.000

Problem 167	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	269	269	182	623	0	0	0	0	-1
normalized size	1	1.00	0.68	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.276	1.706	1.787	0.000	0.480	0.000	0.000	0.000

Problem 168	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	174	483	0	0	0	0	-1
normalized size	1	1.00	0.79	2.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	1.526	1.751	0.000	0.474	0.000	0.000	0.000

Problem 169	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0	-1
normalized size	1	1.00	0.77	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	1.380	1.931	0.000	0.485	0.000	0.000	0.000

Problem 170	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	322	322	214	527	0	0	0	0	-1
normalized size	1	1.00	0.66	1.64	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.411	2.358	2.035	0.000	0.510	0.000	0.000	0.000

Problem 171	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	113	0	0	0	0	0	-1
normalized size	1	1.00	0.93	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.469	2.651	0.000	0.543	0.000	0.000	0.000

Problem 172	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	98	0	0	0	0	0	-1
normalized size	1	1.00	0.45	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.225	0.550	4.564	0.000	0.478	0.000	0.000	0.000

Problem 173	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	98	0	0	0	0	0	-1
normalized size	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.110	0.353	8.256	0.000	0.463	0.000	0.000	0.000

Problem 174	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	74	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.046	0.203	2.605	0.000	0.454	0.000	0.000	0.000

Problem 175	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	4.842	2.433	0.000	0.464	0.000	0.000	0.000

Problem 176	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	92.381	2.212	0.000	0.513	0.000	0.000	0.000

Problem 177	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	100.931	1.332	0.000	0.540	0.000	0.000	0.000

Problem 178	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.544	4.422	0.000	0.465	0.000	0.000	0.000

Problem 179	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	240	0	0	0	0	0	-1
normalized size	1	1.00	2.42	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.166	0.691	5.342	0.000	0.431	0.000	0.000	0.000

Problem 180	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	4.856	1.686	0.000	0.478	0.000	0.000	0.000

Problem 181	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	6.595	1.189	0.000	0.501	0.000	0.000	0.000

Problem 182	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	335	335	219	366	0	0	0	0	1978
normalized size	1	1.00	0.65	1.09	0.00	0.00	0.00	0.00	5.90
time (sec)	N/A	0.699	0.515	0.562	0.000	0.000	0.000	0.000	15.241

Problem 183	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	273	273	255	163	0	0	0	0	1962
normalized size	1	1.00	0.93	0.60	0.00	0.00	0.00	0.00	7.19
time (sec)	N/A	0.567	0.280	0.573	0.000	0.000	0.000	0.000	14.466

Problem 184	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	140	106	0	0	0	0	1672
normalized size	1	1.00	0.54	0.41	0.00	0.00	0.00	0.00	6.46
time (sec)	N/A	0.458	0.181	0.469	0.000	0.000	0.000	0.000	14.901

Problem 185	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	172	78	0	0	0	0	652
normalized size	1	1.00	0.64	0.29	0.00	0.00	0.00	0.00	2.44
time (sec)	N/A	0.262	0.184	0.530	0.000	0.000	0.000	0.000	16.121

Problem 186	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	264	98	0	0	0	0	1439
normalized size	1	1.00	1.00	0.37	0.00	0.00	0.00	0.00	5.45
time (sec)	N/A	0.365	0.274	0.762	0.000	0.000	0.000	0.000	15.656

Problem 187	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	181	144	0	0	0	0	1573
normalized size	1	1.00	0.63	0.50	0.00	0.00	0.00	0.00	5.48
time (sec)	N/A	0.404	0.405	0.803	0.000	0.000	0.000	0.000	15.035

Problem 188	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	290	217	0	0	0	0	1560
normalized size	1	1.00	0.84	0.63	0.00	0.00	0.00	0.00	4.53
time (sec)	N/A	0.478	2.023	0.824	0.000	0.000	0.000	0.000	14.598

Problem 189	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	293	293	164	166	0	0	0	0	1800
normalized size	1	1.00	0.56	0.57	0.00	0.00	0.00	0.00	6.14
time (sec)	N/A	0.394	0.283	0.578	0.000	0.000	0.000	0.000	14.584

Problem 190	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	186	106	0	0	0	0	665
normalized size	1	1.00	0.66	0.38	0.00	0.00	0.00	0.00	2.37
time (sec)	N/A	0.422	0.261	0.561	0.000	0.000	0.000	0.000	15.072

Problem 191	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	231	76	0	0	0	0	590
normalized size	1	1.00	0.96	0.32	0.00	0.00	0.00	0.00	2.46
time (sec)	N/A	0.272	0.186	0.454	0.000	0.000	0.000	0.000	15.652

Problem 192	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	0	0	0	609
normalized size	1	1.00	0.51	0.34	0.00	0.00	0.00	0.00	2.49
time (sec)	N/A	0.261	0.127	0.529	0.000	0.000	0.000	0.000	15.884

Problem 193	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	281	281	196	119	0	0	0	0	697
normalized size	1	1.00	0.70	0.42	0.00	0.00	0.00	0.00	2.48
time (sec)	N/A	0.429	0.312	0.763	0.000	0.000	0.000	0.000	14.417

Problem 194	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	296	296	333	176	0	0	0	0	1503
normalized size	1	1.00	1.12	0.59	0.00	0.00	0.00	0.00	5.08
time (sec)	N/A	0.393	2.150	0.788	0.000	0.000	0.000	0.000	16.383

Problem 195	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	228	159	0	872	0	0	1067
normalized size	1	1.00	1.29	0.90	0.00	4.93	0.00	0.00	6.03
time (sec)	N/A	0.252	0.478	0.367	0.000	0.578	0.000	0.000	14.439

Problem 196	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	310	115	0	849	0	0	1119
normalized size	1	1.00	2.09	0.78	0.00	5.74	0.00	0.00	7.56
time (sec)	N/A	0.176	0.287	0.306	0.000	0.563	0.000	0.000	14.269

Problem 197	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	198	103	0	815	0	0	1001
normalized size	1	1.00	1.43	0.75	0.00	5.91	0.00	0.00	7.25
time (sec)	N/A	0.179	0.251	0.355	0.000	0.543	0.000	0.000	14.260

Problem 198	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	285	78	0	703	0	166	976
normalized size	1	1.00	2.48	0.68	0.00	6.11	0.00	1.44	8.49
time (sec)	N/A	0.117	0.171	0.236	0.000	0.531	0.000	0.735	0.509

Problem 199	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	183	90	0	703	0	183	361
normalized size	1	1.00	1.46	0.72	0.00	5.62	0.00	1.46	2.89
time (sec)	N/A	0.102	0.160	0.365	0.000	0.561	0.000	0.756	15.111

Problem 200	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	318	120	0	773	0	0	2031
normalized size	1	1.00	2.34	0.88	0.00	5.68	0.00	0.00	14.93
time (sec)	N/A	0.178	0.256	0.489	0.000	0.647	0.000	0.000	15.361

Problem 201	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	242	170	0	924	0	0	2779
normalized size	1	1.00	1.32	0.92	0.00	5.02	0.00	0.00	15.10
time (sec)	N/A	0.210	0.340	0.568	0.000	0.683	0.000	0.000	15.186

Problem 202	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	409	232	0	1089	0	0	3692
normalized size	1	1.00	1.79	1.01	0.00	4.76	0.00	0.00	16.12
time (sec)	N/A	0.247	1.174	0.539	0.000	0.773	0.000	0.000	15.517

Problem 203	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	172	605	0	1311	0	461	5022
normalized size	1	1.00	0.93	3.29	0.00	7.12	0.00	2.51	27.29
time (sec)	N/A	0.293	0.920	0.346	0.000	0.675	0.000	1.113	16.867

Problem 204	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	157	551	0	1275	0	695	1273
normalized size	1	1.00	1.01	3.55	0.00	8.23	0.00	4.48	8.21
time (sec)	N/A	0.204	0.832	0.347	0.000	0.638	0.000	1.022	16.401

Problem 205	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	143	517	0	1125	0	912	2991
normalized size	1	1.00	1.13	4.07	0.00	8.86	0.00	7.18	23.55
time (sec)	N/A	0.185	0.417	0.282	0.000	0.590	0.000	0.983	16.275

Problem 206	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	137	492	0	1087	0	398	443
normalized size	1	1.00	1.10	3.94	0.00	8.70	0.00	3.18	3.54
time (sec)	N/A	0.112	0.357	0.431	0.000	0.624	0.000	0.990	16.187

Problem 207	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	128	492	0	1079	0	361	671
normalized size	1	1.00	1.11	4.28	0.00	9.38	0.00	3.14	5.83
time (sec)	N/A	0.090	0.264	0.357	0.000	0.603	0.000	0.420	14.994

Problem 208	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	139	139	143	518	0	1229	0	672	371
normalized size	1	1.00	1.03	3.73	0.00	8.84	0.00	4.83	2.67
time (sec)	N/A	0.178	1.083	0.493	0.000	0.624	0.000	1.159	14.512

Problem 209	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	165	542	0	1365	0	938	1670
normalized size	1	1.00	1.11	3.64	0.00	9.16	0.00	6.30	11.21
time (sec)	N/A	0.193	1.558	0.531	0.000	0.609	0.000	1.040	15.554

Problem 210	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	174	585	0	1477	0	471	416
normalized size	1	1.00	0.98	3.29	0.00	8.30	0.00	2.65	2.34
time (sec)	N/A	0.203	4.674	0.529	0.000	0.643	0.000	0.994	15.256

Problem 211	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	277	624	0	1585	0	467	1704
normalized size	1	1.00	1.41	3.17	0.00	8.05	0.00	2.37	8.65
time (sec)	N/A	0.235	6.320	0.554	0.000	0.647	0.000	1.022	17.078

Problem 212	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	486	482	0	2649	0	0	3941
normalized size	1	1.00	2.06	2.04	0.00	11.22	0.00	0.00	16.70
time (sec)	N/A	0.480	1.192	0.342	0.000	0.898	0.000	0.000	16.004

Problem 213	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	565	394	0	2507	0	0	3612
normalized size	1	1.00	2.69	1.88	0.00	11.94	0.00	0.00	17.20
time (sec)	N/A	0.335	0.606	0.315	0.000	0.843	0.000	0.000	15.870

Problem 214	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	469	440	0	2507	0	0	3839
normalized size	1	1.00	2.16	2.03	0.00	11.55	0.00	0.00	17.69
time (sec)	N/A	0.264	0.664	0.301	0.000	0.817	0.000	0.000	16.630

Problem 215	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	345	213	0	2049	0	605	3060
normalized size	1	1.00	1.85	1.15	0.00	11.02	0.00	3.25	16.45
time (sec)	N/A	0.185	0.341	0.260	0.000	0.687	0.000	1.037	15.603

Problem 216	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	469	488	0	2269	0	693	3507
normalized size	1	1.00	2.12	2.21	0.00	10.27	0.00	3.14	15.87
time (sec)	N/A	0.265	0.454	0.517	0.000	0.813	0.000	0.893	16.794

Problem 217	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	325	325	600	450	0	2711	0	0	7491
normalized size	1	1.00	1.85	1.38	0.00	8.34	0.00	0.00	23.05
time (sec)	N/A	0.336	0.863	0.533	0.000	1.256	0.000	0.000	17.550

Problem 218	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	262	644	0	3544	0	1563	7640
normalized size	1	1.00	0.82	2.01	0.00	11.08	0.00	4.88	23.88
time (sec)	N/A	0.446	5.022	0.314	0.000	1.332	0.000	1.173	17.602

Problem 219	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	233	233	238	674	0	3135	0	1481	3400
normalized size	1	1.00	1.02	2.89	0.00	13.45	0.00	6.36	14.59
time (sec)	N/A	0.351	2.660	0.323	0.000	1.238	0.000	1.202	16.545

Problem 220	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	225	478	0	2796	0	1264	2980
normalized size	1	1.00	1.15	2.45	0.00	14.34	0.00	6.48	15.28
time (sec)	N/A	0.229	4.407	0.269	0.000	0.860	0.000	1.062	15.990

Problem 221	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	255	534	0	3445	0	1407	3842
normalized size	1	1.00	1.16	2.44	0.00	15.73	0.00	6.42	17.54
time (sec)	N/A	0.297	2.142	0.446	0.000	1.176	0.000	1.120	17.349

Problem 222	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	230	618	0	3477	0	1506	3675
normalized size	1	1.00	1.10	2.94	0.00	16.56	0.00	7.17	17.50
time (sec)	N/A	0.260	2.871	0.397	0.000	1.202	0.000	0.438	16.516

Problem 223	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	274	708	0	3648	0	1545	4411
normalized size	1	1.00	1.16	3.00	0.00	15.46	0.00	6.55	18.69
time (sec)	N/A	0.534	2.277	0.531	0.000	1.485	0.000	1.128	18.290

Problem 224	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	785	1164	0	4640	0	0	6675
normalized size	1	1.00	2.49	3.70	0.00	14.73	0.00	0.00	21.19
time (sec)	N/A	0.575	1.590	0.425	0.000	1.539	0.000	0.000	19.294

Problem 225	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	630	814	0	4185	0	0	5824
normalized size	1	1.00	2.17	2.81	0.00	14.43	0.00	0.00	20.08
time (sec)	N/A	0.435	1.148	0.363	0.000	1.130	0.000	0.000	19.618

Problem 226	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	786	1167	0	4524	0	0	6362
normalized size	1	1.00	2.51	3.73	0.00	14.45	0.00	0.00	20.33
time (sec)	N/A	0.472	1.398	0.388	0.000	1.465	0.000	0.000	20.153

Problem 227	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	631	1153	0	4050	0	1076	5566
normalized size	1	1.00	2.19	4.00	0.00	14.06	0.00	3.74	19.33
time (sec)	N/A	0.498	1.122	0.519	0.000	1.187	0.000	1.861	19.278

Problem 228	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	784	1281	0	4160	0	766	5753
normalized size	1	1.00	2.50	4.09	0.00	13.29	0.00	2.45	18.38
time (sec)	N/A	0.459	1.280	0.635	0.000	1.414	0.000	1.754	18.910

Problem 229	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	B	F(-1)	F(-2)	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	920	1139	0	5020	0	0	12247
normalized size	1	1.00	1.49	1.85	0.00	8.14	0.00	0.00	19.85
time (sec)	N/A	0.837	4.285	0.612	0.000	3.463	0.000	0.000	20.835

Problem 230	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	331	1634	0	5219	0	1989	5508
normalized size	1	1.00	1.04	5.12	0.00	16.36	0.00	6.24	17.27
time (sec)	N/A	0.530	3.980	0.377	0.000	1.902	0.000	2.162	19.648

Problem 231	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	350	1909	0	5961	0	2231	6391
normalized size	1	1.00	1.02	5.57	0.00	17.38	0.00	6.50	18.63
time (sec)	N/A	0.765	3.763	0.368	0.000	2.848	0.000	2.237	20.337

Problem 232	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	313	313	316	1624	0	5510	0	1986	5892
normalized size	1	1.00	1.01	5.19	0.00	17.60	0.00	6.35	18.82
time (sec)	N/A	0.695	4.837	0.328	0.000	1.823	0.000	2.225	19.499

Problem 233	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	347	347	457	1906	0	6215	0	1184	6646
normalized size	1	1.00	1.32	5.49	0.00	17.91	0.00	3.41	19.15
time (sec)	N/A	0.724	6.430	0.468	0.000	3.280	0.000	2.291	19.905

Problem 234	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	319	319	333	1803	0	6152	0	1131	6267
normalized size	1	1.00	1.04	5.65	0.00	19.29	0.00	3.55	19.65
time (sec)	N/A	0.652	2.997	0.416	0.000	3.385	0.000	0.676	19.666

Problem 235	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	357	357	357	1959	0	6323	0	2203	7364
normalized size	1	1.00	1.00	5.49	0.00	17.71	0.00	6.17	20.63
time (sec)	N/A	1.294	5.031	0.612	0.000	3.640	0.000	2.270	20.801

Problem 236	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	45	24	18	17	43	724	51	17
normalized size	1	1.80	0.96	0.72	0.68	1.72	28.96	2.04	0.68
time (sec)	N/A	0.019	0.053	0.170	0.443	0.447	84.416	0.118	14.340

Problem 237	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	148	1677	0	823	0	318	407
normalized size	1	1.00	0.30	3.44	0.00	1.69	0.00	0.65	0.84
time (sec)	N/A	1.137	0.310	0.414	0.000	0.570	0.000	0.389	15.178

Problem 238	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	45	239	0	3830	0	170	236
normalized size	1	1.00	0.15	0.77	0.00	12.39	0.00	0.55	0.76
time (sec)	N/A	0.202	0.074	0.454	0.000	32.594	0.000	0.362	14.352

Problem 239	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	47242	439	0	0	0	0	-1
normalized size	1	1.00	99.04	0.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.388	31.595	3.672	0.000	0.467	0.000	0.000	0.000

Problem 240	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	118912	0	0	0	0	0	-1
normalized size	1	1.00	228.24	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.678	31.998	1.539	0.000	0.495	0.000	0.000	0.000

Problem 241	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	47246	837	0	0	0	0	-1
normalized size	1	1.00	97.62	1.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	31.724	1.395	0.000	0.472	0.000	0.000	0.000

Problem 242	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F(-1)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	431	431	89374	398	0	0	0	0	-1
normalized size	1	1.00	207.36	0.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.302	31.828	1.332	0.000	0.457	0.000	0.000	0.000

Problem 243	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	13300	163	0	0	0	0	-1
normalized size	1	1.00	77.78	0.95	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	25.359	0.857	0.000	0.445	0.000	0.000	0.000

Problem 244	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	63281	0	0	0	0	0	-1
normalized size	1	1.00	134.93	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.434	31.471	1.543	0.000	0.000	0.000	0.000	0.000

Problem 245	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	776	776	119171	0	0	0	0	0	-1
normalized size	1	1.00	153.57	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.043	32.628	1.621	0.000	0.485	0.000	0.000	0.000

Problem 246	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	499	499	287	881	0	0	0	0	-1
normalized size	1	1.00	0.58	1.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	2.876	7.356	0.000	0.634	0.000	0.000	0.000

Problem 247	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	0	0	0	-1
normalized size	1	1.00	1.88	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.079	8.701	2.941	0.000	0.488	0.000	0.000	0.000

Problem 248	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	493	493	498	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.419	16.201	1.501	0.000	0.494	0.000	0.000	0.000

Problem 249	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	149	109	0	0	0	0	1515
normalized size	1	1.00	0.39	0.28	0.00	0.00	0.00	0.00	3.95
time (sec)	N/A	0.714	0.213	0.260	0.000	0.000	0.000	0.000	19.752

Problem 250	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	148	68	0	0	0	0	513
normalized size	1	1.00	0.87	0.40	0.00	0.00	0.00	0.00	3.00
time (sec)	N/A	0.257	0.221	1.542	0.000	0.000	0.000	0.000	15.705

Problem 251	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	174	85	0	0	0	0	816
normalized size	1	1.00	0.71	0.35	0.00	0.00	0.00	0.00	3.33
time (sec)	N/A	0.528	0.261	0.262	0.000	0.000	0.000	0.000	16.948

Problem 252	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-2)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	379	379	149	109	0	0	0	0	1515
normalized size	1	1.00	0.39	0.29	0.00	0.00	0.00	0.00	4.00
time (sec)	N/A	0.477	0.191	0.246	0.000	0.000	0.000	0.000	20.141

Problem 253	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	148	71	0	0	0	0	513
normalized size	1	1.00	0.85	0.41	0.00	0.00	0.00	0.00	2.93
time (sec)	N/A	0.260	0.175	1.415	0.000	0.000	0.000	0.000	16.069

Problem 254	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	174	88	0	0	0	0	818
normalized size	1	1.00	0.82	0.41	0.00	0.00	0.00	0.00	3.84
time (sec)	N/A	0.218	0.213	0.263	0.000	0.000	0.000	0.000	16.542

Problem 255	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	411	133	0	0	0	0	3513
normalized size	1	1.00	2.11	0.68	0.00	0.00	0.00	0.00	18.02
time (sec)	N/A	0.383	0.146	0.220	0.000	0.000	0.000	0.000	15.254

Problem 256	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	79	72	71	138	0	185	98
normalized size	1	1.00	0.77	0.70	0.69	1.34	0.00	1.80	0.95
time (sec)	N/A	0.104	0.165	0.179	0.442	0.515	0.000	0.158	14.226

Problem 257	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	129	141	71	0	0	0	0	945
normalized size	1	0.59	0.65	0.33	0.00	0.00	0.00	0.00	4.33
time (sec)	N/A	0.200	0.147	0.204	0.000	0.000	0.000	0.000	14.919

Problem 258	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	413	133	0	0	0	0	3513
normalized size	1	1.00	2.21	0.71	0.00	0.00	0.00	0.00	18.79
time (sec)	N/A	0.269	0.137	0.214	0.000	0.000	0.000	0.000	14.440

Problem 259	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	117	255	0	0	0	197	99
normalized size	1	1.00	1.65	3.59	0.00	0.00	0.00	2.77	1.39
time (sec)	N/A	0.135	0.284	0.452	0.000	0.000	0.000	0.153	14.208

Problem 260	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	64	255	0	3884	0	220	141
normalized size	1	1.00	0.72	2.87	0.00	43.64	0.00	2.47	1.58
time (sec)	N/A	0.077	0.170	0.221	0.000	33.493	0.000	0.351	14.038

Problem 261	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	35	26	28	27	580	28	34
normalized size	1	1.00	0.92	0.68	0.74	0.71	15.26	0.74	0.89
time (sec)	N/A	0.054	0.005	0.167	0.355	0.451	41.264	0.142	0.097

Problem 262	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	22	21	311	22	19
normalized size	1	1.00	0.93	0.69	0.76	0.72	10.72	0.76	0.66
time (sec)	N/A	0.052	0.003	0.151	0.351	0.414	18.896	0.121	0.076

Problem 263	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	14	13	124	14	16
normalized size	1	1.00	1.06	0.78	0.78	0.72	6.89	0.78	0.89
time (sec)	N/A	0.050	0.003	0.149	0.324	0.435	7.946	0.137	0.059

Problem 264	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	15	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	2.50	1.00	1.00
time (sec)	N/A	0.043	0.002	0.129	0.335	0.412	2.860	0.119	13.757

Problem 265	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	21	20	19	23	7
normalized size	1	1.00	5.29	1.14	3.00	2.86	2.71	3.29	1.00
time (sec)	N/A	0.026	0.004	0.104	0.350	0.421	0.290	0.125	13.598

Problem 266	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	66	51	46	0	47	31
normalized size	1	1.00	1.74	1.89	1.46	1.31	0.00	1.34	0.89
time (sec)	N/A	0.058	0.125	0.220	0.341	0.432	0.000	0.123	13.879

Problem 267	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	40	37	23	473	36	25
normalized size	1	1.00	0.79	1.21	1.12	0.70	14.33	1.09	0.76
time (sec)	N/A	0.055	0.003	0.152	0.449	0.423	12.927	0.124	13.614

Problem 268	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	21	12	153	24	13
normalized size	1	1.00	0.90	1.25	1.05	0.60	7.65	1.20	0.65
time (sec)	N/A	0.048	0.003	0.160	1.404	0.421	4.921	0.124	13.813

Problem 269	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	2	14	5
normalized size	1	1.00	1.00	1.60	1.00	1.00	0.40	2.80	1.00
time (sec)	N/A	0.041	0.001	0.194	0.478	0.406	1.599	0.135	13.811

Problem 270	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	45	44	37	37	0	38	25
normalized size	1	1.00	2.05	2.00	1.68	1.68	0.00	1.73	1.14
time (sec)	N/A	0.043	0.041	0.209	0.349	0.452	0.000	0.131	13.873

Problem 271	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	21	14	14	19	0	14	13
normalized size	1	1.00	1.17	0.78	0.78	1.06	0.00	0.78	0.72
time (sec)	N/A	0.049	0.004	0.207	0.376	0.428	0.000	0.142	13.868

Problem 272	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	0	22	21
normalized size	1	1.00	1.07	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.052	0.004	0.212	0.393	0.431	0.000	0.194	13.922

Problem 273	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	27	20	22	21	362	22	19
normalized size	1	1.00	0.93	0.69	0.76	0.72	12.48	0.76	0.66
time (sec)	N/A	0.046	0.004	0.158	0.350	0.418	83.590	0.150	14.000

Problem 274	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	19	14	14	13	144	14	16
normalized size	1	1.00	1.06	0.78	0.78	0.72	8.00	0.78	0.89
time (sec)	N/A	0.044	0.003	0.143	0.349	0.414	40.558	0.142	0.036

Problem 275	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	6	19	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.00	3.17	1.00	1.00
time (sec)	N/A	0.039	0.002	0.143	0.332	0.407	19.210	0.117	0.025

Problem 276	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	7	7	37	8	21	20	22	23	7
normalized size	1	1.00	5.29	1.14	3.00	2.86	3.14	3.29	1.00
time (sec)	N/A	0.039	0.004	0.185	0.330	0.416	7.456	0.132	0.056

Problem 277	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	22	22	45	44	41	37	117	38	30
normalized size	1	1.00	2.05	2.00	1.86	1.68	5.32	1.73	1.36
time (sec)	N/A	0.032	0.006	0.120	0.365	0.424	1.060	0.151	0.076

Problem 278	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	61	66	57	46	0	47	31
normalized size	1	1.00	1.74	1.89	1.63	1.31	0.00	1.34	0.89
time (sec)	N/A	0.048	0.008	0.197	0.372	0.436	0.000	0.142	13.985

Problem 279	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	26	40	43	23	549	31	29
normalized size	1	1.00	0.79	1.21	1.30	0.70	16.64	0.94	0.88
time (sec)	N/A	0.050	0.003	0.161	0.512	0.420	69.897	0.123	13.821

Problem 280	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	20	20	18	25	25	12	178	22	13
normalized size	1	1.00	0.90	1.25	1.25	0.60	8.90	1.10	0.65
time (sec)	N/A	0.043	0.003	0.156	0.446	0.412	31.081	0.120	13.926

Problem 281	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	5	5	5	8	5	5	3	5	5
normalized size	1	1.00	1.00	1.60	1.00	1.00	0.60	1.00	1.00
time (sec)	N/A	0.037	0.000	0.193	0.445	0.392	13.005	0.127	13.943

Problem 282	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	B	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	6	10	20	6	6
normalized size	1	1.00	1.00	1.17	1.00	1.67	3.33	1.00	1.00
time (sec)	N/A	0.043	0.002	0.204	0.366	0.391	5.643	0.127	13.780

Problem 283	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	31	20	22	25	0	22	21
normalized size	1	1.00	1.07	0.69	0.76	0.86	0.00	0.76	0.72
time (sec)	N/A	0.047	0.004	0.236	0.346	0.401	0.000	0.149	13.799

Problem 284	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	37	37	41	24	28	31	0	28	33
normalized size	1	1.00	1.11	0.65	0.76	0.84	0.00	0.76	0.89
time (sec)	N/A	0.048	0.004	0.239	0.358	0.408	0.000	0.128	13.837

Problem 285	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	87	112	122	78	354	87	119
normalized size	1	1.00	0.80	1.03	1.12	0.72	3.25	0.80	1.09
time (sec)	N/A	0.065	0.275	0.537	0.474	0.429	14.472	0.233	15.222

Problem 286	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	64	92	97	63	250	67	91
normalized size	1	1.00	0.77	1.11	1.17	0.76	3.01	0.81	1.10
time (sec)	N/A	0.051	0.151	0.569	0.456	0.413	5.276	0.181	14.203

Problem 287	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	70	69	47	150	41	67
normalized size	1	1.00	0.81	1.23	1.21	0.82	2.63	0.72	1.18
time (sec)	N/A	0.044	0.080	0.352	0.463	0.422	1.416	0.159	13.655

Problem 288	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	33	32	29	29	51	26	27
normalized size	1	1.00	1.10	1.07	0.97	0.97	1.70	0.87	0.90
time (sec)	N/A	0.015	0.029	0.073	0.360	0.425	0.248	0.131	13.646

Problem 289	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	36	30	30	35	0	49	26
normalized size	1	1.00	2.00	1.67	1.67	1.94	0.00	2.72	1.44
time (sec)	N/A	0.033	0.014	0.477	0.457	0.433	0.000	0.178	13.643

Problem 290	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	30	30	41	46	27	38	0	38	31
normalized size	1	1.00	1.37	1.53	0.90	1.27	0.00	1.27	1.03
time (sec)	N/A	0.032	0.073	0.465	0.362	0.412	0.000	0.192	14.162

Problem 291	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	64	76	43	59	0	64	45
normalized size	1	1.00	1.28	1.52	0.86	1.18	0.00	1.28	0.90
time (sec)	N/A	0.044	0.182	0.536	0.347	0.418	0.000	0.202	13.895

Problem 292	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	86	104	60	77	0	88	59
normalized size	1	1.00	1.19	1.44	0.83	1.07	0.00	1.22	0.82
time (sec)	N/A	0.055	0.313	0.504	0.382	0.397	0.000	0.223	13.844

Problem 293	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	96	167	169	114	481	108	160
normalized size	1	1.00	0.62	1.07	1.08	0.73	3.08	0.69	1.03
time (sec)	N/A	0.150	0.323	0.508	0.500	0.423	14.620	0.241	15.490

Problem 294	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	79	134	127	85	314	84	120
normalized size	1	1.00	0.68	1.16	1.09	0.73	2.71	0.72	1.03
time (sec)	N/A	0.146	0.268	0.364	0.494	0.422	4.842	0.184	14.733

Problem 295	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	58	78	68	63	168	60	77
normalized size	1	1.00	0.81	1.08	0.94	0.88	2.33	0.83	1.07
time (sec)	N/A	0.020	0.119	0.352	0.319	0.415	1.339	0.132	14.487

Problem 296	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	48	87	74	68	0	99	74
normalized size	1	1.00	0.94	1.71	1.45	1.33	0.00	1.94	1.45
time (sec)	N/A	0.089	0.313	0.625	0.457	0.422	0.000	0.186	14.088

Problem 297	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	57	76	53	70	0	80	46
normalized size	1	1.00	1.27	1.69	1.18	1.56	0.00	1.78	1.02
time (sec)	N/A	0.062	0.325	0.528	0.468	0.425	0.000	0.221	13.759

Problem 298	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	67	101	55	83	0	86	44
normalized size	1	1.00	1.26	1.91	1.04	1.57	0.00	1.62	0.83
time (sec)	N/A	0.057	0.355	0.602	0.321	0.432	0.000	0.213	15.833

Problem 299	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	92	149	81	108	0	127	72
normalized size	1	1.00	1.15	1.86	1.01	1.35	0.00	1.59	0.90
time (sec)	N/A	0.076	0.479	0.605	0.349	0.410	0.000	0.248	15.137

Problem 300	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	107	195	103	128	0	168	94
normalized size	1	1.00	1.01	1.84	0.97	1.21	0.00	1.58	0.89
time (sec)	N/A	0.093	0.469	0.610	0.350	0.417	0.000	0.289	14.218

Problem 301	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	109	136	86	233	0	98	99
normalized size	1	1.00	1.40	1.74	1.10	2.99	0.00	1.26	1.27
time (sec)	N/A	0.090	0.276	0.202	0.449	0.481	0.000	0.138	0.115

Problem 302	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	79	202	114	312	0	131	1804
normalized size	1	1.00	0.91	2.32	1.31	3.59	0.00	1.51	20.74
time (sec)	N/A	0.187	0.187	0.239	0.467	0.457	0.000	0.159	15.451

Problem 303	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	84	85	52	159	0	58	65
normalized size	1	1.00	1.56	1.57	0.96	2.94	0.00	1.07	1.20
time (sec)	N/A	0.073	0.173	0.195	0.484	0.455	0.000	0.152	14.301

Problem 304	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	111	64	239	0	92	119
normalized size	1	1.00	0.93	1.88	1.08	4.05	0.00	1.56	2.02
time (sec)	N/A	0.108	0.151	0.212	0.464	0.454	0.000	0.151	14.666

Problem 305	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	36	45	30	101	0	30	28
normalized size	1	1.00	1.00	1.25	0.83	2.81	0.00	0.83	0.78
time (sec)	N/A	0.056	0.023	0.240	0.468	0.454	0.000	0.123	0.095

Problem 306	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	58	35	206	0	62	272
normalized size	1	1.00	1.00	1.49	0.90	5.28	0.00	1.59	6.97
time (sec)	N/A	0.060	0.056	0.206	1.439	0.459	0.000	0.132	14.713

Problem 307	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	17	16	78	87	16	17
normalized size	1	1.00	1.00	0.68	0.64	3.12	3.48	0.64	0.68
time (sec)	N/A	0.029	0.007	0.112	0.464	0.453	1.340	0.137	14.636

Problem 308	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	40	40	96	55	47	116	0	49	856
normalized size	1	1.00	2.40	1.38	1.18	2.90	0.00	1.22	21.40
time (sec)	N/A	0.051	0.126	0.207	0.455	0.461	0.000	0.143	14.695

Problem 309	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	39	39	39	38	37	255	0	45	39
normalized size	1	1.00	1.00	0.97	0.95	6.54	0.00	1.15	1.00
time (sec)	N/A	0.061	0.083	0.236	0.455	0.465	0.000	0.132	14.656

Problem 310	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	147	112	104	203	0	102	1139
normalized size	1	1.00	2.41	1.84	1.70	3.33	0.00	1.67	18.67
time (sec)	N/A	0.087	0.320	0.240	0.591	0.515	0.000	0.121	15.357

Problem 311	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	75	72	343	0	134	77
normalized size	1	1.00	1.00	1.27	1.22	5.81	0.00	2.27	1.31
time (sec)	N/A	0.081	0.219	0.256	0.475	0.460	0.000	0.156	15.025

Problem 312	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	214	204	199	327	0	177	832
normalized size	1	1.00	2.30	2.19	2.14	3.52	0.00	1.90	8.95
time (sec)	N/A	0.145	1.245	0.317	0.482	0.555	0.000	0.142	17.450

Problem 313	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	90	147	126	459	0	254	121
normalized size	1	1.00	1.03	1.69	1.45	5.28	0.00	2.92	1.39
time (sec)	N/A	0.113	0.375	0.225	0.458	0.475	0.000	0.131	13.990

Problem 314	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	90	211	150	491	0	175	463
normalized size	1	1.00	0.80	1.87	1.33	4.35	0.00	1.55	4.10
time (sec)	N/A	0.223	0.297	0.230	0.481	0.492	0.000	0.162	14.697

Problem 315	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	118	120	79	296	0	82	96
normalized size	1	1.00	1.64	1.67	1.10	4.11	0.00	1.14	1.33
time (sec)	N/A	0.119	0.340	0.217	0.512	0.460	0.000	0.123	14.402

Problem 316	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	79	132	80	367	0	109	533
normalized size	1	1.00	1.05	1.76	1.07	4.89	0.00	1.45	7.11
time (sec)	N/A	0.106	0.319	0.212	0.947	0.460	0.000	0.159	14.327

Problem 317	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	59	65	53	206	0	56	47
normalized size	1	1.00	1.00	1.10	0.90	3.49	0.00	0.95	0.80
time (sec)	N/A	0.058	0.067	0.233	1.829	0.434	0.000	0.126	0.136

Problem 318	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	59	51	49	313	0	77	50
normalized size	1	1.00	1.09	0.94	0.91	5.80	0.00	1.43	0.93
time (sec)	N/A	0.056	0.133	0.217	0.724	0.474	0.000	0.154	14.305

Problem 319	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	48	48	48	39	38	165	340	38	36
normalized size	1	1.00	1.00	0.81	0.79	3.44	7.08	0.79	0.75
time (sec)	N/A	0.035	0.055	0.108	0.698	0.442	15.379	0.143	15.377

Problem 320	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	130	122	115	354	0	109	2213
normalized size	1	1.00	1.78	1.67	1.58	4.85	0.00	1.49	30.32
time (sec)	N/A	0.090	0.529	0.219	0.778	0.518	0.000	0.130	15.835

Problem 321	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	76	112	119	505	0	113	123
normalized size	1	1.00	1.00	1.47	1.57	6.64	0.00	1.49	1.62
time (sec)	N/A	0.124	0.510	0.254	0.549	0.477	0.000	0.133	14.777

Problem 322	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	183	180	220	560	0	194	2009
normalized size	1	1.00	1.68	1.65	2.02	5.14	0.00	1.78	18.43
time (sec)	N/A	0.163	1.043	0.293	0.651	0.573	0.000	0.130	15.058

Problem 323	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	97	193	170	653	0	270	176
normalized size	1	1.00	1.01	2.01	1.77	6.80	0.00	2.81	1.83
time (sec)	N/A	0.148	0.958	0.268	0.700	0.501	0.000	0.142	14.297

Problem 324	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	125	155	119	511	0	0	-1
normalized size	1	1.00	1.07	1.32	1.02	4.37	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.436	1.535	0.642	0.805	0.000	0.000	0.000

Problem 325	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	72	72	96	62	46	453	0	0	61
normalized size	1	1.00	1.33	0.86	0.64	6.29	0.00	0.00	0.85
time (sec)	N/A	0.055	0.261	0.166	0.338	0.534	0.000	0.000	14.477

Problem 326	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	129	155	126	1246	0	0	-1
normalized size	1	1.00	1.57	1.89	1.54	15.20	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.272	4.140	0.492	0.680	0.000	0.000	0.000

Problem 327	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	164	291	0	337	0	0	-1
normalized size	1	1.00	2.00	3.55	0.00	4.11	0.00	0.00	-0.01
time (sec)	N/A	0.094	2.165	4.431	0.000	0.528	0.000	0.000	0.000

Problem 328	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	669	570	0	443	0	0	-1
normalized size	1	1.00	4.68	3.99	0.00	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.139	13.302	4.852	0.000	0.864	0.000	0.000	0.000

Problem 329	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	260	199	432	0	0	0	0	-1
normalized size	1	1.18	0.90	1.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.275	1.380	1.638	0.000	0.464	0.000	0.000	0.000

Problem 330	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	199	158	265	0	0	0	0	-1
normalized size	1	1.25	0.99	1.67	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.863	1.641	0.000	0.454	0.000	0.000	0.000

Problem 331	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	-1
normalized size	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.078	0.699	0.000	0.434	0.000	0.000	0.000

Problem 332	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	171	134	294	0	0	0	0	-1
normalized size	1	1.31	1.02	2.24	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.472	2.303	0.000	0.434	0.000	0.000	0.000

Problem 333	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	236	187	368	0	0	0	0	-1
normalized size	1	1.20	0.95	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.223	1.830	2.585	0.000	0.436	0.000	0.000	0.000

Problem 334	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	149	277	174	577	0	0	-1
normalized size	1	1.00	0.95	1.76	1.11	3.68	0.00	0.00	-0.01
time (sec)	N/A	0.138	0.811	1.972	0.368	2.133	0.000	0.000	0.000

Problem 335	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	93	90	73	503	0	0	60
normalized size	1	1.00	0.89	0.87	0.70	4.84	0.00	0.00	0.58
time (sec)	N/A	0.069	0.491	0.219	0.361	0.816	0.000	0.000	14.539

Problem 336	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	233	451	168	1381	0	0	-1
normalized size	1	1.00	1.93	3.73	1.39	11.41	0.00	0.00	-0.01
time (sec)	N/A	0.144	0.665	3.708	0.473	0.952	0.000	0.000	0.000

Problem 337	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	210	402	0	1471	0	0	-1
normalized size	1	1.00	1.65	3.17	0.00	11.58	0.00	0.00	-0.01
time (sec)	N/A	0.146	0.950	3.870	0.000	1.038	0.000	0.000	0.000

Problem 338	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	63	406	0	413	0	0	-1
normalized size	1	1.00	0.52	3.33	0.00	3.39	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.127	3.192	0.000	0.977	0.000	0.000	0.000

Problem 339	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	938	693	0	545	0	0	-1
normalized size	1	1.00	4.81	3.55	0.00	2.79	0.00	0.00	-0.01
time (sec)	N/A	0.176	15.217	4.209	0.000	3.561	0.000	0.000	0.000

Problem 340	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	321	321	247	590	0	0	0	0	-1
normalized size	1	1.00	0.77	1.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	2.568	1.694	0.000	0.497	0.000	0.000	0.000

Problem 341	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	259	259	200	429	0	0	0	0	-1
normalized size	1	1.00	0.77	1.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.285	1.342	1.799	0.000	0.473	0.000	0.000	0.000

Problem 342	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0	-1
normalized size	1	1.00	1.01	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.758	1.497	0.000	0.450	0.000	0.000	0.000

Problem 343	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	144	466	0	0	0	0	-1
normalized size	1	1.00	0.79	2.56	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.180	0.855	2.401	0.000	0.448	0.000	0.000	0.000

Problem 344	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	190	375	0	0	0	0	-1
normalized size	1	1.00	0.81	1.59	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.256	2.026	2.688	0.000	0.447	0.000	0.000	0.000

Problem 345	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	79	98	69	461	0	0	-1
normalized size	1	1.00	1.00	1.24	0.87	5.84	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.113	1.491	0.344	0.557	0.000	0.000	0.000

Problem 346	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	34	21	394	0	0	33
normalized size	1	1.00	1.00	0.89	0.55	10.37	0.00	0.00	0.87
time (sec)	N/A	0.046	0.015	0.133	0.349	0.508	0.000	0.000	14.741

Problem 347	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	42	113	105	240	0	0	-1
normalized size	1	1.00	1.00	2.69	2.50	5.71	0.00	0.00	-0.02
time (sec)	N/A	0.072	0.029	3.100	0.457	0.511	0.000	0.000	0.000

Problem 348	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	436	360	0	361	0	0	-1
normalized size	1	1.00	4.79	3.96	0.00	3.97	0.00	0.00	-0.01
time (sec)	N/A	0.111	9.749	3.290	0.000	0.542	0.000	0.000	0.000

Problem 349	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	208	170	316	0	0	0	0	-1
normalized size	1	1.24	1.01	1.88	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.194	0.853	1.542	0.000	0.449	0.000	0.000	0.000

Problem 350	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	153	83	111	0	0	0	0	-1
normalized size	1	1.34	0.73	0.97	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.141	0.196	1.140	0.000	0.425	0.000	0.000	0.000

Problem 351	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0	-1
normalized size	1	1.00	1.18	1.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.035	0.074	0.359	0.000	0.428	0.000	0.000	0.000

Problem 352	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	180	141	278	0	0	0	0	-1
normalized size	1	1.29	1.01	1.99	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.169	0.623	2.815	0.000	0.428	0.000	0.000	0.000

Problem 353	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	252	205	405	0	0	0	0	-1
normalized size	1	1.19	0.97	1.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.248	2.187	3.033	0.000	0.434	0.000	0.000	0.000

Problem 354	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	88	90	74	559	0	0	-1
normalized size	1	1.00	1.17	1.20	0.99	7.45	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.172	1.480	0.339	0.623	0.000	0.000	0.000

Problem 355	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	28	27	49	0	29	117
normalized size	1	1.00	1.00	0.97	0.93	1.69	0.00	1.00	4.03
time (sec)	N/A	0.044	0.027	0.158	0.331	0.444	0.000	0.627	15.613

Problem 356	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	480	397	152	453	0	0	-1
normalized size	1	1.00	6.15	5.09	1.95	5.81	0.00	0.00	-0.01
time (sec)	N/A	0.101	7.409	4.630	0.447	0.564	0.000	0.000	0.000

Problem 357	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	224	3217	0	625	0	0	-1
normalized size	1	1.00	1.67	24.01	0.00	4.66	0.00	0.00	-0.01
time (sec)	N/A	0.176	5.615	11.571	0.000	0.912	0.000	0.000	0.000

Problem 358	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	184	415	0	0	0	0	-1
normalized size	1	1.00	0.67	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	1.121	1.809	0.000	0.498	0.000	0.000	0.000

Problem 359	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	139	274	0	0	0	0	-1
normalized size	1	1.00	0.69	1.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.206	0.558	1.419	0.000	0.457	0.000	0.000	0.000

Problem 360	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	133	145	0	0	0	0	-1
normalized size	1	1.00	0.71	0.77	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.184	0.302	1.277	0.000	0.420	0.000	0.000	0.000

Problem 361	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0	-1
normalized size	1	1.00	0.89	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.060	0.147	1.430	0.000	0.432	0.000	0.000	0.000

Problem 362	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	167	468	0	0	0	0	-1
normalized size	1	1.00	0.70	1.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.239	1.225	2.751	0.000	0.441	0.000	0.000	0.000

Problem 363	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	128	383	207	799	0	0	-1
normalized size	1	1.00	0.98	2.95	1.59	6.15	0.00	0.00	-0.01
time (sec)	N/A	0.135	0.811	4.003	1.360	1.621	0.000	0.000	0.000

Problem 364	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	51	120	107	107	0	58	183
normalized size	1	1.00	0.70	1.64	1.47	1.47	0.00	0.79	2.51
time (sec)	N/A	0.092	0.105	3.887	0.778	0.717	0.000	0.778	23.810

Problem 365	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	47	56	55	104	0	48	164
normalized size	1	1.00	0.72	0.86	0.85	1.60	0.00	0.74	2.52
time (sec)	N/A	0.055	0.045	0.151	0.339	0.610	0.000	0.732	21.839

Problem 366	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	1291	899	272	775	0	0	-1
normalized size	1	1.00	10.25	7.13	2.16	6.15	0.00	0.00	-0.01
time (sec)	N/A	0.159	9.365	5.683	0.577	0.936	0.000	0.000	0.000

Problem 367	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	243	283	194	712	0	0	0	0	-1
normalized size	1	1.16	0.80	2.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.310	2.116	1.898	0.000	0.505	0.000	0.000	0.000

Problem 368	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	263	171	485	0	0	0	0	-1
normalized size	1	1.18	0.77	2.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.283	1.413	1.828	0.000	0.454	0.000	0.000	0.000

Problem 369	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	257	175	552	0	0	0	0	-1
normalized size	1	1.18	0.81	2.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	1.431	1.850	0.000	0.449	0.000	0.000	0.000

Problem 370	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0	-1
normalized size	1	1.00	0.77	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	1.232	1.960	0.000	0.451	0.000	0.000	0.000

Problem 371	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	328	245	1082	0	0	0	0	-1
normalized size	1	1.14	0.85	3.76	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.357	3.348	3.247	0.000	0.496	0.000	0.000	0.000

Problem 372	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	228	0	0	0	0	0	-1
normalized size	1	1.00	1.98	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.900	2.560	0.000	0.526	0.000	0.000	0.000

Problem 373	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	0	0	0	0	0	-1
normalized size	1	1.00	0.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.209	0.397	2.932	0.000	0.444	0.000	0.000	0.000

Problem 374	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	119	120	0	0	0	0	0	-1
normalized size	1	0.96	0.97	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.191	7.105	0.000	0.431	0.000	0.000	0.000

Problem 375	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	64
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.96
time (sec)	N/A	0.044	0.025	2.600	0.000	0.416	0.000	0.000	15.217

Problem 376	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	4.446	1.938	0.000	0.434	0.000	0.000	0.000

Problem 377	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	6.563	1.965	0.000	0.450	0.000	0.000	0.000

Problem 378	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	199	0	0	0	0	0	-1
normalized size	1	1.00	2.21	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.578	4.612	0.000	0.446	0.000	0.000	0.000

Problem 379	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	195	0	0	0	0	0	-1
normalized size	1	1.00	2.17	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.082	0.468	6.166	0.000	0.426	0.000	0.000	0.000

Problem 380	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	145	0	0	0	0	0	-1
normalized size	1	1.00	1.61	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.052	0.503	1.355	0.000	0.437	0.000	0.000	0.000

Problem 381	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	4.555	1.773	0.000	0.433	0.000	0.000	0.000

Problem 382	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.081	6.782	1.236	0.000	0.460	0.000	0.000	0.000

Problem 383	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	219	219	203	278	210	3216	0	221	229
normalized size	1	1.00	0.93	1.27	0.96	14.68	0.00	1.01	1.05
time (sec)	N/A	0.290	0.236	0.919	0.893	1.306	0.000	0.189	15.019

Problem 384	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	139	141	159	2298	0	156	153
normalized size	1	1.00	0.83	0.84	0.95	13.76	0.00	0.93	0.92
time (sec)	N/A	0.148	0.144	0.843	0.450	70.621	0.000	0.185	15.025

Problem 385	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	A	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	116	120	121	401	250	137	123
normalized size	1	1.00	0.81	0.83	0.84	2.78	1.74	0.95	0.85
time (sec)	N/A	0.100	0.051	0.467	0.437	0.506	8.849	0.149	0.282

Problem 386	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	268	374	288	4396	0	309	600
normalized size	1	1.00	0.92	1.29	0.99	15.16	0.00	1.07	2.07
time (sec)	N/A	0.329	0.205	0.959	0.511	1.484	0.000	0.332	0.270

Problem 387	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	385	385	333	668	470	10135	0	510	898
normalized size	1	1.00	0.86	1.74	1.22	26.32	0.00	1.32	2.33
time (sec)	N/A	0.504	2.188	0.973	0.441	2.957	0.000	0.234	15.265

Problem 388	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	300	123	0	0	0	0	2338
normalized size	1	1.00	0.39	0.16	0.00	0.00	0.00	0.00	3.06
time (sec)	N/A	1.530	0.148	1.008	0.000	0.000	0.000	0.000	18.041

Problem 389	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	484	484	231	83	0	0	0	0	951
normalized size	1	1.00	0.48	0.17	0.00	0.00	0.00	0.00	1.96
time (sec)	N/A	0.634	0.099	0.890	0.000	0.000	0.000	0.000	17.401

Problem 390	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	126	83	0	0	0	0	609
normalized size	1	1.00	0.51	0.34	0.00	0.00	0.00	0.00	2.49
time (sec)	N/A	0.251	0.155	0.569	0.000	0.000	0.000	0.000	16.714

Problem 391	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F(-1)	F(-1)	F	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	299	0	432	164	0	0	0	0	19737
normalized size	1	0.00	1.44	0.55	0.00	0.00	0.00	0.00	66.01
time (sec)	N/A	0.045	0.257	0.956	0.000	0.000	0.000	0.000	18.524

Problem 392	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F(-1)	F(-1)	F(-1)	F	B
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	1093	0	679	346	0	0	0	0	323390
normalized size	1	0.00	0.62	0.32	0.00	0.00	0.00	0.00	295.87
time (sec)	N/A	0.046	1.705	1.133	0.000	0.000	0.000	0.000	25.850

Problem 393	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	288	288	402	490	263	6415	0	277	384
normalized size	1	1.00	1.40	1.70	0.91	22.27	0.00	0.96	1.33
time (sec)	N/A	0.335	3.498	0.928	0.449	178.222	0.000	0.214	0.424

Problem 394	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	258	327	213	3878	0	228	203
normalized size	1	1.00	1.08	1.37	0.89	16.29	0.00	0.96	0.85
time (sec)	N/A	0.225	1.087	0.977	0.450	109.777	0.000	0.256	14.982

Problem 395	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	183	183	184	179	163	665	0	169	172
normalized size	1	1.00	1.01	0.98	0.89	3.63	0.00	0.92	0.94
time (sec)	N/A	0.162	0.785	0.929	0.469	0.552	0.000	0.240	0.380

Problem 396	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	152	157	155	655	0	162	165
normalized size	1	1.00	0.86	0.89	0.88	3.72	0.00	0.92	0.94
time (sec)	N/A	0.115	0.467	0.528	0.434	0.534	0.000	0.222	15.004

Problem 397	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	C	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	587	587	503	934	483	10855	0	566	980
normalized size	1	1.00	0.86	1.59	0.82	18.49	0.00	0.96	1.67
time (sec)	N/A	0.688	4.303	0.961	0.452	3.882	0.000	0.279	15.173

Problem 398	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	F(-1)	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	747	747	657	1309	788	0	0	790	1605
normalized size	1	1.00	0.88	1.75	1.05	0.00	0.00	1.06	2.15
time (sec)	N/A	1.022	6.383	1.051	0.450	0.000	0.000	0.329	15.752

Problem 399	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	F(-1)	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	394	550	0	0	0	0	2431
normalized size	1	0.00	15.15	21.15	0.00	0.00	0.00	0.00	93.50
time (sec)	N/A	0.044	0.357	0.965	0.000	0.000	0.000	0.000	15.995

Problem 400	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	273	236	0	0	0	0	1648
normalized size	1	0.00	10.50	9.08	0.00	0.00	0.00	0.00	63.38
time (sec)	N/A	0.043	0.237	0.944	0.000	0.000	0.000	0.000	15.746

Problem 401	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	502	658	0	0	0	0	1567
normalized size	1	0.00	29.53	38.71	0.00	0.00	0.00	0.00	92.18
time (sec)	N/A	0.012	0.476	0.650	0.000	0.000	0.000	0.000	17.893

Problem 402	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A	A
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	845	1276	0	0	0	0	3148
normalized size	1	0.00	32.50	49.08	0.00	0.00	0.00	0.00	121.08
time (sec)	N/A	0.043	1.604	0.956	0.000	0.000	0.000	0.000	21.207

Problem 403	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	F(-1)	F(-1)	F(-1)	A	A
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	1158	1549	0	0	0	0	4657
normalized size	1	0.00	44.54	59.58	0.00	0.00	0.00	0.00	179.12
time (sec)	N/A	0.043	1.744	1.183	0.000	0.000	0.000	0.000	25.437

Problem 404	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	207	350	177	1429	0	360	1931
normalized size	1	1.00	1.58	2.67	1.35	10.91	0.00	2.75	14.74
time (sec)	N/A	0.190	0.263	0.631	0.644	0.745	0.000	0.915	0.774

Problem 405	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	189	252	158	1041	0	311	1097
normalized size	1	1.00	1.67	2.23	1.40	9.21	0.00	2.75	9.71
time (sec)	N/A	0.162	0.200	0.612	0.538	0.619	0.000	0.886	15.784

Problem 406	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	160	160	121	631	0	280	489
normalized size	1	1.00	1.68	1.68	1.27	6.64	0.00	2.95	5.15
time (sec)	N/A	0.102	0.084	0.610	0.544	0.557	0.000	0.771	15.811

Problem 407	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	54	81	100	330	155	224	40
normalized size	1	1.00	0.76	1.14	1.41	4.65	2.18	3.15	0.56
time (sec)	N/A	0.067	0.023	0.227	0.654	110.363	8.905	0.747	0.105

Problem 408	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	184	229	167	1329	0	370	3891
normalized size	1	1.00	1.57	1.96	1.43	11.36	0.00	3.16	33.26
time (sec)	N/A	0.153	0.180	0.613	0.614	0.677	0.000	0.795	17.906

Problem 409	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	255	415	244	2529	0	475	7758
normalized size	1	1.00	1.46	2.37	1.39	14.45	0.00	2.71	44.33
time (sec)	N/A	0.214	1.018	0.704	0.712	1.227	0.000	0.800	19.192

Problem 410	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	249	249	317	660	363	3703	0	630	12217
normalized size	1	1.00	1.27	2.65	1.46	14.87	0.00	2.53	49.06
time (sec)	N/A	0.297	5.602	0.707	1.500	2.594	0.000	0.828	20.572

Problem 411	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	233	880	0	2948	0	896	10319
normalized size	1	1.00	0.92	3.49	0.00	11.70	0.00	3.56	40.95
time (sec)	N/A	0.440	0.907	0.667	0.000	1.965	0.000	1.159	18.799

Problem 412	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	186	186	200	750	0	2433	0	836	8773
normalized size	1	1.00	1.08	4.03	0.00	13.08	0.00	4.49	47.17
time (sec)	N/A	0.326	0.677	0.678	0.000	1.278	0.000	1.101	17.463

Problem 413	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	194	483	0	1751	0	995	3088
normalized size	1	1.00	1.25	3.12	0.00	11.30	0.00	6.42	19.92
time (sec)	N/A	0.286	0.481	0.655	0.000	0.916	0.000	0.997	18.042

Problem 414	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	171	449	0	1197	0	906	4299
normalized size	1	1.00	1.35	3.54	0.00	9.43	0.00	7.13	33.85
time (sec)	N/A	0.235	0.253	0.616	0.000	0.677	0.000	0.973	16.400

Problem 415	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	158	226	0	541	0	559	1409
normalized size	1	1.00	1.26	1.81	0.00	4.33	0.00	4.47	11.27
time (sec)	N/A	0.109	0.283	0.579	0.000	0.564	0.000	0.934	15.661

Problem 416	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	175	393	0	2589	0	1211	2832
normalized size	1	1.00	1.23	2.77	0.00	18.23	0.00	8.53	19.94
time (sec)	N/A	0.232	0.487	0.702	0.000	0.894	0.000	1.002	16.940

Problem 417	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	205	581	0	4113	0	2183	4664
normalized size	1	1.00	1.27	3.61	0.00	25.55	0.00	13.56	28.97
time (sec)	N/A	0.348	1.006	0.762	0.000	1.177	0.000	1.293	17.739

Problem 418	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	253	839	0	5587	0	3106	6534
normalized size	1	1.00	1.24	4.11	0.00	27.39	0.00	15.23	32.03
time (sec)	N/A	0.378	1.320	0.765	0.000	1.964	0.000	1.668	18.226

Problem 419	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.044	8.535	4.550	0.000	0.951	0.000	0.000	0.000

Problem 420	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	191	141	0	0	0	0	0	-1
normalized size	1	0.97	0.72	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.222	0.139	5.240	0.000	0.485	0.000	0.000	0.000

Problem 421	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	0	0	0	0	0	-1
normalized size	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.098	0.051	10.180	0.000	0.521	0.000	0.000	0.000

Problem 422	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	67	0	0	0	0	0	64
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	0.96
time (sec)	N/A	0.042	0.016	4.067	0.000	0.472	0.000	0.000	15.619

Problem 423	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.150	5.928	2.944	0.000	0.471	0.000	0.000	0.000

Problem 424	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	F	F	F	F	F(-1)	F	F
verified	N/A	Yes	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	0	0	0	0	0	0	-1
normalized size	1	1.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.217	9.239	2.916	0.000	0.580	0.000	0.000	0.000

Problem 425	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	5.310	8.284	0.000	0.480	0.000	0.000	0.000

Problem 426	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	17.242	9.685	0.000	0.484	0.000	0.000	0.000

Problem 427	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	1.134	2.142	0.000	0.477	0.000	0.000	0.000

Problem 428	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.043	6.003	2.365	0.000	0.524	0.000	0.000	0.000

Problem 429	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	9.079	1.836	0.000	0.593	0.000	0.000	0.000

Problem 430	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	4.593	2.606	0.000	0.490	0.000	0.000	0.000

Problem 431	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	226	226	155	0	0	0	0	0	-1
normalized size	1	1.00	0.69	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.174	0.216	1.690	0.000	0.474	0.000	0.000	0.000

Problem 432	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	114	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.087	1.722	0.000	0.489	0.000	0.000	0.000

Problem 433	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	69	69	69	0	0	0	0	0	70
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	1.01
time (sec)	N/A	0.049	0.022	1.030	0.000	0.475	0.000	0.000	15.906

Problem 434	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	2.854	0.848	0.000	0.471	0.000	0.000	0.000

Problem 435	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	8.147	0.864	0.000	0.468	0.000	0.000	0.000

Problem 436	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	19.658	1.587	0.000	0.456	0.000	0.000	0.000

Problem 437	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	11.711	1.310	0.000	0.500	0.000	0.000	0.000

Problem 438	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.013	1.573	0.621	0.000	0.484	0.000	0.000	0.000

Problem 439	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.048	4.384	0.818	0.000	0.476	0.000	0.000	0.000

Problem 440	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	8.057	0.885	0.000	0.472	0.000	0.000	0.000

Problem 441	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	113	170	273	179	0	603	115
normalized size	1	1.00	0.88	1.33	2.13	1.40	0.00	4.71	0.90
time (sec)	N/A	0.132	0.284	0.528	0.335	0.827	0.000	7.066	14.340

Problem 442	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	94	94	78	109	159	118	0	393	90
normalized size	1	1.00	0.83	1.16	1.69	1.26	0.00	4.18	0.96
time (sec)	N/A	0.102	0.260	0.459	0.340	0.636	0.000	2.746	14.353

Problem 443	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	52	66	82	78	0	234	52
normalized size	1	1.00	0.81	1.03	1.28	1.22	0.00	3.66	0.81
time (sec)	N/A	0.076	0.094	0.492	0.320	0.526	0.000	0.807	14.282

Problem 444	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	37	47	43	37	0	110	28
normalized size	1	1.00	0.86	1.09	1.00	0.86	0.00	2.56	0.65
time (sec)	N/A	0.039	0.033	0.437	0.324	0.504	0.000	0.232	14.506

Problem 445	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	38	37	37	35	0	38	41
normalized size	1	1.00	1.00	0.97	0.97	0.92	0.00	1.00	1.08
time (sec)	N/A	0.043	0.021	0.303	0.333	0.472	0.000	0.174	14.430

Problem 446	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	50	161	56	91	0	108	69
normalized size	1	1.00	0.79	2.56	0.89	1.44	0.00	1.71	1.10
time (sec)	N/A	0.075	0.148	0.662	0.323	0.487	0.000	0.228	14.457

Problem 447	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	72	302	92	198	0	205	103
normalized size	1	1.00	0.81	3.39	1.03	2.22	0.00	2.30	1.16
time (sec)	N/A	0.089	0.520	0.636	0.328	0.550	0.000	0.276	14.504

Problem 448	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	100	489	137	371	0	353	138
normalized size	1	1.00	0.83	4.04	1.13	3.07	0.00	2.92	1.14
time (sec)	N/A	0.112	0.264	0.643	0.325	0.603	0.000	0.323	15.351

Problem 449	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	147	252	180	602	0	472	141
normalized size	1	1.00	1.22	2.10	1.50	5.02	0.00	3.93	1.18
time (sec)	N/A	0.125	2.387	0.552	0.424	0.561	0.000	10.509	14.780

Problem 450	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	111	161	130	472	0	296	112
normalized size	1	1.00	1.14	1.66	1.34	4.87	0.00	3.05	1.15
time (sec)	N/A	0.107	0.754	0.542	0.424	0.509	0.000	4.321	15.110

Problem 451	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	74	74	75	94	85	366	0	164	83
normalized size	1	1.00	1.01	1.27	1.15	4.95	0.00	2.22	1.12
time (sec)	N/A	0.098	0.304	0.490	0.417	0.492	0.000	1.354	15.184

Problem 452	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	53	53	51	300	0	86	53
normalized size	1	1.00	1.00	1.00	0.96	5.66	0.00	1.62	1.00
time (sec)	N/A	0.076	0.128	0.408	0.423	0.478	0.000	0.488	14.922

Problem 453	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	52	82	50	290	0	85	44
normalized size	1	1.00	1.00	1.58	0.96	5.58	0.00	1.63	0.85
time (sec)	N/A	0.068	0.178	0.470	0.431	0.478	0.000	0.203	14.715

Problem 454	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	72	147	76	402	0	120	64
normalized size	1	1.00	1.01	2.07	1.07	5.66	0.00	1.69	0.90
time (sec)	N/A	0.079	0.292	0.551	0.437	0.490	0.000	0.218	15.090

Problem 455	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	101	239	111	576	0	171	82
normalized size	1	1.00	1.05	2.49	1.16	6.00	0.00	1.78	0.85
time (sec)	N/A	0.096	0.861	0.666	0.416	0.480	0.000	0.261	16.150

Problem 456	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	B	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	135	342	154	834	0	238	100
normalized size	1	1.00	1.15	2.92	1.32	7.13	0.00	2.03	0.85
time (sec)	N/A	0.112	1.077	0.681	0.484	0.490	0.000	0.334	18.908

Problem 457	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	64	64	51	48	69	47	0	0	326
normalized size	1	1.00	0.80	0.75	1.08	0.73	0.00	0.00	5.09
time (sec)	N/A	0.110	0.087	2.511	0.345	0.448	0.000	0.000	19.711

Problem 458	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	46	34	0	0	69
normalized size	1	1.00	0.76	0.92	1.21	0.89	0.00	0.00	1.82
time (sec)	N/A	0.105	0.077	2.102	0.351	0.429	0.000	0.000	0.716

Problem 459	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	19	19	19	21	20	17	0	0	20
normalized size	1	1.00	1.00	1.11	1.05	0.89	0.00	0.00	1.05
time (sec)	N/A	0.063	0.040	0.169	0.429	0.433	0.000	0.000	15.252

Problem 460	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	50	50	55	55	70	57	0	55	-1
normalized size	1	1.00	1.10	1.10	1.40	1.14	0.00	1.10	-0.02
time (sec)	N/A	0.079	0.067	1.635	0.423	0.475	0.000	0.137	0.000

Problem 461	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	88	83	99	88	0	0	-1
normalized size	1	1.00	1.01	0.95	1.14	1.01	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.416	1.850	0.418	0.463	0.000	0.000	0.000

Problem 462	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	75	120	1955	87	0	0	-1
normalized size	1	1.00	0.62	1.00	16.29	0.72	0.00	0.00	-0.01
time (sec)	N/A	0.129	0.328	1.667	1.503	0.471	0.000	0.000	0.000

Problem 463	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	55	84	827	77	0	0	-1
normalized size	1	1.00	0.60	0.92	9.09	0.85	0.00	0.00	-0.01
time (sec)	N/A	0.124	0.191	1.848	0.540	0.468	0.000	0.000	0.000

Problem 464	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	40	54	73	55	0	0	-1
normalized size	1	1.00	0.70	0.95	1.28	0.96	0.00	0.00	-0.02
time (sec)	N/A	0.103	0.047	1.441	0.478	0.462	0.000	0.000	0.000

Problem 465	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	35	43	42	42	0	0	88
normalized size	1	1.00	0.61	0.75	0.74	0.74	0.00	0.00	1.54
time (sec)	N/A	0.112	0.071	0.930	0.422	0.440	0.000	0.000	18.529

Problem 466	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	47	55	57	66	0	0	364
normalized size	1	1.00	0.52	0.60	0.63	0.73	0.00	0.00	4.00
time (sec)	N/A	0.115	0.074	0.900	0.420	0.451	0.000	0.000	18.418

Problem 467	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	67	65	68	86	0	0	555
normalized size	1	1.00	0.54	0.52	0.55	0.69	0.00	0.00	4.48
time (sec)	N/A	0.122	0.182	1.097	0.425	0.463	0.000	0.000	26.643

Problem 468	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	43	51	69	50	0	0	486
normalized size	1	1.00	0.66	0.78	1.06	0.77	0.00	0.00	7.48
time (sec)	N/A	0.115	0.078	2.107	0.345	0.461	0.000	0.000	23.610

Problem 469	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	42	42	31	41	46	40	0	0	100
normalized size	1	1.00	0.74	0.98	1.10	0.95	0.00	0.00	2.38
time (sec)	N/A	0.107	0.064	2.139	0.342	0.442	0.000	0.000	19.497

Problem 470	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	18	18	18	20	65	27	0	0	61
normalized size	1	1.00	1.00	1.11	3.61	1.50	0.00	0.00	3.39
time (sec)	N/A	0.065	0.025	0.137	0.441	0.446	0.000	0.000	0.390

Problem 471	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	31	31	49	40	51	84	0	32	-1
normalized size	1	1.00	1.58	1.29	1.65	2.71	0.00	1.03	-0.03
time (sec)	N/A	0.076	0.042	1.067	0.312	0.461	0.000	0.151	0.000

Problem 472	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	80	69	81	79	0	0	-1
normalized size	1	1.00	1.21	1.05	1.23	1.20	0.00	0.00	-0.02
time (sec)	N/A	0.114	0.173	1.715	0.410	0.447	0.000	0.000	0.000

Problem 473	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	66	103	1518	80	0	0	-1
normalized size	1	1.00	0.73	1.13	16.68	0.88	0.00	0.00	-0.01
time (sec)	N/A	0.139	0.133	1.603	0.733	0.459	0.000	0.000	0.000

Problem 474	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	43	65	527	67	0	0	-1
normalized size	1	1.00	0.69	1.05	8.50	1.08	0.00	0.00	-0.02
time (sec)	N/A	0.119	0.045	1.497	0.502	0.474	0.000	0.000	0.000

Problem 475	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	25	25	25	32	90	36	0	0	37
normalized size	1	1.00	1.00	1.28	3.60	1.44	0.00	0.00	1.48
time (sec)	N/A	0.102	0.028	0.368	0.484	0.506	0.000	0.000	15.063

Problem 476	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	37	44	525	58	0	0	118
normalized size	1	1.00	0.62	0.73	8.75	0.97	0.00	0.00	1.97
time (sec)	N/A	0.117	0.061	0.930	0.501	0.491	0.000	0.000	19.187

Problem 477	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	49	54	1236	79	0	0	491
normalized size	1	1.00	0.51	0.56	12.88	0.82	0.00	0.00	5.11
time (sec)	N/A	0.121	0.069	0.966	0.509	0.543	0.000	0.000	22.470

Problem 478	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	51	51	69	50	0	151	583
normalized size	1	1.00	0.75	0.75	1.01	0.74	0.00	2.22	8.57
time (sec)	N/A	0.127	0.101	2.132	0.333	0.498	0.000	0.768	33.909

Problem 479	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	44	44	34	41	48	40	0	108	389
normalized size	1	1.00	0.77	0.93	1.09	0.91	0.00	2.45	8.84
time (sec)	N/A	0.119	0.110	2.170	0.346	0.453	0.000	0.672	20.209

Problem 480	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	21	21	21	21	95	28	0	64	72
normalized size	1	1.00	1.00	1.00	4.52	1.33	0.00	3.05	3.43
time (sec)	N/A	0.074	0.029	0.108	0.438	0.451	0.000	0.553	18.310

Problem 481	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	53	53	55	75	73	58	0	59	-1
normalized size	1	1.00	1.04	1.42	1.38	1.09	0.00	1.11	-0.02
time (sec)	N/A	0.092	0.068	2.962	0.328	0.452	0.000	0.161	0.000

Problem 482	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	82	69	100	83	0	0	-1
normalized size	1	1.00	1.24	1.05	1.52	1.26	0.00	0.00	-0.02
time (sec)	N/A	0.127	0.173	1.605	0.331	0.460	0.000	0.000	0.000

Problem 483	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	59	104	1532	77	0	0	-1
normalized size	1	1.00	0.56	0.98	14.45	0.73	0.00	0.00	-0.01
time (sec)	N/A	0.157	0.094	1.399	0.727	0.466	0.000	0.000	0.000

Problem 484	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	B	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	44	65	220	66	0	0	-1
normalized size	1	1.00	0.70	1.03	3.49	1.05	0.00	0.00	-0.02
time (sec)	N/A	0.132	0.073	1.365	0.499	0.475	0.000	0.000	0.000

Problem 485	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	38	38	29	35	382	50	0	0	88
normalized size	1	1.00	0.76	0.92	10.05	1.32	0.00	0.00	2.32
time (sec)	N/A	0.126	0.034	0.626	0.492	0.427	0.000	0.000	18.687

Problem 486	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	41	67	1063	75	0	0	393
normalized size	1	1.00	0.53	0.87	13.81	0.97	0.00	0.00	5.10
time (sec)	N/A	0.144	0.091	1.328	0.511	0.437	0.000	0.000	20.606

Problem 487	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F(-2)	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	51	57	2026	100	0	0	589
normalized size	1	1.00	0.44	0.50	17.62	0.87	0.00	0.00	5.12
time (sec)	N/A	0.152	0.130	1.124	0.535	0.515	0.000	0.000	33.014

Problem 488	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	143	721	230	354	0	2790	-1
normalized size	1	1.00	0.81	4.07	1.30	2.00	0.00	15.76	-0.01
time (sec)	N/A	0.211	0.556	5.289	0.438	1.361	0.000	3.632	0.000

Problem 489	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	84	403	127	234	0	1011	-1
normalized size	1	1.00	0.71	3.42	1.08	1.98	0.00	8.57	-0.01
time (sec)	N/A	0.111	0.388	4.796	0.443	0.857	0.000	1.036	0.000

Problem 490	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	58	58	60	134	122	145	0	329	-1
normalized size	1	1.00	1.03	2.31	2.10	2.50	0.00	5.67	-0.02
time (sec)	N/A	0.056	0.057	4.139	0.441	0.667	0.000	0.321	0.000

Problem 491	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	53	61	43	135	0	49	-1
normalized size	1	1.00	0.98	1.13	0.80	2.50	0.00	0.91	-0.02
time (sec)	N/A	0.065	0.047	1.459	0.337	1.133	0.000	0.130	0.000

Problem 492	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	77	130	113	239	0	0	-1
normalized size	1	1.00	0.70	1.18	1.03	2.17	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.200	1.541	0.346	1.504	0.000	0.000	0.000

Problem 493	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	103	230	215	415	0	0	-1
normalized size	1	1.00	0.62	1.39	1.30	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.155	0.569	1.866	0.377	2.063	0.000	0.000	0.000

Problem 494	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	198	380	0	0	0	0	-1
normalized size	1	1.00	0.85	1.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.277	1.993	2.998	0.000	0.523	0.000	0.000	0.000

Problem 495	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	140	294	0	0	0	0	-1
normalized size	1	1.00	0.82	1.72	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.159	0.473	2.532	0.000	0.490	0.000	0.000	0.000

Problem 496	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	61	71	0	0	0	0	-1
normalized size	1	1.00	1.20	1.39	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.033	0.082	0.908	0.000	0.465	0.000	0.000	0.000

Problem 497	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	174	174	143	156	0	0	0	0	-1
normalized size	1	1.00	0.82	0.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.172	0.548	1.397	0.000	0.526	0.000	0.000	0.000

Problem 498	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	232	232	197	351	0	0	0	0	-1
normalized size	1	1.00	0.85	1.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	3.164	1.634	0.000	0.572	0.000	0.000	0.000

Problem 499	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	160	711	238	385	0	3781	-1
normalized size	1	1.00	0.73	3.23	1.08	1.75	0.00	17.19	-0.00
time (sec)	N/A	0.258	2.011	4.035	0.429	1.048	0.000	5.516	0.000

Problem 500	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	116	567	169	265	0	2185	-1
normalized size	1	1.00	0.78	3.83	1.14	1.79	0.00	14.76	-0.01
time (sec)	N/A	0.136	0.507	3.886	0.421	0.819	0.000	2.377	0.000

Problem 501	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	79	423	157	186	0	1338	-1
normalized size	1	1.00	0.94	5.04	1.87	2.21	0.00	15.93	-0.01
time (sec)	N/A	0.078	0.150	3.211	0.459	0.805	0.000	0.881	0.000

Problem 502	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	69	91	61	175	0	71	-1
normalized size	1	1.00	0.88	1.17	0.78	2.24	0.00	0.91	-0.01
time (sec)	N/A	0.079	0.125	1.517	0.314	1.149	0.000	0.136	0.000

Problem 503	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	90	179	148	282	0	0	-1
normalized size	1	1.00	0.64	1.28	1.06	2.01	0.00	0.00	-0.01
time (sec)	N/A	0.127	0.457	1.821	0.333	1.590	0.000	0.000	0.000

Problem 504	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	123	280	272	442	0	0	-1
normalized size	1	1.00	0.59	1.35	1.31	2.12	0.00	0.00	-0.00
time (sec)	N/A	0.192	0.798	1.849	0.332	2.159	0.000	0.000	0.000

Problem 505	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	211	419	0	0	0	0	-1
normalized size	1	1.00	0.77	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.369	2.739	2.839	0.000	0.527	0.000	0.000	0.000

Problem 506	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	174	515	0	0	0	0	-1
normalized size	1	1.00	0.78	2.32	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.231	2.817	2.650	0.000	0.509	0.000	0.000	0.000

Problem 507	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	156	266	0	0	0	0	-1
normalized size	1	1.00	1.01	1.73	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.768	1.387	0.000	0.466	0.000	0.000	0.000

Problem 508	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	173	204	0	0	0	0	-1
normalized size	1	1.00	0.78	0.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.257	2.234	1.651	0.000	0.577	0.000	0.000	0.000

Problem 509	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	218	419	0	0	0	0	-1
normalized size	1	1.00	0.79	1.52	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.351	4.788	1.735	0.000	0.600	0.000	0.000	0.000

Problem 510	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	108	644	248	328	0	2580	-1
normalized size	1	1.00	0.81	4.81	1.85	2.45	0.00	19.25	-0.01
time (sec)	N/A	0.174	0.474	3.429	0.421	0.682	0.000	3.136	0.000

Problem 511	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	81	81	77	353	124	220	0	793	-1
normalized size	1	1.00	0.95	4.36	1.53	2.72	0.00	9.79	-0.01
time (sec)	N/A	0.098	0.219	3.028	0.540	0.593	0.000	1.287	0.000

Problem 512	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	36	36	38	113	106	112	0	98	-1
normalized size	1	1.00	1.06	3.14	2.94	3.11	0.00	2.72	-0.03
time (sec)	N/A	0.051	0.038	2.908	0.415	0.533	0.000	0.576	0.000

Problem 513	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	33	33	33	42	25	100	0	31	-1
normalized size	1	1.00	1.00	1.27	0.76	3.03	0.00	0.94	-0.03
time (sec)	N/A	0.064	0.034	0.982	0.312	0.584	0.000	0.145	0.000

Problem 514	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	71	114	77	220	0	0	-1
normalized size	1	1.00	0.95	1.52	1.03	2.93	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.167	1.706	0.300	0.598	0.000	0.000	0.000

Problem 515	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	101	219	158	388	0	898	-1
normalized size	1	1.00	0.80	1.74	1.25	3.08	0.00	7.13	-0.01
time (sec)	N/A	0.129	0.361	1.970	0.318	0.608	0.000	1.029	0.000

Problem 516	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	188	377	0	0	0	0	-1
normalized size	1	1.00	0.76	1.53	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	2.135	2.831	0.000	0.490	0.000	0.000	0.000

Problem 517	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	109	109	100	222	0	0	0	0	-1
normalized size	1	1.00	0.92	2.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.118	0.384	2.760	0.000	0.497	0.000	0.000	0.000

Problem 518	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	60	60	0	0	0	0	-1
normalized size	1	1.00	1.18	1.18	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.034	0.076	0.366	0.000	0.429	0.000	0.000	0.000

Problem 519	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	101	120	0	0	0	0	-1
normalized size	1	1.00	0.95	1.13	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.364	1.578	0.000	0.476	0.000	0.000	0.000

Problem 520	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	186	351	0	0	0	0	-1
normalized size	1	1.00	0.78	1.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.268	3.969	1.964	0.000	0.485	0.000	0.000	0.000

Problem 521	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	107	3763	334	593	0	2776	-1
normalized size	1	1.00	0.60	21.26	1.89	3.35	0.00	15.68	-0.01
time (sec)	N/A	0.229	0.469	13.202	0.793	0.783	0.000	4.029	0.000

Problem 522	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	75	2194	193	445	0	1011	-1
normalized size	1	1.00	0.64	18.59	1.64	3.77	0.00	8.57	-0.01
time (sec)	N/A	0.122	0.114	10.461	1.066	0.619	0.000	1.614	0.000

Problem 523	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	63	63	54	1317	143	281	0	250	-1
normalized size	1	1.00	0.86	20.90	2.27	4.46	0.00	3.97	-0.02
time (sec)	N/A	0.068	0.071	8.491	0.656	0.565	0.000	0.891	0.000

Problem 524	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	57	57	46	64	46	225	0	57	-1
normalized size	1	1.00	0.81	1.12	0.81	3.95	0.00	1.00	-0.02
time (sec)	N/A	0.076	0.052	1.580	0.302	0.569	0.000	0.155	0.000

Problem 525	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	70	159	117	406	0	525	-1
normalized size	1	1.00	0.64	1.45	1.06	3.69	0.00	4.77	-0.01
time (sec)	N/A	0.123	0.099	1.779	0.506	0.585	0.000	1.008	0.000

Problem 526	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	94	288	219	652	0	1150	-1
normalized size	1	1.00	0.56	1.72	1.31	3.90	0.00	6.89	-0.01
time (sec)	N/A	0.163	0.326	2.008	0.331	0.617	0.000	1.453	0.000

Problem 527	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	197	368	0	0	0	0	-1
normalized size	1	1.00	0.67	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.315	2.278	3.105	0.000	0.511	0.000	0.000	0.000

Problem 528	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	224	224	145	283	0	0	0	0	-1
normalized size	1	1.00	0.65	1.26	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	0.892	2.634	0.000	0.490	0.000	0.000	0.000

Problem 529	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	90	103	0	0	0	0	-1
normalized size	1	1.00	0.89	1.02	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.062	0.148	1.823	0.000	0.463	0.000	0.000	0.000

Problem 530	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	142	141	0	0	0	0	-1
normalized size	1	1.00	0.68	0.67	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.242	0.753	1.611	0.000	0.461	0.000	0.000	0.000

Problem 531	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	199	353	0	0	0	0	-1
normalized size	1	1.00	0.67	1.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.374	3.824	1.910	0.000	0.487	0.000	0.000	0.000

Problem 532	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	107	2139	424	995	0	3826	-1
normalized size	1	1.00	0.49	9.81	1.94	4.56	0.00	17.55	-0.00
time (sec)	N/A	0.277	0.480	8.430	0.657	0.848	0.000	5.035	0.000

Problem 533	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	76	1256	262	769	0	1792	-1
normalized size	1	1.00	0.50	8.21	1.71	5.03	0.00	11.71	-0.01
time (sec)	N/A	0.154	0.119	6.279	0.783	0.680	0.000	2.293	0.000

Problem 534	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	56	898	203	521	0	842	-1
normalized size	1	1.00	0.62	9.87	2.23	5.73	0.00	9.25	-0.01
time (sec)	N/A	0.085	0.082	5.352	0.774	0.600	0.000	1.217	0.000

Problem 535	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	49	271	66	382	0	74	-1
normalized size	1	1.00	0.59	3.27	0.80	4.60	0.00	0.89	-0.01
time (sec)	N/A	0.088	0.056	3.426	0.383	0.597	0.000	0.183	0.000

Problem 536	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	69	1038	156	666	0	751	-1
normalized size	1	1.00	0.48	7.26	1.09	4.66	0.00	5.25	-0.01
time (sec)	N/A	0.137	0.254	4.280	0.441	0.614	0.000	1.355	0.000

Problem 537	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	A	B	F	B	F(-1)
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	117	901	280	984	0	1411	-1
normalized size	1	1.00	0.56	4.33	1.35	4.73	0.00	6.78	-0.00
time (sec)	N/A	0.196	0.840	5.599	0.403	0.674	0.000	1.993	0.000

Problem 538	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	235	667	0	0	0	0	-1
normalized size	1	1.00	0.68	1.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.426	3.420	3.495	0.000	0.572	0.000	0.000	0.000

Problem 539	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	199	851	0	0	0	0	-1
normalized size	1	1.00	0.68	2.91	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.309	2.660	3.007	0.000	0.515	0.000	0.000	0.000

Problem 540	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	172	547	0	0	0	0	-1
normalized size	1	1.00	0.77	2.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.259	1.351	2.089	0.000	0.482	0.000	0.000	0.000

Problem 541	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	209	411	0	0	0	0	-1
normalized size	1	1.00	0.73	1.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	2.661	1.987	0.000	0.525	0.000	0.000	0.000

Problem 542	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	226	633	0	0	0	0	-1
normalized size	1	1.00	0.65	1.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	3.067	1.934	0.000	0.538	0.000	0.000	0.000

Problem 543	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	121	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.122	0.495	4.010	0.000	0.809	0.000	0.000	0.000

Problem 544	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	83	0	0	0	0	0	-1
normalized size	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	0.251	2.011	0.000	0.474	0.000	0.000	0.000

Problem 545	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	61	0	0	0	0	0	-1
normalized size	1	1.00	1.03	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.043	0.062	2.293	0.000	0.455	0.000	0.000	0.000

Problem 546	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.056	2.253	0.000	0.438	0.000	0.000	0.000

Problem 547	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	73	0	0	0	0	0	-1
normalized size	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.429	1.852	0.000	0.483	0.000	0.000	0.000

Problem 548	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.127	4.504	1.089	0.000	0.501	0.000	0.000	0.000

Problem 549	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.105	0.307	1.461	0.000	0.474	0.000	0.000	0.000

Problem 550	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	98	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.203	1.524	0.000	0.483	0.000	0.000	0.000

Problem 551	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	102	0	0	0	0	0	-1
normalized size	1	1.00	1.01	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	3.356	1.109	0.000	0.509	0.000	0.000	0.000

Problem 552	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	C	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	143	126	152	1764	0	144	2003
normalized size	1	1.00	0.93	0.82	0.99	11.53	0.00	0.94	13.09
time (sec)	N/A	0.191	0.312	0.307	1.004	68.267	0.000	0.166	17.542

Problem 553	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-1)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	45	45	45	34	52	0	0	38	-1
normalized size	1	1.00	1.00	0.76	1.16	0.00	0.00	0.84	-0.02
time (sec)	N/A	0.074	0.028	1.934	0.559	0.000	0.000	0.146	0.000

Problem 554	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	F(-2)	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	28	28	28	21	39	0	0	24	-1
normalized size	1	1.00	1.00	0.75	1.39	0.00	0.00	0.86	-0.04
time (sec)	N/A	0.072	0.018	0.277	1.066	0.000	0.000	0.120	0.000

Problem 555	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-1)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	59	59	55	0	68	195	0	0	-1
normalized size	1	1.00	0.93	0.00	1.15	3.31	0.00	0.00	-0.02
time (sec)	N/A	0.089	0.053	1.729	0.611	0.648	0.000	0.000	0.000

Problem 556	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	85	0	0	361	0	0	-1
normalized size	1	1.00	0.96	0.00	0.00	4.06	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.197	1.520	0.000	0.643	0.000	0.000	0.000

Problem 557	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	51	51	65	72	0	240	0	0	-1
normalized size	1	1.00	1.27	1.41	0.00	4.71	0.00	0.00	-0.02
time (sec)	N/A	0.056	0.090	0.514	0.000	0.602	0.000	0.000	0.000

Problem 558	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	35	0	50	140	0	31	-1
normalized size	1	1.00	1.00	0.00	1.43	4.00	0.00	0.89	-0.03
time (sec)	N/A	0.071	0.024	1.546	0.684	0.589	0.000	0.617	0.000

Problem 559	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	66	0	79	247	0	0	-1
normalized size	1	1.00	0.94	0.00	1.13	3.53	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.078	1.811	1.964	0.603	0.000	0.000	0.000

Problem 560	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	141	0	166	371	0	0	-1
normalized size	1	1.00	1.31	0.00	1.54	3.44	0.00	0.00	-0.01
time (sec)	N/A	0.176	2.895	1.881	0.688	0.646	0.000	0.000	0.000

Problem 561	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	291	0	0	0	0	0	-1
normalized size	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.380	6.149	1.540	0.000	0.478	0.000	0.000	0.000

Problem 562	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	304	396	0	0	0	0	-1
normalized size	1	1.00	1.88	2.44	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	2.743	3.392	0.000	0.462	0.000	0.000	0.000

Problem 563	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	477	477	378	0	0	0	0	0	-1
normalized size	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.366	11.300	1.706	0.000	0.476	0.000	0.000	0.000

Problem 564	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	5.751	5.071	0.000	2.703	0.000	0.000	0.000

Problem 565	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	279	279	922	0	0	0	0	0	-1
normalized size	1	1.00	3.30	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.294	18.562	2.622	0.000	0.572	0.000	0.000	0.000

Problem 566	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	463	0	0	0	0	0	-1
normalized size	1	1.00	3.28	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.119	9.901	4.725	0.000	0.486	0.000	0.000	0.000

Problem 567	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	54	54	54	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.065	0.090	4.466	0.000	0.466	0.000	0.000	0.000

Problem 568	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	119	0	0	0	0	0	-1
normalized size	1	1.00	0.94	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.622	2.487	0.000	0.577	0.000	0.000	0.000

Problem 569	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	41.650	1.658	0.000	0.581	0.000	0.000	0.000

Problem 570	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.042	1.826	2.296	0.000	0.533	0.000	0.000	0.000

Problem 571	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.011	0.201	2.498	0.000	0.482	0.000	0.000	0.000

Problem 572	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.041	1.504	2.309	0.000	0.528	0.000	0.000	0.000

Problem 573	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.040	35.145	1.813	0.000	0.562	0.000	0.000	0.000

Problem 574	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	306	306	3544	0	0	0	0	0	-1
normalized size	1	1.00	11.58	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.429	19.601	180.000	0.000	0.493	0.000	0.000	0.000

Problem 575	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	2368	0	0	0	0	0	-1
normalized size	1	1.00	11.01	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.295	15.375	180.000	0.000	0.527	0.000	0.000	0.000

Problem 576	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	F(-1)	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1395	0	0	0	0	0	-1
normalized size	1	1.00	11.25	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.161	13.501	180.000	0.000	0.481	0.000	0.000	0.000

Problem 577	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	2.451	1.567	0.000	0.456	0.000	0.000	0.000

Problem 578	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	F(-1)	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.058	24.109	180.000	0.000	0.452	0.000	0.000	0.000

Problem 579	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	45	38	57	97	0	46	-1
normalized size	1	1.00	0.96	0.81	1.21	2.06	0.00	0.98	-0.02
time (sec)	N/A	0.081	0.028	0.113	0.890	0.458	0.000	0.151	0.000

Problem 580	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	29	29	29	24	41	74	0	27	-1
normalized size	1	1.00	1.00	0.83	1.41	2.55	0.00	0.93	-0.03
time (sec)	N/A	0.079	0.014	0.155	0.810	0.483	0.000	0.134	0.000

Problem 581	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.052	4.097	3.743	0.000	0.481	0.000	0.000	0.000

Problem 582	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	22.875	1.086	0.000	0.473	0.000	0.000	0.000

Problem 583	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	24	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.026	0.964	1.032	0.000	0.473	0.000	0.000	0.000

Problem 584	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	55	55	55	0	0	0	0	0	-1
normalized size	1	1.00	1.00	0.00	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.077	0.050	0.965	0.000	0.464	0.000	0.000	0.000

Problem 585	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	129	0	0	0	0	0	-1
normalized size	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.152	1.032	1.059	0.000	0.463	0.000	0.000	0.000

Problem 586	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	25.254	0.961	0.000	0.500	0.000	0.000	0.000

Problem 587	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.051	2.752	0.906	0.000	0.493	0.000	0.000	0.000

Problem 588	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	17	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.06
time (sec)	N/A	0.013	0.661	0.679	0.000	0.470	0.000	0.000	0.000

Problem 589	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.049	2.092	0.970	0.000	0.474	0.000	0.000	0.000

Problem 590	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	F(-1)	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	26	0	0	0	0	0	0	0	-1
normalized size	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.04
time (sec)	N/A	0.050	36.677	1.076	0.000	0.489	0.000	0.000	0.000

Problem 591	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	114	102	327	0	0	0	0	-1
normalized size	1	1.07	0.95	3.06	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.181	0.183	1.028	0.000	0.471	0.000	0.000	0.000

Problem 592	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	135	0	0	0	0	0	-1
normalized size	1	1.00	0.99	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.843	8.127	0.000	1.839	0.000	0.000	0.000

Problem 593	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	B	B	F	F	F	F	F
verified	N/A	N/A	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	325	4067972	0	0	0	0	-1
normalized size	1	0.00	4.11	51493.32	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.699	1.668	4.679	0.000	0.512	0.000	0.000	0.000

Problem 594	Optimal	Rubi	Mathematica	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	F	C	C	F	F	F	F	F
verified	N/A	N/A	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	79	0	529	258179	0	0	0	0	-1
normalized size	1	0.00	6.70	3268.09	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.697	1.590	3.765	0.000	0.481	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [238] had the largest ratio of [.7500]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	4	3	1.00	10	0.300
2	A	3	3	1.00	10	0.300
3	A	2	2	1.00	10	0.200
4	A	2	2	1.00	10	0.200
5	A	3	3	1.00	10	0.300
6	A	4	3	1.00	10	0.300
7	A	6	3	1.00	10	0.300
8	A	4	3	1.00	10	0.300
9	A	3	3	1.00	10	0.300
10	A	3	3	1.00	10	0.300
11	A	4	3	1.00	10	0.300
12	A	6	3	1.00	10	0.300
13	A	7	3	1.00	10	0.300
14	A	5	3	1.00	10	0.300
15	A	3	3	1.00	10	0.300
16	A	3	3	1.00	10	0.300
17	A	3	2	1.00	10	0.200
18	A	3	2	1.00	10	0.200
19	A	2	2	1.00	14	0.143
20	A	2	2	1.00	14	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
21	A	2	2	1.00	14	0.143
22	A	2	2	1.00	14	0.143
23	A	2	2	1.00	14	0.143
24	A	2	2	1.00	14	0.143
25	A	2	2	1.00	12	0.167
26	A	2	2	1.00	12	0.167
27	A	2	2	1.00	12	0.167
28	A	2	2	1.00	12	0.167
29	A	2	2	1.00	14	0.143
30	A	2	2	1.00	14	0.143
31	A	3	2	1.00	9	0.222
32	A	4	3	1.00	11	0.273
33	A	5	3	1.00	11	0.273
34	A	6	3	1.00	11	0.273
35	A	4	3	1.00	24	0.125
36	A	4	3	1.00	24	0.125
37	A	4	3	1.00	24	0.125
38	A	3	3	1.00	22	0.136
39	A	4	4	1.00	22	0.182
40	A	5	5	1.00	24	0.208
41	A	6	5	1.00	24	0.208
42	A	6	5	1.00	24	0.208
43	A	5	5	1.00	24	0.208
44	A	4	4	1.00	24	0.167
45	A	3	3	1.00	15	0.200
46	A	4	3	1.00	24	0.125
47	A	4	3	1.00	24	0.125
48	A	4	3	1.00	24	0.125
49	A	4	3	1.00	24	0.125
50	A	4	3	1.00	24	0.125
51	A	3	2	1.00	24	0.083
52	A	3	3	1.00	22	0.136
53	A	5	4	1.00	22	0.182
54	A	6	5	1.00	24	0.208
55	A	6	5	1.00	24	0.208
56	A	4	3	1.00	24	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
57	A	3	3	1.00	24	0.125
58	A	3	2	1.00	15	0.133
59	A	4	3	1.00	24	0.125
60	A	4	3	1.00	24	0.125
61	A	3	2	1.00	11	0.182
62	A	3	2	1.00	11	0.182
63	A	3	2	1.00	11	0.182
64	A	3	2	1.00	21	0.095
65	A	2	1	1.00	19	0.053
66	A	2	2	1.00	19	0.105
67	A	2	2	1.00	21	0.095
68	A	4	3	1.00	21	0.143
69	A	3	3	1.00	21	0.143
70	A	3	2	1.00	12	0.167
71	A	2	2	1.00	21	0.095
72	A	3	3	1.00	21	0.143
73	A	3	2	1.00	21	0.095
74	A	3	2	1.00	8	0.250
75	A	1	1	1.00	10	0.100
76	A	2	2	1.00	10	0.200
77	A	3	3	1.00	10	0.300
78	A	4	3	1.00	23	0.130
79	A	4	3	1.00	23	0.130
80	A	3	3	1.00	23	0.130
81	A	2	2	1.00	21	0.095
82	A	4	4	1.00	21	0.190
83	A	5	5	1.00	23	0.217
84	A	6	6	1.00	23	0.261
85	A	7	6	1.00	23	0.261
86	A	6	6	1.00	23	0.261
87	A	5	5	1.00	23	0.217
88	A	3	3	1.00	23	0.130
89	A	2	2	1.00	14	0.143
90	A	3	3	1.00	23	0.130
91	A	4	3	1.00	23	0.130
92	A	4	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
93	A	4	3	1.00	23	0.130
94	A	5	4	1.00	23	0.174
95	A	5	4	1.00	23	0.174
96	A	3	3	1.00	23	0.130
97	A	3	3	1.00	21	0.143
98	A	5	5	1.00	21	0.238
99	A	6	6	1.00	23	0.261
100	A	6	6	1.00	23	0.261
101	A	5	5	1.00	23	0.217
102	A	4	4	1.00	23	0.174
103	A	4	4	1.00	14	0.286
104	A	4	4	1.02	23	0.174
105	A	5	4	1.00	23	0.174
106	A	6	6	1.00	23	0.261
107	A	4	3	1.00	23	0.130
108	A	5	4	1.00	23	0.174
109	A	5	5	1.00	14	0.357
110	A	5	5	1.00	23	0.217
111	A	6	5	1.00	14	0.357
112	A	7	5	1.00	14	0.357
113	A	2	2	1.00	13	0.154
114	A	3	3	1.00	13	0.231
115	A	2	2	1.00	21	0.095
116	A	5	4	1.00	13	0.308
117	A	4	4	1.00	13	0.308
118	A	3	3	1.00	13	0.231
119	A	3	3	1.00	13	0.231
120	A	4	4	1.00	13	0.308
121	A	5	4	1.00	13	0.308
122	A	5	5	1.00	25	0.200
123	A	4	4	1.00	23	0.174
124	A	6	6	1.00	23	0.261
125	A	4	4	1.00	25	0.160
126	A	5	5	1.00	25	0.200
127	A	8	8	1.00	25	0.320
128	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
129	A	2	2	1.00	16	0.125
130	A	8	8	1.00	25	0.320
131	A	8	8	1.00	25	0.320
132	A	6	5	1.00	25	0.200
133	A	5	4	1.00	23	0.174
134	A	7	7	1.00	23	0.304
135	A	7	7	1.00	25	0.280
136	A	5	4	1.00	25	0.160
137	A	6	5	1.00	25	0.200
138	A	9	8	1.00	25	0.320
139	A	7	6	1.00	25	0.240
140	A	6	6	1.00	16	0.375
141	A	7	7	1.00	25	0.280
142	A	8	8	1.00	25	0.320
143	A	7	7	1.00	16	0.438
144	A	4	4	1.00	25	0.160
145	A	3	3	1.00	23	0.130
146	A	3	3	1.00	23	0.130
147	A	4	4	1.00	25	0.160
148	A	7	7	1.00	25	0.280
149	A	5	5	1.00	25	0.200
150	A	2	2	1.00	16	0.125
151	A	8	8	1.00	25	0.320
152	A	8	8	1.00	25	0.320
153	A	4	4	1.00	25	0.160
154	A	2	2	1.00	23	0.087
155	A	4	4	1.00	23	0.174
156	A	6	6	1.00	25	0.240
157	A	8	8	1.00	25	0.320
158	A	7	7	1.00	25	0.280
159	A	6	6	1.00	25	0.240
160	A	4	4	1.00	16	0.250
161	A	8	8	1.00	25	0.320
162	A	5	5	1.00	25	0.200
163	A	3	3	1.00	25	0.120
164	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
165	A	6	6	1.00	23	0.261
166	A	8	8	1.00	25	0.320
167	A	8	8	1.00	25	0.320
168	A	7	6	1.00	25	0.240
169	A	7	7	1.00	16	0.438
170	A	9	9	1.00	25	0.360
171	A	3	3	1.00	25	0.120
172	A	5	5	1.00	23	0.217
173	A	4	4	1.00	23	0.174
174	A	3	3	1.00	21	0.143
175	A	3	3	1.00	21	0.143
176	A	3	3	1.00	23	0.130
177	A	3	3	1.00	23	0.130
178	A	3	3	1.00	23	0.130
179	A	3	3	1.00	23	0.130
180	A	3	3	1.00	23	0.130
181	A	3	3	1.00	23	0.130
182	A	17	7	1.00	23	0.304
183	A	15	7	1.00	23	0.304
184	A	13	5	1.00	23	0.217
185	A	11	4	1.00	21	0.190
186	A	14	6	1.00	21	0.286
187	A	15	7	1.00	23	0.304
188	A	18	8	1.00	23	0.348
189	A	15	6	1.00	23	0.261
190	A	14	5	1.00	23	0.217
191	A	11	5	1.00	23	0.217
192	A	11	4	1.00	14	0.286
193	A	15	6	1.00	23	0.261
194	A	16	7	1.00	23	0.304
195	A	6	5	1.00	24	0.208
196	A	6	5	1.00	24	0.208
197	A	6	5	1.00	24	0.208
198	A	4	4	1.00	24	0.167
199	A	4	4	1.00	22	0.182
200	A	7	6	1.00	22	0.273

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
201	A	7	6	1.00	24	0.250
202	A	7	6	1.00	24	0.250
203	A	12	6	1.00	24	0.250
204	A	9	6	1.00	24	0.250
205	A	7	5	1.00	24	0.208
206	A	4	3	1.00	24	0.125
207	A	4	3	1.00	15	0.200
208	A	6	4	1.00	24	0.167
209	A	6	4	1.00	24	0.167
210	A	6	4	1.00	24	0.167
211	A	6	4	1.00	24	0.167
212	A	7	6	1.00	24	0.250
213	A	5	5	1.00	24	0.208
214	A	5	5	1.00	24	0.208
215	A	5	5	1.00	24	0.208
216	A	5	5	1.00	22	0.227
217	A	11	7	1.00	22	0.318
218	A	14	9	1.00	24	0.375
219	A	6	5	1.00	24	0.208
220	A	7	6	1.00	24	0.250
221	A	5	4	1.00	24	0.167
222	A	5	4	1.00	15	0.267
223	A	7	5	1.00	24	0.208
224	A	6	6	1.00	24	0.250
225	A	6	6	1.00	24	0.250
226	A	6	6	1.00	24	0.250
227	A	6	5	1.00	24	0.208
228	A	6	6	1.00	22	0.273
229	A	16	7	1.00	22	0.318
230	A	9	7	1.00	24	0.292
231	A	6	5	1.00	24	0.208
232	A	6	5	1.00	24	0.208
233	A	6	5	1.00	24	0.208
234	A	6	5	1.00	15	0.333
235	A	8	6	1.00	24	0.250
236	A	3	3	1.80	10	0.300

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
237	A	10	6	1.00	10	0.600
238	A	10	6	1.00	8	0.750
239	A	5	5	1.00	23	0.217
240	A	8	7	1.00	23	0.304
241	A	5	5	1.00	25	0.200
242	A	4	4	1.00	25	0.160
243	A	2	2	1.00	23	0.087
244	A	4	4	1.00	23	0.174
245	A	7	7	1.00	25	0.280
246	A	4	4	1.00	25	0.160
247	A	2	2	1.00	16	0.125
248	A	5	5	1.00	25	0.200
249	A	17	4	1.00	10	0.400
250	A	7	3	1.00	10	0.300
251	A	9	3	1.00	10	0.300
252	A	17	4	1.00	11	0.364
253	A	7	3	1.00	11	0.273
254	A	9	3	1.00	11	0.273
255	A	15	5	1.00	8	0.625
256	A	7	3	1.00	8	0.375
257	A	9	3	0.59	8	0.375
258	A	15	5	1.00	10	0.500
259	A	8	6	1.00	10	0.600
260	A	10	6	1.00	10	0.600
261	A	3	2	1.00	16	0.125
262	A	3	2	1.00	16	0.125
263	A	3	2	1.00	16	0.125
264	A	2	2	1.00	16	0.125
265	A	2	2	1.00	14	0.143
266	A	4	3	1.00	16	0.188
267	A	4	3	1.00	16	0.188
268	A	3	3	1.00	16	0.188
269	A	2	2	1.00	16	0.125
270	A	3	3	1.00	14	0.214
271	A	3	2	1.00	16	0.125
272	A	3	2	1.00	16	0.125

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
273	A	3	2	1.00	16	0.125
274	A	3	2	1.00	16	0.125
275	A	2	2	1.00	16	0.125
276	A	2	2	1.00	16	0.125
277	A	3	3	1.00	14	0.214
278	A	4	3	1.00	14	0.214
279	A	4	3	1.00	16	0.188
280	A	3	3	1.00	16	0.188
281	A	2	2	1.00	16	0.125
282	A	3	3	1.00	16	0.188
283	A	3	2	1.00	16	0.125
284	A	3	2	1.00	16	0.125
285	A	6	4	1.00	21	0.190
286	A	5	4	1.00	21	0.190
287	A	4	4	1.00	21	0.190
288	A	3	2	1.00	12	0.167
289	A	3	3	1.00	21	0.143
290	A	2	1	1.00	21	0.048
291	A	3	2	1.00	21	0.095
292	A	3	2	1.00	21	0.095
293	A	6	5	1.00	23	0.217
294	A	5	5	1.00	23	0.217
295	A	1	1	1.00	14	0.071
296	A	5	4	1.00	23	0.174
297	A	4	3	1.00	23	0.130
298	A	3	2	1.00	23	0.087
299	A	3	2	1.00	23	0.087
300	A	3	2	1.00	23	0.087
301	A	4	3	1.00	15	0.200
302	A	6	6	1.00	15	0.400
303	A	4	3	1.00	15	0.200
304	A	5	5	1.00	15	0.333
305	A	3	3	1.00	15	0.200
306	A	4	4	1.00	15	0.267
307	A	2	2	1.00	13	0.154
308	A	4	4	1.00	13	0.308

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
309	A	3	3	1.00	15	0.200
310	A	5	5	1.00	15	0.333
311	A	4	3	1.00	15	0.200
312	A	6	6	1.00	15	0.400
313	A	4	3	1.00	15	0.200
314	A	6	6	1.00	15	0.400
315	A	5	4	1.00	15	0.267
316	A	5	5	1.00	15	0.333
317	A	3	3	1.00	15	0.200
318	A	3	3	1.00	15	0.200
319	A	3	3	1.00	13	0.231
320	A	5	5	1.00	13	0.385
321	A	5	4	1.00	15	0.267
322	A	6	6	1.00	15	0.400
323	A	5	4	1.00	15	0.267
324	A	5	5	1.00	25	0.200
325	A	4	4	1.00	23	0.174
326	A	6	5	1.00	23	0.217
327	A	4	4	1.00	25	0.160
328	A	5	5	1.00	25	0.200
329	A	8	8	1.18	25	0.320
330	A	7	7	1.25	25	0.280
331	A	2	2	1.00	16	0.125
332	A	8	8	1.31	25	0.320
333	A	8	8	1.20	25	0.320
334	A	6	5	1.00	25	0.200
335	A	5	4	1.00	23	0.174
336	A	7	6	1.00	23	0.261
337	A	7	6	1.00	25	0.240
338	A	5	4	1.00	25	0.160
339	A	6	5	1.00	25	0.200
340	A	9	8	1.00	25	0.320
341	A	8	8	1.00	25	0.320
342	A	6	6	1.00	16	0.375
343	A	7	7	1.00	25	0.280
344	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
345	A	4	4	1.00	25	0.160
346	A	3	3	1.00	23	0.130
347	A	3	3	1.00	23	0.130
348	A	4	4	1.00	25	0.160
349	A	7	7	1.24	25	0.280
350	A	6	6	1.34	25	0.240
351	A	2	2	1.00	16	0.125
352	A	8	8	1.29	25	0.320
353	A	8	8	1.19	25	0.320
354	A	4	4	1.00	25	0.160
355	A	2	2	1.00	23	0.087
356	A	4	4	1.00	23	0.174
357	A	6	6	1.00	25	0.240
358	A	8	8	1.00	25	0.320
359	A	7	7	1.00	25	0.280
360	A	7	7	1.00	25	0.280
361	A	4	4	1.00	16	0.250
362	A	8	8	1.00	25	0.320
363	A	5	5	1.00	25	0.200
364	A	3	3	1.00	25	0.120
365	A	3	3	1.00	23	0.130
366	A	6	6	1.00	23	0.261
367	A	8	8	1.16	25	0.320
368	A	8	8	1.18	25	0.320
369	A	8	8	1.18	25	0.320
370	A	7	7	1.00	16	0.438
371	A	9	8	1.14	25	0.320
372	A	3	3	1.00	25	0.120
373	A	5	5	1.00	23	0.217
374	A	4	4	0.96	23	0.174
375	A	3	3	1.00	21	0.143
376	A	3	3	1.00	21	0.143
377	A	3	3	1.00	23	0.130
378	A	3	3	1.00	23	0.130
379	A	3	3	1.00	23	0.130
380	A	3	3	1.00	14	0.214

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
381	A	3	3	1.00	23	0.130
382	A	3	3	1.00	23	0.130
383	A	11	10	1.00	23	0.435
384	A	9	9	1.00	23	0.391
385	A	7	7	1.00	21	0.333
386	A	11	10	1.00	21	0.476
387	A	11	10	1.00	23	0.435
388	A	38	8	1.00	23	0.348
389	A	24	7	1.00	23	0.304
390	A	11	4	1.00	14	0.286
391	F	0	0	N/A	0	N/A
392	F	0	0	N/A	0	N/A
393	A	10	9	1.00	23	0.391
394	A	8	8	1.00	23	0.348
395	A	9	9	1.00	23	0.391
396	A	8	8	1.00	21	0.381
397	A	18	11	1.00	21	0.524
398	A	18	11	1.00	23	0.478
399	A	0	0	0.00	0	0.000
400	A	0	0	0.00	0	0.000
401	A	0	0	0.00	0	0.000
402	A	0	0	0.00	0	0.000
403	A	0	0	0.00	0	0.000
404	A	6	5	1.00	24	0.208
405	A	6	5	1.00	24	0.208
406	A	4	4	1.00	24	0.167
407	A	4	4	1.00	22	0.182
408	A	7	6	1.00	22	0.273
409	A	7	6	1.00	24	0.250
410	A	7	6	1.00	24	0.250
411	A	16	6	1.00	24	0.250
412	A	12	6	1.00	24	0.250
413	A	9	6	1.00	24	0.250
414	A	7	5	1.00	24	0.208
415	A	4	3	1.00	24	0.125
416	A	6	4	1.00	24	0.167

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	number of rules / integrand leaf size
417	A	6	4	1.00	24	0.167
418	A	6	4	1.00	24	0.167
419	A	0	0	0.00	0	0.000
420	A	8	7	0.97	23	0.304
421	A	7	6	1.00	23	0.261
422	A	3	3	1.00	21	0.143
423	A	7	6	1.00	21	0.286
424	A	9	6	1.00	23	0.261
425	A	0	0	0.00	0	0.000
426	A	0	0	0.00	0	0.000
427	A	0	0	0.00	0	0.000
428	A	0	0	0.00	0	0.000
429	A	0	0	0.00	0	0.000
430	A	0	0	0.00	0	0.000
431	A	9	6	1.00	23	0.261
432	A	7	6	1.00	23	0.261
433	A	3	3	1.00	21	0.143
434	A	0	0	0.00	0	0.000
435	A	0	0	0.00	0	0.000
436	A	0	0	0.00	0	0.000
437	A	0	0	0.00	0	0.000
438	A	0	0	0.00	0	0.000
439	A	0	0	0.00	0	0.000
440	A	0	0	0.00	0	0.000
441	A	3	2	1.00	23	0.087
442	A	3	2	1.00	23	0.087
443	A	3	2	1.00	23	0.087
444	A	4	3	1.00	21	0.143
445	A	4	4	1.00	21	0.190
446	A	3	2	1.00	23	0.087
447	A	3	2	1.00	23	0.087
448	A	3	2	1.00	23	0.087
449	A	4	3	1.00	23	0.130
450	A	4	3	1.00	23	0.130
451	A	4	3	1.00	23	0.130
452	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
453	A	3	3	1.00	23	0.130
454	A	4	3	1.00	23	0.130
455	A	5	3	1.00	23	0.130
456	A	6	3	1.00	23	0.130
457	A	5	4	1.00	26	0.154
458	A	5	4	1.00	26	0.154
459	A	4	4	1.00	24	0.167
460	A	5	5	1.00	24	0.208
461	A	7	7	1.00	26	0.269
462	A	7	6	1.00	26	0.231
463	A	6	6	1.00	26	0.231
464	A	5	5	1.00	26	0.192
465	A	5	4	1.00	26	0.154
466	A	5	4	1.00	26	0.154
467	A	5	4	1.00	26	0.154
468	A	5	4	1.00	26	0.154
469	A	5	4	1.00	26	0.154
470	A	4	4	1.00	24	0.167
471	A	4	4	1.00	24	0.167
472	A	6	6	1.00	26	0.231
473	A	5	4	1.00	26	0.154
474	A	4	4	1.00	26	0.154
475	A	4	4	1.00	26	0.154
476	A	4	3	1.00	26	0.115
477	A	5	4	1.00	26	0.154
478	A	5	4	1.00	26	0.154
479	A	5	4	1.00	26	0.154
480	A	4	4	1.00	24	0.167
481	A	5	5	1.00	24	0.208
482	A	6	6	1.00	26	0.231
483	A	5	5	1.00	26	0.192
484	A	5	5	1.00	26	0.192
485	A	4	4	1.00	26	0.154
486	A	5	4	1.00	26	0.154
487	A	5	4	1.00	26	0.154
488	A	6	6	1.00	25	0.240

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
489	A	5	5	1.00	25	0.200
490	A	4	4	1.00	23	0.174
491	A	4	4	1.00	23	0.174
492	A	5	5	1.00	25	0.200
493	A	6	6	1.00	25	0.240
494	A	8	8	1.00	25	0.320
495	A	7	7	1.00	25	0.280
496	A	2	2	1.00	16	0.125
497	A	7	7	1.00	25	0.280
498	A	8	8	1.00	25	0.320
499	A	7	6	1.00	25	0.240
500	A	6	5	1.00	25	0.200
501	A	5	4	1.00	23	0.174
502	A	5	4	1.00	23	0.174
503	A	6	5	1.00	25	0.200
504	A	7	6	1.00	25	0.240
505	A	9	9	1.00	25	0.360
506	A	8	8	1.00	25	0.320
507	A	6	6	1.00	16	0.375
508	A	8	8	1.00	25	0.320
509	A	9	9	1.00	25	0.360
510	A	5	5	1.00	25	0.200
511	A	4	4	1.00	25	0.160
512	A	3	3	1.00	23	0.130
513	A	3	3	1.00	23	0.130
514	A	4	4	1.00	25	0.160
515	A	5	5	1.00	25	0.200
516	A	8	8	1.00	25	0.320
517	A	4	4	1.00	25	0.160
518	A	2	2	1.00	16	0.125
519	A	5	5	1.00	25	0.200
520	A	8	8	1.00	25	0.320
521	A	6	6	1.00	25	0.240
522	A	5	5	1.00	25	0.200
523	A	4	4	1.00	23	0.174
524	A	4	4	1.00	23	0.174

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
525	A	5	5	1.00	25	0.200
526	A	6	6	1.00	25	0.240
527	A	9	8	1.00	25	0.320
528	A	8	8	1.00	25	0.320
529	A	4	4	1.00	16	0.250
530	A	8	8	1.00	25	0.320
531	A	9	8	1.00	25	0.320
532	A	7	6	1.00	25	0.240
533	A	6	5	1.00	25	0.200
534	A	5	4	1.00	23	0.174
535	A	5	4	1.00	23	0.174
536	A	6	5	1.00	25	0.200
537	A	7	6	1.00	25	0.240
538	A	10	8	1.00	25	0.320
539	A	9	8	1.00	25	0.320
540	A	7	7	1.00	16	0.438
541	A	9	9	1.00	25	0.360
542	A	10	9	1.00	25	0.360
543	A	3	3	1.00	25	0.120
544	A	3	3	1.00	23	0.130
545	A	2	2	1.00	21	0.095
546	A	2	2	1.00	21	0.095
547	A	3	3	1.00	23	0.130
548	A	3	3	1.00	23	0.130
549	A	3	3	1.00	23	0.130
550	A	3	3	1.00	23	0.130
551	A	3	3	1.00	23	0.130
552	A	11	10	1.00	15	0.667
553	A	5	5	1.00	15	0.333
554	A	4	4	1.00	15	0.267
555	A	5	5	1.00	23	0.217
556	A	4	4	1.00	25	0.160
557	A	3	3	1.00	23	0.130
558	A	4	4	1.00	23	0.174
559	A	5	5	1.00	25	0.200
560	A	6	6	1.00	25	0.240

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
561	A	4	4	1.00	25	0.160
562	A	2	2	1.00	16	0.125
563	A	6	6	1.00	25	0.240
564	A	0	0	0.00	0	0.000
565	A	11	10	1.00	23	0.435
566	A	7	6	1.00	21	0.286
567	A	3	3	1.00	21	0.143
568	A	6	6	1.00	23	0.261
569	A	0	0	0.00	0	0.000
570	A	0	0	0.00	0	0.000
571	A	0	0	0.00	0	0.000
572	A	0	0	0.00	0	0.000
573	A	0	0	0.00	0	0.000
574	A	10	5	1.00	23	0.217
575	A	8	5	1.00	23	0.217
576	A	6	5	1.00	21	0.238
577	A	0	0	0.00	0	0.000
578	A	0	0	0.00	0	0.000
579	A	5	5	1.00	15	0.333
580	A	4	4	1.00	15	0.267
581	A	0	0	0.00	0	0.000
582	A	0	0	0.00	0	0.000
583	A	0	0	0.00	0	0.000
584	A	3	3	1.00	21	0.143
585	A	7	6	1.00	23	0.261
586	A	0	0	0.00	0	0.000
587	A	0	0	0.00	0	0.000
588	A	0	0	0.00	0	0.000
589	A	0	0	0.00	0	0.000
590	A	0	0	0.00	0	0.000
591	A	7	7	1.07	37	0.189
592	A	3	3	1.00	35	0.086
593	F	0	0	N/A	0	N/A
594	F	0	0	N/A	0	N/A

Chapter 3

Listing of integrals

3.1 $\int \left(a \sin^2(x)\right)^{5/2} dx$

Optimal. Leaf size=53

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

```
[Out] -4/15*a*cot(x)*(a*sin(x)^2)^(3/2)-1/5*cot(x)*(a*sin(x)^2)^(5/2)-8/15*a^2*co
t(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2638}

$$-\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(5/2),x]
```

```
[Out] (-8*a^2*Cot[x]*Sqrt[a*Sin[x]^2])/15 - (4*a*Cot[x]*(a*Sin[x]^2)^(3/2))/15 -
(Cot[x]*(a*Sin[x]^2)^(5/2))/5
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a\sin^2(x)\right)^{5/2}\,dx &= -\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{5}(4a)\int \left(a\sin^2(x)\right)^{3/2}\,dx\\ &= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\sin^2(x)}\,dx\\ &= -\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}+\frac{1}{15}\left(8a^2\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\,d\\ &= -\frac{8}{15}a^2\cot(x)\sqrt{a\sin^2(x)}-\frac{4}{15}a\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{1}{5}\cot(x)\left(a\sin^2(x)\right)^{5/2}\end{aligned}$$

Mathematica [A] time = 0.03, size = 36, normalized size = 0.68

$$-\frac{1}{240}a^2(150\cos(x)-25\cos(3x)+3\cos(5x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(5/2),x]
```

```
[Out] -1/240*(a^2*(150*Cos[x] - 25*Cos[3*x] + 3*Cos[5*x])*Csc[x]*Sqrt[a*Sin[x]^2]
)
```

fricas [A] time = 0.43, size = 43, normalized size = 0.81

$$-\frac{\left(3\,a^2\cos(x)^5-10\,a^2\cos(x)^3+15\,a^2\cos(x)\right)\sqrt{-a\cos(x)^2+a}}{15\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x))*sqrt(-a*cos(x)^2 +
 a)/sin(x)
```

giac [A] time = 3.74, size = 45, normalized size = 0.85

$$\frac{1}{15}\left(8\,a^2\mathrm{sgn}\left(\sin(x)\right)-\left(3\,a^2\cos(x)^5-10\,a^2\cos(x)^3+15\,a^2\cos(x)\right)\mathrm{sgn}\left(\sin(x)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/15*(8*a^2*sgn(sin(x)) - (3*a^2*cos(x)^5 - 10*a^2*cos(x)^3 + 15*a^2*cos(x)
)*sgn(sin(x)))*sqrt(a)
```

maple [A] time = 0.71, size = 32, normalized size = 0.60

$$-\frac{a^3\sin(x)\cos(x)\left(3\left(\sin^4(x)\right)+4\left(\sin^2(x)\right)+8\right)}{15\sqrt{a\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(5/2),x)
```

```
[Out] -1/15*a^3*sin(x)*cos(x)*(3*sin(x)^4+4*sin(x)^2+8)/(a*sin(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin(x)^2\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((a*sin(x)^2)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a \sin(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(5/2),x)
[Out] int((a*sin(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin^2(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(5/2),x)
[Out] Integral((a*sin(x)**2)**(5/2), x)
```

3.2 $\int \left(a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

```
[Out] -1/3*cot(x)*(a*sin(x)^2)^(3/2)-2/3*a*cot(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3203, 3207, 2638}

$$-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}-\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(3/2),x]
```

```
[Out] (-2*a*Cot[x]*Sqrt[a*Sin[x]^2])/3 - (Cot[x]*(a*Sin[x]^2)^(3/2))/3
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] && !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(a\sin^2(x)\right)^{3/2}\,dx &= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}(2a)\int \sqrt{a\sin^2(x)}\,dx\\
&= -\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}+\frac{1}{3}\left(2a\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\,dx\\
&= -\frac{2}{3}a\cot(x)\sqrt{a\sin^2(x)}-\frac{1}{3}\cot(x)\left(a\sin^2(x)\right)^{3/2}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.76

$$\frac{1}{12}a(\cos(3x)-9\cos(x))\csc(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(3/2),x]
[Out] (a*(-9*Cos[x] + Cos[3*x])*Csc[x]*Sqrt[a*Sin[x]^2])/12
```

fricas [A] time = 0.42, size = 29, normalized size = 0.85

$$\frac{\left(a\cos(x)^3 - 3\,a\cos(x)\right)\sqrt{-a\cos(x)^2 + a}}{3\,\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="fricas")
[Out] 1/3*(a*cos(x)^3 - 3*a*cos(x))*sqrt(-a*cos(x)^2 + a)/sin(x)
```

giac [A] time = 0.13, size = 24, normalized size = 0.71

$$\frac{1}{3}\left(\left(\cos(x)^3 - 3\,\cos(x)\right)\operatorname{sgn}(\sin(x)) + 2\operatorname{sgn}(\sin(x))\right)a^{\frac{3}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="giac")
[Out] 1/3*((cos(x)^3 - 3*cos(x))*sgn(sin(x)) + 2*sgn(sin(x)))*a^(3/2)
```

maple [A] time = 0.82, size = 24, normalized size = 0.71

$$-\frac{a^2\sin(x)\cos(x)\left(2+\sin^2(x)\right)}{3\sqrt{a\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(3/2),x)
[Out] -1/3*a^2*sin(x)*cos(x)*(2+sin(x)^2)/(a*sin(x)^2)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin(x)^2\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((a*sin(x)^2)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int {\left(a\,{\sin(x)}^2\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(3/2),x)
[Out] int((a*sin(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin^2(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(3/2),x)
[Out] Integral((a*sin(x)**2)**(3/2), x)
```

3.3 $\int \sqrt{a \sin^2(x)} \, dx$

Optimal. Leaf size=14

$$-\cot(x)\sqrt{a\sin^2(x)}$$

```
[Out] -cot(x)*(a*sin(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 14, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{3207, 2638\}$

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a\sin^2(x)}\,dx &= \left(\csc(x)\sqrt{a\sin^2(x)}\right)\int \sin(x)\,dx\\ &= -\cot(x)\sqrt{a\sin^2(x)}\end{aligned}$$

Mathematica [A] time = 0.00, size = 14, normalized size = 1.00

$$-\cot(x)\sqrt{a\sin^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^2],x]
```

```
[Out] -(Cot[x]*Sqrt[a*Sin[x]^2])
```

fricas [A] time = 0.44, size = 19, normalized size = 1.36

$$-\frac{\sqrt{-a\cos(x)^2 + a}\,\cos(x)}{\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(-a*cos(x)^2 + a)*cos(x)/sin(x)
```

giac [A] time = 0.13, size = 17, normalized size = 1.21

$$-\left(\cos(x)\mathrm{sgn}\left(\sin(x)\right)-\mathrm{sgn}\left(\sin(x)\right)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="giac")
[Out] -(cos(x)*sgn(sin(x)) - sgn(sin(x)))*sqrt(a)
```

maple [A] time = 0.61, size = 16, normalized size = 1.14

$$-\frac{a\cos(x)\sin(x)}{\sqrt{a\left(\sin^{2}(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(1/2),x)
[Out] -1/(a*sin(x)^2)^(1/2)*a*cos(x)*sin(x)
```

maxima [A] time = 0.71, size = 13, normalized size = 0.93

$$-\frac{\sqrt{a}}{\sqrt{\tan(x)^{2}+1}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] -sqrt(a)/sqrt(tan(x)^2 + 1)
```

mupad [B] time = 13.64, size = 40, normalized size = 2.86

$$-\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{2\sin(x)^{2}}\,\left(-\sin(x)^{2}+\frac{\sin(2\,x)\,1\mathrm{i}}{2}+1\right)}{\sin(x)^{2}\,2\mathrm{i}+\sin\left(2\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^2)^(1/2),x)
[Out] -(2^(1/2)*a^(1/2)*(2*sin(x)^2)^(1/2)*((sin(2*x)*1i)/2 - sin(x)^2 + 1))/(sin
(2*x) + sin(x)^2*2i)
```

sympy [A] time = 0.58, size = 20, normalized size = 1.43

$$-\frac{\sqrt{a}\,\sqrt{\sin^{2}\left(x\right)}\,\cos\left(x\right)}{\sin\left(x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**2)**(1/2),x)
[Out] -sqrt(a)*sqrt(sin(x)**2)*cos(x)/sin(x)
```

3.4 $\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx$

Optimal. Leaf size=17

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

```
[Out] -arctanh(cos(x))*sin(x)/(a*sin(x)^2)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 17, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3207, 3770}

$$-\frac{\sin(x)\tanh^{-1}(\cos(x))}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] -((ArcTanh[Cos[x]]*Sin[x])/Sqrt[a*Sin[x]^2])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx &= \frac{\sin(x)\int \csc(x)\,dx}{\sqrt{a\sin^2(x)}}\\
&= -\frac{\tanh^{-1}(\cos(x))\sin(x)}{\sqrt{a\sin^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 30, normalized size = 1.76

$$\frac{\sin(x)\left(\log\left(\sin\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^2],x]
```

```
[Out] ((-Log[Cos[x/2]] + Log[Sin[x/2]])*Sin[x])/Sqrt[a*Sin[x]^2]
```

fricas [B] time = 0.45, size = 70, normalized size = 4.12

$$\left[\frac{\sqrt{-a\cos(x)^2+a}\,\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)}{2\,a\sin(x)}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-a\cos(x)^2+a}\,\sqrt{-a}\,\cos(x)}{a\sin(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*sqrt(-a*cos(x)^2 + a)*log(-(cos(x) - 1)/(cos(x) + 1))/(a*sin(x)), sqrt
(-a)*arctan(sqrt(-a*cos(x)^2 + a)*sqrt(-a)*cos(x)/(a*sin(x)))/a]
```

giac [A] time = 0.16, size = 15, normalized size = 0.88

$$\frac{\log\left(\left|\tan\left(\frac{1}{2}\,x\right)\right|\right)}{\sqrt{a}\,\mathrm{sgn}\left(\sin(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] log(abs(tan(1/2*x)))/(sqrt(a)*sgn(sin(x)))
```

maple [B] time = 0.90, size = 49, normalized size = 2.88

$$-\frac{\sin(x)\sqrt{a\left(\cos^2(x)\right)}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)}{\sqrt{a}\,\cos(x)\sqrt{a\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(1/2),x)
```

```
[Out] -sin(x)*(a*cos(x)^2)^(1/2)/a^(1/2)*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)/sin(
x))/cos(x)/(a*sin(x)^2)^(1/2)
```

maxima [A] time = 0.63, size = 26, normalized size = 1.53

$$\frac{\sqrt{-a}\left(\arctan\left(\sin(x),\cos(x)+1\right)-\arctan\left(\sin(x),\cos(x)-1\right)\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(-a)*(arctan2(sin(x), cos(x) + 1) - arctan2(sin(x), cos(x) - 1))/a
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a\sin(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(1/2),x)
```

```
[Out] int(1/(a*sin(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**2), x)
```

3.5 $\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2a\sqrt{a\sin^2(x)}}$$

```
[Out] -1/2*cot(x)/a/(a*sin(x)^2)^(1/2)-1/2*arctanh(cos(x))*sin(x)/a/(a*sin(x)^2)^
(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$-\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\sin(x)\tanh^{-1}(\cos(x))}{2a\sqrt{a\sin^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -Cot[x]/(2*a*Sqrt[a*Sin[x]^2]) - (ArcTanh[Cos[x]]*Sin[x])/(2*a*Sqrt[a*Sin[x
]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx &= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}+\frac{\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{2a}\\
&= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}+\frac{\sin(x)\int \csc(x)\,dx}{2a\sqrt{a\sin^2(x)}}\\
&= -\frac{\cot(x)}{2a\sqrt{a\sin^2(x)}}-\frac{\tanh^{-1}(\cos(x))\sin(x)}{2a\sqrt{a\sin^2(x)}}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 55, normalized size = 1.31

$$-\frac{\sin^3(x)\left(\csc^2\left(\frac{x}{2}\right)-\sec^2\left(\frac{x}{2}\right)-4\log\left(\sin\left(\frac{x}{2}\right)\right)+4\log\left(\cos\left(\frac{x}{2}\right)\right)\right)}{8\left(a\sin^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -1/8*((Csc[x/2]^2 + 4*Log[Cos[x/2]] - 4*Log[Sin[x/2]] - Sec[x/2]^2)*Sin[x]^
3)/(a*Sin[x]^2)^(3/2)
```

fricas [A] time = 0.45, size = 58, normalized size = 1.38

$$\frac{\sqrt{-a\cos(x)^2+a}\left(\left(\cos(x)^2-1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)+2\cos(x)\right)}{4\left(a^2\cos(x)^2-a^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-a*cos(x)^2 + a)*((cos(x)^2 - 1)*log(-(cos(x) - 1)/(cos(x) + 1)) +
 2*cos(x))/((a^2*cos(x)^2 - a^2)*sin(x))
```

giac [A] time = 0.23, size = 61, normalized size = 1.45

$$\frac{\frac{\tan\left(\frac{1}{2}x\right)^2}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}+\frac{2\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}-\frac{2\tan\left(\frac{1}{2}x\right)^2+1}{\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2}}{8a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*(tan(1/2*x)^2/sgn(tan(1/2*x)) + 2*log(tan(1/2*x)^2)/sgn(tan(1/2*x)) - (
2*tan(1/2*x)^2 + 1)/(sgn(tan(1/2*x))*tan(1/2*x)^2))/a^(3/2)
```

maple [B] time = 1.56, size = 70, normalized size = 1.67

$$-\frac{\sqrt{a\left(\cos^2(x)\right)}\left(\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)\left(\sin^2(x)\right)a+\sqrt{a}\sqrt{a\left(\cos^2(x)\right)}\right)}{2a^{\frac{5}{2}}\sin(x)\cos(x)\sqrt{a\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(3/2),x)
```

```
[Out] -1/2/a^(5/2)/sin(x)*(a*cos(x)^2)^(1/2)*(ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2)+a)
/sin(x))*sin(x)^2*a+a^(1/2)*(a*cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

maxima [B] time = 0.69, size = 314, normalized size = 7.48

$$-\frac{\left(\left(2\left(2\cos\left(2x\right)-1\right)\cos\left(4x\right)-\cos\left(4x\right)^2-4\cos\left(2x\right)^2-\sin\left(4x\right)^2+4\sin\left(4x\right)\sin\left(2x\right)-4\sin\left(2x\right)^2+4\cos\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*((2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4*x)^2
 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x), cos
(x) + 1) - (2*(2*cos(2*x) - 1)*cos(4*x) - cos(4*x)^2 - 4*cos(2*x)^2 - sin(4
*x)^2 + 4*sin(4*x)*sin(2*x) - 4*sin(2*x)^2 + 4*cos(2*x) - 1)*arctan2(sin(x)
, cos(x) - 1) + 2*(sin(3*x) + sin(x))*cos(4*x) - 2*(cos(3*x) + cos(x))*sin(
4*x) - 2*(2*cos(2*x) - 1)*sin(3*x) + 4*cos(3*x)*sin(2*x) + 4*cos(x)*sin(2*x
) - 4*cos(2*x)*sin(x) + 2*sin(x))*sqrt(-a)/(a^2*cos(4*x)^2 + 4*a^2*cos(2*x)
^2 + a^2*sin(4*x)^2 - 4*a^2*sin(4*x)*sin(2*x) + 4*a^2*sin(2*x)^2 - 4*a^2*co
s(2*x) + a^2 - 2*(2*a^2*cos(2*x) - a^2)*cos(4*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\sin(x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(3/2),x)
```

```
[Out] int(1/(a*sin(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^2(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(-3/2), x)
```

3.6 $\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\sin(x)\tanh^{-1}(\cos(x))}{8a^2\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

```
[Out] -1/4*cot(x)/a/(a*sin(x)^2)^(3/2)-3/8*cot(x)/a^2/(a*sin(x)^2)^(1/2)-3/8*arct
anh(cos(x))*sin(x)/a^2/(a*sin(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3204, 3207, 3770}

$$-\frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}}-\frac{3\sin(x)\tanh^{-1}(\cos(x))}{8a^2\sqrt{a\sin^2(x)}}-\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -Cot[x]/(4*a*(a*Sin[x]^2)^(3/2)) - (3*Cot[x])/(8*a^2*Sqrt[a*Sin[x]^2]) - (3
*ArcTanh[Cos[x]]*Sin[x])/(8*a^2*Sqrt[a*Sin[x]^2])
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a\sin^2(x)\right)^{5/2}}\,dx = -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\sin^2(x)\right)^{3/2}}\,dx}{4a}$$

$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\sin^2(x)}}\,dx}{8a^2}$$

$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} + \frac{(3\sin(x))\int \csc(x)\,dx}{8a^2\sqrt{a\sin^2(x)}}$$

$$= -\frac{\cot(x)}{4a\left(a\sin^2(x)\right)^{3/2}} - \frac{3\cot(x)}{8a^2\sqrt{a\sin^2(x)}} - \frac{3\tanh^{-1}(\cos(x))\sin(x)}{8a^2\sqrt{a\sin^2(x)}}$$

Mathematica [A] time = 0.21, size = 77, normalized size = 1.26

$$-\frac{\csc(x)\sqrt{a\sin^2(x)}\left(\csc^4\left(\frac{x}{2}\right)+6\csc^2\left(\frac{x}{2}\right)-\sec^4\left(\frac{x}{2}\right)-6\sec^2\left(\frac{x}{2}\right)+24\left(\log\left(\cos\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)\right)\right)\right)}{64a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^2)^(-5/2),x]
```

```
[Out] -1/64*(Csc[x]*(6*Csc[x/2]^2 + Csc[x/2]^4 + 24*(Log[Cos[x/2]] - Log[Sin[x/2]
]) - 6*Sec[x/2]^2 - Sec[x/2]^4)*Sqrt[a*Sin[x]^2])/a^3
```

fricas [A] time = 0.45, size = 78, normalized size = 1.28

$$\frac{\sqrt{-a\cos(x)^2+a}\left(6\cos(x)^3+3\left(\cos(x)^4-2\cos(x)^2+1\right)\log\left(-\frac{\cos(x)-1}{\cos(x)+1}\right)-10\cos(x)\right)}{16\left(a^3\cos(x)^4-2a^3\cos(x)^2+a^3\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/16*sqrt(-a*cos(x)^2 + a)*(6*cos(x)^3 + 3*(cos(x)^4 - 2*cos(x)^2 + 1)*log(
-(cos(x) - 1)/(cos(x) + 1)) - 10*cos(x))/((a^3*cos(x)^4 - 2*a^3*cos(x)^2 +
a^3)*sin(x))
```

giac [A] time = 0.25, size = 80, normalized size = 1.31

$$\frac{\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^4+8\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^2+\frac{12\log\left(\tan\left(\frac{1}{2}x\right)^2\right)}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)}-\frac{18\tan\left(\frac{1}{2}x\right)^4+8\tan\left(\frac{1}{2}x\right)^2+1}{\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)\right)\tan\left(\frac{1}{2}x\right)^4}}{64\,a^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/64*(sgn(tan(1/2*x))*tan(1/2*x)^4 + 8*sgn(tan(1/2*x))*tan(1/2*x)^2 + 12*lo
g(tan(1/2*x)^2)/sgn(tan(1/2*x)) - (18*tan(1/2*x)^4 + 8*tan(1/2*x)^2 + 1)/(s
gn(tan(1/2*x))*tan(1/2*x)^4))/a^(5/2)
```

maple [A] time = 1.45, size = 89, normalized size = 1.46

$$-\frac{\sqrt{a\left(\cos^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}+2a}{\sin(x)}\right)a\left(\sin^4(x)\right)+3\sqrt{a\left(\cos^2(x)\right)}\,\left(\sin^2(x)\right)\sqrt{a}\,+2\sqrt{a}\,\sqrt{a\left(\cos^2(x)\right)}\right)}{8a^{\frac{7}{2}}\sin(x)^3\cos(x)\sqrt{a\left(\sin^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(5/2),x)
```

```
[Out] -1/8/a^(7/2)/sin(x)^3*(a*cos(x)^2)^(1/2)*(3*ln(2*(a^(1/2)*(a*cos(x)^2)^(1/2
)+a)/sin(x))*a*sin(x)^4+3*(a*cos(x)^2)^(1/2)*sin(x)^2*a^(1/2)+2*a^(1/2)*(a*
cos(x)^2)^(1/2))/cos(x)/(a*sin(x)^2)^(1/2)
```

maxima [B] time = 0.87, size = 931, normalized size = 15.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*(2*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2
 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x
) - 1)*cos(4*x) - 36*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x
) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*
x) - 16*sin(6*x)^2 - 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 +
 8*cos(2*x) - 1)*arctan2(sin(x), cos(x) + 1) - 3*(2*(4*cos(6*x) - 6*cos(4*x
) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)^2 + 8*(6*cos(4*x) - 4*cos(2*x) + 1)
*cos(6*x) - 16*cos(6*x)^2 + 12*(4*cos(2*x) - 1)*cos(4*x) - 36*cos(4*x)^2 -
16*cos(2*x)^2 + 4*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x
)^2 + 16*(3*sin(4*x) - 2*sin(2*x))*sin(6*x) - 16*sin(6*x)^2 - 36*sin(4*x)^2
 + 48*sin(4*x)*sin(2*x) - 16*sin(2*x)^2 + 8*cos(2*x) - 1)*arctan2(sin(x), c
os(x) - 1) + 2*(3*sin(7*x) - 11*sin(5*x) - 11*sin(3*x) + 3*sin(x))*cos(8*x)
 + 12*(2*sin(6*x) - 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 8*(11*sin(5*x) + 11
*sin(3*x) - 3*sin(x))*cos(6*x) + 44*(3*sin(4*x) - 2*sin(2*x))*cos(5*x) - 12
*(11*sin(3*x) - 3*sin(x))*cos(4*x) - 2*(3*cos(7*x) - 11*cos(5*x) - 11*cos(3
*x) + 3*cos(x))*sin(8*x) - 6*(4*cos(6*x) - 6*cos(4*x) + 4*cos(2*x) - 1)*sin
(7*x) - 8*(11*cos(5*x) + 11*cos(3*x) - 3*cos(x))*sin(6*x) - 22*(6*cos(4*x)
- 4*cos(2*x) + 1)*sin(5*x) + 12*(11*cos(3*x) - 3*cos(x))*sin(4*x) + 22*(4*c
os(2*x) - 1)*sin(3*x) - 88*cos(3*x)*sin(2*x) + 24*cos(x)*sin(2*x) - 24*cos(
2*x)*sin(x) + 6*sin(x))*sqrt(-a)/(a^3*cos(8*x)^2 + 16*a^3*cos(6*x)^2 + 36*a
^3*cos(4*x)^2 + 16*a^3*cos(2*x)^2 + a^3*sin(8*x)^2 + 16*a^3*sin(6*x)^2 + 36
*a^3*sin(4*x)^2 - 48*a^3*sin(4*x)*sin(2*x) + 16*a^3*sin(2*x)^2 - 8*a^3*cos(
2*x) + a^3 - 2*(4*a^3*cos(6*x) - 6*a^3*cos(4*x) + 4*a^3*cos(2*x) - a^3)*cos
(8*x) - 8*(6*a^3*cos(4*x) - 4*a^3*cos(2*x) + a^3)*cos(6*x) - 12*(4*a^3*cos(
2*x) - a^3)*cos(4*x) - 4*(2*a^3*sin(6*x) - 3*a^3*sin(4*x) + 2*a^3*sin(2*x))
*sin(8*x) - 16*(3*a^3*sin(4*x) - 2*a^3*sin(2*x))*sin(6*x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a\sin(x)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^2)^(5/2),x)
```

```
[Out] int(1/(a*sin(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^{2}(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**2)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**2)**(-5/2), x)
```

3.7 $\int \left(a\sin^3(x)\right)^{5/2}\,dx$

Optimal. Leaf size=123

$$-\frac{26}{165}a^2\sin^3(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{2}{15}a^2\sin^5(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{77}a^2\cot(x)\sqrt{}$$

```
[Out] -26/77*a^2*cot(x)*(a*sin(x)^3)^(1/2)-26/77*a^2*(sin(1/4*Pi+1/2*x)^2)^(1/2)/
sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/s
in(x)^(3/2)-78/385*a^2*cos(x)*sin(x)*(a*sin(x)^3)^(1/2)-26/165*a^2*cos(x)*s
in(x)^3*(a*sin(x)^3)^(1/2)-2/15*a^2*cos(x)*sin(x)^5*(a*sin(x)^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2641}

$$-\frac{2}{15}a^2\sin^5(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{165}a^2\sin^3(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{78}{385}a^2\sin(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{26}{77}a^2\cot(x)\sqrt{}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (-26*a^2*Cot[x]*Sqrt[a*Sin[x]^3])/77 - (26*a^2*EllipticF[Pi/4 - x/2, 2]*Sqr
t[a*Sin[x]^3])/(77*Sin[x]^(3/2)) - (78*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^3])/
385 - (26*a^2*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^3])/165 - (2*a^2*Cos[x]*Sin[x]^
5*Sqrt[a*Sin[x]^3])/15
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^3(x)\right)^{5/2} dx = \frac{\left(a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{15}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(13a^2\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{11}{2}}(x)\,dx}{15\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5(x)\sqrt{a\sin^3(x)} + \frac{\left(39a^2\sqrt{a\sin^3(x)}\right.}{55\sin^{\frac{3}{2}}}$$
$$= -\frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\sin^3(x)} - \frac{2}{15}a^2\cos(x)\sin^5($$
$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(x)} - \frac{26}{165}a^2\cos(x)\sin^3(x)\sqrt{a\,si}$$
$$= -\frac{26}{77}a^2\cot(x)\sqrt{a\sin^3(x)} - \frac{26a^2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{77\sin^{\frac{3}{2}}(x)} - \frac{78}{385}a^2\cos(x)\sin(x)\sqrt{a\sin^3(}$$

Mathematica [A] time = 0.17, size = 65, normalized size = 0.53

$$\frac{a\left(a\sin^3(x)\right)^{3/2}\left(\sqrt{\sin(x)}\left(-15465\cos(x)+3657\cos(3x)-749\cos(5x)+77\cos(7x)\right)-12480F\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right.}{36960\sin^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(5/2),x]
```

```
[Out] (a*(-12480*EllipticF[(Pi - 2*x)/4, 2] + (-15465*Cos[x] + 3657*Cos[3*x] - 74
9*Cos[5*x] + 77*Cos[7*x])*Sqrt[Sin[x]])*(a*Sin[x]^3)^(3/2))/(36960*Sin[x]^(
9/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(a^2\cos(x)^6 - 3\,a^2\cos(x)^4 + 3\,a^2\cos(x)^2 - a^2\right)\sqrt{-\left(a\cos(x)^2 - a\right)\sin(x)}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sqrt(-(a*c
os(x)^2 - a)*sin(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin(x)^3\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

maple [C] time = 0.43, size = 155, normalized size = 1.26

$$-\frac{\left(-154\left(\cos^{8}(x)\right)+195i\sqrt{2}\ \sin(x)\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\ \text{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\ \sqrt{\frac{i\cos(x)+\sin(x)}{\sin(x)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(5/2),x)
```

```
[Out] -1/1155*(-154*cos(x)^8+195*I*2^(1/2)*sin(x)*(-(I*cos(x)-sin(x)-I)/sin(x))^(
1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*(-I*(-1+cos(
x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)+154*cos(x)^7+644*cos(x
)^6-644*cos(x)^5-1060*cos(x)^4+1060*cos(x)^3+960*cos(x)^2-960*cos(x))*(a*(1
-cos(x)^2)*sin(x))^(5/2)/sin(x)^7/(-1+cos(x))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin(x)^{3}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\sin\left(x\right)}^3\right)}^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(5/2),x)
```

```
[Out] int((a*sin(x)^3)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(5/2),x)
```

```
[Out] Timed out
```

3.8 $\int \left(a\sin^3(x)\right)^{3/2}\,dx$

Optimal. Leaf size=73

$$-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{2}{9}a\sin^2(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}$$

```
[Out] -14/45*a*cos(x)*(a*sin(x)^3)^(1/2)-14/15*a*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(
1/4*Pi+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x
)^(3/2)-2/9*a*cos(x)*sin(x)^2*(a*sin(x)^3)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2639}

$$-\frac{2}{9}a\sin^2(x)\cos(x)\sqrt{a\sin^3(x)}-\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)}-\frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(3/2),x]
```

```
[Out] (-14*a*Cos[x]*Sqrt[a*Sin[x]^3])/45 - (14*a*EllipticE[Pi/4 - x/2, 2]*Sqrt[a*
Sin[x]^3])/(15*Sin[x]^(3/2)) - (2*a*Cos[x]*Sin[x]^2*Sqrt[a*Sin[x]^3])/9
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^3(x)\right)^{3/2}\,dx = \frac{\left(a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{9}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)} + \frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sin^{\frac{5}{2}}(x)\,dx}{9\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)} + \frac{\left(7a\sqrt{a\sin^3(x)}\right)\int \sqrt{\sin(x)}\,dx}{15\sin^{\frac{3}{2}}(x)}$$
$$= -\frac{14}{45}a\cos(x)\sqrt{a\sin^3(x)} - \frac{14aE\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{15\sin^{\frac{3}{2}}(x)} - \frac{2}{9}a\cos(x)\sin^2(x)\sqrt{a\sin^3(x)}$$

Mathematica [A] time = 0.10, size = 54, normalized size = 0.74

$$\frac{\left(a\sin^3(x)\right)^{3/2}\left(\sqrt{\sin(x)}\,(5\sin(4x)-38\sin(2x))-168E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{180\sin^{\frac{9}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(3/2),x]
[Out] ((a*Sin[x]^3)^(3/2)*(-168*EllipticE[(Pi - 2*x)/4, 2] + Sqrt[Sin[x]]*(-38*Si
n[2*x] + 5*Sin[4*x])))/(180*Sin[x]^(9/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(a\cos(x)^2-a\right)\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}\,\sin(x),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="fricas")
[Out] integral(-(a*cos(x)^2 - a)*sqrt(-(a*cos(x)^2 - a)*sin(x))*sin(x), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin(x)^3\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="giac")
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

maple [C] time = 0.51, size = 337, normalized size = 4.62

$$\left(21\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\ \mathrm{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)\sqrt{2}\,\cos(x)-42\sqrt{\frac{i\cos(x)+\sin(x)}{\sin(x)}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(3/2),x)
```

```
[Out] 1/45*(21*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(
1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(
1/2),1/2*2^(1/2))*2^(1/2)*cos(x)-42*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I
*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticE(((I
*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)+21*((I*cos(x)+s
in(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/
sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(
1/2)-42*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1
/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1
/2),1/2*2^(1/2))*2^(1/2)-10*cos(x)^5+34*cos(x)^3-66*cos(x)+42)*(a*sin(x)^3)
^(3/2)/sin(x)^5
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \sin(x)^3\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(a\,{\sin\left(x\right)}^3\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(3/2),x)
```

```
[Out] int((a*sin(x)^3)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(3/2),x)
```

```
[Out] Timed out
```

3.9 $\int \sqrt{a \sin^3(x)}\, dx$

Optimal. Leaf size=50

$$-\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}-\frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

```
[Out] -2/3*cot(x)*(a*sin(x)^3)^(1/2)-2/3*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1
/2*x)*EllipticF(cos(1/4*Pi+1/2*x),2^(1/2))*(a*sin(x)^3)^(1/2)/sin(x)^(3/2)
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 2641}

$$-\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}-\frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cot[x]*Sqrt[a*Sin[x]^3])/3 - (2*EllipticF[Pi/4 - x/2, 2]*Sqrt[a*Sin[x]^
3])/(3*Sin[x]^(3/2))
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a\sin^3(x)}\,dx &= \frac{\sqrt{a\sin^3(x)}\int \sin^{\frac{3}{2}}(x)\,dx}{\sin^{\frac{3}{2}}(x)}\\
&= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}+\frac{\sqrt{a\sin^3(x)}\int \frac{1}{\sqrt{\sin(x)}}\,dx}{3\sin^{\frac{3}{2}}(x)}\\
&= -\frac{2}{3}\cot(x)\sqrt{a\sin^3(x)}-\frac{2F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sqrt{a\sin^3(x)}}{3\sin^{\frac{3}{2}}(x)}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 41, normalized size = 0.82

$$-\frac{2\sqrt{a\sin^3(x)}\left(F\left(\frac{1}{4}(\pi-2x)\middle|2\right)+\sqrt{\sin(x)}\cos(x)\right)}{3\sin^{\frac{3}{2}}(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*(EllipticF[(Pi - 2*x)/4, 2] + Cos[x]*Sqrt[Sin[x]])*Sqrt[a*Sin[x]^3])/(3
*Sin[x]^(3/2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-(a*cos(x)^2 - a)*sin(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

maple [C] time = 0.48, size = 124, normalized size = 2.48

$$-\frac{\left(i\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\,\sin(x)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\,\text{EllipticF}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)+\left(\cos^2(x)\right)\sqrt{2}-\right.}{6\sin(x)\left(-1+\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(1/2),x)
```

```
[Out] -1/6*(I*(-I*(-1+cos(x))/sin(x))^(1/2)*sin(x)*((I*cos(x)+sin(x)-I)/sin(x))^(
1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin
(x))^(1/2),1/2*2^(1/2))+cos(x)^2*2^(1/2)-cos(x)*2^(1/2))*(a*(1-cos(x)^2)*si
n(x))^(1/2)/sin(x)/(-1+cos(x))*8^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\sin(x)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(a*sin(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\sqrt{a\,{\sin(x)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^3)^(1/2),x)
```

```
[Out] int((a*sin(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a \sin^3(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a*sin(x)**3), x)
```

3.10 $\int \frac{1}{\sqrt{a \sin^3(x)}} \, dx$

Optimal. Leaf size=48

$$\frac{2 \sin^{\frac{3}{2}}(x) E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{\sqrt{a \sin^3(x)}}-\frac{2 \sin(x) \cos(x)}{\sqrt{a \sin^3(x)}}$$

```
[Out] -2*cos(x)*sin(x)/(a*sin(x)^3)^(1/2)+2*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*P
i+1/2*x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/(a*sin(x)^3)^(1/
2)
```

Rubi [A] time = 0.02, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2636, 2639}

$$\frac{2 \sin^{\frac{3}{2}}(x) E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{\sqrt{a \sin^3(x)}}-\frac{2 \sin(x) \cos(x)}{\sqrt{a \sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^3],x]
```

```
[Out] (-2*Cos[x]*Sin[x])/Sqrt[a*Sin[x]^3] + (2*EllipticE[Pi/4 - x/2, 2]*Sin[x]^(3
/2))/Sqrt[a*Sin[x]^3]
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\sqrt{a\sin^3(x)}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{3}{2}}(x)}\,dx}{\sqrt{a\sin^3(x)}}$$
$$= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} - \frac{\sin^{\frac{3}{2}}(x)\int \sqrt{\sin(x)}\,dx}{\sqrt{a\sin^3(x)}}$$
$$= -\frac{2\cos(x)\sin(x)}{\sqrt{a\sin^3(x)}} + \frac{2E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{\sqrt{a\sin^3(x)}}$$

Mathematica [A] time = 0.02, size = 37, normalized size = 0.77

$$\frac{2\sin^{\frac{3}{2}}(x)E\left(\frac{1}{4}(\pi-2x)\middle|2\right)-\sin(2x)}{\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^3],x]
[Out] (2*EllipticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2) - Sin[2*x])/Sqrt[a*Sin[x]^3]
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}}{\left(a\cos(x)^2-a\right)\sin(x)}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="fricas")
[Out] integral(-sqrt(-(a*cos(x)^2 - a)*sin(x))/((a*cos(x)^2 - a)*sin(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin(x)^3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="giac")
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

maple [C] time = 0.39, size = 330, normalized size = 6.88

$$\left(2\sqrt{2}\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\cos(x)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\sqrt{-\frac{i\cos(x)-\sin(x)-i}{\sin(x)}}\,\mathrm{EllipticE}\left(\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}},\frac{\sqrt{2}}{2}\right)-\sqrt{2}\sqrt{-\frac{i(-1+\cos}{\sin(x}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(1/2),x)
[Out] (2*2^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*cos(x)*((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/
```

```
sin(x))^(1/2),1/2*2^(1/2))-2^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*cos(x)*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/sin(x))^(1/2)*Ellipti
cF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+2*2^(1/2)*(-I*(-1+cos(x)
)/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*cos(x)-sin(x)-I)/si
n(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-2^(1/
2)*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*(-(I*co
s(x)-sin(x)-I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1
/2*2^(1/2))-2)*sin(x)/(a*sin(x)^3)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sin(x)^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(a*sin(x)^3), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{a\,{\sin(x)}^3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(1/2),x)
[Out] int(1/(a*sin(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a \sin^{3}(x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(1/2),x)
[Out] Integral(1/sqrt(a*sin(x)**3), x)
```

3.11 $\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{10\sin^{\frac{3}{2}}(x)F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}$$

```
[Out] -10/21*cos(x)/a/(a*sin(x)^3)^(1/2)-2/7*cot(x)*csc(x)/a/(a*sin(x)^3)^(1/2)-1
0/21*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1/2*x)*EllipticF(cos(1/4*Pi+1/2
*x),2^(1/2))*sin(x)^(3/2)/a/(a*sin(x)^3)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2636, 2641}

$$-\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}}-\frac{10\sin^{\frac{3}{2}}(x)F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{21a\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-3/2),x]
```

```
[Out] (-10*Cos[x])/(21*a*Sqrt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x])/(7*a*Sqrt[a*Sin[x]
^3]) - (10*EllipticF[Pi/4 - x/2, 2]*Sin[x]^(3/2))/(21*a*Sqrt[a*Sin[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2641

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticF[(1*(c -
 Pi/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sin^3(x)\right)^{3/2}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{9}{2}}(x)}\,dx}{a\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} + \frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{5}{2}}(x)}\,dx}{7a\sqrt{a\sin^3(x)}}$$

$$= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} + \frac{\left(5\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sqrt{\sin(x)}}\,dx}{21a\sqrt{a\sin^3(x)}}$$

$$= -\frac{10\cos(x)}{21a\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc(x)}{7a\sqrt{a\sin^3(x)}} - \frac{10F\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)\sin^{\frac{3}{2}}(x)}{21a\sqrt{a\sin^3(x)}}$$

Mathematica [A] time = 0.07, size = 48, normalized size = 0.62

$$-\frac{2\sin^2(x)\left(3\cot(x)+5\sin(x)\cos(x)+5\sin^{\frac{5}{2}}(x)F\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{21\left(a\sin^3(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-3/2),x]
```

```
[Out] (-2*Sin[x]^2*(3*Cot[x] + 5*Cos[x]*Sin[x] + 5*EllipticF[(Pi - 2*x)/4, 2]*Sin
[x]^(5/2)))/(21*(a*Sin[x]^3)^(3/2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}}{a^2\cos(x)^6-3\,a^2\cos(x)^4+3\,a^2\cos(x)^2-a^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-(a*cos(x)^2 - a)*sin(x))/(a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3
*a^2*cos(x)^2 - a^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

maple [C] time = 0.46, size = 360, normalized size = 4.68

$$-\frac{(\cos(x)+1)^2(-1+\cos(x))^2\left(5i\sqrt{2}\,\sin(x)\left(\cos^3(x)\right)\sqrt{\frac{i\cos(x)+\sin(x)-i}{\sin(x)}}\,\sqrt{\frac{-i\cos(x)+\sin(x)+i}{\sin(x)}}\,\sqrt{-\frac{i(-1+\cos(x))}{\sin(x)}}\ \mathrm{Ellipt}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(3/2),x)
```

```
[Out] -1/21*(cos(x)+1)^2*(-1+cos(x))^2*(5*I*2^(1/2)*sin(x)*cos(x)^3*((I*cos(x)+si
n(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/s
in(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))+5*I*
2^(1/2)*sin(x)*cos(x)^2*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(
x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(
x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*2^(1/2)*sin(x)*cos(x)*((I*cos(x)+sin(x
)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(
x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))-5*I*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*2^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)
*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
,1/2*2^(1/2))*sin(x)-10*cos(x)^3+16*cos(x))/(a*sin(x)^3)^(3/2)/sin(x)^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin(x)^3\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sin(x)^3\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(3/2),x)
```

```
[Out] int(1/(a*sin(x)^3)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^{3}(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**3)**(-3/2), x)
```

3.12 $\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=123

$$-\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\sin^3(x)}}-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}+\frac{154\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}$$

```
[Out] -154/585*cot(x)/a^2/(a*sin(x)^3)^(1/2)-22/117*cot(x)*csc(x)^2/a^2/(a*sin(x)
^3)^(1/2)-2/13*cot(x)*csc(x)^4/a^2/(a*sin(x)^3)^(1/2)-154/195*cos(x)*sin(x)
/a^2/(a*sin(x)^3)^(1/2)+154/195*(sin(1/4*Pi+1/2*x)^2)^(1/2)/sin(1/4*Pi+1/2*
x)*EllipticE(cos(1/4*Pi+1/2*x),2^(1/2))*sin(x)^(3/2)/a^2/(a*sin(x)^3)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3207, 2636, 2639}

$$-\frac{154\sin(x)\cos(x)}{195a^2\sqrt{a\sin^3(x)}}-\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}}+\frac{154\sin^{\frac{3}{2}}(x)E\left(\frac{\pi}{4}-\frac{x}{2}\middle|2\right)}{195a^2\sqrt{a\sin^3(x)}}-\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}}-\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-154*Cot[x])/(585*a^2*Sqrt[a*Sin[x]^3]) - (22*Cot[x]*Csc[x]^2)/(117*a^2*Sq
rt[a*Sin[x]^3]) - (2*Cot[x]*Csc[x]^4)/(13*a^2*Sqrt[a*Sin[x]^3]) - (154*Cos[
x]*Sin[x])/(195*a^2*Sqrt[a*Sin[x]^3]) + (154*EllipticE[Pi/4 - x/2, 2]*Sin[x
]^(3/2))/(195*a^2*Sqrt[a*Sin[x]^3])
```

Rule 2636

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1))/(b*d*(n + 1)), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2639

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2*EllipticE[(1*(c - P
i/2 + d*x))/2, 2])/d, x] /; FreeQ[{c, d}, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{1}{\left(a\sin^3(x)\right)^{5/2}}\,dx = \frac{\sin^{\frac{3}{2}}(x)\int \frac{1}{\sin^{\frac{15}{2}}(x)}\,dx}{a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(11\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{11}{2}}(x)}\,dx}{13a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{7}{2}}(x)}\,dx}{117a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} + \frac{\left(77\sin^{\frac{3}{2}}(x)\right)\int \frac{1}{\sin^{\frac{3}{2}}(x)}\,dx}{195a^2\sqrt{a\sin^3(x)}}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} - \frac{\left(77\sin^{\frac{3}{2}}(\ldots\right.}{195a\ldots}$$

$$= -\frac{154\cot(x)}{585a^2\sqrt{a\sin^3(x)}} - \frac{22\cot(x)\csc^2(x)}{117a^2\sqrt{a\sin^3(x)}} - \frac{2\cot(x)\csc^4(x)}{13a^2\sqrt{a\sin^3(x)}} - \frac{154\cos(x)\sin(x)}{195a^2\sqrt{a\sin^3(x)}} + \frac{154E\left(\frac{\pi}{4}\ldots\right.}{195a^2\ldots}$$

Mathematica [A] time = 0.22, size = 60, normalized size = 0.49

$$-\frac{2\left(231\sin(x)\cos(x)+\cot(x)\left(45\csc^4(x)+55\csc^2(x)+77\right)-231\sin^{\frac{3}{2}}(x)E\left(\frac{1}{4}(\pi-2x)\middle|2\right)\right)}{585a^2\sqrt{a\sin^3(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^3)^(-5/2),x]
```

```
[Out] (-2*(Cot[x]*(77 + 55*Csc[x]^2 + 45*Csc[x]^4) + 231*Cos[x]*Sin[x] - 231*Elli
pticE[(Pi - 2*x)/4, 2]*Sin[x]^(3/2)))/(585*a^2*Sqrt[a*Sin[x]^3])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-\left(a\cos(x)^2-a\right)\sin(x)}}{\left(a^3\cos(x)^8-4\,a^3\cos(x)^6+6\,a^3\cos(x)^4-4\,a^3\cos(x)^2+a^3\right)\sin(x)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-(a*cos(x)^2 - a)*sin(x))/((a^3*cos(x)^8 - 4*a^3*cos(x)^6 + 6
*a^3*cos(x)^4 - 4*a^3*cos(x)^2 + a^3)*sin(x)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

maple [C] time = 0.40, size = 1301, normalized size = 10.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(5/2),x)
```

```
[Out] 1/585*(231*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)
*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))
^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^7-462*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*
cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*Elliptic
E(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^7+231*(-I*
(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+si
n(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1
/2))*2^(1/2)*cos(x)^6-462*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I
)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+si
n(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^6-693*(-I*(-1+cos(x))/sin
(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))
^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*co
s(x)^5+1386*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2
)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x)
)^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^5-693*(-I*(-1+cos(x))/sin(x))^(1/2)*((I
*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*Ellipti
cF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^4+1386*(-
I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+
sin(x)+I)/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^
(1/2))*2^(1/2)*cos(x)^4+693*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)
-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+
sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^3-1386*(-I*(-1+cos(x))/
sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(
x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)
*cos(x)^3+693*(-I*(-1+cos(x))/sin(x))^(1/2)*((I*cos(x)+sin(x)-I)/sin(x))^(1
/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(
x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^2-1386*(-I*(-1+cos(x))/sin(x))^(1/2)*
((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*Elli
pticE(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)^2-231*
((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*
(-1+cos(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*
2^(1/2))*2^(1/2)*cos(x)+462*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+
sin(x)+I)/sin(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticE(((I*cos(x)+
sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2))*2^(1/2)*cos(x)+462*cos(x)^6-231*((I*co
s(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin(x))^(1/2)*(-I*(-1+co
s(x))/sin(x))^(1/2)*EllipticF(((I*cos(x)+sin(x)-I)/sin(x))^(1/2),1/2*2^(1/2
))*2^(1/2)+462*((I*cos(x)+sin(x)-I)/sin(x))^(1/2)*((-I*cos(x)+sin(x)+I)/sin
(x))^(1/2)*(-I*(-1+cos(x))/sin(x))^(1/2)*EllipticE(((I*cos(x)+sin(x)-I)/sin
(x))^(1/2),1/2*2^(1/2))*2^(1/2)-154*cos(x)^5-1386*cos(x)^4+418*cos(x)^3+138
6*cos(x)^2-354*cos(x)-462)*sin(x)/(a*sin(x)^3)^(5/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin(x)^3\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^3)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((a*sin(x)^3)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(a\sin(x)^3\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^3)^(5/2),x)
[Out] int(1/(a*sin(x)^3)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^3(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**3)**(5/2),x)
[Out] Integral((a*sin(x)**3)**(-5/2), x)
```

3.13 $\int \left(a\sin^4(x)\right)^{5/2}\,dx$

Optimal. Leaf size=132

$$-\frac{21}{128}a^2\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{1}{10}a^2\sin^7(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\sin^5(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\sin^3$$

```
[Out] -63/256*a^2*cot(x)*(a*sin(x)^4)^(1/2)+63/256*a^2*x*csc(x)^2*(a*sin(x)^4)^(1
/2)-21/128*a^2*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-21/160*a^2*cos(x)*sin(x)^3*
(a*sin(x)^4)^(1/2)-9/80*a^2*cos(x)*sin(x)^5*(a*sin(x)^4)^(1/2)-1/10*a^2*cos
(x)*sin(x)^7*(a*sin(x)^4)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$-\frac{1}{10}a^2\sin^7(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{9}{80}a^2\sin^5(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{160}a^2\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{21}{128}a^2\sin$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(5/2),x]
```

```
[Out] (-63*a^2*Cot[x]*Sqrt[a*Sin[x]^4])/256 + (63*a^2*x*Csc[x]^2*Sqrt[a*Sin[x]^4]
)/256 - (21*a^2*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/128 - (21*a^2*Cos[x]*Sin[x]
^3*Sqrt[a*Sin[x]^4])/160 - (9*a^2*Cos[x]*Sin[x]^5*Sqrt[a*Sin[x]^4])/80 - (a
^2*Cos[x]*Sin[x]^7*Sqrt[a*Sin[x]^4])/10
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \left(a\sin^4(x)\right)^{5/2} dx = \left(a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^{10}(x)\,dx$$
$$= -\frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)} + \frac{1}{10}\left(9a^2\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^8(x)\,dx$$
$$= -\frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)} - \frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{a\sin^4(x)} + \frac{1}{80}\left(63a^2\csc^2(x)\sqrt{a\,\mathrm{s}}\right.$$
$$= -\frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} - \frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{a\sin^4(x)} - \frac{1}{10}a^2\cos(x)\sin^7(x)\sqrt{}$$
$$= -\frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4(x)} - \frac{9}{80}a^2\cos(x)\sin^5(x)\sqrt{}$$
$$= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)} - \frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} - \frac{21}{160}a^2\cos(x)\sin^3(x)\sqrt{a\sin^4}$$
$$= -\frac{63}{256}a^2\cot(x)\sqrt{a\sin^4(x)} + \frac{63}{256}a^2x\csc^2(x)\sqrt{a\sin^4(x)} - \frac{21}{128}a^2\cos(x)\sin(x)\sqrt{a\sin^4(x)} -$$

Mathematica [A] time = 0.18, size = 53, normalized size = 0.40

$$\frac{a(2520x - 2100\sin(2x) + 600\sin(4x) - 150\sin(6x) + 25\sin(8x) - 2\sin(10x))\csc^6(x)\left(a\sin^4(x)\right)^{3/2}}{10240}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(5/2),x]
```

```
[Out] (a*Csc[x]^6*(a*Sin[x]^4)^(3/2)*(2520*x - 2100*Sin[2*x] + 600*Sin[4*x] - 150
*Sin[6*x] + 25*Sin[8*x] - 2*Sin[10*x]))/10240
```

fricas [A] time = 0.45, size = 82, normalized size = 0.62

$$-\frac{\sqrt{a\cos(x)^4 - 2a\cos(x)^2 + a}\left(315a^2x - \left(128a^2\cos(x)^9 - 656a^2\cos(x)^7 + 1368a^2\cos(x)^5 - 1490a^2\cos(x)^3 + 9\right.\right.}{1280\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/1280*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(315*a^2*x - (128*a^2*cos(x)^9
- 656*a^2*cos(x)^7 + 1368*a^2*cos(x)^5 - 1490*a^2*cos(x)^3 + 965*a^2*cos(x)
)*sin(x))/(cos(x)^2 - 1)
```

giac [A] time = 0.15, size = 57, normalized size = 0.43

$$\frac{1}{10240}\left(2520a^2x - 2a^2\sin(10x) + 25a^2\sin(8x) - 150a^2\sin(6x) + 600a^2\sin(4x) - 2100a^2\sin(2x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] 1/10240*(2520*a^2*x - 2*a^2*sin(10*x) + 25*a^2*sin(8*x) - 150*a^2*sin(6*x)
+ 600*a^2*sin(4*x) - 2100*a^2*sin(2*x))*sqrt(a)
```

maple [A] time = 0.47, size = 63, normalized size = 0.48

$$-\frac{\left(a\left(1-\left(\cos^2(x)\right)\right)^2\right)^{\frac{5}{2}}\left(128\left(\cos^9(x)\right)\sin(x) - 656\left(\cos^7(x)\right)\sin(x) + 1368\left(\cos^5(x)\right)\sin(x) - 1490\left(\cos^3(x)\right)\sin(\right.}{1280\sin(x)^{10}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(5/2),x)
```

```
[Out] -1/1280*(a*(1-cos(x)^2)^2)^(5/2)*(128*cos(x)^9*sin(x)-656*cos(x)^7*sin(x)+1
368*cos(x)^5*sin(x)-1490*cos(x)^3*sin(x)+965*sin(x)*cos(x)-315*x)/sin(x)^10
```

maxima [A] time = 0.54, size = 85, normalized size = 0.64

$$\frac{63}{256}\,a^{\frac{5}{2}}x - \frac{965\,a^{\frac{5}{2}}\tan(x)^9 + 2370\,a^{\frac{5}{2}}\tan(x)^7 + 2688\,a^{\frac{5}{2}}\tan(x)^5 + 1470\,a^{\frac{5}{2}}\tan(x)^3 + 315\,a^{\frac{5}{2}}\tan(x)}{1280\left(\tan(x)^{10} + 5\,\tan(x)^8 + 10\,\tan(x)^6 + 10\,\tan(x)^4 + 5\,\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] 63/256*a^(5/2)*x - 1/1280*(965*a^(5/2)*tan(x)^9 + 2370*a^(5/2)*tan(x)^7 + 2
688*a^(5/2)*tan(x)^5 + 1470*a^(5/2)*tan(x)^3 + 315*a^(5/2)*tan(x))/(tan(x)^
10 + 5*tan(x)^8 + 10*tan(x)^6 + 10*tan(x)^4 + 5*tan(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\sin(x)^4\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(5/2),x)
```

```
[Out] int((a*sin(x)^4)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin^4(x)\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(5/2), x)
```

3.14 $\int \left(a\sin^4(x)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}$$

```
[Out] -5/16*a*cot(x)*(a*sin(x)^4)^(1/2)+5/16*a*x*csc(x)^2*(a*sin(x)^4)^(1/2)-5/24
*a*cos(x)*sin(x)*(a*sin(x)^4)^(1/2)-1/6*a*cos(x)*sin(x)^3*(a*sin(x)^4)^(1/2
)
```

Rubi [A] time = 0.03, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$-\frac{1}{6}a\sin^3(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\sin(x)\cos(x)\sqrt{a\sin^4(x)}-\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] (-5*a*Cot[x]*Sqrt[a*Sin[x]^4])/16 + (5*a*x*Csc[x]^2*Sqrt[a*Sin[x]^4])/16 -
(5*a*Cos[x]*Sin[x]*Sqrt[a*Sin[x]^4])/24 - (a*Cos[x]*Sin[x]^3*Sqrt[a*Sin[x]^
4])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \left(a\sin^4(x)\right)^{3/2}\,dx &= \left(a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^6(x)\,dx\\
&= -\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}+\frac{1}{6}\left(5a\csc^2(x)\sqrt{a\sin^4(x)}\right)\int \sin^4(x)\,dx\\
&= -\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}+\frac{1}{8}\left(5a\csc^2(x)\sqrt{a\sin^4(x)}\right)\\
&= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\cos(x)\sin^3(x)\sqrt{a\sin^4(x)}+\frac{1}{1}\\
&= -\frac{5}{16}a\cot(x)\sqrt{a\sin^4(x)}+\frac{5}{16}ax\csc^2(x)\sqrt{a\sin^4(x)}-\frac{5}{24}a\cos(x)\sin(x)\sqrt{a\sin^4(x)}-\frac{1}{6}a\,\mathrm{co}
\end{aligned}$$

Mathematica [A] time = 0.10, size = 38, normalized size = 0.49

$$-\frac{1}{192}(-60x+45\sin(2x)-9\sin(4x)+\sin(6x))\csc^6(x)\left(a\sin^4(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(3/2),x]
```

```
[Out] -1/192*(Csc[x]^6*(a*Sin[x]^4)^(3/2)*(-60*x + 45*Sin[2*x] - 9*Sin[4*x] + Sin
[6*x]))
```

fricas [A] time = 0.44, size = 56, normalized size = 0.72

$$-\frac{\sqrt{a\cos(x)^4-2\,a\cos(x)^2+a}\left(15\,ax-\left(8\,a\cos(x)^5-26\,a\cos(x)^3+33\,a\cos(x)\right)\sin(x)\right)}{48\left(\cos(x)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/48*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(15*a*x - (8*a*cos(x)^5 - 26*a*co
s(x)^3 + 33*a*cos(x))*sin(x))/(cos(x)^2 - 1)
```

giac [A] time = 0.13, size = 27, normalized size = 0.35

$$\frac{1}{192}\,a^{\frac{3}{2}}(60\,x-\sin(6\,x)+9\,\sin(4\,x)-45\,\sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] 1/192*a^(3/2)*(60*x - sin(6*x) + 9*sin(4*x) - 45*sin(2*x))
```

maple [A] time = 0.28, size = 47, normalized size = 0.60

$$-\frac{\left(a\left(1-\left(\cos^2(x)\right)\right)^2\right)^{\frac{3}{2}}\left(8\left(\cos^5(x)\right)\sin(x)-26\left(\cos^3(x)\right)\sin(x)+33\sin(x)\cos(x)-15x\right)}{48\sin(x)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(3/2),x)
```

```
[Out] -1/48*(a*(1-cos(x)^2)^2)^(3/2)*(8*cos(x)^5*sin(x)-26*cos(x)^3*sin(x)+33*sin
(x)*cos(x)-15*x)/sin(x)^6
```

maxima [A] time = 0.61, size = 55, normalized size = 0.71

$$\frac{5}{16}\,a^{\frac{3}{2}}x-\frac{33\,a^{\frac{3}{2}}\tan(x)^5+40\,a^{\frac{3}{2}}\tan(x)^3+15\,a^{\frac{3}{2}}\tan(x)}{48\left(\tan(x)^6+3\,\tan(x)^4+3\,\tan(x)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] 5/16*a^(3/2)*x - 1/48*(33*a^(3/2)*tan(x)^5 + 40*a^(3/2)*tan(x)^3 + 15*a^(3/
2)*tan(x))/(tan(x)^6 + 3*tan(x)^4 + 3*tan(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\left(a\sin(x)^4\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(3/2),x)
[Out] int((a*sin(x)^4)^(3/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\sin^4(x)\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(3/2),x)
[Out] Integral((a*sin(x)**4)**(3/2), x)
```

3.15 $\int \sqrt{a \sin^4(x)}\, dx$

Optimal. Leaf size=36

$$\frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} - \frac{1}{2} \cot(x) \sqrt{a \sin^4(x)}$$

[Out] -1/2*cot(x)*(a*sin(x)^4)^(1/2)+1/2*x*csc(x)^2*(a*sin(x)^4)^(1/2)

Rubi [A] time = 0.01, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 2635, 8}

$$\frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} - \frac{1}{2} \cot(x) \sqrt{a \sin^4(x)}$$

Antiderivative was successfully verified.

[In] Int[Sqrt[a*Sin[x]^4],x]

[Out] -(Cot[x]*Sqrt[a*Sin[x]^4])/2 + (x*Csc[x]^2*Sqrt[a*Sin[x]^4])/2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \sqrt{a \sin^4(x)}\, dx &= \left(\csc^2(x) \sqrt{a \sin^4(x)}\right) \int \sin^2(x)\, dx \\ &= -\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} \left(\csc^2(x) \sqrt{a \sin^4(x)}\right) \int 1\, dx \\ &= -\frac{1}{2} \cot(x) \sqrt{a \sin^4(x)} + \frac{1}{2} x \csc^2(x) \sqrt{a \sin^4(x)} \end{aligned}$$

Mathematica [A] time = 0.02, size = 25, normalized size = 0.69

$$\frac{1}{2} \csc(x) \sqrt{a \sin^4(x)} (x \csc(x) - \cos(x))$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a*Sin[x]^4],x]
[Out] (Csc[x]*(-Cos[x] + x*Csc[x])*Sqrt[a*Sin[x]^4])/2
```

fricas [A] time = 0.44, size = 36, normalized size = 1.00

$$\frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\,(\cos(x)\sin(x) - x)}{2\left(\cos(x)^2 - 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(cos(x)*sin(x) - x)/(cos(x)^2 - 1)
```

giac [A] time = 0.14, size = 15, normalized size = 0.42

$$\frac{1}{4}\,\sqrt{a}\,(2\,x - \sin(2\,x))$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="giac")
[Out] 1/4*sqrt(a)*(2*x - sin(2*x))
```

maple [A] time = 0.28, size = 33, normalized size = 0.92

$$-\frac{\sqrt{a\left(1-\left(\cos^2(x)\right)\right)^2}\;(\sin(x)\cos(x) - x)\,\sqrt{16}}{8\sin(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(1/2),x)
[Out] -1/8*(a*(1-cos(x)^2)^2)^(1/2)*(sin(x)*cos(x)-x)/sin(x)^2*16^(1/2)
```

maxima [A] time = 0.48, size = 22, normalized size = 0.61

$$\frac{1}{2}\,\sqrt{a}\,x - \frac{\sqrt{a}\,\tan(x)}{2\left(\tan(x)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)^4)^(1/2),x, algorithm="maxima")
[Out] 1/2*sqrt(a)*x - 1/2*sqrt(a)*tan(x)/(tan(x)^2 + 1)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sqrt{a\sin(x)^4}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*sin(x)^4)^(1/2),x)
[Out] int((a*sin(x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\sin^4(x)}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*sin(x)**4)**(1/2),x)
[Out] Integral(sqrt(a*sin(x)**4), x)
```

3.16 $\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$

Optimal. Leaf size=16

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

```
[Out] -cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3207, 3767, 8}

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a*Sin[x]^4],x]
```

```
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx &= \frac{\sin^2(x)\int \csc^2(x)\,dx}{\sqrt{a\sin^4(x)}}\\ &= -\frac{\sin^2(x)\,\mathrm{Subst}\left(\int 1\,dx, x, \cot(x)\right)}{\sqrt{a\sin^4(x)}}\\ &= -\frac{\cos(x)\sin(x)}{\sqrt{a\sin^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 16, normalized size = 1.00

$$-\frac{\sin(x)\cos(x)}{\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a*Sin[x]^4],x]
[Out] -((Cos[x]*Sin[x])/Sqrt[a*Sin[x]^4])
```

fricas [B] time = 0.42, size = 36, normalized size = 2.25

$$\frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\,\cos(x)}{\left(a\cos(x)^2 - a\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*cos(x)/((a*cos(x)^2 - a)*sin(x))
```

giac [A] time = 0.13, size = 9, normalized size = 0.56

$$-\frac{1}{\sqrt{a}\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="giac")
[Out] -1/(sqrt(a)*tan(x))
```

maple [A] time = 0.26, size = 15, normalized size = 0.94

$$-\frac{\cos(x)\sin(x)}{\sqrt{a\left(\sin^4(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(1/2),x)
[Out] -cos(x)*sin(x)/(a*sin(x)^4)^(1/2)
```

maxima [A] time = 0.52, size = 9, normalized size = 0.56

$$-\frac{1}{\sqrt{a}\,\tan(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(1/2),x, algorithm="maxima")
[Out] -1/(sqrt(a)*tan(x))
```

mupad [B] time = 13.71, size = 7, normalized size = 0.44

$$-\frac{\cot(x)}{\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(1/2),x)
[Out] -cot(x)/a^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a\sin^4(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a*sin(x)**4), x)
```

3.17 $\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$-\frac{\sin(x)\cos(x)}{a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}$$

```
[Out] -2/3*cos(x)^2*cot(x)/a/(a*sin(x)^4)^(1/2)-1/5*cos(x)^2*cot(x)^3/a/(a*sin(x)
^4)^(1/2)-cos(x)*sin(x)/a/(a*sin(x)^4)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$-\frac{\sin(x)\cos(x)}{a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-3/2),x]
```

```
[Out] (-2*Cos[x]^2*Cot[x])/(3*a*Sqrt[a*Sin[x]^4]) - (Cos[x]^2*Cot[x]^3)/(5*a*Sqrt
[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a*Sqrt[a*Sin[x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\sin^4(x)\right)^{3/2}}\,dx &= \frac{\sin^2(x)\int \csc^6(x)\,dx}{a\sqrt{a\sin^4(x)}}\\ &= -\frac{\sin^2(x)\,\text{Subst}\left(\int \left(1+2x^2+x^4\right)\,dx,x,\cot(x)\right)}{a\sqrt{a\sin^4(x)}}\\ &= -\frac{2\cos^2(x)\cot(x)}{3a\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^3(x)}{5a\sqrt{a\sin^4(x)}}-\frac{\cos(x)\sin(x)}{a\sqrt{a\sin^4(x)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 34, normalized size = 0.50

$$-\frac{\sin^5(x)\cos(x)\left(3\csc^4(x)+4\csc^2(x)+8\right)}{15\left(a\sin^4(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-3/2),x]
```

```
[Out] -1/15*(Cos[x]*(8 + 4*Csc[x]^2 + 3*Csc[x]^4)*Sin[x]^5)/(a*Sin[x]^4)^(3/2)
```

fricas [A] time = 0.42, size = 74, normalized size = 1.09

$$\frac{\sqrt{a\cos(x)^4 - 2\,a\cos(x)^2 + a}\left(8\,\cos(x)^5 - 20\,\cos(x)^3 + 15\,\cos(x)\right)}{15\left(a^2\cos(x)^6 - 3\,a^2\cos(x)^4 + 3\,a^2\cos(x)^2 - a^2\right)\sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/15*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)*(8*cos(x)^5 - 20*cos(x)^3 + 15*cos
(x))/((a^2*cos(x)^6 - 3*a^2*cos(x)^4 + 3*a^2*cos(x)^2 - a^2)*sin(x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.23, size = 29, normalized size = 0.43

$$-\frac{\left(8\left(\cos^4(x)\right) - 20\left(\cos^2(x)\right) + 15\right)\sin(x)\cos(x)}{15\left(a\left(\sin^4(x)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(3/2),x)
```

```
[Out] -1/15*(8*cos(x)^4-20*cos(x)^2+15)*sin(x)*cos(x)/(a*sin(x)^4)^(3/2)
```

maxima [A] time = 0.72, size = 23, normalized size = 0.34

$$-\frac{15\,\tan(x)^4 + 10\,\tan(x)^2 + 3}{15\,a^{\frac{3}{2}}\,\tan(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*tan(x)^4 + 10*tan(x)^2 + 3)/(a^(3/2)*tan(x)^5)
```

mupad [B] time = 14.28, size = 44, normalized size = 0.65

$$\frac{\frac{8\mathrm{i}}{15\,a^{3/2}} - \frac{4\left(2\sin(2\,x)^3 - 9\,\sin(2\,x) + 3\,\sin(4\,x) + 2\mathrm{i}\right)}{15\,a^{3/2}}}{\left(\cos\left(2\,x\right) - 1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(3/2),x)
```

```
[Out] (8i/(15*a^(3/2)) - (4*(3*sin(4*x) - 9*sin(2*x) + 2*sin(2*x)^3 + 2i))/(15*a^
(3/2)))/(cos(2*x) - 1)^3
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^4(x)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(3/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(-3/2), x)
```

3.18 $\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sin(x)\cos(x)}{a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}$$

```
[Out] -4/3*cos(x)^2*cot(x)/a^2/(a*sin(x)^4)^(1/2)-6/5*cos(x)^2*cot(x)^3/a^2/(a*si
n(x)^4)^(1/2)-4/7*cos(x)^2*cot(x)^5/a^2/(a*sin(x)^4)^(1/2)-1/9*cos(x)^2*cot
(x)^7/a^2/(a*sin(x)^4)^(1/2)-cos(x)*sin(x)/a^2/(a*sin(x)^4)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3207, 3767}

$$-\frac{\sin(x)\cos(x)}{a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*Sin[x]^4)^(-5/2),x]
```

```
[Out] (-4*Cos[x]^2*Cot[x])/(3*a^2*Sqrt[a*Sin[x]^4]) - (6*Cos[x]^2*Cot[x]^3)/(5*a^
2*Sqrt[a*Sin[x]^4]) - (4*Cos[x]^2*Cot[x]^5)/(7*a^2*Sqrt[a*Sin[x]^4]) - (Cos
[x]^2*Cot[x]^7)/(9*a^2*Sqrt[a*Sin[x]^4]) - (Cos[x]*Sin[x])/(a^2*Sqrt[a*Sin[
x]^4])
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a\sin^4(x)\right)^{5/2}}\,dx &= \frac{\sin^2(x)\int \csc^{10}(x)\,dx}{a^2\sqrt{a\sin^4(x)}}\\ &= -\frac{\sin^2(x)\,\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,\cot(x)\right)}{a^2\sqrt{a\sin^4(x)}}\\ &= -\frac{4\cos^2(x)\cot(x)}{3a^2\sqrt{a\sin^4(x)}}-\frac{6\cos^2(x)\cot^3(x)}{5a^2\sqrt{a\sin^4(x)}}-\frac{4\cos^2(x)\cot^5(x)}{7a^2\sqrt{a\sin^4(x)}}-\frac{\cos^2(x)\cot^7(x)}{9a^2\sqrt{a\sin^4(x)}}-\frac{\cos(x)\,\mathrm{si}}{a^2\sqrt{a\,\mathrm{sin}}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 47, normalized size = 0.40

$$-\frac{\sin(x)\cos(x)\left(35\csc^8(x)+40\csc^6(x)+48\csc^4(x)+64\csc^2(x)+128\right)}{315a^2\sqrt{a\sin^4(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*Sin[x]^4)^(-5/2),x]
```

```
[Out] -1/315*(Cos[x]*(128 + 64*Csc[x]^2 + 48*Csc[x]^4 + 40*Csc[x]^6 + 35*Csc[x]^8
)*Sin[x])/(a^2*Sqrt[a*Sin[x]^4])
```

fricas [A] time = 0.43, size = 104, normalized size = 0.88

$$\frac{\left(128 \cos(x)^{9} - 576 \cos(x)^{7} + 1008 \cos(x)^{5} - 840 \cos(x)^{3} + 315 \cos(x)\right)\sqrt{a \cos(x)^{4} - 2\, a \cos(x)^{2} + a}}{315 \left(a^{3} \cos(x)^{10} - 5\, a^{3} \cos(x)^{8} + 10\, a^{3} \cos(x)^{6} - 10\, a^{3} \cos(x)^{4} + 5\, a^{3} \cos(x)^{2} - a^{3}\right) \sin(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^9 - 576*cos(x)^7 + 1008*cos(x)^5 - 840*cos(x)^3 + 315*cos
(x))*sqrt(a*cos(x)^4 - 2*a*cos(x)^2 + a)/((a^3*cos(x)^10 - 5*a^3*cos(x)^8 +
 10*a^3*cos(x)^6 - 10*a^3*cos(x)^4 + 5*a^3*cos(x)^2 - a^3)*sin(x))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:sym2poly/r2sym(const gen & e,const index_m &
 i,const vecteur & l) Error: Bad Argument Value
```

maple [A] time = 0.30, size = 41, normalized size = 0.35

$$-\frac{\left(128 \left(\cos^{8}(x)\right) - 576 \left(\cos^{6}(x)\right) + 1008 \left(\cos^{4}(x)\right) - 840 \left(\cos^{2}(x)\right) + 315\right) \sin(x) \cos(x)}{315 \left(a \left(\sin^{4}(x)\right)\right)^{\frac{5}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(5/2),x)
```

```
[Out] -1/315*(128*cos(x)^8-576*cos(x)^6+1008*cos(x)^4-840*cos(x)^2+315)*sin(x)*co
s(x)/(a*sin(x)^4)^(5/2)
```

maxima [A] time = 0.59, size = 35, normalized size = 0.30

$$-\frac{315 \tan(x)^{8} + 420 \tan(x)^{6} + 378 \tan(x)^{4} + 180 \tan(x)^{2} + 35}{315\, a^{\frac{5}{2}} \tan(x)^{9}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)^4)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/315*(315*tan(x)^8 + 420*tan(x)^6 + 378*tan(x)^4 + 180*tan(x)^2 + 35)/(a^
(5/2)*tan(x)^9)
```

mupad [B] time = 16.56, size = 117, normalized size = 0.99

$$\frac{256 \left(\mathrm{e}^{x\,46\mathrm{i}}\,1\mathrm{i} - \mathrm{e}^{x\,48\mathrm{i}}\,9\mathrm{i} + \mathrm{e}^{x\,50\mathrm{i}}\,36\mathrm{i} - \mathrm{e}^{x\,52\mathrm{i}}\,84\mathrm{i} + \mathrm{e}^{x\,54\mathrm{i}}\,126\mathrm{i}\right)}{315\, a^{5/2} \left(\mathrm{e}^{x\,46\mathrm{i}} - 9\,\mathrm{e}^{x\,48\mathrm{i}} + 36\,\mathrm{e}^{x\,50\mathrm{i}} - 84\,\mathrm{e}^{x\,52\mathrm{i}} + 126\,\mathrm{e}^{x\,54\mathrm{i}} - 126\,\mathrm{e}^{x\,56\mathrm{i}} + 84\,\mathrm{e}^{x\,58\mathrm{i}} - 36\,\mathrm{e}^{x\,60\mathrm{i}} + 9\,\mathrm{e}^{x\,62\mathrm{i}} - \mathrm{e}^{x\,64\mathrm{i}}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a*sin(x)^4)^(5/2),x)
```

```
[Out] (256*(exp(x*46i)*1i - exp(x*48i)*9i + exp(x*50i)*36i - exp(x*52i)*84i + exp
(x*54i)*126i))/(315*a^(5/2)*(exp(x*46i) - 9*exp(x*48i) + 36*exp(x*50i) - 84
*exp(x*52i) + 126*exp(x*54i) - 126*exp(x*56i) + 84*exp(x*58i) - 36*exp(x*60
i) + 9*exp(x*62i) - exp(x*64i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a\sin^4(x)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a*sin(x)**4)**(5/2),x)
```

```
[Out] Integral((a*sin(x)**4)**(-5/2), x)
```

3.19 $\int (c\sin^m(a+bx))^{5/2}\,dx$

Optimal. Leaf size=89

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*c^2*cos(b*x+a)*hypergeom([1/2, 1/2+5/4*m],[3/2+5/4*m],sin(b*x+a)^2)*sin(b
*x+a)^(1+2*m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+5*m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3208, 2643}

$$\frac{2c^2\cos(a+bx)\sin^{2m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\;{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*c^2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + 2*m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 5*m)*Sqrt[
Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (c\sin^m(a+bx))^{5/2}\,dx &= \left(c^2\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{5m}{2}}(a+bx)\,dx\\ &= \frac{2c^2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+5m);\frac{1}{4}(6+5m);\sin^2(a+bx)\right)\sin^{1+2m}(a+bx)\sqrt{c\sin^m(a}}{b(2+5m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.17, size = 74, normalized size = 0.83

$$\frac{2\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\,(c\sin^m(a+bx))^{5/2}\;{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5m+2);\frac{1}{4}(5m+6);\sin^2(a+bx)\right)}{b(5m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(5/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 5*m)/4, (6 + 5*m)/4, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^m)^(5/2)*Tan[a + b*x])/(b*(2 + 5*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin \left(bx + a\right)^{m}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

maple [F] time = 6.45, size = 0, normalized size = 0.00

$$\int \left(c \left(\sin^{m} \left(bx + a\right)\right)\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(5/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin \left(bx + a\right)^{m}\right)^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(5/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(5/2),x)
```

```
[Out] Timed out
```

3.20 $\int (c\sin^m(a+bx))^{3/2}\,dx$

Optimal. Leaf size=83

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*c*cos(b*x+a)*hypergeom([1/2, 1/2+3/4*m],[3/2+3/4*m],sin(b*x+a)^2)*sin(b*x
+a)^(1+m)*(c*sin(b*x+a)^m)^(1/2)/b/(2+3*m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3208, 2643}

$$\frac{2c\cos(a+bx)\sin^{m+1}(a+bx)\sqrt{c\sin^m(a+bx)}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(3/2),x]
```

```
[Out] (2*c*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4, Sin[a
+ b*x]^2]*Sin[a + b*x]^(1 + m)*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + 3*m)*Sqrt[Co
s[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (c\sin^m(a+bx))^{3/2}\,dx &= \left(c\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{3m}{2}}(a+bx)\,dx\\ &= \frac{2c\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2+3m);\frac{3(2+m)}{4};\sin^2(a+bx)\right)\sin^{1+m}(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+3m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 72, normalized size = 0.87

$$\frac{2\sqrt{\cos^2(a+bx)}\,\tan(a+bx)\,(c\sin^m(a+bx))^{3/2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3m+2);\frac{3(m+2)}{4};\sin^2(a+bx)\right)}{b(3m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(3/2),x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + 3*m)/4, (3*(2 + m))/4,
Sin[a + b*x]^2]*(c*Sin[a + b*x]^m)^(3/2)*Tan[a + b*x])/(b*(2 + 3*m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin \left(bx + a\right)^{m}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \left(c \left(\sin^{m} \left(bx + a\right)\right)\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(3/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin \left(bx + a\right)^{m}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(3/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c \sin^{m}{\left(a + bx \right)}\right)^{\frac{3}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(3/2), x)
```

3.21 $\int \sqrt{c \sin^m(a+bx)}\,dx$

Optimal. Leaf size=74

$$\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\sin^m(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2+1/4*m],[3/2+1/4*m],sin(b*x+a)^2)*sin(b*x+a
)*(c*sin(b*x+a)^m)^(1/2)/b/(2+m)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2\sin(a+bx)\cos(a+bx)\sqrt{c\sin^m(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a + b*x]^2
]*Sin[a + b*x]*Sqrt[c*Sin[a + b*x]^m])/(b*(2 + m)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{c\sin^m(a+bx)}\,dx &= \left(\sin^{-\frac{m}{2}}(a+bx)\sqrt{c\sin^m(a+bx)}\right)\int \sin^{\frac{m}{2}}(a+bx)\,dx\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{4};\frac{6+m}{4};\sin^2(a+bx)\right)\sin(a+bx)\sqrt{c\sin^m(a+bx)}}{b(2+m)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 68, normalized size = 0.92

$$\frac{2\sqrt{\cos^2(a+bx)}\tan(a+bx)\sqrt{c\sin^m(a+bx)}\ {}_2F_1\left(\frac{1}{2},\frac{m+2}{4};\frac{m+6}{4};\sin^2(a+bx)\right)}{b(m+2)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 + m)/4, (6 + m)/4, Sin[a
+ b*x]^2]*Sqrt[c*Sin[a + b*x]^m]*Tan[a + b*x])/(b*(2 + m))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin(bx+a)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

maple [F] time = 0.66, size = 0, normalized size = 0.00

$$\int \sqrt{c\left(\sin^{m}(bx+a)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^m)^(1/2),x)
```

```
[Out] int((c*sin(b*x+a)^m)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin(bx+a)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(c*sin(b*x + a)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{c \sin(a+b\,x)^m}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^m)^(1/2),x)
```

```
[Out] int((c*sin(a + b*x)^m)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{c \sin^{m}(a+bx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(sqrt(c*sin(a + b*x)**m), x)
```

3.22 $\int \frac{1}{\sqrt{c \sin^m(a+bx)}} \, dx$

Optimal. Leaf size=80

$$\frac{2\sin(a+bx)\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-1/4*m],[3/2-1/4*m],sin(b*x+a)^2)*sin(b*x+a
)/b/(2-m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3208, 2643}

$$\frac{2\sin(a+bx)\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a + b*x]^2
]*Sin[a + b*x])/(b*(2 - m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Sin[a + b*x]^m])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\sqrt{c\sin^m(a+bx)}}\,dx &= \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{m}{2}}(a+bx)\,dx}{\sqrt{c\sin^m(a+bx)}}\\
&= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)\sin(a+bx)}{b(2-m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}
\end{aligned}$$

Mathematica [A] time = 0.08, size = 72, normalized size = 0.90

$$-\frac{2\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{2-m}{4};\frac{6-m}{4};\sin^2(a+bx)\right)}{b(m-2)\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[c*Sin[a + b*x]^m],x]
```

```
[Out] (-2*Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - m)/4, (6 - m)/4, Sin[a
 + b*x]^2]*Tan[a + b*x])/(b*(-2 + m)*Sqrt[c*Sin[a + b*x]^m])
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >> Error detected within library code: integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \sin \left(bx+a\right)^{m}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \left(\sin^{m}\left(bx+a\right)\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \sin \left(bx+a\right)^{m}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(c*sin(b*x + a)^m), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{c\,{\sin\left(a+b\,x\right)}^m}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(1/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{c \sin^{m}{\left(a + bx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(1/2),x)
```

```
[Out] Integral(1/sqrt(c*sin(a + b*x)**m), x)
```

3.23 $\int \frac{1}{\left(c\sin^m(a+bx)\right)^{3/2}}\,dx$

Optimal. Leaf size=89

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-3/4*m],[3/2-3/4*m],sin(b*x+a)^2)*sin(b*x+a
)^(1-m)/b/c/(2-3*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{2\cos(a+bx)\sin^{1-m}(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-3/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (3*(2 - m))/4, Sin[a +
b*x]^2]*Sin[a + b*x]^(1 - m))/(b*c*(2 - 3*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*Si
n[a + b*x]^m])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \frac{1}{(c\sin^m(a+bx))^{3/2}}\,dx &= \frac{\sin^{\frac{m}{2}}(a+bx)\int \sin^{-\frac{3m}{2}}(a+bx)\,dx}{c\sqrt{c\sin^m(a+bx)}}\\ &= \frac{2\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);\frac{3(2-m)}{4};\sin^2(a+bx)\right)\sin^{1-m}(a+bx)}{bc(2-3m)\sqrt{\cos^2(a+bx)}\sqrt{c\sin^m(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.11, size = 71, normalized size = 0.80

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(2-3m);-\frac{3}{4}(m-2);\sin^2(a+bx)\right)}{\left(b-\frac{3bm}{2}\right)(c\sin^m(a+bx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-3/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 3*m)/4, (-3*(-2 + m))/4,
Sin[a + b*x]^2]*Tan[a + b*x])/((b - (3*b*m)/2)*(c*Sin[a + b*x]^m)^(3/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \sin \left(bx + a\right)^{m}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

maple [F] time = 0.55, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(\sin^{m} \left(bx + a\right)\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(3/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \sin \left(bx + a\right)^{m}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(3/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sin^{m}\left(a+bx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(3/2),x)
```

```
[Out] Integral((c*sin(a + b*x)**m)**(-3/2), x)
```

3.24 $\int \frac{1}{\left(c \sin^m(a+bx)\right)^{5/2}} \, dx$

Optimal. Leaf size=89

$$\frac{2 \cos(a+bx) \sin^{1-2m}(a+bx)\, {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2-5m); \frac{1}{4}(6-5m); \sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\, \sqrt{c \sin^m(a+bx)}}$$

```
[Out] 2*cos(b*x+a)*hypergeom([1/2, 1/2-5/4*m],[3/2-5/4*m],sin(b*x+a)^2)*sin(b*x+a
)^(1-2*m)/b/c^2/(2-5*m)/(cos(b*x+a)^2)^(1/2)/(c*sin(b*x+a)^m)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3208, 2643}

$$\frac{2 \cos(a+bx) \sin^{1-2m}(a+bx)\, {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2-5m); \frac{1}{4}(6-5m); \sin^2(a+bx)\right)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\, \sqrt{c \sin^m(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^m)^(-5/2),x]
```

```
[Out] (2*Cos[a + b*x]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[a + b*
x]^2]*Sin[a + b*x]^(1 - 2*m))/(b*c^2*(2 - 5*m)*Sqrt[Cos[a + b*x]^2]*Sqrt[c*
Sin[a + b*x]^m])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned} \int \frac{1}{\left(c \sin^m(a+bx)\right)^{5/2}} \, dx &= \frac{\sin^{\frac{m}{2}}(a+bx) \int \sin^{-\frac{5m}{2}}(a+bx) \, dx}{c^2\sqrt{c \sin^m(a+bx)}} \\ &= \frac{2 \cos(a+bx)\, {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2-5m); \frac{1}{4}(6-5m); \sin^2(a+bx)\right) \sin^{1-2m}(a+bx)}{bc^2(2-5m)\sqrt{\cos^2(a+bx)}\, \sqrt{c \sin^m(a+bx)}} \end{aligned}$$

Mathematica [A] time = 0.11, size = 73, normalized size = 0.82

$$\frac{\sqrt{\cos^2(a+bx)}\, \tan(a+bx)\, {}_2F_1\left(\frac{1}{2}, \frac{1}{4}(2-5m); \frac{1}{4}(6-5m); \sin^2(a+bx)\right)}{\left(b - \frac{5bm}{2}\right)\left(c \sin^m(a+bx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^m)^(-5/2),x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (2 - 5*m)/4, (6 - 5*m)/4, Sin[
a + b*x]^2]*Tan[a + b*x])/((b - (5*b*m)/2)*(c*Sin[a + b*x]^m)^(5/2))
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   integ
rate: implementation incomplete (has polynomial part)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \sin \left(bx + a\right)^{m}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

maple [F] time = 0.54, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \left(\sin^{m}\left(bx + a\right)\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

```
[Out] int(1/(c*sin(b*x+a)^m)^(5/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c \sin \left(bx + a\right)^{m}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)^m)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^m)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{{\left(c\,{\sin\left(a+b\,x\right)}^m\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(c*sin(a + b*x)^m)^(5/2),x)
```

```
[Out] int(1/(c*sin(a + b*x)^m)^(5/2), x)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(c\sin^m\left(a+bx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(c*sin(b*x+a)**m)**(5/2),x)
[Out] Integral((c*sin(a + b*x)**m)**(-5/2), x)
```

3.25 $\int (b\sin^n(c+dx))^p \, dx$

Optimal. Leaf size=77

$$\frac{\sin(c+dx)\cos(c+dx)\,(b\sin^n(c+dx))^p\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*x
+c)*(b*sin(d*x+c)^n)^p/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3208, 2643}

$$\frac{\sin(c+dx)\cos(c+dx)\,(b\sin^n(c+dx))^p\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(b*Sin[c + d*x]^n)^p)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (b\sin^n(c+dx))^p\,dx &= \left(\sin^{-np}(c+dx)\,(b\sin^n(c+dx))^p\right)\int \sin^{np}(c+dx)\,dx\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\,(b\sin^n(c+dx))^p}{d(1+np)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.06, size = 71, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\,(b\sin^n(c+dx))^p\;{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(b*Sin[c + d*x]^n)^p*Tan[c + d*x])/(d*(1 + n*p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^{n}\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

maple [F] time = 0.86, size = 0, normalized size = 0.00

$$\int \left(b\left(\sin^{n}\left(dx+c\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(d*x+c)^n)^p,x)
```

```
[Out] int((b*sin(d*x+c)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(c+d\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sin(c + d*x)^n)^p,x)
```

```
[Out] int((b*sin(c + d*x)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin^{n}\left(c+dx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sin(d*x+c)**n)**p,x)
```

```
[Out] Integral((b*sin(c + d*x)**n)**p, x)
```

3.26 $\int \left(c \sin^2(a+bx)\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+p],[3/2+p],sin(b*x+a)^2)*sin(b*x+a)*(c*sin(b
*x+a)^2)^p/b/(1+2*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^2(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},\frac{1}{2}(2p+1);\frac{1}{2}(2p+3);\sin^2(a+bx)\right)}{b(2p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^2)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 2*p)/2, (3 + 2*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^2)^p)/(b*(1 + 2*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^2(a+bx)\right)^p dx &= \left(\sin^{-2p}(a+bx)\left(c\sin^2(a+bx)\right)^p\right)\int \sin^{2p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+2p);\frac{1}{2}(3+2p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^2(a+bx)\right)^p}{b(1+2p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 61, normalized size = 0.79

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^2(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},p+\frac{1}{2};p+\frac{3}{2};\sin^2(a+bx)\right)}{2bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^2)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + p, 3/2 + p, Sin[a + b*x]
^2]*(c*Sin[a + b*x]^2)^p*Tan[a + b*x])/(b + 2*b*p)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-c\cos\left(bx+a\right)^2+c\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-c*cos(b*x + a)^2 + c)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

maple [F] time = 1.84, size = 0, normalized size = 0.00

$$\int\left(c\left(\sin^2\left(bx+a\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^2)^p,x)
```

```
[Out] int((c*sin(b*x+a)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^2\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^2)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(c\,{\sin\left(a+b\,x\right)}^2\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^2)^p,x)
```

```
[Out] int((c*sin(a + b*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin^2\left(a+bx\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**2)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**2)**p, x)
```

3.27 $\int \left(c\sin^3(a+bx)\right)^p\,dx$

Optimal. Leaf size=75

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+3/2*p],[3/2+3/2*p],sin(b*x+a)^2)*sin(b*x+a)*
(c*sin(b*x+a)^3)^p/b/(1+3*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{b(3p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^3)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Sin[a + b*
x]^2]*Sin[a + b*x]*(c*Sin[a + b*x]^3)^p)/(b*(1 + 3*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^3(a+bx)\right)^p\,dx &= \left(\sin^{-3p}(a+bx)\left(c\sin^3(a+bx)\right)^p\right)\int \sin^{3p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+3p);\frac{3(1+p)}{2};\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^3(a+bx)\right)^p}{b(1+3p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 67, normalized size = 0.89

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^3(a+bx)\right)^p\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(3p+1);\frac{3(p+1)}{2};\sin^2(a+bx)\right)}{3bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^3)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, (1 + 3*p)/2, (3*(1 + p))/2, Si
n[a + b*x]^2]*(c*Sin[a + b*x]^3)^p*Tan[a + b*x])/(b + 3*b*p)
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-\left(c\cos\left(bx+a\right)^{2}-c\right)\sin\left(bx+a\right)\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="fricas")
```

```
[Out] integral((-(c*cos(b*x + a)^2 - c)*sin(b*x + a))^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^{3}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

maple [F] time = 1.71, size = 0, normalized size = 0.00

$$\int\left(c\left(\sin^{3}\left(bx+a\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^3)^p,x)
```

```
[Out] int((c*sin(b*x+a)^3)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^{3}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^3)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^3)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(c\,{\sin\left(a+b\,x\right)}^3\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^3)^p,x)
```

```
[Out] int((c*sin(a + b*x)^3)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin^{3}\left(a+bx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**3)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**3)**p, x)
```

3.28 $\int \left(c \sin^4(a+bx)\right)^p \, dx$

Optimal. Leaf size=77

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

```
[Out] cos(b*x+a)*hypergeom([1/2, 1/2+2*p],[3/2+2*p],sin(b*x+a)^2)*sin(b*x+a)*(c*s
in(b*x+a)^4)^p/b/(1+4*p)/(cos(b*x+a)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3207, 2643}

$$\frac{\sin(a+bx)\cos(a+bx)\left(c\sin^4(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},\frac{1}{2}(4p+1);\frac{1}{2}(4p+3);\sin^2(a+bx)\right)}{b(4p+1)\sqrt{\cos^2(a+bx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^4)^p,x]
```

```
[Out] (Cos[a + b*x]*Hypergeometric2F1[1/2, (1 + 4*p)/2, (3 + 4*p)/2, Sin[a + b*x]
^2]*Sin[a + b*x]*(c*Sin[a + b*x]^4)^p)/(b*(1 + 4*p)*Sqrt[Cos[a + b*x]^2])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(c\sin^4(a+bx)\right)^p \, dx &= \left(\sin^{-4p}(a+bx)\left(c\sin^4(a+bx)\right)^p\right)\int \sin^{4p}(a+bx)\,dx\\ &= \frac{\cos(a+bx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+4p);\frac{1}{2}(3+4p);\sin^2(a+bx)\right)\sin(a+bx)\left(c\sin^4(a+bx)\right)^p}{b(1+4p)\sqrt{\cos^2(a+bx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 65, normalized size = 0.84

$$\frac{\sqrt{\cos^2(a+bx)}\tan(a+bx)\left(c\sin^4(a+bx)\right)^p {}_2F_1\left(\frac{1}{2},2p+\frac{1}{2};2p+\frac{3}{2};\sin^2(a+bx)\right)}{4bp+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^4)^p,x]
```

```
[Out] (Sqrt[Cos[a + b*x]^2]*Hypergeometric2F1[1/2, 1/2 + 2*p, 3/2 + 2*p, Sin[a +
b*x]^2]*(c*Sin[a + b*x]^4)^p*Tan[a + b*x])/(b + 4*b*p)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(c\cos\left(bx+a\right)^{4}-2\,c\cos\left(bx+a\right)^{2}+c\right)^{p},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((c*cos(b*x + a)^4 - 2*c*cos(b*x + a)^2 + c)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^{4}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

maple [F] time = 2.09, size = 0, normalized size = 0.00

$$\int\left(c\left(\sin^{4}\left(bx+a\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^4)^p,x)
```

```
[Out] int((c*sin(b*x+a)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin\left(bx+a\right)^{4}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^4)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int{\left(c\,{\sin\left(a+b\,x\right)}^4\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^4)^p,x)
```

```
[Out] int((c*sin(a + b*x)^4)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(c\sin^{4}\left(a+bx\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**4)**p,x)
```

```
[Out] Integral((c*sin(a + b*x)**4)**p, x)
```

3.29 $\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

```
[Out] -cot(b*x+a)*(c*sin(b*x+a)^n)^(1/n)/b
```

Rubi [A] time = 0.02, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2638}

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] && !IntegerQ[p] && !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (c\sin^n(a+bx))^{\frac{1}{n}}\,dx &= \left(\csc(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}\right)\int \sin(a+bx)\,dx\\ &= -\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 1.00

$$-\frac{\cot(a+bx)\,(c\sin^n(a+bx))^{\frac{1}{n}}}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Sin[a + b*x]^n)^n^(-1),x]
```

```
[Out] -((Cot[a + b*x]*(c*Sin[a + b*x]^n)^n^(-1))/b)
```

fricas [A] time = 0.44, size = 16, normalized size = 0.64

$$-\frac{c^{\left(\frac{1}{n}\right)}\cos\left(bx+a\right)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="fricas")
```

```
[Out] -c^(1/n)*cos(b*x + a)/b
```

giac [B] time = 3.43, size = 384, normalized size = 15.36

$$\frac{|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)^4-2\,|c|^{\left(\frac{1}{n}\right)}\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a\right)}{b\tan\left(\frac{1}{2}\,bx+\frac{1}{2}\,a+\frac{\pi\mathrm{sgn}(c)}{4\,n}-\frac{\pi}{4\,n}\right)^2\tan\left(\frac{1}{2}\,bx\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="giac")
```

```
[Out] (abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b
*x + 1/2*a)^4 - 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*
pi/n)^2*tan(1/2*b*x + 1/2*a)^2 + 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*p
i*sgn(c)/n - 1/4*pi/n)*tan(1/2*b*x + 1/2*a)^3 - abs(c)^(1/n)*tan(1/2*b*x +
1/2*a)^4 + abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2
 - 4*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)*tan(1/2
*b*x + 1/2*a) + 2*abs(c)^(1/n)*tan(1/2*b*x + 1/2*a)^2 - abs(c)^(1/n))/(b*ta
n(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a)^4 +
2*b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n - 1/4*pi/n)^2*tan(1/2*b*x + 1/2*a
)^2 + b*tan(1/2*b*x + 1/2*a)^4 + b*tan(1/2*b*x + 1/2*a + 1/4*pi*sgn(c)/n -
1/4*pi/n)^2 + 2*b*tan(1/2*b*x + 1/2*a)^2 + b)
```

maple [F] time = 0.85, size = 0, normalized size = 0.00

$$\int \left(c\left(\sin^{n}\left(bx+a\right)\right)\right)^{\frac{1}{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(b*x+a)^n)^(1/n),x)
```

```
[Out] int((c*sin(b*x+a)^n)^(1/n),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(c\sin\left(bx+a\right)^{n}\right)^{\left(\frac{1}{n}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)^n)^(1/n),x, algorithm="maxima")
```

```
[Out] integrate((c*sin(b*x + a)^n)^(1/n), x)
```

mupad [B] time = 13.75, size = 36, normalized size = 1.44

$$-\frac{\sin\left(2\,a+2\,b\,x\right)\,{\left(c\,\sin\left(a+b\,x\right)^n\right)}^{1/n}}{2\,b\,\sin\left(a+b\,x\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sin(a + b*x)^n)^(1/n),x)
```

```
[Out] -(sin(2*a + 2*b*x)*(c*sin(a + b*x)^n)^(1/n))/(2*b*sin(a + b*x)^2)
```

sympy [A] time = 2.11, size = 61, normalized size = 2.44

$$\begin{cases} x\left(c\sin^{n}\left(a\right)\right)^{\frac{1}{n}} & \text{for } b=0 \\ x\left(0^{n}c\right)^{\frac{1}{n}} & \text{for } a=-bx\vee a=-bx+\pi \\ -\frac{c^{\frac{1}{n}}\left(\sin^{n}\left(a+bx\right)\right)^{\frac{1}{n}}\cos\left(a+bx\right)}{b\sin\left(a+bx\right)} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sin(b*x+a)**n)**(1/n),x)
```

```
[Out] Piecewise((x*(c*sin(a)**n)**(1/n), Eq(b, 0)), (x*(0**n*c)**(1/n), Eq(a, -b*
x) | Eq(a, -b*x + pi)), (-c**(1/n)*(sin(a + b*x)**n)**(1/n)*cos(a + b*x)/(b
*sin(a + b*x)), True))
```

3.30 $\int (a(b\sin(c+dx))^p)^n \, dx$

Optimal. Leaf size=79

$$\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

```
[Out] cos(d*x+c)*hypergeom([1/2, 1/2*n*p+1/2],[1/2*n*p+3/2],sin(d*x+c)^2)*sin(d*x
+c)*(a*(b*sin(d*x+c))^p)^n/d/(n*p+1)/(cos(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3208, 2643}

$$\frac{\sin(c+dx)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)\sqrt{\cos^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a*(b*Sin[c + d*x])^p)^n,x]
```

```
[Out] (Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[c + d*x]
^2]*Sin[c + d*x]*(a*(b*Sin[c + d*x])^p)^n)/(d*(1 + n*p)*Sqrt[Cos[c + d*x]^2
])
```

Rule 2643

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(Cos[c + d*x]*(
b*Sin[c + d*x])^(n + 1)*Hypergeometric2F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c
+ d*x]^2])/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]), x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3208

```
Int[(u_.)*((b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :> D
ist[(b^IntPart[p]*(b*(c*Sin[e + f*x])^n)^FracPart[p])/(c*Sin[e + f*x])^(n*F
racPart[p]), Int[ActivateTrig[u]*(c*Sin[e + f*x])^(n*p), x], x] /; FreeQ[{b
, c, e, f, n, p}, x] &&  !IntegerQ[p] &&  !IntegerQ[n] && (EqQ[u, 1] || Mat
chQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /; FreeQ[{d, m}, x] && MemberQ[{sin,
cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int (a(b\sin(c+dx))^p)^n \, dx &= \left((b\sin(c+dx))^{-np}(a(b\sin(c+dx))^p)^n\right)\int (b\sin(c+dx))^{np}\,dx\\ &= \frac{\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+np);\frac{1}{2}(3+np);\sin^2(c+dx)\right)\sin(c+dx)\,(a(b\sin(c+dx))^p)}{d(1+np)\sqrt{\cos^2(c+dx)}}\end{aligned}$$

Mathematica [A] time = 0.05, size = 73, normalized size = 0.92

$$\frac{\sqrt{\cos^2(c+dx)}\tan(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(np+1);\frac{1}{2}(np+3);\sin^2(c+dx)\right)(a(b\sin(c+dx))^p)^n}{d(np+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a*(b*Sin[c + d*x])^p)^n,x]
```

```
[Out] (Sqrt[Cos[c + d*x]^2]*Hypergeometric2F1[1/2, (1 + n*p)/2, (3 + n*p)/2, Sin[
c + d*x]^2]*(a*(b*Sin[c + d*x])^p)^n*Tan[c + d*x])/(d*(1 + n*p))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left((b\sin(dx+c))^p a\right)^n, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="fricas")
```

```
[Out] integral(((b*sin(d*x + c))^p*a)^n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((b\sin(dx+c))^p a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="giac")
```

```
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

maple [F] time = 1.02, size = 0, normalized size = 0.00

$$\int \left(a(b\sin(dx+c))^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sin(d*x+c))^p)^n,x)
```

```
[Out] int((a*(b*sin(d*x+c))^p)^n,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left((b\sin(dx+c))^p a\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))^p)^n,x, algorithm="maxima")
```

```
[Out] integrate(((b*sin(d*x + c))^p*a)^n, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(a\,(b\,\sin(c+d\,x))^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*(b*sin(c + d*x))^p)^n,x)
```

```
[Out] int((a*(b*sin(c + d*x))^p)^n, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a(b\sin(c+dx))^p\right)^n dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*(b*sin(d*x+c))**p)**n,x)
```

```
[Out] Integral((a*(b*sin(c + d*x))**p)**n, x)
```

3.31 $\int \left(a - a\sin^2(x)\right)\,dx$

Optimal. Leaf size=16

$$\frac{ax}{2} + \frac{1}{2}a\sin(x)\cos(x)$$

```
[Out] 1/2*a*x+1/2*a*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 9, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.222, Rules used = {2635, 8}

$$\frac{ax}{2} + \frac{1}{2}a\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a - a*Sin[x]^2,x]
```

```
[Out] (a*x)/2 + (a*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(a - a\sin^2(x)\right)\,dx &= ax - a\int \sin^2(x)\,dx\\ &= ax + \frac{1}{2}a\cos(x)\sin(x) - \frac{1}{2}a\int 1\,dx\\ &= \frac{ax}{2} + \frac{1}{2}a\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 16, normalized size = 1.00

$$a\left(\frac{x}{2} + \frac{1}{4}\sin(2x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[a - a*Sin[x]^2,x]
```

```
[Out] a*(x/2 + Sin[2*x]/4)
```

fricas [A] time = 0.42, size = 12, normalized size = 0.75

$$\frac{1}{2}\,a\cos(x)\sin(x) + \frac{1}{2}\,ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="fricas")
```

```
[Out] 1/2*a*cos(x)*sin(x) + 1/2*a*x
```

giac [A] time = 0.12, size = 17, normalized size = 1.06

$$-\frac{1}{4}\,a(2\,x-\sin\left(2\,x\right))+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="giac")
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

maple [A] time = 0.07, size = 18, normalized size = 1.12

$$ax-a\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a-a*sin(x)^2,x)
[Out] a*x-a*(-1/2*sin(x)*cos(x)+1/2*x)
```

maxima [A] time = 0.32, size = 17, normalized size = 1.06

$$-\frac{1}{4}\,a(2\,x-\sin\left(2\,x\right))+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)^2,x, algorithm="maxima")
[Out] -1/4*a*(2*x - sin(2*x)) + a*x
```

mupad [B] time = 13.58, size = 11, normalized size = 0.69

$$\frac{a\,\left(2\,x+\sin\left(2\,x\right)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a - a*sin(x)^2,x)
[Out] (a*(2*x + sin(2*x)))/4
```

sympy [A] time = 0.10, size = 15, normalized size = 0.94

$$ax-a\left(\frac{x}{2}-\frac{\sin\left(x\right)\cos\left(x\right)}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a-a*sin(x)**2,x)
[Out] a*x - a*(x/2 - sin(x)*cos(x)/2)
```

3.32 $\int \left(a - a\sin^2(x)\right)^2 dx$

Optimal. Leaf size=33

$$\frac{3a^2x}{8} + \frac{1}{4}a^2\sin(x)\cos^3(x) + \frac{3}{8}a^2\sin(x)\cos(x)$$

```
[Out] 3/8*a^2*x+3/8*a^2*cos(x)*sin(x)+1/4*a^2*cos(x)^3*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{3a^2x}{8} + \frac{1}{4}a^2\sin(x)\cos^3(x) + \frac{3}{8}a^2\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*a^2*x)/8 + (3*a^2*Cos[x]*Sin[x])/8 + (a^2*Cos[x]^3*Sin[x])/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \left(a - a\sin^2(x)\right)^2 dx &= a^2\int \cos^4(x)\,dx\\ &= \frac{1}{4}a^2\cos^3(x)\sin(x) + \frac{1}{4}\left(3a^2\right)\int \cos^2(x)\,dx\\ &= \frac{3}{8}a^2\cos(x)\sin(x) + \frac{1}{4}a^2\cos^3(x)\sin(x) + \frac{1}{8}\left(3a^2\right)\int 1\,dx\\ &= \frac{3a^2x}{8} + \frac{3}{8}a^2\cos(x)\sin(x) + \frac{1}{4}a^2\cos^3(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.79

$$a^2\left(\frac{3x}{8} + \frac{1}{4}\sin(2x) + \frac{1}{32}\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^2,x]
```

```
[Out] a^2*((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)
```

fricas [A] time = 0.42, size = 28, normalized size = 0.85

$$\frac{3}{8}\,a^{2}x+\frac{1}{8}\left(2\,a^{2}\cos(x)^{3}+3\,a^{2}\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] 3/8*a^2*x + 1/8*(2*a^2*cos(x)^3 + 3*a^2*cos(x))*sin(x)
```

giac [A] time = 0.14, size = 25, normalized size = 0.76

$$\frac{3}{8}\,a^{2}x+\frac{1}{32}\,a^{2}\sin\left(4\,x\right)+\frac{1}{4}\,a^{2}\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 3/8*a^2*x + 1/32*a^2*sin(4*x) + 1/4*a^2*sin(2*x)
```

maple [A] time = 0.34, size = 43, normalized size = 1.30

$$a^{2}\left(-\frac{\left(\sin^{3}(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-2a^{2}\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^2,x)
[Out] a^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-2*a^2*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

maxima [A] time = 0.35, size = 40, normalized size = 1.21

$$\frac{1}{32}\,a^{2}(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))-\frac{1}{2}\,a^{2}(2\,x-\sin\left(2\,x\right))+a^{2}x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/32*a^2*(12*x + sin(4*x) - 8*sin(2*x)) - 1/2*a^2*(2*x - sin(2*x)) + a^2*x
```

mupad [B] time = 13.76, size = 33, normalized size = 1.00

$$\frac{\frac{3\,a^{2}\tan(x)^{3}}{8}+\frac{5\,a^{2}\tan(x)}{8}}{\left(\tan(x)^{2}+1\right)^{2}}+\frac{3\,a^{2}\,x}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^2,x)
[Out] ((5*a^2*tan(x))/8 + (3*a^2*tan(x)^3)/8)/(tan(x)^2 + 1)^2 + (3*a^2*x)/8
```

sympy [B] time = 1.12, size = 110, normalized size = 3.33

$$\frac{3a^{2}x\sin^{4}(x)}{8}+\frac{3a^{2}x\sin^{2}(x)\cos^{2}(x)}{4}-a^{2}x\sin^{2}(x)+\frac{3a^{2}x\cos^{4}(x)}{8}-a^{2}x\cos^{2}(x)+a^{2}x-\frac{5a^{2}\sin^{3}(x)\cos(x)}{8}-\frac{3a^{2}\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**2,x)
```

```
[Out] 3*a**2*x*sin(x)**4/8 + 3*a**2*x*sin(x)**2*cos(x)**2/4 - a**2*x*sin(x)**2 +
3*a**2*x*cos(x)**4/8 - a**2*x*cos(x)**2 + a**2*x - 5*a**2*sin(x)**3*cos(x)/
8 - 3*a**2*sin(x)*cos(x)**3/8 + a**2*sin(x)*cos(x)
```

3.33 $\int \left(a - a\sin^2(x)\right)^3 \, dx$

Optimal. Leaf size=46

$$\frac{5a^3x}{16} + \frac{1}{6}a^3\sin(x)\cos^5(x) + \frac{5}{24}a^3\sin(x)\cos^3(x) + \frac{5}{16}a^3\sin(x)\cos(x)$$

```
[Out] 5/16*a^3*x+5/16*a^3*cos(x)*sin(x)+5/24*a^3*cos(x)^3*sin(x)+1/6*a^3*cos(x)^5
*sin(x)
```

Rubi [A] time = 0.03, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{5a^3x}{16} + \frac{1}{6}a^3\sin(x)\cos^5(x) + \frac{5}{24}a^3\sin(x)\cos^3(x) + \frac{5}{16}a^3\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^3,x]
```

```
[Out] (5*a^3*x)/16 + (5*a^3*Cos[x]*Sin[x])/16 + (5*a^3*Cos[x]^3*Sin[x])/24 + (a^3
*Cos[x]^5*Sin[x])/6
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \left(a - a\sin^2(x)\right)^3 \, dx &= a^3 \int \cos^6(x) \, dx \\
&= \frac{1}{6}a^3\cos^5(x)\sin(x) + \frac{1}{6}\left(5a^3\right)\int \cos^4(x) \, dx \\
&= \frac{5}{24}a^3\cos^3(x)\sin(x) + \frac{1}{6}a^3\cos^5(x)\sin(x) + \frac{1}{8}\left(5a^3\right)\int \cos^2(x) \, dx \\
&= \frac{5}{16}a^3\cos(x)\sin(x) + \frac{5}{24}a^3\cos^3(x)\sin(x) + \frac{1}{6}a^3\cos^5(x)\sin(x) + \frac{1}{16}\left(5a^3\right)\int 1 \, dx \\
&= \frac{5a^3x}{16} + \frac{5}{16}a^3\cos(x)\sin(x) + \frac{5}{24}a^3\cos^3(x)\sin(x) + \frac{1}{6}a^3\cos^5(x)\sin(x)
\end{aligned}$$

Mathematica [A] time = 0.00, size = 34, normalized size = 0.74

$$a^3\left(\frac{5x}{16} + \frac{15}{64}\sin(2x) + \frac{3}{64}\sin(4x) + \frac{1}{192}\sin(6x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^3,x]
[Out] a^3*((5*x)/16 + (15*Sin[2*x])/64 + (3*Sin[4*x])/64 + Sin[6*x]/192)
```

fricas [A] time = 0.45, size = 37, normalized size = 0.80

$$\frac{5}{16}\,a^3x+\frac{1}{48}\left(8\,a^3\cos(x)^5+10\,a^3\cos(x)^3+15\,a^3\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="fricas")
[Out] 5/16*a^3*x + 1/48*(8*a^3*cos(x)^5 + 10*a^3*cos(x)^3 + 15*a^3*cos(x))*sin(x)
```

giac [A] time = 0.13, size = 34, normalized size = 0.74

$$\frac{5}{16}\,a^3x+\frac{1}{192}\,a^3\sin\left(6\,x\right)+\frac{3}{64}\,a^3\sin\left(4\,x\right)+\frac{15}{64}\,a^3\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="giac")
[Out] 5/16*a^3*x + 1/192*a^3*sin(6*x) + 3/64*a^3*sin(4*x) + 15/64*a^3*sin(2*x)
```

maple [A] time = 0.44, size = 72, normalized size = 1.57

$$-a^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a^3\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)-3a^3\left(-\frac{\sin(x)\cos(x)}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^3,x)
[Out] -a^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a^3*(-1/4*(
sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-3*a^3*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*x
```

maxima [A] time = 0.32, size = 69, normalized size = 1.50

$$-\frac{1}{192}\left(4\,\sin\left(2\,x\right)^3+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)a^3+\frac{3}{32}\,a^3(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))-\frac{3}{4}\,a^3(2\,x-\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^3,x, algorithm="maxima")
[Out] -1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^3 + 3/32*a^3*(12*
x + sin(4*x) - 8*sin(2*x)) - 3/4*a^3*(2*x - sin(2*x)) + a^3*x
```

mupad [B] time = 13.68, size = 42, normalized size = 0.91

$$\frac{11\,a^3\cos(x)^5\,\sin(x)}{16}+\frac{5\,a^3\cos(x)^3\,\sin(x)^3}{6}+\frac{5\,a^3\,\cos(x)\sin(x)^5}{16}+\frac{5\,x\,a^3}{16}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^3,x)
[Out] (5*a^3*x)/16 + (5*a^3*cos(x)*sin(x)^5)/16 + (11*a^3*cos(x)^5*sin(x))/16 + (
5*a^3*cos(x)^3*sin(x)^3)/6
```

sympy [B] time = 2.66, size = 233, normalized size = 5.07

$$-\frac{5a^3x\sin^6(x)}{16}-\frac{15a^3x\sin^4(x)\cos^2(x)}{16}+\frac{9a^3x\sin^4(x)}{8}-\frac{15a^3x\sin^2(x)\cos^4(x)}{16}+\frac{9a^3x\sin^2(x)\cos^2(x)}{4}-\frac{3a^3x\sin}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**3,x)
```

```
[Out] -5*a**3*x*sin(x)**6/16 - 15*a**3*x*sin(x)**4*cos(x)**2/16 + 9*a**3*x*sin(x)
**4/8 - 15*a**3*x*sin(x)**2*cos(x)**4/16 + 9*a**3*x*sin(x)**2*cos(x)**2/4 -
 3*a**3*x*sin(x)**2/2 - 5*a**3*x*cos(x)**6/16 + 9*a**3*x*cos(x)**4/8 - 3*a*
*3*x*cos(x)**2/2 + a**3*x + 11*a**3*sin(x)**5*cos(x)/16 + 5*a**3*sin(x)**3*
cos(x)**3/6 - 15*a**3*sin(x)**3*cos(x)/8 + 5*a**3*sin(x)*cos(x)**5/16 - 9*a
**3*sin(x)*cos(x)**3/8 + 3*a**3*sin(x)*cos(x)/2
```

3.34 $\int \left(a-a\sin^2(x)\right)^4\,dx$

Optimal. Leaf size=59

$$\frac{35a^4x}{128}+\frac{1}{8}a^4\sin(x)\cos^7(x)+\frac{7}{48}a^4\sin(x)\cos^5(x)+\frac{35}{192}a^4\sin(x)\cos^3(x)+\frac{35}{128}a^4\sin(x)\cos(x)$$

```
[Out] 35/128*a^4*x+35/128*a^4*cos(x)*sin(x)+35/192*a^4*cos(x)^3*sin(x)+7/48*a^4*c
os(x)^5*sin(x)+1/8*a^4*cos(x)^7*sin(x)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3175, 2635, 8}

$$\frac{35a^4x}{128}+\frac{1}{8}a^4\sin(x)\cos^7(x)+\frac{7}{48}a^4\sin(x)\cos^5(x)+\frac{35}{192}a^4\sin(x)\cos^3(x)+\frac{35}{128}a^4\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^4,x]
```

```
[Out] (35*a^4*x)/128 + (35*a^4*Cos[x]*Sin[x])/128 + (35*a^4*Cos[x]^3*Sin[x])/192
+ (7*a^4*Cos[x]^5*Sin[x])/48 + (a^4*Cos[x]^7*Sin[x])/8
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$
\begin{aligned}
\int \left(a-a\sin^2(x)\right)^4\,dx &= a^4\int \cos^8(x)\,dx\\
&=\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{8}\left(7a^4\right)\int \cos^6(x)\,dx\\
&=\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{48}\left(35a^4\right)\int \cos^4(x)\,dx\\
&=\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)+\frac{1}{64}\left(35a^4\right)\int \cos^2(x)\\
&=\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(x)\sin(x)\\
&=\frac{35a^4x}{128}+\frac{35}{128}a^4\cos(x)\sin(x)+\frac{35}{192}a^4\cos^3(x)\sin(x)+\frac{7}{48}a^4\cos^5(x)\sin(x)+\frac{1}{8}a^4\cos^7(
\end{aligned}
$$

Mathematica [A] time = 0.00, size = 42, normalized size = 0.71

$$a^4\left(\frac{35x}{128}+\frac{7}{32}\sin(2x)+\frac{7}{128}\sin(4x)+\frac{1}{96}\sin(6x)+\frac{\sin(8x)}{1024}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^4,x]
```

```
[Out] a^4*((35*x)/128 + (7*Sin[2*x])/32 + (7*Sin[4*x])/128 + Sin[6*x]/96 + Sin[8*
x]/1024)
```

fricas [A] time = 0.45, size = 46, normalized size = 0.78

$$\frac{35}{128}\,a^4x + \frac{1}{384}\left(48\,a^4\cos(x)^7 + 56\,a^4\cos(x)^5 + 70\,a^4\cos(x)^3 + 105\,a^4\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 35/128*a^4*x + 1/384*(48*a^4*cos(x)^7 + 56*a^4*cos(x)^5 + 70*a^4*cos(x)^3 +
 105*a^4*cos(x))*sin(x)
```

giac [A] time = 0.12, size = 43, normalized size = 0.73

$$\frac{35}{128}\,a^4x + \frac{1}{1024}\,a^4\sin(8\,x) + \frac{1}{96}\,a^4\sin(6\,x) + \frac{7}{128}\,a^4\sin(4\,x) + \frac{7}{32}\,a^4\sin(2\,x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 35/128*a^4*x + 1/1024*a^4*sin(8*x) + 1/96*a^4*sin(6*x) + 7/128*a^4*sin(4*x)
 + 7/32*a^4*sin(2*x)
```

maple [B] time = 0.54, size = 105, normalized size = 1.78

$$a^4\left(-\frac{\left(\sin^7(x) + \frac{7\left(\sin^5(x)\right)}{6} + \frac{35\left(\sin^3(x)\right)}{24} + \frac{35\sin(x)}{16}\right)\cos(x)}{8} + \frac{35x}{128}\right) - 4a^4\left(-\frac{\left(\sin^5(x) + \frac{5\left(\sin^3(x)\right)}{4} + \frac{15\sin(x)}{8}\right)\cos(x)}{6} +\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^4,x)
```

```
[Out] a^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)-4*a^4*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^4*(-
1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)-4*a^4*(-1/2*sin(x)*cos(x)+1/2*x)+a^
4*x
```

maxima [B] time = 0.32, size = 104, normalized size = 1.76

$$\frac{1}{3072}\left(128\,\sin(2\,x)^3 + 840\,x + 3\,\sin(8\,x) + 168\,\sin(4\,x) - 768\,\sin(2\,x)\right)a^4 - \frac{1}{48}\left(4\,\sin(2\,x)^3 + 60\,x + 9\,\sin(4\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/3072*(128*sin(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*
a^4 - 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a^4 + 3/16*a^4*
(12*x + sin(4*x) - 8*sin(2*x)) - a^4*(2*x - sin(2*x)) + a^4*x
```

mupad [B] time = 13.69, size = 51, normalized size = 0.86

$$\frac{\frac{35\,a^4\tan(x)^7}{128} + \frac{385\,a^4\tan(x)^5}{384} + \frac{511\,a^4\tan(x)^3}{384} + \frac{93\,a^4\tan(x)}{128}}{\left(\tan(x)^2 + 1\right)^4} + \frac{35\,a^4\,x}{128}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^4,x)
```

```
[Out] ((93*a^4*tan(x))/128 + (511*a^4*tan(x)^3)/384 + (385*a^4*tan(x)^5)/384 + (3
5*a^4*tan(x)^7)/128)/(tan(x)^2 + 1)^4 + (35*a^4*x)/128
```

sympy [B] time = 7.32, size = 376, normalized size = 6.37

$$\frac{35a^4x\sin^8(x)}{128}+\frac{35a^4x\sin^6(x)\cos^2(x)}{32}-\frac{5a^4x\sin^6(x)}{4}+\frac{105a^4x\sin^4(x)\cos^4(x)}{64}-\frac{15a^4x\sin^4(x)\cos^2(x)}{4}+\frac{9a^4x\sin}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**4,x)
```

```
[Out] 35*a**4*x*sin(x)**8/128 + 35*a**4*x*sin(x)**6*cos(x)**2/32 - 5*a**4*x*sin(x
)**6/4 + 105*a**4*x*sin(x)**4*cos(x)**4/64 - 15*a**4*x*sin(x)**4*cos(x)**2/
4 + 9*a**4*x*sin(x)**4/4 + 35*a**4*x*sin(x)**2*cos(x)**6/32 - 15*a**4*x*sin
(x)**2*cos(x)**4/4 + 9*a**4*x*sin(x)**2*cos(x)**2/2 - 2*a**4*x*sin(x)**2 +
35*a**4*x*cos(x)**8/128 - 5*a**4*x*cos(x)**6/4 + 9*a**4*x*cos(x)**4/4 - 2*a
**4*x*cos(x)**2 + a**4*x - 93*a**4*sin(x)**7*cos(x)/128 - 511*a**4*sin(x)**
5*cos(x)**3/384 + 11*a**4*sin(x)**5*cos(x)/4 - 385*a**4*sin(x)**3*cos(x)**5
/384 + 10*a**4*sin(x)**3*cos(x)**3/3 - 15*a**4*sin(x)**3*cos(x)/4 - 35*a**4
*sin(x)*cos(x)**7/128 + 5*a**4*sin(x)*cos(x)**5/4 - 9*a**4*sin(x)*cos(x)**3
/4 + 2*a**4*sin(x)*cos(x)
```

3.35 $\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{ad}+\frac{3\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] 3*cos(d*x+c)/a/d-cos(d*x+c)^3/a/d+1/5*cos(d*x+c)^5/a/d+sec(d*x+c)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3175, 2590, 270}

$$\frac{\cos^5(c+dx)}{5ad}-\frac{\cos^3(c+dx)}{ad}+\frac{3\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (3*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(a*d) + Cos[c + d*x]^5/(5*a*d) + Se
c[c + d*x]/(a*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^5(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(-3+\frac{1}{x^2}+3x^2-x^4\right)dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{3\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{ad}+\frac{\cos^5(c+dx)}{5ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.06, size = 58, normalized size = 0.94

$$\frac{\frac{19\cos(c+dx)}{8d}-\frac{3\cos(3(c+dx))}{16d}+\frac{\cos(5(c+dx))}{80d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((19*Cos[c + d*x])/(8*d) - (3*Cos[3*(c + d*x)])/(16*d) + Cos[5*(c + d*x)]/(
80*d) + Sec[c + d*x]/d)/a
```

fricas [A] time = 0.44, size = 46, normalized size = 0.74

$$\frac{\cos (dx+c)^6 - 5 \cos (dx+c)^4 + 15 \cos (dx+c)^2 + 5}{5\, ad \cos (dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/5*(cos(d*x + c)^6 - 5*cos(d*x + c)^4 + 15*cos(d*x + c)^2 + 5)/(a*d*cos(d*
x + c))
```

giac [B] time = 0.15, size = 149, normalized size = 2.40

$$\frac{2\left(\frac{5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)} + \frac{\frac{50\,(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{80\,(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + \frac{30\,(\cos(dx+c)-1)^3}{(\cos(dx+c)+1)^3} - \frac{5\,(\cos(dx+c)-1)^4}{(\cos(dx+c)+1)^4} - 11}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^5}\right)}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 2/5*(5/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (50*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 80*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 30
*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 - 5*(cos(d*x + c) - 1)^4/(cos(d*
x + c) + 1)^4 - 11)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

maple [A] time = 0.33, size = 45, normalized size = 0.73

$$\frac{\frac{\left(\cos^5(dx+c)\right)}{5} - \left(\cos^3(dx+c)\right) + 3\cos(dx+c) + \frac{1}{\cos(dx+c)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x)
```

```
[Out] 1/d/a*(1/5*cos(d*x+c)^5-cos(d*x+c)^3+3*cos(d*x+c)+1/cos(d*x+c))
```

maxima [A] time = 0.36, size = 50, normalized size = 0.81

$$\frac{\frac{\cos(dx+c)^5 - 5\cos(dx+c)^3 + 15\cos(dx+c)}{a} + \frac{5}{a\cos(dx+c)}}{5\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*((cos(d*x + c)^5 - 5*cos(d*x + c)^3 + 15*cos(d*x + c))/a + 5/(a*cos(d*x
 + c)))/d
```

mupad [B] time = 13.65, size = 54, normalized size = 0.87

$$\frac{\frac{3\cos(c+d\,x)}{a} + \frac{1}{a\cos(c+d\,x)} - \frac{\cos(c+d\,x)^3}{a} + \frac{\cos(c+d\,x)^5}{5\,a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2),x)
```

```
[Out] ((3*cos(c + d*x))/a + 1/(a*cos(c + d*x)) - cos(c + d*x)^3/a + cos(c + d*x)^
5/(5*a))/d
```

sympy [A] time = 35.44, size = 314, normalized size = 5.06

$$\begin{cases} -\frac{160\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\frac{dx}{2}\right)+25ad\tan^{8}\left(\frac{c}{2}+\frac{dx}{2}\right)-25ad\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)-20ad\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-5ad} - \frac{}{5ad\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)+20ad\tan^{10}\left(\frac{c}{2}+\right.} \\ \frac{x\sin^{7}(c)}{-a\sin^{2}(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-160*tan(c/2 + d*x/2)**4/(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*ta
n(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**
4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 128*tan(c/2 + d*x/2)**2/(5*a*d*ta
n(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2 + d*x/2)*
*8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a*d) - 32/
(5*a*d*tan(c/2 + d*x/2)**12 + 20*a*d*tan(c/2 + d*x/2)**10 + 25*a*d*tan(c/2
+ d*x/2)**8 - 25*a*d*tan(c/2 + d*x/2)**4 - 20*a*d*tan(c/2 + d*x/2)**2 - 5*a
*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a), True))
```

3.36 $\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{2\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] 2*cos(d*x+c)/a/d-1/3*cos(d*x+c)^3/a/d+sec(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3175, 2590, 270}

$$-\frac{\cos^3(c+dx)}{3ad}+\frac{2\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (2*Cos[c + d*x])/(a*d) - Cos[c + d*x]^3/(3*a*d) + Sec[c + d*x]/(a*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin^3(c+dx)\tan^2(c+dx)\,dx}{a}\\
&= -\frac{\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\
&= -\frac{\operatorname{Subst}\left(\int \left(-2+\frac{1}{x^2}+x^2\right)\,dx, x, \cos(c+dx)\right)}{ad}\\
&= \frac{2\cos(c+dx)}{ad}-\frac{\cos^3(c+dx)}{3ad}+\frac{\sec(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 43, normalized size = 0.93

$$\frac{\frac{7\cos(c+dx)}{4d}-\frac{\cos(3(c+dx))}{12d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
[Out] ((7*Cos[c + d*x])/(4*d) - Cos[3*(c + d*x)]/(12*d) + Sec[c + d*x]/d)/a
```

fricas [A] time = 0.44, size = 36, normalized size = 0.78

$$-\frac{\cos\left(dx+c\right)^{4}-6\,\cos\left(dx+c\right)^{2}-3}{3\,ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -1/3*(cos(d*x + c)^4 - 6*cos(d*x + c)^2 - 3)/(a*d*cos(d*x + c))
```

giac [B] time = 0.14, size = 105, normalized size = 2.28

$$\frac{2\left(\frac{3}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}+\frac{\frac{12\,(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3\,(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}-5}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^{3}}\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] 2/3*(3/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)) + (12*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 5)/
(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))/d
```

maple [A] time = 0.31, size = 35, normalized size = 0.76

$$\frac{-\frac{\left(\cos^{3}(dx+c)\right)}{3}+2\cos\left(dx+c\right)+\frac{1}{\cos(dx+c)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x)
[Out] 1/d/a*(-1/3*cos(d*x+c)^3+2*cos(d*x+c)+1/cos(d*x+c))
```

maxima [A] time = 0.36, size = 40, normalized size = 0.87

$$-\frac{\frac{\cos(dx+c)^{3}-6\,\cos(dx+c)}{a}-\frac{3}{a\cos(dx+c)}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] -1/3*((cos(d*x + c)^3 - 6*cos(d*x + c))/a - 3/(a*cos(d*x + c)))/d
```

mupad [B] time = 0.06, size = 38, normalized size = 0.83

$$\frac{-\cos\left(c+d\,x\right)^{4}+6\cos\left(c+d\,x\right)^{2}+3}{3\,a\,d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2),x)
```

```
[Out] (6*cos(c + d*x)^2 - cos(c + d*x)^4 + 3)/(3*a*d*cos(c + d*x))
```

sympy [A] time = 14.35, size = 143, normalized size = 3.11

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} - \frac{16}{3ad\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+6ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-6ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3ad} & \text{for } d \neq 0 \\ \frac{x\sin^5(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a*d*tan(c/2 + d*x/2)**8 + 6*a*d*tan(c
/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d) - 16/(3*a*d*tan(c/2 + d
*x/2)**8 + 6*a*d*tan(c/2 + d*x/2)**6 - 6*a*d*tan(c/2 + d*x/2)**2 - 3*a*d),
Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a), True))
```

3.37 $\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=27

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

```
[Out] cos(d*x+c)/a/d+sec(d*x+c)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 27, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 14}

$$\frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] Cos[c + d*x]/(a*d) + Sec[c + d*x]/(a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sin(c+dx)\tan^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, \cos(c+dx)\right)}{ad}\\ &= -\frac{\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, \cos(c+dx)\right)}{ad}\\ &= \frac{\cos(c+dx)}{ad}+\frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.03, size = 25, normalized size = 0.93

$$\frac{\frac{\cos(c+dx)}{d}+\frac{\sec(c+dx)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (Cos[c + d*x]/d + Sec[c + d*x]/d)/a
```

fricas [A] time = 0.44, size = 25, normalized size = 0.93

$$\frac{\cos\left(dx+c\right)^2+1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (cos(d*x + c)^2 + 1)/(a*d*cos(d*x + c))
```

giac [A] time = 0.13, size = 29, normalized size = 1.07

$$\frac{\cos\left(dx+c\right)}{ad}+\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] cos(d*x + c)/(a*d) + 1/(a*d*cos(d*x + c))
```

maple [A] time = 0.30, size = 23, normalized size = 0.85

$$\frac{\cos\left(dx+c\right)+\frac{1}{\cos(dx+c)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x)
```

```
[Out] 1/d/a*(cos(d*x+c)+1/cos(d*x+c))
```

maxima [A] time = 0.34, size = 27, normalized size = 1.00

$$\frac{\frac{\cos(dx+c)}{a}+\frac{1}{a\cos(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] (cos(d*x + c)/a + 1/(a*cos(d*x + c)))/d
```

mupad [B] time = 0.04, size = 25, normalized size = 0.93

$$\frac{\cos\left(c+d\,x\right)^2+1}{a\,d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2),x)
```

```
[Out] (cos(c + d*x)^2 + 1)/(a*d*cos(c + d*x))
```

sympy [A] time = 6.93, size = 36, normalized size = 1.33

$$\begin{cases} -\frac{4}{ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d \neq 0 \\ \frac{x\sin^3(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-4/(a*d*tan(c/2 + d*x/2)**4 - a*d), Ne(d, 0)), (x*sin(c)**3/(-a*
sin(c)**2 + a), True))
```

3.38 $\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=13

$$\frac{\sec(c+dx)}{ad}$$

```
[Out] sec(d*x+c)/a/d
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.136, Rules used = {3175, 2606, 8}

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
[Out] Sec[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec(c+dx)\tan(c+dx)\,dx}{a}\\ &= \frac{\text{Subst}\left(\int 1\,dx, x, \sec(c+dx)\right)}{ad}\\ &= \frac{\sec(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\sec(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2),x]
[Out] Sec[c + d*x]/(a*d)
```

fricas [A] time = 0.39, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] 1/(a*d*cos(d*x + c))
```

giac [A] time = 0.13, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/(a*d*cos(d*x + c))
```

maple [A] time = 0.19, size = 16, normalized size = 1.23

$$\frac{1}{da\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-a*sin(d*x+c)^2),x)
[Out] 1/d/a/cos(d*x+c)
```

maxima [A] time = 0.36, size = 15, normalized size = 1.15

$$\frac{1}{ad\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] 1/(a*d*cos(d*x + c))
```

mupad [B] time = 13.60, size = 15, normalized size = 1.15

$$\frac{1}{a\,d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - a*sin(c + d*x)^2),x)
[Out] 1/(a*d*cos(c + d*x))
```

sympy [A] time = 2.48, size = 34, normalized size = 2.62

$$\begin{cases} -\frac{2}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin\left(c\right)}{-a\sin^2\left(c\right)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2),x)
[Out] Piecewise((-2/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)), (x*sin(c)/(-a*sin
(c)**2 + a), True))
```

3.39 $\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=29

$$\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+sec(d*x+c)/a/d
```

Rubi [A] time = 0.06, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 2622, 321, 207}

$$\frac{\sec(c+dx)}{ad}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + Sec[c + d*x]/(a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= \frac{\sec(c+dx)}{ad} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sec(c+dx)}{ad}$$

Mathematica [A] time = 0.04, size = 46, normalized size = 1.59

$$\frac{\frac{\sec(c+dx)}{d} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2),x]
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d)/a
```

fricas [A] time = 0.44, size = 55, normalized size = 1.90

$$-\frac{\cos(dx+c)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - \cos(dx+c)\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 2}{2\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(cos(d*x + c)*log(1/2*cos(d*x + c) + 1/2) - cos(d*x + c)*log(-1/2*cos(
d*x + c) + 1/2) - 2)/(a*d*cos(d*x + c))
```

giac [B] time = 0.15, size = 62, normalized size = 2.14

$$\frac{\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} + \frac{4}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + 4/(a*((cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.44, size = 51, normalized size = 1.76

$$\frac{\ln(\cos(dx+c)-1)}{2ad} + \frac{1}{da\cos(dx+c)} - \frac{\ln(1+\cos(dx+c))}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-a*sin(d*x+c)^2),x)
[Out] 1/2/a/d*ln(cos(d*x+c)-1)+1/d/a/cos(d*x+c)-1/2/a/d*ln(1+cos(d*x+c))
```

maxima [A] time = 0.35, size = 46, normalized size = 1.59

$$-\frac{\frac{\log(\cos(dx+c)+1)}{a} - \frac{\log(\cos(dx+c)-1)}{a} - \frac{2}{a\cos(dx+c)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(cos(d*x + c) + 1)/a - log(cos(d*x + c) - 1)/a - 2/(a*cos(d*x + c)
))/d
```

mupad [B] time = 0.08, size = 31, normalized size = 1.07

$$\frac{1}{a\,d\,\cos(c+d\,x)} - \frac{\operatorname{atanh}(\cos(c+d\,x))}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] 1/(a*d*cos(c + d*x)) - atanh(cos(c + d*x))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)/(sin(c + d*x)**2 - 1), x)/a
```

3.40 $\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=58

$$\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

```
[Out] -3/2*arctanh(cos(d*x+c))/a/d+3/2*sec(d*x+c)/a/d-1/2*csc(d*x+c)^2*sec(d*x+c)
/a/d
```

Rubi [A] time = 0.10, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 321, 207}

$$\frac{3\sec(c+dx)}{2ad}-\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*ArcTanh[Cos[c + d*x]])/(2*a*d) + (3*Sec[c + d*x])/(2*a*d) - (Csc[c + d*
x]^2*Sec[c + d*x])/(2*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] && !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] && !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^3(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^3(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{ad}\\
&= -\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= \frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}+\frac{3\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2ad}\\
&= -\frac{3\tanh^{-1}(\cos(c+dx))}{2ad}+\frac{3\sec(c+dx)}{2ad}-\frac{\csc^2(c+dx)\sec(c+dx)}{2ad}
\end{aligned}$$

Mathematica [B] time = 0.26, size = 146, normalized size = 2.52

$$\frac{\csc^4(c+dx)\left(-6\cos(2(c+dx))+2\cos(3(c+dx))+3\cos(3(c+dx))\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)-3\cos(3(c+dx))\log\ldots\right.}{2ad\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\ldots\right.\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (Csc[c + d*x]^4*(2 - 6*Cos[2*(c + d*x)] + 2*Cos[3*(c + d*x)] + 3*Cos[3*(c +
 d*x)]*Log[Cos[(c + d*x)/2]] - 3*Cos[3*(c + d*x)]*Log[Sin[(c + d*x)/2]] + C
os[c + d*x]*(-2 - 3*Log[Cos[(c + d*x)/2]] + 3*Log[Sin[(c + d*x)/2]])))/(2*a
*d*(Csc[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2))
```

fricas [A] time = 0.43, size = 98, normalized size = 1.69

$$\frac{6\cos(dx+c)^2-3\left(\cos(dx+c)^3-\cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+3\left(\cos(dx+c)^3-\cos(dx+c)\right)\log\left(\ldots\right.}{4\left(ad\cos(dx+c)^3-ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(6*cos(d*x + c)^2 - 3*(cos(d*x + c)^3 - cos(d*x + c))*log(1/2*cos(d*x +
 c) + 1/2) + 3*(cos(d*x + c)^3 - cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2)
 - 4)/(a*d*cos(d*x + c)^3 - a*d*cos(d*x + c))
```

giac [B] time = 0.21, size = 149, normalized size = 2.57

$$\frac{\frac{6\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}+\frac{\frac{14(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+1}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(6*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a + (14*(cos(d*x +
 c) - 1)/(cos(d*x + c) + 1) - 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 +
```

```
1)/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + (cos(d*x + c) - 1)^2/(cos(d
*x + c) + 1)^2)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.51, size = 87, normalized size = 1.50

$$\frac{1}{4ad\left(\cos\left(dx+c\right)-1\right)}+\frac{3\ln\left(\cos\left(dx+c\right)-1\right)}{4ad}+\frac{1}{da\cos\left(dx+c\right)}+\frac{1}{4ad\left(1+\cos\left(dx+c\right)\right)}-\frac{3\ln\left(1+\cos\left(dx+c\right)\right)}{4ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x)
```

```
[Out] 1/4/a/d/(cos(d*x+c)-1)+3/4/a/d*ln(cos(d*x+c)-1)+1/d/a/cos(d*x+c)+1/4/a/d/(1
+cos(d*x+c))-3/4/a/d*ln(1+cos(d*x+c))
```

maxima [A] time = 0.36, size = 70, normalized size = 1.21

$$\frac{\frac{2\left(3\cos\left(dx+c\right)^{2}-2\right)}{a\cos\left(dx+c\right)^{3}-a\cos\left(dx+c\right)}-\frac{3\log\left(\cos\left(dx+c\right)+1\right)}{a}+\frac{3\log\left(\cos\left(dx+c\right)-1\right)}{a}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(3*cos(d*x + c)^2 - 2)/(a*cos(d*x + c)^3 - a*cos(d*x + c)) - 3*log(c
os(d*x + c) + 1)/a + 3*log(cos(d*x + c) - 1)/a)/d
```

mupad [B] time = 0.09, size = 55, normalized size = 0.95

$$-\frac{\frac{3\cos\left(c+d\,x\right)^{2}}{2}-1}{d\,\left(a\,\cos\left(c+d\,x\right)-a\cos\left(c+d\,x\right)^{3}\right)}-\frac{3\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] - ((3*cos(c + d*x)^2)/2 - 1)/(d*(a*cos(c + d*x) - a*cos(c + d*x)^3)) - (3*a
tanh(cos(c + d*x)))/(2*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^{3}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**3/(sin(c + d*x)**2 - 1), x)/a
```

3.41 $\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{15\sec(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}$$

```
[Out] -15/8*arctanh(cos(d*x+c))/a/d+15/8*sec(d*x+c)/a/d-5/8*csc(d*x+c)^2*sec(d*x+
c)/a/d-1/4*csc(d*x+c)^4*sec(d*x+c)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 321, 207}

$$\frac{15\sec(c+dx)}{8ad}-\frac{15\tanh^{-1}(\cos(c+dx))}{8ad}-\frac{\csc^4(c+dx)\sec(c+dx)}{4ad}-\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*ArcTanh[Cos[c + d*x]])/(8*a*d) + (15*Sec[c + d*x])/(8*a*d) - (5*Csc[c
+ d*x]^2*Sec[c + d*x])/(8*a*d) - (Csc[c + d*x]^4*Sec[c + d*x])/(4*a*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \csc^5(c+dx)\sec^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^3}\,dx, x, \sec(c+dx)\right)}{ad}$$
$$= -\frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{4ad}$$
$$= -\frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx, x, \text{s}\right.}{8ad}$$
$$= \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+dx)\sec(c+dx)}{4ad} + \frac{15\,\text{Subst}}{}$$
$$= -\frac{15\tanh^{-1}(\cos(c+dx))}{8ad} + \frac{15\sec(c+dx)}{8ad} - \frac{5\csc^2(c+dx)\sec(c+dx)}{8ad} - \frac{\csc^4(c+}{}$$

Mathematica [A] time = 4.26, size = 132, normalized size = 1.61

$$-\frac{\csc^4\left(\frac{1}{2}(c+dx)\right) + 14\csc^2\left(\frac{1}{2}(c+dx)\right) + \frac{\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-14\tan^2\left(\frac{1}{2}(c+dx)\right)+\cos(c+dx)\left(\sec^4\left(\frac{1}{2}(c+dx)\right)-8\left(-15\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\right.\right.\right.}{\tan^2\left(\frac{1}{2}(c+dx)\right)-1}}{64ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -1/64*(14*Csc[(c + d*x)/2]^2 + Csc[(c + d*x)/2]^4 + (Sec[(c + d*x)/2]^2*(78
 + Cos[c + d*x]*(-8*(8 + 15*Log[Cos[(c + d*x)/2]] - 15*Log[Sin[(c + d*x)/2]
]) + Sec[(c + d*x)/2]^4) - 14*Tan[(c + d*x)/2]^2))/(-1 + Tan[(c + d*x)/2]^2
))/(a*d)
```

fricas [A] time = 0.43, size = 135, normalized size = 1.65

$$\frac{30\cos(dx+c)^4 - 50\cos(dx+c)^2 - 15\left(\cos(dx+c)^5 - 2\cos(dx+c)^3 + \cos(dx+c)\right)\log\left(\frac{1}{2}\cos(dx+c) +\right.}{16\left(ad\cos(dx+c)^5 - 2ad\cos(dx+c)^3\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(30*cos(d*x + c)^4 - 50*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - 2*cos(d*
x + c)^3 + cos(d*x + c))*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 -
 2*cos(d*x + c)^3 + cos(d*x + c))*log(-1/2*cos(d*x + c) + 1/2) + 16)/(a*d*c
os(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))
```

giac [B] time = 0.17, size = 181, normalized size = 2.21

$$\frac{\frac{\left(\frac{16(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{90(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} - 1\right)(\cos(dx+c)+1)^2}{a(\cos(dx+c)-1)^2} + \frac{60\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a} - \frac{\frac{16a(\cos(dx+c)-1)}{\cos(dx+c)+1} - \frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2} + \frac{128}{a\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/64*((16*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 90*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 1)*(cos(d*x + c) + 1)^2/(a*(cos(d*x + c) - 1)^2) + 60
*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a - (16*a*(cos(d*x + c)
- 1)/(cos(d*x + c) + 1) - a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2
+ 128/(a*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)))/d
```

maple [A] time = 0.49, size = 123, normalized size = 1.50

$$-\frac{1}{16ad\left(\cos\left(dx+c\right)-1\right)^{2}}+\frac{7}{16ad\left(\cos\left(dx+c\right)-1\right)}+\frac{15\ln\left(\cos\left(dx+c\right)-1\right)}{16ad}+\frac{1}{da\cos\left(dx+c\right)}+\frac{1}{16ad\left(1+\cos\left(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x)
```

```
[Out] -1/16/a/d/(cos(d*x+c)-1)^2+7/16/a/d/(cos(d*x+c)-1)+15/16/a/d*ln(cos(d*x+c)-
1)+1/d/a/cos(d*x+c)+1/16/a/d/(1+cos(d*x+c))^2+7/16/a/d/(1+cos(d*x+c))-15/16
/a/d*ln(1+cos(d*x+c))
```

maxima [A] time = 0.34, size = 90, normalized size = 1.10

$$\frac{\frac{2\left(15\cos(dx+c)^{4}-25\cos(dx+c)^{2}+8\right)}{a\cos(dx+c)^{5}-2a\cos(dx+c)^{3}+a\cos(dx+c)}-\frac{15\log(\cos(dx+c)+1)}{a}+\frac{15\log(\cos(dx+c)-1)}{a}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/16*(2*(15*cos(d*x + c)^4 - 25*cos(d*x + c)^2 + 8)/(a*cos(d*x + c)^5 - 2*a
*cos(d*x + c)^3 + a*cos(d*x + c)) - 15*log(cos(d*x + c) + 1)/a + 15*log(cos
(d*x + c) - 1)/a)/d
```

mupad [B] time = 0.10, size = 74, normalized size = 0.90

$$\frac{\frac{15\cos(c+d\,x)^{4}}{8}-\frac{25\cos(c+d\,x)^{2}}{8}+1}{d\,\left(a\cos\left(c+d\,x\right)^{5}-2\,a\cos\left(c+d\,x\right)^{3}+a\,\cos\left(c+d\,x\right)\right)}-\frac{15\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{8\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] ((15*cos(c + d*x)^4)/8 - (25*cos(c + d*x)^2)/8 + 1)/(d*(a*cos(c + d*x) - 2*
a*cos(c + d*x)^3 + a*cos(c + d*x)^5)) - (15*atanh(cos(c + d*x)))/(8*a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\csc^{5}(c+dx)}{\sin^{2}(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**5/(sin(c + d*x)**2 - 1), x)/a
```

3.42 $\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=73

$$\frac{15\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{15x}{8a}$$

```
[Out] -15/8*x/a+15/8*tan(d*x+c)/a/d-5/8*sin(d*x+c)^2*tan(d*x+c)/a/d-1/4*sin(d*x+c
)^4*tan(d*x+c)/a/d
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 321, 203}

$$\frac{15\tan(c+dx)}{8ad}-\frac{\sin^4(c+dx)\tan(c+dx)}{4ad}-\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad}-\frac{15x}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-15*x)/(8*a) + (15*Tan[c + d*x])/(8*a*d) - (5*Sin[c + d*x]^2*Tan[c + d*x])
/(8*a*d) - (Sin[c + d*x]^4*Tan[c + d*x])/(4*a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \sin^4(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{4ad}$$
$$= -\frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} + \frac{15\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(\right.}{8ad}$$
$$= \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad} - \frac{15\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{15x}{8a} + \frac{15\tan(c+dx)}{8ad} - \frac{5\sin^2(c+dx)\tan(c+dx)}{8ad} - \frac{\sin^4(c+dx)\tan(c+dx)}{4ad}$$

Mathematica [A] time = 0.19, size = 44, normalized size = 0.60

$$-\frac{-16\sin(2(c+dx)) + \sin(4(c+dx)) - 32\tan(c+dx) + 60c + 60dx}{32ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -1/32*(60*c + 60*d*x - 16*Sin[2*(c + d*x)] + Sin[4*(c + d*x)] - 32*Tan[c +
d*x])/(a*d)
```

fricas [A] time = 0.43, size = 56, normalized size = 0.77

$$-\frac{15\,dx\cos(dx+c) + \left(2\,\cos(dx+c)^4 - 9\,\cos(dx+c)^2 - 8\right)\sin(dx+c)}{8\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/8*(15*d*x*cos(d*x + c) + (2*cos(d*x + c)^4 - 9*cos(d*x + c)^2 - 8)*sin(d
*x + c))/(a*d*cos(d*x + c))
```

giac [A] time = 0.15, size = 63, normalized size = 0.86

$$-\frac{\frac{15\,(dx+c)}{a} - \frac{8\,\tan(dx+c)}{a} - \frac{9\,\tan(dx+c)^3 + 7\,\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2 a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(15*(d*x + c)/a - 8*tan(d*x + c)/a - (9*tan(d*x + c)^3 + 7*tan(d*x + c
))/((tan(d*x + c)^2 + 1)^2*a))/d
```

maple [A] time = 0.32, size = 84, normalized size = 1.15

$$\frac{\tan(dx+c)}{ad} + \frac{9\left(\tan^3(dx+c)\right)}{8ad\left(\tan^2(dx+c)+1\right)^2} + \frac{7\tan(dx+c)}{8ad\left(\tan^2(dx+c)+1\right)^2} - \frac{15\arctan(\tan(dx+c))}{8ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x)
```

```
[Out] tan(d*x+c)/a/d+9/8/a/d/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3+7/8/a/d/(tan(d*x+c)^
2+1)^2*tan(d*x+c)-15/8/a/d*arctan(tan(d*x+c))
```

maxima [A] time = 0.43, size = 72, normalized size = 0.99

$$\frac{\frac{9\,\tan(dx+c)^3+7\,\tan(dx+c)}{a\tan(dx+c)^4+2\,a\tan(dx+c)^2+a}-\frac{15\,(dx+c)}{a}+\frac{8\,\tan(dx+c)}{a}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((9*tan(d*x + c)^3 + 7*tan(d*x + c))/(a*tan(d*x + c)^4 + 2*a*tan(d*x +
c)^2 + a) - 15*(d*x + c)/a + 8*tan(d*x + c)/a)/d
```

mupad [B] time = 13.72, size = 68, normalized size = 0.93

$$\frac{\tan(c+d\,x)}{a\,d}-\frac{15\,x}{8\,a}+\frac{\frac{9\tan(c+d\,x)^3}{8}+\frac{7\tan(c+d\,x)}{8}}{d\,\left(a\tan(c+d\,x)^4+2\,a\tan(c+d\,x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(a*d) - (15*x)/(8*a) + ((7*tan(c + d*x))/8 + (9*tan(c + d*x)^3
)/8)/(d*(a + 2*a*tan(c + d*x)^2 + a*tan(c + d*x)^4))
```

sympy [A] time = 28.49, size = 1161, normalized size = 15.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-15*d*x*tan(c/2 + d*x/2)**10/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*
d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2
)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 45*d*x*tan(c/2 + d*x/2)**8/(8*
a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*
x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d)
- 30*d*x*tan(c/2 + d*x/2)**6/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 +
 d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a
*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 30*d*x*tan(c/2 + d*x/2)**4/(8*a*d*tan(c/2
 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 1
6*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) + 45*d*x*ta
n(c/2 + d*x/2)**2/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8
+ 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2
+ d*x/2)**2 - 8*a*d) + 15*d*x/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2
+ d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*
a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*tan(c/2 + d*x/2)**9/(8*a*d*tan(c/2 +
d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a
*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 +
 d*x/2)**7/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*
d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2
)**2 - 8*a*d) - 36*tan(c/2 + d*x/2)**5/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d
*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)
**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 80*tan(c/2 + d*x/2)**3/(8*a*d*t
```

```
an(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 + 16*a*d*tan(c/2 + d*x/2)*
*6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 + d*x/2)**2 - 8*a*d) - 30*
tan(c/2 + d*x/2)/(8*a*d*tan(c/2 + d*x/2)**10 + 24*a*d*tan(c/2 + d*x/2)**8 +
 16*a*d*tan(c/2 + d*x/2)**6 - 16*a*d*tan(c/2 + d*x/2)**4 - 24*a*d*tan(c/2 +
 d*x/2)**2 - 8*a*d), Ne(d, 0)), (x*sin(c)**6/(-a*sin(c)**2 + a), True))
```

3.43 $\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=49

$$\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3x}{2a}$$

```
[Out] -3/2*x/a+3/2*tan(d*x+c)/a/d-1/2*sin(d*x+c)^2*tan(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 49, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 321, 203}

$$\frac{3\tan(c+dx)}{2ad}-\frac{\sin^2(c+dx)\tan(c+dx)}{2ad}-\frac{3x}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*x)/(2*a) + (3*Tan[c + d*x])/(2*a*d) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
a*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a-a\sin^2(c+dx)}\,dx = \frac{\int \sin^2(c+dx)\tan^2(c+dx)\,dx}{a}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\sin^2(c+dx)\tan(c+dx)}{2ad} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2ad}$$
$$= \frac{3\tan(c+dx)}{2ad} - \frac{\sin^2(c+dx)\tan(c+dx)}{2ad} - \frac{3\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2ad}$$
$$= -\frac{3x}{2a} + \frac{3\tan(c+dx)}{2ad} - \frac{\sin^2(c+dx)\tan(c+dx)}{2ad}$$

Mathematica [A] time = 0.13, size = 34, normalized size = 0.69

$$\frac{-6(c+dx)+\sin(2(c+dx))+4\tan(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
[Out] (-6*(c + d*x) + Sin[2*(c + d*x)] + 4*Tan[c + d*x])/(4*a*d)
```

fricas [A] time = 0.42, size = 45, normalized size = 0.92

$$-\frac{3\,dx\cos(dx+c)-\left(\cos(dx+c)^2+2\right)\sin(dx+c)}{2\,ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(3*d*x*cos(d*x + c) - (cos(d*x + c)^2 + 2)*sin(d*x + c))/(a*d*cos(d*x
+ c))
```

giac [A] time = 0.14, size = 50, normalized size = 1.02

$$-\frac{\frac{3\,(dx+c)}{a} - \frac{2\,\tan(dx+c)}{a} - \frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] -1/2*(3*(d*x + c)/a - 2*tan(d*x + c)/a - tan(d*x + c)/((tan(d*x + c)^2 + 1)
*a))/d
```

maple [A] time = 0.32, size = 56, normalized size = 1.14

$$\frac{\tan(dx+c)}{ad} + \frac{\tan(dx+c)}{2ad\left(\tan^2(dx+c)+1\right)} - \frac{3\arctan(\tan(dx+c))}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x)
```

```
[Out] tan(d*x+c)/a/d+1/2/a/d*tan(d*x+c)/(tan(d*x+c)^2+1)-3/2/a/d*arctan(tan(d*x+c
))
```

maxima [A] time = 0.44, size = 49, normalized size = 1.00

$$-\frac{\frac{3(dx+c)}{a} - \frac{\tan(dx+c)}{a\tan(dx+c)^2+a} - \frac{2\tan(dx+c)}{a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*(d*x + c)/a - tan(d*x + c)/(a*tan(d*x + c)^2 + a) - 2*tan(d*x + c)/
a)/d
```

mupad [B] time = 13.53, size = 45, normalized size = 0.92

$$\frac{\tan(c+d\,x)}{2\,d\,\left(a\tan(c+d\,x)^2+a\right)} - \frac{3\,x}{2\,a} + \frac{\tan(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(2*d*(a + a*tan(c + d*x)^2)) - (3*x)/(2*a) + tan(c + d*x)/(a*d
)
```

sympy [A] time = 12.75, size = 502, normalized size = 10.24

$$\begin{cases} -\frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad} - \frac{3dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+2ad\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-2ad} + \frac{}{2ad\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right. } \\ \frac{x\sin^4(c)}{-a\sin^2(c)+a} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-3*d*x*tan(c/2 + d*x/2)**6/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*ta
n(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 3*d*x*tan(c/2 + d*
x/2)**4/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(
c/2 + d*x/2)**2 - 2*a*d) + 3*d*x*tan(c/2 + d*x/2)**2/(2*a*d*tan(c/2 + d*x/2
)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) + 3*d
*x/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 +
 d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)**5/(2*a*d*tan(c/2 + d*x/2)**6 + 2*
a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d) - 4*tan(c/2 +
d*x/2)**3/(2*a*d*tan(c/2 + d*x/2)**6 + 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*ta
n(c/2 + d*x/2)**2 - 2*a*d) - 6*tan(c/2 + d*x/2)/(2*a*d*tan(c/2 + d*x/2)**6
+ 2*a*d*tan(c/2 + d*x/2)**4 - 2*a*d*tan(c/2 + d*x/2)**2 - 2*a*d), Ne(d, 0))
, (x*sin(c)**4/(-a*sin(c)**2 + a), True))
```

3.44 $\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=20

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

```
[Out] -x/a+tan(d*x+c)/a/d
```

Rubi [A] time = 0.06, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3171, 3175, 3767, 8}

$$\frac{\tan(c+dx)}{ad}-\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(x/a) + Tan[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{a-a\sin^2(c+dx)}\,dx &= -\frac{x}{a}+\int \frac{1}{a-a\sin^2(c+dx)}\,dx\\
&= -\frac{x}{a}+\frac{\int \sec^2(c+dx)\,dx}{a}\\
&= -\frac{x}{a}-\frac{\text{Subst}\left(\int 1\,dx,x,-\tan(c+dx)\right)}{ad}\\
&= -\frac{x}{a}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 27, normalized size = 1.35

$$\frac{\frac{\tan(c+dx)}{d}-\frac{\tan^{-1}(\tan(c+dx))}{d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
[Out] (-(ArcTan[Tan[c + d*x]]/d) + Tan[c + d*x]/d)/a
```

fricas [A] time = 0.43, size = 34, normalized size = 1.70

$$-\frac{dx\cos(dx+c)-\sin(dx+c)}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -(d*x*cos(d*x + c) - sin(d*x + c))/(a*d*cos(d*x + c))
```

giac [A] time = 0.15, size = 26, normalized size = 1.30

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

maple [A] time = 0.23, size = 30, normalized size = 1.50

$$\frac{\tan(dx+c)}{ad}-\frac{\arctan(\tan(dx+c))}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x)
[Out] tan(d*x+c)/a/d-1/a/d*arctan(tan(d*x+c))
```

maxima [A] time = 0.45, size = 26, normalized size = 1.30

$$-\frac{\frac{dx+c}{a}-\frac{\tan(dx+c)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] -((d*x + c)/a - tan(d*x + c)/a)/d
```

mupad [B] time = 13.74, size = 20, normalized size = 1.00

$$\frac{\tan(c+d\,x)}{a\,d}-\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2),x)
[Out] tan(c + d*x)/(a*d) - x/a
```

sympy [**A**] time = 4.57, size = 100, normalized size = 5.00

$$\begin{cases} -\frac{dx\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}+\frac{dx}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad}-\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x\sin^2(c)}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-d*x*tan(c/2 + d*x/2)**2/(a*d*tan(c/2 + d*x/2)**2 - a*d) + d*x/(
a*d*tan(c/2 + d*x/2)**2 - a*d) - 2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**
2 - a*d), Ne(d, 0)), (x*sin(c)**2/(-a*sin(c)**2 + a), True))
```

3.45 $\int \frac{1}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=13

$$\frac{\tan(c+dx)}{ad}$$

```
[Out] tan(d*x+c)/a/d
```

Rubi [A] time = 0.02, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3175, 3767, 8}

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] Tan[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \sec^2(c+dx)\,dx}{a}\\ &= -\frac{\text{Subst}(\int 1\,dx, x, -\tan(c+dx))}{ad}\\ &= \frac{\tan(c+dx)}{ad}\end{aligned}$$

Mathematica [A] time = 0.01, size = 13, normalized size = 1.00

$$\frac{\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-1),x]
```

```
[Out] Tan[c + d*x]/(a*d)
```

fricas [A] time = 0.40, size = 21, normalized size = 1.62

$$\frac{\sin(dx+c)}{ad\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] sin(d*x + c)/(a*d*cos(d*x + c))
```

giac [A] time = 0.14, size = 13, normalized size = 1.00

$$\frac{\tan(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] tan(d*x + c)/(a*d)
```

maple [A] time = 0.32, size = 14, normalized size = 1.08

$$\frac{\tan(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(d*x+c)^2),x)
[Out] tan(d*x+c)/a/d
```

maxima [A] time = 0.36, size = 13, normalized size = 1.00

$$\frac{\tan(dx+c)}{ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] tan(d*x + c)/(a*d)
```

mupad [B] time = 13.59, size = 13, normalized size = 1.00

$$\frac{\tan(c+d\,x)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(c + d*x)^2),x)
[Out] tan(c + d*x)/(a*d)
```

sympy [A] time = 1.89, size = 41, normalized size = 3.15

$$\begin{cases} -\frac{2\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{ad\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-ad} & \text{for } d\neq 0 \\ \frac{x}{-a\sin^2(c)+a} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2),x)
[Out] Piecewise((-2*tan(c/2 + d*x/2)/(a*d*tan(c/2 + d*x/2)**2 - a*d), Ne(d, 0)),
(x/(-a*sin(c)**2 + a), True))
```

3.46 $\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=28

$$\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d+tan(d*x+c)/a/d
```

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 14}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] -(Cot[c + d*x]/(a*d)) + Tan[c + d*x]/(a*d)
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] &&  !LinearQ[u, x] &&  !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^2(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2}\,dx,x,\tan(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^2}\right)\,dx,x,\tan(c+dx)\right)}{ad}\\
&= -\frac{\cot(c+dx)}{ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 16, normalized size = 0.57

$$-\frac{2\cot(2(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2),x]
[Out] (-2*Cot[2*(c + d*x)])/(a*d)
```

fricas [A] time = 0.40, size = 36, normalized size = 1.29

$$-\frac{2 \cos\left(dx+c\right)^{2}-1}{ad \cos\left(dx+c\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -(2*cos(d*x + c)^2 - 1)/(a*d*cos(d*x + c)*sin(d*x + c))
```

giac [A] time = 0.16, size = 19, normalized size = 0.68

$$-\frac{2}{ad \tan\left(2\,dx+2\,c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] -2/(a*d*tan(2*d*x + 2*c))
```

maple [A] time = 0.46, size = 25, normalized size = 0.89

$$\frac{\tan\left(dx+c\right)-\frac{1}{\tan(dx+c)}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x)
[Out] 1/d/a*(tan(d*x+c)-1/tan(d*x+c))
```

maxima [A] time = 0.35, size = 28, normalized size = 1.00

$$\frac{\frac{\tan(dx+c)}{a}-\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] (tan(d*x + c)/a - 1/(a*tan(d*x + c)))/d
```

mupad [B] time = 13.50, size = 17, normalized size = 0.61

$$-\frac{2\cot\left(2\,c+2\,d\,x\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)),x)
[Out] -(2*cot(2*c + 2*d*x))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^{2}\left(c+dx\right)}{\sin^{2}\left(c+dx\right)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2),x)
[Out] -Integral(csc(c + d*x)**2/(sin(c + d*x)**2 - 1), x)/a
```

3.47 $\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{2\cot(c+dx)}{ad}$$

```
[Out] -2*cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+tan(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{2\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(3*a*d) + Tan[c + d*x]/(a*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^4(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^4}\,dx, x, \tan(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}+\frac{2}{x^2}\right)dx, x, \tan(c+dx)\right)}{ad}\\
&= -\frac{2\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.07

$$\frac{\frac{\tan(c+dx)}{d}-\frac{5\cot(c+dx)}{3d}-\frac{\cot(c+dx)\csc^2(c+dx)}{3d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2),x]
[Out] ((-5*Cot[c + d*x])/(3*d) - (Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) + Tan[c + d*
x]/d)/a
```

fricas [A] time = 0.42, size = 56, normalized size = 1.22

$$-\frac{8\cos(dx+c)^4-12\cos(dx+c)^2+3}{3\left(ad\cos(dx+c)^3-ad\cos(dx+c)\right)\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -1/3*(8*cos(d*x + c)^4 - 12*cos(d*x + c)^2 + 3)/((a*d*cos(d*x + c)^3 - a*d*
cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.15, size = 42, normalized size = 0.91

$$\frac{\frac{3\tan(dx+c)}{a}-\frac{6\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

maple [A] time = 0.47, size = 35, normalized size = 0.76

$$\frac{\tan(dx+c)-\frac{2}{\tan(dx+c)}-\frac{1}{3\tan(dx+c)^3}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x)
[Out] 1/d/a*(tan(d*x+c)-2/tan(d*x+c)-1/3/tan(d*x+c)^3)
```

maxima [A] time = 0.33, size = 42, normalized size = 0.91

$$\frac{\frac{3\tan(dx+c)}{a}-\frac{6\tan(dx+c)^2+1}{a\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*(3*tan(d*x + c)/a - (6*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

mupad [B] time = 13.73, size = 38, normalized size = 0.83

$$-\frac{-\tan(c+d\,x)^4+2\tan(c+d\,x)^2+\frac{1}{3}}{a\,d\tan(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] -(2*tan(c + d*x)^2 - tan(c + d*x)^4 + 1/3)/(a*d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^4(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2),x)
[Out] -Integral(csc(c + d*x)**4/(sin(c + d*x)**2 - 1), x)/a
```

3.48 $\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{ad}-\frac{3\cot(c+dx)}{ad}$$

```
[Out] -3*cot(d*x+c)/a/d-cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+tan(d*x+c)/a/d
```

Rubi [A] time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}-\frac{\cot^3(c+dx)}{ad}-\frac{3\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] (-3*Cot[c + d*x])/(a*d) - Cot[c + d*x]^3/(a*d) - Cot[c + d*x]^5/(5*a*d) + T
an[c + d*x]/(a*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^6(c+dx)}{a-a\sin^2(c+dx)}\,dx &= \frac{\int \csc^6(c+dx)\sec^2(c+dx)\,dx}{a}\\
&= \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6}\,dx, x, \tan(c+dx)\right)}{ad}\\
&= \frac{\text{Subst}\left(\int \left(1+\frac{1}{x^6}+\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x, \tan(c+dx)\right)}{ad}\\
&= -\frac{3\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\tan(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 70, normalized size = 1.13

$$\frac{\frac{\tan(c+dx)}{d}-\frac{11\cot(c+dx)}{5d}-\frac{\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{3\cot(c+dx)\csc^2(c+dx)}{5d}}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - a*Sin[c + d*x]^2),x]
```

```
[Out] ((-11*Cot[c + d*x])/(5*d) - (3*Cot[c + d*x]*Csc[c + d*x]^2)/(5*d) - (Cot[c
+ d*x]*Csc[c + d*x]^4)/(5*d) + Tan[c + d*x]/d)/a
```

fricas [A] time = 0.40, size = 77, normalized size = 1.24

$$-\frac{16 \cos\left(dx+c\right)^{6}-40 \cos\left(dx+c\right)^{4}+30 \cos\left(dx+c\right)^{2}-5}{5 \left(ad \cos\left(dx+c\right)^{5}-2 \, ad \cos\left(dx+c\right)^{3}+ad \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/5*(16*cos(d*x + c)^6 - 40*cos(d*x + c)^4 + 30*cos(d*x + c)^2 - 5)/((a*d*
cos(d*x + c)^5 - 2*a*d*cos(d*x + c)^3 + a*d*cos(d*x + c))*sin(d*x + c))
```

giac [A] time = 0.17, size = 52, normalized size = 0.84

$$\frac{\frac{5 \tan\left(dx+c\right)}{a}-\frac{15 \tan\left(dx+c\right)^{4}+5 \tan\left(dx+c\right)^{2}+1}{a \tan\left(dx+c\right)^{5}}}{5 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

maple [A] time = 0.51, size = 45, normalized size = 0.73

$$\frac{\tan\left(dx+c\right)-\frac{3}{\tan\left(dx+c\right)}-\frac{1}{5 \tan\left(dx+c\right)^{5}}-\frac{1}{\tan\left(dx+c\right)^{3}}}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x)
```

```
[Out] 1/d/a*(tan(d*x+c)-3/tan(d*x+c)-1/5/tan(d*x+c)^5-1/tan(d*x+c)^3)
```

maxima [A] time = 0.33, size = 52, normalized size = 0.84

$$\frac{\frac{5 \tan\left(dx+c\right)}{a}-\frac{15 \tan\left(dx+c\right)^{4}+5 \tan\left(dx+c\right)^{2}+1}{a \tan\left(dx+c\right)^{5}}}{5 \, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-a*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/5*(5*tan(d*x + c)/a - (15*tan(d*x + c)^4 + 5*tan(d*x + c)^2 + 1)/(a*tan(d
*x + c)^5))/d
```

mupad [B] time = 13.96, size = 50, normalized size = 0.81

$$\frac{\tan\left(c+d\,x\right)}{a\,d}-\frac{3\,\tan\left(c+d\,x\right)^4+\tan\left(c+d\,x\right)^2+\frac{1}{5}}{a\,d\,\tan\left(c+d\,x\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a - a*sin(c + d*x)^2)),x)
```

```
[Out] tan(c + d*x)/(a*d) - (tan(c + d*x)^2 + 3*tan(c + d*x)^4 + 1/5)/(a*d*tan(c +
 d*x)^5)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\csc^6(c+dx)}{\sin^2(c+dx)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-a*sin(d*x+c)**2),x)
```

```
[Out] -Integral(csc(c + d*x)**6/(sin(c + d*x)**2 - 1), x)/a
```

3.49 $\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=65

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}$$

```
[Out] -3*cos(d*x+c)/a^2/d+1/3*cos(d*x+c)^3/a^2/d-3*sec(d*x+c)/a^2/d+1/3*sec(d*x+c
)^3/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cos[c + d*x])/(a^2*d) + Cos[c + d*x]^3/(3*a^2*d) - (3*Sec[c + d*x])/(a^
2*d) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^7(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sin^3(c+dx)\tan^4(c+dx)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{x^4}\,dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}-\frac{3}{x^2}-x^2\right)dx,x,\cos(c+dx)\right)}{a^2d}\\ &= -\frac{3\cos(c+dx)}{a^2d}+\frac{\cos^3(c+dx)}{3a^2d}-\frac{3\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.05, size = 59, normalized size = 0.91

$$\frac{-\frac{11\cos(c+dx)}{4d}+\frac{\cos(3(c+dx))}{12d}+\frac{\sec^3(c+dx)}{3d}-\frac{3\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-11*Cos[c + d*x])/(4*d) + Cos[3*(c + d*x)]/(12*d) - (3*Sec[c + d*x])/d +
Sec[c + d*x]^3/(3*d))/a^2
```

fricas [A] time = 0.43, size = 46, normalized size = 0.71

$$\frac{\cos(dx+c)^6-9\cos(dx+c)^4-9\cos(dx+c)^2+1}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(d*x + c)^6 - 9*cos(d*x + c)^4 - 9*cos(d*x + c)^2 + 1)/(a^2*d*cos(d
*x + c)^3)
```

giac [A] time = 0.18, size = 57, normalized size = 0.88

$$-\frac{32\left(\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)}{3a^2d\left(\frac{(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-1\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -32/3*(3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)/(a^2*d*((cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2 - 1)^3)
```

maple [A] time = 0.30, size = 47, normalized size = 0.72

$$\frac{\frac{\left(\cos^3(dx+c)\right)}{3}-3\cos(dx+c)-\frac{3}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*(1/3*cos(d*x+c)^3-3*cos(d*x+c)-3/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

maxima [A] time = 0.34, size = 52, normalized size = 0.80

$$\frac{\frac{\cos(dx+c)^3-9\cos(dx+c)}{a^2}-\frac{9\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((cos(d*x + c)^3 - 9*cos(d*x + c))/a^2 - (9*cos(d*x + c)^2 - 1)/(a^2*co
s(d*x + c)^3))/d
```

mupad [B] time = 13.64, size = 48, normalized size = 0.74

$$-\frac{-\cos\left(c+d\,x\right)^6+9\cos\left(c+d\,x\right)^4+9\cos\left(c+d\,x\right)^2-1}{3\,a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] -(9*cos(c + d*x)^2 + 9*cos(c + d*x)^4 - cos(c + d*x)^6 - 1)/(3*a^2*d*cos(c
+ d*x)^3)
```

sympy [A] time = 106.84, size = 156, normalized size = 2.40

$$\begin{cases} -\frac{96\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d}+\frac{32}{3a^2d\tan^{12}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } \\ \frac{x\sin^7(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{othe} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-96*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**12 - 9*a**2*
d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(c/2 + d*x/2)**4 - 3*a**2*d) + 32/(3*a*
*2*d*tan(c/2 + d*x/2)**12 - 9*a**2*d*tan(c/2 + d*x/2)**8 + 9*a**2*d*tan(c/2
 + d*x/2)**4 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**7/(-a*sin(c)**2 + a)**2, Tr
ue))
```

3.50 $\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$-\frac{\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}$$

```
[Out] -cos(d*x+c)/a^2/d-2*sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2590, 270}

$$-\frac{\cos(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}-\frac{2\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Cos[c + d*x]/(a^2*d)) - (2*Sec[c + d*x])/(a^2*d) + Sec[c + d*x]^3/(3*a^2*
d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^5(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sin(c+dx)\tan^4(c+dx)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{x^4}\,dx, x, \cos(c+dx)\right)}{a^2d}\\
&= -\frac{\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, \cos(c+dx)\right)}{a^2d}\\
&= -\frac{\cos(c+dx)}{a^2d}-\frac{2\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 0.89

$$\frac{-\frac{\cos(c+dx)}{d}+\frac{\sec^3(c+dx)}{3d}-\frac{2\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - a*Sin[c + d*x]^2)^2,x]
[Out] (-(Cos[c + d*x]/d) - (2*Sec[c + d*x])/d + Sec[c + d*x]^3/(3*d))/a^2
```

fricas [A] time = 0.43, size = 38, normalized size = 0.81

$$-\frac{3\cos(dx+c)^4+6\cos(dx+c)^2-1}{3a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/3*(3*cos(d*x + c)^4 + 6*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

giac [B] time = 0.16, size = 106, normalized size = 2.26

$$\frac{2\left(\frac{3}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)}-\frac{\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+5}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] 2/3*(3/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)) - (12*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 5
)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

maple [A] time = 0.28, size = 37, normalized size = 0.79

$$\frac{-\cos(dx+c)-\frac{2}{\cos(dx+c)}+\frac{1}{3\cos(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x)
[Out] 1/d/a^2*(-cos(d*x+c)-2/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

maxima [A] time = 0.34, size = 41, normalized size = 0.87

$$-\frac{\frac{3\cos(dx+c)}{a^2}+\frac{6\cos(dx+c)^2-1}{a^2\cos(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/3*(3*cos(d*x + c)/a^2 + (6*cos(d*x + c)^2 - 1)/(a^2*cos(d*x + c)^3))/d
```

mupad [B] time = 0.05, size = 36, normalized size = 0.77

$$-\frac{\cos(c+d\,x)^4+2\cos(c+d\,x)^2-\frac{1}{3}}{a^2\,d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] -(2*cos(c + d*x)^2 + cos(c + d*x)^4 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

sympy [A] time = 49.03, size = 156, normalized size = 3.32

$$\begin{cases} -\frac{32\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{16}{3a^2d\tan^8\left(\frac{c}{2}+\frac{dx}{2}\right)-6a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)+6a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \neq (\\ \frac{x\sin^5(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-32*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d
*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 16/(3*a**
2*d*tan(c/2 + d*x/2)**8 - 6*a**2*d*tan(c/2 + d*x/2)**6 + 6*a**2*d*tan(c/2 +
 d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**5/(-a*sin(c)**2 + a)**2, True
))
```

3.51 $\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

```
[Out] -sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.083, Rules used = {3175, 2606}

$$\frac{\sec^3(c+dx)}{3a^2d}-\frac{\sec(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Sec[c + d*x]/(a^2*d)) + Sec[c + d*x]^3/(3*a^2*d)
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec(c+dx)\tan^3(c+dx)\,dx}{a^2}\\ &= \frac{\text{Subst}\left(\int \left(-1+x^2\right)\,dx, x, \sec(c+dx)\right)}{a^2d}\\ &= -\frac{\sec(c+dx)}{a^2d}+\frac{\sec^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 31, normalized size = 0.94

$$\frac{\frac{\sec^3(c+dx)}{3d}-\frac{\sec(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Sec[c + d*x]/d) + Sec[c + d*x]^3/(3*d))/a^2
```

fricas [A] time = 0.42, size = 28, normalized size = 0.85

$$-\frac{3 \cos(dx+c)^2 - 1}{3 a^2 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.14, size = 28, normalized size = 0.85

$$-\frac{3 \cos(dx+c)^2 - 1}{3 a^2 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

maple [A] time = 0.24, size = 29, normalized size = 0.88

$$\frac{-\frac{1}{\cos(dx+c)} + \frac{1}{3\cos(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x)
[Out] 1/d/a^2*(-1/cos(d*x+c)+1/3/cos(d*x+c)^3)
```

maxima [A] time = 0.33, size = 28, normalized size = 0.85

$$-\frac{3 \cos(dx+c)^2 - 1}{3 a^2 d \cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] -1/3*(3*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3)
```

mupad [B] time = 13.56, size = 26, normalized size = 0.79

$$-\frac{\cos(c+d\,x)^2 - \frac{1}{3}}{a^2\,d\cos(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - a*sin(c + d*x)^2)^2,x)
[Out] -(cos(c + d*x)^2 - 1/3)/(a^2*d*cos(c + d*x)^3)
```

sympy [A] time = 21.00, size = 156, normalized size = 4.73

$$\begin{cases} -\frac{12\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \neq (\\ \frac{x\sin^3(c)}{\left(-a\sin^2(c)+a\right)^2} & \text{otherwi} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-12*tan(c/2 + d*x/2)**2/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d
*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4/(3*a**2
*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 +
d*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)**3/(-a*sin(c)**2 + a)**2, True)
)
```

3.52 $\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{\sec^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.136$, Rules used = {3175, 2606, 30}

$$\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^3(c+dx)\tan(c+dx)\,dx}{a^2}\\
&= \frac{\text{Subst}\left(\int x^2\,dx, x, \sec(c+dx)\right)}{a^2d}\\
&= \frac{\sec^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 18, normalized size = 1.00

$$\frac{\sec^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Sec[c + d*x]^3/(3*a^2*d)
```

fricas [A] time = 0.42, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.14, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

maple [A] time = 0.15, size = 17, normalized size = 0.94

$$\frac{1}{3d\,a^2\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x)
[Out] 1/3/d/a^2/cos(d*x+c)^3
```

maxima [A] time = 0.34, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2 d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/3/(a^2*d*cos(d*x + c)^3)
```

mupad [B] time = 13.59, size = 16, normalized size = 0.89

$$\frac{1}{3\,a^2\,d\cos\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - a*sin(c + d*x)^2)^2,x)
[Out] 1/(3*a^2*d*cos(c + d*x)^3)
```

sympy [A] time = 9.72, size = 156, normalized size = 8.67

$$\begin{cases} -\frac{6\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{2}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} & \text{for } d \\ \frac{x\sin\left(c\right)}{\left(-a\sin^2\left(c\right)+a\right)^2} & \text{other} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 2/(3*a**2*
d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d
*x/2)**2 - 3*a**2*d), Ne(d, 0)), (x*sin(c)/(-a*sin(c)**2 + a)**2, True))
```

3.53 $\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+sec(d*x+c)/a^2/d+1/3*sec(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 2622, 302, 207}

$$\frac{\sec^3(c+dx)}{3a^2d}+\frac{\sec(c+dx)}{a^2d}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + Sec[c + d*x]/(a^2*d) + Sec[c + d*x]^3/(3
*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \csc(c+dx)\sec^4(c+dx)\,dx}{a^2}$$
$$= \frac{\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d} + \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2 d} + \frac{\sec(c+dx)}{a^2 d} + \frac{\sec^3(c+dx)}{3a^2 d}$$

Mathematica [A] time = 0.04, size = 61, normalized size = 1.30

$$\frac{\frac{\sec^3(c+dx)}{3d} + \frac{\sec(c+dx)}{d} + \frac{\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} - \frac{\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Log[Cos[(c + d*x)/2]]/d) + Log[Sin[(c + d*x)/2]]/d + Sec[c + d*x]/d + Se
c[c + d*x]^3/(3*d))/a^2
```

fricas [A] time = 0.44, size = 70, normalized size = 1.49

$$-\frac{3\cos(dx+c)^3\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 3\cos(dx+c)^3\log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) - 6\cos(dx+c)^2 - 2}{6a^2 d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/6*(3*cos(d*x + c)^3*log(1/2*cos(d*x + c) + 1/2) - 3*cos(d*x + c)^3*log(-
1/2*cos(d*x + c) + 1/2) - 6*cos(d*x + c)^2 - 2)/(a^2*d*cos(d*x + c)^3)
```

giac [B] time = 0.18, size = 107, normalized size = 2.28

$$\frac{\frac{3\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{8\left(\frac{3(\cos(dx+c)-1)}{\cos(dx+c)+1} + \frac{3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2} + 2\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2 + 8*(3*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + 3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2 + 2)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

maple [A] time = 0.43, size = 67, normalized size = 1.43

$$\frac{\ln(\cos(dx+c)-1)}{2d\,a^2} + \frac{1}{3d\,a^2\cos(dx+c)^3} + \frac{1}{d\,a^2\cos(dx+c)} - \frac{\ln(1+\cos(dx+c))}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2*ln(cos(d*x+c)-1)+1/3/d/a^2/cos(d*x+c)^3+1/d/a^2/cos(d*x+c)-1/2/d/
a^2*ln(1+cos(d*x+c))
```

maxima [A] time = 0.33, size = 59, normalized size = 1.26

$$-\frac{\frac{3 \log(\cos(dx+c)+1)}{a^2} - \frac{3 \log(\cos(dx+c)-1)}{a^2} - \frac{2\left(3 \cos(dx+c)^2+1\right)}{a^2 \cos(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*log(cos(d*x + c) + 1)/a^2 - 3*log(cos(d*x + c) - 1)/a^2 - 2*(3*cos(
d*x + c)^2 + 1)/(a^2*cos(d*x + c)^3))/d
```

mupad [B] time = 0.09, size = 41, normalized size = 0.87

$$\frac{\cos\left(c+d\,x\right)^2+\frac{1}{3}}{a^2\,d\cos\left(c+d\,x\right)^3}-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] (cos(c + d*x)^2 + 1/3)/(a^2*d*cos(c + d*x)^3) - atanh(cos(c + d*x))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc\left(c+dx\right)}{\sin^4\left(c+dx\right)-2\sin^2\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

3.54 $\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{5\sec^3(c+dx)}{6a^2d}+\frac{5\sec(c+dx)}{2a^2d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

```
[Out] -5/2*arctanh(cos(d*x+c))/a^2/d+5/2*sec(d*x+c)/a^2/d+5/6*sec(d*x+c)^3/a^2/d-
1/2*csc(d*x+c)^2*sec(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2622, 288, 302, 207}

$$\frac{5\sec^3(c+dx)}{6a^2d}+\frac{5\sec(c+dx)}{2a^2d}-\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-5*ArcTanh[Cos[c + d*x]])/(2*a^2*d) + (5*Sec[c + d*x])/(2*a^2*d) + (5*Sec[
c + d*x]^3)/(6*a^2*d) - (Csc[c + d*x]^2*Sec[c + d*x]^3)/(2*a^2*d)
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2622

```
Int[csc[(e_.) + (f_.)*(x_)]^(n_.)*((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_), x_S
ymbol] :> Dist[1/(f*a^n), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n + 1)/2
), x], x, a*Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)
/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \csc^3(c+dx)\sec^4(c+dx)\,dx}{a^2}$$

$$= \frac{\text{Subst}\left(\int \frac{x^6}{\left(-1+x^2\right)^2}\,dx, x, \sec(c+dx)\right)}{a^2 d}$$

$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2 d} + \frac{5\,\text{Subst}\left(\int \frac{x^4}{-1+x^2}\,dx, x, \sec(c+dx)\right)}{2a^2 d}$$

$$= -\frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2 d} + \frac{5\,\text{Subst}\left(\int \left(1+x^2+\frac{1}{-1+x^2}\right)dx, x, \sec(c+dx)\right)}{2a^2 d}$$

$$= \frac{5\sec(c+dx)}{2a^2 d} + \frac{5\sec^3(c+dx)}{6a^2 d} - \frac{\csc^2(c+dx)\sec^3(c+dx)}{2a^2 d} + \frac{5\,\text{Subst}\left(\int \frac{1}{-1+x^2}\right.}{2a}$$

$$= -\frac{5\tanh^{-1}(\cos(c+dx))}{2a^2 d} + \frac{5\sec(c+dx)}{2a^2 d} + \frac{5\sec^3(c+dx)}{6a^2 d} - \frac{\csc^2(c+dx)\sec^3(c}{2a^2 d}$$

Mathematica [B] time = 0.44, size = 208, normalized size = 2.67

$$\frac{2\csc^8(c+dx)\left(-40\cos(2(c+dx))+13\cos(3(c+dx))-30\cos(4(c+dx))+13\cos(5(c+dx))+15\cos(3(c+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*Csc[c + d*x]^8*(22 - 40*Cos[2*(c + d*x)] + 13*Cos[3*(c + d*x)] - 30*Cos[
4*(c + d*x)] + 13*Cos[5*(c + d*x)] + 15*Cos[3*(c + d*x)]*Log[Cos[(c + d*x)/
2]] + 15*Cos[5*(c + d*x)]*Log[Cos[(c + d*x)/2]] - 15*Cos[3*(c + d*x)]*Log[S
in[(c + d*x)/2]] - 15*Cos[5*(c + d*x)]*Log[Sin[(c + d*x)/2]] + Cos[c + d*x]
*(-26 - 30*Log[Cos[(c + d*x)/2]] + 30*Log[Sin[(c + d*x)/2]])))/(3*a^2*d*(Cs
c[(c + d*x)/2]^2 - Sec[(c + d*x)/2]^2)^3)
```

fricas [A] time = 0.44, size = 118, normalized size = 1.51

$$\frac{30\cos(dx+c)^4-20\cos(dx+c)^2-15\left(\cos(dx+c)^5-\cos(dx+c)^3\right)\log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)+15\left(\cos(dx\right.}{12\left(a^2 d\cos(dx+c)^5-a^2 d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/12*(30*cos(d*x + c)^4 - 20*cos(d*x + c)^2 - 15*(cos(d*x + c)^5 - cos(d*x
+ c)^3)*log(1/2*cos(d*x + c) + 1/2) + 15*(cos(d*x + c)^5 - cos(d*x + c)^3)*
log(-1/2*cos(d*x + c) + 1/2) - 4)/(a^2*d*cos(d*x + c)^5 - a^2*d*cos(d*x + c
)^3)
```

giac [B] time = 0.17, size = 175, normalized size = 2.24

$$-\frac{\frac{3\left(\frac{10(\cos(dx+c)-1)}{\cos(dx+c)+1}-1\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)} - \frac{30\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2} + \frac{3(\cos(dx+c)-1)}{a^2(\cos(dx+c)+1)} - \frac{16\left(\frac{12(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{9(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+7\right)}{a^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/24*(3*(10*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)*(cos(d*x + c) + 1)/
(a^2*(cos(d*x + c) - 1)) - 30*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) +
 1))/a^2 + 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c) + 1)) - 16*(12*(cos(d*x
+ c) - 1)/(cos(d*x + c) + 1) + 9*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
+ 7)/(a^2*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)^3))/d
```

maple [A] time = 0.52, size = 104, normalized size = 1.33

$$\frac{1}{4d\,a^{2}\left(\cos\left(dx+c\right)-1\right)}+\frac{5\ln\left(\cos\left(dx+c\right)-1\right)}{4d\,a^{2}}+\frac{1}{3d\,a^{2}\cos\left(dx+c\right)^{3}}+\frac{2}{d\,a^{2}\cos\left(dx+c\right)}+\frac{1}{4d\,a^{2}\left(1+\cos\left(dx+c\right)\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d/a^2/(cos(d*x+c)-1)+5/4/d/a^2*ln(cos(d*x+c)-1)+1/3/d/a^2/cos(d*x+c)^3+
2/d/a^2/cos(d*x+c)+1/4/d/a^2/(1+cos(d*x+c))-5/4/d/a^2*ln(1+cos(d*x+c))
```

maxima [A] time = 0.34, size = 86, normalized size = 1.10

$$\frac{\frac{2\left(15\cos\left(dx+c\right)^{4}-10\cos\left(dx+c\right)^{2}-2\right)}{a^{2}\cos\left(dx+c\right)^{5}-a^{2}\cos\left(dx+c\right)^{3}}-\frac{15\log\left(\cos\left(dx+c\right)+1\right)}{a^{2}}+\frac{15\log\left(\cos\left(dx+c\right)-1\right)}{a^{2}}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(2*(15*cos(d*x + c)^4 - 10*cos(d*x + c)^2 - 2)/(a^2*cos(d*x + c)^5 - a
^2*cos(d*x + c)^3) - 15*log(cos(d*x + c) + 1)/a^2 + 15*log(cos(d*x + c) - 1
)/a^2)/d
```

mupad [B] time = 13.76, size = 70, normalized size = 0.90

$$\frac{-\frac{5\cos\left(c+d\,x\right)^{4}}{2}+\frac{5\cos\left(c+d\,x\right)^{2}}{3}+\frac{1}{3}}{d\,\left(a^{2}\cos\left(c+d\,x\right)^{3}-a^{2}\cos\left(c+d\,x\right)^{5}\right)}-\frac{5\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{2\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] ((5*cos(c + d*x)^2)/3 - (5*cos(c + d*x)^4)/2 + 1/3)/(d*(a^2*cos(c + d*x)^3
- a^2*cos(c + d*x)^5)) - (5*atanh(cos(c + d*x)))/(2*a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{\csc^{3}\left(c+dx\right)}{\sin^{4}\left(c+dx\right)-2\sin^{2}\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**3/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

3.55 $\int \frac{\sin^6(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=69

$$\frac{5\tan^3(c+dx)}{6a^2d}-\frac{5\tan(c+dx)}{2a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5x}{2a^2}$$

```
[Out] 5/2*x/a^2-5/2*tan(d*x+c)/a^2/d+5/6*tan(d*x+c)^3/a^2/d-1/2*sin(d*x+c)^2*tan(
d*x+c)^3/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3175, 2591, 288, 302, 203}

$$\frac{5\tan^3(c+dx)}{6a^2d}-\frac{5\tan(c+dx)}{2a^2d}-\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}+\frac{5x}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (5*x)/(2*a^2) - (5*Tan[c + d*x])/(2*a^2*d) + (5*Tan[c + d*x]^3)/(6*a^2*d) -
 (Sin[c + d*x]^2*Tan[c + d*x]^3)/(2*a^2*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 2591

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(b*ff)/f, Subst[In
t[(ff*x)^(m + n)/(b^2 + ff^2*x^2)^(m/2 + 1), x], x, (b*Tan[e + f*x])/ff], x
]] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx = \frac{\int \sin^2(c+dx)\tan^4(c+dx)\,dx}{a^2}$$
$$= \frac{\operatorname{Subst}\left(\int \frac{x^6}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\operatorname{Subst}\left(\int \frac{x^4}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2a^2d}$$
$$= -\frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\operatorname{Subst}\left(\int \left(-1+x^2+\frac{1}{1+x^2}\right)dx, x, \tan(c+dx)\right)}{2a^2d}$$
$$= -\frac{5\tan(c+dx)}{2a^2d} + \frac{5\tan^3(c+dx)}{6a^2d} - \frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d} + \frac{5\operatorname{Subst}\left(\int \frac{1}{1+x^2}\,dx,\right.}{2a^2c}$$
$$= \frac{5x}{2a^2} - \frac{5\tan(c+dx)}{2a^2d} + \frac{5\tan^3(c+dx)}{6a^2d} - \frac{\sin^2(c+dx)\tan^3(c+dx)}{2a^2d}$$

Mathematica [A] time = 0.22, size = 46, normalized size = 0.67

$$\frac{30(c+dx) - 3\sin(2(c+dx)) + 4\tan(c+dx)\left(\sec^2(c+dx)-7\right)}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (30*(c + d*x) - 3*Sin[2*(c + d*x)] + 4*(-7 + Sec[c + d*x]^2)*Tan[c + d*x])/
(12*a^2*d)
```

fricas [A] time = 0.43, size = 59, normalized size = 0.86

$$\frac{15\,dx\cos(dx+c)^3 - \left(3\cos(dx+c)^4 + 14\cos(dx+c)^2 - 2\right)\sin(dx+c)}{6\,a^2d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/6*(15*d*x*cos(d*x + c)^3 - (3*cos(d*x + c)^4 + 14*cos(d*x + c)^2 - 2)*sin
(d*x + c))/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.17, size = 68, normalized size = 0.99

$$\frac{\frac{15(dx+c)}{a^2} - \frac{3\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)a^2} + \frac{2\left(a^4\tan(dx+c)^3 - 6a^4\tan(dx+c)\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(15*(d*x + c)/a^2 - 3*tan(d*x + c)/((tan(d*x + c)^2 + 1)*a^2) + 2*(a^4*
tan(d*x + c)^3 - 6*a^4*tan(d*x + c))/a^6)/d
```

maple [A] time = 0.35, size = 73, normalized size = 1.06

$$\frac{\tan^3(dx+c)}{3a^2d} - \frac{2\tan(dx+c)}{a^2d} - \frac{\tan(dx+c)}{2d\,a^2\left(\tan^2(dx+c)+1\right)} + \frac{5\arctan(\tan(dx+c))}{2d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/3*tan(d*x+c)^3/a^2/d-2*tan(d*x+c)/a^2/d-1/2/d/a^2*tan(d*x+c)/(tan(d*x+c)^
2+1)+5/2/d/a^2*arctan(tan(d*x+c))
```

maxima [A] time = 0.45, size = 64, normalized size = 0.93

$$-\frac{\frac{3 \tan(dx+c)}{a^2 \tan(dx+c)^2+a^2} - \frac{2\left(\tan(dx+c)^3-6 \tan(dx+c)\right)}{a^2} - \frac{15\,(dx+c)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/6*(3*tan(d*x + c)/(a^2*tan(d*x + c)^2 + a^2) - 2*(tan(d*x + c)^3 - 6*tan
(d*x + c))/a^2 - 15*(d*x + c)/a^2)/d
```

mupad [B] time = 13.81, size = 66, normalized size = 0.96

$$\frac{5\,x}{2\,a^2}-\frac{\tan\left(c+d\,x\right)}{2\,d\,\left(a^2\,\tan\left(c+d\,x\right)^2+a^2\right)}-\frac{2\,\tan\left(c+d\,x\right)}{a^2\,d}+\frac{\tan\left(c+d\,x\right)^3}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] (5*x)/(2*a^2) - tan(c + d*x)/(2*d*(a^2 + a^2*tan(c + d*x)^2)) - (2*tan(c +
d*x))/(a^2*d) + tan(c + d*x)^3/(3*a^2*d)
```

sympy [A] time = 72.43, size = 1275, normalized size = 18.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((15*d*x*tan(c/2 + d*x/2)**10/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a
**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c
/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 15*d*x*tan(c/2
+ d*x/2)**8/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 -
 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*t
an(c/2 + d*x/2)**2 - 6*a**2*d) - 30*d*x*tan(c/2 + d*x/2)**6/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*d*x*tan(c/2 + d*x/2)**4/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*
tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d
*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 15*d*x*tan(c/2 + d*x/
2)**2/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a*
*2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2
 + d*x/2)**2 - 6*a**2*d) - 15*d*x/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d
*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 +
d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) + 30*tan(c/2 + d*x/2)*
*9/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*
d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 +
d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**7/(6*a**2*d*tan(c/2 + d*x/2)**
10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2
*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 44*tan(
c/2 + d*x/2)**5/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)*
```

```
*8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2
*d*tan(c/2 + d*x/2)**2 - 6*a**2*d) - 40*tan(c/2 + d*x/2)**3/(6*a**2*d*tan(c
/2 + d*x/2)**10 - 6*a**2*d*tan(c/2 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)
**6 + 12*a**2*d*tan(c/2 + d*x/2)**4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2
*d) + 30*tan(c/2 + d*x/2)/(6*a**2*d*tan(c/2 + d*x/2)**10 - 6*a**2*d*tan(c/2
 + d*x/2)**8 - 12*a**2*d*tan(c/2 + d*x/2)**6 + 12*a**2*d*tan(c/2 + d*x/2)**
4 + 6*a**2*d*tan(c/2 + d*x/2)**2 - 6*a**2*d), Ne(d, 0)), (x*sin(c)**6/(-a*s
in(c)**2 + a)**2, True))
```

3.56 $\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=38

$$\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{x}{a^2}$$

```
[Out] x/a^2-tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.06, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3473, 8}

$$\frac{\tan^3(c+dx)}{3a^2d}-\frac{\tan(c+dx)}{a^2d}+\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/a^2 - Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3473

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(b*(b*Tan[c + d
*x])^(n - 1))/(d*(n - 1)), x] - Dist[b^2, Int[(b*Tan[c + d*x])^(n - 2), x],
 x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \tan^4(c+dx)\,dx}{a^2}\\
&= \frac{\tan^3(c+dx)}{3a^2d}-\frac{\int \tan^2(c+dx)\,dx}{a^2}\\
&= -\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}+\frac{\int 1\,dx}{a^2}\\
&= \frac{x}{a^2}-\frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 42, normalized size = 1.11

$$\frac{\frac{\tan^{-1}(\tan(c+dx))}{d}+\frac{\tan^3(c+dx)}{3d}-\frac{\tan(c+dx)}{d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
[Out] (ArcTan[Tan[c + d*x]]/d - Tan[c + d*x]/d + Tan[c + d*x]^3/(3*d))/a^2
```

fricas [A] time = 0.42, size = 49, normalized size = 1.29

$$\frac{3\,dx\cos\left(dx+c\right)^{3}-\left(4\,\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
[Out] 1/3*(3*d*x*cos(d*x + c)^3 - (4*cos(d*x + c)^2 - 1)*sin(d*x + c))/(a^2*d*cos
(d*x + c)^3)
```

giac [A] time = 0.15, size = 44, normalized size = 1.16

$$\frac{\frac{3\,(dx+c)}{a^{2}}+\frac{a^{4}\tan(dx+c)^{3}-3\,a^{4}\tan(dx+c)}{a^{6}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/3*(3*(d*x + c)/a^2 + (a^4*tan(d*x + c)^3 - 3*a^4*tan(d*x + c))/a^6)/d
```

maple [A] time = 0.29, size = 46, normalized size = 1.21

$$\frac{\tan^{3}\left(dx+c\right)}{3a^{2}d}-\frac{\tan\left(dx+c\right)}{a^{2}d}+\frac{\arctan\left(\tan\left(dx+c\right)\right)}{d\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x)
[Out] 1/3*tan(d*x+c)^3/a^2/d-tan(d*x+c)/a^2/d+1/d/a^2*arctan(tan(d*x+c))
```

maxima [A] time = 0.43, size = 37, normalized size = 0.97

$$\frac{\frac{\tan(dx+c)^{3}-3\,\tan(dx+c)}{a^{2}}+\frac{3\,(dx+c)}{a^{2}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/3*((tan(d*x + c)^3 - 3*tan(d*x + c))/a^2 + 3*(d*x + c)/a^2)/d
```

mupad [B] time = 13.48, size = 31, normalized size = 0.82

$$\frac{x}{a^{2}}-\frac{\tan\left(c+d\,x\right)-\frac{\tan(c+d\,x)^{3}}{3}}{a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - a*sin(c + d*x)^2)^2,x)
[Out] x/a^2 - (tan(c + d*x) - tan(c + d*x)^3/3)/(a^2*d)
```

sympy [A] time = 29.68, size = 551, normalized size = 14.50

$$\begin{cases} \frac{3dx\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{9dx\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{}{3a^2d\tan^6} \\ \frac{x\sin^4(c)}{\left(-a\sin^2(c)+a\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((3*d*x*tan(c/2 + d*x/2)**6/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2
*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 9*d*x*t
an(c/2 + d*x/2)**4/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2
)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 9*d*x*tan(c/2 + d*x/2)**2
/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*ta
n(c/2 + d*x/2)**2 - 3*a**2*d) - 3*d*x/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**
2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 6*tan(
c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**
4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 20*tan(c/2 + d*x/2)**3/(3*a*
*2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2
+ d*x/2)**2 - 3*a**2*d) + 6*tan(c/2 + d*x/2)/(3*a**2*d*tan(c/2 + d*x/2)**6
- 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d),
Ne(d, 0)), (x*sin(c)**4/(-a*sin(c)**2 + a)**2, True))
```

3.57 $\int \frac{\sin^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^3(c+dx)}{3a^2d}$$

```
[Out] 1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3175, 2607, 30}

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2607

```
Int[sec[(e_.) + (f_.)*(x_)]^(m_)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(n_.), x_S
ymbol] :> Dist[1/f, Subst[Int[(b*x)^n*(1 + x^2)^(m/2 - 1), x], x, Tan[e + f
*x]], x] /; FreeQ[{b, e, f, n}, x] && IntegerQ[m/2] &&  !(IntegerQ[(n - 1)/
2] && LtQ[0, n, m - 1])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^2(c+dx)\tan^2(c+dx)\,dx}{a^2}\\ &= \frac{\text{Subst}\left(\int x^2\,dx, x, \tan(c+dx)\right)}{a^2d}\\ &= \frac{\tan^3(c+dx)}{3a^2d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 18, normalized size = 1.00

$$\frac{\tan^3(c+dx)}{3a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
[Out] Tan[c + d*x]^3/(3*a^2*d)
```

fricas [A] time = 0.40, size = 32, normalized size = 1.78

$$-\frac{\left(\cos\left(dx+c\right)^{2}-1\right)\sin\left(dx+c\right)}{3\,a^{2}d\cos\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
[Out] -1/3*(cos(d*x + c)^2 - 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.15, size = 16, normalized size = 0.89

$$\frac{\tan\left(dx+c\right)^{3}}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

maple [A] time = 0.20, size = 17, normalized size = 0.94

$$\frac{\tan^{3}\left(dx+c\right)}{3a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x)
[Out] 1/3*tan(d*x+c)^3/a^2/d
```

maxima [A] time = 0.33, size = 16, normalized size = 0.89

$$\frac{\tan\left(dx+c\right)^{3}}{3\,a^{2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
[Out] 1/3*tan(d*x + c)^3/(a^2*d)
```

mupad [B] time = 13.38, size = 16, normalized size = 0.89

$$\frac{\tan\left(c+d\,x\right)^{3}}{3\,a^{2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - a*sin(c + d*x)^2)^2,x)
[Out] tan(c + d*x)^3/(3*a^2*d)
```

sympy [A] time = 12.97, size = 94, normalized size = 5.22

$$\begin{cases} -\frac{8\tan^{3}\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^{2}d\tan^{6}\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^{2}d\tan^{4}\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^{2}d\tan^{2}\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^{2}d} & \text{for}\ d\neq 0\\ \frac{x\sin^{2}\left(c\right)}{\left(-a\sin^{2}\left(c\right)+a\right)^{2}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-8*tan(c/2 + d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d), Ne(d, 0)),
(x*sin(c)**2/(-a*sin(c)**2 + a)**2, True))
```

3.58 $\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=32

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}$$

```
[Out] tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.133, Rules used = {3175, 3767}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] Tan[c + d*x]/(a^2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \sec^4(c+dx)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx, x, -\tan(c+dx)\right)}{a^2d}\\
&= \frac{\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 26, normalized size = 0.81

$$\frac{\frac{1}{3}\tan^3(c+dx)+\tan(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] (Tan[c + d*x] + Tan[c + d*x]^3/3)/(a^2*d)
```

fricas [A] time = 0.41, size = 34, normalized size = 1.06

$$\frac{\left(2\cos\left(dx+c\right)^2+1\right)\sin\left(dx+c\right)}{3\,a^2d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(2*cos(d*x + c)^2 + 1)*sin(d*x + c)/(a^2*d*cos(d*x + c)^3)
```

giac [A] time = 0.13, size = 25, normalized size = 0.78

$$\frac{\tan(dx+c)^3 + 3\tan(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

maple [A] time = 0.30, size = 25, normalized size = 0.78

$$\frac{\frac{\left(\tan^3(dx+c)\right)}{3} + \tan(dx+c)}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+tan(d*x+c))
```

maxima [A] time = 0.35, size = 25, normalized size = 0.78

$$\frac{\tan(dx+c)^3 + 3\tan(dx+c)}{3a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*(tan(d*x + c)^3 + 3*tan(d*x + c))/(a^2*d)
```

mupad [B] time = 13.45, size = 24, normalized size = 0.75

$$\frac{\tan(c+d\,x)\,\left(\tan(c+d\,x)^2+3\right)}{3\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(c + d*x)^2)^2,x)
```

```
[Out] (tan(c + d*x)*(tan(c + d*x)^2 + 3))/(3*a^2*d)
```

sympy [A] time = 5.17, size = 238, normalized size = 7.44

$$\begin{cases} -\frac{6\tan^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} + \frac{4\tan^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{3a^2d\tan^6\left(\frac{c}{2}+\frac{dx}{2}\right)-9a^2d\tan^4\left(\frac{c}{2}+\frac{dx}{2}\right)+9a^2d\tan^2\left(\frac{c}{2}+\frac{dx}{2}\right)-3a^2d} - \frac{}{3a^2d\tan^6(} \\ \frac{x}{\left(-a\sin^2(c)+a\right)^2} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Piecewise((-6*tan(c/2 + d*x/2)**5/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*
tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) + 4*tan(c/2
+ d*x/2)**3/(3*a**2*d*tan(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 +
9*a**2*d*tan(c/2 + d*x/2)**2 - 3*a**2*d) - 6*tan(c/2 + d*x/2)/(3*a**2*d*tan
(c/2 + d*x/2)**6 - 9*a**2*d*tan(c/2 + d*x/2)**4 + 9*a**2*d*tan(c/2 + d*x/2)
**2 - 3*a**2*d), Ne(d, 0)), (x/(-a*sin(c)**2 + a)**2, True))
```

3.59 $\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=47

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

```
[Out] -cot(d*x+c)/a^2/d+2*tan(d*x+c)/a^2/d+1/3*tan(d*x+c)^3/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{2\tan(c+dx)}{a^2d}-\frac{\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(Cot[c + d*x]/(a^2*d)) + (2*Tan[c + d*x])/(a^2*d) + Tan[c + d*x]^3/(3*a^2*
d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^2(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2}\,dx &= \frac{\int \csc^2(c+dx)\sec^4(c+dx)\,dx}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(2+\frac{1}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{a^2d}\\
&= -\frac{\cot(c+dx)}{a^2d}+\frac{2\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 50, normalized size = 1.06

$$\frac{\frac{5\tan(c+dx)}{3d}-\frac{\cot(c+dx)}{d}+\frac{\tan(c+dx)\sec^2(c+dx)}{3d}}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-(Cot[c + d*x]/d) + (5*Tan[c + d*x])/(3*d) + (Sec[c + d*x]^2*Tan[c + d*x])
/(3*d))/a^2
```

fricas [A] time = 0.40, size = 46, normalized size = 0.98

$$-\frac{8\cos(dx+c)^4-4\cos(dx+c)^2-1}{3a^2d\cos(dx+c)^3\sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(8*cos(d*x + c)^4 - 4*cos(d*x + c)^2 - 1)/(a^2*d*cos(d*x + c)^3*sin(d*
x + c))
```

giac [A] time = 0.15, size = 48, normalized size = 1.02

$$-\frac{\frac{3}{a^2\tan(dx+c)}-\frac{a^4\tan(dx+c)^3+6a^4\tan(dx+c)}{a^6}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(3/(a^2*tan(d*x + c)) - (a^4*tan(d*x + c)^3 + 6*a^4*tan(d*x + c))/a^6)
/d
```

maple [A] time = 0.45, size = 37, normalized size = 0.79

$$\frac{\frac{(\tan^3(dx+c))}{3}+2\tan(dx+c)-\frac{1}{\tan(dx+c)}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+2*tan(d*x+c)-1/tan(d*x+c))
```

maxima [A] time = 0.34, size = 40, normalized size = 0.85

$$\frac{\frac{\tan(dx+c)^3+6\tan(dx+c)}{a^2}-\frac{3}{a^2\tan(dx+c)}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 6*tan(d*x + c))/a^2 - 3/(a^2*tan(d*x + c)))/d
```

mupad [B] time = 13.58, size = 36, normalized size = 0.77

$$\frac{\tan(c+d\,x)^4+6\tan(c+d\,x)^2-3}{3\,a^2\,d\tan(c+d\,x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] (6*tan(c + d*x)^2 + tan(c + d*x)^4 - 3)/(3*a^2*d*tan(c + d*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^2(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**2/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

3.60 $\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} dx$

Optimal. Leaf size=65

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}$$

```
[Out] -3*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a^2/d+3*tan(d*x+c)/a^2/d+1/3*tan(d*x+c
)^3/a^2/d
```

Rubi [A] time = 0.08, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2620, 270}

$$\frac{\tan^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}-\frac{3\cot(c+dx)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-3*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a^2*d) + (3*Tan[c + d*x])/(a^
2*d) + Tan[c + d*x]^3/(3*a^2*d)
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2620

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> Dist[1/f, Subst[Int[(1 + x^2)^((m + n)/2 - 1)/x^m, x], x, Tan[e + f*x]],
 x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n)/2]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc^4(c+dx)}{\left(a-a\sin^2(c+dx)\right)^2} dx &= \frac{\int \csc^4(c+dx)\sec^4(c+dx)\,dx}{a^2}\\
&= \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4}\,dx, x, \tan(c+dx)\right)}{a^2d}\\
&= \frac{\text{Subst}\left(\int \left(3+\frac{1}{x^4}+\frac{3}{x^2}+x^2\right)dx, x, \tan(c+dx)\right)}{a^2d}\\
&= -\frac{3\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3a^2d}+\frac{3\tan(c+dx)}{a^2d}+\frac{\tan^3(c+dx)}{3a^2d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 46, normalized size = 0.71

$$\frac{16\left(-\frac{\cot(2(c+dx))}{3d}-\frac{\cot(2(c+dx))\csc^2(2(c+dx))}{6d}\right)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - a*Sin[c + d*x]^2)^2,x]
```

```
[Out] (16*(-1/3*Cot[2*(c + d*x)]/d - (Cot[2*(c + d*x)]*Csc[2*(c + d*x)]^2)/(6*d))
)/a^2
```

fricas [A] time = 0.41, size = 72, normalized size = 1.11

$$-\frac{16 \cos(dx+c)^6 - 24 \cos(dx+c)^4 + 6 \cos(dx+c)^2 + 1}{3\left(a^2 d \cos(dx+c)^5 - a^2 d \cos(dx+c)^3\right) \sin(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/3*(16*cos(d*x + c)^6 - 24*cos(d*x + c)^4 + 6*cos(d*x + c)^2 + 1)/((a^2*d
*cos(d*x + c)^5 - a^2*d*cos(d*x + c)^3)*sin(d*x + c))
```

giac [A] time = 0.15, size = 34, normalized size = 0.52

$$-\frac{8\left(3 \tan(2\,dx+2\,c)^2+1\right)}{3\,a^2 d \tan(2\,dx+2\,c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -8/3*(3*tan(2*d*x + 2*c)^2 + 1)/(a^2*d*tan(2*d*x + 2*c)^3)
```

maple [A] time = 0.51, size = 47, normalized size = 0.72

$$\frac{\frac{\left(\tan^3(dx+c)\right)}{3}+3\tan(dx+c)-\frac{3}{\tan(dx+c)}-\frac{1}{3\tan(dx+c)^3}}{d\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d/a^2*(1/3*tan(d*x+c)^3+3*tan(d*x+c)-3/tan(d*x+c)-1/3/tan(d*x+c)^3)
```

maxima [A] time = 0.34, size = 52, normalized size = 0.80

$$\frac{\frac{\tan(dx+c)^3+9\tan(dx+c)}{a^2}-\frac{9\tan(dx+c)^2+1}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-a*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((tan(d*x + c)^3 + 9*tan(d*x + c))/a^2 - (9*tan(d*x + c)^2 + 1)/(a^2*ta
n(d*x + c)^3))/d
```

mupad [B] time = 13.68, size = 48, normalized size = 0.74

$$-\frac{-\tan(c+d\,x)^6-9\tan(c+d\,x)^4+9\tan(c+d\,x)^2+1}{3\,a^2\,d\tan(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - a*sin(c + d*x)^2)^2),x)
```

```
[Out] -(9*tan(c + d*x)^2 - 9*tan(c + d*x)^4 - tan(c + d*x)^6 + 1)/(3*a^2*d*tan(c
+ d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\csc^4(c+dx)}{\sin^4(c+dx)-2\sin^2(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-a*sin(d*x+c)**2)**2,x)
```

```
[Out] Integral(csc(c + d*x)**4/(sin(c + d*x)**4 - 2*sin(c + d*x)**2 + 1), x)/a**2
```

3.61 $\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan(x)}{a^3}$$

```
[Out] tan(x)/a^3+2/3*tan(x)^3/a^3+1/5*tan(x)^5/a^3
```

Rubi [A] time = 0.02, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3),x]
```

```
[Out] Tan[x]/a^3 + (2*Tan[x]^3)/(3*a^3) + Tan[x]^5/(5*a^3)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^3}\,dx &= \frac{\int \sec^6(x)\,dx}{a^3}\\ &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(x)\right)}{a^3}\\ &= \frac{\tan(x)}{a^3}+\frac{2\tan^3(x)}{3a^3}+\frac{\tan^5(x)}{5a^3}\end{aligned}$$

Mathematica [A] time = 0.01, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3),x]
```

```
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^3
```

fricas [A] time = 0.40, size = 25, normalized size = 0.86

$$\frac{\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15\,a^3\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^3*cos(x)^5)
```

giac [A] time = 0.13, size = 22, normalized size = 0.76

$$\frac{3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)}{15 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

maple [A] time = 0.16, size = 20, normalized size = 0.69

$$\frac{\frac{(\tan^5(x))}{5} + \frac{2(\tan^3(x))}{3} + \tan(x)}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^3,x)
```

```
[Out] 1/a^3*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

maxima [A] time = 0.37, size = 22, normalized size = 0.76

$$\frac{3 \tan(x)^5 + 10 \tan(x)^3 + 15 \tan(x)}{15 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^3
```

mupad [B] time = 13.41, size = 21, normalized size = 0.72

$$\frac{\tan(x) \left(3 \tan(x)^4 + 10 \tan(x)^2 + 15\right)}{15 a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^3,x)
```

```
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^3)
```

sympy [B] time = 7.35, size = 362, normalized size = 12.48

$$-\frac{30 \tan^9\left(\frac{x}{2}\right)}{15a^3 \tan^{10}\left(\frac{x}{2}\right) - 75a^3 \tan^8\left(\frac{x}{2}\right) + 150a^3 \tan^6\left(\frac{x}{2}\right) - 150a^3 \tan^4\left(\frac{x}{2}\right) + 75a^3 \tan^2\left(\frac{x}{2}\right) - 15a^3} + \frac{}{15a^3 \tan^{10}\left(\frac{x}{2}\right) - 7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**3,x)
```

```
[Out] -30*tan(x/2)**9/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(
x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/
2)**7/(15*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 -
150*a**3*tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 116*tan(x/2)**5/(15
```

```
*a**3*tan(x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*
tan(x/2)**4 + 75*a**3*tan(x/2)**2 - 15*a**3) + 40*tan(x/2)**3/(15*a**3*tan(
x/2)**10 - 75*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**
4 + 75*a**3*tan(x/2)**2 - 15*a**3) - 30*tan(x/2)/(15*a**3*tan(x/2)**10 - 75
*a**3*tan(x/2)**8 + 150*a**3*tan(x/2)**6 - 150*a**3*tan(x/2)**4 + 75*a**3*t
an(x/2)**2 - 15*a**3)
```

3.62 $\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^7(x)}{7a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^3(x)}{a^4}+\frac{\tan(x)}{a^4}$$

```
[Out] tan(x)/a^4+tan(x)^3/a^4+3/5*tan(x)^5/a^4+1/7*tan(x)^7/a^4
```

Rubi [A] time = 0.02, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 3767}

$$\frac{\tan^7(x)}{7a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^3(x)}{a^4}+\frac{\tan(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-4),x]
```

```
[Out] Tan[x]/a^4 + Tan[x]^3/a^4 + (3*Tan[x]^5)/(5*a^4) + Tan[x]^7/(7*a^4)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^4}\,dx &= \frac{\int \sec^8(x)\,dx}{a^4}\\ &= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)dx, x, -\tan(x)\right)}{a^4}\\ &= \frac{\tan(x)}{a^4}+\frac{\tan^3(x)}{a^4}+\frac{3\tan^5(x)}{5a^4}+\frac{\tan^7(x)}{7a^4}\end{aligned}$$

Mathematica [A] time = 0.01, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)+\frac{6}{35}\tan(x)\sec^4(x)+\frac{8}{35}\tan(x)\sec^2(x)}{a^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-4),x]
```

```
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^4
```

fricas [A] time = 0.40, size = 31, normalized size = 0.84

$$\frac{\left(16 \cos(x)^{6} + 8 \cos(x)^{4} + 6 \cos(x)^{2} + 5\right) \sin(x)}{35 a^{4} \cos(x)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="fricas")
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^4*cos(x)^7)
```

giac [A] time = 0.12, size = 28, normalized size = 0.76

$$\frac{5 \tan(x)^{7} + 21 \tan(x)^{5} + 35 \tan(x)^{3} + 35 \tan(x)}{35 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="giac")
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

maple [A] time = 0.16, size = 24, normalized size = 0.65

$$\frac{\frac{\left(\tan^{7}(x)\right)}{7} + \frac{3\left(\tan^{5}(x)\right)}{5} + \tan^{3}(x) + \tan(x)}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^4,x)
[Out] 1/a^4*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

maxima [A] time = 0.34, size = 28, normalized size = 0.76

$$\frac{5 \tan(x)^{7} + 21 \tan(x)^{5} + 35 \tan(x)^{3} + 35 \tan(x)}{35 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^4,x, algorithm="maxima")
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^4
```

mupad [B] time = 13.41, size = 33, normalized size = 0.89

$$\frac{\tan(x)}{a^{4}} + \frac{\tan(x)^{3}}{a^{4}} + \frac{3 \tan(x)^{5}}{5 a^{4}} + \frac{\tan(x)^{7}}{7 a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^4,x)
[Out] tan(x)/a^4 + tan(x)^3/a^4 + (3*tan(x)^5)/(5*a^4) + tan(x)^7/(7*a^4)
```

sympy [B] time = 22.20, size = 675, normalized size = 18.24

$$-\frac{70 \tan^{13}\left(\frac{x}{2}\right)}{35a^{4} \tan^{14}\left(\frac{x}{2}\right) - 245a^{4} \tan^{12}\left(\frac{x}{2}\right) + 735a^{4} \tan^{10}\left(\frac{x}{2}\right) - 1225a^{4} \tan^{8}\left(\frac{x}{2}\right) + 1225a^{4} \tan^{6}\left(\frac{x}{2}\right) - 735a^{4} \tan^{4}\left(\frac{x}{2}\right) + 24}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**4,x)
```

```
[Out] -70*tan(x/2)**13/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*t
an(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(
x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(x/2)**11/(35*a**4*tan(x
/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2
)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2
- 35*a**4) - 602*tan(x/2)**9/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12
+ 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 7
35*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 424*tan(x/2)**7/(35
*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a
**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*t
an(x/2)**2 - 35*a**4) - 602*tan(x/2)**5/(35*a**4*tan(x/2)**14 - 245*a**4*ta
n(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(
x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4) + 140*tan(
x/2)**3/(35*a**4*tan(x/2)**14 - 245*a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**
10 - 1225*a**4*tan(x/2)**8 + 1225*a**4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 +
 245*a**4*tan(x/2)**2 - 35*a**4) - 70*tan(x/2)/(35*a**4*tan(x/2)**14 - 245*
a**4*tan(x/2)**12 + 735*a**4*tan(x/2)**10 - 1225*a**4*tan(x/2)**8 + 1225*a*
*4*tan(x/2)**6 - 735*a**4*tan(x/2)**4 + 245*a**4*tan(x/2)**2 - 35*a**4)
```

3.63 $\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx$

Optimal. Leaf size=51

$$\frac{\tan^9(x)}{9a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{\tan(x)}{a^5}$$

```
[Out] tan(x)/a^5+4/3*tan(x)^3/a^5+6/5*tan(x)^5/a^5+4/7*tan(x)^7/a^5+1/9*tan(x)^9/
a^5
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3175, 3767}

$$\frac{\tan^9(x)}{9a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{\tan(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] Tan[x]/a^5 + (4*Tan[x]^3)/(3*a^5) + (6*Tan[x]^5)/(5*a^5) + (4*Tan[x]^7)/(7*
a^5) + Tan[x]^9/(9*a^5)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\left(a-a\sin^2(x)\right)^5}\,dx&=\frac{\int \sec^{10}(x)\,dx}{a^5}\\&=-\frac{\text{Subst}\left(\int \left(1+4x^2+6x^4+4x^6+x^8\right)\,dx,x,-\tan(x)\right)}{a^5}\\&=\frac{\tan(x)}{a^5}+\frac{4\tan^3(x)}{3a^5}+\frac{6\tan^5(x)}{5a^5}+\frac{4\tan^7(x)}{7a^5}+\frac{\tan^9(x)}{9a^5}\end{aligned}$$

Mathematica [A] time = 0.01, size = 51, normalized size = 1.00

$$\frac{\frac{128\tan(x)}{315}+\frac{1}{9}\tan(x)\sec^8(x)+\frac{8}{63}\tan(x)\sec^6(x)+\frac{16}{105}\tan(x)\sec^4(x)+\frac{64}{315}\tan(x)\sec^2(x)}{a^5}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5),x]
```

```
[Out] ((128*Tan[x])/315 + (64*Sec[x]^2*Tan[x])/315 + (16*Sec[x]^4*Tan[x])/105 + (
8*Sec[x]^6*Tan[x])/63 + (Sec[x]^8*Tan[x])/9)/a^5
```

fricas [A] time = 0.43, size = 37, normalized size = 0.73

$$\frac{\left(128\,\cos(x)^8+64\,\cos(x)^6+48\,\cos(x)^4+40\,\cos(x)^2+35\right)\sin(x)}{315\,a^5\cos(x)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="fricas")
```

```
[Out] 1/315*(128*cos(x)^8 + 64*cos(x)^6 + 48*cos(x)^4 + 40*cos(x)^2 + 35)*sin(x)/
(a^5*cos(x)^9)
```

giac [A] time = 0.12, size = 34, normalized size = 0.67

$$\frac{35\,\tan(x)^9+180\,\tan(x)^7+378\,\tan(x)^5+420\,\tan(x)^3+315\,\tan(x)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="giac")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

maple [A] time = 0.16, size = 32, normalized size = 0.63

$$\frac{\frac{\left(\tan^9(x)\right)}{9}+\frac{4\left(\tan^7(x)\right)}{7}+\frac{6\left(\tan^5(x)\right)}{5}+\frac{4\left(\tan^3(x)\right)}{3}+\tan(x)}{a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^5,x)
```

```
[Out] 1/a^5*(1/9*tan(x)^9+4/7*tan(x)^7+6/5*tan(x)^5+4/3*tan(x)^3+tan(x))
```

maxima [A] time = 0.34, size = 34, normalized size = 0.67

$$\frac{35\,\tan(x)^9+180\,\tan(x)^7+378\,\tan(x)^5+420\,\tan(x)^3+315\,\tan(x)}{315\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/315*(35*tan(x)^9 + 180*tan(x)^7 + 378*tan(x)^5 + 420*tan(x)^3 + 315*tan(x
))/a^5
```

mupad [B] time = 13.34, size = 43, normalized size = 0.84

$$\frac{\tan(x)}{a^5}+\frac{4\,{\tan(x)}^3}{3\,a^5}+\frac{6\,{\tan(x)}^5}{5\,a^5}+\frac{4\,{\tan(x)}^7}{7\,a^5}+\frac{{\tan(x)}^9}{9\,a^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^5,x)
```

```
[Out] tan(x)/a^5 + (4*tan(x)^3)/(3*a^5) + (6*tan(x)^5)/(5*a^5) + (4*tan(x)^7)/(7*
a^5) + tan(x)^9/(9*a^5)
```

sympy [B] time = 69.51, size = 1083, normalized size = 21.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**5,x)
```

```
[Out] -630*tan(x/2)**17/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a
**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 3969
0*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835
*a**5*tan(x/2)**2 - 315*a**5) + 1680*tan(x/2)**15/(315*a**5*tan(x/2)**18 -
2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12
+ 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6
 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576*tan(x/
2)**13/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2
)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(
x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x
/2)**2 - 315*a**5) + 10224*tan(x/2)**11/(315*a**5*tan(x/2)**18 - 2835*a**5*
tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a*
*5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a
**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 21316*tan(x/2)**9/(31
5*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26
460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 +
26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 3
15*a**5) + 10224*tan(x/2)**7/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**1
6 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)
**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2
)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) - 9576*tan(x/2)**5/(315*a**5*tan(x
/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan
(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*t
an(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5) + 1
680*tan(x/2)**3/(315*a**5*tan(x/2)**18 - 2835*a**5*tan(x/2)**16 + 11340*a**
5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690*a**5*tan(x/2)**10 - 39690*
a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 11340*a**5*tan(x/2)**4 + 2835*a
**5*tan(x/2)**2 - 315*a**5) - 630*tan(x/2)/(315*a**5*tan(x/2)**18 - 2835*a*
*5*tan(x/2)**16 + 11340*a**5*tan(x/2)**14 - 26460*a**5*tan(x/2)**12 + 39690
*a**5*tan(x/2)**10 - 39690*a**5*tan(x/2)**8 + 26460*a**5*tan(x/2)**6 - 1134
0*a**5*tan(x/2)**4 + 2835*a**5*tan(x/2)**2 - 315*a**5)
```

3.64 $\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=51

$$\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

```
[Out] -(a+b)*cos(d*x+c)/d+1/3*(a+2*b)*cos(d*x+c)^3/d-1/5*b*cos(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3013, 373}

$$\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}-\frac{b\cos^5(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + ((a + 2*b)*Cos[c + d*x]^3)/(3*d) - (b*Cos[c +
 d*x]^5)/(5*d)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin^3(c+dx)\left(a+b\sin^2(c+dx)\right)dx&=-\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{\text{Subst}\left(\int\left(a\left(1+\frac{b}{a}\right)-(a+2b)x^2+bx^4\right)dx,x,\cos(c+dx)\right)}{d}\\&=-\frac{(a+b)\cos(c+dx)}{d}+\frac{(a+2b)\cos^3(c+dx)}{3d}-\frac{b\cos^5(c+dx)}{5d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 77, normalized size = 1.51

$$-\frac{3a\cos(c+dx)}{4d}+\frac{a\cos(3(c+dx))}{12d}-\frac{5b\cos(c+dx)}{8d}+\frac{5b\cos(3(c+dx))}{48d}-\frac{b\cos(5(c+dx))}{80d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-3*a*Cos[c + d*x])/(4*d) - (5*b*Cos[c + d*x])/(8*d) + (a*Cos[3*(c + d*x)])
/(12*d) + (5*b*Cos[3*(c + d*x)])/(48*d) - (b*Cos[5*(c + d*x)])/(80*d)
```

fricas [A] time = 0.41, size = 43, normalized size = 0.84

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-5\,(a+2\,b)\cos\left(dx+c\right)^{3}+15\,(a+b)\cos\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

giac [A] time = 0.13, size = 67, normalized size = 1.31

$$-\frac{b\cos\left(dx+c\right)^{5}}{5\,d}+\frac{a\cos\left(dx+c\right)^{3}}{3\,d}+\frac{2\,b\cos\left(dx+c\right)^{3}}{3\,d}-\frac{a\cos\left(dx+c\right)}{d}-\frac{b\cos\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/5*b*cos(d*x + c)^5/d + 1/3*a*cos(d*x + c)^3/d + 2/3*b*cos(d*x + c)^3/d -
 a*cos(d*x + c)/d - b*cos(d*x + c)/d
```

maple [A] time = 0.44, size = 54, normalized size = 1.06

$$\frac{-\frac{b\left(\frac{8}{3}+\sin^{4}(dx+c)+\frac{4\left(\sin^{2}(dx+c)\right)}{3}\right)\cos(dx+c)}{5}-\frac{a\left(2+\sin^{2}(dx+c)\right)\cos(dx+c)}{3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/5*b*(8/3+sin(d*x+c)^4+4/3*sin(d*x+c)^2)*cos(d*x+c)-1/3*a*(2+sin(d*x
+c)^2)*cos(d*x+c))
```

maxima [A] time = 0.34, size = 43, normalized size = 0.84

$$-\frac{3\,b\cos\left(dx+c\right)^{5}-5\,(a+2\,b)\cos\left(dx+c\right)^{3}+15\,(a+b)\cos\left(dx+c\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(3*b*cos(d*x + c)^5 - 5*(a + 2*b)*cos(d*x + c)^3 + 15*(a + b)*cos(d*x
 + c))/d
```

mupad [B] time = 13.37, size = 44, normalized size = 0.86

$$-\frac{\frac{b\cos(c+d\,x)^{5}}{5}+\left(-\frac{a}{3}-\frac{2\,b}{3}\right)\,\cos\left(c+d\,x\right)^{3}+(a+b)\,\cos\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3*(a + b*sin(c + d*x)^2),x)
```

```
[Out] -((b*cos(c + d*x)^5)/5 - cos(c + d*x)^3*(a/3 + (2*b)/3) + cos(c + d*x)*(a +
 b))/d
```

sympy [A] time = 3.32, size = 107, normalized size = 2.10

$$\begin{cases} -\frac{a\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2a\cos^3(c+dx)}{3d} - \frac{b\sin^4(c+dx)\cos(c+dx)}{d} - \frac{4b\sin^2(c+dx)\cos^3(c+dx)}{3d} - \frac{8b\cos^5(c+dx)}{15d} & \text{for } d \neq 0 \\ x\left(a+b\sin^2(c)\right)\sin^3(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((-a*sin(c + d*x)**2*cos(c + d*x)/d - 2*a*cos(c + d*x)**3/(3*d) -
b*sin(c + d*x)**4*cos(c + d*x)/d - 4*b*sin(c + d*x)**2*cos(c + d*x)**3/(3*d
) - 8*b*cos(c + d*x)**5/(15*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**3,
True))
```

3.65 $\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=31

$$\frac{b\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}$$

```
[Out] -(a+b)*cos(d*x+c)/d+1/3*b*cos(d*x+c)^3/d
```

Rubi [A] time = 0.02, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.053, Rules used = {3013}

$$\frac{b\cos^3(c+dx)}{3d}-\frac{(a+b)\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)*Cos[c + d*x])/d) + (b*Cos[c + d*x]^3)/(3*d)
```

Rule 3013

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \sin(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{\text{Subst}\left(\int \left(a+b-bx^2\right)\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{(a+b)\cos(c+dx)}{d}+\frac{b\cos^3(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 1.74

$$\frac{a\sin(c)\sin(dx)}{d}-\frac{a\cos(c)\cos(dx)}{d}-\frac{3b\cos(c+dx)}{4d}+\frac{b\cos(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*Cos[c]*Cos[d*x])/d) - (3*b*Cos[c + d*x])/(4*d) + (b*Cos[3*(c + d*x)])/
(12*d) + (a*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.42, size = 27, normalized size = 0.87

$$\frac{b\cos\left(dx+c\right)^3-3\,(a+b)\cos\left(dx+c\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(b*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/d
```

giac [A] time = 0.13, size = 40, normalized size = 1.29

$$\frac{1}{3}\left(\frac{\cos(dx+c)^3}{d} - \frac{3\cos(dx+c)}{d}\right)b - \frac{a\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/3*(cos(d*x + c)^3/d - 3*cos(d*x + c)/d)*b - a*cos(d*x + c)/d
```

maple [A] time = 0.33, size = 34, normalized size = 1.10

$$\frac{-\frac{b\left(2+\sin^2(dx+c)\right)\cos(dx+c)}{3} - a\cos(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c)^2),x)
[Out] 1/d*(-1/3*b*(2+sin(d*x+c)^2)*cos(d*x+c)-a*cos(d*x+c))
```

maxima [A] time = 0.33, size = 34, normalized size = 1.10

$$\frac{\left(\cos(dx+c)^3 - 3\cos(dx+c)\right)b - 3a\cos(dx+c)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] 1/3*((cos(d*x + c)^3 - 3*cos(d*x + c))*b - 3*a*cos(d*x + c))/d
```

mupad [B] time = 13.31, size = 27, normalized size = 0.87

$$\frac{\frac{b\cos(c+d\,x)^3}{3} - \cos(c+d\,x)\,(a+b)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x)^2),x)
[Out] ((b*cos(c + d*x)^3)/3 - cos(c + d*x)*(a + b))/d
```

sympy [A] time = 1.00, size = 58, normalized size = 1.87

$$\begin{cases} -\frac{a\cos(c+dx)}{d} - \frac{b\sin^2(c+dx)\cos(c+dx)}{d} - \frac{2b\cos^3(c+dx)}{3d} & \text{for } d \neq 0 \\ x\left(a+b\sin^2(c)\right)\sin(c) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**2),x)
[Out] Piecewise((-a*cos(c + d*x)/d - b*sin(c + d*x)**2*cos(c + d*x)/d - 2*b*cos(c
 + d*x)**3/(3*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c), True))
```

3.66 $\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=26

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

```
[Out] -a*arctanh(cos(d*x+c))/d-b*cos(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 26, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 19, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.105, Rules used = {3014, 3770}

$$-\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTanh[Cos[c + d*x]])/d) - (b*Cos[c + d*x])/d
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)}{d}+a\int \csc(c+dx)\,dx\\ &= -\frac{a\tanh^{-1}(\cos(c+dx))}{d}-\frac{b\cos(c+dx)}{d}\end{aligned}$$

Mathematica [B] time = 0.02, size = 63, normalized size = 2.42

$$\frac{a\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-\frac{a\log\left(\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}+\frac{b\sin(c)\sin(dx)}{d}-\frac{b\cos(c)\cos(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*Cos[c]*Cos[d*x])/d) - (a*Log[Cos[c/2 + (d*x)/2]])/d + (a*Log[Sin[c/2 +
 (d*x)/2]])/d + (b*Sin[c]*Sin[d*x])/d
```

fricas [A] time = 0.46, size = 42, normalized size = 1.62

$$-\frac{2\,b\cos\left(dx+c\right)+a\log\left(\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)-a\log\left(-\frac{1}{2}\,\cos\left(dx+c\right)+\frac{1}{2}\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(1/2*cos(d*x + c) + 1/2) - a*log(-1/2*cos(d*x
 + c) + 1/2))/d
```

giac [B] time = 0.16, size = 58, normalized size = 2.23

$$\frac{a \log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right) + \frac{4\,b}{\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(a*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + 4*b/((cos(d*x +
c) - 1)/(cos(d*x + c) + 1) - 1))/d
```

maple [A] time = 0.31, size = 35, normalized size = 1.35

$$-\frac{b\cos\left(dx+c\right)}{d}+\frac{a\ln\left(\csc\left(dx+c\right)-\cot\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c)^2),x)
```

```
[Out] -b*cos(d*x+c)/d+1/d*a*ln(csc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.34, size = 38, normalized size = 1.46

$$-\frac{2\,b\cos\left(dx+c\right)+a\log\left(\cos\left(dx+c\right)+1\right)-a\log\left(\cos\left(dx+c\right)-1\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*b*cos(d*x + c) + a*log(cos(d*x + c) + 1) - a*log(cos(d*x + c) - 1))
/d
```

mupad [B] time = 13.37, size = 23, normalized size = 0.88

$$-\frac{b\,\cos\left(c+d\,x\right)+a\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x),x)
```

```
[Out] -(b*cos(c + d*x) + a*atanh(cos(c + d*x)))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^{2}\left(c+dx\right)\right)\csc\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x), x)
```

3.67 $\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=40

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(d*x+c))/d-1/2*a*cot(d*x+c)*csc(d*x+c)/d
```

Rubi [A] time = 0.03, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3770}

$$-\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a + 2*b)*ArcTanh[Cos[c + d*x]])/(2*d) - (a*Cot[c + d*x]*Csc[c + d*x])/(2
*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \csc^3(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc(c+dx)}{2d}+\frac{1}{2}(a+2b)\int \csc(c+dx)\,dx\\ &= -\frac{(a+2b)\tanh^{-1}(\cos(c+dx))}{2d}-\frac{a\cot(c+dx)\csc(c+dx)}{2d}\end{aligned}$$

Mathematica [B] time = 0.04, size = 118, normalized size = 2.95

$$-\frac{a\csc^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)}{8d}+\frac{a\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}-\frac{a\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2d}+\frac{b\log\left(\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\right)}{d}-$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/8*(a*Csc[(c + d*x)/2]^2)/d - (b*Log[Cos[c/2 + (d*x)/2]])/d - (a*Log[Cos[
(c + d*x)/2]])/(2*d) + (b*Log[Sin[c/2 + (d*x)/2]])/d + (a*Log[Sin[(c + d*x)
/2]])/(2*d) + (a*Sec[(c + d*x)/2]^2)/(8*d)
```

fricas [B] time = 0.46, size = 95, normalized size = 2.38

$$\frac{2\,a\cos(dx+c)-\left((a+2\,b)\cos(dx+c)^2-a-2\,b\right)\log\left(\frac{1}{2}\,\cos(dx+c)+\frac{1}{2}\right)+\left((a+2\,b)\cos(dx+c)^2-a-2\,b\right)1}{4\left(d\cos(dx+c)^2-d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a*cos(d*x + c) - ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(1/2*cos(d*
x + c) + 1/2) + ((a + 2*b)*cos(d*x + c)^2 - a - 2*b)*log(-1/2*cos(d*x + c)
+ 1/2))/(d*cos(d*x + c)^2 - d)
```

giac [B] time = 0.16, size = 121, normalized size = 3.02

$$\frac{2\,(a+2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{\cos(dx+c)-1}-\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(2*(a + 2*b)*log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1)) + (a - 2
*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1))*(cos(d*x + c) + 1)/(cos(d*x + c) - 1) - a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1))/d
```

maple [A] time = 0.52, size = 63, normalized size = 1.58

$$-\frac{a\cot(dx+c)\csc(dx+c)}{2d}+\frac{a\ln(\csc(dx+c)-\cot(dx+c))}{2d}+\frac{b\ln(\csc(dx+c)-\cot(dx+c))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/2*a*cot(d*x+c)*csc(d*x+c)/d+1/2/d*a*ln(csc(d*x+c)-cot(d*x+c))+1/d*b*ln(c
sc(d*x+c)-cot(d*x+c))
```

maxima [A] time = 0.33, size = 58, normalized size = 1.45

$$-\frac{(a+2\,b)\log(\cos(dx+c)+1)-(a+2\,b)\log(\cos(dx+c)-1)-\frac{2\,a\cos(dx+c)}{\cos(dx+c)^2-1}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*((a + 2*b)*log(cos(d*x + c) + 1) - (a + 2*b)*log(cos(d*x + c) - 1) - 2
*a*cos(d*x + c)/(cos(d*x + c)^2 - 1))/d
```

mupad [B] time = 13.39, size = 42, normalized size = 1.05

$$\frac{a\,\cos(c+d\,x)}{2\,d\,\left(\cos(c+d\,x)^2-1\right)}-\frac{\mathrm{atanh}(\cos(c+d\,x))\,\left(\frac{a}{2}+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^3,x)
```

```
[Out] (a*cos(c + d*x))/(2*d*(cos(c + d*x)^2 - 1)) - (atanh(cos(c + d*x))*(a/2 + b
))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^2(c+dx)\right)\csc^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**3, x)
```

3.68 $\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=89

$$-\frac{(6a+5b)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(6a+5b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a+5b)-\frac{b\sin^5(c+dx)\cos(c+d}{6d}$$

```
[Out] 1/16*(6*a+5*b)*x-1/16*(6*a+5*b)*cos(d*x+c)*sin(d*x+c)/d-1/24*(6*a+5*b)*cos(
d*x+c)*sin(d*x+c)^3/d-1/6*b*cos(d*x+c)*sin(d*x+c)^5/d
```

Rubi [A] time = 0.05, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3014, 2635, 8}

$$-\frac{(6a+5b)\sin^3(c+dx)\cos(c+dx)}{24d}-\frac{(6a+5b)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x(6a+5b)-\frac{b\sin^5(c+dx)\cos(c+d}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((6*a + 5*b)*x)/16 - ((6*a + 5*b)*Cos[c + d*x]*Sin[c + d*x])/(16*d) - ((6*a
 + 5*b)*Cos[c + d*x]*Sin[c + d*x]^3)/(24*d) - (b*Cos[c + d*x]*Sin[c + d*x]^
5)/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] && !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}
\int \sin^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{1}{6}(6a+5b)\int \sin^4(c+dx)\,dx\\
&= -\frac{(6a+5b)\cos(c+dx)\sin^3(c+dx)}{24d}-\frac{b\cos(c+dx)\sin^5(c+dx)}{6d}+\frac{1}{8}\\
&= -\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c+dx)\sin^3(c+}{24d}\\
&= \frac{1}{16}(6a+5b)x-\frac{(6a+5b)\cos(c+dx)\sin(c+dx)}{16d}-\frac{(6a+5b)\cos(c-}{2}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 70, normalized size = 0.79

$$\frac{-3(16a+15b)\sin(2(c+dx))+(6a+9b)\sin(4(c+dx))+72ac+72adx-b\sin(6(c+dx))+60bc+60bdx}{192d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (72*a*c + 60*b*c + 72*a*d*x + 60*b*d*x - 3*(16*a + 15*b)*Sin[2*(c + d*x)] +
 (6*a + 9*b)*Sin[4*(c + d*x)] - b*Sin[6*(c + d*x)])/(192*d)
```

fricas [A] time = 0.45, size = 69, normalized size = 0.78

$$\frac{3\,(6\,a+5\,b)dx-\left(8\,b\cos\left(dx+c\right)^{5}-2\,(6\,a+13\,b)\cos\left(dx+c\right)^{3}+3\,(10\,a+11\,b)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + 5*b)*d*x - (8*b*cos(d*x + c)^5 - 2*(6*a + 13*b)*cos(d*x + c)
^3 + 3*(10*a + 11*b)*cos(d*x + c))*sin(d*x + c))/d
```

giac [A] time = 0.13, size = 68, normalized size = 0.76

$$\frac{1}{16}\,(6\,a+5\,b)x-\frac{b\sin\left(6\,dx+6\,c\right)}{192\,d}+\frac{(2\,a+3\,b)\sin\left(4\,dx+4\,c\right)}{64\,d}-\frac{(16\,a+15\,b)\sin\left(2\,dx+2\,c\right)}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + 5*b)*x - 1/192*b*sin(6*d*x + 6*c)/d + 1/64*(2*a + 3*b)*sin(4*d*
x + 4*c)/d - 1/64*(16*a + 15*b)*sin(2*d*x + 2*c)/d
```

maple [A] time = 0.46, size = 86, normalized size = 0.97

$$\frac{b\left(-\frac{\left(\sin^{5}(dx+c)+\frac{5\left(\sin^{3}(dx+c)\right)}{4}+\frac{15\sin(dx+c)}{8}\right)\cos(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+a\left(-\frac{\left(\sin^{3}(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/6*(sin(d*x+c)^5+5/4*sin(d*x+c)^3+15/8*sin(d*x+c))*cos(d*x+c)+5/1
6*d*x+5/16*c)+a*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*
c))
```

maxima [A] time = 0.44, size = 104, normalized size = 1.17

$$\frac{3\,(dx+c)(6\,a+5\,b)-\frac{3\,(10\,a+11\,b)\tan(dx+c)^{5}+8\,(6\,a+5\,b)\tan(dx+c)^{3}+3\,(6\,a+5\,b)\tan(dx+c)}{\tan(dx+c)^{6}+3\,\tan(dx+c)^{4}+3\,\tan(dx+c)^{2}+1}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(d*x + c)*(6*a + 5*b) - (3*(10*a + 11*b)*tan(d*x + c)^5 + 8*(6*a +
5*b)*tan(d*x + c)^3 + 3*(6*a + 5*b)*tan(d*x + c))/(tan(d*x + c)^6 + 3*tan(d
*x + c)^4 + 3*tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 13.94, size = 92, normalized size = 1.03

$$x\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)-\frac{\left(\frac{5\,a}{8}+\frac{11\,b}{16}\right)\,\tan\left(c+d\,x\right)^5+\left(a+\frac{5\,b}{6}\right)\,\tan\left(c+d\,x\right)^3+\left(\frac{3\,a}{8}+\frac{5\,b}{16}\right)\,\tan\left(c+d\,x\right)}{d\,\left(\tan\left(c+d\,x\right)^6+3\,\tan\left(c+d\,x\right)^4+3\,\tan\left(c+d\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4*(a + b*sin(c + d*x)^2),x)
```

```
[Out] x*((3*a)/8 + (5*b)/16) - (tan(c + d*x)^5*((5*a)/8 + (11*b)/16) + tan(c + d*
x)*((3*a)/8 + (5*b)/16) + tan(c + d*x)^3*(a + (5*b)/6))/(d*(3*tan(c + d*x)^
2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1))
```

sympy [A] time = 5.37, size = 258, normalized size = 2.90

$$\begin{cases} \frac{3ax\sin^4(c+dx)}{8} + \frac{3ax\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3ax\cos^4(c+dx)}{8} - \frac{5a\sin^3(c+dx)\cos(c+dx)}{8d} - \frac{3a\sin(c+dx)\cos^3(c+dx)}{8d} + \frac{5bx\sin^6(c+dx)}{16} \\ x\left(a + b\sin^2(c)\right)\sin^4(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(c + d*x)**4/8 + 3*a*x*sin(c + d*x)**2*cos(c + d*x)**2/
4 + 3*a*x*cos(c + d*x)**4/8 - 5*a*sin(c + d*x)**3*cos(c + d*x)/(8*d) - 3*a*
sin(c + d*x)*cos(c + d*x)**3/(8*d) + 5*b*x*sin(c + d*x)**6/16 + 15*b*x*sin(
c + d*x)**4*cos(c + d*x)**2/16 + 15*b*x*sin(c + d*x)**2*cos(c + d*x)**4/16
+ 5*b*x*cos(c + d*x)**6/16 - 11*b*sin(c + d*x)**5*cos(c + d*x)/(16*d) - 5*b
*sin(c + d*x)**3*cos(c + d*x)**3/(6*d) - 5*b*sin(c + d*x)*cos(c + d*x)**5/(
16*d), Ne(d, 0)), (x*(a + b*sin(c)**2)*sin(c)**4, True))
```

3.69 $\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=61

$$-\frac{(4a+3b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3b)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

```
[Out] 1/8*(4*a+3*b)*x-1/8*(4*a+3*b)*cos(d*x+c)*sin(d*x+c)/d-1/4*b*cos(d*x+c)*sin(
d*x+c)^3/d
```

Rubi [A] time = 0.04, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3014, 2635, 8}

$$-\frac{(4a+3b)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(4a+3b)-\frac{b\sin^3(c+dx)\cos(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((4*a + 3*b)*x)/8 - ((4*a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(8*d) - (b*Cos[
c + d*x]*Sin[c + d*x]^3)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3014

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(
x_)]^2), x_Symbol] :> -Simp[(C*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*
(m + 2)), x] + Dist[(A*(m + 2) + C*(m + 1))/(m + 2), Int[(b*Sin[e + f*x])^m
, x], x] /; FreeQ[{b, e, f, A, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sin^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{4}(4a+3b)\int \sin^2(c+dx)\,dx\\ &= -\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c+dx)}{4d}+\frac{1}{8}(4a\\ &= \frac{1}{8}(4a+3b)x-\frac{(4a+3b)\cos(c+dx)\sin(c+dx)}{8d}-\frac{b\cos(c+dx)\sin^3(c\\ &\quad \\ &\quad\frac{}{4d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 45, normalized size = 0.74

$$\frac{4(4a+3b)(c+dx)-8(a+b)\sin(2(c+dx))+b\sin(4(c+dx))}{32d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(4*a + 3*b)*(c + d*x) - 8*(a + b)*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])
/(32*d)
```

fricas [A] time = 0.44, size = 50, normalized size = 0.82

$$\frac{(4a + 3b)dx + \left(2b\cos(dx + c)^3 - (4a + 5b)\cos(dx + c)\right)\sin(dx + c)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + 3*b)*d*x + (2*b*cos(d*x + c)^3 - (4*a + 5*b)*cos(d*x + c))*sin(
d*x + c))/d
```

giac [A] time = 0.13, size = 43, normalized size = 0.70

$$\frac{1}{8}(4a + 3b)x + \frac{b\sin(4dx + 4c)}{32d} - \frac{(a + b)\sin(2dx + 2c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + 3*b)*x + 1/32*b*sin(4*d*x + 4*c)/d - 1/4*(a + b)*sin(2*d*x + 2*c
)/d
```

maple [A] time = 0.34, size = 65, normalized size = 1.07

$$\frac{b\left(-\frac{\left(\sin^3(dx+c)+\frac{3\sin(dx+c)}{2}\right)\cos(dx+c)}{4} + \frac{3dx}{8} + \frac{3c}{8}\right) + a\left(-\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(b*(-1/4*(sin(d*x+c)^3+3/2*sin(d*x+c))*cos(d*x+c)+3/8*d*x+3/8*c)+a*(-1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

maxima [A] time = 0.46, size = 74, normalized size = 1.21

$$\frac{(dx + c)(4a + 3b) - \frac{(4a+5b)\tan(dx+c)^3+(4a+3b)\tan(dx+c)}{\tan(dx+c)^4+2\tan(dx+c)^2+1}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((d*x + c)*(4*a + 3*b) - ((4*a + 5*b)*tan(d*x + c)^3 + (4*a + 3*b)*tan(
d*x + c))/(tan(d*x + c)^4 + 2*tan(d*x + c)^2 + 1))/d
```

mupad [B] time = 13.55, size = 68, normalized size = 1.11

$$x\left(\frac{a}{2} + \frac{3b}{8}\right) - \frac{\left(\frac{a}{2} + \frac{5b}{8}\right)\tan(c + dx)^3 + \left(\frac{a}{2} + \frac{3b}{8}\right)\tan(c + dx)}{d\left(\tan(c + dx)^4 + 2\tan(c + dx)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2*(a + b*sin(c + d*x)^2),x)
```

```
[Out] x*(a/2 + (3*b)/8) - (tan(c + d*x)^3*(a/2 + (5*b)/8) + tan(c + d*x)*(a/2 + (
3*b)/8))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1))
```

sympy [A] time = 1.93, size = 158, normalized size = 2.59

$$\begin{cases} \frac{ax\sin^2(c+dx)}{2} + \frac{ax\cos^2(c+dx)}{2} - \frac{a\sin(c+dx)\cos(c+dx)}{2d} + \frac{3bx\sin^4(c+dx)}{8} + \frac{3bx\sin^2(c+dx)\cos^2(c+dx)}{4} + \frac{3bx\cos^4(c+dx)}{8} - \frac{5b\sin^3(c+dx)}{8d} \\ x\left(a + b\sin^2(c)\right)\sin^2(c) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Piecewise((a*x*sin(c + d*x)**2/2 + a*x*cos(c + d*x)**2/2 - a*sin(c + d*x)*c
os(c + d*x)/(2*d) + 3*b*x*sin(c + d*x)**4/8 + 3*b*x*sin(c + d*x)**2*cos(c +
 d*x)**2/4 + 3*b*x*cos(c + d*x)**4/8 - 5*b*sin(c + d*x)**3*cos(c + d*x)/(8*
d) - 3*b*sin(c + d*x)*cos(c + d*x)**3/(8*d), Ne(d, 0)), (x*(a + b*sin(c)**2
)*sin(c)**2, True))
```

3.70 $\int \left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A] time = 0.01, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax-\frac{b\sin(c+dx)\cos(c+dx)}{2d}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(c+dx)\right)\,dx &= ax+b\int \sin^2(c+dx)\,dx\\ &= ax-\frac{b\cos(c+dx)\sin(c+dx)}{2d}+\frac{1}{2}b\int 1\,dx\\ &= ax+\frac{bx}{2}-\frac{b\cos(c+dx)\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.10

$$ax+\frac{b(c+dx)}{2d}-\frac{b\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[c + d*x]^2,x]
```

```
[Out] a*x + (b*(c + d*x))/(2*d) - (b*Sin[2*(c + d*x)])/(4*d)
```

fricas [A] time = 0.43, size = 29, normalized size = 0.97

$$\frac{(2\,a+b)dx-b\cos\left(dx+c\right)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + b)*d*x - b*cos(d*x + c)*sin(d*x + c))/d
```

giac [A] time = 0.12, size = 25, normalized size = 0.83

$$\frac{1}{4}b\left(2x - \frac{\sin(2dx + 2c)}{d}\right) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*d*x + 2*c)/d) + a*x
```

maple [A] time = 0.07, size = 32, normalized size = 1.07

$$ax + \frac{b\left(-\frac{\cos(dx+c)\sin(dx+c)}{2} + \frac{dx}{2} + \frac{c}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(d*x+c)^2,x)
```

```
[Out] a*x+b/d*(-1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)
```

maxima [A] time = 0.33, size = 29, normalized size = 0.97

$$ax + \frac{(2dx + 2c - \sin(2dx + 2c))b}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] a*x + 1/4*(2*d*x + 2*c - sin(2*d*x + 2*c))*b/d
```

mupad [B] time = 13.40, size = 27, normalized size = 0.90

$$-\frac{\frac{b\sin(2c+2dx)}{4} - dx\left(a + \frac{b}{2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(c + d*x)^2,x)
```

```
[Out] -((b*sin(2*c + 2*d*x))/4 - d*x*(a + b/2))/d
```

sympy [A] time = 0.35, size = 51, normalized size = 1.70

$$ax + b\left(\begin{cases} \frac{x\sin^2(c+dx)}{2} + \frac{x\cos^2(c+dx)}{2} - \frac{\sin(c+dx)\cos(c+dx)}{2d} & \text{for}\ d \neq 0 \\ x\sin^2(c) & \text{otherwise} \end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(d*x+c)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sin(c + d*x)**2/2 + x*cos(c + d*x)**2/2 - sin(c + d*x)
*cos(c + d*x)/(2*d), Ne(d, 0)), (x*sin(c)**2, True))
```

3.71 $\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=16

$$bx-\frac{a\cot(c+dx)}{d}$$

```
[Out] b*x-a*cot(d*x+c)/d
```

Rubi [A] time = 0.02, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 8}

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] b*x - (a*Cot[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx&=-\frac{a\cot(c+dx)}{d}+b\int 1\,dx\\&=bx-\frac{a\cot(c+dx)}{d}\end{aligned}$$

Mathematica [A] time = 0.02, size = 16, normalized size = 1.00

$$bx-\frac{a\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] b*x - (a*Cot[c + d*x])/d
```

fricas [A] time = 0.43, size = 32, normalized size = 2.00

$$\frac{bdx\sin\left(dx+c\right)-a\cos\left(dx+c\right)}{d\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] (b*d*x*sin(d*x + c) - a*cos(d*x + c))/(d*sin(d*x + c))
```

giac [B] time = 0.13, size = 39, normalized size = 2.44

$$\frac{2\,(dx+c)b + a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{a}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(2*(d*x + c)*b + a*tan(1/2*d*x + 1/2*c) - a/tan(1/2*d*x + 1/2*c))/d
```

maple [A] time = 0.43, size = 22, normalized size = 1.38

$$\frac{-\cot\left(dx+c\right)a + \left(dx+c\right)b}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x)
[Out] 1/d*(-cot(d*x+c)*a+(d*x+c)*b)
```

maxima [A] time = 0.46, size = 23, normalized size = 1.44

$$\frac{(dx+c)b - \frac{a}{\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] ((d*x + c)*b - a/tan(d*x + c))/d
```

mupad [B] time = 13.36, size = 16, normalized size = 1.00

$$b\,x - \frac{a\cot\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^2,x)
[Out] b*x - (a*cot(c + d*x))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b\sin^2\left(c+dx\right)\right)\csc^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2*(a+b*sin(d*x+c)**2),x)
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**2, x)
```

3.72 $\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=43

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

```
[Out] -1/3*(2*a+3*b)*cot(d*x+c)/d-1/3*a*cot(d*x+c)*csc(d*x+c)^2/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3012, 3767, 8}

$$-\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a + 3*b)*Cot[c + d*x])/(3*d) - (a*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^4(c+dx)\left(a+b\sin^2(c+dx)\right)dx &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}+\frac{1}{3}(2a+3b)\int \csc^2(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{(2a+3b)\,\text{Subst}(\int 1\,dx,x,\cot(c+dx))}{3d}\\ &= -\frac{(2a+3b)\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}\end{aligned}$$

Mathematica [A] time = 0.03, size = 49, normalized size = 1.14

$$-\frac{2a\cot(c+dx)}{3d}-\frac{a\cot(c+dx)\csc^2(c+dx)}{3d}-\frac{b\cot(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-2*a*Cot[c + d*x])/(3*d) - (b*Cot[c + d*x])/d - (a*Cot[c + d*x]*Csc[c + d*
x]^2)/(3*d)
```

fricas [A] time = 0.40, size = 54, normalized size = 1.26

$$-\frac{(2\,a+3\,b)\cos\left(dx+c\right)^{3}-3\,(a+b)\cos\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^{2}-d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/3*((2*a + 3*b)*cos(d*x + c)^3 - 3*(a + b)*cos(d*x + c))/((d*cos(d*x + c)
^2 - d)*sin(d*x + c))
```

giac [A] time = 0.15, size = 37, normalized size = 0.86

$$-\frac{3\,a\tan\left(dx+c\right)^{2}+3\,b\tan\left(dx+c\right)^{2}+a}{3\,d\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/3*(3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

maple [A] time = 0.59, size = 35, normalized size = 0.81

$$\frac{a\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot\left(dx+c\right)-b\cot\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-2/3-1/3*csc(d*x+c)^2)*cot(d*x+c)-b*cot(d*x+c))
```

maxima [A] time = 0.33, size = 28, normalized size = 0.65

$$-\frac{3\,(a+b)\tan\left(dx+c\right)^{2}+a}{3\,d\tan\left(dx+c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(a + b)*tan(d*x + c)^2 + a)/(d*tan(d*x + c)^3)
```

mupad [B] time = 13.37, size = 29, normalized size = 0.67

$$-\frac{a\cot\left(c+d\,x\right)^{3}}{3\,d}-\frac{\cot\left(c+d\,x\right)\,\left(a+b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^4,x)
```

```
[Out] - (a*cot(c + d*x)^3)/(3*d) - (cot(c + d*x)*(a + b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{2}\left(c+dx\right)\right)\csc^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4*(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)*csc(c + d*x)**4, x)
```

3.73 $\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx$

Optimal. Leaf size=65

$$-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

```
[Out] -1/5*(4*a+5*b)*cot(d*x+c)/d-1/15*(4*a+5*b)*cot(d*x+c)^3/d-1/5*a*cot(d*x+c)*
csc(d*x+c)^4/d
```

Rubi [A] time = 0.04, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3012, 3767}

$$-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((4*a + 5*b)*Cot[c + d*x])/(5*d) - ((4*a + 5*b)*Cot[c + d*x]^3)/(15*d) - (
a*Cot[c + d*x]*Csc[c + d*x]^4)/(5*d)
```

Rule 3012

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x
_)]^2), x_Symbol] :> Simp[(A*Cos[e + f*x]*(b*Sin[e + f*x])^(m + 1))/(b*f*(m
 + 1)), x] + Dist[(A*(m + 2) + C*(m + 1))/(b^2*(m + 1)), Int[(b*Sin[e + f*x
])^(m + 2), x], x] /; FreeQ[{b, e, f, A, C}, x] && LtQ[m, -1]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \csc^6(c+dx)\left(a+b\sin^2(c+dx)\right)\,dx &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}+\frac{1}{5}(4a+5b)\int \csc^4(c+dx)\,dx\\ &= -\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{(4a+5b)\operatorname{Subst}\left(\int\left(1+x^2\right)\,dx,x,\cot(c+\\ &= -\frac{(4a+5b)\cot(c+dx)}{5d}-\frac{(4a+5b)\cot^3(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c\end{aligned}$$

Mathematica [A] time = 0.03, size = 95, normalized size = 1.46

$$-\frac{8a\cot(c+dx)}{15d}-\frac{a\cot(c+dx)\csc^4(c+dx)}{5d}-\frac{4a\cot(c+dx)\csc^2(c+dx)}{15d}-\frac{2b\cot(c+dx)}{3d}-\frac{b\cot(c+dx)\csc^2(c+d}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6*(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-8*a*Cot[c + d*x])/(15*d) - (2*b*Cot[c + d*x])/(3*d) - (4*a*Cot[c + d*x]*C
sc[c + d*x]^2)/(15*d) - (b*Cot[c + d*x]*Csc[c + d*x]^2)/(3*d) - (a*Cot[c +
d*x]*Csc[c + d*x]^4)/(5*d)
```

fricas [A] time = 0.41, size = 81, normalized size = 1.25

$$-\frac{2\,(4\,a+5\,b)\cos\left(dx+c\right)^{5}-5\,(4\,a+5\,b)\cos\left(dx+c\right)^{3}+15\,(a+b)\cos\left(dx+c\right)}{15\,\left(d\cos\left(dx+c\right)^{4}-2\,d\cos\left(dx+c\right)^{2}+d\right)\sin\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/15*(2*(4*a + 5*b)*cos(d*x + c)^5 - 5*(4*a + 5*b)*cos(d*x + c)^3 + 15*(a
+ b)*cos(d*x + c))/((d*cos(d*x + c)^4 - 2*d*cos(d*x + c)^2 + d)*sin(d*x + c
))
```

giac [A] time = 0.15, size = 61, normalized size = 0.94

$$-\frac{15\,a\tan\left(dx+c\right)^{4}+15\,b\tan\left(dx+c\right)^{4}+10\,a\tan\left(dx+c\right)^{2}+5\,b\tan\left(dx+c\right)^{2}+3\,a}{15\,d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a*tan(d*x + c)^2 + 5*
b*tan(d*x + c)^2 + 3*a)/(d*tan(d*x + c)^5)
```

maple [A] time = 0.58, size = 56, normalized size = 0.86

$$\frac{a\left(-\frac{8}{15}-\frac{\left(\csc^{4}(dx+c)\right)}{5}-\frac{4\left(\csc^{2}(dx+c)\right)}{15}\right)\cot\left(dx+c\right)+b\left(-\frac{2}{3}-\frac{\left(\csc^{2}(dx+c)\right)}{3}\right)\cot\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(a*(-8/15-1/5*csc(d*x+c)^4-4/15*csc(d*x+c)^2)*cot(d*x+c)+b*(-2/3-1/3*cs
c(d*x+c)^2)*cot(d*x+c))
```

maxima [A] time = 0.38, size = 45, normalized size = 0.69

$$-\frac{15\,(a+b)\tan\left(dx+c\right)^{4}+5\,(2\,a+b)\tan\left(dx+c\right)^{2}+3\,a}{15\,d\tan\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6*(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a + b)*tan(d*x + c)^4 + 5*(2*a + b)*tan(d*x + c)^2 + 3*a)/(d*tan
(d*x + c)^5)
```

mupad [B] time = 13.38, size = 49, normalized size = 0.75

$$-\frac{a\cot\left(c+d\,x\right)^{5}}{5\,d}-\frac{\cot\left(c+d\,x\right)\,\left(a+b\right)}{d}-\frac{\cot\left(c+d\,x\right)^{3}\,\left(\frac{2\,a}{3}+\frac{b}{3}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)/sin(c + d*x)^6,x)
```

```
[Out] - (a*cot(c + d*x)^5)/(5*d) - (cot(c + d*x)*(a + b))/d - (cot(c + d*x)^3*((2
*a)/3 + b/3))/d
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6*(a+b*sin(d*x+c)**2),x)
[Out] Timed out
```

3.74 $\int \left(a+b\sin^2(x)\right)\,dx$

Optimal. Leaf size=19

$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(x)*sin(x)
```

Rubi [A] time = 0.01, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {2635, 8}

$$ax+\frac{bx}{2}-\frac{1}{2}b\sin(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[x]^2,x]
[Out] a*x + (b*x)/2 - (b*Cos[x]*Sin[x])/2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)\,dx &= ax+b\int \sin^2(x)\,dx\\ &= ax-\frac{1}{2}b\cos(x)\sin(x)+\frac{1}{2}b\int 1\,dx\\ &= ax+\frac{bx}{2}-\frac{1}{2}b\cos(x)\sin(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.00

$$ax+\frac{bx}{2}-\frac{1}{4}b\sin(2x)$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[x]^2,x]
[Out] a*x + (b*x)/2 - (b*Sin[2*x])/4
```

fricas [A] time = 0.43, size = 16, normalized size = 0.84

$$-\frac{1}{2}\,b\cos(x)\sin(x)+\frac{1}{2}\,(2\,a+b)x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="fricas")
```

```
[Out] -1/2*b*cos(x)*sin(x) + 1/2*(2*a + b)*x
```

giac [A] time = 0.13, size = 17, normalized size = 0.89

$$\frac{1}{4}\,b(2\,x - \sin{(2\,x)}) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="giac")
[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

maple [A] time = 0.06, size = 17, normalized size = 0.89

$$ax + b\left(-\frac{\sin(x)\cos(x)}{2} + \frac{x}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(x)^2,x)
[Out] a*x+b*(-1/2*sin(x)*cos(x)+1/2*x)
```

maxima [A] time = 0.34, size = 17, normalized size = 0.89

$$\frac{1}{4}\,b(2\,x - \sin{(2\,x)}) + ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)^2,x, algorithm="maxima")
[Out] 1/4*b*(2*x - sin(2*x)) + a*x
```

mupad [B] time = 13.31, size = 15, normalized size = 0.79

$$x\,\left(a + \frac{b}{2}\right) - \frac{b\,\sin{(2\,x)}}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(x)^2,x)
[Out] x*(a + b/2) - (b*sin(2*x))/4
```

sympy [A] time = 0.06, size = 15, normalized size = 0.79

$$ax + b\left(\frac{x}{2} - \frac{\sin{(x)}\cos{(x)}}{2}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(x)**2,x)
[Out] a*x + b*(x/2 - sin(x)*cos(x)/2)
```

3.75 $\int \left(a+b\sin^2(x)\right)^2\,dx$

Optimal. Leaf size=50

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(x)*sin(x)-1/4*b^2*cos(x)*sin(
x)^3
```

Rubi [A] time = 0.02, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.100, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{1}{8}b(8a+3b)\sin(x)\cos(x)-\frac{1}{4}b^2\sin^3(x)\cos(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[x]*Sin[x])/8 - (b^2*Cos[
x]*Sin[x]^3)/4
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^2\,dx=\frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{1}{8}b(8a+3b)\cos(x)\sin(x)-\frac{1}{4}b^2\cos(x)\sin^3(x)$$

Mathematica [A] time = 0.06, size = 43, normalized size = 0.86

$$\frac{1}{32}\left(4x\left(8a^2+8ab+3b^2\right)-8b(2a+b)\sin(2x)+b^2\sin(4x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*x - 8*b*(2*a + b)*Sin[2*x] + b^2*Sin[4*x])/32
```

fricas [A] time = 0.41, size = 47, normalized size = 0.94

$$\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x+\frac{1}{8}\left(2\,b^2\cos(x)^3-\left(8\,ab+5\,b^2\right)\cos(x)\right)\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/8*(2*b^2*cos(x)^3 - (8*a*b + 5*b^2)*cos(x
))*sin(x)
```

giac [A] time = 0.13, size = 42, normalized size = 0.84

$$\frac{1}{32}\,b^2\sin\left(4\,x\right)+\frac{1}{8}\left(8\,a^2+8\,ab+3\,b^2\right)x-\frac{1}{4}\left(2\,ab+b^2\right)\sin\left(2\,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/32*b^2*sin(4*x) + 1/8*(8*a^2 + 8*a*b + 3*b^2)*x - 1/4*(2*a*b + b^2)*sin(2
*x)
```

maple [A] time = 0.34, size = 42, normalized size = 0.84

$$b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+2ab\left(-\frac{\sin(x)\cos(x)}{2}+\frac{x}{2}\right)+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^2,x)
```

```
[Out] b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+2*a*b*(-1/2*sin(x)*cos(x)+1/2
*x)+a^2*x
```

maxima [A] time = 0.33, size = 39, normalized size = 0.78

$$\frac{1}{32}\,b^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))+\frac{1}{2}\,ab(2\,x-\sin\left(2\,x\right))+a^2x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/32*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + 1/2*a*b*(2*x - sin(2*x)) + a^2*x
```

mupad [B] time = 13.52, size = 44, normalized size = 0.88

$$x\,a^2-\sin(x)\,a\,b\,\cos(x)+x\,a\,b+\frac{\sin(x)\,b^2\,{\cos(x)}^3}{4}-\frac{5\,\sin(x)\,b^2\,\cos(x)}{8}+\frac{3\,x\,b^2}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^2,x)
```

```
[Out] a^2*x + (3*b^2*x)/8 + (b^2*cos(x)^3*sin(x))/4 + a*b*x - (5*b^2*cos(x)*sin(x
))/8 - a*b*cos(x)*sin(x)
```

sympy [B] time = 0.76, size = 110, normalized size = 2.20

$$a^2x+abx\sin^2\left(x\right)+abx\cos^2\left(x\right)-ab\sin\left(x\right)\cos\left(x\right)+\frac{3b^2x\sin^4\left(x\right)}{8}+\frac{3b^2x\sin^2\left(x\right)\cos^2\left(x\right)}{4}+\frac{3b^2x\cos^4\left(x\right)}{8}-\frac{5b^2\sin^3\left(x\right)\cos\left(x\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**2,x)
```

```
[Out] a**2*x + a*b*x*sin(x)**2 + a*b*x*cos(x)**2 - a*b*sin(x)*cos(x) + 3*b**2*x*s
in(x)**4/8 + 3*b**2*x*sin(x)**2*cos(x)**2/4 + 3*b**2*x*cos(x)**4/8 - 5*b**2
*sin(x)**3*cos(x)/8 - 3*b**2*sin(x)*cos(x)**3/8
```

3.76 $\int \left(a+b\sin^2(x)\right)^3\,dx$

Optimal. Leaf size=87

$$\frac{1}{16}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{24}b^2(2a+b)\sin^3(x)\cos(x)-\frac{1}{6}b\sin(x)$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*x-1/48*b*(64*a^2+54*a*b+15*b^2)*cos(x)*sin
(x)-5/24*b^2*(2*a+b)*cos(x)*sin(x)^3-1/6*b*cos(x)*sin(x)*(a+b*sin(x)^2)^2
```

Rubi [A] time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3180, 3169}

$$\frac{1}{16}x(2a+b)\left(8a^2+8ab+5b^2\right)-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\sin(x)\cos(x)-\frac{5}{24}b^2(2a+b)\sin^3(x)\cos(x)-\frac{1}{6}b\sin(x)$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^3,x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x)/16 - (b*(64*a^2 + 54*a*b + 15*b^2)*Co
s[x]*Sin[x])/48 - (5*b^2*(2*a + b)*Cos[x]*Sin[x]^3)/24 - (b*Cos[x]*Sin[x]*(
a + b*Sin[x]^2)^2)/6
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(x)\right)^3\,dx &= -\frac{1}{6}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2+\frac{1}{6}\int \left(a+b\sin^2(x)\right)\left(a(6a+b)+5b(2a+b)\sin^2\right.\\ &= \frac{1}{16}(2a+b)\left(8a^2+8ab+5b^2\right)x-\frac{1}{48}b\left(64a^2+54ab+15b^2\right)\cos(x)\sin(x)-\frac{5}{24}b^2(2a\end{aligned}$$

Mathematica [C] time = 0.10, size = 80, normalized size = 0.92

$$\frac{1}{192}\left(12x(2a+b)\left(8a^2+8ab+5b^2\right)+9b^2(2a+b)\sin(4x)+9ib(4ia+(1+2i)b)(4a+(2+i)b)\sin(2x)+b^3(-\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^3,x]
```

```
[Out] (12*(2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*x + (9*I)*b*((4*I)*a + (1 + 2*I)*b)*(
4*a + (2 + I)*b)*Sin[2*x] + 9*b^2*(2*a + b)*Sin[4*x] - b^3*Sin[6*x])/192
```

fricas [A] time = 0.43, size = 81, normalized size = 0.93

$$\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x-\frac{1}{48}\left(8\,b^3\cos(x)^5-2\left(18\,ab^2+13\,b^3\right)\cos(x)^3+3\left(24\,a^2b+30\,ab^2+11\,b^3\right)\mathrm{c}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="fricas")
```

```
[Out] 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x - 1/48*(8*b^3*cos(x)^5 - 2*(1
8*a*b^2 + 13*b^3)*cos(x)^3 + 3*(24*a^2*b + 30*a*b^2 + 11*b^3)*cos(x))*sin(x
)
```

giac [A] time = 0.15, size = 76, normalized size = 0.87

$$-\frac{1}{192}\,b^3\sin\left(6\,x\right)+\frac{1}{16}\left(16\,a^3+24\,a^2b+18\,ab^2+5\,b^3\right)x+\frac{3}{64}\left(2\,ab^2+b^3\right)\sin\left(4\,x\right)-\frac{3}{64}\left(16\,a^2b+16\,ab^2+5\,b^3\right)\sin$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="giac")
```

```
[Out] -1/192*b^3*sin(6*x) + 1/16*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*x + 3/64*
(2*a*b^2 + b^3)*sin(4*x) - 3/64*(16*a^2*b + 16*a*b^2 + 5*b^3)*sin(2*x)
```

maple [A] time = 0.47, size = 73, normalized size = 0.84

$$b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}+\frac{5x}{16}\right)+3a\,b^2\left(-\frac{\left(\sin^3(x)+\frac{3\sin(x)}{2}\right)\cos(x)}{4}+\frac{3x}{8}\right)+3a^2b\left(-\frac{\sin(x)\cos(x)}{2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^3,x)
```

```
[Out] b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+3*a*b^2*(-1/4*
(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+3*a^2*b*(-1/2*sin(x)*cos(x)+1/2*x)+a^3*
x
```

maxima [A] time = 0.35, size = 71, normalized size = 0.82

$$\frac{1}{192}\left(4\,\sin\left(2\,x\right)^3+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)b^3+\frac{3}{32}\,ab^2(12\,x+\sin\left(4\,x\right)-8\,\sin\left(2\,x\right))+\frac{3}{4}\,a^2b(2\,x-\sin\left(2\,x\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/192*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*b^3 + 3/32*a*b^2*(12
*x + sin(4*x) - 8*sin(2*x)) + 3/4*a^2*b*(2*x - sin(2*x)) + a^3*x
```

mupad [B] time = 14.13, size = 118, normalized size = 1.36

$$a^3\,x+\frac{5\,b^3\,x}{16}-\frac{\left(72\,a^2\,b+90\,a\,b^2+33\,b^3\right)\,\tan(x)^5+\left(144\,a^2\,b+144\,a\,b^2+40\,b^3\right)\,\tan(x)^3+\left(72\,a^2\,b+54\,a\,b^2+15\,b\right.}{48\,\tan(x)^6+144\,\tan(x)^4+144\,\tan(x)^2+48}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^3,x)
```

```
[Out] a^3*x + (5*b^3*x)/16 - (tan(x)^5*(90*a*b^2 + 72*a^2*b + 33*b^3) + tan(x)^3*
(144*a*b^2 + 144*a^2*b + 40*b^3) + tan(x)*(54*a*b^2 + 72*a^2*b + 15*b^3))/(
```

```
144*tan(x)^2 + 144*tan(x)^4 + 48*tan(x)^6 + 48) + (9*a*b^2*x)/8 + (3*a^2*b*
x)/2
```

sympy [B] time = 2.76, size = 246, normalized size = 2.83

$$a^3x+\frac{3a^2bx\sin^2(x)}{2}+\frac{3a^2bx\cos^2(x)}{2}-\frac{3a^2b\sin(x)\cos(x)}{2}+\frac{9ab^2x\sin^4(x)}{8}+\frac{9ab^2x\sin^2(x)\cos^2(x)}{4}+\frac{9ab^2x\cos^4}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**3,x)
```

```
[Out] a**3*x + 3*a**2*b*x*sin(x)**2/2 + 3*a**2*b*x*cos(x)**2/2 - 3*a**2*b*sin(x)*
cos(x)/2 + 9*a*b**2*x*sin(x)**4/8 + 9*a*b**2*x*sin(x)**2*cos(x)**2/4 + 9*a*
b**2*x*cos(x)**4/8 - 15*a*b**2*sin(x)**3*cos(x)/8 - 9*a*b**2*sin(x)*cos(x)*
*3/8 + 5*b**3*x*sin(x)**6/16 + 15*b**3*x*sin(x)**4*cos(x)**2/16 + 15*b**3*x
*sin(x)**2*cos(x)**4/16 + 5*b**3*x*cos(x)**6/16 - 11*b**3*sin(x)**5*cos(x)/
16 - 5*b**3*sin(x)**3*cos(x)**3/6 - 5*b**3*sin(x)*cos(x)**5/16
```

3.77 $\int \left(a + b \sin^2(x)\right)^4 dx$

Optimal. Leaf size=140

$$-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{384}b\left(608a^3+808a^2b+480ab^2+105b^3\right)\sin(x)\cos(x)+\frac{1}{128}x\,(1$$

```
[Out] 1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*x-1/384*b*(608*a^3+8
08*a^2*b+480*a*b^2+105*b^3)*cos(x)*sin(x)-1/192*b^2*(104*a^2+104*a*b+35*b^2
)*cos(x)*sin(x)^3-7/48*b*(2*a+b)*cos(x)*sin(x)*(a+b*sin(x)^2)^2-1/8*b*cos(x
)*sin(x)*(a+b*sin(x)^2)^3
```

Rubi [A] time = 0.17, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3180, 3170, 3169}

$$\frac{1}{128}x\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)-\frac{1}{192}b^2\left(104a^2+104ab+35b^2\right)\sin^3(x)\cos(x)-\frac{1}{384}b\left(808a^2b\right.$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^2)^4,x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x)/128 - (b*(608*
a^3 + 808*a^2*b + 480*a*b^2 + 105*b^3)*Cos[x]*Sin[x])/384 - (b^2*(104*a^2 +
 104*a*b + 35*b^2)*Cos[x]*Sin[x]^3)/192 - (7*b*(2*a + b)*Cos[x]*Sin[x]*(a +
 b*Sin[x]^2)^2)/48 - (b*Cos[x]*Sin[x]*(a + b*Sin[x]^2)^3)/8
```

Rule 3169

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)*((A_.) + (B_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[((4*A*(2*a + b) + B*(4*a + 3*b))*x)/8, x] +
(-Simp[(b*B*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f), x] - Simp[((4*A*b + B*(4*a
+ 3*b))*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a, b, e, f, A, B},
x]
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rubi steps

$$\int \left(a+b\sin^2(x)\right)^4 dx = -\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3+\frac{1}{8}\int \left(a+b\sin^2(x)\right)^2\left(a(8a+b)+7b(2a+b)\sin\right.$$
$$= -\frac{7}{48}b(2a+b)\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^2-\frac{1}{8}b\cos(x)\sin(x)\left(a+b\sin^2(x)\right)^3+\frac{1}{48}$$
$$= \frac{1}{128}\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)x-\frac{1}{384}b\left(608a^3+808a^2b+480a\right.$$

Mathematica [A] time = 0.16, size = 113, normalized size = 0.81

$$\frac{24b^2\left(24a^2+24ab+7b^2\right)\sin(4x)-96b(2a+b)\left(16a^2+16ab+7b^2\right)\sin(2x)+24x\left(128a^4+256a^3b+288a^2b\right.}{3072}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^2)^4,x]
```

```
[Out] (24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x - 96*b*(2*a
+ b)*(16*a^2 + 16*a*b + 7*b^2)*Sin[2*x] + 24*b^2*(24*a^2 + 24*a*b + 7*b^2)*
Sin[4*x] - 32*b^3*(2*a + b)*Sin[6*x] + 3*b^4*Sin[8*x])/3072
```

fricas [A] time = 0.44, size = 123, normalized size = 0.88

$$\frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x+\frac{1}{384}\left(48\,b^4\cos(x)^7-8\left(32\,ab^3+25\,b^4\right)\cos(x)^5+2\left(28\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="fricas")
```

```
[Out] 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*x + 1/384*(4
8*b^4*cos(x)^7 - 8*(32*a*b^3 + 25*b^4)*cos(x)^5 + 2*(288*a^2*b^2 + 416*a*b^
3 + 163*b^4)*cos(x)^3 - 3*(256*a^3*b + 480*a^2*b^2 + 352*a*b^3 + 93*b^4)*co
s(x))*sin(x)
```

giac [A] time = 0.14, size = 118, normalized size = 0.84

$$\frac{1}{1024}\,b^4\sin(8\,x)+\frac{1}{128}\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)x-\frac{1}{96}\left(2\,ab^3+b^4\right)\sin(6\,x)+\frac{1}{128}\left(24\,a^2\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="giac")
```

```
[Out] 1/1024*b^4*sin(8*x) + 1/128*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3
+ 35*b^4)*x - 1/96*(2*a*b^3 + b^4)*sin(6*x) + 1/128*(24*a^2*b^2 + 24*a*b^3
+ 7*b^4)*sin(4*x) - 1/32*(32*a^3*b + 48*a^2*b^2 + 30*a*b^3 + 7*b^4)*sin(2*x
)
```

maple [A] time = 0.56, size = 110, normalized size = 0.79

$$b^4\left(-\frac{\left(\sin^7(x)+\frac{7\left(\sin^5(x)\right)}{6}+\frac{35\left(\sin^3(x)\right)}{24}+\frac{35\sin(x)}{16}\right)\cos(x)}{8}+\frac{35x}{128}\right)+4a\,b^3\left(-\frac{\left(\sin^5(x)+\frac{5\left(\sin^3(x)\right)}{4}+\frac{15\sin(x)}{8}\right)\cos(x)}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(x)^2)^4,x)
```

```
[Out] b^4*(-1/8*(sin(x)^7+7/6*sin(x)^5+35/24*sin(x)^3+35/16*sin(x))*cos(x)+35/128
*x)+4*a*b^3*(-1/6*(sin(x)^5+5/4*sin(x)^3+15/8*sin(x))*cos(x)+5/16*x)+6*a^2*
b^2*(-1/4*(sin(x)^3+3/2*sin(x))*cos(x)+3/8*x)+4*a^3*b*(-1/2*sin(x)*cos(x)+1
/2*x)+a^4*x
```

maxima [A] time = 0.34, size = 108, normalized size = 0.77

$$\frac{1}{48}\left(4\,\sin\left(2\,x\right)^{3}+60\,x+9\,\sin\left(4\,x\right)-48\,\sin\left(2\,x\right)\right)ab^{3}+\frac{1}{3072}\left(128\,\sin\left(2\,x\right)^{3}+840\,x+3\,\sin\left(8\,x\right)+168\,\sin\left(4\,x\right)\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(4*sin(2*x)^3 + 60*x + 9*sin(4*x) - 48*sin(2*x))*a*b^3 + 1/3072*(128*s
in(2*x)^3 + 840*x + 3*sin(8*x) + 168*sin(4*x) - 768*sin(2*x))*b^4 + 3/16*a^
2*b^2*(12*x + sin(4*x) - 8*sin(2*x)) + a^3*b*(2*x - sin(2*x)) + a^4*x
```

mupad [B] time = 13.63, size = 147, normalized size = 1.05

$$x\,a^4-2\,\sin(x)\,a^3\,b\,\cos(x)+2\,x\,a^3\,b+\frac{3\,\sin(x)\,a^2\,b^2\,\cos(x)^3}{2}-\frac{15\,\sin(x)\,a^2\,b^2\,\cos(x)}{4}+\frac{9\,x\,a^2\,b^2}{4}-\frac{2\,\sin(x)\,a\,b^3\,\cos(x)^5}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(x)^2)^4,x)
```

```
[Out] a^4*x + (35*b^4*x)/128 + (163*b^4*cos(x)^3*sin(x))/192 - (25*b^4*cos(x)^5*s
in(x))/48 + (b^4*cos(x)^7*sin(x))/8 + (9*a^2*b^2*x)/4 - (93*b^4*cos(x)*sin(
x))/128 + (5*a*b^3*x)/4 + 2*a^3*b*x + (3*a^2*b^2*cos(x)^3*sin(x))/2 - (11*a
*b^3*cos(x)*sin(x))/4 - 2*a^3*b*cos(x)*sin(x) - (15*a^2*b^2*cos(x)*sin(x))/
4 + (13*a*b^3*cos(x)^3*sin(x))/6 - (2*a*b^3*cos(x)^5*sin(x))/3
```

sympy [B] time = 7.52, size = 410, normalized size = 2.93

$$a^4x+2a^3bx\sin^2(x)+2a^3bx\cos^2(x)-2a^3b\sin(x)\cos(x)+\frac{9a^2b^2x\sin^4(x)}{4}+\frac{9a^2b^2x\sin^2(x)\cos^2(x)}{2}+\frac{9a^2b^2x\cos^4(x)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(x)**2)**4,x)
```

```
[Out] a**4*x + 2*a**3*b*x*sin(x)**2 + 2*a**3*b*x*cos(x)**2 - 2*a**3*b*sin(x)*cos(
x) + 9*a**2*b**2*x*sin(x)**4/4 + 9*a**2*b**2*x*sin(x)**2*cos(x)**2/2 + 9*a*
*2*b**2*x*cos(x)**4/4 - 15*a**2*b**2*sin(x)**3*cos(x)/4 - 9*a**2*b**2*sin(x
)*cos(x)**3/4 + 5*a*b**3*x*sin(x)**6/4 + 15*a*b**3*x*sin(x)**4*cos(x)**2/4
+ 15*a*b**3*x*sin(x)**2*cos(x)**4/4 + 5*a*b**3*x*cos(x)**6/4 - 11*a*b**3*si
n(x)**5*cos(x)/4 - 10*a*b**3*sin(x)**3*cos(x)**3/3 - 5*a*b**3*sin(x)*cos(x)
**5/4 + 35*b**4*x*sin(x)**8/128 + 35*b**4*x*sin(x)**6*cos(x)**2/32 + 105*b*
*4*x*sin(x)**4*cos(x)**4/64 + 35*b**4*x*sin(x)**2*cos(x)**6/32 + 35*b**4*x*
cos(x)**8/128 - 93*b**4*sin(x)**7*cos(x)/128 - 511*b**4*sin(x)**5*cos(x)**3
/384 - 385*b**4*sin(x)**3*cos(x)**5/384 - 35*b**4*sin(x)*cos(x)**7/128
```

3.78 $\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=106

$$\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}d\sqrt{a+b}}-\frac{\left(a^2-ab+b^2\right)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

```
[Out] -(a^2-a*b+b^2)*cos(d*x+c)/b^3/d-1/3*(a-2*b)*cos(d*x+c)^3/b^2/d-1/5*cos(d*x+
c)^5/b/d+a^3*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 390, 208}

$$-\frac{\left(a^2-ab+b^2\right)\cos(c+dx)}{b^3d}+\frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}d\sqrt{a+b}}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(7/2)*Sqrt[a + b]*d) -
 ((a^2 - a*b + b^2)*Cos[c + d*x])/(b^3*d) - ((a - 2*b)*Cos[c + d*x]^3)/(3*b
^2*d) - Cos[c + d*x]^5/(5*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(\frac{a^2-ab+b^2}{b^3}+\frac{(a-2b)x^2}{b^2}+\frac{x^4}{b}-\frac{a^3}{b^3(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c+dx)}{5bd}+\frac{a^3\,\text{Subst}\left(\int \frac{}{a+b}\right.}{}$$
$$= \frac{a^3\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{7/2}\sqrt{a+b}\,d}-\frac{(a^2-ab+b^2)\cos(c+dx)}{b^3d}-\frac{(a-2b)\cos^3(c+dx)}{3b^2d}-\frac{\cos^5(c}{5b}$$

Mathematica [C] time = 1.44, size = 180, normalized size = 1.70

$$\frac{-240a^3\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-240a^3\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-2\sqrt{b}\sqrt{-a-b}\cos(c+dx)\left(120a^2+4b(5a\right.}{240b^{7/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-240*a^3*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 240
*a^3*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] - 2*Sqrt[-
a - b]*Sqrt[b]*Cos[c + d*x]*(120*a^2 - 100*a*b + 89*b^2 + 4*(5*a - 7*b)*b*C
os[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)]))/(240*Sqrt[-a - b]*b^(7/2)*d)
```

fricas [A] time = 0.46, size = 272, normalized size = 2.57

$$\left[-\frac{6(ab^3+b^4)\cos(dx+c)^5-15\sqrt{ab+b^2}\,a^3\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+10(a^2b^2-ab^3-2b^4)\cos(\right.}{30(ab^4+b^5)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(6*(a*b^3 + b^4)*cos(d*x + c)^5 - 15*sqrt(a*b + b^2)*a^3*log((b*cos(
d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a
- b)) + 10*(a^2*b^2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 30*(a^3*b + b^4)*cos(
d*x + c))/((a*b^4 + b^5)*d), -1/15*(3*(a*b^3 + b^4)*cos(d*x + c)^5 + 15*sqr
t(-a*b - b^2)*a^3*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + 5*(a^2*b^
2 - a*b^3 - 2*b^4)*cos(d*x + c)^3 + 15*(a^3*b + b^4)*cos(d*x + c))/((a*b^4
+ b^5)*d)]
```

giac [B] time = 0.18, size = 332, normalized size = 3.13

$$-\frac{\frac{15a^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^3}-\frac{2\left(15a^2-10ab+8b^2-\frac{60a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{50ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{40b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{90a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{70ab(c}{(\cos}\right.}{b^3\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right.}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*a^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^3) - 2*(15*a^2 - 10*a*b + 8*b^2
- 60*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 50*a*b*(cos(d*x + c) - 1)/
(cos(d*x + c) + 1) - 40*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 90*a^2*
(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 70*a*b*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 80*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 60*a
^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 30*a*b*(cos(d*x + c) - 1)^3/
(cos(d*x + c) + 1)^3 + 15*a^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4)/(b
^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^5))/d
```

maple [A] time = 0.32, size = 110, normalized size = 1.04

$$-\frac{\frac{\frac{b^2\left(\cos^5(dx+c)\right)}{5}+\frac{ab\left(\cos^3(dx+c)\right)}{3}-\frac{2\left(\cos^3(dx+c)\right)b^2}{3}+a^2\cos(dx+c)-ab\cos(dx+c)+\cos(dx+c)b^2}{b^3}+\frac{a^3\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{b^3\sqrt{(a+b)b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/b^3*(1/5*b^2*cos(d*x+c)^5+1/3*a*b*cos(d*x+c)^3-2/3*cos(d*x+c)^3*b^2
+a^2*cos(d*x+c)-a*b*cos(d*x+c)+cos(d*x+c)*b^2)+a^3/b^3/((a+b)*b)^(1/2)*arct
anh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

maxima [A] time = 0.45, size = 116, normalized size = 1.09

$$-\frac{\frac{15\,a^3\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^3}+\frac{2\left(3\,b^2\cos(dx+c)^5+5\left(ab-2\,b^2\right)\cos(dx+c)^3+15\left(a^2-ab+b^2\right)\cos(dx+c)\right)}{b^3}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/30*(15*a^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt
((a + b)*b)))/(sqrt((a + b)*b)*b^3) + 2*(3*b^2*cos(d*x + c)^5 + 5*(a*b - 2*
b^2)*cos(d*x + c)^3 + 15*(a^2 - a*b + b^2)*cos(d*x + c))/b^3)/d
```

mupad [B] time = 0.16, size = 112, normalized size = 1.06

$$\frac{a^3\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{7/2}\,d\,\sqrt{a+b}}-\frac{\cos(c+d\,x)^5}{5\,b\,d}-\frac{\cos(c+d\,x)^3\,\left(\frac{a+b}{3\,b^2}-\frac{1}{b}\right)}{d}-\frac{\cos(c+d\,x)\,\left(\frac{3}{b}+\frac{(a+b)\,\left(\frac{a+b}{b^2}-\frac{3}{b}\right)}{b}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a^3*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(7/2)*d*(a + b)^(1/2))
 - cos(c + d*x)^5/(5*b*d) - (cos(c + d*x)^3*((a + b)/(3*b^2) - 1/b))/d - (c
os(c + d*x)*(3/b + ((a + b)*((a + b)/b^2 - 3/b))/b))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.79 $\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

```
[Out] (a-b)*cos(d*x+c)/b^2/d+1/3*cos(d*x+c)^3/b/d-a^2*arctanh(cos(d*x+c)*b^(1/2)/
(a+b)^(1/2))/b^(5/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 390, 208}

$$-\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}d\sqrt{a+b}}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^2*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(5/2)*Sqrt[a + b]*d)
) + ((a - b)*Cos[c + d*x])/(b^2*d) + Cos[c + d*x]^3/(3*b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a-b}{b^2}-\frac{x^2}{b}+\frac{a^2}{b^2(a+b-bx^2)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{a+b-bx^2}\,dx,x,\cos(c+dx)\right)}{b^2d}$$

$$= -\frac{a^2\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{5/2}\sqrt{a+b}\,d}+\frac{(a-b)\cos(c+dx)}{b^2d}+\frac{\cos^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.51, size = 150, normalized size = 1.95

$$\frac{6a^2\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+6a^2\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\sqrt{b}\sqrt{-a-b}\cos(c+dx)(6a+b\cos(2(c+d}{6b^{5/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (6*a^2*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + 6*a^2*
ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*
Sqrt[b]*Cos[c + d*x]*(6*a - 5*b + b*Cos[2*(c + d*x)]))/(6*Sqrt[-a - b]*b^(5
/2)*d)
```

fricas [A] time = 0.48, size = 218, normalized size = 2.83

$$\left[\frac{2\left(ab^2+b^3\right)\cos(dx+c)^3+3\sqrt{ab+b^2}\,a^2\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+6\left(a^2b-b^3\right)\cos(dx+c)}{6\left(ab^3+b^4\right)d},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/6*(2*(a*b^2 + b^3)*cos(d*x + c)^3 + 3*sqrt(a*b + b^2)*a^2*log(-(b*cos(d*
x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a -
b)) + 6*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d), 1/3*((a*b^2 + b^3)*c
os(d*x + c)^3 + 3*sqrt(-a*b - b^2)*a^2*arctan(sqrt(-a*b - b^2)*cos(d*x + c)
/(a + b)) + 3*(a^2*b - b^3)*cos(d*x + c))/((a*b^3 + b^4)*d)]
```

giac [B] time = 0.15, size = 173, normalized size = 2.25

$$\frac{\frac{3\,a^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,b^2}-\frac{2\left(3\,a-2\,b-\frac{6\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{6\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{b^2\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*b^2) - 2*(3*a - 2*b - 6*a*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + 6*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*
a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(b^2*((cos(d*x + c) - 1)/(cos(
d*x + c) + 1) - 1)^3))/d
```

maple [A] time = 0.30, size = 70, normalized size = 0.91

$$\frac{\frac{\frac{\left(\cos^{3}(dx+c)\right)b}{3}+a\cos(dx+c)-b\cos(dx+c)}{b^{2}}-\frac{a^{2}\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{b^{2}\sqrt{(a+b)b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(1/b^2*(1/3*cos(d*x+c)^3*b+a*cos(d*x+c)-b*cos(d*x+c))-a^2/b^2/((a+b)*b)
^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

maxima [A] time = 0.44, size = 88, normalized size = 1.14

$$\frac{\frac{3\,a^{2}\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b^{2}}+\frac{2\left(b\cos(dx+c)^{3}+3\,(a-b)\cos(dx+c)\right)}{b^{2}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/6*(3*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a
 + b)*b)))/(sqrt((a + b)*b)*b^2) + 2*(b*cos(d*x + c)^3 + 3*(a - b)*cos(d*x
+ c))/b^2)/d
```

mupad [B] time = 0.11, size = 72, normalized size = 0.94

$$\frac{\cos\left(c+d\,x\right)\,\left(\frac{a+b}{b^2}-\frac{2}{b}\right)}{d}+\frac{{\cos\left(c+d\,x\right)}^3}{3\,b\,d}-\frac{a^2\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)}{b^{5/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (cos(c + d*x)*((a + b)/b^2 - 2/b))/d + cos(c + d*x)^3/(3*b*d) - (a^2*atanh(
(b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(5/2)*d*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.80 $\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}-\frac{\cos(c+dx)}{bd}$$

```
[Out] -cos(d*x+c)/b/d+a*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/d/(a+b)^(
1/2)
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 388, 208}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}d\sqrt{a+b}}-\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(b^(3/2)*Sqrt[a + b]*d) - C
os[c + d*x]/(b*d)
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin^3(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1-x^2}{a+b-bx^2}\,dx,x,\cos(c+dx)\right)}{d}\\ &= -\frac{\cos(c+dx)}{bd}+\frac{a\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx,x,\cos(c+dx)\right)}{bd}\\ &= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{b^{3/2}\sqrt{a+b}\,d}-\frac{\cos(c+dx)}{bd}\end{aligned}$$

Mathematica [C] time = 0.24, size = 125, normalized size = 2.40

$$-\frac{\sqrt{b}\sqrt{-a-b}\cos(c+dx)+a\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+a\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{b^{3/2}d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + a*ArcTan
[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + Sqrt[-a - b]*Sqrt[b
]*Cos[c + d*x])/(Sqrt[-a - b]*b^(3/2)*d))
```

fricas [A] time = 0.47, size = 165, normalized size = 3.17

$$\left[\frac{\sqrt{ab+b^2}\,a\log\left(\frac{b\cos(dx+c)^2+2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)-2\left(ab+b^2\right)\cos\left(dx+c\right)}{2\left(ab^2+b^3\right)d},-\frac{\sqrt{-ab-b^2}\,a\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+}{a+b}\right.}{\left(ab^2+b^3\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a*b + b^2)*a*log((b*cos(d*x + c)^2 + 2*sqrt(a*b + b^2)*cos(d*x +
 c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a*b + b^2)*cos(d*x + c))/((a*
b^2 + b^3)*d), -(sqrt(-a*b - b^2)*a*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a
 + b)) + (a*b + b^2)*cos(d*x + c))/((a*b^2 + b^3)*d)]
```

giac [A] time = 0.17, size = 57, normalized size = 1.10

$$-\frac{a\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,bd}-\frac{\cos\left(dx+c\right)}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -a*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*b*d) - cos(d*x
 + c)/(b*d)
```

maple [A] time = 0.27, size = 45, normalized size = 0.87

$$\frac{-\frac{\cos(dx+c)}{b}+\frac{a\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{b\sqrt{(a+b)b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*(-1/b*cos(d*x+c)+a/b/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/
2)))
```

maxima [A] time = 0.43, size = 67, normalized size = 1.29

$$-\frac{\frac{a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,b}+\frac{2\cos(dx+c)}{b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
b)*b)))/(sqrt((a + b)*b)*b) + 2*cos(d*x + c)/b)/d
```

mupad [B] time = 0.10, size = 44, normalized size = 0.85

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{b^{3/2}\,d\,\sqrt{a+b}}-\frac{\cos\left(c+d\,x\right)}{b\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2)))/(b^(3/2)*d*(a + b)^(1/2)) -
 cos(c + d*x)/(b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.81 $\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=37

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

```
[Out] -arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/d/b^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(Sqrt[b]*Sqrt[a + b]*d))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(c+dx)}{a+b\sin^2(c+dx)}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{d}\\ &= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{\sqrt{b}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [C] time = 0.14, size = 97, normalized size = 2.62

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)+\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}\,d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]] + ArcTan[(Sqrt
[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/(Sqrt[-a - b]*Sqrt[b]*d)
```

fricas [**A**] time = 0.44, size = 117, normalized size = 3.16

$$\left[\frac{\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)}{2\sqrt{ab+b^2}d}, \frac{\sqrt{-ab-b^2}\arctan\left(\frac{\sqrt{-ab-b^2}\cos(dx+c)}{a+b}\right)}{(ab+b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*co
s(d*x + c)^2 - a - b))/(sqrt(a*b + b^2)*d), sqrt(-a*b - b^2)*arctan(sqrt(-a
*b - b^2)*cos(d*x + c)/(a + b))/((a*b + b^2)*d)]
```

giac [**A**] time = 0.17, size = 37, normalized size = 1.00

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*d)
```

maple [**A**] time = 0.20, size = 29, normalized size = 0.78

$$-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{d\sqrt{(a+b)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))
```

maxima [**A**] time = 0.43, size = 50, normalized size = 1.35

$$\frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{2\sqrt{(a+b)b}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*b
)))/(sqrt((a + b)*b)*d)
```

mupad [**B**] time = 0.09, size = 29, normalized size = 0.78

$$-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\cos(c+d\,x)}{\sqrt{a+b}}\right)}{\sqrt{b}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))/(b^(1/2)*d*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.82 $\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=55

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d+arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a/
d/(a+b)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3186, 391, 206, 208}

$$\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{ad\sqrt{a+b}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a*d)) + (Sqrt[b]*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sq
rt[a + b]])/(a*Sqrt[a + b]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a\sqrt{a+b}\,d}$$

Mathematica [C] time = 0.29, size = 143, normalized size = 2.60

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} - \log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right) + \log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((Sqrt[b]*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]])/Sq
rt[-a - b] + (Sqrt[b]*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/Sqrt[-a - b] + Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]])/(a*d)
)
```

fricas [A] time = 0.46, size = 161, normalized size = 2.93

$$\left[\frac{\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right) - \log\left(\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right) + \log\left(-\frac{1}{2}\cos(dx+c)+\frac{1}{2}\right)}{2\,ad}, -\frac{2\sqrt{-\frac{b}{a+b}}\ldots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))*cos
(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - log(1/2*cos(d*x + c) + 1/2
) + log(-1/2*cos(d*x + c) + 1/2))/(a*d), -1/2*(2*sqrt(-b/(a + b))*arctan(sq
rt(-b/(a + b))*cos(d*x + c)) + log(1/2*cos(d*x + c) + 1/2) - log(-1/2*cos(d
*x + c) + 1/2))/(a*d)]
```

giac [B] time = 0.16, size = 100, normalized size = 1.82

$$-\frac{\frac{2\,b\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a} - \frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*b*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a) - log(abs(-cos(d*x + c) + 1)/abs(co
s(d*x + c) + 1))/a)/d
```

maple [A] time = 0.46, size = 67, normalized size = 1.22

$$\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2ad}+\frac{b\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{da\sqrt{(a+b)\,b}}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{2ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/2/a/d*ln(cos(d*x+c)-1)+1/d/a*b/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b
)*b)^(1/2))-1/2/a/d*ln(1+cos(d*x+c))
```

maxima [A] time = 0.43, size = 83, normalized size = 1.51

$$-\frac{\frac{b\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a}+\frac{\log(\cos(dx+c)+1)}{a}-\frac{\log(\cos(dx+c)-1)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(b*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a +
b)*b)))/(sqrt((a + b)*b)*a) + log(cos(d*x + c) + 1)/a - log(cos(d*x + c) -
1)/a)/d
```

mupad [B] time = 13.73, size = 457, normalized size = 8.31

$$-\frac{\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)}{a\,d}-\frac{\mathrm{atan}\left(\frac{\frac{\left(2\,b^3\,\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{b\,(a+b)}\,1\mathrm{i}}{a^2+b\,a}+\frac{\left(2\,b^3\,\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos\ldots}{}\right)}{}\right)\ldots}{}}{\frac{\left(2\,b^3\,\cos(c+d\,x)+\frac{\left(2\,a^2\,b^2-\frac{\cos(c+d\,x)\left(8\,a^3\,b^2+16\,a^2\,b^3\right)\sqrt{b\,(a+b)}}{4\left(a^2+b\,a\right)}\right)\sqrt{b\,(a+b)}}{2\left(a^2+b\,a\right)}\right)\sqrt{b\,(a+b)}}{a^2+b\,a}-\frac{\left(2\,b^3\,\cos(c+d\,x)-\frac{\left(2\,a^2\,b^2+\frac{\cos\ldots}{}\right)}{}\right)\ldots}{}}\right)\ldots}{d\left(a^2+b\,a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - atanh(cos(c + d*x))/(a*d) - (atan((((2*b^3*cos(c + d*x) + ((2*a^2*b^2 - (
cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(
b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(a*b + a^2) + ((2*
b^3*cos(c + d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*
(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a
+ b))^(1/2)*1i)/(a*b + a^2))/(((2*b^3*cos(c + d*x) + ((2*a^2*b^2 - (cos(c +
 d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(1/2))/(4*(a*b + a^2)))*(b*(a +
b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2))/(a*b + a^2) - ((2*b^3*cos(c
+ d*x) - ((2*a^2*b^2 + (cos(c + d*x)*(16*a^2*b^3 + 8*a^3*b^2)*(b*(a + b))^(
1/2))/(4*(a*b + a^2)))*(b*(a + b))^(1/2))/(2*(a*b + a^2)))*(b*(a + b))^(1/2
))/(a*b + a^2)))*(b*(a + b))^(1/2)*1i)/(d*(a*b + a^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.83 $\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=85

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

```
[Out] -1/2*(a-2*b)*arctanh(cos(d*x+c))/a^2/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-b^(3/2
)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/a^2/d/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 85, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 414, 522, 206, 208}

$$-\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a+b}}-\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d}-\frac{\cot(c+dx)\csc(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a - 2*b)*ArcTanh[Cos[c + d*x]])/(2*a^2*d) - (b^(3/2)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(a^2*Sqrt[a + b]*d) - (Cot[c + d*x]*Csc[c + d*x])
/(2*a*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{a-b-bx^2}{(1-x^2)(a+b-bx^2)}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(a-2b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{2a^2d} - \frac{b^2\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{(a-2b)\tanh^{-1}(\cos(c+dx))}{2a^2d} - \frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^2\sqrt{a+b}\,d} - \frac{\cot(c+dx)\csc(c+d}{2ad}$$

Mathematica [C] time = 2.21, size = 224, normalized size = 2.64

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)-8b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)\right.}{16a^2d\sqrt{-a-b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/16*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-8*b^(3/2)*ArcTan[(Sq
rt[b] - I*Sqrt[a]*Tan[(c + d*x)/2]])/Sqrt[-a - b]] - 8*b^(3/2)*ArcTan[(Sqrt[
b] + I*Sqrt[a]*Tan[(c + d*x)/2]])/Sqrt[-a - b]] + Sqrt[-a - b]*(a*Csc[(c + d
*x)/2]^2 + 4*(a - 2*b)*(Log[Cos[(c + d*x)/2]] - Log[Sin[(c + d*x)/2]]) - a*
Sec[(c + d*x)/2]^2)))/(a^2*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2))
```

fricas [A] time = 0.49, size = 327, normalized size = 3.85

$$\left[\frac{2\left(b\cos(dx+c)^2-b\right)\sqrt{\frac{b}{a+b}}\log\left(-\frac{b\cos(dx+c)^2-2(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2a\cos(dx+c)-\left((a-2b)\cos(dx+\right.}{4\left(a^2d\cos(dx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*(b*cos(d*x + c)^2 - b)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(
a + b)*sqrt(b/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) +
2*a*cos(d*x + c) - ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c
) + 1/2) + ((a - 2*b)*cos(d*x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2
))/(a^2*d*cos(d*x + c)^2 - a^2*d), 1/4*(4*(b*cos(d*x + c)^2 - b)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + 2*a*cos(d*x + c) - ((a - 2*b
)*cos(d*x + c)^2 - a + 2*b)*log(1/2*cos(d*x + c) + 1/2) + ((a - 2*b)*cos(d*
x + c)^2 - a + 2*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*cos(d*x + c)^2 - a
^2*d)]
```

giac [B] time = 0.21, size = 196, normalized size = 2.31

$$\frac{\frac{8\,b^2\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^2}+\frac{2\,(a-2\,b)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}+\frac{\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)(\cos(dx+c)+1)}{a^2(\cos(dx+c)-1)}-\frac{\cos(dx+c)-1}{a(\cos(dx+c)+1)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(8*b^2*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c) +
 sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^2) + 2*(a - 2*b)*log(abs(-cos(d*x +
 c) + 1)/abs(cos(d*x + c) + 1))/a^2 + (a - 2*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) + 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1))*(cos(d*x + c) + 1)/(
a^2*(cos(d*x + c) - 1)) - (cos(d*x + c) - 1)/(a*(cos(d*x + c) + 1)))/d
```

maple [A] time = 0.55, size = 142, normalized size = 1.67

$$\frac{1}{4ad\left(\cos\left(dx+c\right)-1\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{4ad}-\frac{\ln\left(\cos\left(dx+c\right)-1\right)b}{2d\,a^2}-\frac{b^2\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{d\,a^2\sqrt{(a+b)\,b}}+\frac{1}{4ad\left(1+\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/4/a/d/(cos(d*x+c)-1)+1/4/a/d*ln(cos(d*x+c)-1)-1/2/d/a^2*ln(cos(d*x+c)-1)*
b-1/d*b^2/a^2/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))+1/4/a/d
/(1+cos(d*x+c))-1/4/a/d*ln(1+cos(d*x+c))+1/2/d/a^2*ln(1+cos(d*x+c))*b
```

maxima [A] time = 0.43, size = 120, normalized size = 1.41

$$\frac{\frac{2\,b^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^2}+\frac{2\,\cos(dx+c)}{a\cos(dx+c)^2-a}-\frac{(a-2\,b)\log(\cos(dx+c)+1)}{a^2}+\frac{(a-2\,b)\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*b^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a
 + b)*b)))/(sqrt((a + b)*b)*a^2) + 2*cos(d*x + c)/(a*cos(d*x + c)^2 - a) -
(a - 2*b)*log(cos(d*x + c) + 1)/a^2 + (a - 2*b)*log(cos(d*x + c) - 1)/a^2)/
d
```

mupad [B] time = 13.91, size = 592, normalized size = 6.96

$$-\frac{a\,\left(b\,\cos\left(c+d\,x\right)-b\,\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)+b\cos\left(c+d\,x\right)^2\mathrm{atanh}\left(\cos\left(c+d\,x\right)\right)\right)+a^2\,\left(\cos\left(c+d\,x\right)+\mathrm{atanh}\left(\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] -(a*(b*cos(c + d*x) - b*atanh(cos(c + d*x)) + b*cos(c + d*x)^2*atanh(cos(c
+ d*x))) + a^2*(cos(c + d*x) + atanh(cos(c + d*x)) - cos(c + d*x)^2*atanh(c
os(c + d*x))) - 2*b^2*atanh(cos(c + d*x)) + atan((b^5*cos(c + d*x)*(a*b^3 +
 b^4)^(1/2)*8i - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)*(a*
b^3 + b^4)^(3/2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2
*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^4)^(1/
```

```
2)*12i + a^4*b*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a^3*b^4
+ a^4*b^3 - a^5*b^2))*(a*b^3 + b^4)^(1/2)*2i + 2*b^2*cos(c + d*x)^2*atanh(c
os(c + d*x)) - cos(c + d*x)^2*atan((b^5*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*8i
 - b*cos(c + d*x)*(a*b^3 + b^4)^(3/2)*8i - a*cos(c + d*x)*(a*b^3 + b^4)^(3/
2)*4i + a^2*b^3*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i - a^3*b^2*cos(c + d*x)*
(a*b^3 + b^4)^(1/2)*2i + a*b^4*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*12i + a^4*b
*cos(c + d*x)*(a*b^3 + b^4)^(1/2)*1i)/(3*a^2*b^5 + 5*a^3*b^4 + a^4*b^3 - a^
5*b^2))*(a*b^3 + b^4)^(1/2)*2i)/(d*(2*a^2*b + 2*a^3 - 2*a^3*cos(c + d*x)^2
- 2*a^2*b*cos(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.84 $\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\cos(c+dx))}{8a^3d}-\frac{\cot(c+dx)}{4}$$

```
[Out] -1/8*(3*a^2-4*a*b+8*b^2)*arctanh(cos(d*x+c))/a^3/d-1/8*(3*a-4*b)*cot(d*x+c)
*csc(d*x+c)/a^2/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d+b^(5/2)*arctanh(cos(d*x+c
)*b^(1/2)/(a+b)^(1/2))/a^3/d/(a+b)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 208}

$$-\frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\cos(c+dx))}{8a^3d}+\frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a+b}}-\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d}-\frac{\cot(c+d}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((3*a^2 - 4*a*b + 8*b^2)*ArcTanh[Cos[c + d*x]])/(8*a^3*d) + (b^(5/2)*ArcTa
nh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(a^3*Sqrt[a + b]*d) - ((3*a - 4*b)*
Cot[c + d*x]*Csc[c + d*x])/(8*a^2*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d
)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sin^2(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+b-bx^2\right)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a-b-3bx^2}{\left(1-x^2\right)^2\left(a+b-bx^2\right)}\,dx, x, \cos(c+dx)\right)}{4ad}$$

$$= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} - \frac{\text{Subst}\left(\int \frac{3a^2-ab+4b^{\cdots}}{\left(1-x^2\right)(\cdots}\right.}{\cdots}$$

$$= -\frac{(3a-4b)\cot(c+dx)\csc(c+dx)}{8a^2d} - \frac{\cot(c+dx)\csc^3(c+dx)}{4ad} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+b-bx^{\cdots}}\right.}{a^{\cdots}}$$

$$= -\frac{\left(3a^2-4ab+8b^2\right)\tanh^{-1}(\cos(c+dx))}{8a^3d} + \frac{b^{5/2}\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{a^3\sqrt{a+b}\,d} - \frac{(3a-4b)\,\text{c}\cdots}{\cdots}$$

Mathematica [C] time = 6.30, size = 657, normalized size = 5.26

$$\frac{b^{5/2}\csc^2(c+dx)(-2a+b\cos(2(c+dx))-b)\tan^{-1}\left(\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(\sqrt{b}\cos\left(\frac{1}{2}(c+dx)\right)-i\sqrt{a}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{\sqrt{-a-b}}\right)}{2a^3d\sqrt{-a-b}\left(a\csc^2(c+dx)+b\right)} + \frac{b^{5/2}\csc^2(c+d\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sec[(c + d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] - I*Sqrt[a]*Sin
[(c + d*x)/2]))/Sqrt[-a - b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^
2)/(2*a^3*Sqrt[-a - b]*d*(b + a*Csc[c + d*x]^2)) + (b^(5/2)*ArcTan[(Sec[(c
+ d*x)/2]*(Sqrt[b]*Cos[(c + d*x)/2] + I*Sqrt[a]*Sin[(c + d*x)/2]))/Sqrt[-a
- b]]*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2)/(2*a^3*Sqrt[-a - b]*d
*(b + a*Csc[c + d*x]^2)) + ((3*a - 4*b)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc
[(c + d*x)/2]^2*Csc[c + d*x]^2)/(64*a^2*d*(b + a*Csc[c + d*x]^2)) + ((-2*a
- b + b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^4*Csc[c + d*x]^2)/(128*a*d*(b +
a*Csc[c + d*x]^2)) + ((3*a^2 - 4*a*b + 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)
])*Csc[c + d*x]^2*Log[Cos[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2))
+ ((-3*a^2 + 4*a*b - 8*b^2)*(-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*
Log[Sin[(c + d*x)/2]])/(16*a^3*d*(b + a*Csc[c + d*x]^2)) + ((-3*a + 4*b)*(-
```

```
2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*Sec[(c + d*x)/2]^2)/(64*a^2*d*
(b + a*Csc[c + d*x]^2)) - ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*S
ec[(c + d*x)/2]^4)/(128*a*d*(b + a*Csc[c + d*x]^2))
```

fricas [B] time = 0.51, size = 612, normalized size = 4.90

$$\left[\frac{2\left(3a^2-4ab\right)\cos(dx+c)^3+8\left(b^2\cos(dx+c)^4-2b^2\cos(dx+c)^2+b^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2+2(a+b)\sqrt{\frac{b}{a+b}}\cos(dx}{b\cos(dx+c)^2-a-b}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(2*(3*a^2 - 4*a*b)*cos(d*x + c)^3 + 8*(b^2*cos(d*x + c)^4 - 2*b^2*cos
(d*x + c)^2 + b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b
/(a + b))*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(5*a^2 - 4*
a*b)*cos(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*
a*b + 8*b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) +
 1/2) + ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)
*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3
*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d), 1/16*(2*(3*a^2 - 4*a*b
)*cos(d*x + c)^3 - 16*(b^2*cos(d*x + c)^4 - 2*b^2*cos(d*x + c)^2 + b^2)*sqr
t(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(5*a^2 - 4*a*b)*cos
(d*x + c) - ((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*
b^2)*cos(d*x + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(1/2*cos(d*x + c) + 1/2) +
((3*a^2 - 4*a*b + 8*b^2)*cos(d*x + c)^4 - 2*(3*a^2 - 4*a*b + 8*b^2)*cos(d*x
 + c)^2 + 3*a^2 - 4*a*b + 8*b^2)*log(-1/2*cos(d*x + c) + 1/2))/(a^3*d*cos(d
*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)]
```

giac [B] time = 0.25, size = 334, normalized size = 2.67

$$-\frac{\frac{64b^3\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\sqrt{-ab-b^2}\,a^3}+\frac{\frac{8a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{a^2}-\frac{4\left(3a^2-4ab+8b^2\right)\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^3}+\frac{\left(a^2-\frac{8a^2}{c}\right.}{}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*(64*b^3*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos(d*x + c
) + sqrt(-a*b - b^2)))/(sqrt(-a*b - b^2)*a^3) + (8*a*(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 8*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - a*(cos(d*x +
c) - 1)^2/(cos(d*x + c) + 1)^2)/a^2 - 4*(3*a^2 - 4*a*b + 8*b^2)*log(abs(-co
s(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^3 + (a^2 - 8*a^2*(cos(d*x + c) - 1
)/(cos(d*x + c) + 1) + 8*a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 18*a^2
*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 24*a*b*(cos(d*x + c) - 1)^2/(c
os(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)*(cos
(d*x + c) + 1)^2/(a^3*(cos(d*x + c) - 1)^2))/d
```

maple [B] time = 0.52, size = 255, normalized size = 2.04

$$-\frac{1}{16ad\left(\cos(dx+c)-1\right)^2}+\frac{3}{16ad\left(\cos(dx+c)-1\right)}-\frac{b}{4d\,a^2\left(\cos(dx+c)-1\right)}+\frac{3\ln\left(\cos(dx+c)-1\right)}{16ad}-\frac{\ln(\cos(dx+}{4d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/16/a/d/(cos(d*x+c)-1)^2+3/16/a/d/(cos(d*x+c)-1)-1/4/d/a^2/(cos(d*x+c)-1)
*b+3/16/a/d*ln(cos(d*x+c)-1)-1/4/d/a^2*ln(cos(d*x+c)-1)*b+1/2/d/a^3*ln(cos(
d*x+c)-1)*b^2+1/d*b^3/a^3/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1
/2))+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*x+c))-1/4/d/a^2/(1+cos(d*x
+c))*b-3/16/a/d*ln(1+cos(d*x+c))+1/4/d/a^2*ln(1+cos(d*x+c))*b-1/2/d/a^3*ln(
1+cos(d*x+c))*b^2
```

maxima [A] time = 0.43, size = 181, normalized size = 1.45

$$-\frac{\frac{8\,b^3\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,a^3}-\frac{2\left((3\,a-4\,b)\cos(dx+c)^3-(5\,a-4\,b)\cos(dx+c)\right)}{a^2\cos(dx+c)^4-2\,a^2\cos(dx+c)^2+a^2}+\frac{\left(3\,a^2-4\,ab+8\,b^2\right)\log(\cos(dx+c)+1)}{a^3}-\frac{\left(3\,a^2-4\,ab+8\,b^2\right)\log(\cos(dx+c)-1)}{a^3}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/16*(8*b^3*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt(
(a + b)*b)))/(sqrt((a + b)*b)*a^3) - 2*((3*a - 4*b)*cos(d*x + c)^3 - (5*a -
 4*b)*cos(d*x + c))/(a^2*cos(d*x + c)^4 - 2*a^2*cos(d*x + c)^2 + a^2) + (3*
a^2 - 4*a*b + 8*b^2)*log(cos(d*x + c) + 1)/a^3 - (3*a^2 - 4*a*b + 8*b^2)*lo
g(cos(d*x + c) - 1)/a^3)/d
```

mupad [B] time = 13.93, size = 1105, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - ((cos(c + d*x)*(5*a - 4*b))/(8*a^2) - (cos(c + d*x)^3*(3*a - 4*b))/(8*a^2
))/(d*(cos(c + d*x)^4 - cos(c + d*x)^2 + sin(c + d*x)^2)) - (atanh((63*b^4*
cos(c + d*x))/(64*((63*b^4)/64 - (81*a*b^3)/256 + (27*a^2*b^2)/256 - (35*b^
5)/(32*a) + (5*b^6)/(4*a^2))) - (81*b^3*cos(c + d*x))/(256*((27*a*b^2)/256
- (81*b^3)/256 + (63*b^4)/(64*a) - (35*b^5)/(32*a^2) + (5*b^6)/(4*a^3))) -
(35*b^5*cos(c + d*x))/(32*((63*a*b^4)/64 - (35*b^5)/32 - (81*a^2*b^3)/256 +
 (27*a^3*b^2)/256 + (5*b^6)/(4*a))) + (5*b^6*cos(c + d*x))/(4*((5*b^6)/4 -
(35*a*b^5)/32 + (63*a^2*b^4)/64 - (81*a^3*b^3)/256 + (27*a^4*b^2)/256)) + (
27*b^2*cos(c + d*x))/(256*((27*b^2)/256 - (81*b^3)/(256*a) + (63*b^4)/(64*a
^2) - (35*b^5)/(32*a^3) + (5*b^6)/(4*a^4))))*(3*a^2 - 4*a*b + 8*b^2))/(8*a^
3*d) - (atan((((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b^6 + 64*
a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((b^5*(a + b))^(1/2)*((2*a^6*
b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (cos(c + d*x)*(512*a^6*b^3 + 2
56*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^4)
))*1i)/(a^3*b + a^4) + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*
b^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((b^5*(a + b))^(1/2)
*((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (cos(c + d*x)*(512*a^
6*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3
*b + a^4)))*1i)/(a^3*b + a^4))/((((5*a*b^7)/4 - b^8 - (3*a^2*b^6)/4 + (9*a^3
*b^5)/32)/a^6 + ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b^6 + 6
4*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) + ((b^5*(a + b))^(1/2)*((2*a^
6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) - (cos(c + d*x)*(512*a^6*b^3 +
 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*b + a^
4))))/(a^3*b + a^4) - ((b^5*(a + b))^(1/2)*((cos(c + d*x)*(128*b^7 - 64*a*b
^6 + 64*a^2*b^5 - 24*a^3*b^4 + 9*a^4*b^3))/(64*a^4) - ((b^5*(a + b))^(1/2)*
((2*a^6*b^4 - (a^7*b^3)/2 + (3*a^8*b^2)/2)/(2*a^6) + (cos(c + d*x)*(512*a^6
*b^3 + 256*a^7*b^2)*(b^5*(a + b))^(1/2))/(128*a^4*(a^3*b + a^4))))/(2*(a^3*
b + a^4))))/(a^3*b + a^4)))*(b^5*(a + b))^(1/2)*1i)/(d*(a^3*b + a^4))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.85 $\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=163

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4d\sqrt{a+b}}-\frac{(8a^2-6ab+5b^2)\sin(c+dx)\cos(c+dx)}{16b^3d}-\frac{x(16a^3-8a^2b+6ab^2-5b^3)}{16b^4}+\frac{(6a-5b}{}$$

```
[Out] -1/16*(16*a^3-8*a^2*b+6*a*b^2-5*b^3)*x/b^4-1/16*(8*a^2-6*a*b+5*b^2)*cos(d*x
+c)*sin(d*x+c)/b^3/d+1/24*(6*a-5*b)*cos(d*x+c)*sin(d*x+c)^3/b^2/d-1/6*cos(d
*x+c)*sin(d*x+c)^5/b/d+a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/b^4/d
/(a+b)^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 470, 578, 522, 203, 205}

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4d\sqrt{a+b}}-\frac{(8a^2-6ab+5b^2)\sin(c+dx)\cos(c+dx)}{16b^3d}-\frac{x(-8a^2b+16a^3+6ab^2-5b^3)}{16b^4}+\frac{(6a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*x)/(16*b^4) + (a^(7/2)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(b^4*Sqrt[a + b]*d) - ((8*a^2 - 6*a*b + 5*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(16*b^3*d) + ((6*a - 5*b)*Cos[c + d*x]*Sin[c
+ d*x]^3)/(24*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^5)/(6*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^5(c+dx)}{6bd} + \frac{\text{Subst}\left(\int \frac{x^4\left(5a+(-a+5b)x^2\right)}{\left(1+x^2\right)^3\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{6bd}$$

$$= \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos(c+dx)\sin^5(c+dx)}{6bd} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a(6a-5b)-\cdots\right.}{\left(1+x^2\right)^2\left(\cdots\right.}\right.}{\cdots}$$

$$= -\frac{\left(8a^2-6ab+5b^2\right)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos\cdots}{\cdots}$$

$$= -\frac{\left(8a^2-6ab+5b^2\right)\cos(c+dx)\sin(c+dx)}{16b^3d} + \frac{(6a-5b)\cos(c+dx)\sin^3(c+dx)}{24b^2d} - \frac{\cos\cdots}{\cdots}$$

$$= -\frac{\left(16a^3-8a^2b+6ab^2-5b^3\right)x}{16b^4} + \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^4\sqrt{a+b}\,d} - \frac{\left(8a^2-6ab+5b^2\right)\cos(c\cdots}{16b^3d\cdots}$$

Mathematica [A] time = 1.18, size = 133, normalized size = 0.82

$$-\frac{-\frac{192a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 3b\left(16a^2-16ab+15b^2\right)\sin(2(c+dx)) + 12\left(16a^3-8a^2b+6ab^2-5b^3\right)(c+dx) + 3\cdots}{192b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/192*(12*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(c + d*x) - (192*a^(7/2)*Ar
cTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b] + 3*b*(16*a^2 - 16*a*
b + 15*b^2)*Sin[2*(c + d*x)] + 3*(2*a - 3*b)*b^2*Sin[4*(c + d*x)] + b^3*Sin
[6*(c + d*x)])/(b^4*d)
```

fricas [A] time = 0.49, size = 453, normalized size = 2.78

$$\left[\frac{12\,a^{3}\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{\left(8\,a^{2}+8\,ab+b^{2}\right)\cos(dx+c)^{4}-2\left(4\,a^{2}+5\,ab+b^{2}\right)\cos(dx+c)^{2}-4\left(\left(2\,a^{2}+3\,ab+b^{2}\right)\cos(dx+c)^{3}-\left(a^{2}+2\,ab+b^{2}\right)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}}{b^{2}\cos(dx+c)^{4}-2\left(ab+b^{2}\right)\cos(dx+c)^{2}+a^{2}+2\,ab+b^{2}}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/48*(12*a^3*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
+ 2*a*b + b^2)) - 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x - (8*b^3*cos(d
*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 + 1
1*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d), -1/48*(24*a^3*sqrt(a/(a + b))*a
rctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c
)*sin(d*x + c))) + 3*(16*a^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*d*x + (8*b^3*cos(
d*x + c)^5 + 2*(6*a*b^2 - 13*b^3)*cos(d*x + c)^3 + 3*(8*a^2*b - 10*a*b^2 +
11*b^3)*cos(d*x + c))*sin(d*x + c))/(b^4*d)]
```

giac [A] time = 0.18, size = 233, normalized size = 1.43

$$\frac{\frac{48\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^{2}+ab}}\right)\right)a^{4}}{\sqrt{a^{2}+ab}\,b^{4}}-\frac{3\left(16\,a^{3}-8\,a^{2}b+6\,ab^{2}-5\,b^{3}\right)(dx+c)}{b^{4}}-\frac{24\,a^{2}\tan(dx+c)^{5}-30\,ab\tan(dx+c)^{5}+33\,b^{2}}{}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/48*(48*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
 c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/(sqrt(a^2 + a*b)*b^4) - 3*(16*a
^3 - 8*a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4 - (24*a^2*tan(d*x + c)^5 - 30
*a*b*tan(d*x + c)^5 + 33*b^2*tan(d*x + c)^5 + 48*a^2*tan(d*x + c)^3 - 48*a*
b*tan(d*x + c)^3 + 40*b^2*tan(d*x + c)^3 + 24*a^2*tan(d*x + c) - 18*a*b*tan
(d*x + c) + 15*b^2*tan(d*x + c))/((tan(d*x + c)^2 + 1)^3*b^3))/d
```

maple [B] time = 0.31, size = 361, normalized size = 2.21

$$\frac{a^{4}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,b^{4}\sqrt{a\,(a+b)}}-\frac{\left(\tan^{5}(dx+c)\right)a^{2}}{2d\,b^{3}\left(\tan^{2}(dx+c)+1\right)^{3}}+\frac{5\left(\tan^{5}(dx+c)\right)a}{8d\,b^{2}\left(\tan^{2}(dx+c)+1\right)^{3}}-\frac{11\left(\tan^{5}(dx+c)\right)}{16db\left(\tan^{2}(dx+c)+1\right)^{3}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*a^4/b^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/2/d/
b^3/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5*a^2+5/8/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*
x+c)^5*a-11/16/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5-1/d/b^3/(tan(d*x+c)^2+1)
^3*tan(d*x+c)^3*a^2+1/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)^3*a-5/6/d/b/(tan(
d*x+c)^2+1)^3*tan(d*x+c)^3-1/2/d/b^3/(tan(d*x+c)^2+1)^3*tan(d*x+c)*a^2+3/8/
d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)*a-5/16/d/b/(tan(d*x+c)^2+1)^3*tan(d*x+c
)-1/d/b^4*arctan(tan(d*x+c))*a^3+1/2/d/b^3*arctan(tan(d*x+c))*a^2-3/8/d/b^2
*arctan(tan(d*x+c))*a+5/16/d/b*arctan(tan(d*x+c))
```

maxima [A] time = 0.44, size = 192, normalized size = 1.18

$$\frac{\frac{48\,a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^4}-\frac{3\left(8\,a^2-10\,ab+11\,b^2\right)\tan(dx+c)^5+8\left(6\,a^2-6\,ab+5\,b^2\right)\tan(dx+c)^3+3\left(8\,a^2-6\,ab+5\,b^2\right)\tan(dx+c)}{b^3\tan(dx+c)^6+3\,b^3\tan(dx+c)^4+3\,b^3\tan(dx+c)^2+b^3}-\frac{3\left(16\,a^3-8\,a^2b+6\,ab\right.}{b^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(48*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*
b^4) - (3*(8*a^2 - 10*a*b + 11*b^2)*tan(d*x + c)^5 + 8*(6*a^2 - 6*a*b + 5*b
^2)*tan(d*x + c)^3 + 3*(8*a^2 - 6*a*b + 5*b^2)*tan(d*x + c))/(b^3*tan(d*x +
 c)^6 + 3*b^3*tan(d*x + c)^4 + 3*b^3*tan(d*x + c)^2 + b^3) - 3*(16*a^3 - 8*
a^2*b + 6*a*b^2 - 5*b^3)*(d*x + c)/b^4)/d
```

mupad [B] time = 15.32, size = 2244, normalized size = 13.77

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan(((((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^
7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((((
5*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 - (t
an(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^
11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*
16i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)/(32*b
^4) + (((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7
 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) + ((((5
*a*b^12)/4 + a^2*b^11 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 + (ta
n(c + d*x)*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^1
1 + 5120*a^2*b^9 + 2048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*1
6i - b^3*5i))/(32*b^4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i)*1i)/(32*b^
4))/(((a^10*b)/4 + a^11 + (25*a^4*b^7)/128 - (5*a^5*b^6)/64 + (21*a^6*b^5)/
128 - (21*a^7*b^4)/32 - (15*a^8*b^3)/32 - (a^9*b^2)/8)/b^9 - (((tan(c + d*x
)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*
a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((((5*a*b^12)/4 + a^2*b^1
1 + (a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 - (tan(c + d*x)*(a*b^2*6i
 - a^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2
048*a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^
4))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4) + (((tan(c + d*x)*(1
5*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*
b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) + ((((5*a*b^12)/4 + a^2*b^11 +
(a^3*b^10)/4 + (5*a^4*b^9)/2 + 2*a^5*b^8)/b^9 + (tan(c + d*x)*(a*b^2*6i - a
^2*b*8i + a^3*16i - b^3*5i)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*
a^3*b^8))/(4096*b^10))*(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4))*
(a*b^2*6i - a^2*b*8i + a^3*16i - b^3*5i))/(32*b^4)))*(a*b^2*6i - a^2*b*8i +
 a^3*16i - b^3*5i)*1i)/(16*b^4*d) - ((tan(c + d*x)*(8*a^2 - 6*a*b + 5*b^2))
/(16*b^3) + (tan(c + d*x)^3*(6*a^2 - 6*a*b + 5*b^2))/(6*b^3) + (tan(c + d*x
)^5*(8*a^2 - 10*a*b + 11*b^2))/(16*b^3))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d
*x)^4 + tan(c + d*x)^6 + 1)) + (atan((((-a^7*(a + b))^(1/2)*((tan(c + d*x)*
(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^
4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((-a^7*(a + b))^(1/2)*((320
*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9)
 - (tan(c + d*x)*(-a^7*(a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b
^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)))*1i)/(2*(a*
b^4 + b^5)) + ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b +
512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 25
```

```
6*a^7*b^2))/(128*b^6) + ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 +
 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) + (tan(c + d*x)*(-a^7*(
a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256
*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)))*1i)/(2*(a*b^4 + b^5)))/((32*a^10*b
 + 128*a^11 + 25*a^4*b^7 - 10*a^5*b^6 + 21*a^6*b^5 - 84*a^7*b^4 - 60*a^8*b^
3 - 16*a^9*b^2)/(128*b^9) - ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8
+ 768*a^8*b + 512*a^9 + 25*b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 14
0*a^5*b^4 + 256*a^7*b^2))/(128*b^6) - ((-a^7*(a + b))^(1/2)*((320*a*b^12 +
256*a^2*b^11 + 64*a^3*b^10 + 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) - (tan(c
+ d*x)*(-a^7*(a + b))^(1/2)*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*
a^3*b^8))/(256*b^6*(a*b^4 + b^5))))/(2*(a*b^4 + b^5))))/(2*(a*b^4 + b^5)) +
 ((-a^7*(a + b))^(1/2)*((tan(c + d*x)*(15*a*b^8 + 768*a^8*b + 512*a^9 + 25*
b^9 + 11*a^2*b^7 - 63*a^3*b^6 - 224*a^4*b^5 - 140*a^5*b^4 + 256*a^7*b^2))/(
128*b^6) + ((-a^7*(a + b))^(1/2)*((320*a*b^12 + 256*a^2*b^11 + 64*a^3*b^10
+ 640*a^4*b^9 + 512*a^5*b^8)/(256*b^9) + (tan(c + d*x)*(-a^7*(a + b))^(1/2)
*(4096*a*b^10 + 1024*b^11 + 5120*a^2*b^9 + 2048*a^3*b^8))/(256*b^6*(a*b^4 +
 b^5))))/(2*(a*b^4 + b^5))))/(2*(a*b^4 + b^5))))*(-a^7*(a + b))^(1/2)*1i)/(
d*(a*b^4 + b^5))
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.86 $\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}+\frac{(4a-3b)\sin(c+dx)\cos(c+dx)}{8b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

```
[Out] 1/8*(8*a^2-4*a*b+3*b^2)*x/b^3+1/8*(4*a-3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-1/4
*cos(d*x+c)*sin(d*x+c)^3/b/d-a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))
/b^3/d/(a+b)^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3187, 470, 578, 522, 203, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3d\sqrt{a+b}}+\frac{x\left(8a^2-4ab+3b^2\right)}{8b^3}+\frac{(4a-3b)\sin(c+dx)\cos(c+dx)}{8b^2d}-\frac{\sin^3(c+dx)\cos(c+dx)}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2)*x)/(8*b^3) - (a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c +
d*x])/Sqrt[a]])/(b^3*Sqrt[a + b]*d) + ((4*a - 3*b)*Cos[c + d*x]*Sin[c + d*x
])/(8*b^2*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^3(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin^3(c+dx)}{4bd} + \frac{\text{Subst}\left(\int \frac{x^2(3a+(-a+3b)x^2)}{(1+x^2)^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{4bd}$$

$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{\text{Subst}\left(\int \frac{a(4a-3b)+(-}{(1+x^2)(}\right.}{}$$

$$= \frac{(4a-3b)\cos(c+dx)\sin(c+dx)}{8b^2d} - \frac{\cos(c+dx)\sin^3(c+dx)}{4bd} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x}\right.}{b^3}$$

$$= \frac{(8a^2-4ab+3b^2)x}{8b^3} - \frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^3\sqrt{a+b}\,d} + \frac{(4a-3b)\cos(c+dx)\sin(c+d}{8b^2d}$$

Mathematica [A] time = 0.46, size = 95, normalized size = 0.81

$$\frac{-\frac{32a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 4\left(8a^2-4ab+3b^2\right)(c+dx) + 8b(a-b)\sin(2(c+dx)) + b^2\sin(4(c+dx))}{32b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (4*(8*a^2 - 4*a*b + 3*b^2)*(c + d*x) - (32*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[
c + d*x])/Sqrt[a]])/Sqrt[a + b] + 8*(a - b)*b*Sin[2*(c + d*x)] + b^2*Sin[4*
(c + d*x)])/(32*b^3*d)
```

fricas [A] time = 0.50, size = 372, normalized size = 3.18

$$\left[\frac{2a^2\sqrt{-\frac{a}{a+b}}\log\left(\frac{(8a^2+8ab+b^2)\cos(dx+c)^4-2(4a^2+5ab+b^2)\cos(dx+c)^2+4\left((2a^2+3ab+b^2)\cos(dx+c)^3-(a^2+2ab+b^2)\cos(dx+c)\right)\sqrt{-\frac{a}{a+b}}}{b^2\cos(dx+c)^4-2(ab+b^2)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{8b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*a^2*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*
(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c
)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2
 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
2*a*b + b^2)) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (4*a*
b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d), 1/8*(4*a^2*sqrt(a/(a + b))*
arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x +
c)*sin(d*x + c))) + (8*a^2 - 4*a*b + 3*b^2)*d*x + (2*b^2*cos(d*x + c)^3 + (
4*a*b - 5*b^2)*cos(d*x + c))*sin(d*x + c))/(b^3*d)]
```

giac [A] time = 0.16, size = 157, normalized size = 1.34

$$-\frac{\frac{8\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\sqrt{a^2+ab}\,b^3}-\frac{(8a^2-4ab+3b^2)(dx+c)}{b^3}-\frac{4a\tan(dx+c)^3-5b\tan(dx+c)^3+4a\tan(dx+c)-3b\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)^2b^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x +
c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/(sqrt(a^2 + a*b)*b^3) - (8*a^2 -
 4*a*b + 3*b^2)*(d*x + c)/b^3 - (4*a*tan(d*x + c)^3 - 5*b*tan(d*x + c)^3 +
4*a*tan(d*x + c) - 3*b*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b^2))/d
```

maple [A] time = 0.32, size = 196, normalized size = 1.68

$$-\frac{a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,b^3\sqrt{a(a+b)}}+\frac{\left(\tan^3(dx+c)\right)a}{2d\,b^2\left(\tan^2(dx+c)+1\right)^2}-\frac{5\left(\tan^3(dx+c)\right)}{8db\left(\tan^2(dx+c)+1\right)^2}+\frac{\tan(dx+c)\,a}{2d\,b^2\left(\tan^2(dx+c)+1\right)^2}-\frac{}{8db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d*a^3/b^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/2/d
/b^2/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3*a-5/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+c
)^3+1/2/d/b^2/(tan(d*x+c)^2+1)^2*tan(d*x+c)*a-3/8/d/b/(tan(d*x+c)^2+1)^2*ta
n(d*x+c)+1/d/b^3*arctan(tan(d*x+c))*a^2-1/2/d/b^2*arctan(tan(d*x+c))*a+3/8/
d/b*arctan(tan(d*x+c))
```

maxima [A] time = 0.45, size = 128, normalized size = 1.09

$$-\frac{\frac{8a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}-\frac{(4a-5b)\tan(dx+c)^3+(4a-3b)\tan(dx+c)}{b^2\tan(dx+c)^4+2b^2\tan(dx+c)^2+b^2}-\frac{(8a^2-4ab+3b^2)(dx+c)}{b^3}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b
^3) - ((4*a - 5*b)*tan(d*x + c)^3 + (4*a - 3*b)*tan(d*x + c))/(b^2*tan(d*x
+ c)^4 + 2*b^2*tan(d*x + c)^2 + b^2) - (8*a^2 - 4*a*b + 3*b^2)*(d*x + c)/b^
3)/d
```

mupad [B] time = 14.82, size = 1892, normalized size = 16.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] ((tan(c + d*x)*(4*a - 3*b))/(8*b^2) + (tan(c + d*x)^3*(4*a - 5*b))/(8*b^2))
/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 + 1)) - (atan(((((((3*a*b^9)/2 + a^
2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c + d*x)*(-a^5*(a + b))^(
1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*(a*b^3 +
 b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(3*a*b^6 +
192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5
*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4) - ((((((3*a*b^9)/2
+ a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) + (tan(c + d*x)*(-a^5*(a + b
))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*(a*b
^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) + (tan(c + d*x)*(3*a*b^
6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64
*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2)*1i)/(a*b^3 + b^4))/((((((3*a*b^9
)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) - (tan(c + d*x)*(-a^5*(a
 + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*b^4*
(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(3*
a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3
+ 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2))/(a*b^3 + b^4) - ((a^7*b)/4 +
 a^8 + (9*a^3*b^5)/32 - (3*a^4*b^4)/16 + (25*a^5*b^3)/32 + (a^6*b^2)/2)/b^6
 + ((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/(2*b^6) + (tan(c
 + d*x)*(-a^5*(a + b))^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3
*b^6))/(128*b^4*(a*b^3 + b^4)))*(-a^5*(a + b))^(1/2))/(2*(a*b^3 + b^4)) + (
tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b
^4 + 40*a^4*b^3 + 64*a^5*b^2))/(64*b^4))*(-a^5*(a + b))^(1/2))/(a*b^3 + b^4
)))*(-a^5*(a + b))^(1/2)*1i)/(d*(a*b^3 + b^4)) - (atan(((((((3*a*b^9)/2 +
a^2*b^8 - (5*a^3*b^7)/2 - 2*a^4*b^6)/b^6 - (tan(c + d*x)*(a^2*8i - a*b*4i +
 b^2*3i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a
^2*8i - a*b*4i + b^2*3i))/(16*b^3) - (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 1
28*a^7 + 9*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^
4))*(a^2*8i - a*b*4i + b^2*3i)*1i)/(16*b^3) - ((((((3*a*b^9)/2 + a^2*b^8 -
(5*a^3*b^7)/2 - 2*a^4*b^6)/b^6 + (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(
1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*
b*4i + b^2*3i))/(16*b^3) + (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9
*b^7 + 19*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8
i - a*b*4i + b^2*3i)*1i)/(16*b^3))/(((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b^7
)/2 - 2*a^4*b^6)/b^6 - (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(1024*a*b^8
 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*b*4i + b^2
*3i))/(16*b^3) - (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 19*
a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8i - a*b*4i
 + b^2*3i))/(16*b^3) - ((a^7*b)/4 + a^8 + (9*a^3*b^5)/32 - (3*a^4*b^4)/16 +
 (25*a^5*b^3)/32 + (a^6*b^2)/2)/b^6 + ((((((3*a*b^9)/2 + a^2*b^8 - (5*a^3*b
^7)/2 - 2*a^4*b^6)/b^6 + (tan(c + d*x)*(a^2*8i - a*b*4i + b^2*3i)*(1024*a*b
^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a^2*8i - a*b*4i + b
^2*3i))/(16*b^3) + (tan(c + d*x)*(3*a*b^6 + 192*a^6*b + 128*a^7 + 9*b^7 + 1
9*a^2*b^5 + 65*a^3*b^4 + 40*a^4*b^3 + 64*a^5*b^2))/(32*b^4))*(a^2*8i - a*b*
4i + b^2*3i))/(16*b^3)))*(a^2*8i - a*b*4i + b^2*3i)*1i)/(8*b^3*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.87 $\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

```
[Out] -1/2*(2*a-b)*x/b^2-1/2*cos(d*x+c)*sin(d*x+c)/b/d+a^(3/2)*arctan((a+b)^(1/2)
*tan(d*x+c)/a^(1/2))/b^2/d/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 470, 522, 203, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{b^2d\sqrt{a+b}}-\frac{x(2a-b)}{2b^2}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((2*a - b)*x)/(2*b^2) + (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]
])/(b^2*Sqrt[a + b]*d) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
```

```
x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{a+(-a+b)x^2}{(1+x^2)\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{2bd}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{b^2 d} - \frac{(2a-b)\,\text{Sub}}{}$$

$$= -\frac{(2a-b)x}{2b^2} + \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b^2\sqrt{a+b}\,d} - \frac{\cos(c+dx)\sin(c+dx)}{2bd}$$

Mathematica [A] time = 0.32, size = 69, normalized size = 0.90

$$-\frac{-\frac{4a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} + 2(2a-b)(c+dx) + b\sin(2(c+dx))}{4b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/4*(2*(2*a - b)*(c + d*x) - (4*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/
Sqrt[a]])/Sqrt[a + b] + b*Sin[2*(c + d*x)])/(b^2*d)
```

fricas [A] time = 0.48, size = 305, normalized size = 3.96

$$\left[-\frac{2\,(2\,a-b)dx + 2\,b\cos(dx+c)\sin(dx+c) - a\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4 - 2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2 - 4\left((2\,a\right.}{b^2\cos(dx+c)^4-2}\right.}{4\,b^2 d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*(2*a - b)*d*x + 2*b*cos(d*x + c)*sin(d*x + c) - a*sqrt(-a/(a + b))
*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*
x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*co
s(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
 + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/(b^2*d), -1/2
*((2*a - b)*d*x + b*cos(d*x + c)*sin(d*x + c) + a*sqrt(a/(a + b))*arctan(1/
2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*
x + c))))/(b^2*d)]
```

giac [A] time = 0.15, size = 114, normalized size = 1.48

$$\frac{\frac{2\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{\sqrt{a^2+ab}\,b^2} - \frac{(dx+c)(2\,a-b)}{b^2} - \frac{\tan(dx+c)}{\left(\tan(dx+c)^2+1\right)b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(2*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/(sqrt(a^2 + a*b)*b^2) - (d*x + c)
*(2*a - b)/b^2 - tan(d*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

maple [A] time = 0.25, size = 94, normalized size = 1.22

$$\frac{a^{2}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,b^{2}\sqrt{a\,(a+b)}}-\frac{\tan\left(dx+c\right)}{2db\left(\tan^{2}\left(dx+c\right)+1\right)}+\frac{\arctan\left(\tan\left(dx+c\right)\right)}{2db}-\frac{\arctan\left(\tan\left(dx+c\right)\right)a}{d\,b^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*a^2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/2/d/
b*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2/d/b*arctan(tan(d*x+c))-1/d/b^2*arctan(tan
(d*x+c))*a
```

maxima [A] time = 0.44, size = 78, normalized size = 1.01

$$\frac{\frac{2\,a^{2}\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^{2}}-\frac{(dx+c)(2\,a-b)}{b^{2}}-\frac{\tan(dx+c)}{b\tan(dx+c)^{2}+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^
2) - (d*x + c)*(2*a - b)/b^2 - tan(d*x + c)/(b*tan(d*x + c)^2 + b))/d
```

mupad [B] time = 13.97, size = 481, normalized size = 6.25

$$\frac{b^{2}\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^{3}+2\,a\,b^{2}\right)}-\frac{2\,a^{2}\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^{3}+2\,a\,b^{2}\right)}-\frac{b^{2}\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^{3}+2\,a\,b^{2}\right)}-\frac{a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\left(2\,b^{3}+2\,a\,b^{2}\right)}-\frac{a\,b\,\mathrm{atan}\left(\frac{\sin(c+d\,x)}{\cos(c+d\,x)}\right)}{d\,\left(2\,b^{3}+2\,a\,b^{2}\right)}-\mathrm{atan}\left(\cdots\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (b^2*atan(sin(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (2*a^2*atan(s
in(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3)) - (b^2*sin(2*c + 2*d*x))/(
2*d*(2*a*b^2 + 2*b^3)) - (atan((a*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*8i + b
*sin(c + d*x)*(- a^3*b - a^4)^(3/2)*4i + a^5*sin(c + d*x)*(- a^3*b - a^4)^(
1/2)*8i + b^5*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*1i - a*b^4*sin(c + d*x)*(-
 a^3*b - a^4)^(1/2)*1i + a^4*b*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*12i - a^2
*b^3*sin(c + d*x)*(- a^3*b - a^4)^(1/2)*5i + a^3*b^2*sin(c + d*x)*(- a^3*b
- a^4)^(1/2)*1i)/(a^3*b^4*cos(c + d*x) - a^2*b^5*cos(c + d*x) + 5*a^4*b^3*c
os(c + d*x) + 3*a^5*b^2*cos(c + d*x)))*(- a^3*b - a^4)^(1/2)*2i)/(d*(2*a*b^
2 + 2*b^3)) - (a*b*sin(2*c + 2*d*x))/(2*d*(2*a*b^2 + 2*b^3)) - (a*b*atan(si
n(c + d*x)/cos(c + d*x)))/(d*(2*a*b^2 + 2*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.88 $\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=46

$$\frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

```
[Out] x/b-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b/d/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 46, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3171, 3181, 205}

$$\frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] x/b - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(b*Sqrt[a + b]*d
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3171

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/((a_) + (b_.)*sin[(e_.) + (f_
.)*(x_)]^2), x_Symbol] :> Simp[(B*x)/b, x] + Dist[(A*b - a*B)/b, Int[1/(a +
 b*Sin[e + f*x]^2), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{x}{b}-\frac{a\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{b}\\
&= \frac{x}{b}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{bd}\\
&= \frac{x}{b}-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{b\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 46, normalized size = 1.00

$$\frac{-\frac{\sqrt{a}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}}+c+dx}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (c + d*x - (Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]
)/(b*d)
```

fricas [A] time = 0.47, size = 260, normalized size = 5.65

$$\left[\frac{4\,dx+\sqrt{-\frac{a}{a+b}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2+4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx+c)\right)\sqrt{-}}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{4\,bd}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(4*d*x + sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 -
2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
 c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a
^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
+ 2*a*b + b^2)))/(b*d), 1/2*(2*d*x + sqrt(a/(a + b))*arctan(1/2*((2*a + b)*
cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/(b*
d)]
```

giac [B] time = 0.15, size = 81, normalized size = 1.76

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*b) - (d*x + c)/b)/d
```

maple [A] time = 0.24, size = 50, normalized size = 1.09

$$-\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{db\sqrt{a\,(a+b)}}+\frac{\arctan\left(\tan\left(dx+c\right)\right)}{db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d*a/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/d/b*arc
tan(tan(d*x+c))
```

maxima [A] time = 0.45, size = 46, normalized size = 1.00

$$-\frac{\frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{dx+c}{b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - (d*x
 + c)/b)/d
```

mupad [B] time = 13.48, size = 104, normalized size = 2.26

$$\frac{\operatorname{atan}\left(\frac{2\,a\,b^2\,\tan(c+d\,x)}{2\,a^2\,b+2\,a\,b^2}+\frac{2\,a^2\,b\,\tan(c+d\,x)}{2\,a^2\,b+2\,a\,b^2}\right)}{b\,d}+\frac{\operatorname{atanh}\left(\frac{\tan(c+d\,x)\,\sqrt{-a\,(a+b)}}{a}\right)\sqrt{-a\,(a+b)}}{d\,\left(b^2+a\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] atan((2*a*b^2*tan(c + d*x))/(2*a*b^2 + 2*a^2*b) + (2*a^2*b*tan(c + d*x))/(2
*a*b^2 + 2*a^2*b))/(b*d) + (atanh((tan(c + d*x)*(-a*(a + b))^(1/2))/a)*(-a*
(a + b))^(1/2))/(d*(a*b + b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.89 $\int \frac{1}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=36

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

```
[Out] arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/d/a^(1/2)/(a+b)^(1/2)
```

Rubi [A] time = 0.02, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3181, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-1),x]
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A] time = 0.08, size = 36, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-1),x]
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*Sqrt[a + b]*d)
```

fricas [B] time = 0.45, size = 236, normalized size = 6.56

$$\left[-\frac{\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2+4\left((2\,a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+a^2+2}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right)}{4\left(a^2+ab\right)d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(
d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a^2 + a*b)*d),
 -1/2*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*
x + c)*sin(d*x + c)))/(sqrt(a^2 + a*b)*d)]
```

giac [B] time = 0.13, size = 64, normalized size = 1.78

$$\frac{\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] (pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*t
an(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*d)
```

maple [A] time = 0.38, size = 30, normalized size = 0.83

$$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a\left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.43, size = 29, normalized size = 0.81

$$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{\left(a+b\right)a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*d)
```

mupad [B] time = 13.53, size = 33, normalized size = 0.92

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(a+b)}{\sqrt{a^2+b\,a}}\right)}{d\,\sqrt{a^2+b\,a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2),x)
```

```
[Out] atan((tan(c + d*x)*(a + b))/(a*b + a^2)^(1/2))/(d*(a*b + a^2)^(1/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.90 $\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a+b}}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-b*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/d/(a+b)^(1
/2)
```

Rubi [A] time = 0.07, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 453, 205}

$$-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d\sqrt{a+b}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*Sqrt[a + b]*d)) -
 Cot[c + d*x]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{x^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad} - \frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}\,d} - \frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 0.31, size = 53, normalized size = 1.00

$$\frac{-\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a+b}} - \sqrt{a}\,\cot(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-((b*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/Sqrt[a + b]) - Sqrt[a]*Co
t[c + d*x])/(a^(3/2)*d)
```

fricas [B] time = 0.46, size = 313, normalized size = 5.91

$$\left[-\frac{\sqrt{-a^2-ab}\,b\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2-4\left((2a+b)\cos(dx+c)^3-(a+b)\cos(dx+c)\right)\sqrt{-a^2-ab}\,\sin(dx+c)+}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4\left(a^3+a^2b\right)d\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*
a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*c
os(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*
x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c)
+ 4*(a^2 + a*b)*cos(d*x + c))/((a^3 + a^2*b)*d*sin(d*x + c)), 1/2*(sqrt(a^2
 + a*b)*b*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*co
s(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(a^2 + a*b)*cos(d*x + c))/((a^3
+ a^2*b)*d*sin(d*x + c))]
```

giac [A] time = 0.19, size = 83, normalized size = 1.57

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b}{\sqrt{a^2+ab}\,a} + \frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*b/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x + c))
)/d
```

maple [A] time = 0.54, size = 52, normalized size = 0.98

$$-\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{da\sqrt{a\,(a+b)}}-\frac{1}{da\tan\,(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d/a*b/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/d/a/tan
(d*x+c)
```

maxima [A] time = 1.34, size = 48, normalized size = 0.91

$$-\frac{\frac{b\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(b*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a) + 1/(a
*tan(d*x + c)))/d
```

mupad [B] time = 13.48, size = 45, normalized size = 0.85

$$-\frac{\cot\,(c+d\,x)}{a\,d}-\frac{b\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{3/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - cot(c + d*x)/(a*d) - (b*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(3
/2)*d*(a + b)^(1/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\,(c+dx)}{a+b\sin^2\,(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.91 $\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=77

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] -(a-b)*cot(d*x+c)/a^2/d-1/3*cot(d*x+c)^3/a/d+b^2*arctan((a+b)^(1/2)*tan(d*x
+c)/a^(1/2))/a^(5/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 461, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d\sqrt{a+b}}-\frac{(a-b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*Sqrt[a + b]*d) -
((a - b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^2}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{a-b}{a^2x^2}+\frac{b^2}{a^2(a+(a+b)x^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a-b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{b^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}\sqrt{a+b}\,d} - \frac{(a-b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.67, size = 119, normalized size = 1.55

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(a\csc^2(c+dx)+2a-3b\right)-3b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)\right)}{6a^{5/2}d\sqrt{a+b}\left(a\csc^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/6*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(-3*b^2*ArcTan[(Sqrt[a
+ b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(2*a - 3*b +
 a*Csc[c + d*x]^2)))/(a^(5/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

fricas [B] time = 0.48, size = 451, normalized size = 5.86

$$\left[-\frac{4\left(2a^3-a^2b-3ab^2\right)\cos(dx+c)^3+3\left(b^2\cos(dx+c)^2-b^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b\right.}{b^2}\right.}{12\left(\left(a^4+a^3b\right)d\cos(dx+c)^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(4*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3*(b^2*cos(d*x + c)^2
- b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) -
12*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^
3*b)*d)*sin(d*x + c)), -1/6*(2*(2*a^3 - a^2*b - 3*a*b^2)*cos(d*x + c)^3 + 3
*(b^2*cos(d*x + c)^2 - b^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x +
 c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) -
6*(a^3 - a*b^2)*cos(d*x + c))/(((a^4 + a^3*b)*d*cos(d*x + c)^2 - (a^4 + a^3
*b)*d)*sin(d*x + c))]
```

giac [A] time = 0.19, size = 111, normalized size = 1.44

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^2}{\sqrt{a^2+ab}\,a^2}-\frac{3a\tan(dx+c)^2-3b\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^2/(sqrt(a^2 + a*b)*a^2) - (3*a*tan(
d*x + c)^2 - 3*b*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

maple [A] time = 0.57, size = 85, normalized size = 1.10

$$\frac{b^2 \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,a^2\sqrt{a\,(a+b)}} - \frac{1}{3da\tan(dx+c)^3} - \frac{1}{da\tan(dx+c)} + \frac{b}{d\,a^2\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*b^2/a^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/3/d/
a/tan(d*x+c)^3-1/d/a/tan(d*x+c)+1/d/a^2/tan(d*x+c)*b
```

maxima [A] time = 0.43, size = 69, normalized size = 0.90

$$\frac{\frac{3\,b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2} - \frac{3\,(a-b)\tan(dx+c)^2+a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*b^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^
2) - (3*(a - b)*tan(d*x + c)^2 + a)/(a^2*tan(d*x + c)^3))/d
```

mupad [B] time = 13.43, size = 68, normalized size = 0.88

$$\frac{b^2\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{5/2}\,d\,\sqrt{a+b}} - \frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(a-b)}{a^2}}{d\,\tan\left(c+d\,x\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] (b^2*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(5/2)*d*(a + b)^(1/2))
- (1/(3*a) + (tan(c + d*x)^2*(a - b))/a^2)/(d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.92 $\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=109

$$-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a+b}}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\left(a^2-ab+b^2\right)\cot(c+dx)}{a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -(a^2-a*b+b^2)*cot(d*x+c)/a^3/d-1/3*(2*a-b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+
c)^5/a/d-b^3*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3187, 461, 205}

$$-\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d\sqrt{a+b}}-\frac{\left(a^2-ab+b^2\right)\cot(c+dx)}{a^3d}-\frac{(2a-b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((b^3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*Sqrt[a + b]*d))
 - ((a^2 - a*b + b^2)*Cot[c + d*x])/(a^3*d) - ((2*a - b)*Cot[c + d*x]^3)/(3
*a^2*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^3}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6} + \frac{2a-b}{a^2x^4} + \frac{a^2-ab+b^2}{a^3x^2} + \frac{b^3}{a^3(-a-(a+b)x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d} - \frac{(2a-b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} + \frac{b^3\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}\sqrt{a+b}\,d} - \frac{(a^2-ab+b^2)\cot(c+dx)}{a^3d} - \frac{(2a-b)\cot^3(c+dx)}{3a^2d} - \text{c}$$

Mathematica [A] time = 1.52, size = 147, normalized size = 1.35

$$-\frac{\csc^2(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(3a^2\csc^4(c+dx)+8a^2+a(4a-5b)\csc^2(c+\right.\right.}{30a^{7/2}d\sqrt{a+b}\left(a\csc^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/30*((2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]^2*(15*b^3*ArcTan[(Sqrt[a
 + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Sqrt[a + b]*Cot[c + d*x]*(8*a^2 - 10
*a*b + 15*b^2 + a*(4*a - 5*b)*Csc[c + d*x]^2 + 3*a^2*Csc[c + d*x]^4)))/(a^(
7/2)*Sqrt[a + b]*d*(b + a*Csc[c + d*x]^2))
```

fricas [B] time = 0.46, size = 595, normalized size = 5.46

$$\left[-\frac{4\left(8a^4-2a^3b+5a^2b^2+15ab^3\right)\cos(dx+c)^5-20\left(4a^4-a^3b+a^2b^2+6ab^3\right)\cos(dx+c)^3+15\left(b^3\cos(d\right.}{6}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 20*(4*a
^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 + 15*(b^3*cos(d*x + c)^4 - 2
*b^3*cos(d*x + c)^2 + b^3)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(
d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x
+ c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b
+ b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b
^2))*sin(d*x + c) + 60*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*
x + c)^4 - 2*(a^5 + a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c)
), -1/30*(2*(8*a^4 - 2*a^3*b + 5*a^2*b^2 + 15*a*b^3)*cos(d*x + c)^5 - 10*(4
*a^4 - a^3*b + a^2*b^2 + 6*a*b^3)*cos(d*x + c)^3 - 15*(b^3*cos(d*x + c)^4 -
 2*b^3*cos(d*x + c)^2 + b^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x
+ c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) +
 30*(a^4 + a*b^3)*cos(d*x + c))/(((a^5 + a^4*b)*d*cos(d*x + c)^4 - 2*(a^5 +
 a^4*b)*d*cos(d*x + c)^2 + (a^5 + a^4*b)*d)*sin(d*x + c))]
```

giac [A] time = 0.17, size = 155, normalized size = 1.42

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^3}{\sqrt{a^2+ab}\,a^3} + \frac{15a^2\tan(dx+c)^4-15ab\tan(dx+c)^4+15b^2\tan(dx+c)^4+10a^2\tan(dx+c)^2-5ab}{a^3\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^3/(sqrt(a^2 + a*b)*a^3) + (15*a^
2*tan(d*x + c)^4 - 15*a*b*tan(d*x + c)^4 + 15*b^2*tan(d*x + c)^4 + 10*a^2*t
an(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

maple [A] time = 0.58, size = 138, normalized size = 1.27

$$-\frac{b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,a^3\sqrt{a\,(a+b)}}-\frac{1}{5da\tan\left(dx+c\right)^5}-\frac{2}{3da\tan\left(dx+c\right)^3}+\frac{b}{3d\,a^2\tan\left(dx+c\right)^3}-\frac{1}{da\tan\left(dx+c\right)}+\frac{b}{d\,a^2\tan(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d*b^3/a^3/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/5/d
/a/tan(d*x+c)^5-2/3/d/a/tan(d*x+c)^3+1/3/d/a^2/tan(d*x+c)^3*b-1/d/a/tan(d*x
+c)+1/d/a^2/tan(d*x+c)*b-1/d/a^3/tan(d*x+c)*b^2
```

maxima [A] time = 0.46, size = 98, normalized size = 0.90

$$-\frac{\frac{15\,b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{15\left(a^2-ab+b^2\right)\tan(dx+c)^4+5\left(2\,a^2-ab\right)\tan(dx+c)^2+3\,a^2}{a^3\tan(dx+c)^5}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*b^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)
*a^3) + (15*(a^2 - a*b + b^2)*tan(d*x + c)^4 + 5*(2*a^2 - a*b)*tan(d*x + c)
^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

mupad [B] time = 13.80, size = 95, normalized size = 0.87

$$-\frac{\tan\left(c+d\,x\right)^4\,\left(a^2-a\,b+b^2\right)+\frac{a^2}{5}-\tan\left(c+d\,x\right)^2\,\left(\frac{a\,b}{3}-\frac{2\,a^2}{3}\right)}{a^3\,d\,\tan\left(c+d\,x\right)^5}-\frac{b^3\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{7/2}\,d\,\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] - (tan(c + d*x)^4*(a^2 - a*b + b^2) + a^2/5 - tan(c + d*x)^2*((a*b)/3 - (2*
a^2)/3))/(a^3*d*tan(c + d*x)^5) - (b^3*atan((tan(c + d*x)*(a + b)^(1/2))/a^
(1/2)))/(a^(7/2)*d*(a + b)^(1/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.93 $\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=140

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}}-\frac{(3a-b)\cot^5(c+dx)}{5a^2d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-$$

```
[Out] -(a-b)*(a^2+b^2)*cot(d*x+c)/a^4/d-1/3*(3*a^2-2*a*b+b^2)*cot(d*x+c)^3/a^3/d-
1/5*(3*a-b)*cot(d*x+c)^5/a^2/d-1/7*cot(d*x+c)^7/a/d+b^4*arctan((a+b)^(1/2)*
tan(d*x+c)/a^(1/2))/a^(9/2)/d/(a+b)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3187, 461, 205}

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}}-\frac{\left(3a^2-2ab+b^2\right)\cot^3(c+dx)}{3a^3d}-\frac{(a-b)\left(a^2+b^2\right)\cot(c+dx)}{a^4d}-\frac{(3a-b)\cot^5(c+dx)}{5a^2d}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
((a - b)*(a^2 + b^2)*Cot[c + d*x])/(a^4*d) - ((3*a^2 - 2*a*b + b^2)*Cot[c +
 d*x]^3)/(3*a^3*d) - ((3*a - b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/
(7*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 461

```
Int[(((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_))/((c_) + (d_.)*(x_)^(
n_)), x_Symbol] :> Int[ExpandIntegrand[((e*x)^m*(a + b*x^n)^p)/(c + d*x^n),
 x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0]
&& IGtQ[p, 0] && (IntegerQ[m] || IGtQ[2*(m + 1), 0] ||  !RationalQ[m])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^4}{x^8(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3a-b}{a^2x^6}+\frac{3a^2-2ab+b^2}{a^3x^4}+\frac{(a-b)(a^2+b^2)}{a^4x^2}+\frac{b^4}{a^4(a+(a+b)x^2)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a-b)(a^2+b^2)\cot(c+dx)}{a^4d} - \frac{(3a^2-2ab+b^2)\cot^3(c+dx)}{3a^3d} - \frac{(3a-b)\cot^5(c+dx)}{5a^2d}$$

$$= \frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}\sqrt{a+b}\,d} - \frac{(a-b)(a^2+b^2)\cot(c+dx)}{a^4d} - \frac{(3a^2-2ab+b^2)\cot^3(c+d}{3a^3d}$$

Mathematica [A] time = 1.71, size = 137, normalized size = 0.98

$$\frac{b^4\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d\sqrt{a+b}} - \frac{\cot(c+dx)\left(15a^3\csc^6(c+dx)+48a^3+a\left(24a^2-28ab+35b^2\right)\csc^2(c+dx)+3a^2(6a-}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (b^4*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*Sqrt[a + b]*d) -
(Cot[c + d*x]*(48*a^3 - 56*a^2*b + 70*a*b^2 - 105*b^3 + a*(24*a^2 - 28*a*b
+ 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(6*a - 7*b)*Csc[c + d*x]^4 + 15*a^3*Csc[c
+ d*x]^6))/(105*a^4*d)
```

fricas [B] time = 0.48, size = 789, normalized size = 5.64

$$\left[-\frac{4\left(48a^5-8a^4b+14a^3b^2-35a^2b^3-105ab^4\right)\cos(dx+c)^7-28\left(24a^5-4a^4b+7a^3b^2-10a^2b^3-45ab^4\right)\text{co}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/420*(4*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105*a*b^4)*cos(d*x
 + c)^7 - 28*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45*a*b^4)*cos(d*x
 + c)^5 + 140*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*cos(d*x + c)^3
+ 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(d*x + c)^2 - b
^4)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 +
 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*
x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c
)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 420
*(a^5 - a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^
5*b)*d*cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)
*sin(d*x + c)), -1/210*(2*(48*a^5 - 8*a^4*b + 14*a^3*b^2 - 35*a^2*b^3 - 105
*a*b^4)*cos(d*x + c)^7 - 14*(24*a^5 - 4*a^4*b + 7*a^3*b^2 - 10*a^2*b^3 - 45
*a*b^4)*cos(d*x + c)^5 + 70*(6*a^5 - a^4*b + a^3*b^2 - a^2*b^3 - 9*a*b^4)*c
os(d*x + c)^3 + 105*(b^4*cos(d*x + c)^6 - 3*b^4*cos(d*x + c)^4 + 3*b^4*cos(
d*x + c)^2 - b^4)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a
- b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^5 -
 a*b^4)*cos(d*x + c))/(((a^6 + a^5*b)*d*cos(d*x + c)^6 - 3*(a^6 + a^5*b)*d*
```

```
cos(d*x + c)^4 + 3*(a^6 + a^5*b)*d*cos(d*x + c)^2 - (a^6 + a^5*b)*d)*sin(d*
x + c))]
```

giac [A] time = 0.18, size = 215, normalized size = 1.54

$$\frac{\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)b^4}{\sqrt{a^2+ab}\,a^4}-\frac{105a^3\tan(dx+c)^6-105a^2b\tan(dx+c)^6+105ab^2\tan(dx+c)^6-105b^3\tan(dx+c)^6}{}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*b^4/(sqrt(a^2 + a*b)*a^4) - (105*
a^3*tan(d*x + c)^6 - 105*a^2*b*tan(d*x + c)^6 + 105*a*b^2*tan(d*x + c)^6 -
105*b^3*tan(d*x + c)^6 + 105*a^3*tan(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 +
 35*a*b^2*tan(d*x + c)^4 + 63*a^3*tan(d*x + c)^2 - 21*a^2*b*tan(d*x + c)^2
+ 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

maple [A] time = 0.60, size = 207, normalized size = 1.48

$$\frac{b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,a^4\sqrt{a\,(a+b)}}-\frac{1}{7da\tan(dx+c)^7}-\frac{3}{5da\tan(dx+c)^5}+\frac{b}{5d\,a^2\tan(dx+c)^5}-\frac{1}{da\tan(dx+c)^3}+\frac{}{3d\,a^2\,\mathrm{t}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d*b^4/a^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/7/d/
a/tan(d*x+c)^7-3/5/d/a/tan(d*x+c)^5+1/5/d/a^2/tan(d*x+c)^5*b-1/d/a/tan(d*x+
c)^3+2/3/d/a^2/tan(d*x+c)^3*b-1/3/d/a^3/tan(d*x+c)^3*b^2-1/d/a/tan(d*x+c)+1
/d/a^2/tan(d*x+c)*b-1/d/a^3/tan(d*x+c)*b^2+1/d/a^4/tan(d*x+c)*b^3
```

maxima [A] time = 0.56, size = 137, normalized size = 0.98

$$\frac{\frac{105\,b^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^4}-\frac{105\left(a^3-a^2b+ab^2-b^3\right)\tan(dx+c)^6+35\left(3a^3-2a^2b+ab^2\right)\tan(dx+c)^4+15a^3+21\left(3a^3-a^2b\right)\tan(dx+c)^2}{a^4\tan(dx+c)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*b^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a
)*a^4) - (105*(a^3 - a^2*b + a*b^2 - b^3)*tan(d*x + c)^6 + 35*(3*a^3 - 2*a^
2*b + a*b^2)*tan(d*x + c)^4 + 15*a^3 + 21*(3*a^3 - a^2*b)*tan(d*x + c)^2)/(
a^4*tan(d*x + c)^7))/d
```

mupad [B] time = 15.07, size = 130, normalized size = 0.93

$$\frac{b^4\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)}{a^{9/2}\,d\,\sqrt{a+b}}-\frac{\tan(c+d\,x)^4\,\left(a^3-\frac{2\,a^2\,b}{3}+\frac{a\,b^2}{3}\right)-\tan(c+d\,x)^2\,\left(\frac{a^2\,b}{5}-\frac{3\,a^3}{5}\right)+\tan(c+d\,x)^6\,\left(a^3-a^2\right.}{a^4\,d\tan(c+d\,x)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a + b*sin(c + d*x)^2)),x)
```

```
[Out] (b^4*atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2)))/(a^(9/2)*d*(a + b)^(1/2))
- (tan(c + d*x)^4*((a*b^2)/3 - (2*a^2*b)/3 + a^3) - tan(c + d*x)^2*((a^2*b)
```

```
/5 - (3*a^3)/5) + tan(c + d*x)^6*(a*b^2 - a^2*b + a^3 - b^3) + a^3/7)/(a^4*
d*tan(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.94 $\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=128

$$\frac{a^3\cos(c+dx)}{2b^3d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}d(a+b)^{3/2}}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

```
[Out] -1/2*a^2*(5*a+6*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(7/2)/(a+b)^(3
/2)/d+(2*a-b)*cos(d*x+c)/b^3/d+1/3*cos(d*x+c)^3/b^2/d+1/2*a^3*cos(d*x+c)/b^
3/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A] time = 0.19, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 208}

$$\frac{a^3\cos(c+dx)}{2b^3d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}d(a+b)^{3/2}}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(a^2*(5*a + 6*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(7/2)*(
a + b)^(3/2)*d) + ((2*a - b)*Cos[c + d*x])/(b^3*d) + Cos[c + d*x]^3/(3*b^2*
d) + (a^3*Cos[c + d*x])/(2*b^3*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{2a-b}{b^3}-\frac{x^2}{b^2}+\frac{a^2(2a+3b)-3a^2bx^2}{b^3(a+b-bx^2)^2}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}-\frac{\text{Subst}\left(\int \frac{a^2(2a+3b)-3a^2bx^2}{(a+b-bx^2)^2}\,dx, x, \cos(c+dx)\right)}{b^3d}$$

$$= \frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{a^3\cos(c+dx)}{2b^3(a+b)d\left(a+b-b\cos^2(c+dx)\right)}-\frac{\left(a^2(5\right.}{}$$

$$= -\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{7/2}(a+b)^{3/2}d}+\frac{(2a-b)\cos(c+dx)}{b^3d}+\frac{\cos^3(c+dx)}{3b^2d}+\frac{}{2b^3(a}$$

Mathematica [C] time = 1.57, size = 194, normalized size = 1.52

$$\frac{\sqrt{b}\left(\cos(c+dx)\left(\frac{12a^3}{(a+b)(2a-b\cos(2(c+dx))+b)}+24a-9b\right)+b\cos(3(c+dx))\right)-\frac{6a^2(5a+6b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}-\frac{6a^2(5a}{}}{12b^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
- b]])/(-a - b)^(3/2) - (6*a^2*(5*a + 6*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[
(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + Sqrt[b]*(Cos[c + d*x]*(24*a -
 9*b + (12*a^3)/((a + b)*(2*a + b - b*Cos[2*(c + d*x)]))) + b*Cos[3*(c + d*
x)]))/(12*b^(7/2)*d)
```

fricas [B] time = 0.51, size = 529, normalized size = 4.13

$$\left[\frac{4\left(a^2b^3+2ab^4+b^5\right)\cos(dx+c)^5+4\left(5a^3b^2+6a^2b^3-3ab^4-4b^5\right)\cos(dx+c)^3-3\left(5a^4+11a^3b+6a^2b^2-\right.}{12\left(\left(a^2b^5+2ab^6+b^7\right)\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/12*(4*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 4*(5*a^3*b^2 + 6*a^2*b^
3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5*
a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 -
 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 6*(5
*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2*b^5)*cos(d*x + c))/((a^2*b^5
+ 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d
), 1/6*(2*(a^2*b^3 + 2*a*b^4 + b^5)*cos(d*x + c)^5 + 2*(5*a^3*b^2 + 6*a^2*b
^3 - 3*a*b^4 - 4*b^5)*cos(d*x + c)^3 - 3*(5*a^4 + 11*a^3*b + 6*a^2*b^2 - (5
*a^3*b + 6*a^2*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^2
)*cos(d*x + c)/(a + b)) - 3*(5*a^4*b + 11*a^3*b^2 + 6*a^2*b^3 - 2*a*b^4 - 2
```

```
*b^5)*cos(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^2 - (a^3*b^4
+ 3*a^2*b^5 + 3*a*b^6 + b^7)*d)]
```

giac [B] time = 0.21, size = 322, normalized size = 2.52

$$\frac{\frac{3\left(5a^3+6a^2b\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\left(ab^3+b^4\right)\sqrt{-ab-b^2}}+\frac{6\left(a^3-\frac{a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(ab^3+b^4\right)\left(a-\frac{2a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{8\left(3a-b-\frac{6a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\cdots\right.}{b^3\left(\cdots\right.}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*(5*a^3 + 6*a^2*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*
cos(d*x + c) + sqrt(-a*b - b^2)))/((a*b^3 + b^4)*sqrt(-a*b - b^2)) + 6*(a^3
 - a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a^2*b*(cos(d*x + c) - 1)/(
cos(d*x + c) + 1))/((a*b^3 + b^4)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c)
 + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/
(cos(d*x + c) + 1)^2)) - 8*(3*a - b - 6*a*(cos(d*x + c) - 1)/(cos(d*x + c)
+ 1) + 3*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/(b^3*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1)^3))
/d
```

maple [A] time = 0.33, size = 118, normalized size = 0.92

$$\frac{\frac{\frac{\left(\cos^3(dx+c)\right)b}{3}+2a\cos(dx+c)-b\cos(dx+c)}{b^3}+\frac{a^2\left(-\frac{a\cos(dx+c)}{2(a+b)\left(b\left(\cos^2(dx+c)\right)-a-b\right)}-\frac{(5a+6b)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/b^3*(1/3*cos(d*x+c)^3*b+2*a*cos(d*x+c)-b*cos(d*x+c))+a^2/b^3*(-1/2*a
/(a+b)*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-1/2*(5*a+6*b)/(a+b)/((a+b)*b)^(1/2)*
arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))))
```

maxima [A] time = 0.72, size = 154, normalized size = 1.20

$$\frac{\frac{6a^3\cos(dx+c)}{a^2b^3+2ab^4+b^5-\left(ab^4+b^5\right)\cos(dx+c)^2}+\frac{3(5a+6b)a^2\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\left(ab^3+b^4\right)\sqrt{(a+b)b}}+\frac{4\left(b\cos(dx+c)^3+3(2a-b)\cos(dx+c)\right)}{b^3}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/12*(6*a^3*cos(d*x + c)/(a^2*b^3 + 2*a*b^4 + b^5 - (a*b^4 + b^5)*cos(d*x +
 c)^2) + 3*(5*a + 6*b)*a^2*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*
x + c) + sqrt((a + b)*b)))/((a*b^3 + b^4)*sqrt((a + b)*b)) + 4*(b*cos(d*x +
 c)^3 + 3*(2*a - b)*cos(d*x + c))/b^3)/d
```

mupad [B] time = 0.22, size = 123, normalized size = 0.96

$$\frac{\cos(c+dx)\left(\frac{2(a+b)}{b^3}-\frac{3}{b^2}\right)}{d}+\frac{\cos(c+dx)^3}{3b^2d}+\frac{a^3\cos(c+dx)}{2d(a+b)\left(-b^4\cos(c+dx)^2+b^4+ab^3\right)}-\frac{a^2\operatorname{atanh}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right.\cdots}{2b^{7/2}d(a+b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (cos(c + d*x)*((2*(a + b))/b^3 - 3/b^2))/d + cos(c + d*x)^3/(3*b^2*d) + (a^
3*cos(c + d*x))/(2*d*(a + b)*(a*b^3 + b^4 - b^4*cos(c + d*x)^2)) - (a^2*ata
nh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(5*a + 6*b))/(2*b^(7/2)*d*(a + b)^
(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.95 $\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2} \, dx$

Optimal. Leaf size=102

$$-\frac{a^2\cos(c+dx)}{2b^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}-\frac{\cos(c+dx)}{b^2d}$$

```
[Out] 1/2*a*(3*a+4*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(5/2)/(a+b)^(3/2)
/d-cos(d*x+c)/b^2/d-1/2*a^2*cos(d*x+c)/b^2/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 390, 385, 208}

$$-\frac{a^2\cos(c+dx)}{2b^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}+\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}d(a+b)^{3/2}}-\frac{\cos(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(5/2)*(a +
 b)^(3/2)*d) - Cos[c + d*x]/(b^2*d) - (a^2*Cos[c + d*x])/(2*b^2*(a + b)*d*(
a + b - b*Cos[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{b^2}-\frac{a(a+2b)-2abx^2}{b^2\left(a+b-bx^2\right)^2}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)}{b^2d}+\frac{\text{Subst}\left(\int \frac{a(a+2b)-2abx^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{b^2d}$$

$$= -\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}+\frac{(a(3a+4b))\,\text{Subst}\left(\int \frac{1}{a+b-b\ldots}\right.}{2b^2(a+b\ldots}$$

$$= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{5/2}(a+b)^{3/2}d}-\frac{\cos(c+dx)}{b^2d}-\frac{a^2\cos(c+dx)}{2b^2(a+b)d\left(a+b-b\cos^2(c+dx\ldots\right.}$$

Mathematica [C] time = 0.92, size = 172, normalized size = 1.69

$$\frac{2\sqrt{b}\cos(c+dx)\left(-\frac{a^2}{(a+b)(2a-b\cos(2(c+dx))+b)}-1\right)+\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}+\frac{a(3a+4b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}}}{2b^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((a*(3*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]
)/(-a - b)^(3/2) + (a*(3*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)
/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*Sqrt[b]*Cos[c + d*x]*(-1 - a^2/((a +
 b)*(2*a + b - b*Cos[2*(c + d*x)]))))/(2*b^(5/2)*d)
```

fricas [B] time = 0.47, size = 427, normalized size = 4.19

$$\left[-\frac{4\left(a^2b^2+2ab^3+b^4\right)\cos(dx+c)^3+\left(3a^3+7a^2b+4ab^2-\left(3a^2b+4ab^2\right)\cos(dx+c)^2\right)\sqrt{ab+b^2}\,\log\left(\frac{b\cos(d\ldots}{\ldots}\right.}{4\left(\left(a^2b^4+2ab^5+b^6\right)d\cos(dx+c)^2-\left(a^3b^3+3a^2b^4\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(4*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 + (3*a^3 + 7*a^2*b + 4*a*
b^2 - (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(a*b + b^2)*log((b*cos(d*x +
c)^2 + 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b))
- 2*(3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x + c))/((a^2*b^4 + 2*a*b
^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 + 3*a*b^5 + b^6)*d), -1/2
*(2*(a^2*b^2 + 2*a*b^3 + b^4)*cos(d*x + c)^3 - (3*a^3 + 7*a^2*b + 4*a*b^2 -
 (3*a^2*b + 4*a*b^2)*cos(d*x + c)^2)*sqrt(-a*b - b^2)*arctan(sqrt(-a*b - b^
2)*cos(d*x + c)/(a + b)) - (3*a^3*b + 7*a^2*b^2 + 6*a*b^3 + 2*b^4)*cos(d*x
+ c))/((a^2*b^4 + 2*a*b^5 + b^6)*d*cos(d*x + c)^2 - (a^3*b^3 + 3*a^2*b^4 +
3*a*b^5 + b^6)*d)]
```

giac [B] time = 0.21, size = 342, normalized size = 3.35

$$-\frac{\frac{\left(3\,a^2+4\,ab\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\left(ab^2+b^3\right)\sqrt{-ab-b^2}}+\frac{2\left(3\,a^2+2\,ab-\frac{6\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{14\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{8\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a^2(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,ab(\cos\cdots}{(\cos\cdots}\right.}{\left(ab^2+b^3\right)\left(a-\frac{3\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}+\frac{4\,b(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a(\cos(dx\cdots}{(\cos(dx+\cdots}\right.}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a^2 + 4*a*b)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a*b^2 + b^3)*sqrt(-a*b - b^2)) + 2*(3*a^2
+ 2*a*b - 6*a^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 14*a*b*(cos(d*x + c
) - 1)/(cos(d*x + c) + 1) - 8*b^2*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3
*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4*a*b*(cos(d*x + c) - 1)^2
/(cos(d*x + c) + 1)^2)/((a*b^2 + b^3)*(a - 3*a*(cos(d*x + c) - 1)/(cos(d*x
+ c) + 1) - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) -
 1)^2/(cos(d*x + c) + 1)^2 + 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2
- a*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)))/d
```

maple [A] time = 0.32, size = 94, normalized size = 0.92

$$\frac{-\frac{\cos(dx+c)}{b^2}-\frac{a\left(-\frac{a\cos(dx+c)}{2(a+b)\left(b\left(\cos^2(dx+c)\right)-a-b\right)}-\frac{(3a+4b)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}\right)}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/b^2*cos(d*x+c)-a/b^2*(-1/2*a/(a+b)*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-
1/2*(3*a+4*b)/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))))
```

maxima [A] time = 0.90, size = 131, normalized size = 1.28

$$-\frac{\frac{2\,a^2\cos(dx+c)}{a^2b^2+2\,ab^3+b^4-\left(ab^3+b^4\right)\cos(dx+c)^2}+\frac{(3\,a+4\,b)a\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\left(ab^2+b^3\right)\sqrt{(a+b)b}}+\frac{4\cos(dx+c)}{b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/4*(2*a^2*cos(d*x + c)/(a^2*b^2 + 2*a*b^3 + b^4 - (a*b^3 + b^4)*cos(d*x +
 c)^2) + (3*a + 4*b)*a*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
c) + sqrt((a + b)*b)))/((a*b^2 + b^3)*sqrt((a + b)*b)) + 4*cos(d*x + c)/b^2
)/d
```

mupad [B] time = 0.16, size = 95, normalized size = 0.93

$$\frac{a\,\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)\,(3\,a+4\,b)}{2\,b^{5/2}\,d\,(a+b)^{3/2}}-\frac{a^2\,\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b^3\,\cos\left(c+d\,x\right)^2+b^3+a\,b^2\right)}-\frac{\cos\left(c+d\,x\right)}{b^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*atanh((b^(1/2)*cos(c + d*x))/(a + b)^(1/2))*(3*a + 4*b))/(2*b^(5/2)*d*(a
 + b)^(3/2)) - (a^2*cos(c + d*x))/(2*d*(a + b)*(a*b^2 + b^3 - b^3*cos(c + d
*x)^2)) - cos(c + d*x)/(b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.96 $\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=83

$$\frac{a\cos(c+dx)}{2bd(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}$$

```
[Out] -1/2*(a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))/b^(3/2)/(a+b)^(3/2)/d+
1/2*a*cos(d*x+c)/b/(a+b)/d/(a+b-b*cos(d*x+c)^2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 385, 208}

$$\frac{a\cos(c+dx)}{2bd(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -((a + 2*b)*ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*b^(3/2)*(a + b)
^(3/2)*d) + (a*Cos[c + d*x])/(2*b*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2b(a+b)d}$$

$$= -\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2b^{3/2}(a+b)^{3/2}d} + \frac{a\cos(c+dx)}{2b(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [C] time = 0.48, size = 160, normalized size = 1.93

$$\frac{\frac{2a\sqrt{b}\cos(c+dx)}{2a-b\cos(2(c+dx))+b} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}} + \frac{(a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{-a-b}}}{2b^{3/2}d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (((a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2]])/Sqrt[-a - b]])/Sq
rt[-a - b] + ((a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[
-a - b]])/Sqrt[-a - b] + (2*a*Sqrt[b]*Cos[c + d*x])/(2*a + b - b*Cos[2*(c +
 d*x)]))/(2*b^(3/2)*(a + b)*d)
```

fricas [B] time = 0.46, size = 327, normalized size = 3.94

$$\left[\frac{\left(\left(ab+2\,b^2\right)\cos\left(dx+c\right)^2 - a^2 - 3\,ab - 2\,b^2\right)\sqrt{ab+b^2}\,\log\left(-\frac{b\cos(dx+c)^2-2\,\sqrt{ab+b^2}\,\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right) - 2\left(a^2b+ab^2\right)\cos}{4\left(\left(a^2b^3+2\,ab^4+b^5\right)d\cos\left(dx+c\right)^2 - \left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)d\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(a*b + b^2)*
log(-(b*cos(d*x + c)^2 - 2*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x
 + c)^2 - a - b)) - 2*(a^2*b + a*b^2)*cos(d*x + c))/((a^2*b^3 + 2*a*b^4 + b
^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5)*d), 1/2*(((a*b
 + 2*b^2)*cos(d*x + c)^2 - a^2 - 3*a*b - 2*b^2)*sqrt(-a*b - b^2)*arctan(sqr
t(-a*b - b^2)*cos(d*x + c)/(a + b)) - (a^2*b + a*b^2)*cos(d*x + c))/((a^2*b
^3 + 2*a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 + b^5
)*d)]
```

giac [A] time = 0.18, size = 93, normalized size = 1.12

$$\frac{(a+2\,b)\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\left(ab+b^2\right)\sqrt{-ab-b^2}\,d} - \frac{a\cos\left(dx+c\right)}{2\left(b\cos\left(dx+c\right)^2-a-b\right)\left(ab+b^2\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(a + 2*b)*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/((a*b + b^2)*sqrt(-a*
b - b^2)*d) - 1/2*a*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a*b + b^2)*d)
```

maple [A] time = 0.29, size = 80, normalized size = 0.96

$$\frac{-\frac{a\cos(dx+c)}{2(a+b)b\left(b\left(\cos^2(dx+c)\right)-a-b\right)}-\frac{(a+2b)\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{2(a+b)b\sqrt{(a+b)b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(-1/2*a/(a+b)/b*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-1/2*(a+2*b)/(a+b)/b/((a
+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

maxima [A] time = 0.42, size = 111, normalized size = 1.34

$$\frac{\frac{2\,a\cos(dx+c)}{a^2b+2\,ab^2+b^3-\left(ab^2+b^3\right)\cos(dx+c)^2}+\frac{(a+2\,b)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\left(ab+b^2\right)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*a*cos(d*x + c)/(a^2*b + 2*a*b^2 + b^3 - (a*b^2 + b^3)*cos(d*x + c)^2
) + (a + 2*b)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt
((a + b)*b)))/(sqrt((a + b)*b)*(a*b + b^2)))/d
```

mupad [B] time = 13.49, size = 71, normalized size = 0.86

$$\frac{a\,\cos\left(c+d\,x\right)}{2\,b\,d\,\left(a+b\right)\,\left(-b\,{\cos\left(c+d\,x\right)}^2+a+b\right)}-\frac{\mathrm{atanh}\left(\frac{\sqrt{b}\,\cos\left(c+d\,x\right)}{\sqrt{a+b}}\right)\,\left(a+2\,b\right)}{2\,b^{3/2}\,d\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*cos(c + d*x))/(2*b*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - (atanh((b^(1/
2)*cos(c + d*x))/(a + b)^(1/2))*(a + 2*b))/(2*b^(3/2)*d*(a + b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.97 $\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(c+dx)}{2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}$$

```
[Out] -1/2*cos(d*x+c)/(a+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*arctanh(cos(d*x+c)*b^(1/2)
/(a+b)^(1/2))/(a+b)^(3/2)/d/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 199, 208}

$$-\frac{\cos(c+dx)}{2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -ArcTanh[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]]/(2*Sqrt[b]*(a + b)^(3/2)*d) -
Cos[c + d*x]/(2*(a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{a+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2(a+b)d}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2\sqrt{b}\,(a+b)^{3/2}d} - \frac{\cos(c+dx)}{2(a+b)d\left(a+b-b\cos^2(c+dx)\right)}$$

Mathematica [**C**] time = 0.29, size = 149, normalized size = 2.01

$$\frac{-\frac{2\cos(c+dx)}{2a-b\cos(2(c+dx))+b} + \frac{\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}\sqrt{-a-b}} + \frac{\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{\sqrt{b}\sqrt{-a-b}}}{2d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqrt[-a - b]*
Sqrt[b]) + ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a - b]]/(Sqr
t[-a - b]*Sqrt[b]) - (2*Cos[c + d*x])/(2*a + b - b*Cos[2*(c + d*x)]))/(2*(a
 + b)*d)
```

fricas [**A**] time = 0.45, size = 282, normalized size = 3.81

$$\left[\frac{\left(b\cos\left(dx+c\right)^2-a-b\right)\sqrt{ab+b^2}\,\log\left(-\frac{b\cos(dx+c)^2-2\sqrt{ab+b^2}\cos(dx+c)+a+b}{b\cos(dx+c)^2-a-b}\right)+2\left(ab+b^2\right)\cos\left(dx+c\right)}{4\left(\left(a^2b^2+2\,ab^3+b^4\right)d\cos\left(dx+c\right)^2-\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)d\right)}, \frac{\left(b\cos\left(d\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*((b*cos(d*x + c)^2 - a - b)*sqrt(a*b + b^2)*log(-(b*cos(d*x + c)^2 - 2
*sqrt(a*b + b^2)*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*(a*b
 + b^2)*cos(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b
+ 3*a^2*b^2 + 3*a*b^3 + b^4)*d), 1/2*((b*cos(d*x + c)^2 - a - b)*sqrt(-a*b
- b^2)*arctan(sqrt(-a*b - b^2)*cos(d*x + c)/(a + b)) + (a*b + b^2)*cos(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^3*b + 3*a^2*b^2 + 3*
a*b^3 + b^4)*d)]
```

giac [**A**] time = 0.16, size = 79, normalized size = 1.07

$$\frac{\arctan\left(\frac{b\cos(dx+c)}{\sqrt{-ab-b^2}}\right)}{2\sqrt{-ab-b^2}\,(a+b)d} + \frac{\cos\left(dx+c\right)}{2\left(b\cos\left(dx+c\right)^2-a-b\right)(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*arctan(b*cos(d*x + c)/sqrt(-a*b - b^2))/(sqrt(-a*b - b^2)*(a + b)*d) +
1/2*cos(d*x + c)/((b*cos(d*x + c)^2 - a - b)*(a + b)*d)
```

maple [A] time = 0.22, size = 68, normalized size = 0.92

$$\frac{\frac{\cos(dx+c)}{2(a+b)\left(b\left(\cos^{2}(dx+c)\right)-a-b\right)}-\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{2(a+b)\sqrt{(a+b)b}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/d*(1/2*cos(d*x+c)/(a+b)/(b*cos(d*x+c)^2-a-b)-1/2/(a+b)/((a+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/((a+b)*b)^(1/2)))
```

maxima [A] time = 0.46, size = 98, normalized size = 1.32

$$\frac{\frac{2\cos(dx+c)}{\left(ab+b^{2}\right)\cos(dx+c)^{2}-a^{2}-2\,ab-b^{2}}+\frac{\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\sqrt{(a+b)b}\,(a+b)}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*cos(d*x + c)/((a*b + b^2)*cos(d*x + c)^2 - a^2 - 2*a*b - b^2) + log(
(b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c) + sqrt((a + b)*b)))/(sqr
t((a + b)*b)*(a + b)))/d
```

mupad [B] time = 0.11, size = 62, normalized size = 0.84

$$-\frac{\cos\left(c+d\,x\right)}{2\,d\,\left(a+b\right)\,\left(-b\cos\left(c+d\,x\right)^{2}+a+b\right)}-\frac{\operatorname{atanh}\left(\frac{\sqrt{b}\,\cos(c+d\,x)}{\sqrt{a+b}}\right)}{2\,\sqrt{b}\,d\,\left(a+b\right)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] - cos(c + d*x)/(2*d*(a + b)*(a + b - b*cos(c + d*x)^2)) - atanh((b^(1/2)*co
s(c + d*x))/(a + b)^(1/2))/(2*b^(1/2)*d*(a + b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.98 $\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=103

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b\cos(c+dx)}{2ad(a+b)\left(a-b\cos^2(c+dx)+b\right)}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d+1/2*b*cos(d*x+c)/a/(a+b)/d/(a+b-b*cos(d*x+c)^2)+
1/2*(3*a+2*b)*arctanh(cos(d*x+c)*b^(1/2)/(a+b)^(1/2))*b^(1/2)/a^2/(a+b)^(3/
2)/d
```

Rubi [A] time = 0.13, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3186, 414, 522, 206, 208}

$$\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2d(a+b)^{3/2}}-\frac{\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b\cos(c+dx)}{2ad(a+b)\left(a-b\cos^2(c+dx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(ArcTanh[Cos[c + d*x]]/(a^2*d)) + (Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Co
s[c + d*x])/Sqrt[a + b]])/(2*a^2*(a + b)^(3/2)*d) + (b*Cos[c + d*x])/(2*a*(
a + b)*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-2a-b-bx^2}{(1-x^2)\left(a+b-bx^2\right)}\,dx, x, \cos(c+dx)\right)}{2a(a+b)d}$$

$$= \frac{b\cos(c+dx)}{2a(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \cos(c+dx)\right)}{a^2d} + \frac{(b(3a+2\ldots}{\ldots}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^2(a+b)^{3/2}d} + \frac{b\cos(c+\ldots}{2a(a+b)d\left(a+b-b\ldots\right.}$$

Mathematica [C] time = 0.80, size = 194, normalized size = 1.88

$$\frac{\frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{\sqrt{b}(3a+2b)\tan^{-1}\left(\frac{\sqrt{b}+i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + 2\left(\frac{ab\cos(c+dx)}{(a+b)(2a-b\cos(2(c+dx))+b)} + \log\left(\sin\left(\frac{1}{2}(c+dx\ldots\right.\right.\right.}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt[-a
 - b]])/(-a - b)^(3/2) + (Sqrt[b]*(3*a + 2*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*T
an[(c + d*x)/2])/Sqrt[-a - b]])/(-a - b)^(3/2) + 2*((a*b*Cos[c + d*x])/((a
+ b)*(2*a + b - b*Cos[2*(c + d*x)])) - Log[Cos[(c + d*x)/2]] + Log[Sin[(c +
 d*x)/2]]))/(2*a^2*d)
```

fricas [B] time = 0.54, size = 455, normalized size = 4.42

$$\left[-\frac{2ab\cos(dx+c) - \left(\left(3ab+2b^2\right)\cos(dx+c)^2 - 3a^2 - 5ab - 2b^2\right)\sqrt{\frac{b}{a+b}}\log\left(\frac{b\cos(dx+c)^2 + 2(a+b)\sqrt{\frac{b}{a+b}}\cos(dx+c) + a+\ldots}{b\cos(dx+c)^2 - a - b}\right.}{4\left(\left(a^3b+\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(2*a*b*cos(d*x + c) - ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b
 - 2*b^2)*sqrt(b/(a + b))*log((b*cos(d*x + c)^2 + 2*(a + b)*sqrt(b/(a + b))
*cos(d*x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) + 2*((a*b + b^2)*cos(d*x
 + c)^2 - a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - 2*((a*b + b^2)*c
os(d*x + c)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b +
a^2*b^2)*d*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d), -1/2*(a*b*cos(d*x
 + c) + ((3*a*b + 2*b^2)*cos(d*x + c)^2 - 3*a^2 - 5*a*b - 2*b^2)*sqrt(-b/(a
 + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) + ((a*b + b^2)*cos(d*x + c)^2
```

```
- a^2 - 2*a*b - b^2)*log(1/2*cos(d*x + c) + 1/2) - ((a*b + b^2)*cos(d*x + c
)^2 - a^2 - 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3*b + a^2*b^2)*d
*cos(d*x + c)^2 - (a^4 + 2*a^3*b + a^2*b^2)*d)]
```

giac [B] time = 0.19, size = 246, normalized size = 2.39

$$-\frac{\frac{\left(3\,ab+2\,b^2\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\,\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\left(a^3+a^2b\right)\sqrt{-ab-b^2}}-\frac{2\left(ab-\frac{ab(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,b^2(\cos(dx+c)-1)}{\cos(dx+c)+1}\right)}{\left(a^3+a^2b\right)\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}-\frac{\log\left(\frac{|-\cos(dx+c)+1|}{|\cos(dx+c)+1|}\right)}{a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((3*a*b + 2*b^2)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2)*cos
(d*x + c) + sqrt(-a*b - b^2)))/((a^3 + a^2*b)*sqrt(-a*b - b^2)) - 2*(a*b -
a*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*b^2*(cos(d*x + c) - 1)/(cos(d
*x + c) + 1))/((a^3 + a^2*b)*(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
 - 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(
d*x + c) + 1)^2)) - log(abs(-cos(d*x + c) + 1)/abs(cos(d*x + c) + 1))/a^2)/
d
```

maple [A] time = 0.51, size = 150, normalized size = 1.46

$$\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2d\,a^2}-\frac{b\cos\left(dx+c\right)}{2da\left(a+b\right)\left(b\left(\cos^2\left(dx+c\right)\right)-a-b\right)}+\frac{3b\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{2da\left(a+b\right)\sqrt{\left(a+b\right)b}}+\frac{b^2\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{(a+b)b}}\right)}{d\,a^2\left(a+b\right)\sqrt{\left(a+b\right)b}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d/a^2*ln(cos(d*x+c)-1)-1/2/d/a*b/(a+b)*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)+
3/2/d/a*b/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))+1/d/a
^2*b^2/(a+b)/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))-1/2/d/a^
2*ln(1+cos(d*x+c))
```

maxima [A] time = 0.57, size = 149, normalized size = 1.45

$$\frac{\frac{2\,b\cos(dx+c)}{a^3+2\,a^2b+ab^2-\left(a^2b+ab^2\right)\cos(dx+c)^2}-\frac{\left(3\,ab+2\,b^2\right)\log\left(\frac{b\cos(dx+c)-\sqrt{(a+b)b}}{b\cos(dx+c)+\sqrt{(a+b)b}}\right)}{\left(a^3+a^2b\right)\sqrt{(a+b)b}}-\frac{2\,\log(\cos(dx+c)+1)}{a^2}+\frac{2\,\log(\cos(dx+c)-1)}{a^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(2*b*cos(d*x + c)/(a^3 + 2*a^2*b + a*b^2 - (a^2*b + a*b^2)*cos(d*x + c)
^2) - (3*a*b + 2*b^2)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x + c
) + sqrt((a + b)*b)))/((a^3 + a^2*b)*sqrt((a + b)*b)) - 2*log(cos(d*x + c)
+ 1)/a^2 + 2*log(cos(d*x + c) - 1)/a^2)/d
```

mupad [B] time = 14.63, size = 2039, normalized size = 19.80

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] (atan(((((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 +
 a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a
```

```
^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*(32*a
^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2
)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3*a^4*b + a^5 +
a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b)*1i)/(4*(3*a^4*b + a^
5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)
)/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*
b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) + (cos(c + d*x)*(b*(a + b)^3)^(1
/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*
a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))
/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b)
*1i)/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))/(((3*a*b^3)/2 + b^4)/(2*a^4
*b + a^5 + a^3*b^2) - (((cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(2*(
2*a^3*b + a^4 + a^2*b^2)) + ((b*(a + b)^3)^(1/2)*((2*a^4*b^4 + 6*a^5*b^3 +
4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) - (cos(c + d*x)*(b*(a + b)^3)^(1/2)*(3
*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*(2*a^3*b
+ a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(3*a + 2*b))/(4*(3
*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)*(3*a + 2*b))/(4*(
3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (((cos(c + d*x)*(20*a*b^4 + 8*b^5 +
 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((b*(a + b)^3)^(1/2)*((2*a^4*
b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*a^4*b + a^5 + a^3*b^2) + (cos(c + d*x)*(b*(
a + b)^3)^(1/2)*(3*a + 2*b)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*
b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*
(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)))*(b*(a + b)^3)^(1/2)
*(3*a + 2*b))/(4*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2))))*(b*(a + b)^3)^(1/
2)*(3*a + 2*b)*1i)/(2*d*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (atan(((((
(2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) - (cos(c
+ d*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b
 + a^4 + a^2*b^2)))*1i)/(2*a^2) + (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*
b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 - ((((2*a^4*b^4 + 6*a^5*b^3 + 4
*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (cos(c + d*x)*(32*a^4*b^5 + 80*a^
5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2
*a^2) - (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4
 + a^2*b^2)))/a^2)/((((2*a^4*b^4 + 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5
 + a^3*b^2)) - (cos(c + d*x)*(32*a^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7
*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*a^2) + (cos(c + d*x)*(20*a*b^4
 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/a^2 + (((2*a^4*b^4 +
 6*a^5*b^3 + 4*a^6*b^2)/(2*(2*a^4*b + a^5 + a^3*b^2)) + (cos(c + d*x)*(32*a
^4*b^5 + 80*a^5*b^4 + 64*a^6*b^3 + 16*a^7*b^2))/(8*a^2*(2*a^3*b + a^4 + a^2
*b^2)))/(2*a^2) - (cos(c + d*x)*(20*a*b^4 + 8*b^5 + 13*a^2*b^3))/(4*(2*a^3*
b + a^4 + a^2*b^2)))/a^2 - ((3*a*b^3)/2 + b^4)/(2*a^4*b + a^5 + a^3*b^2)))*
1i)/(a^2*d) + (b*cos(c + d*x))/(2*a*d*(a + b)*(a + b - b*cos(c + d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.99 $\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=153

$$-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{b(a+2b)\cos(c+dx)}{2a^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{c}{2ad}$$

```
[Out] -1/2*(a-4*b)*arctanh(cos(d*x+c))/a^3/d-1/2*b^(3/2)*(5*a+4*b)*arctanh(cos(d*
x+c)*b^(1/2)/(a+b)^(1/2))/a^3/(a+b)^(3/2)/d-1/2*b*(a+2*b)*cos(d*x+c)/a^2/(a
+b)/d/(a+b-b*cos(d*x+c)^2)-1/2*cot(d*x+c)*csc(d*x+c)/a/d/(a+b-b*cos(d*x+c)^
2)
```

Rubi [A] time = 0.24, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 522, 206, 208}

$$-\frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3d(a+b)^{3/2}}-\frac{b(a+2b)\cos(c+dx)}{2a^2d(a+b)\left(a-b\cos^2(c+dx)+b\right)}-\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d}-\frac{c}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -((a - 4*b)*ArcTanh[Cos[c + d*x]])/(2*a^3*d) - (b^(3/2)*(5*a + 4*b)*ArcTanh
[(Sqrt[b]*Cos[c + d*x])/Sqrt[a + b]])/(2*a^3*(a + b)^(3/2)*d) - (b*(a + 2*b
)*Cos[c + d*x])/(2*a^2*(a + b)*d*(a + b - b*Cos[c + d*x]^2)) - (Cot[c + d*x
]*Csc[c + d*x])/(2*a*d*(a + b - b*Cos[c + d*x]^2))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a-b-3bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^2}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{b(a+2b)\cos(c+dx)}{2a^2(a+b)d\left(a+b-b\cos^2(c+dx)\right)} - \frac{\cot(c+dx)\csc(c+dx)}{2ad\left(a+b-b\cos^2(c+dx)\right)} - \frac{(a-4b)\,\text{Su}}{}$$

$$= -\frac{(a-4b)\tanh^{-1}(\cos(c+dx))}{2a^3d} - \frac{b^{3/2}(5a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\cos(c+dx)}{\sqrt{a+b}}\right)}{2a^3(a+b)^{3/2}d} - \frac{b(a+}{2a^2(a+b)d\,(}$$

Mathematica [C] time = 1.57, size = 390, normalized size = 2.55

$$\frac{\csc^3(c+dx)(-2a+b\cos(2(c+dx))-b)\left(\frac{4b^{3/2}(5a+4b)\csc(c+dx)(2a-b\cos(2(c+dx))+b)\tan^{-1}\left(\frac{\sqrt{b}-i\sqrt{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a-b}}\right)}{(-a-b)^{3/2}} + \frac{4b^{3/2}(5a+4b)\,\text{cs}}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^3*((8*a*b^2*Cot[c + d*x])/(a
+ b) + (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] - I*Sqrt[a]*Tan[(c + d*x)/2])
/Sqrt[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2)
+ (4*b^(3/2)*(5*a + 4*b)*ArcTan[(Sqrt[b] + I*Sqrt[a]*Tan[(c + d*x)/2])/Sqrt
[-a - b]]*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x])/(-a - b)^(3/2) + a*(
2*a + b - b*Cos[2*(c + d*x)])*Csc[(c + d*x)/2]^2*Csc[c + d*x] + 4*(a - 4*b)
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Cos[(c + d*x)/2]] - 4*(a -
 4*b)*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Log[Sin[(c + d*x)/2]] - a
```

```
*(2*a + b - b*Cos[2*(c + d*x)])*Csc[c + d*x]*Sec[(c + d*x)/2]^2))/(32*a^3*d
*(b + a*Csc[c + d*x]^2)^2)
```

fricas [B] time = 0.57, size = 838, normalized size = 5.48

$$\left[\frac{2\left(a^2b+2\,ab^2\right)\cos\left(dx+c\right)^3+\left(\left(5\,ab^2+4\,b^3\right)\cos\left(dx+c\right)^4+5\,a^2b+9\,ab^2+4\,b^3-\left(5\,a^2b+14\,ab^2+8\,b^3\right)\mathrm{co}\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(2*(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + ((5*a*b^2 + 4*b^3)*cos(d*x + c)^
4 + 5*a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2
)*sqrt(b/(a + b))*log(-(b*cos(d*x + c)^2 - 2*(a + b)*sqrt(b/(a + b))*cos(d*
x + c) + a + b)/(b*cos(d*x + c)^2 - a - b)) - 2*(a^3 + 2*a^2*b + 2*a*b^2)*c
os(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^2*b - 7
*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*log(1/2*c
os(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a^5 + 3*
a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d), 1/4*(2*
(a^2*b + 2*a*b^2)*cos(d*x + c)^3 + 2*((5*a*b^2 + 4*b^3)*cos(d*x + c)^4 + 5*
a^2*b + 9*a*b^2 + 4*b^3 - (5*a^2*b + 14*a*b^2 + 8*b^3)*cos(d*x + c)^2)*sqrt
(-b/(a + b))*arctan(sqrt(-b/(a + b))*cos(d*x + c)) - 2*(a^3 + 2*a^2*b + 2*a
*b^2)*cos(d*x + c) - ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3 - 2*a^
2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c)^2)*lo
g(1/2*cos(d*x + c) + 1/2) + ((a^2*b - 3*a*b^2 - 4*b^3)*cos(d*x + c)^4 + a^3
 - 2*a^2*b - 7*a*b^2 - 4*b^3 - (a^3 - a^2*b - 10*a*b^2 - 8*b^3)*cos(d*x + c
)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^4*b + a^3*b^2)*d*cos(d*x + c)^4 - (a
^5 + 3*a^4*b + 2*a^3*b^2)*d*cos(d*x + c)^2 + (a^5 + 2*a^4*b + a^3*b^2)*d)]
```

giac [B] time = 0.20, size = 512, normalized size = 3.35

$$\frac{\frac{12\left(5\,ab^2+4\,b^3\right)\arctan\left(\frac{b\cos(dx+c)+a+b}{\sqrt{-ab-b^2}\,\cos(dx+c)+\sqrt{-ab-b^2}}\right)}{\left(a^4+a^3b\right)\sqrt{-ab-b^2}}+\frac{3\,a^3+3\,a^2b-\frac{8\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{12\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{28\,ab^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{7\,a^3(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}-\frac{a^2b\cdots}{\cdots}}{\left(a^4+a^3b\right)\left(\frac{a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{2\,a\cdots}{\cdots}\right.}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/24*(12*(5*a*b^2 + 4*b^3)*arctan((b*cos(d*x + c) + a + b)/(sqrt(-a*b - b^2
)*cos(d*x + c) + sqrt(-a*b - b^2)))/((a^4 + a^3*b)*sqrt(-a*b - b^2)) + (3*a
^3 + 3*a^2*b - 8*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 12*a^2*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) - 28*a*b^2*(cos(d*x + c) - 1)/(cos(d*x + c
) + 1) + 7*a^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - a^2*b*(cos(d*x +
 c) - 1)^2/(cos(d*x + c) + 1)^2 - 16*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x +
c) + 1)^2 + 16*b^3*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 - 2*a^3*(cos(d
*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 6*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x
 + c) + 1)^3 + 8*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)/((a^4 + a
^3*b)*(a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 2*a*(cos(d*x + c) - 1)^2/(
cos(d*x + c) + 1)^2 - 4*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + a*(co
s(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3)) + 6*(a - 4*b)*log(abs(-cos(d*x + c
) + 1)/abs(cos(d*x + c) + 1))/a^3 - 3*(cos(d*x + c) - 1)/(a^2*(cos(d*x + c)
 + 1)))/d
```

maple [A] time = 0.62, size = 226, normalized size = 1.48

$$\frac{1}{4d\,a^{2}\left(\cos\left(dx+c\right)-1\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{4d\,a^{2}}-\frac{\ln\left(\cos\left(dx+c\right)-1\right)b}{d\,a^{3}}+\frac{b^{2}\cos\left(dx+c\right)}{2d\,a^{2}\left(a+b\right)\left(b\left(\cos^{2}\left(dx+c\right)\right)-a-b\right)}-\frac{5}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/4/d/a^2/(cos(d*x+c)-1)+1/4/d/a^2*ln(cos(d*x+c)-1)-1/d/a^3*ln(cos(d*x+c)-1
)*b+1/2/d/a^2*b^2/(a+b)*cos(d*x+c)/(b*cos(d*x+c)^2-a-b)-5/2/d/a^2*b^2/(a+b)
/((a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))-2/d/a^3*b^3/(a+b)/((
a+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/((a+b)*b)^(1/2))+1/4/d/a^2/(1+cos(d*x+c)
)-1/4/d/a^2*ln(1+cos(d*x+c))+1/d/a^3*ln(1+cos(d*x+c))*b
```

maxima [A] time = 0.48, size = 223, normalized size = 1.46

$$\frac{\frac{\left(5\,ab^{2}+4\,b^{3}\right)\log\left(\frac{b\cos\left(dx+c\right)-\sqrt{\left(a+b\right)b}}{b\cos\left(dx+c\right)+\sqrt{\left(a+b\right)b}}\right)}{\left(a^{4}+a^{3}b\right)\sqrt{\left(a+b\right)b}}+\frac{2\left(\left(ab+2\,b^{2}\right)\cos\left(dx+c\right)^{3}-\left(a^{2}+2\,ab+2\,b^{2}\right)\cos\left(dx+c\right)\right)}{\left(a^{3}b+a^{2}b^{2}\right)\cos\left(dx+c\right)^{4}+a^{4}+2\,a^{3}b+a^{2}b^{2}-\left(a^{4}+3\,a^{3}b+2\,a^{2}b^{2}\right)\cos\left(dx+c\right)^{2}}-\frac{\left(a-4\,b\right)\log\left(\cos\left(dx+c\right)+1\right)}{a^{3}}+\frac{(a-}{}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*((5*a*b^2 + 4*b^3)*log((b*cos(d*x + c) - sqrt((a + b)*b))/(b*cos(d*x +
c) + sqrt((a + b)*b)))/((a^4 + a^3*b)*sqrt((a + b)*b)) + 2*((a*b + 2*b^2)*c
os(d*x + c)^3 - (a^2 + 2*a*b + 2*b^2)*cos(d*x + c))/((a^3*b + a^2*b^2)*cos(
d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 - (a^4 + 3*a^3*b + 2*a^2*b^2)*cos(d*x
+ c)^2) - (a - 4*b)*log(cos(d*x + c) + 1)/a^3 + (a - 4*b)*log(cos(d*x + c)
- 1)/a^3)/d
```

mupad [B] time = 14.95, size = 2338, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] - ((cos(c + d*x)*(2*a*b + a^2 + 2*b^2))/(2*a^2*(a + b)) - (b*cos(c + d*x)^3
*(a + 2*b))/(2*a^2*(a + b)))/(d*(a + b + b*cos(c + d*x)^4 - cos(c + d*x)^2*
(a + 2*b))) - (atan((((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*
b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 +
8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x
)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*
a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3))*1i)/(4*a^3) + ((a - 4*b)*((cos
(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5
*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2
*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4
+ 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(
4*a^3))*1i)/(4*a^3))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/(2
*a^7*b + a^8 + a^6*b^2) - ((a - 4*b)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26
*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b
^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c
+ d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^
3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))/(4*a^3)))/(4*a^3) + ((a - 4*b)*((c
os(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a
^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/
(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*(a - 4*b)*(32*a^6*b^5 + 80*a^7*b^
```

```
4 + 64*a^8*b^3 + 16*a^9*b^2))/(8*a^3*(2*a^5*b + a^6 + a^4*b^2)))*(a - 4*b))
/(4*a^3)))/(4*a^3)))*(a - 4*b)*1i)/(2*a^3*d) - (atan(((((5*a)/4 + b)*(b^3*(
a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4
+ a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^
8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2) - (cos(c + d*x)*((5*a)/4 + b)*
(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/
(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)
/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(
3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2) + (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)
*((cos(c + d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*
(2*a^5*b + a^6 + a^4*b^2)) - (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b
^2)/(2*a^7*b + a^8 + a^6*b^2) + (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)
^(1/2)*(32*a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a
^6 + a^4*b^2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a
 + b)^3)^(1/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))*1i)/(3*a^5*b + a^6 +
 a^3*b^3 + 3*a^4*b^2))/((12*a*b^6 + 8*b^7 + (3*a^2*b^5)/2 - (5*a^3*b^4)/4)/
(2*a^7*b + a^8 + a^6*b^2) - (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c +
d*x)*(64*a*b^6 + 32*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b +
a^6 + a^4*b^2)) + (((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*
b + a^8 + a^6*b^2) - (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*
a^6*b^5 + 80*a^7*b^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^
2)*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1
/2))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a
^4*b^2) + (((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*((cos(c + d*x)*(64*a*b^6 + 3
2*b^7 + 26*a^2*b^5 - 6*a^3*b^4 + a^4*b^3))/(2*(2*a^5*b + a^6 + a^4*b^2)) -
(((4*a^6*b^5 + 8*a^7*b^4 + 2*a^8*b^3 - 2*a^9*b^2)/(2*a^7*b + a^8 + a^6*b^2)
 + (cos(c + d*x)*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2)*(32*a^6*b^5 + 80*a^7*b
^4 + 64*a^8*b^3 + 16*a^9*b^2))/(2*(2*a^5*b + a^6 + a^4*b^2)*(3*a^5*b + a^6
+ a^3*b^3 + 3*a^4*b^2)))*((5*a)/4 + b)*(b^3*(a + b)^3)^(1/2))/(3*a^5*b + a^
6 + a^3*b^3 + 3*a^4*b^2)))/(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*((5*a)/4
 + b)*(b^3*(a + b)^3)^(1/2)*2i)/(d*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**2)**2,x)
[Out] Timed out
```

3.100 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=148

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3d(a+b)^{3/2}}-\frac{x(4a-b)}{2b^3}-\frac{a(2a+b)\tan(c+dx)}{2b^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left((a+b)\tan^2(c+dx)\right)}$$

```
[Out] -1/2*(4*a-b)*x/b^3+1/2*a^(3/2)*(4*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1
/2))/b^3/(a+b)^(3/2)/d-1/2*a*(2*a+b)*tan(d*x+c)/b^2/(a+b)/d/(a+(a+b)*tan(d*
x+c)^2)-1/2*sin(d*x+c)^2*tan(d*x+c)/b/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A] time = 0.29, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$\frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2b^3d(a+b)^{3/2}}-\frac{a(2a+b)\tan(c+dx)}{2b^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{x(4a-b)}{2b^3}-\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left((a+b)\tan^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -((4*a - b)*x)/(2*b^3) + (a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d
*x])/Sqrt[a]])/(2*b^3*(a + b)^(3/2)*d) - (a*(2*a + b)*Tan[c + d*x])/(2*b^2*
(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)) - (Sin[c + d*x]^2*Tan[c + d*x])/(2*
b*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a+b)x^2\right)}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{2bd}$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Sub}}{}$$

$$= -\frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\sin^2(c+dx)\tan(c+dx)}{2bd\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{(4a-}{}$$

$$= -\frac{(4a-b)x}{2b^3} + \frac{a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^3(a+b)^{3/2}d} - \frac{a(2a+b)\tan(c+dx)}{2b^2(a+b)d\left(a+(a+b)\tan^2\right.}$$

Mathematica [A] time = 1.53, size = 106, normalized size = 0.72

$$\frac{\frac{2a^{3/2}(4a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + b\sin(2(c+dx))\left(-\frac{2a^2}{(a+b)(2a-b\cos(2(c+dx))+b)}-1\right) - 2(4a-b)(c+dx)}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (-2*(4*a - b)*(c + d*x) + (2*a^(3/2)*(4*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c
+ d*x])/Sqrt[a]])/(a + b)^(3/2) + b*(-1 - (2*a^2)/((a + b)*(2*a + b - b*Cos
[2*(c + d*x)])))*Sin[2*(c + d*x)])/(4*b^3*d)
```

fricas [A] time = 0.49, size = 623, normalized size = 4.21

$$\left[-\frac{4\left(4a^2b+3ab^2-b^3\right)dx\cos\left(dx+c\right)^2-4\left(4a^3+7a^2b+2ab^2-b^3\right)dx+\left(4a^3+9a^2b+5ab^2-\left(4a^2b+5ab^2\right.\right.\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 4*(4*a^3 + 7*a^2*b
+ 2*a*b^2 - b^3)*d*x + (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^2*b + 5*a*b^2)*cos
(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2
*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^2)*cos(d*x +
c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^
2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 +
 2*a*b + b^2)) + 4*((a*b^2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3
)*cos(d*x + c))*sin(d*x + c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 +
2*a*b^4 + b^5)*d), -1/4*(2*(4*a^2*b + 3*a*b^2 - b^3)*d*x*cos(d*x + c)^2 - 2
*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*d*x - (4*a^3 + 9*a^2*b + 5*a*b^2 - (4*a^
2*b + 5*a*b^2)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*
x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) + 2*((a*b^
2 + b^3)*cos(d*x + c)^3 - (2*a^2*b + 2*a*b^2 + b^3)*cos(d*x + c))*sin(d*x +
 c))/((a*b^4 + b^5)*d*cos(d*x + c)^2 - (a^2*b^3 + 2*a*b^4 + b^5)*d)]
```

giac [A] time = 0.18, size = 223, normalized size = 1.51

$$\frac{\frac{\left(4a^3+5a^2b\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2a+2b\right)+\arctan\left(\frac{a\tan\left(dx+c\right)+b\tan\left(dx+c\right)}{\sqrt{a^2+ab}}\right)\right)}{\left(ab^3+b^4\right)\sqrt{a^2+ab}}-\frac{2a^2\tan\left(dx+c\right)^3+2ab\tan\left(dx+c\right)^3+b^2\tan\left(dx+c\right)^3+2a^2\tan\left(dx+c\right)+ab\tan\left(dx\cdots\right.}{\left(a\tan\left(dx+c\right)^4+b\tan\left(dx+c\right)^4+2a\tan\left(dx+c\right)^2+b\tan\left(dx+c\right)^2+a\right)\left(ab^2+b^3\right)}\cdots}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((4*a^3 + 5*a^2*b)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arcta
n((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a*b^3 + b^4)*sqrt(a
^2 + a*b)) - (2*a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c
)^3 + 2*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a*tan(d*x + c)^4 + b*tan(d*x
 + c)^4 + 2*a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b^2 + b^3)) - (d*x
+ c)*(4*a - b)/b^3)/d
```

maple [A] time = 0.34, size = 187, normalized size = 1.26

$$-\frac{a^2\tan\left(dx+c\right)}{2d\,b^2\left(a+b\right)\left(a\left(\tan^2\left(dx+c\right)\right)+\left(\tan^2\left(dx+c\right)\right)b+a\right)}+\frac{2a^3\arctan\left(\frac{\left(a+b\right)\tan\left(dx+c\right)}{\sqrt{a\left(a+b\right)}}\right)}{d\,b^3\left(a+b\right)\sqrt{a\left(a+b\right)}}+\frac{5a^2\arctan\left(\frac{\left(a+b\right)\tan\left(dx+c\right)}{\sqrt{a\left(a+b\right)}}\right)}{2d\,b^2\left(a+b\right)\sqrt{a\left(a+b\right)}}\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d*a^2/b^2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+2/d*a^3/b
^3/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+5/2/d*a^2
/b^2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/2/d/b
^2*tan(d*x+c)/(tan(d*x+c)^2+1)+1/2/d/b^2*arctan(tan(d*x+c))-2/d/b^3*arctan(
tan(d*x+c))*a
```

maxima [A] time = 0.45, size = 181, normalized size = 1.22

$$\frac{\frac{\left(4\,a^3+5\,a^2b\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(ab^3+b^4\right)\sqrt{(a+b)a}}-\frac{\left(2\,a^2+2\,ab+b^2\right)\tan(dx+c)^3+\left(2\,a^2+ab\right)\tan(dx+c)}{\left(a^2b^2+2\,ab^3+b^4\right)\tan(dx+c)^4+a^2b^2+ab^3+\left(2\,a^2b^2+3\,ab^3+b^4\right)\tan(dx+c)^2}-\frac{(dx+c)(4\,a-b)}{b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((4*a^3 + 5*a^2*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^3
 + b^4)*sqrt((a + b)*a)) - ((2*a^2 + 2*a*b + b^2)*tan(d*x + c)^3 + (2*a^2 +
 a*b)*tan(d*x + c))/((a^2*b^2 + 2*a*b^3 + b^4)*tan(d*x + c)^4 + a^2*b^2 + a
*b^3 + (2*a^2*b^2 + 3*a*b^3 + b^4)*tan(d*x + c)^2) - (d*x + c)*(4*a - b)/b^
3)/d
```

mupad [B] time = 16.06, size = 2295, normalized size = 15.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (atan((((a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a
*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^
5)) + (((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan
(c + d*x)*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^
2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*
b^4 + a^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a
^2*b^4 + a^3*b^3))*1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3) + ((a + (5*b)/
4)*(-a^3*(a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6
 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) - (((2*a*b^9 +
8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a + (5*b
)/4)*(-a^3*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7
 + 32*a^4*b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a
 + (5*b)/4)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3))*
1i)/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3))/((16*a^5*b + 8*a^6 + (5*a^2*b^4)
/4 - (13*a^3*b^3)/2 + (3*a^4*b^2)/2)/(a*b^6 + b^7) + ((a + (5*b)/4)*(-a^3*(
a + b)^3)^(1/2)*((tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*
b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)) + (((2*a*b^9 + 8*a^2*b^8
+ 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan(c + d*x)*(a + (5*b)/4)*(-a^3
*(a + b)^3)^(1/2)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*
b^6))/(2*(a*b^4 + b^5)*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a + (5*b)/4
)*(-a^3*(a + b)^3)^(1/2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))/(3*a*b^5
+ b^6 + 3*a^2*b^4 + a^3*b^3) - ((a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*((tan(
c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*
a^4*b^2))/(2*(a*b^4 + b^5)) - (((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b
^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*(80*
a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*(a*b^4 + b^5)
*(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/
2))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^3*b^3)))/(3*a*b^5 + b^6 + 3*a^2*b^4 + a^
3*b^3)))*(a + (5*b)/4)*(-a^3*(a + b)^3)^(1/2)*2i)/(d*(3*a*b^5 + b^6 + 3*a^2
*b^4 + a^3*b^3)) - (atan((((((((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6
)/(a*b^6 + b^7) - (tan(c + d*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144
*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)
/4))/b^3 - (tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 +
20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)*1i)/b^3 - ((
(((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) + (tan(c + d
*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*
a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3 + (tan(c + d*x)*(96
```

```
*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2
*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)*1i)/b^3)/((16*a^5*b + 8*a^6 + (5*a^2*b^4
)/4 - (13*a^3*b^3)/2 + (3*a^4*b^2)/2)/(a*b^6 + b^7) + (((((2*a*b^9 + 8*a^2*
b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(a*b^6 + b^7) - (tan(c + d*x)*(a*1i - (b*1i)/
4)*(80*a*b^9 + 16*b^10 + 144*a^2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a
*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3 - (tan(c + d*x)*(96*a^5*b - 4*a*b^5 +
32*a^6 + b^6 - 10*a^2*b^4 + 20*a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a
*1i - (b*1i)/4))/b^3 + (((((2*a*b^9 + 8*a^2*b^8 + 10*a^3*b^7 + 4*a^4*b^6)/(
a*b^6 + b^7) + (tan(c + d*x)*(a*1i - (b*1i)/4)*(80*a*b^9 + 16*b^10 + 144*a^
2*b^8 + 112*a^3*b^7 + 32*a^4*b^6))/(2*b^3*(a*b^4 + b^5)))*(a*1i - (b*1i)/4)
)/b^3 + (tan(c + d*x)*(96*a^5*b - 4*a*b^5 + 32*a^6 + b^6 - 10*a^2*b^4 + 20*
a^3*b^3 + 90*a^4*b^2))/(2*(a*b^4 + b^5)))*(a*1i - (b*1i)/4))/b^3))*(a*1i -
(b*1i)/4)*2i)/(b^3*d) - ((tan(c + d*x)^3*(2*a*b + 2*a^2 + b^2))/(2*b^2*(a +
 b)) + (a*tan(c + d*x)*(2*a + b))/(2*b^2*(a + b)))/(d*(a + tan(c + d*x)^4*(
a + b) + tan(c + d*x)^2*(2*a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.101 $\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=93

$$-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2d(a+b)^{3/2}}+\frac{a\tan(c+dx)}{2bd(a+b)\left((a+b)\tan^2(c+dx)+a\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2-1/2*(2*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/b^2/(a+b
)^(3/2)/d+1/2*a*tan(d*x+c)/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A] time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3187, 470, 522, 203, 205}

$$-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2d(a+b)^{3/2}}+\frac{a\tan(c+dx)}{2bd(a+b)\left((a+b)\tan^2(c+dx)+a\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] x/b^2 - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2
*b^2*(a + b)^(3/2)*d) + (a*Tan[c + d*x])/(2*b*(a + b)*d*(a + (a + b)*Tan[c
+ d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{a+(-a-2b)x^2}{(1+x^2)\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{2b(a+b)d}$$

$$= \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d} - (a(2a+$$

$$= \frac{x}{b^2} - \frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2b^2(a+b)^{3/2}d} + \frac{a\tan(c+dx)}{2b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)}$$

Mathematica [A] time = 0.86, size = 93, normalized size = 1.00

$$\frac{-\frac{\sqrt{a}\,(2a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{ab\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)} + 2(c+dx)}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (2*(c + d*x) - (Sqrt[a]*(2*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[
a]])/(a + b)^(3/2) + (a*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c
+ d*x)])))/(2*b^2*d)
```

fricas [B] time = 0.48, size = 492, normalized size = 5.29

$$\left[\frac{8\left(ab+b^2\right)dx\cos\left(dx+c\right)^2 - 4\,ab\cos\left(dx+c\right)\sin\left(dx+c\right) - 8\left(a^2+2\,ab+b^2\right)dx + \left(\left(2\,ab+3\,b^2\right)\cos\left(dx+c\right)^2 - \right.}{8\left(\left(ab^3+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(8*(a*b + b^2)*d*x*cos(d*x + c)^2 - 4*a*b*cos(d*x + c)*sin(d*x + c) -
8*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*b
 - 3*b^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(d*x + c)^
3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c) + a^2 +
 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2)))/((a*b^3 + b^4)*d*cos(d*x + c)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)
, 1/4*(4*(a*b + b^2)*d*x*cos(d*x + c)^2 - 2*a*b*cos(d*x + c)*sin(d*x + c) -
```

```
4*(a^2 + 2*a*b + b^2)*d*x + ((2*a*b + 3*b^2)*cos(d*x + c)^2 - 2*a^2 - 5*a*
b - 3*b^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sq
rt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))))/((a*b^3 + b^4)*d*cos(d*x + c)
^2 - (a^2*b^2 + 2*a*b^3 + b^4)*d)]
```

giac [A] time = 0.19, size = 140, normalized size = 1.51

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2a^2+3ab)}{(ab^2+b^3)\sqrt{a^2+ab}}-\frac{a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(ab+b^2)}-\frac{2(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a^2 + 3*a*b)/((a*b^2 + b^3)*sqrt(a^
2 + a*b)) - a*tan(d*x + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a*b
+ b^2)) - 2*(d*x + c)/b^2)/d
```

maple [A] time = 0.27, size = 140, normalized size = 1.51

$$\frac{a\tan(dx+c)}{2db(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}-\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,b^2(a+b)\sqrt{a(a+b)}}-\frac{3a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2db(a+b)\sqrt{a(a+b)}}+\frac{a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*a/b/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)-1/d*a^2/b^2/(a
+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-3/2/d*a/b/(a+b
)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/d/b^2*arctan(t
an(d*x+c))
```

maxima [A] time = 0.48, size = 109, normalized size = 1.17

$$\frac{\frac{a\tan(dx+c)}{a^2b+ab^2+(a^2b+2ab^2+b^3)\tan(dx+c)^2}-\frac{(2a^2+3ab)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(ab^2+b^3)\sqrt{(a+b)a}}+\frac{2(dx+c)}{b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a*tan(d*x + c)/(a^2*b + a*b^2 + (a^2*b + 2*a*b^2 + b^3)*tan(d*x + c)^2
) - (2*a^2 + 3*a*b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a*b^2 +
b^3)*sqrt((a + b)*a)) + 2*(d*x + c)/b^2)/d
```

mupad [B] time = 15.23, size = 1959, normalized size = 21.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (a*tan(c + d*x))/(2*d*(a + tan(c + d*x)^2*(a + b))*(a*b + b^2)) - atan((((
(2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(80
*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 +
b^3)))/(2*b^2) + (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^
2*b^2))/(4*(a*b^2 + b^3)))/b^2 - ((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(
```

```
2*(a*b^3 + b^4)) + (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3
*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))/(2*b^2) - (tan(c + d*x)*(16*a*b^
3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2))/(4*(a*b^2 + b^3)))/b^2)/((3*a*b
^2 + (7*a^2*b)/2 + a^3)/(a*b^3 + b^4) + (((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^
4)*1i)/(2*(a*b^3 + b^4)) - (tan(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 +
 112*a^3*b^5 + 32*a^4*b^4))/(8*b^2*(a*b^2 + b^3)))*1i)/(2*b^2) + (tan(c + d
*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(a*b^2 + b^3)
))/b^2 + (((((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)*1i)/(2*(a*b^3 + b^4)) + (tan
(c + d*x)*(80*a*b^7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*
b^2*(a*b^2 + b^3)))*1i)/(2*b^2) - (tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^
4 + 4*b^4 + 33*a^2*b^2)*1i)/(4*(a*b^2 + b^3)))/b^2))/(b^2*d) - (atan((((-a*
(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a
^2*b^2))/(2*(a*b^2 + b^3)) - (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/(a*b^3 + b
^4) - (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 + 16*b^8 + 1
44*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4 + b^5 + 3
*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 +
 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3
*b^2)) + ((-a*(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4
+ 4*b^4 + 33*a^2*b^2))/(2*(a*b^2 + b^3)) + (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b
^4)/(a*b^3 + b^4) + (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^
7 + 16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a
*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3
*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b)*1i)/(4*(3*a*b^4 + b^5 + 3
*a^2*b^3 + a^3*b^2)))/((3*a*b^2 + (7*a^2*b)/2 + a^3)/(a*b^3 + b^4) - ((-a*(
a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*b^4 + 33*a^
2*b^2))/(2*(a*b^2 + b^3)) - (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/(a*b^3 + b^
4) - (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 + 16*b^8 + 14
4*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4 + b^5 + 3*
a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b^4 + b^5 +
3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^3 + a^3*b^2
)) + ((-a*(a + b)^3)^(1/2)*((tan(c + d*x)*(16*a*b^3 + 28*a^3*b + 8*a^4 + 4*
b^4 + 33*a^2*b^2))/(2*(a*b^2 + b^3)) + (((2*a*b^6 + 4*a^2*b^5 + 2*a^3*b^4)/
(a*b^3 + b^4) + (tan(c + d*x)*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*(80*a*b^7 +
16*b^8 + 144*a^2*b^6 + 112*a^3*b^5 + 32*a^4*b^4))/(8*(a*b^2 + b^3)*(3*a*b^4
 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b))/(4*(3*a*b
^4 + b^5 + 3*a^2*b^3 + a^3*b^2)))*(2*a + 3*b))/(4*(3*a*b^4 + b^5 + 3*a^2*b^
3 + a^3*b^2))))*(-a*(a + b)^3)^(1/2)*(2*a + 3*b)*1i)/(2*d*(3*a*b^4 + b^5 +
3*a^2*b^3 + a^3*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.102 $\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=78

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] -1/2*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)+1/2*arctan((a+b)^(1/2
)*tan(d*x+c)/a^(1/2))/(a+b)^(3/2)/d/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3173, 12, 3181, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,d(a+b)^{3/2}}-\frac{\sin(c+dx)\cos(c+dx)}{2d(a+b)\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(2*Sqrt[a]*(a + b)^(3/2)*d) - (C
os[c + d*x]*Sin[c + d*x])/(2*(a + b)*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{a}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2(a+b)}$$
$$= -\frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{2(a+b)d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2\sqrt{a}\,(a+b)^{3/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{2(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.53, size = 74, normalized size = 0.95

$$\frac{\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}} - \frac{\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]/(Sqrt[a]*(a + b)^(3/2)) - Sin[2
*(c + d*x)]/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*d)
```

fricas [B] time = 0.46, size = 419, normalized size = 5.37

$$\left[\frac{4\left(a^2+ab\right)\cos(dx+c)\sin(dx+c) - \left(b\cos(dx+c)^2 - a - b\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4 - 2\left(4a^2+5ab+b^2\right.}{b^2 c}\right.}{8\left(\left(a^3b+2a^2b^2+ab^3\right)d\cos(dx+c)^2 - \left(a^4+3a^3b+3a^2\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*
sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x +
c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4
- 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)))/((a^3*b + 2*a^2*b^2 +
 a*b^3)*d*cos(d*x + c)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d), 1/4*(2*(
a^2 + a*b)*cos(d*x + c)*sin(d*x + c) - (b*cos(d*x + c)^2 - a - b)*sqrt(a^2
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))))/((a^3*b + 2*a^2*b^2 + a*b^3)*d*cos(d*x + c)^2 - (a^
4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*d)]
```

giac [A] time = 0.18, size = 109, normalized size = 1.40

$$\frac{\frac{\pi\left\lfloor \frac{dx+c}{\pi} + \frac{1}{2}\right\rfloor \mathrm{sgn}(2a+2b) + \arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,(a+b)} - \frac{\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)(a+b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c
)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a + b)))/d
```

maple [A] time = 0.25, size = 77, normalized size = 0.99

$$-\frac{\tan(dx+c)}{2d(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2d(a+b)\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/2/d/(a+b)/(a*(a
+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.47, size = 74, normalized size = 0.95

$$-\frac{\frac{\tan(dx+c)}{(a^2+2ab+b^2)\tan(dx+c)^2+a^2+ab}-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(tan(d*x + c)/((a^2 + 2*a*b + b^2)*tan(d*x + c)^2 + a^2 + a*b) - arcta
n((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)))/d
```

mupad [B] time = 13.43, size = 72, normalized size = 0.92

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)^2}{4\,\sqrt{a}\,(a+b)^{3/2}}\right)}{2\,\sqrt{a}\,d\,(a+b)^{3/2}}-\frac{\tan(c+d\,x)}{2\,d\,\left((a+b)\,\tan(c+d\,x)^2+a\right)\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] atan((tan(c + d*x)*(2*a + 2*b)^2)/(4*a^(1/2)*(a + b)^(3/2)))/(2*a^(1/2)*d*(
a + b)^(3/2)) - tan(c + d*x)/(2*d*(a + tan(c + d*x)^2*(a + b))*(a + b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.103 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=87

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] 1/2*(2*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(3/2)/d+1/
2*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)
```

Rubi [A] time = 0.06, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3184, 12, 3181, 205}

$$\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}d(a+b)^{3/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{2ad(a+b)\left(a+b\sin^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] ((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(
3/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a + b)*d*(a + b*Sin[c + d*x]^
2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} - \frac{\int \frac{-2a-b}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$
$$= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{(2a+b)\int \frac{1}{a+b\sin^2(c+dx)}\,dx}{2a(a+b)}$$
$$= \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)} + \frac{(2a+b)\,\mathrm{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{2a(a+b)d}$$
$$= \frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{3/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{2a(a+b)d\left(a+b\sin^2(c+dx)\right)}$$

Mathematica [A] time = 0.41, size = 84, normalized size = 0.97

$$\frac{\frac{(2a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}} + \frac{\sqrt{a}\,b\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)}}{2a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-2),x]
```

```
[Out] (((2*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(3/2) + (Sq
rt[a]*b*Sin[2*(c + d*x)])/((a + b)*(2*a + b - b*Cos[2*(c + d*x)])))/(2*a^(3
/2)*d)
```

fricas [B] time = 0.47, size = 463, normalized size = 5.32

$$\left[-\frac{4\left(a^2b+ab^2\right)\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(2\,ab+b^2\right)\cos\left(dx+c\right)^2-2\,a^2-3\,ab-b^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\right.}{}\right.}{8\left(\left(a^4b+2\,a^3b^2+a^2b^3\right)d\cos\left(dx+c\right)^2-\left(\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c) + ((2*a*b + b^2)*cos(d*x
 + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(
d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*
a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
 + b^2)))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a^4*b
+ 3*a^3*b^2 + a^2*b^3)*d), -1/4*(2*(a^2*b + a*b^2)*cos(d*x + c)*sin(d*x + c
) + ((2*a*b + b^2)*cos(d*x + c)^2 - 2*a^2 - 3*a*b - b^2)*sqrt(a^2 + a*b)*ar
ctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*s
in(d*x + c))))/((a^4*b + 2*a^3*b^2 + a^2*b^3)*d*cos(d*x + c)^2 - (a^5 + 3*a
^4*b + 3*a^3*b^2 + a^2*b^3)*d)]
```

giac [A] time = 0.15, size = 113, normalized size = 1.30

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(2\,a+b)}{\left(a^2+ab\right)^{\frac{3}{2}}} + \frac{b\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)\left(a^2+ab\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(2*a + b)/(a^2 + a*b)^(3/2) + b*tan(d*x
 + c)/((a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)*(a^2 + a*b)))/d
```

maple [A] time = 0.36, size = 119, normalized size = 1.37

$$\frac{b\tan\left(dx+c\right)}{2da\left(a+b\right)\left(a\left(\tan^{2}\left(dx+c\right)\right)+\left(\tan^{2}\left(dx+c\right)\right)b+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\left(a+b\right)\sqrt{a\left(a+b\right)}}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{2da\left(a+b\right)\sqrt{a\left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b/a/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+1/d/(a+b)/(a*(
a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/2/d/a/(a+b)/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

maxima [A] time = 0.48, size = 89, normalized size = 1.02

$$\frac{\frac{b\tan(dx+c)}{a^{3}+a^{2}b+\left(a^{3}+2\,a^{2}b+ab^{2}\right)\tan(dx+c)^{2}}+\frac{(2\,a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\left(a^{2}+ab\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(b*tan(d*x + c)/(a^3 + a^2*b + (a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^2)
+ (2*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(
a^2 + a*b)))/d
```

mupad [B] time = 13.43, size = 79, normalized size = 0.91

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)\,(2\,a+b)}{2\,a^{3/2}\,d\,(a+b)^{3/2}}+\frac{b\tan\left(c+d\,x\right)}{2\,a\,d\,\left(\left(a+b\right)\,\tan\left(c+d\,x\right)^{2}+a\right)\,\left(a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^2,x)
```

```
[Out] (atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))*(2*a + b))/(2*a
^(3/2)*d*(a + b)^(3/2)) + (b*tan(c + d*x))/(2*a*d*(a + tan(c + d*x)^2*(a +
b))*(a + b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.104 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=127

$$-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^{3/2}}-\frac{\left(2ab+3b^2\right)\sin(c+dx)\cos(c+dx)}{2a^2d(a+b)\left(a+b\sin^2(c+dx)\right)}-\frac{\cot(c+dx)}{ad\left(a+b\sin^2(c+dx)\right)}$$

```
[Out] -1/2*b*(4*a+3*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+b)^(3/2)
/d-cot(d*x+c)/a/d/(a+b*sin(d*x+c)^2)-1/2*(2*a*b+3*b^2)*cos(d*x+c)*sin(d*x+c
)/a^2/(a+b)/d/(a+b*sin(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 130, normalized size of antiderivative = 1.02, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3187, 462, 385, 205}

$$-\frac{\left(2a^2+4ab+3b^2\right)\tan(c+dx)}{2a^2d(a+b)\left((a+b)\tan^2(c+dx)+a\right)}-\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}d(a+b)^{3/2}}-\frac{\cot(c+dx)}{ad\left((a+b)\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(5/2)*(a +
 b)^(3/2)*d) - Cot[c + d*x]/(a*d*(a + (a + b)*Tan[c + d*x]^2)) - ((2*a^2 +
4*a*b + 3*b^2)*Tan[c + d*x])/(2*a^2*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 462

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^2, x_Symbol] :> Simp[(c^2*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)
), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^p*Simp[b*c^2*
n*(p + 1) + c*(b*c - 2*a*d)*(m + 1) - a*(m + 1)*d^2*x^n, x], x], x] /; Free
Q[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] &
& GtQ[n, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-a-3b+ax^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= -\frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\left(2a^2+4ab+3b^2\right)\tan(c+dx)}{2a^2(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{(b(4a+3\ldots}{\ldots}$$

$$= -\frac{b(4a+3b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{5/2}(a+b)^{3/2}d} - \frac{\cot(c+dx)}{ad\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\left(2a^2+4ab\ldots\right.}{2a^2(a+b)d\left(a\ldots\right.}$$

Mathematica [A] time = 1.19, size = 155, normalized size = 1.22

$$-\frac{\csc^4(c+dx)(2a-b\cos(2(c+dx))+b)\left(\sqrt{a}\sqrt{a+b}\cot(c+dx)\left(4a^2-b(2a+3b)\cos(2(c+dx))+6ab+3b^2\right)+\ldots\right.}{8a^{5/2}d(a+b)^{3/2}\left(a\csc^2(c+dx)+b\right)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] -1/8*((2*a + b - b*Cos[2*(c + d*x)])*(b*(4*a + 3*b)*ArcTan[(Sqrt[a + b]*Tan
[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)]) + Sqrt[a]*Sqrt[a + b]*(4
*a^2 + 6*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(c + d*x)])*Cot[c + d*x])*Csc[c
+ d*x]^4)/(a^(5/2)*(a + b)^(3/2)*d*(b + a*Csc[c + d*x]^2)^2)
```

fricas [B] time = 0.49, size = 588, normalized size = 4.63

$$\left[-\frac{4\left(2a^3b+5a^2b^2+3ab^3\right)\cos(dx+c)^3-\left(4a^2b+7ab^2+3b^3-\left(4ab^2+3b^3\right)\cos(dx+c)^2\right)\sqrt{-a^2-ab}\log\left(\ldots\right.}{8\left(\left(a^5b+2a^4b^2+\ldots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(2*a^3*b + 5*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^2*b + 7*a*b^
2 + 3*b^3 - (4*a*b^2 + 3*b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2
+ 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*
((2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x
+ c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^
2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 4*(2*a^4 + 6*a^3*b + 7*a^2*b^2 + 3*a
*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^3*b^3)*d*cos(d*x + c)^2 - (a^6
 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x + c)), -1/4*(2*(2*a^3*b + 5*a^
2*b^2 + 3*a*b^3)*cos(d*x + c)^3 + (4*a^2*b + 7*a*b^2 + 3*b^3 - (4*a*b^2 + 3
*b^3)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2
- a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 2*(2*a
^4 + 6*a^3*b + 7*a^2*b^2 + 3*a*b^3)*cos(d*x + c))/(((a^5*b + 2*a^4*b^2 + a^
3*b^3)*d*cos(d*x + c)^2 - (a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*d)*sin(d*x
+ c))]
```

giac [A] time = 0.18, size = 179, normalized size = 1.41

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)\left(4\,ab+3\,b^2\right)}{\left(a^3+a^2b\right)\sqrt{a^2+ab}}+\frac{2\,a^2\tan(dx+c)^2+4\,ab\tan(dx+c)^2+3\,b^2\tan(dx+c)^2+2\,a^2+2\,ab}{\left(a\tan(dx+c)^3+b\tan(dx+c)^3+a\tan(dx+c)\right)\left(a^3+a^2b\right)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
 + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a*b + 3*b^2)/((a^3 + a^2*b)*sqrt(a^
2 + a*b)) + (2*a^2*tan(d*x + c)^2 + 4*a*b*tan(d*x + c)^2 + 3*b^2*tan(d*x +
c)^2 + 2*a^2 + 2*a*b)/((a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x + c
))*(a^3 + a^2*b)))/d
```

maple [A] time = 0.54, size = 144, normalized size = 1.13

$$-\frac{b^2\tan(dx+c)}{2d\,a^2\,(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}-\frac{2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{da\,(a+b)\sqrt{a\,(a+b)}}-\frac{3b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2d\,a^2\,(a+b)\sqrt{a\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] -1/2/d/a^2*b^2/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)-2/d/a/(a+
b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-3/2/d/a^2*b^2
/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/d/a^2/tan
(d*x+c)
```

maxima [A] time = 0.47, size = 133, normalized size = 1.05

$$-\frac{\frac{\left(4\,ab+3\,b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^3+a^2b\right)\sqrt{(a+b)a}}+\frac{\left(2\,a^2+4\,ab+3\,b^2\right)\tan(dx+c)^2+2\,a^2+2\,ab}{\left(a^4+2\,a^3b+a^2b^2\right)\tan(dx+c)^3+\left(a^4+a^3b\right)\tan(dx+c)}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*((4*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 +
a^2*b)*sqrt((a + b)*a)) + ((2*a^2 + 4*a*b + 3*b^2)*tan(d*x + c)^2 + 2*a^2 +
 2*a*b)/((a^4 + 2*a^3*b + a^2*b^2)*tan(d*x + c)^3 + (a^4 + a^3*b)*tan(d*x +
 c)))/d
```

mupad [B] time = 13.60, size = 132, normalized size = 1.04

$$-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)^2\,\left(2\,a^2+4\,a\,b+3\,b^2\right)}{2\,a^2\,(a+b)}}{d\,\left((a+b)\,\tan(c+d\,x)^3+a\tan(c+d\,x)\right)}-\frac{b\,\mathrm{atan}\left(\frac{b\tan(c+d\,x)\,\left(a^3+b\,a^2\right)\,(4\,a+3\,b)}{a^{5/2}\,\sqrt{a+b}\,\left(3\,b^2+4\,a\,b\right)}\right)\,(4\,a+3\,b)}{2\,a^{5/2}\,d\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] - (1/a + (tan(c + d*x)^2*(4*a*b + 2*a^2 + 3*b^2))/(2*a^2*(a + b)))/(d*(a*ta
n(c + d*x) + tan(c + d*x)^3*(a + b))) - (b*atan((b*tan(c + d*x)*(a^2*b + a^
3)*(4*a + 3*b))/(a^(5/2)*(a + b)^(1/2)*(4*a*b + 3*b^2)))*(4*a + 3*b))/(2*a^
(5/2)*d*(a + b)^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.105 $\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx$

Optimal. Leaf size=162

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2d(a+b)}-\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3d(a+b)}+\frac{b\csc^3(c+dx)\,\mathrm{s}}{2ad(a+b)\left((a+b)\,\mathrm{t}\right.}$$

```
[Out] 1/2*b^2*(6*a+5*b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/(a+b)^(3/2
)/d-1/2*(2*a^2-a*b-5*b^2)*cot(d*x+c)/a^3/(a+b)/d-1/6*(2*a+5*b)*cot(d*x+c)^3
/a^2/(a+b)/d+1/2*b*csc(d*x+c)^3*sec(d*x+c)/a/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A] time = 0.20, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3187, 468, 570, 205}

$$\frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}d(a+b)^{3/2}}-\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3d(a+b)}-\frac{(2a+5b)\cot^3(c+dx)}{6a^2d(a+b)}+\frac{b\csc^3(c+dx)\,\mathrm{s}}{2ad(a+b)\left((a+b)\,\mathrm{t}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] (b^2*(6*a + 5*b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(2*a^(7/2)*(a
+ b)^(3/2)*d) - ((2*a^2 - a*b - 5*b^2)*Cot[c + d*x])/(2*a^3*(a + b)*d) - ((
2*a + 5*b)*Cot[c + d*x]^3)/(6*a^2*(a + b)*d) + (b*Csc[c + d*x]^3*Sec[c + d*
x])/(2*a*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 570

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_))^(r_.), x_Symbol] :> Int[ExpandIntegrand[
(g*x)^m*(a + b*x^n)^p*(c + d*x^n)^q*(e + f*x^n)^r, x], x] /; FreeQ[{a, b, c
, d, e, f, g, m, n}, x] && IGtQ[p, -2] && IGtQ[q, 0] && IGtQ[r, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-2a-5b+(-2a-b)x^2\right)}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c\right.}{2a(a+b)d}$$

$$= \frac{b\csc^3(c+dx)\sec(c+dx)}{2a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(\frac{-2a-5b}{ax^4} + \frac{-2a^2+ab+5b^2}{a^2x^2} + \frac{(-6a-5b)b}{a^2\left(a+(a+b)\right.}\right.\right.}{2a(a+b)d}$$

$$= -\frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c+dx)}{6a^2(a+b)d} + \frac{b\csc^3(c+dx)\sec(c}{2a(a+b)d\left(a+(a+b)\,\text{ta}\right.}$$

$$= \frac{b^2(6a+5b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{2a^{7/2}(a+b)^{3/2}d} - \frac{\left(2a^2-ab-5b^2\right)\cot(c+dx)}{2a^3(a+b)d} - \frac{(2a+5b)\cot^3(c}{6a^2(a+b)}$$

Mathematica [A] time = 1.30, size = 202, normalized size = 1.25

$$\frac{\csc^4(c+dx)(-2a+b\cos(2(c+dx))-b)\left(2a^{3/2}\cot(c+dx)\csc^2(c+dx)(2a-b\cos(2(c+dx))+b) - \frac{3\sqrt{a}\,b^3\sin(2(c+}{a+b}\right.}{24a^{7/2}d\left(a\csc^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^2)^2,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^4*((3*b^2*(6*a + 5*b)*ArcTan[
(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(-2*a - b + b*Cos[2*(c + d*x)]))/(a + b
)^(3/2) + 4*Sqrt[a]*(a - 3*b)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x] +
 2*a^(3/2)*(2*a + b - b*Cos[2*(c + d*x)])*Cot[c + d*x]*Csc[c + d*x]^2 - (3*
Sqrt[a]*b^3*Sin[2*(c + d*x)])/(a + b)))/(24*a^(7/2)*d*(b + a*Csc[c + d*x]^2
)^2)
```

fricas [B] time = 0.50, size = 843, normalized size = 5.20

$$\left[-\frac{4\left(4a^4b-4a^3b^2-23a^2b^3-15ab^4\right)\cos(dx+c)^5 - 8\left(2a^5+3a^4b-12a^3b^2-28a^2b^3-15ab^4\right)\cos(dx+c)^3}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(4*(4*a^4*b - 4*a^3*b^2 - 23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 8*
(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 28*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*(
(6*a*b^3 + 5*b^4)*cos(d*x + c)^4 + 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^
2 + 17*a*b^3 + 10*b^4)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b
 + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a +
 b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) +
```

```
a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*sin(d*x + c) + 12*(2*a^5 + 2*a^4*b - 6*a^3*b^2 - 11*a^2*b^
3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b^3)*d*cos(d*x + c)^4
 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x + c)^2 + (a^7 + 3*a^6*
b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c)), -1/12*(2*(4*a^4*b - 4*a^3*b^2 -
23*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^5 - 4*(2*a^5 + 3*a^4*b - 12*a^3*b^2 - 2
8*a^2*b^3 - 15*a*b^4)*cos(d*x + c)^3 + 3*((6*a*b^3 + 5*b^4)*cos(d*x + c)^4
+ 6*a^2*b^2 + 11*a*b^3 + 5*b^4 - (6*a^2*b^2 + 17*a*b^3 + 10*b^4)*cos(d*x +
c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a
^2 + a*b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) + 6*(2*a^5 + 2*a^4*b - 6
*a^3*b^2 - 11*a^2*b^3 - 5*a*b^4)*cos(d*x + c))/(((a^6*b + 2*a^5*b^2 + a^4*b
^3)*d*cos(d*x + c)^4 - (a^7 + 4*a^6*b + 5*a^5*b^2 + 2*a^4*b^3)*d*cos(d*x +
c)^2 + (a^7 + 3*a^6*b + 3*a^5*b^2 + a^4*b^3)*d)*sin(d*x + c))]
```

giac [A] time = 0.19, size = 174, normalized size = 1.07

$$\frac{\frac{3\,b^3\tan(dx+c)}{(a^4+a^3b)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)}+\frac{3\left(6\,ab^2+5\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^4+a^3b)\sqrt{a^2+ab}}-\frac{2\left(3\,a\tan(dx+c)^2-6\,b\tan\right.}{a^3\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="giac")
```

```
[Out] 1/6*(3*b^3*tan(d*x + c)/((a^4 + a^3*b)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2
 + a)) + 3*(6*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) +
 arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^4 + a^3*b)*
sqrt(a^2 + a*b)) - 2*(3*a*tan(d*x + c)^2 - 6*b*tan(d*x + c)^2 + a)/(a^3*tan
(d*x + c)^3))/d
```

maple [A] time = 0.56, size = 179, normalized size = 1.10

$$\frac{b^3\tan(dx+c)}{2d\,a^3\,(a+b)\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)}+\frac{3b^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,a^2\,(a+b)\sqrt{a\,(a+b)}}+\frac{5b^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{2d\,a^3\,(a+b)\sqrt{a\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x)
```

```
[Out] 1/2/d*b^3/a^3/(a+b)*tan(d*x+c)/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)+3/d/a^2*b^
2/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+5/2/d*b^3/
a^3/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-1/3/d/a^
2/tan(d*x+c)^3-1/d/a^2/tan(d*x+c)+2/d/a^3/tan(d*x+c)*b
```

maxima [A] time = 0.56, size = 172, normalized size = 1.06

$$\frac{\frac{3\left(6\,ab^2+5\,b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+a^3b)\sqrt{(a+b)a}}-\frac{3\left(2\,a^3-6\,ab^2-5\,b^3\right)\tan(dx+c)^4+2\,a^3+2\,a^2b+2\left(4\,a^3-a^2b-5\,ab^2\right)\tan(dx+c)^2}{\left(a^5+2\,a^4b+a^3b^2\right)\tan(dx+c)^5+\left(a^5+a^4b\right)\tan(dx+c)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/6*(3*(6*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^4
 + a^3*b)*sqrt((a + b)*a)) - (3*(2*a^3 - 6*a*b^2 - 5*b^3)*tan(d*x + c)^4 +
2*a^3 + 2*a^2*b + 2*(4*a^3 - a^2*b - 5*a*b^2)*tan(d*x + c)^2)/((a^5 + 2*a^4
*b + a^3*b^2)*tan(d*x + c)^5 + (a^5 + a^4*b)*tan(d*x + c)^3))/d
```

mupad [B] time = 14.40, size = 164, normalized size = 1.01

$$\frac{b^2\,\mathrm{atan}\left(\frac{b^2\tan(c+d\,x)\left(a^4+b\,a^3\right)(6\,a+5\,b)}{a^{7/2}\left(5\,b^3+6\,a\,b^2\right)\sqrt{a+b}}\right)(6\,a+5\,b)}{2\,a^{7/2}\,d\,(a+b)^{3/2}}-\frac{\frac{1}{3\,a}+\frac{\tan(c+d\,x)^2\,(4\,a-5\,b)}{3\,a^2}-\frac{\tan(c+d\,x)^4\left(-2\,a^3+6\,a\,b^2+5\,b^3\right)}{2\,a^3\,(a+b)}}{d\,\left((a+b)\,\tan\left(c+d\,x\right)^5+a\tan\left(c+d\,x\right)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^2)^2),x)
```

```
[Out] (b^2*atan((b^2*tan(c + d*x)*(a^3*b + a^4)*(6*a + 5*b))/(a^(7/2)*(6*a*b^2 +
5*b^3)*(a + b)^(1/2)))*(6*a + 5*b))/(2*a^(7/2)*d*(a + b)^(3/2)) - (1/(3*a)
+ (tan(c + d*x)^2*(4*a - 5*b))/(3*a^2) - (tan(c + d*x)^4*(6*a*b^2 - 2*a^3 +
 5*b^3))/(2*a^3*(a + b)))/(d*(tan(c + d*x)^5*(a + b) + a*tan(c + d*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**2)**2,x)
```

```
[Out] Timed out
```

3.106 $\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=148

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3d(a+b)^{5/2}}+\frac{a(4a+7b)\tan(c+dx)}{8b^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{a\tan^3(c\ldots}{4bd(a+b)\left((a+b)\ldots\right.}$$

```
[Out] x/b^3-1/8*(8*a^2+20*a*b+15*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1
/2)/b^3/(a+b)^(5/2)/d+1/4*a*tan(d*x+c)^3/b/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2
+1/8*a*(4*a+7*b)*tan(d*x+c)/b^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A] time = 0.28, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3187, 470, 578, 522, 203, 205}

$$-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3d(a+b)^{5/2}}+\frac{a(4a+7b)\tan(c+dx)}{8b^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{a\tan^3(c\ldots}{4bd(a+b)\left((a+b)\ldots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] x/b^3 - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(8*b^3*(a + b)^(5/2)*d) + (a*Tan[c + d*x]^3)/(4*b*(a + b)*d*(a
+ (a + b)*Tan[c + d*x]^2)^2) + (a*(4*a + 7*b)*Tan[c + d*x])/(8*b^2*(a + b)^
2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^2\left(3a+(-a-4b)x^2\right)}{(1+x^2)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+d\right.}{4b(a+b)d}$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} -$$

$$= \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{a(4a+7b)\tan(c+dx)}{8b^2(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} +$$

$$= \frac{x}{b^3} - \frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8b^3(a+b)^{5/2}d} + \frac{a\tan^3(c+dx)}{4b(a+b)d\left(a+(a+b)\tan^2\right.}$$

Mathematica [A] time = 2.70, size = 134, normalized size = 0.91

$$\frac{-\frac{\sqrt{a}\left(8a^2+20ab+15b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}} + \frac{ab\sin(2(c+dx))\left(8a^2-3b(2a+3b)\cos(2(c+dx))+20ab+9b^2\right)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2} + 8(c+dx)}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(8*a^2 + 20*a*b + 15*b^2)*ArcTan[(Sqrt[a + b]*Tan[c
 + d*x])/Sqrt[a]])/(a + b)^(5/2) + (a*b*(8*a^2 + 20*a*b + 9*b^2 - 3*b*(2*a
+ 3*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c
 + d*x)])^2))/(8*b^3*d)
```

fricas [B] time = 0.55, size = 950, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [1/32*(32*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 64*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 32*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(-a/(a + b))*log(((8*a^2 + 8*a*b + b^2
)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a
*b + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b
))*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*co
s(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*(3*(2*a^2*b^2 + 3*a*b^3)*cos(d*x + c
)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^2*b^
5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^
7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6 + b^7)*d),
 1/16*(16*(a^2*b^2 + 2*a*b^3 + b^4)*d*x*cos(d*x + c)^4 - 32*(a^3*b + 3*a^2*
b^2 + 3*a*b^3 + b^4)*d*x*cos(d*x + c)^2 + 16*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*x + ((8*a^2*b^2 + 20*a*b^3 + 15*b^4)*cos(d*x + c)^4 + 8*a^4
 + 36*a^3*b + 63*a^2*b^2 + 50*a*b^3 + 15*b^4 - 2*(8*a^3*b + 28*a^2*b^2 + 35
*a*b^3 + 15*b^4)*cos(d*x + c)^2)*sqrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(
d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x + c))) - 2*(3*(
2*a^2*b^2 + 3*a*b^3)*cos(d*x + c)^3 - (4*a^3*b + 13*a^2*b^2 + 9*a*b^3)*cos(
d*x + c))*sin(d*x + c))/((a^2*b^5 + 2*a*b^6 + b^7)*d*cos(d*x + c)^4 - 2*(a^
3*b^4 + 3*a^2*b^5 + 3*a*b^6 + b^7)*d*cos(d*x + c)^2 + (a^4*b^3 + 4*a^3*b^4
+ 6*a^2*b^5 + 4*a*b^6 + b^7)*d)]
```

giac [A] time = 0.24, size = 224, normalized size = 1.51

$$-\frac{\frac{\left(8\,a^{3}+20\,a^{2}b+15\,ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^{2}+ab}}\right)\right)}{\left(a^{2}b^{3}+2\,ab^{4}+b^{5}\right)\sqrt{a^{2}+ab}}-\frac{4\,a^{3}\tan(dx+c)^{3}+13\,a^{2}b\tan(dx+c)^{3}+9\,ab^{2}\tan(dx+c)^{3}+4\,a^{3}\tan}{\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\left(a\tan(dx+c)^{2}+b\tan(dx+c\right.}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a +
 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2*b^
3 + 2*a*b^4 + b^5)*sqrt(a^2 + a*b)) - (4*a^3*tan(d*x + c)^3 + 13*a^2*b*tan(
d*x + c)^3 + 9*a*b^2*tan(d*x + c)^3 + 4*a^3*tan(d*x + c) + 7*a^2*b*tan(d*x
+ c))/((a^2*b^2 + 2*a*b^3 + b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^
2) - 8*(d*x + c)/b^3)/d
```

maple [B] time = 0.33, size = 363, normalized size = 2.45

$$\frac{a^{2}\left(\tan^{3}\left(dx+c\right)\right)}{2d\,b^{2}\left(a\left(\tan^{2}\left(dx+c\right)\right)+\left(\tan^{2}\left(dx+c\right)\right)b+a\right)^{2}\left(a+b\right)}+\frac{9a\left(\tan^{3}\left(dx+c\right)\right)}{8db\left(a\left(\tan^{2}\left(dx+c\right)\right)+\left(\tan^{2}\left(dx+c\right)\right)b+a\right)^{2}\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x)
```

```
[Out] 1/2/d*a^2/b^2/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3+9/8/d*
a/b/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3+1/2/d*a^3/b^2/(a
*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)+7/8/d*a^2/b/(a
*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)-1/d*a^3/b^3/(a
^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-5/2/
d*a^2/b^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))
```

```
^(1/2))-15/8/d*a/b/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/
(a*(a+b))^(1/2))+1/d/b^3*arctan(tan(d*x+c))
```

maxima [A] time = 0.52, size = 234, normalized size = 1.58

$$-\frac{\frac{\left(8\,a^{3}+20\,a^{2}b+15\,ab^{2}\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^{2}b^{3}+2\,ab^{4}+b^{5}\right)\sqrt{(a+b)a}}-\frac{\left(4\,a^{3}+13\,a^{2}b+9\,ab^{2}\right)\tan(dx+c)^{3}+\left(4\,a^{3}+7\,a^{2}b\right)\tan(dx+c)}{a^{4}b^{2}+2\,a^{3}b^{3}+a^{2}b^{4}+\left(a^{4}b^{2}+4\,a^{3}b^{3}+6\,a^{2}b^{4}+4\,ab^{5}+b^{6}\right)\tan(dx+c)^{4}+2\left(a^{4}b^{2}+3\,a^{3}b^{3}+3\,a^{2}b^{4}+ab^{5}\right)\tan(\cdots}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((8*a^3 + 20*a^2*b + 15*a*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b
)*a))/((a^2*b^3 + 2*a*b^4 + b^5)*sqrt((a + b)*a)) - ((4*a^3 + 13*a^2*b + 9*
a*b^2)*tan(d*x + c)^3 + (4*a^3 + 7*a^2*b)*tan(d*x + c))/(a^4*b^2 + 2*a^3*b^
3 + a^2*b^4 + (a^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*tan(d*x + c
)^4 + 2*(a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*tan(d*x + c)^2) - 8*(d*x
+ c)/b^3)/d
```

mupad [B] time = 17.96, size = 3189, normalized size = 21.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] ((tan(c + d*x)^3*(9*a*b + 4*a^2))/(8*(a*b^2 + b^3)) + (a*tan(c + d*x)*(7*a*
b + 4*a^2))/(8*(2*a*b^3 + b^4 + a^2*b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^2
+ b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) - atan((((((7*a*b^10)/2 +
(25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b
^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^12 +
5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/
(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3) + (tan(c + d*x)*(3
84*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 160
0*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3 - (((((7*a*b^10
)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*
(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6)) + (tan(c + d*x)*(1792*a*b^11 + 256*b
^12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*
b^6))/(128*b^3*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/(2*b^3) - (tan(c + d
*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3
 + 1600*a^4*b^2))/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3)/(((15*a*
b^4)/4 + (19*a^4*b)/4 + a^5 + (295*a^2*b^3)/32 + (19*a^3*b^2)/2)/(3*a*b^8 +
 b^9 + 3*a^2*b^7 + a^3*b^6) + ((((((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*
b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b^8 + b^9 + 3*a^2*b^7 + a^
3*b^6)) - (tan(c + d*x)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*
b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(128*b^3*(3*a*b^6 + b^7 +
 3*a^2*b^5 + a^3*b^4)))*1i)/(2*b^3) + (tan(c + d*x)*(384*a*b^5 + 704*a^5*b
+ 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2)*1i)/(64*(3
*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3 + ((((((7*a*b^10)/2 + (25*a^2*b^9
)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)*1i)/(2*(3*a*b^8 + b^9 +
3*a^2*b^7 + a^3*b^6)) + (tan(c + d*x)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^
10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(128*b^3*(3
*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))*1i)/(2*b^3) - (tan(c + d*x)*(384*a*b^
5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b
^2)*1i)/(64*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)))/b^3))/(b^3*d) - (atan((
((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*
b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2))/(32*(3*a*b^6 + b^7 + 3*a
^2*b^5 + a^3*b^4)) - ((-a*(a + b)^5)^(1/2)*(((7*a*b^10)/2 + (25*a^2*b^9)/2
+ (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)/(3*a*b^8 + b^9 + 3*a^2*b^7 +
```

```
a^3*b^6) - (tan(c + d*x)*(-a*(a + b)^5)^(1/2)*(20*a*b + 8*a^2 + 15*b^2)*(1
792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*
a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)*(5*a*b^7
 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 +
 15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^
3)))*(20*a*b + 8*a^2 + 15*b^2)*1i)/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3
*b^5 + 5*a^4*b^4 + a^5*b^3)) + ((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*
b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4
*b^2))/(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) + ((-a*(a + b)^5)^(1/2)*(
((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^
6)/(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) + (tan(c + d*x)*(-a*(a + b)^5)^(1/
2)*(20*a*b + 8*a^2 + 15*b^2)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680
*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7
+ 3*a^2*b^5 + a^3*b^4)*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4
 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 +
10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2)*1i)/(16*(5*a*
b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))/(((15*a*b^4)/4
 + (19*a^4*b)/4 + a^5 + (295*a^2*b^3)/32 + (19*a^3*b^2)/2)/(3*a*b^8 + b^9 +
 3*a^2*b^7 + a^3*b^6) - ((-a*(a + b)^5)^(1/2)*((tan(c + d*x)*(384*a*b^5 + 7
04*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 1880*a^3*b^3 + 1600*a^4*b^2))/
(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) - ((-a*(a + b)^5)^(1/2)*(((7*a*b
^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*a^4*b^7)/2 + 2*a^5*b^6)/(3*a
*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) - (tan(c + d*x)*(-a*(a + b)^5)^(1/2)*(20*
a*b + 8*a^2 + 15*b^2)*(1792*a*b^11 + 256*b^12 + 5120*a^2*b^10 + 7680*a^3*b^
9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))/(512*(3*a*b^6 + b^7 + 3*a^2
*b^5 + a^3*b^4)*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*
b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*
b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7 + b^8
+ 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)) + ((-a*(a + b)^5)^(1/2)*(
(tan(c + d*x)*(384*a*b^5 + 704*a^5*b + 128*a^6 + 64*b^6 + 1185*a^2*b^4 + 18
80*a^3*b^3 + 1600*a^4*b^2))/(32*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)) + ((
-a*(a + b)^5)^(1/2)*(((7*a*b^10)/2 + (25*a^2*b^9)/2 + (33*a^3*b^8)/2 + (19*
a^4*b^7)/2 + 2*a^5*b^6)/(3*a*b^8 + b^9 + 3*a^2*b^7 + a^3*b^6) + (tan(c + d*
x)*(-a*(a + b)^5)^(1/2)*(20*a*b + 8*a^2 + 15*b^2)*(1792*a*b^11 + 256*b^12 +
 5120*a^2*b^10 + 7680*a^3*b^9 + 6400*a^4*b^8 + 2816*a^5*b^7 + 512*a^6*b^6))
/(512*(3*a*b^6 + b^7 + 3*a^2*b^5 + a^3*b^4)*(5*a*b^7 + b^8 + 10*a^2*b^6 + 1
0*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 + 15*b^2))/(16*(5*a*b^7
+ b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3)))*(20*a*b + 8*a^2 +
15*b^2))/(16*(5*a*b^7 + b^8 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3
))))*(-a*(a + b)^5)^(1/2)*(20*a*b + 8*a^2 + 15*b^2)*1i)/(8*d*(5*a*b^7 + b^8
 + 10*a^2*b^6 + 10*a^3*b^5 + 5*a^4*b^4 + a^5*b^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**2)**3,x)
[Out] Timed out
```

3.107 $\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=110

$$\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a+b)^{5/2}}-\frac{3\tan(c+dx)}{8d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}-\frac{\tan^3(c+dx)}{4d(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}$$

```
[Out] 3/8*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d/a^(1/2)-1/4*tan(d*
x+c)^3/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2-3/8*tan(d*x+c)/(a+b)^2/d/(a+(a+b)*t
an(d*x+c)^2)
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3187, 288, 205\}$

$$-\frac{\tan^3(c+dx)}{4d(a+b)\left((a+b)\tan^2(c+dx)+a\right)^2}-\frac{3\tan(c+dx)}{8d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}+\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*Sqrt[a]*(a + b)^(5/2)*d)
- Tan[c + d*x]^3/(4*(a + b)*d*(a + (a + b)*Tan[c + d*x]^2)^2) - (3*Tan[c +
d*x])/(8*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{3\,\text{Subst}\left(\int \frac{x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(c+dx\right.}{4(a+b)d}$$

$$= -\frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{3\tan(c+dx)}{8(a+b)^2d\left(a+(a+b)\tan^2(c+dx)\right)} +$$

$$= \frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8\sqrt{a}\,(a+b)^{5/2}d} - \frac{\tan^3(c+dx)}{4(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{3\,\text{t}}{8(a+b)^2d\,(a\;}$$

Mathematica [A] time = 1.32, size = 97, normalized size = 0.88

$$\frac{\frac{3\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}} + \frac{\sin(2(c+dx))((2a+5b)\cos(2(c+dx))-8a-5b)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((3*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + (
(-8*a - 5*b + (2*a + 5*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2
*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

fricas [B] time = 0.49, size = 683, normalized size = 6.21

$$\left[-\frac{3\left(b^2\cos(dx+c)^4 - 2\left(ab+b^2\right)\cos(dx+c)^2 + a^2 + 2ab + b^2\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4 - 2\left(4a^2+5\right.}{}\right.}{32\left(\left(a^4b^2+3a^3b^3+3a^2b^4+ab^5\right)d\cos(dx+c)^4 - 2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(3*(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b
+ b^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^
2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a + b)*cos
(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x
+ c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((2*a^3 + 7
*a^2*b + 5*a*b^2)*cos(d*x + c)^3 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*
sin(d*x + c))/((a^4*b^2 + 3*a^3*b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 -
 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (
a^6 + 5*a^5*b + 10*a^4*b^2 + 10*a^3*b^3 + 5*a^2*b^4 + a*b^5)*d), -1/16*(3*(
b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt
(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*
cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3 + 7*a^2*b + 5*a*b^2)*cos(d*x + c)^3
 - 5*(a^3 + 2*a^2*b + a*b^2)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 + 3*a^3*
b^3 + 3*a^2*b^4 + a*b^5)*d*cos(d*x + c)^4 - 2*(a^5*b + 4*a^4*b^2 + 6*a^3*b^
3 + 4*a^2*b^4 + a*b^5)*d*cos(d*x + c)^2 + (a^6 + 5*a^5*b + 10*a^4*b^2 + 10*
a^3*b^3 + 5*a^2*b^4 + a*b^5)*d)]
```

giac [A] time = 0.20, size = 152, normalized size = 1.38

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{(a^2+2\,ab+b^2)\sqrt{a^2+ab}}-\frac{5\,a\tan(dx+c)^3+5\,b\tan(dx+c)^3+3\,a\tan(dx+c)}{\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2(a^2+2\,ab+b^2)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b))
 - (5*a*tan(d*x + c)^3 + 5*b*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a*tan(d*x
 + c)^2 + b*tan(d*x + c)^2 + a)^2*(a^2 + 2*a*b + b^2)))/d
```

maple [A] time = 0.26, size = 136, normalized size = 1.24

$$-\frac{5\left(\tan^3(dx+c)\right)}{8d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2(a+b)}-\frac{3a\tan(dx+c)}{8d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2(a^2+2ab$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x)
```

```
[Out] -5/8/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3-3/8/d/(a*tan(
d*x+c)^2+tan(d*x+c)^2*b+a)^2*a/(a^2+2*a*b+b^2)*tan(d*x+c)+3/8/d/(a^2+2*a*b+
b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.51, size = 158, normalized size = 1.44

$$-\frac{\frac{5\,(a+b)\tan(dx+c)^3+3\,a\tan(dx+c)}{(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4)\tan(dx+c)^4+a^4+2\,a^3b+a^2b^2+2\,(a^4+3\,a^3b+3\,a^2b^2+ab^3)\tan(dx+c)^2}-\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*((5*(a + b)*tan(d*x + c)^3 + 3*a*tan(d*x + c))/((a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4)*tan(d*x + c)^4 + a^4 + 2*a^3*b + a^2*b^2 + 2*(a^4 + 3
*a^3*b + 3*a^2*b^2 + a*b^3)*tan(d*x + c)^2) - 3*arctan((a + b)*tan(d*x + c)
/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b + b^2)))/d
```

mupad [B] time = 13.69, size = 149, normalized size = 1.35

$$\frac{3\,\mathrm{atan}\left(\frac{3\tan(c+d\,x)\,(2\,a+2\,b)\left(\frac{8\,a^2}{3}+\frac{16\,a\,b}{3}+\frac{8\,b^2}{3}\right)}{16\,\sqrt{a}\,(a+b)^{5/2}}\right)}{8\,\sqrt{a}\,d\,(a+b)^{5/2}}-\frac{\frac{5\tan(c+d\,x)^3}{8\,(a+b)}+\frac{3\,a\tan(c+d\,x)}{8\,(a^2+2\,a\,b+b^2)}}{d\left(\tan(c+d\,x)^4\,(a^2+2\,a\,b+b^2)+a^2+\tan(c+d\,x)^2\,(2\,a^2+2\,b\,a)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] (3*atan((3*tan(c + d*x)*(2*a + 2*b)*((16*a*b)/3 + (8*a^2)/3 + (8*b^2)/3))/(
16*a^(1/2)*(a + b)^(5/2))))/(8*a^(1/2)*d*(a + b)^(5/2)) - ((5*tan(c + d*x)^
3)/(8*(a + b)) + (3*a*tan(c + d*x))/(8*(2*a*b + a^2 + b^2)))/(d*(tan(c + d*
x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.108 $\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=131

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^{5/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{8ad(a+b)^2\left(a+b\sin^2(c+dx)\right)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

```
[Out] 1/8*(4*a+b)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(3/2)/(a+b)^(5/2)/d-1/
4*cos(d*x+c)*sin(d*x+c)/(a+b)/d/(a+b*sin(d*x+c)^2)^2-1/8*(2*a-b)*cos(d*x+c)
*sin(d*x+c)/a/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3173, 12, 3181, 205}

$$\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}d(a+b)^{5/2}}-\frac{(2a-b)\sin(c+dx)\cos(c+dx)}{8ad(a+b)^2\left(a+b\sin^2(c+dx)\right)}-\frac{\sin(c+dx)\cos(c+dx)}{4d(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(3/2)*(a + b)^(
5/2)*d) - (Cos[c + d*x]*Sin[c + d*x])/(4*(a + b)*d*(a + b*Sin[c + d*x]^2)^2
) - ((2*a - b)*Cos[c + d*x]*Sin[c + d*x])/(8*a*(a + b)^2*d*(a + b*Sin[c + d
*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} + \frac{\int \frac{a+2a\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx}{4a(a+b)}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{\int \frac{a(4a+}{a+b\sin^2(}}{8a^2(a-}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(4a+b)\int}{8a}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(c+dx)\sin(c+dx)}{8a(a+b)^2d\left(a+b\sin^2(c+dx)\right)} + \frac{(4a+b)\,S}{}$$

$$= \frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{8a^{3/2}(a+b)^{5/2}d} - \frac{\cos(c+dx)\sin(c+dx)}{4(a+b)d\left(a+b\sin^2(c+dx)\right)^2} - \frac{(2a-b)\cos(}{8a(a+b)^2d\,(}$$

Mathematica [A] time = 1.39, size = 112, normalized size = 0.85

$$\frac{\frac{(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} - \frac{\sin(2(c+dx))\left(8a^2+b(b-2a)\cos(2(c+dx))+4ab-b^2\right)}{a(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (((4*a + b)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5
/2)) - ((8*a^2 + 4*a*b - b^2 + b*(-2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*
x)])/(a*(a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*d)
```

fricas [B] time = 0.50, size = 771, normalized size = 5.89

$$\left[-\frac{\left(\left(4\,ab^2+b^3\right)\cos\left(dx+c\right)^4+4\,a^3+9\,a^2b+6\,ab^2+b^3-2\left(4\,a^2b+5\,ab^2+b^3\right)\cos\left(dx+c\right)^2\right)\sqrt{-a^2-ab}\,\log}{32\left(\left(a^5b^2+3\,a^4b^3+3\,a^3b^4+a^2b^5\right)c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((4*a*b^2 + b^3)*cos(d*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 -
 2*(4*a^2*b + 5*a*b^2 + b^3)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 +
 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*(
(2*a + b)*cos(d*x + c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x +
 c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2
 + a^2 + 2*a*b + b^2)) - 4*((2*a^3*b + a^2*b^2 - a*b^3)*cos(d*x + c)^3 - (4
*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*sin(d*x + c))/((a^5*b^2 +
 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4 - 2*(a^6*b + 4*a^5*b^2 +
 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2 + (a^7 + 5*a^6*b + 10*a^
5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d), -1/16*(((4*a*b^2 + b^3)*cos(d
*x + c)^4 + 4*a^3 + 9*a^2*b + 6*a*b^2 + b^3 - 2*(4*a^2*b + 5*a*b^2 + b^3)*c
os(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) - 2*((2*a^3*b + a^2*b^2 - a*
```

```
b^3)*cos(d*x + c)^3 - (4*a^4 + 7*a^3*b + 2*a^2*b^2 - a*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^4
- 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*d*cos(d*x + c)^2
+ (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*d)]
```

giac [A] time = 0.20, size = 191, normalized size = 1.46

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(4a+b)}{\left(a^3+2a^2b+ab^2\right)\sqrt{a^2+ab}}-\frac{4a^2\tan(dx+c)^3+3ab\tan(dx+c)^3-b^2\tan(dx+c)^3+4a^2\tan(dx+c)+ab\tan(dx+c)}{\left(a^3+2a^2b+ab^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(4*a + b)/((a^3 + 2*a^2*b + a*b^2)*sqrt
(a^2 + a*b)) - (4*a^2*tan(d*x + c)^3 + 3*a*b*tan(d*x + c)^3 - b^2*tan(d*x +
 c)^3 + 4*a^2*tan(d*x + c) + a*b*tan(d*x + c))/((a^3 + 2*a^2*b + a*b^2)*(a*
tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

maple [B] time = 0.26, size = 278, normalized size = 2.12

$$-\frac{\tan^3(dx+c)}{2d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2(a+b)}+\frac{\left(\tan^3(dx+c)\right)b}{8d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2a(a+b)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x)
```

```
[Out] -1/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3+1/8/d/(a*tan(
d*x+c)^2+tan(d*x+c)^2*b+a)^2/a/(a+b)*tan(d*x+c)^3*b-1/2/d/(a*tan(d*x+c)^2+t
an(d*x+c)^2*b+a)^2*a/(a^2+2*a*b+b^2)*tan(d*x+c)-1/8/d/(a*tan(d*x+c)^2+tan(d
*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)*b+1/2/d/(a^2+2*a*b+b^2)/(a*(a+b))
^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/8/d/(a^2+2*a*b+b^2)/a/(a*
(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

maxima [A] time = 0.95, size = 191, normalized size = 1.46

$$\frac{\frac{(4a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^3+2a^2b+ab^2\right)\sqrt{(a+b)a}}-\frac{\left(4a^2+3ab-b^2\right)\tan(dx+c)^3+\left(4a^2+ab\right)\tan(dx+c)}{a^5+2a^4b+a^3b^2+\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)\tan(dx+c)^4+2\left(a^5+3a^4b+3a^3b^2+a^2b^3\right)\tan(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((4*a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 2*a^2*b
 + a*b^2)*sqrt((a + b)*a)) - ((4*a^2 + 3*a*b - b^2)*tan(d*x + c)^3 + (4*a^2
 + a*b)*tan(d*x + c))/(a^5 + 2*a^4*b + a^3*b^2 + (a^5 + 4*a^4*b + 6*a^3*b^2
 + 4*a^2*b^3 + a*b^4)*tan(d*x + c)^4 + 2*(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b
^3)*tan(d*x + c)^2))/d
```

mupad [B] time = 13.85, size = 159, normalized size = 1.21

$$\frac{\operatorname{atan}\left(\frac{\tan(c+dx)(2a+2b)\left(a^2+2ab+b^2\right)}{2\sqrt{a}(a+b)^{5/2}}\right)(4a+b)}{8a^{3/2}d(a+b)^{5/2}}-\frac{\frac{\tan(c+dx)(4a+b)}{8\left(a^2+2ab+b^2\right)}+\frac{\tan(c+dx)^3(4a-b)}{8a(a+b)}}{d\left(\tan(c+dx)^4\left(a^2+2ab+b^2\right)+a^2+\tan(c+dx)^2\left(2a^2+2ba\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] (atan((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/
2)))*(4*a + b))/(8*a^(3/2)*d*(a + b)^(5/2)) - ((tan(c + d*x)*(4*a + b))/(8*
(2*a*b + a^2 + b^2)) + (tan(c + d*x)^3*(4*a - b))/(8*a*(a + b)))/(d*(tan(c
+ d*x)^4*(2*a*b + a^2 + b^2) + a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.109 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=144

$$\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{8a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)}+\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/(a+b
)^(5/2)/d+1/4*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)^2+3/8*b*
(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)
```

Rubi [A] time = 0.15, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.357, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}d(a+b)^{5/2}}+\frac{3b(2a+b)\sin(c+dx)\cos(c+dx)}{8a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)}+\frac{b\sin(c+dx)\cos(c+dx)}{4ad(a+b)\left(a+b\sin^2(c+dx)\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(8*a^(
5/2)*(a + b)^(5/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(4*a*(a + b)*d*(a + b
*Sin[c + d*x]^2)^2) + (3*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/(8*a^2*(a +
 b)^2*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
```

```
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^3}\,dx=\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}-\frac{\int\frac{-4a-3b+2b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^2}\,dx}{4a(a+b)}$$

$$=\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}+\frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)}-\frac{\int\frac{-8a^2-}{a+b\sin}}{8a^2(}$$

$$=\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}+\frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)}+\frac{\left(8a^2+\right.}{}$$

$$=\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}+\frac{3b(2a+b)\cos(c+dx)\sin(c+dx)}{8a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)}+\frac{\left(8a^2+\right.}{}$$

$$=\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{5/2}(a+b)^{5/2}d}+\frac{b\cos(c+dx)\sin(c+dx)}{4a(a+b)d\left(a+b\sin^2(c+dx)\right)^2}+\frac{3}{8}$$

Mathematica [A] time = 1.27, size = 125, normalized size = 0.87

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}}+\frac{\sqrt{a}\,b\sin(2(c+dx))\left(16a^2-3b(2a+b)\cos(2(c+dx))+16ab+3b^2\right)}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2}}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-3),x]
```

```
[Out] (((8*a^2 + 8*a*b + 3*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a +
b)^(5/2) + (Sqrt[a]*b*(16*a^2 + 16*a*b + 3*b^2 - 3*b*(2*a + b)*Cos[2*(c + d
*x)])*Sin[2*(c + d*x)])/((a + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2))/(8*a^
(5/2)*d)
```

fricas [B] time = 0.54, size = 843, normalized size = 5.85

$$\left[-\frac{\left(\left(8a^2b^2+8ab^3+3b^4\right)\cos(dx+c)^4+8a^4+24a^3b+27a^2b^2+14ab^3+3b^4-2\left(8a^3b+16a^2b^2+11ab^3\right.\right.}{32\left(\left(a^6b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d*x + c)^4 + 8*a^4 + 24*a^3*b +
27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b + 16*a^2*b^2 + 11*a*b^3 + 3*b^4)
*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (
a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^
2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(
3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*a^3*b^2 +
```

```
14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a^5*b^3 + 3
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a^5*b^3 + 4
*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^2 + 10*a^5
*b^3 + 5*a^4*b^4 + a^3*b^5)*d), -1/16*(((8*a^2*b^2 + 8*a*b^3 + 3*b^4)*cos(d
*x + c)^4 + 8*a^4 + 24*a^3*b + 27*a^2*b^2 + 14*a*b^3 + 3*b^4 - 2*(8*a^3*b +
 16*a^2*b^2 + 11*a*b^3 + 3*b^4)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x +
c))) + 2*(3*(2*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(d*x + c)^3 - (8*a^4*b + 19*
a^3*b^2 + 14*a^2*b^3 + 3*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^6*b^2 + 3*a
^5*b^3 + 3*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^4 - 2*(a^7*b + 4*a^6*b^2 + 6*a
^5*b^3 + 4*a^4*b^4 + a^3*b^5)*d*cos(d*x + c)^2 + (a^8 + 5*a^7*b + 10*a^6*b^
2 + 10*a^5*b^3 + 5*a^4*b^4 + a^3*b^5)*d)]
```

giac [A] time = 0.14, size = 211, normalized size = 1.47

$$\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)\left(8a^2+8ab+3b^2\right)}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{a^2+ab}}+\frac{8a^2b\tan(dx+c)^3+11ab^2\tan(dx+c)^3+3b^3\tan(dx+c)^3+8a^2b\tan(dx+c)+\cdots}{\left(a^4+2a^3b+a^2b^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c)^2+a\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] 1/8*((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c)
+ b*tan(d*x + c))/sqrt(a^2 + a*b)))*(8*a^2 + 8*a*b + 3*b^2)/((a^4 + 2*a^3*b
 + a^2*b^2)*sqrt(a^2 + a*b)) + (8*a^2*b*tan(d*x + c)^3 + 11*a*b^2*tan(d*x +
 c)^3 + 3*b^3*tan(d*x + c)^3 + 8*a^2*b*tan(d*x + c) + 5*a*b^2*tan(d*x + c))
/((a^4 + 2*a^3*b + a^2*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^2))/d
```

maple [B] time = 0.40, size = 334, normalized size = 2.32

$$\frac{\left(\tan^3(dx+c)\right)b}{d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2a(a+b)}+\frac{3b^2\left(\tan^3(dx+c)\right)}{8d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2a^2(a+b)}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^2)^3,x)
```

```
[Out] 1/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a/(a+b)*tan(d*x+c)^3*b+3/8/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a^2*b^2/(a+b)*tan(d*x+c)^3+1/d/(a*tan(d*x+c)^
2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)*b+5/8/d/(a*tan(d*x+c)^2+ta
n(d*x+c)^2*b+a)^2*b^2/a/(a^2+2*a*b+b^2)*tan(d*x+c)+1/d/(a^2+2*a*b+b^2)/(a*(
a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+1/d/(a^2+2*a*b+b^2)/a/
(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+3/8/d/a^2/(a^2+2
*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2
```

maxima [A] time = 0.55, size = 211, normalized size = 1.47

$$\frac{\frac{\left(8a^2+8ab+3b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^4+2a^3b+a^2b^2\right)\sqrt{(a+b)a}}+\frac{\left(8a^2b+11ab^2+3b^3\right)\tan(dx+c)^3+\left(8a^2b+5ab^2\right)\tan(dx+c)}{a^6+2a^5b+a^4b^2+\left(a^6+4a^5b+6a^4b^2+4a^3b^3+a^2b^4\right)\tan(dx+c)^4+2\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\tan(dx+c)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] 1/8*((8*a^2 + 8*a*b + 3*b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(
(a^4 + 2*a^3*b + a^2*b^2)*sqrt((a + b)*a)) + ((8*a^2*b + 11*a*b^2 + 3*b^3)*
tan(d*x + c)^3 + (8*a^2*b + 5*a*b^2)*tan(d*x + c))/(a^6 + 2*a^5*b + a^4*b^2
```

```
 + (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*tan(d*x + c)^4 + 2*(a^
6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*tan(d*x + c)^2))/d
```

mupad [**B**] time = 13.82, size = 176, normalized size = 1.22

$$\frac{\frac{\tan(c+d\,x)^3\,\left(3\,b^2+8\,a\,b\right)}{8\,a^2\,(a+b)}+\frac{\tan(c+d\,x)\,\left(5\,b^2+8\,a\,b\right)}{8\,a\,\left(a^2+2\,a\,b+b^2\right)}}{d\,\left(\tan\left(c+d\,x\right)^4\,\left(a^2+2\,a\,b+b^2\right)+a^2+\tan\left(c+d\,x\right)^2\,\left(2\,a^2+2\,b\,a\right)\right)}+\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,(a+b)^{5/2}}\right)\,\left(8\,a\right.}{8\,a^{5/2}\,d\,\left(a+b\right)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^3,x)
```

```
[Out] ((tan(c + d*x)^3*(8*a*b + 3*b^2))/(8*a^2*(a + b)) + (tan(c + d*x)*(8*a*b +
5*b^2))/(8*a*(2*a*b + a^2 + b^2)))/(d*(tan(c + d*x)^4*(2*a*b + a^2 + b^2) +
 a^2 + tan(c + d*x)^2*(2*a*b + 2*a^2))) + (atan((tan(c + d*x)*(2*a + 2*b)*(
2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)))*(8*a*b + 8*a^2 + 3*b^2))/(8*
a^(5/2)*d*(a + b)^(5/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.110 $\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx$

Optimal. Leaf size=196

$$-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3d(a+b)^2}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+dx)+4a+5b\right)}{8a^2d(a+b)^2\left((a+b)\tan^2(c+dx)+a\right)}-\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\right.}{8a^{7/2}d(a+b)^{5/2}}$$

```
[Out] -3/8*b*(8*a^2+12*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/
(a+b)^(5/2)/d-1/8*(2*a+3*b)*(4*a+5*b)*cot(d*x+c)/a^3/(a+b)^2/d+1/4*b*csc(d*
x+c)*sec(d*x+c)^3/a/(a+b)/d/(a+(a+b)*tan(d*x+c)^2)^2+1/8*b*cot(d*x+c)*(4*a+
5*b+(4*a+b)*tan(d*x+c)^2)/a^2/(a+b)^2/d/(a+(a+b)*tan(d*x+c)^2)
```

Rubi [A] time = 0.25, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3187, 468, 577, 453, 205}

$$-\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}d(a+b)^{5/2}}-\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3d(a+b)^2}+\frac{b\cot(c+dx)\left((4a+b)\tan^2(c+d\right.}{8a^2d(a+b)^2\left((a+b)\tan^2(c+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] (-3*b*(8*a^2 + 12*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/
(8*a^(7/2)*(a + b)^(5/2)*d) - ((2*a + 3*b)*(4*a + 5*b)*Cot[c + d*x])/(8*a^3
*(a + b)^2*d) + (b*Csc[c + d*x]*Sec[c + d*x]^3)/(4*a*(a + b)*d*(a + (a + b)
*Tan[c + d*x]^2)^2) + (b*Cot[c + d*x]*(4*a + 5*b + (4*a + b)*Tan[c + d*x]^2
))/(8*a^2*(a + b)^2*d*(a + (a + b)*Tan[c + d*x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 453

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*e*(m + 1)),
 x] + Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*e^n*(m + 1)), Int[(e*
x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c
 - a*d, 0] && (IntegerQ[n] || GtQ[e, 0]) && ((GtQ[n, 0] && LtQ[m, -1]) || (
LtQ[n, 0] && GtQ[m + n, -1])) &&  !ILtQ[p, -1]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
```

```
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 3187

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(m/2 + p + 1),
 x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] &&
 IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^2\left(a+(a+b)x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)\left(-4a-5b+(-4a-b)x^2\right)}{x^2\left(a+(a+b)x^2\right)^2}\,dx, x, \text{t}\right.}{4a(a+b)d}$$

$$= \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\cot(c+dx)\left(4a+5b+(4a+b)\tan^2(c\right.}{8a^2(a+b)^2d\left(a+(a+b)\tan^2(c+\right.}$$

$$= -\frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d} + \frac{b\csc(c+dx)\sec^3(c+dx)}{4a(a+b)d\left(a+(a+b)\tan^2(c+dx)\right)^2} + \frac{b\,\text{cc}}{}$$

$$= -\frac{3b\left(8a^2+12ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{8a^{7/2}(a+b)^{5/2}d} - \frac{(2a+3b)(4a+5b)\cot(c+dx)}{8a^3(a+b)^2d}$$

Mathematica [A] time = 1.72, size = 214, normalized size = 1.09

$$\frac{\csc^6(c+dx)(-2a+b\cos(2(c+dx))-b)\left(\frac{4a^{3/2}b^2\sin(2(c+dx))}{a+b} + \frac{3b\left(8a^2+12ab+5b^2\right)(2a-b\cos(2(c+dx))+b)^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}}\right.}{64a^{7/2}d\left(a\csc^2(c+dx)+\right.}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^2)^3,x]
```

```
[Out] ((-2*a - b + b*Cos[2*(c + d*x)])*Csc[c + d*x]^6*((3*b*(8*a^2 + 12*a*b + 5*b
^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]*(2*a + b - b*Cos[2*(c + d*x)
])^2)/(a + b)^(5/2) + 8*Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])^2*Cot[c + d*
x] + (4*a^(3/2)*b^2*Sin[2*(c + d*x)])/(a + b) + (Sqrt[a]*b^2*(10*a + 7*b)*(
2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a + b)^2))/(64*a^(7/2)*d*(
b + a*Csc[c + d*x]^2)^3)
```

fricas [B] time = 0.53, size = 1003, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="fricas")
```

```
[Out] [-1/32*(4*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 + 15*a*b^5)*cos(d*x + c)^5 -
 4*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2*b^4 + 30*a*b^5)*cos(d*x +
 c)^3 + 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22*a*b^4 + 5*b^5 + (8*a^2*b^
3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b^2 + 20*a^2*b^3 + 17*a*b^4
 + 5*b^5)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a + b)*cos(d*x +
 c)^3 - (a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b +
 b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2))*sin(d*x + c) + 4*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66*a^2*
b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4*b^5)
*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4*b^5)
*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b^4 +
a^4*b^5)*d)*sin(d*x + c)), -1/16*(2*(8*a^4*b^2 + 34*a^3*b^3 + 41*a^2*b^4 +
15*a*b^5)*cos(d*x + c)^5 - 2*(16*a^5*b + 76*a^4*b^2 + 137*a^3*b^3 + 107*a^2
*b^4 + 30*a*b^5)*cos(d*x + c)^3 - 3*(8*a^4*b + 28*a^3*b^2 + 37*a^2*b^3 + 22
*a*b^4 + 5*b^5 + (8*a^2*b^3 + 12*a*b^4 + 5*b^5)*cos(d*x + c)^4 - 2*(8*a^3*b
^2 + 20*a^2*b^3 + 17*a*b^4 + 5*b^5)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(
1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*
x + c)))*sin(d*x + c) + 2*(8*a^6 + 40*a^5*b + 92*a^4*b^2 + 111*a^3*b^3 + 66
*a^2*b^4 + 15*a*b^5)*cos(d*x + c))/(((a^7*b^2 + 3*a^6*b^3 + 3*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^4 - 2*(a^8*b + 4*a^7*b^2 + 6*a^6*b^3 + 4*a^5*b^4 + a^4
*b^5)*d*cos(d*x + c)^2 + (a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*a^5*b
^4 + a^4*b^5)*d)*sin(d*x + c))]
```

giac [A] time = 0.21, size = 232, normalized size = 1.18

$$-\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{a^2+ab}}+\frac{12a^2b^2\tan(dx+c)^3+19ab^3\tan(dx+c)^3+7b^4\tan(dx+c)^3+12a^2b^2}{\left(a^5+2a^4b+a^3b^2\right)\left(a\tan(dx+c)^2+b\tan(dx+c\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="giac")
```

```
[Out] -1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a
+ 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/((a^5 +
 2*a^4*b + a^3*b^2)*sqrt(a^2 + a*b)) + (12*a^2*b^2*tan(d*x + c)^3 + 19*a*b^
3*tan(d*x + c)^3 + 7*b^4*tan(d*x + c)^3 + 12*a^2*b^2*tan(d*x + c) + 9*a*b^3
*tan(d*x + c))/((a^5 + 2*a^4*b + a^3*b^2)*(a*tan(d*x + c)^2 + b*tan(d*x + c
)^2 + a)^2) + 8/(a^3*tan(d*x + c)))/d
```

maple [B] time = 0.58, size = 367, normalized size = 1.87

$$-\frac{3b^2\left(\tan^3(dx+c)\right)}{2d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2a^2(a+b)}-\frac{7b^3\left(\tan^3(dx+c)\right)}{8d\,a^3\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^2(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x)
```

```
[Out] -3/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/a^2*b^2/(a+b)*tan(d*x+c)^3-7/8/d
*b^3/a^3/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a+b)*tan(d*x+c)^3-3/2/d/(a*ta
```

```
n(d*x+c)^2+tan(d*x+c)^2*b+a)^2*b^2/a/(a^2+2*a*b+b^2)*tan(d*x+c)-9/8/d*b^3/a
^2/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^2/(a^2+2*a*b+b^2)*tan(d*x+c)-3/d/(a^2+
2*a*b+b^2)/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-9/2
/d/a^2/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1
/2))*b^2-15/8/d*b^3/a^3/(a^2+2*a*b+b^2)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*
x+c)/(a*(a+b))^(1/2))-1/d/a^3/tan(d*x+c)
```

maxima [A] time = 0.51, size = 270, normalized size = 1.38

$$-\frac{\frac{3\left(8a^2b+12ab^2+5b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^5+2a^4b+a^3b^2\right)\sqrt{(a+b)a}}+\frac{\left(8a^4+32a^3b+60a^2b^2+51ab^3+15b^4\right)\tan(dx+c)^4+8a^4+16a^3b+8a^2b^2+\left(16a^4+48a^3b+60a^2b^2\cdots\right.}{\left(a^7+4a^6b+6a^5b^2+4a^4b^3+a^3b^4\right)\tan(dx+c)^5+2\left(a^7+3a^6b+3a^5b^2+a^4b^3\right)\tan(dx+c)^3+\left(a^7+2a^6\cdots\right.}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^2)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*(8*a^2*b + 12*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/sqrt((a +
b)*a))/((a^5 + 2*a^4*b + a^3*b^2)*sqrt((a + b)*a)) + ((8*a^4 + 32*a^3*b + 6
0*a^2*b^2 + 51*a*b^3 + 15*b^4)*tan(d*x + c)^4 + 8*a^4 + 16*a^3*b + 8*a^2*b^
2 + (16*a^4 + 48*a^3*b + 60*a^2*b^2 + 25*a*b^3)*tan(d*x + c)^2)/((a^7 + 4*a
^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*tan(d*x + c)^5 + 2*(a^7 + 3*a^6*b +
 3*a^5*b^2 + a^4*b^3)*tan(d*x + c)^3 + (a^7 + 2*a^6*b + a^5*b^2)*tan(d*x +
c)))/d
```

mupad [B] time = 15.24, size = 251, normalized size = 1.28

$$-\frac{\frac{1}{a}+\frac{\tan(c+d\,x)^4\left(8\,a^3+24\,a^2\,b+36\,a\,b^2+15\,b^3\right)}{8\,a^3\,(a+b)}+\frac{\tan(c+d\,x)^2\left(16\,a^3+48\,a^2\,b+60\,a\,b^2+25\,b^3\right)}{8\,a^2\left(a^2+2\,a\,b+b^2\right)}}{d\left(\tan\left(c+d\,x\right)^5\,\left(a^2+2\,a\,b+b^2\right)+a^2\tan\left(c+d\,x\right)+\tan\left(c+d\,x\right)^3\,\left(2\,a^2+2\,b\,a\right)\right)}-\frac{3\,b\,\mathrm{atan}\left(\frac{3\,b\tan(c+d\,x)\left(a^5+\cdots\right.}{a^{7/2}\,(a+b)^{3/2}\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^2)^3),x)
```

```
[Out] - (1/a + (tan(c + d*x)^4*(36*a*b^2 + 24*a^2*b + 8*a^3 + 15*b^3))/(8*a^3*(a
+ b)) + (tan(c + d*x)^2*(60*a*b^2 + 48*a^2*b + 16*a^3 + 25*b^3))/(8*a^2*(2*
a*b + a^2 + b^2)))/(d*(tan(c + d*x)^5*(2*a*b + a^2 + b^2) + a^2*tan(c + d*x
) + tan(c + d*x)^3*(2*a*b + 2*a^2))) - (3*b*atan((3*b*tan(c + d*x)*(2*a^4*b
 + a^5 + a^3*b^2)*(12*a*b + 8*a^2 + 5*b^2))/(a^(7/2)*(a + b)^(3/2)*(36*a*b^
2 + 24*a^2*b + 15*b^3)))*(12*a*b + 8*a^2 + 5*b^2))/(8*a^(7/2)*d*(a + b)^(5/
2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**2)**3,x)
```

```
[Out] Timed out
```

3.111 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx$

Optimal. Leaf size=206

$$\frac{5b(2a+b)\sin(c+dx)\cos(c+dx)}{24a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)^2}+\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^{7/2}}+\frac{b\left(44a^2+44ab+15b^2\right)\text{si}}{48a^3d(a+b)^3\left(a+\right.}$$

```
[Out] 1/16*(2*a+b)*(8*a^2+8*a*b+5*b^2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(
7/2)/(a+b)^(7/2)/d+1/6*b*cos(d*x+c)*sin(d*x+c)/a/(a+b)/d/(a+b*sin(d*x+c)^2)
^3+5/24*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d/(a+b*sin(d*x+c)^2)^2+
1/48*b*(44*a^2+44*a*b+15*b^2)*cos(d*x+c)*sin(d*x+c)/a^3/(a+b)^3/d/(a+b*sin(
d*x+c)^2)
```

Rubi [A] time = 0.30, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}d(a+b)^{7/2}}+\frac{b\left(44a^2+44ab+15b^2\right)\sin(c+dx)\cos(c+dx)}{48a^3d(a+b)^3\left(a+b\sin^2(c+dx)\right)}+\frac{5b(2a+b)\text{si}}{24a^2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((2*a + b)*(8*a^2 + 8*a*b + 5*b^2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a
]])/(16*a^(7/2)*(a + b)^(7/2)*d) + (b*Cos[c + d*x]*Sin[c + d*x])/(6*a*(a +
b)*d*(a + b*Sin[c + d*x]^2)^3) + (5*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x])/
(24*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^2) + (b*(44*a^2 + 44*a*b + 15*b^
2)*Cos[c + d*x]*Sin[c + d*x])/(48*a^3*(a + b)^3*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^4}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} - \frac{\int \frac{-6a-5b+4b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^3}\,dx}{6a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} - \frac{\int \frac{-24a^2 \cdots}{\cdots}}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^{\cdots}\right.}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^{\cdots}\right.}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+dx)\right)^3} + \frac{5b(2a+b)\cos(c+dx)\sin(c+dx)}{24a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^2} + \frac{b\left(44a^{\cdots}\right.}{\cdots}$$

$$= \frac{(2a+b)\left(8a^2+8ab+5b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{16a^{7/2}(a+b)^{7/2}d} + \frac{b\cos(c+dx)\sin(c+dx)}{6a(a+b)d\left(a+b\sin^2(c+d\cdots\right.}$$

Mathematica [A] time = 1.47, size = 201, normalized size = 0.98

$$\frac{\frac{32a^{5/2}b\sin(2(c+dx))}{(a+b)(2a-b\cos(2(c+dx))+b)^3} + \frac{20a^{3/2}b(2a+b)\sin(2(c+dx))}{(a+b)^2(2a-b\cos(2(c+dx))+b)^2} + \frac{\sqrt{a}\,b\left(44a^2+44ab+15b^2\right)\sin(2(c+dx))}{(a+b)^3(2a-b\cos(2(c+dx))+b)} + \frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\tan^{-1}\left(\cdots\right.}{(a+b)^{7/2}}}{48a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-4),x]
```

```
[Out] ((3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*ArcTan[(Sqrt[a + b]*Tan[c + d*x]
)/Sqrt[a]])/(a + b)^(7/2) + (32*a^(5/2)*b*Sin[2*(c + d*x)])/((a + b)*(2*a +
 b - b*Cos[2*(c + d*x)])^3) + (20*a^(3/2)*b*(2*a + b)*Sin[2*(c + d*x)])/((a
 + b)^2*(2*a + b - b*Cos[2*(c + d*x)])^2) + (Sqrt[a]*b*(44*a^2 + 44*a*b + 1
5*b^2)*Sin[2*(c + d*x)])/((a + b)^3*(2*a + b - b*Cos[2*(c + d*x)])))/(48*a^
(7/2)*d)
```

fricas [B] time = 0.54, size = 1361, normalized size = 6.61

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6)*cos(d*x + c)^6 - 1
6*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b^4 - 33*a*b^5 - 5*b^
```

```
6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5 + 5*b^6)*cos(d*x + c
)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^4 + 28*a*b^5 + 5*b^6
)*cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 -
(a + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b
^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*
((44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a
^5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3
+ 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^
6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^
6 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^
6*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*
a^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
 (a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a
^5*b^6 + a^4*b^7)*d), -1/96*(3*((16*a^3*b^3 + 24*a^2*b^4 + 18*a*b^5 + 5*b^6
)*cos(d*x + c)^6 - 16*a^6 - 72*a^5*b - 138*a^4*b^2 - 147*a^3*b^3 - 93*a^2*b
^4 - 33*a*b^5 - 5*b^6 - 3*(16*a^4*b^2 + 40*a^3*b^3 + 42*a^2*b^4 + 23*a*b^5
+ 5*b^6)*cos(d*x + c)^4 + 3*(16*a^5*b + 56*a^4*b^2 + 82*a^3*b^3 + 65*a^2*b^
4 + 28*a*b^5 + 5*b^6)*cos(d*x + c)^2)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)
*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d*x + c)*sin(d*x + c))) + 2*(
(44*a^4*b^3 + 88*a^3*b^4 + 59*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^5 - 2*(54*a^
5*b^2 + 157*a^4*b^3 + 167*a^3*b^4 + 79*a^2*b^5 + 15*a*b^6)*cos(d*x + c)^3 +
 3*(24*a^6*b + 90*a^5*b^2 + 131*a^4*b^3 + 93*a^3*b^4 + 33*a^2*b^5 + 5*a*b^6
)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 + 4*a^7*b^4 + 6*a^6*b^5 + 4*a^5*b^6
 + a^4*b^7)*d*cos(d*x + c)^6 - 3*(a^9*b^2 + 5*a^8*b^3 + 10*a^7*b^4 + 10*a^6
*b^5 + 5*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^4 + 3*(a^10*b + 6*a^9*b^2 + 15*a
^8*b^3 + 20*a^7*b^4 + 15*a^6*b^5 + 6*a^5*b^6 + a^4*b^7)*d*cos(d*x + c)^2 -
(a^11 + 7*a^10*b + 21*a^9*b^2 + 35*a^8*b^3 + 35*a^7*b^4 + 21*a^6*b^5 + 7*a^
5*b^6 + a^4*b^7)*d)]
```

giac [A] time = 0.17, size = 344, normalized size = 1.67

$$\frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{a^2+ab}}+\frac{72a^4b\tan(dx+c)^5+198a^3b^2\tan(dx+c)^5+195a^2b^3\tan(dx+}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="giac")
```

```
[Out] 1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*(pi*floor((d*x + c)/pi + 1/2
)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)
))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt(a^2 + a*b)) + (72*a^4*b*tan(
d*x + c)^5 + 198*a^3*b^2*tan(d*x + c)^5 + 195*a^2*b^3*tan(d*x + c)^5 + 84*a
*b^4*tan(d*x + c)^5 + 15*b^5*tan(d*x + c)^5 + 144*a^4*b*tan(d*x + c)^3 + 28
8*a^3*b^2*tan(d*x + c)^3 + 184*a^2*b^3*tan(d*x + c)^3 + 40*a*b^4*tan(d*x +
c)^3 + 72*a^4*b*tan(d*x + c) + 90*a^3*b^2*tan(d*x + c) + 33*a^2*b^3*tan(d*x
 + c))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*(a*tan(d*x + c)^2 + b*tan(d*x
 + c)^2 + a)^3))/d
```

maple [B] time = 0.40, size = 705, normalized size = 3.42

$$\frac{3b\left(\tan^5(dx+c)\right)}{2d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^3 a(a+b)}+\frac{9b^2\left(\tan^5(dx+c)\right)}{8d\left(a\left(\tan^2(dx+c)\right)+\left(\tan^2(dx+c)\right)b+a\right)^3 a^2(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^2)^4,x)
```

```
[Out] 3/2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^3/a*b/(a+b)*tan(d*x+c)^5+9/8/d/(a*t
an(d*x+c)^2+tan(d*x+c)^2*b+a)^3/a^2*b^2/(a+b)*tan(d*x+c)^5+5/16/d/(a*tan(d*
x+c)^2+tan(d*x+c)^2*b+a)^3/a^3*b^3/(a+b)*tan(d*x+c)^5+3/d/(a*tan(d*x+c)^2+t
an(d*x+c)^2*b+a)^3*b/(a^2+2*a*b+b^2)*tan(d*x+c)^3+3/d/(a*tan(d*x+c)^2+tan(d
*x+c)^2*b+a)^3/a*b^2/(a^2+2*a*b+b^2)*tan(d*x+c)^3+5/6/d/(a*tan(d*x+c)^2+tan
(d*x+c)^2*b+a)^3/a^2*b^3/(a^2+2*a*b+b^2)*tan(d*x+c)^3+3/2/d/(a*tan(d*x+c)^2
+tan(d*x+c)^2*b+a)^3*b*a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)+15/8/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^3*b^2/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)+11/1
6/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^3*b^3/a/(a^3+3*a^2*b+3*a*b^2+b^3)*tan
(d*x+c)+1/d/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+
c)/(a*(a+b))^(1/2))+3/2/d/a/(a^3+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+9/8/d/a^2/(a^3+3*a^2*b+3*a*b^2+b^3)/(
a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2+5/16/d/a^3/(a^3
+3*a^2*b+3*a*b^2+b^3)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))*b^3
```

maxima [A] time = 0.67, size = 378, normalized size = 1.83

$$\frac{\frac{3\left(16a^3+24a^2b+18ab^2+5b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^6+3a^5b+3a^4b^2+a^3b^3\right)\sqrt{(a+b)a}}+\frac{3\left(24a^4b+66a^3b^2+65a^2b^3+28ab^4+5b^5\right)\tan(dx+c)^5+8\left(18a^4b\right.}{a^9+3a^8b+3a^7b^2+a^6b^3+\left(a^9+6a^8b+15a^7b^2+20a^6b^3+15a^5b^4+6a^4b^5+a^3b^6\right)\tan(dx+c)^6+}}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^4,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(16*a^3 + 24*a^2*b + 18*a*b^2 + 5*b^3)*arctan((a + b)*tan(d*x + c)/
sqrt((a + b)*a))/((a^6 + 3*a^5*b + 3*a^4*b^2 + a^3*b^3)*sqrt((a + b)*a)) +
(3*(24*a^4*b + 66*a^3*b^2 + 65*a^2*b^3 + 28*a*b^4 + 5*b^5)*tan(d*x + c)^5 +
 8*(18*a^4*b + 36*a^3*b^2 + 23*a^2*b^3 + 5*a*b^4)*tan(d*x + c)^3 + 3*(24*a^
4*b + 30*a^3*b^2 + 11*a^2*b^3)*tan(d*x + c))/(a^9 + 3*a^8*b + 3*a^7*b^2 + a
^6*b^3 + (a^9 + 6*a^8*b + 15*a^7*b^2 + 20*a^6*b^3 + 15*a^5*b^4 + 6*a^4*b^5
+ a^3*b^6)*tan(d*x + c)^6 + 3*(a^9 + 5*a^8*b + 10*a^7*b^2 + 10*a^6*b^3 + 5*
a^5*b^4 + a^4*b^5)*tan(d*x + c)^4 + 3*(a^9 + 4*a^8*b + 6*a^7*b^2 + 4*a^6*b^
3 + a^5*b^4)*tan(d*x + c)^2))/d
```

mupad [B] time = 15.44, size = 339, normalized size = 1.65

$$\frac{\frac{\tan(c+d\,x)\left(24\,a^2\,b+30\,a\,b^2+11\,b^3\right)}{16\,a\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\tan(c+d\,x)^3\left(18\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{6\,a^2\left(a^2+2\,a\,b+b^2\right)}+\frac{\tan(c+d\,x)^5\left(24\,a^2\,b+18\,a\,b^2+5\,b^3\right)}{16\,a^3\,(a+b)}}{d\left(\tan(c+d\,x)^6\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)+\tan(c+d\,x)^2\left(3\,a^3+3\,b\,a^2\right)+\tan(c+d\,x)^4\left(3\,a^3+6\,a^2\,b+3\,a\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^4,x)
```

```
[Out] ((tan(c + d*x)*(30*a*b^2 + 24*a^2*b + 11*b^3))/(16*a*(3*a*b^2 + 3*a^2*b + a
^3 + b^3)) + (tan(c + d*x)^3*(18*a*b^2 + 18*a^2*b + 5*b^3))/(6*a^2*(2*a*b +
 a^2 + b^2)) + (tan(c + d*x)^5*(18*a*b^2 + 24*a^2*b + 5*b^3))/(16*a^3*(a +
b)))/(d*(tan(c + d*x)^6*(3*a*b^2 + 3*a^2*b + a^3 + b^3) + tan(c + d*x)^2*(3
*a^2*b + 3*a^3) + tan(c + d*x)^4*(3*a*b^2 + 6*a^2*b + 3*a^3) + a^3)) + (ata
n((tan(c + d*x)*(2*a + b)*(2*a + 2*b)*(8*a*b + 8*a^2 + 5*b^2)*(3*a*b^2 + 3*
a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(18*a*b^2 + 24*a^2*b + 16*a^3
+ 5*b^3)))*(2*a + b)*(8*a*b + 8*a^2 + 5*b^2))/(16*a^(7/2)*d*(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**4,x)
```

```
[Out] Timed out
```

3.112 $\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx$

Optimal. Leaf size=279

$$\frac{7b(2a+b)\sin(c+dx)\cos(c+dx)}{48a^2d(a+b)^2\left(a+b\sin^2(c+dx)\right)^3}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx)\cos(c+dx)}{384a^4d(a+b)^4\left(a+b\sin^2(c+dx)\right)}+\frac{b\left(104a^2+104ab\right.}{192a^3d(a+}$$

```
[Out] 1/128*(128*a^4+256*a^3*b+288*a^2*b^2+160*a*b^3+35*b^4)*arctan((a+b)^(1/2)*t
an(d*x+c)/a^(1/2))/a^(9/2)/(a+b)^(9/2)/d+1/8*b*cos(d*x+c)*sin(d*x+c)/a/(a+b
)/d/(a+b*sin(d*x+c)^2)^4+7/48*b*(2*a+b)*cos(d*x+c)*sin(d*x+c)/a^2/(a+b)^2/d
/(a+b*sin(d*x+c)^2)^3+1/192*b*(104*a^2+104*a*b+35*b^2)*cos(d*x+c)*sin(d*x+c
)/a^3/(a+b)^3/d/(a+b*sin(d*x+c)^2)^2+5/384*b*(2*a+b)*(40*a^2+40*a*b+21*b^2)
*cos(d*x+c)*sin(d*x+c)/a^4/(a+b)^4/d/(a+b*sin(d*x+c)^2)
```

Rubi [A] time = 0.53, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}} = 0.357$, Rules used = {3184, 3173, 12, 3181, 205}

$$\frac{\left(288a^2b^2+256a^3b+128a^4+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}d(a+b)^{9/2}}+\frac{5b(2a+b)\left(40a^2+40ab+21b^2\right)\sin(c+dx}{384a^4d(a+b)^4\left(a+b\sin^2(c+dx\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[a +
b]*Tan[c + d*x])/Sqrt[a]])/(128*a^(9/2)*(a + b)^(9/2)*d) + (b*Cos[c + d*x]*
Sin[c + d*x])/(8*a*(a + b)*d*(a + b*Sin[c + d*x]^2)^4) + (7*b*(2*a + b)*Cos
[c + d*x]*Sin[c + d*x])/(48*a^2*(a + b)^2*d*(a + b*Sin[c + d*x]^2)^3) + (b*
(104*a^2 + 104*a*b + 35*b^2)*Cos[c + d*x]*Sin[c + d*x])/(192*a^3*(a + b)^3*
d*(a + b*Sin[c + d*x]^2)^2) + (5*b*(2*a + b)*(40*a^2 + 40*a*b + 21*b^2)*Cos
[c + d*x]*Sin[c + d*x])/(384*a^4*(a + b)^4*d*(a + b*Sin[c + d*x]^2))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
```

```
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(c+dx)\right)^5}\,dx = \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} - \frac{\int \frac{-8a-7b+6b\sin^2(c+dx)}{\left(a+b\sin^2(c+dx)\right)^4}\,dx}{8a(a+b)}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} - \frac{\int \frac{-48a^2\cdots}{\cdots}}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104\cdots\right.}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104\cdots\right.}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104\cdots\right.}{\cdots}$$

$$= \frac{b\cos(c+dx)\sin(c+dx)}{8a(a+b)d\left(a+b\sin^2(c+dx)\right)^4} + \frac{7b(2a+b)\cos(c+dx)\sin(c+dx)}{48a^2(a+b)^2d\left(a+b\sin^2(c+dx)\right)^3} + \frac{b\left(104\cdots\right.}{\cdots}$$

$$= \frac{\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{128a^{9/2}(a+b)^{9/2}d} + \frac{b\cos\cdots}{8a(a+b\cdots}$$

Mathematica [A] time = 1.93, size = 312, normalized size = 1.12

$$\frac{\frac{24\left(128a^4+256a^3b+288a^2b^2+160ab^3+35b^4\right)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}} + \frac{2\sqrt{a}\,b\sin(2(c+dx))\left(24576a^6+73728a^5b+97280a^4b^2-400a^3b^3\cos(6(c+dx))\cdots\right.}{\cdots}}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(-5),x]
```

```
[Out] ((24*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*ArcTan[(Sqrt[
a + b]*Tan[c + d*x])/Sqrt[a]])/(a + b)^(9/2) + (2*Sqrt[a]*b*(24576*a^6 + 73
728*a^5*b + 97280*a^4*b^2 + 71680*a^3*b^3 + 32272*a^2*b^4 + 8720*a*b^5 + 10
50*b^6 - b*(27648*a^5 + 69120*a^4*b + 73616*a^3*b^2 + 41304*a^2*b^3 + 12310
*a*b^4 + 1575*b^5)*Cos[2*(c + d*x)] + 2*b^2*(2816*a^4 + 5632*a^3*b + 4816*a
^2*b^2 + 2000*a*b^3 + 315*b^4)*Cos[4*(c + d*x)] - 400*a^3*b^3*Cos[6*(c + d*
x)] - 600*a^2*b^4*Cos[6*(c + d*x)] - 410*a*b^5*Cos[6*(c + d*x)] - 105*b^6*C
```

```
os[6*(c + d*x)])*Sin[2*(c + d*x)])/((a + b)^4*(2*a + b - b*Cos[2*(c + d*x)]
)^4))/(3072*a^(9/2)*d)
```

fricas [B] time = 0.60, size = 2017, normalized size = 7.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="fricas")
```

```
[Out] [-1/1536*(3*((128*a^4*b^4 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)
*cos(d*x + c)^8 + 128*a^8 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*
a^4*b^4 + 2508*a^3*b^5 + 1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3
 + 384*a^4*b^4 + 544*a^3*b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x +
c)^6 + 6*(128*a^6*b^2 + 512*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b
^6 + 230*a*b^7 + 35*b^8)*cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440
*a^5*b^3 + 1920*a^4*b^4 + 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*
cos(d*x + c)^2)*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4
- 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a + b)*cos(d*x + c)^3 - (a
 + b)*cos(d*x + c))*sqrt(-a^2 - a*b)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2
*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)) + 4*(5
*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103*a^2*b^7 + 21*a*b^8)*cos(d*x
+ c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a^4*b^5 + 4923*a^3*b^6 + 1930*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^7*b^2 + 7456*a^6*b^3 + 13370*
a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*a^2*b^7 + 315*a*b^8)*cos(d*x
+ c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^6*b^3 + 3427*a^5*b^4 + 2508*a
^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8)*cos(d*x + c))*sin(d*x + c))
/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*b^7 + 5*a^6*b^8 + a^5*b^9)*d*
cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15*a^9*b^5 + 20*a^8*b^6 + 15*a^
7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6 + 6*(a^12*b^2 + 7*a^11*b^3 +
21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7*b^7 + 7*a^6*b^8 + a^5*b^9)*d
*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*a^11*b^3 + 56*a^10*b^4 + 70*a
^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^2 +
(a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 + 126*a^10*b^4 + 126*a^9*b^5 +
 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9)*d), -1/768*(3*((128*a^4*b^4
 + 256*a^3*b^5 + 288*a^2*b^6 + 160*a*b^7 + 35*b^8)*cos(d*x + c)^8 + 128*a^8
 + 768*a^7*b + 2080*a^6*b^2 + 3360*a^5*b^3 + 3555*a^4*b^4 + 2508*a^3*b^5 +
1138*a^2*b^6 + 300*a*b^7 + 35*b^8 - 4*(128*a^5*b^3 + 384*a^4*b^4 + 544*a^3*
b^5 + 448*a^2*b^6 + 195*a*b^7 + 35*b^8)*cos(d*x + c)^6 + 6*(128*a^6*b^2 + 5
12*a^5*b^3 + 928*a^4*b^4 + 992*a^3*b^5 + 643*a^2*b^6 + 230*a*b^7 + 35*b^8)*
cos(d*x + c)^4 - 4*(128*a^7*b + 640*a^6*b^2 + 1440*a^5*b^3 + 1920*a^4*b^4 +
 1635*a^3*b^5 + 873*a^2*b^6 + 265*a*b^7 + 35*b^8)*cos(d*x + c)^2)*sqrt(a^2
+ a*b)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)/(sqrt(a^2 + a*b)*cos(d
*x + c)*sin(d*x + c))) + 2*(5*(80*a^5*b^4 + 200*a^4*b^5 + 202*a^3*b^6 + 103
*a^2*b^7 + 21*a*b^8)*cos(d*x + c)^7 - (1408*a^6*b^3 + 4824*a^5*b^4 + 6724*a
^4*b^5 + 4923*a^3*b^6 + 1930*a^2*b^7 + 315*a*b^8)*cos(d*x + c)^5 + (1728*a^
7*b^2 + 7456*a^6*b^3 + 13370*a^5*b^4 + 12969*a^4*b^5 + 7327*a^3*b^6 + 2315*
a^2*b^7 + 315*a*b^8)*cos(d*x + c)^3 - 3*(256*a^8*b + 1312*a^7*b^2 + 2848*a^
6*b^3 + 3427*a^5*b^4 + 2508*a^4*b^5 + 1138*a^3*b^6 + 300*a^2*b^7 + 35*a*b^8
)*cos(d*x + c))*sin(d*x + c))/((a^10*b^4 + 5*a^9*b^5 + 10*a^8*b^6 + 10*a^7*
b^7 + 5*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^8 - 4*(a^11*b^3 + 6*a^10*b^4 + 15
*a^9*b^5 + 20*a^8*b^6 + 15*a^7*b^7 + 6*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^6
+ 6*(a^12*b^2 + 7*a^11*b^3 + 21*a^10*b^4 + 35*a^9*b^5 + 35*a^8*b^6 + 21*a^7
*b^7 + 7*a^6*b^8 + a^5*b^9)*d*cos(d*x + c)^4 - 4*(a^13*b + 8*a^12*b^2 + 28*
a^11*b^3 + 56*a^10*b^4 + 70*a^9*b^5 + 56*a^8*b^6 + 28*a^7*b^7 + 8*a^6*b^8 +
 a^5*b^9)*d*cos(d*x + c)^2 + (a^14 + 9*a^13*b + 36*a^12*b^2 + 84*a^11*b^3 +
 126*a^10*b^4 + 126*a^9*b^5 + 84*a^8*b^6 + 36*a^7*b^7 + 9*a^6*b^8 + a^5*b^9
)*d)]
```

giac [B] time = 0.17, size = 524, normalized size = 1.88

$$\frac{3\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\left(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4\right)\sqrt{a^2+ab}}+\frac{768\,a^6b\tan(dx+c)^7+3168\,a^5b^2\tan(dx+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="giac")
```

```
[Out] 1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*(pi*floor
((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x +
c))/sqrt(a^2 + a*b)))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3 + a^4*b^4)*sq
rt(a^2 + a*b)) + (768*a^6*b*tan(d*x + c)^7 + 3168*a^5*b^2*tan(d*x + c)^7 +
5376*a^4*b^3*tan(d*x + c)^7 + 4905*a^3*b^4*tan(d*x + c)^7 + 2619*a^2*b^5*ta
n(d*x + c)^7 + 795*a*b^6*tan(d*x + c)^7 + 105*b^7*tan(d*x + c)^7 + 2304*a^6
*b*tan(d*x + c)^5 + 7776*a^5*b^2*tan(d*x + c)^5 + 10400*a^4*b^3*tan(d*x + c
)^5 + 7073*a^3*b^4*tan(d*x + c)^5 + 2530*a^2*b^5*tan(d*x + c)^5 + 385*a*b^6
*tan(d*x + c)^5 + 2304*a^6*b*tan(d*x + c)^3 + 6048*a^5*b^2*tan(d*x + c)^3 +
 6080*a^4*b^3*tan(d*x + c)^3 + 2847*a^3*b^4*tan(d*x + c)^3 + 511*a^2*b^5*ta
n(d*x + c)^3 + 768*a^6*b*tan(d*x + c) + 1440*a^5*b^2*tan(d*x + c) + 1056*a^
4*b^3*tan(d*x + c) + 279*a^3*b^4*tan(d*x + c))/((a^8 + 4*a^7*b + 6*a^6*b^2
+ 4*a^5*b^3 + a^4*b^4)*(a*tan(d*x + c)^2 + b*tan(d*x + c)^2 + a)^4))/d
```

maple [B] time = 0.41, size = 1249, normalized size = 4.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^2)^5,x)
```

```
[Out] 2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b/a/(a+b)*tan(d*x+c)^7+9/4/d/(a*tan
(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^2/a^2/(a+b)*tan(d*x+c)^7+5/4/d/(a*tan(d*x+c
)^2+tan(d*x+c)^2*b+a)^4*b^3/a^3/(a+b)*tan(d*x+c)^7+35/128/d/(a*tan(d*x+c)^2
+tan(d*x+c)^2*b+a)^4*b^4/a^4/(a+b)*tan(d*x+c)^7+6/d/(a*tan(d*x+c)^2+tan(d*x
+c)^2*b+a)^4*b/(a^2+2*a*b+b^2)*tan(d*x+c)^5+33/4/d/(a*tan(d*x+c)^2+tan(d*x+
c)^2*b+a)^4/a*b^2/(a^2+2*a*b+b^2)*tan(d*x+c)^5+55/12/d/(a*tan(d*x+c)^2+tan(
d*x+c)^2*b+a)^4/a^2*b^3/(a^2+2*a*b+b^2)*tan(d*x+c)^5+385/384/d/(a*tan(d*x+c
)^2+tan(d*x+c)^2*b+a)^4/a^3*b^4/(a^2+2*a*b+b^2)*tan(d*x+c)^5+6/d/(a*tan(d*x
+c)^2+tan(d*x+c)^2*b+a)^4*a*b/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^3+39/4/d
/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^2/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+
c)^3+73/12/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4/a*b^3/(a^3+3*a^2*b+3*a*b^2
+b^3)*tan(d*x+c)^3+511/384/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4/a^2*b^4/(a
^3+3*a^2*b+3*a*b^2+b^3)*tan(d*x+c)^3+2/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^
4*b*a^2/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)+15/4/d/(a*tan(d*x+c)
^2+tan(d*x+c)^2*b+a)^4*b^2*a/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)
+11/4/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^3/(a^4+4*a^3*b+6*a^2*b^2+4*a*
b^3+b^4)*tan(d*x+c)+93/128/d/(a*tan(d*x+c)^2+tan(d*x+c)^2*b+a)^4*b^4/a/(a^4
+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*tan(d*x+c)+1/d/(a^4+4*a^3*b+6*a^2*b^2+4*a*b
^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+2/d/a/(a^4
+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*
(a+b))^(1/2))*b+9/4/d/a^2/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/
2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2+5/4/d/a^3/(a^4+4*a^3*b+6*a^
2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))
*b^3+35/128/d/a^4/(a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^4
```

maxima [B] time = 0.53, size = 588, normalized size = 2.11

$$\frac{\frac{3\left(128\,a^4+256\,a^3b+288\,a^2b^2+160\,ab^3+35\,b^4\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^8+4\,a^7b+6\,a^6b^2+4\,a^5b^3+a^4b^4\right)\sqrt{(a+b)a}}+\frac{3\left(256\,a^6b+1056\,a^5b^2+1792\,a^4b^3+1635\,a^3b^4+873\,a^2b^5+\right.}{a^{12}+4\,a^{11}b+6\,a^{10}b^2+4\,a^9b^3+a^8b^4+\left(a^{12}+8\,a^{11}b+28\,a^{10}b^2+56\,a^9b^3+70\,a^8b^4+5\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^2)^5,x, algorithm="maxima")
```

```
[Out] 1/384*(3*(128*a^4 + 256*a^3*b + 288*a^2*b^2 + 160*a*b^3 + 35*b^4)*arctan((a
 + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^8 + 4*a^7*b + 6*a^6*b^2 + 4*a^5*b^3
 + a^4*b^4)*sqrt((a + b)*a)) + (3*(256*a^6*b + 1056*a^5*b^2 + 1792*a^4*b^3
+ 1635*a^3*b^4 + 873*a^2*b^5 + 265*a*b^6 + 35*b^7)*tan(d*x + c)^7 + (2304*a
^6*b + 7776*a^5*b^2 + 10400*a^4*b^3 + 7073*a^3*b^4 + 2530*a^2*b^5 + 385*a*b
^6)*tan(d*x + c)^5 + (2304*a^6*b + 6048*a^5*b^2 + 6080*a^4*b^3 + 2847*a^3*b
^4 + 511*a^2*b^5)*tan(d*x + c)^3 + 3*(256*a^6*b + 480*a^5*b^2 + 352*a^4*b^3
 + 93*a^3*b^4)*tan(d*x + c))/(a^12 + 4*a^11*b + 6*a^10*b^2 + 4*a^9*b^3 + a^
8*b^4 + (a^12 + 8*a^11*b + 28*a^10*b^2 + 56*a^9*b^3 + 70*a^8*b^4 + 56*a^7*b
^5 + 28*a^6*b^6 + 8*a^5*b^7 + a^4*b^8)*tan(d*x + c)^8 + 4*(a^12 + 7*a^11*b
+ 21*a^10*b^2 + 35*a^9*b^3 + 35*a^8*b^4 + 21*a^7*b^5 + 7*a^6*b^6 + a^5*b^7)
*tan(d*x + c)^6 + 6*(a^12 + 6*a^11*b + 15*a^10*b^2 + 20*a^9*b^3 + 15*a^8*b^
4 + 6*a^7*b^5 + a^6*b^6)*tan(d*x + c)^4 + 4*(a^12 + 5*a^11*b + 10*a^10*b^2
+ 10*a^9*b^3 + 5*a^8*b^4 + a^7*b^5)*tan(d*x + c)^2))/d
```

mupad [B] time = 16.10, size = 450, normalized size = 1.61

$$\frac{\frac{\tan(c+d\,x)\left(256\,a^3\,b+480\,a^2\,b^2+352\,a\,b^3+93\,b^4\right)}{128\,a\left(a^4+4\,a^3\,b+6\,a^2\,b^2+4\,a\,b^3+b^4\right)}+\frac{\tan(c+d\,x)^3\left(2304\,a^3\,b+3744\,a^2\,b^2+2336\,a\,b^3+511\,b^4\right)}{384\,a^2\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}+\frac{\tan(c+d\,x)^5\left(2304\,a^3\,b+316\right.}{384\,a^3\left(a^2+\right.}}{d\left(\tan\left(c+d\,x\right)^4\left(6\,a^4+12\,a^3\,b+6\,a^2\,b^2\right)+\tan\left(c+d\,x\right)^2\left(4\,a^4+4\,b\,a^3\right)+\tan\left(c+d\,x\right)^6\left(4\,a^4+12\,a^3\,b+12\,a^2\,b\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^2)^5,x)
```

```
[Out] ((tan(c + d*x)*(352*a*b^3 + 256*a^3*b + 93*b^4 + 480*a^2*b^2))/(128*a*(4*a*
b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2)) + (tan(c + d*x)^3*(2336*a*b^3 + 230
4*a^3*b + 511*b^4 + 3744*a^2*b^2))/(384*a^2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) + (tan(c + d*x)^5*(1760*a*b^3 + 2304*a^3*b + 385*b^4 + 3168*a^2*b^2))/(38
4*a^3*(2*a*b + a^2 + b^2)) + (tan(c + d*x)^7*(160*a*b^3 + 256*a^3*b + 35*b^
4 + 288*a^2*b^2))/(128*a^4*(a + b)))/(d*(tan(c + d*x)^4*(12*a^3*b + 6*a^4 +
 6*a^2*b^2) + tan(c + d*x)^2*(4*a^3*b + 4*a^4) + tan(c + d*x)^6*(4*a*b^3 +
12*a^3*b + 4*a^4 + 12*a^2*b^2) + a^4 + tan(c + d*x)^8*(4*a*b^3 + 4*a^3*b +
a^4 + b^4 + 6*a^2*b^2))) + (atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^3 + 4*a^3
*b + a^4 + b^4 + 6*a^2*b^2))/(2*a^(1/2)*(a + b)^(9/2)))*(160*a*b^3 + 256*a^
3*b + 128*a^4 + 35*b^4 + 288*a^2*b^2))/(128*a^(9/2)*d*(a + b)^(9/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**2)**5,x)
```

```
[Out] Timed out
```

3.113 $\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx$

Optimal. Leaf size=11

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

```
[Out] -arcsin(1/2*cos(x)*2^(1/2))
```

Rubi [A] time = 0.03, antiderivative size = 11, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3186, 216}

$$-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]/Sqrt[1 + Sin[x]^2],x]
[Out] -ArcSin[Cos[x]/Sqrt[2]]
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(x)}{\sqrt{1+\sin^2(x)}}\,dx &= -\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx, x, \cos(x)\right)\\ &= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)\end{aligned}$$

Mathematica [C] time = 0.06, size = 29, normalized size = 2.64

$$i\log\left(\sqrt{3-\cos(2x)}+i\sqrt{2}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]/Sqrt[1 + Sin[x]^2],x]
[Out] I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3 - Cos[2*x]]]
```

fricas [B] time = 0.43, size = 57, normalized size = 5.18

$$\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-\left(\cos(x)^3-\cos(x)\right)\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 - cos(x))*sqrt(-cos(x)^2 + 2))/(cos(
x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(x)/cos(x))
```

giac [A] time = 0.20, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -arcsin(1/2*sqrt(2)*cos(x))
```

maple [B] time = 0.76, size = 33, normalized size = 3.00

$$\frac{\sqrt{\left(1+\sin^2(x)\right)\left(\cos^2(x)\right)}\ \arcsin\left(\sin^2(x)\right)}{2\cos(x)\sqrt{1+\sin^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(1+sin(x)^2)^(1/2),x)
```

```
[Out] 1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*arcsin(sin(x)^2)/cos(x)/(1+sin(x)^2)^(1/2
)
```

maxima [A] time = 0.51, size = 10, normalized size = 0.91

$$-\arcsin\left(\frac{1}{2}\sqrt{2}\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(1/2*sqrt(2)*cos(x))
```

mupad [B] time = 13.38, size = 18, normalized size = 1.64

$$\ln\left(\sqrt{\sin(x)^2+1}+\cos(x)\,1\mathrm{i}\right)\,1\mathrm{i}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)/(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] log(cos(x)*1i + (sin(x)^2 + 1)^(1/2))*1i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(x)}{\sqrt{\sin^2(x)+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)/(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sin(x)/sqrt(sin(x)**2 + 1), x)
```

3.114 $\int \sin(x)\sqrt{1+\sin^2(x)}\,dx$

Optimal. Leaf size=30

$$-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

```
[Out] -arcsin(1/2*cos(x)*2^(1/2))-1/2*cos(x)*(2-cos(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3186, 195, 216}

$$-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -ArcSin[Cos[x]/Sqrt[2]] - (Cos[x]*Sqrt[2 - Cos[x]^2])/2
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \sin(x)\sqrt{1+\sin^2(x)}\,dx &= -\text{Subst}\left(\int \sqrt{2-x^2}\,dx,x,\cos(x)\right)\\ &= -\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}-\text{Subst}\left(\int \frac{1}{\sqrt{2-x^2}}\,dx,x,\cos(x)\right)\\ &= -\sin^{-1}\left(\frac{\cos(x)}{\sqrt{2}}\right)-\frac{1}{2}\cos(x)\sqrt{2-\cos^2(x)}\end{aligned}$$

Mathematica [C] time = 0.05, size = 53, normalized size = 1.77

$$-\frac{\cos(x)\sqrt{3-\cos(2x)}}{2\sqrt{2}}+i\log\left(\sqrt{3-\cos(2x)}+i\sqrt{2}\cos(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[x]*Sqrt[1 + Sin[x]^2],x]
```

```
[Out] -1/2*(Cos[x]*Sqrt[3 - Cos[2*x]])/Sqrt[2] + I*Log[I*Sqrt[2]*Cos[x] + Sqrt[3
- Cos[2*x]]]
```

fricas [B] time = 0.46, size = 71, normalized size = 2.37

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\,\cos(x)+\frac{1}{2}\arctan\left(-\frac{\cos(x)\sin(x)-\left(\cos(x)^3-\cos(x)\right)\sqrt{-\cos(x)^2+2}}{\cos(x)^4-3\,\cos(x)^2+1}\right)-\frac{1}{2}\arctan\left(\frac{\sin(x)}{\cos(x)}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) + 1/2*arctan(-(cos(x)*sin(x) - (cos(x)^3 -
cos(x))*sqrt(-cos(x)^2 + 2))/(cos(x)^4 - 3*cos(x)^2 + 1)) - 1/2*arctan(sin(
x)/cos(x))
```

giac [A] time = 0.15, size = 25, normalized size = 0.83

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\,\cos(x)-\arcsin\left(\frac{1}{2}\sqrt{2}\,\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

maple [A] time = 1.11, size = 51, normalized size = 1.70

$$-\frac{\sqrt{\left(1+\sin^2(x)\right)\left(\cos^2(x)\right)}\,\left(\sqrt{-\left(\cos^4(x)\right)+2\left(\cos^2(x)\right)}+\arcsin\left(\cos^2(x)-1\right)\right)}{2\cos(x)\sqrt{1+\sin^2(x)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(1+sin(x)^2)^(1/2),x)
```

```
[Out] -1/2*((1+sin(x)^2)*cos(x)^2)^(1/2)*((-cos(x)^4+2*cos(x)^2)^(1/2)+arcsin(cos
(x)^2-1))/cos(x)/(1+sin(x)^2)^(1/2)
```

maxima [A] time = 0.54, size = 25, normalized size = 0.83

$$-\frac{1}{2}\sqrt{-\cos(x)^2+2}\,\cos(x)-\arcsin\left(\frac{1}{2}\sqrt{2}\,\cos(x)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*sqrt(-cos(x)^2 + 2)*cos(x) - arcsin(1/2*sqrt(2)*cos(x))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \sin(x)\,\sqrt{{\sin(x)}^2+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(x)*(sin(x)^2 + 1)^(1/2),x)
```

```
[Out] int(sin(x)*(sin(x)^2 + 1)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{\sin^2(x)+1}\,\sin(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(x)*(1+sin(x)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(sin(x)**2 + 1)*sin(x), x)
```

3.115 $\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}} \, dx$

Optimal. Leaf size=15

$$-\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(3x+7)\right)$$

```
[Out] -1/3*arcsin(1/2*cos(7+3*x))
```

Rubi [A] time = 0.03, antiderivative size = 15, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3186, 216}

$$-\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Int[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

```
[Out] -ArcSin[Cos[7 + 3*x]/2]/3
```

Rule 216

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSin[(Rt[-b, 2]*x)/Sqr
t[a]]/Rt[-b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && NegQ[b]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(7+3x)}{\sqrt{3+\sin^2(7+3x)}} \, dx &= -\left(\frac{1}{3}\text{Subst}\left(\int \frac{1}{\sqrt{4-x^2}} \, dx, x, \cos(7+3x)\right)\right)\\ &= -\frac{1}{3}\sin^{-1}\left(\frac{1}{2}\cos(7+3x)\right)\end{aligned}$$

Mathematica [C] time = 0.09, size = 39, normalized size = 2.60

$$\frac{1}{3}i\log\left(\sqrt{7-\cos(2(3x+7))}+i\sqrt{2}\cos(3x+7)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[7 + 3*x]/Sqrt[3 + Sin[7 + 3*x]^2],x]
```

```
[Out] (I/3)*Log[I*Sqrt[2]*Cos[7 + 3*x] + Sqrt[7 - Cos[2*(7 + 3*x)]]]
```

fricas [B] time = 0.45, size = 94, normalized size = 6.27

$$\frac{1}{6}\arctan\left(-\frac{4\cos(3x+7)\sin(3x+7)-\left(\cos(3x+7)^3-2\cos(3x+7)\right)\sqrt{-\cos(3x+7)^2+4}}{\cos(3x+7)^4-8\cos(3x+7)^2+4}\right)-\frac{1}{6}\arctan\left(\frac{\text{s}}{\text{c}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/6*arctan(-(4*cos(3*x + 7)*sin(3*x + 7) - (cos(3*x + 7)^3 - 2*cos(3*x + 7)
)*sqrt(-cos(3*x + 7)^2 + 4))/(cos(3*x + 7)^4 - 8*cos(3*x + 7)^2 + 4)) - 1/6
*arctan(sin(3*x + 7)/cos(3*x + 7))
```

giac [A] time = 0.38, size = 11, normalized size = 0.73

$$-\frac{1}{3} \arcsin\left(\frac{1}{2} \cos\left(3 x + 7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/3*arcsin(1/2*cos(3*x + 7))
```

maple [B] time = 1.16, size = 57, normalized size = 3.80

$$-\frac{\sqrt{\left(3+\sin^{2}\left(7+3 x\right)\right) \left(\cos^{2}\left(7+3 x\right)\right)}\, \arcsin\left(-1+\frac{\left(\cos^{2}\left(7+3 x\right)\right)}{2}\right)}{6 \cos\left(7+3 x\right) \sqrt{3+\sin^{2}\left(7+3 x\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x)
```

```
[Out] -1/6*((3+sin(7+3*x)^2)*cos(7+3*x)^2)^(1/2)*arcsin(-1+1/2*cos(7+3*x)^2)/cos(
7+3*x)/(3+sin(7+3*x)^2)^(1/2)
```

maxima [A] time = 0.44, size = 11, normalized size = 0.73

$$-\frac{1}{3} \arcsin\left(\frac{1}{2} \cos\left(3 x + 7\right)\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/3*arcsin(1/2*cos(3*x + 7))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.07

$$\int \frac{\sin\left(3 x + 7\right)}{\sqrt{{\sin\left(3 x + 7\right)}^2 + 3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2),x)
```

```
[Out] int(sin(3*x + 7)/(sin(3*x + 7)^2 + 3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(3x + 7\right)}{\sqrt{\sin^{2}\left(3x + 7\right) + 3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(7+3*x)/(3+sin(7+3*x)**2)**(1/2),x)
```

```
[Out] Integral(sin(3*x + 7)/sqrt(sin(3*x + 7)**2 + 3), x)
```

3.116 $\int \left(a-a\sin^2(x)\right)^{5/2}\,dx$

Optimal. Leaf size=53

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

```
[Out] 4/15*a*(a*cos(x)^2)^(3/2)*tan(x)+1/5*(a*cos(x)^2)^(5/2)*tan(x)+8/15*a^2*(a*
cos(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.05, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3203, 3207, 2637}

$$\frac{8}{15}a^2\tan(x)\sqrt{a\cos^2(x)}+\frac{1}{5}\tan(x)\left(a\cos^2(x)\right)^{5/2}+\frac{4}{15}a\tan(x)\left(a\cos^2(x)\right)^{3/2}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(5/2),x]
```

```
[Out] (8*a^2*Sqrt[a*Cos[x]^2]*Tan[x])/15 + (4*a*(a*Cos[x]^2)^(3/2)*Tan[x])/15 + (
(a*Cos[x]^2)^(5/2)*Tan[x])/5
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$
\begin{aligned}
\int \left(a-a\sin^2(x)\right)^{5/2}\,dx &= \int \left(a\cos^2(x)\right)^{5/2}\,dx \\
&= \frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{5}(4a)\int \left(a\cos^2(x)\right)^{3/2}\,dx \\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\right)\int \sqrt{a\cos^2(x)}\,dx \\
&= \frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)+\frac{1}{15}\left(8a^2\sqrt{a\cos^2(x)}\,\sec(x)\right)\int c \\
&= \frac{8}{15}a^2\sqrt{a\cos^2(x)}\,\tan(x)+\frac{4}{15}a\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{5}\left(a\cos^2(x)\right)^{5/2}\tan(x)
\end{aligned}
$$

Mathematica [A] time = 0.02, size = 36, normalized size = 0.68

$$\frac{1}{240}a^2(150\sin(x)+25\sin(3x)+3\sin(5x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(5/2),x]
[Out] (a^2*Sqrt[a*Cos[x]^2]*Sec[x]*(150*Sin[x] + 25*Sin[3*x] + 3*Sin[5*x]))/240
```

fricas [A] time = 0.45, size = 40, normalized size = 0.75

$$\frac{\left(3\,a^2\cos(x)^4+4\,a^2\cos(x)^2+8\,a^2\right)\sqrt{a\cos(x)^2}\,\sin(x)}{15\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
[Out] 1/15*(3*a^2*cos(x)^4 + 4*a^2*cos(x)^2 + 8*a^2)*sqrt(a*cos(x)^2)*sin(x)/cos(
x)
```

giac [B] time = 0.21, size = 84, normalized size = 1.58

$$-\frac{2\left(15\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^4\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)-40\,a^{\frac{5}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)+48\,a\right.}{15\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="giac")
[Out] -2/15*(15*a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^4*sgn(tan(1/2*x)^4 - 1) - 40*
a^(5/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) + 48*a^(5/2)*sg
n(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^5
```

maple [A] time = 0.88, size = 32, normalized size = 0.60

$$\frac{a^3\cos(x)\sin(x)\left(3\left(\cos^4(x)\right)+4\left(\cos^2(x)\right)+8\right)}{15\sqrt{a\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(5/2),x)
[Out] 1/15*a^3*cos(x)*sin(x)*(3*cos(x)^4+4*cos(x)^2+8)/(a*cos(x)^2)^(1/2)
```

maxima [A] time = 0.52, size = 31, normalized size = 0.58

$$\frac{1}{240}\left(3\,a^2\sin\left(5\,x\right)+25\,a^2\sin\left(3\,x\right)+150\,a^2\sin(x)\right)\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
[Out] 1/240*(3*a^2*sin(5*x) + 25*a^2*sin(3*x) + 150*a^2*sin(x))*sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \left(a-a\sin(x)^2\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(5/2),x)
[Out] int((a - a*sin(x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(5/2),x)
[Out] Timed out
```

3.117 $\int \left(a-a\sin^2(x)\right)^{3/2}\,dx$

Optimal. Leaf size=34

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] 1/3*(a*cos(x)^2)^(3/2)*tan(x)+2/3*a*(a*cos(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.04, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3203, 3207, 2637}

$$\frac{1}{3}\tan(x)\left(a\cos^2(x)\right)^{3/2}+\frac{2}{3}a\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(3/2),x]
```

```
[Out] (2*a*Sqrt[a*Cos[x]^2]*Tan[x])/3 + ((a*Cos[x]^2)^(3/2)*Tan[x])/3
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3203

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(Cot[e + f*x
]*(b*Sin[e + f*x]^2)^p)/(2*f*p), x] + Dist[(b*(2*p - 1))/(2*p), Int[(b*Sin[
e + f*x]^2)^(p - 1), x], x] /; FreeQ[{b, e, f}, x] &&  !IntegerQ[p] && GtQ[
p, 1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \left(a-a\sin^2(x)\right)^{3/2}\,dx &= \int \left(a\cos^2(x)\right)^{3/2}\,dx\\ &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}(2a)\int \sqrt{a\cos^2(x)}\,dx\\ &= \frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)+\frac{1}{3}\left(2a\sqrt{a\cos^2(x)}\sec(x)\right)\int \cos(x)\,dx\\ &= \frac{2}{3}a\sqrt{a\cos^2(x)}\tan(x)+\frac{1}{3}\left(a\cos^2(x)\right)^{3/2}\tan(x)\end{aligned}$$

Mathematica [A] time = 0.01, size = 26, normalized size = 0.76

$$\frac{1}{12}a(9\sin(x)+\sin(3x))\sec(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(3/2),x]
```

```
[Out] (a*Sqrt[a*Cos[x]^2]*Sec[x]*(9*Sin[x] + Sin[3*x]))/12
```

fricas [A] time = 0.43, size = 26, normalized size = 0.76

$$\frac{\left(a\cos(x)^2+2\,a\right)\sqrt{a\cos(x)^2}\,\sin(x)}{3\,\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(a*cos(x)^2 + 2*a)*sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

giac [B] time = 0.18, size = 57, normalized size = 1.68

$$-\frac{2\left(3\,a^{\frac{3}{2}}\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^2\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)-4\,a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)\right)}{3\left(\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -2/3*(3*a^(3/2)*(1/tan(1/2*x) + tan(1/2*x))^2*sgn(tan(1/2*x)^4 - 1) - 4*a^(
3/2)*sgn(tan(1/2*x)^4 - 1))/(1/tan(1/2*x) + tan(1/2*x))^3
```

maple [A] time = 0.85, size = 24, normalized size = 0.71

$$\frac{a^2\cos(x)\sin(x)\left(\cos^2(x)+2\right)}{3\sqrt{a\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(3/2),x)
```

```
[Out] 1/3*a^2*cos(x)*sin(x)*(cos(x)^2+2)/(a*cos(x)^2)^(1/2)
```

maxima [A] time = 0.49, size = 17, normalized size = 0.50

$$\frac{1}{12}\,(a\sin(3\,x)+9\,a\sin(x))\sqrt{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/12*(a*sin(3*x) + 9*a*sin(x))*sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int\left(a-a\sin(x)^2\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(3/2),x)
```

```
[Out] int((a - a*sin(x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.118 $\int \sqrt{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=13

$$\tan(x)\sqrt{a\cos^2(x)}$$

```
[Out] (a*cos(x)^2)^(1/2)*tan(x)
```

Rubi [A] time = 0.03, antiderivative size = 13, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}} = 0.231$, Rules used = {3176, 3207, 2637}

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(x)}\,dx &= \int \sqrt{a\cos^2(x)}\,dx\\ &= \left(\sqrt{a\cos^2(x)}\,\sec(x)\right)\int \cos(x)\,dx\\ &= \sqrt{a\cos^2(x)}\,\tan(x)\end{aligned}$$

Mathematica [A] time = 0.00, size = 13, normalized size = 1.00

$$\tan(x)\sqrt{a\cos^2(x)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] Sqrt[a*Cos[x]^2]*Tan[x]
```

fricas [**A**] time = 0.42, size = 15, normalized size = 1.15

$$\frac{\sqrt{a\cos(x)^2}\,\sin(x)}{\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(x)^2)*sin(x)/cos(x)
```

giac [**B**] time = 0.15, size = 27, normalized size = 2.08

$$-\frac{2\,\sqrt{a}\,\mathrm{sgn}\left(\tan\left(\frac{1}{2}\,x\right)^4-1\right)}{\frac{1}{\tan\left(\frac{1}{2}\,x\right)}+\tan\left(\frac{1}{2}\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="giac")
[Out] -2*sqrt(a)*sgn(tan(1/2*x)^4 - 1)/(1/tan(1/2*x) + tan(1/2*x))
```

maple [**A**] time = 0.52, size = 15, normalized size = 1.15

$$\frac{a\cos(x)\sin(x)}{\sqrt{a\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(x)^2)^(1/2),x)
[Out] a*cos(x)*sin(x)/(a*cos(x)^2)^(1/2)
```

maxima [**A**] time = 0.48, size = 6, normalized size = 0.46

$$\sqrt{a}\,\sin(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
[Out] sqrt(a)*sin(x)
```

mupad [**B**] time = 0.22, size = 46, normalized size = 3.54

$$\frac{\sqrt{2}\,\sqrt{a}\,\sqrt{\cos(2\,x)+1}\,\left(\cos(2\,x)-1+\sin(2\,x)\,1\mathrm{i}\right)}{2\,\left(\cos(2\,x)\,1\mathrm{i}-\sin(2\,x)+1\mathrm{i}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a - a*sin(x)^2)^(1/2),x)
[Out] (2^(1/2)*a^(1/2)*(cos(2*x) + 1)^(1/2)*(cos(2*x) + sin(2*x)*1i - 1))/(2*(cos
(2*x)*1i - sin(2*x) + 1i))
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\sin^2(x)+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(x)**2)**(1/2),x)
[Out] Integral(sqrt(-a*sin(x)**2 + a), x)
```

3.119 $\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx$

Optimal. Leaf size=16

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

```
[Out] arctanh(sin(x))*cos(x)/(a*cos(x)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 16, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 3770}

$$\frac{\cos(x)\tanh^{-1}(\sin(x))}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/Sqrt[a*Cos[x]^2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a-a\sin^2(x)}}\,dx &= \int \frac{1}{\sqrt{a\cos^2(x)}}\,dx\\ &= \frac{\cos(x)\int \sec(x)\,dx}{\sqrt{a\cos^2(x)}}\\ &= \frac{\tanh^{-1}(\sin(x))\cos(x)}{\sqrt{a\cos^2(x)}}\end{aligned}$$

Mathematica [B] time = 0.02, size = 46, normalized size = 2.88

$$\frac{\cos(x)\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a - a*Sin[x]^2],x]
```

```
[Out] (Cos[x]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]]))/Sqrt[a*Cos[
x]^2]
```

fricas [B] time = 0.43, size = 65, normalized size = 4.06

$$\left[-\frac{\sqrt{a\cos(x)^2}\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)}{2\,a\cos(x)}, -\frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos(x)^2}\sqrt{-a}\sin(x)}{a\cos(x)}\right)}{a}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(x)^2)*log(-(sin(x) - 1)/(sin(x) + 1))/(a*cos(x)), -sqrt(-a
)*arctan(sqrt(a*cos(x)^2)*sqrt(-a)*sin(x)/(a*cos(x)))/a]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sin(x)^2 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(-a*sin(x)^2 + a), x)
```

maple [C] time = 0.12, size = 20, normalized size = 1.25

$$\frac{\cos(x)\mathrm{am}^{-1}(x|1)}{\sqrt{a\left(\cos^2(x)\right)}\,\mathrm{csgn}\left(\cos(x)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(1/2),x)
```

```
[Out] 1/(a*cos(x)^2)^(1/2)/csgn(cos(x))*cos(x)*InverseJacobiAM(x,1)
```

maxima [B] time = 0.51, size = 38, normalized size = 2.38

$$\frac{\log\left(\cos(x)^2 + \sin(x)^2 + 2\sin(x) + 1\right) - \log\left(\cos(x)^2 + \sin(x)^2 - 2\sin(x) + 1\right)}{2\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - log(cos(x)^2 + sin(x)^2 - 2*
sin(x) + 1))/sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.06

$$\int \frac{1}{\sqrt{a - a\sin(x)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(1/2),x)
```

```
[Out] int(1/(a - a*sin(x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{-a\sin^2(x)+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(-a*sin(x)**2 + a), x)
```

3.120 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx$

Optimal. Leaf size=42

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

```
[Out] 1/2*arctanh(sin(x))*cos(x)/a/(a*cos(x)^2)^(1/2)+1/2*tan(x)/a/(a*cos(x)^2)^(
1/2)
```

Rubi [A] time = 0.04, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3204, 3207, 3770}

$$\frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}+\frac{\cos(x)\tanh^{-1}(\sin(x))}{2a\sqrt{a\cos^2(x)}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-3/2),x]
```

```
[Out] (ArcTanh[Sin[x]]*Cos[x])/(2*a*Sqrt[a*Cos[x]^2]) + Tan[x]/(2*a*Sqrt[a*Cos[x]
^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{1}{\left(a-a\sin^2(x)\right)^{3/2}}\,dx &= \int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx \\
&= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{2a} \\
&= \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\cos(x)\int \sec(x)\,dx}{2a\sqrt{a\cos^2(x)}} \\
&= \frac{\tanh^{-1}(\sin(x))\cos(x)}{2a\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{2a\sqrt{a\cos^2(x)}}
\end{aligned}
$$

Mathematica [B] time = 0.07, size = 91, normalized size = 2.17

$$
-\frac{\cos(x)\left(-2\sin(x)+\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\cos(2x)\left(\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)-\log\right.}{4\left(a\cos^2(x)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-3/2),x]
```

```
[Out] -1/4*(Cos[x]*(Log[Cos[x/2] - Sin[x/2]] + Cos[2*x]*(Log[Cos[x/2] - Sin[x/2]]
 - Log[Cos[x/2] + Sin[x/2]]) - Log[Cos[x/2] + Sin[x/2]] - 2*Sin[x]))/(a*Cos
[x]^2)^(3/2)
```

fricas [A] time = 0.43, size = 40, normalized size = 0.95

$$
-\frac{\sqrt{a\cos(x)^2}\left(\cos(x)^2\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\,\sin(x)\right)}{4\,a^2\cos(x)^3}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(x)^2)*(cos(x)^2*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*sin(x))
/(a^2*cos(x)^3)
```

giac [A] time = 0.23, size = 44, normalized size = 1.05

$$
-\frac{\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)}{\left(\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2-4\right)a^{\frac{3}{2}}\operatorname{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(1/tan(1/2*x) + tan(1/2*x))/(((1/tan(1/2*x) + tan(1/2*x))^2 - 4)*a^(3/2)*s
gn(tan(1/2*x)^4 - 1))
```

maple [B] time = 1.34, size = 70, normalized size = 1.67

$$
\frac{\sqrt{a\left(\sin^2(x)\right)}\left(\ln\left(\frac{2\sqrt{a}\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)\left(\cos^2(x)\right)a+\sqrt{a}\sqrt{a\left(\sin^2(x)\right)}\right)}{2a^{\frac{5}{2}}\cos(x)\sin(x)\sqrt{a\left(\cos^2(x)\right)}}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(3/2),x)
```

```
[Out] 1/2/a^(5/2)/cos(x)*(a*sin(x)^2)^(1/2)*(ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2)^(1
/2)+a))*cos(x)^2*a+a^(1/2)*(a*sin(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

maxima [B] time = 0.54, size = 304, normalized size = 7.24

$$\frac{4\,(\sin(3\,x) - \sin(x))\cos(4\,x) + \left(2\,(2\,\cos(2\,x) + 1)\cos(4\,x) + \cos(4\,x)^2 + 4\,\cos(2\,x)^2 + \sin(4\,x)^2 + 4\,\sin(4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*x) - sin(x))*cos(4*x) + (2*(2*cos(2*x) + 1)*cos(4*x) + cos(4*
x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4*sin(2*x)^2 + 4*c
os(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1) - (2*(2*cos(2*x) + 1)*
cos(4*x) + cos(4*x)^2 + 4*cos(2*x)^2 + sin(4*x)^2 + 4*sin(4*x)*sin(2*x) + 4
*sin(2*x)^2 + 4*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(
cos(3*x) - cos(x))*sin(4*x) + 4*(2*cos(2*x) + 1)*sin(3*x) - 8*cos(3*x)*sin(
2*x) + 8*cos(x)*sin(2*x) - 8*cos(2*x)*sin(x) - 4*sin(x))/((a*cos(4*x)^2 + 4
*a*cos(2*x)^2 + a*sin(4*x)^2 + 4*a*sin(4*x)*sin(2*x) + 4*a*sin(2*x)^2 + 2*(
2*a*cos(2*x) + a)*cos(4*x) + 4*a*cos(2*x) + a)*sqrt(a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a - a\sin(x)^2\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(3/2),x)
```

```
[Out] int(1/(a - a*sin(x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\sin^2(x) + a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(3/2),x)
```

```
[Out] Integral((-a*sin(x)**2 + a)**(-3/2), x)
```

3.121 $\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx$

Optimal. Leaf size=61

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

```
[Out] 3/8*arctanh(sin(x))*cos(x)/a^2/(a*cos(x)^2)^(1/2)+1/4*tan(x)/a/(a*cos(x)^2)
^(3/2)+3/8*tan(x)/a^2/(a*cos(x)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3176, 3204, 3207, 3770}

$$\frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}+\frac{3\cos(x)\tanh^{-1}(\sin(x))}{8a^2\sqrt{a\cos^2(x)}}+\frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - a*Sin[x]^2)^(-5/2),x]
```

```
[Out] (3*ArcTanh[Sin[x]]*Cos[x])/(8*a^2*Sqrt[a*Cos[x]^2]) + Tan[x]/(4*a*(a*Cos[x]
^2)^(3/2)) + (3*Tan[x])/(8*a^2*Sqrt[a*Cos[x]^2])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3204

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Simp[(Cot[e + f*x]
*(b*Sin[e + f*x]^2)^(p + 1))/(b*f*(2*p + 1)), x] + Dist[(2*(p + 1))/(b*(2*p
 + 1)), Int[(b*Sin[e + f*x]^2)^(p + 1), x], x] /; FreeQ[{b, e, f}, x] &&  !
IntegerQ[p] && LtQ[p, -1]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{1}{\left(a-a\sin^2(x)\right)^{5/2}}\,dx = \int \frac{1}{\left(a\cos^2(x)\right)^{5/2}}\,dx$$
$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\int \frac{1}{\left(a\cos^2(x)\right)^{3/2}}\,dx}{4a}$$
$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{3\int \frac{1}{\sqrt{a\cos^2(x)}}\,dx}{8a^2}$$
$$= \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{(3\cos(x))\int \sec(x)\,dx}{8a^2\sqrt{a\cos^2(x)}}$$
$$= \frac{3\tanh^{-1}(\sin(x))\cos(x)}{8a^2\sqrt{a\cos^2(x)}} + \frac{\tan(x)}{4a\left(a\cos^2(x)\right)^{3/2}} + \frac{3\tan(x)}{8a^2\sqrt{a\cos^2(x)}}$$

Mathematica [A] time = 0.16, size = 72, normalized size = 1.18

$$\frac{\cos^5(x)\left(\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)\right)}{16\left(a\cos^2(x)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - a*Sin[x]^2)^(-5/2),x]
```

```
[Out] (Cos[x]^5*(-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[
x]^4*(11*Sin[x] + 3*Sin[3*x]))/2))/(16*(a*Cos[x]^2)^(5/2))
```

fricas [A] time = 0.42, size = 49, normalized size = 0.80

$$-\frac{\left(3\cos(x)^4\log\left(-\frac{\sin(x)-1}{\sin(x)+1}\right)-2\left(3\cos(x)^2+2\right)\sin(x)\right)\sqrt{a\cos(x)^2}}{16\,a^3\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(x)^4*log(-(sin(x) - 1)/(sin(x) + 1)) - 2*(3*cos(x)^2 + 2)*sin(
x))*sqrt(a*cos(x)^2)/(a^3*cos(x)^5)
```

giac [B] time = 0.27, size = 129, normalized size = 2.11

$$-\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)+2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}+\frac{3\log\left(\left|\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)-2\right|\right)}{16\,a^{\frac{5}{2}}\mathrm{sgn}\left(\tan\left(\frac{1}{2}x\right)^4-1\right)}-\frac{5\sqrt{a}\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^3-12\sqrt{a}\left(\cdots\right.}{4\left(\left(\frac{1}{\tan\left(\frac{1}{2}x\right)}+\tan\left(\frac{1}{2}x\right)\right)^2-4\right)^2a^3\mathrm{sgn}\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) + 2))/(a^(5/2)*sgn(tan(1/2*x)^4 - 1
)) + 3/16*log(abs(1/tan(1/2*x) + tan(1/2*x) - 2))/(a^(5/2)*sgn(tan(1/2*x)^4
```

```
 - 1)) - 1/4*(5*sqrt(a)*(1/tan(1/2*x) + tan(1/2*x))^3 - 12*sqrt(a)*(1/tan(1
/2*x) + tan(1/2*x)))/(((1/tan(1/2*x) + tan(1/2*x))^2 - 4)^2*a^3*sgn(tan(1/2
*x)^4 - 1))
```

maple [A] time = 1.39, size = 89, normalized size = 1.46

$$\frac{\sqrt{a\left(\sin^2(x)\right)}\left(3\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}+2a}{\cos(x)}\right)a\left(\cos^4(x)\right)+3\sqrt{a\left(\sin^2(x)\right)}\,\left(\cos^2(x)\right)\sqrt{a}\,+2\sqrt{a}\,\sqrt{a\left(\sin^2(x)\right)}\right)}{8a^{\frac{7}{2}}\cos(x)^3\sin(x)\sqrt{a\left(\cos^2(x)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-a*sin(x)^2)^(5/2),x)
```

```
[Out] 1/8/a^(7/2)/cos(x)^3*(a*sin(x)^2)^(1/2)*(3*ln(2/cos(x)*(a^(1/2)*(a*sin(x)^2
)^(1/2)+a))*a*cos(x)^4+3*(a*sin(x)^2)^(1/2)*cos(x)^2*a^(1/2)+2*a^(1/2)*(a*s
in(x)^2)^(1/2))/sin(x)/(a*cos(x)^2)^(1/2)
```

maxima [B] time = 0.71, size = 933, normalized size = 15.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/16*(4*(3*sin(7*x) + 11*sin(5*x) - 11*sin(3*x) - 3*sin(x))*cos(8*x) - 24*(
2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*cos(7*x) + 16*(11*sin(5*x) - 11*sin(3
*x) - 3*sin(x))*cos(6*x) - 88*(3*sin(4*x) + 2*sin(2*x))*cos(5*x) - 24*(11*s
in(3*x) + 3*sin(x))*cos(4*x) + 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) +
 1)*cos(8*x) + cos(8*x)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*c
os(6*x)^2 + 12*(4*cos(2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 +
4*(2*sin(6*x) + 3*sin(4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(
4*x) + 2*sin(2*x))*sin(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*s
in(2*x) + 16*sin(2*x)^2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 + 2*sin(x
) + 1) - 3*(2*(4*cos(6*x) + 6*cos(4*x) + 4*cos(2*x) + 1)*cos(8*x) + cos(8*x
)^2 + 8*(6*cos(4*x) + 4*cos(2*x) + 1)*cos(6*x) + 16*cos(6*x)^2 + 12*(4*cos(
2*x) + 1)*cos(4*x) + 36*cos(4*x)^2 + 16*cos(2*x)^2 + 4*(2*sin(6*x) + 3*sin(
4*x) + 2*sin(2*x))*sin(8*x) + sin(8*x)^2 + 16*(3*sin(4*x) + 2*sin(2*x))*sin
(6*x) + 16*sin(6*x)^2 + 36*sin(4*x)^2 + 48*sin(4*x)*sin(2*x) + 16*sin(2*x)^
2 + 8*cos(2*x) + 1)*log(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1) - 4*(3*cos(7*x)
 + 11*cos(5*x) - 11*cos(3*x) - 3*cos(x))*sin(8*x) + 12*(4*cos(6*x) + 6*cos(
4*x) + 4*cos(2*x) + 1)*sin(7*x) - 16*(11*cos(5*x) - 11*cos(3*x) - 3*cos(x))
*sin(6*x) + 44*(6*cos(4*x) + 4*cos(2*x) + 1)*sin(5*x) + 24*(11*cos(3*x) + 3
*cos(x))*sin(4*x) - 44*(4*cos(2*x) + 1)*sin(3*x) + 176*cos(3*x)*sin(2*x) +
48*cos(x)*sin(2*x) - 48*cos(2*x)*sin(x) - 12*sin(x))/((a^2*cos(8*x)^2 + 16*
a^2*cos(6*x)^2 + 36*a^2*cos(4*x)^2 + 16*a^2*cos(2*x)^2 + a^2*sin(8*x)^2 + 1
6*a^2*sin(6*x)^2 + 36*a^2*sin(4*x)^2 + 48*a^2*sin(4*x)*sin(2*x) + 16*a^2*si
n(2*x)^2 + 8*a^2*cos(2*x) + a^2 + 2*(4*a^2*cos(6*x) + 6*a^2*cos(4*x) + 4*a^
2*cos(2*x) + a^2)*cos(8*x) + 8*(6*a^2*cos(4*x) + 4*a^2*cos(2*x) + a^2)*cos(
6*x) + 12*(4*a^2*cos(2*x) + a^2)*cos(4*x) + 4*(2*a^2*sin(6*x) + 3*a^2*sin(4
*x) + 2*a^2*sin(2*x))*sin(8*x) + 16*(3*a^2*sin(4*x) + 2*a^2*sin(2*x))*sin(6
*x))*sqrt(a))
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\left(a-a\sin(x)^2\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - a*sin(x)^2)^(5/2),x)
```

```
[Out] int(1/(a - a*sin(x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(-a\sin^2\left(x\right)+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-a*sin(x)**2)**(5/2),x)
```

```
[Out] Integral((-a*sin(x)**2 + a)**(-5/2), x)
```

3.122 $\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=125

$$\frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4bf}+\frac{(a-3b)\cos(e+fx)\sqrt{a-b\cos}}{8bf}$$

```
[Out] 1/8*(a-3*b)*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(
3/2)/f-1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f+1/8*(a-3*b)*cos(f*x+e)
*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.13, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 388, 195, 217, 203}

$$\frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4bf}+\frac{(a-3b)\cos(e+fx)\sqrt{a-b\cos}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - 3*b)*(a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x
]^2]])/(8*b^(3/2)*f) + ((a - 3*b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^
2])/(8*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
```

```
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a-3b)\,\text{Subst}\left(\int \sqrt{a+b}\right.}{4}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\left(a+b-\right.}{4bf}$$

$$= \frac{(a-3b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8bf} - \frac{\cos(e+fx)\left(a+b-\right.}{4bf}$$

$$= \frac{(a-3b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a-3b)\cos(e+fx)\sqrt{a+}}{8bf}$$

Mathematica [A] time = 0.43, size = 119, normalized size = 0.95

$$\frac{\frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\,(-a+b\cos(2(e+fx))-4b)}{\sqrt{2}b} + \frac{(a+b)(3b-a)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{(-b)^{3/2}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(-a - 4*b + b*Cos[2*(e +
f*x)]))/(Sqrt[2]*b) + ((a + b)*(-a + 3*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x]
 + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(8*f)
```

fricas [A] time = 0.81, size = 501, normalized size = 4.01

$$\left[\frac{\left(a^2-2ab-3b^2\right)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2ab^3+b^4\right)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 - 2*a*b - 3*b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b
^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a
^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3
+ b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x
 + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*b
^2*cos(f*x + e)^3 - (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b))/(b^2*f), -1/32*((a^2 - 2*a*b - 3*b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f
*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x
```

```
+ e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*(2*b^2*cos(f*x + e)^3 -
 (a*b + 5*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
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nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
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 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)-2*(-8*b^2*f^6/64/b^2/f^4*cos(f*x+exp(1))/f*cos(f*x+exp(1))/f+(20*b^
2*f^4+4*b*f^4*a)/64/b^2/f^4)*cos(f*x+exp(1))/f*sqrt(a-b*f^2*(-cos(f*x+exp(1
))/f)^2+b)+2*(a^2-2*a*b-3*b^2)/16/b/sqrt(-b)/abs(f)*ln(abs(sqrt(a-b*f^2*(-c
os(f*x+exp(1))/f)^2+b)+sqrt(-b*f^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.90, size = 311, normalized size = 2.49

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(-4b^{\frac{5}{2}}\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\left(\cos^2\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(-4*b^(5/2)*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2)*cos(f*x+e)^2+10*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)*b^(5/2)+2*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+ar
ctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2))*a^2*b-2*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+
```

```
e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*b^2-3*b^3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b
)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)/(
a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.46, size = 176, normalized size = 1.41

$$\frac{\frac{(a+b)a\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{(a+b)\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{4\,a\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-4\,\sqrt{b}\,\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)-4\,\sqrt{-b\cos\left(fx+e\right.}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*((a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + (a + b)*arc
sin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 4*a*arcsin(b*cos(f*x + e)/sqr
t((a + b)*b))/sqrt(b) - 4*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b)) -
4*sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e) - 2*(-b*cos(f*x + e)^2 + a +
 b)^(3/2)*cos(f*x + e)/b + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x
+ e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.123 $\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=78

$$-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

```
[Out] -1/2*(a+b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)-
1/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 203}

$$-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2
*Sqrt[b]*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-bx^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x,$$

$$= -\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{b}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+f$$

Mathematica [A] time = 0.26, size = 93, normalized size = 1.19

$$-\frac{\sqrt{2}\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b} + \frac{2(a+b)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/4*(Sqrt[2]*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + (2*(a + b)*
Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sq
rt[-b])/f
```

fricas [B] time = 0.55, size = 433, normalized size = 5.55

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)+(a+b)\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (a + b)*sqrt(-b)
*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b
^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos
(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*
cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-b)))/(b*f), 1/8*((a + b)*sqrt(b)*arctan(1/4*(8*b^
2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*c
os(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*cos(f*x + e))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
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/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check
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to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
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2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
ign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unab
le to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-
2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
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x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)-1/2*cos(f*x+exp(1))/f*sqrt(a-b*f^2*(-cos(f*x+exp(1))/f)^2+b)+2*(-a-
b)/4/sqrt(-b)/abs(f)*ln(abs(sqrt(a-b*f^2*(-cos(f*x+exp(1))/f)^2+b)+sqrt(-b*
f^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.33, size = 182, normalized size = 2.33

$$\frac{\sqrt{\left(\cos^2(fx+e)\right)\left(a+b\left(\sin^2(fx+e)\right)\right)}\left(b\arctan\left(\frac{-2b\left(\cos^2(fx+e)\right)+a+b}{2\sqrt{b}\,\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}}\right)+a\arctan\left(\frac{-2b(}{2\sqrt{b}\,\sqrt{-b(\cos^4}}\right.\right.}{4\sqrt{b}\,\cos(fx+e)\sqrt{a+b\left(\sin^2(fx+e\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b*arctan(1/2*(-2*b*cos(f*x+e)^
2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+a*arctan(1/2*(-2
*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*
b^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))/b^(1/2)/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.44, size = 70, normalized size = 0.90

$$-\frac{\frac{a\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)+\sqrt{-b\cos(fx+e)^2+a+b}\,\cos(fx+e)}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + sqrt(b)*arcsin(b*c
os(f*x + e)/sqrt((a + b)*b)) + sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)
)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\sin\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x), x)
```

3.124 $\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=83

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-arctan(co
s(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 402, 217, 203, 377, 206}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\operatorname{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$
$$= -\frac{a\operatorname{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f} - \frac{b\operatorname{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\right.}{f}$$
$$= -\frac{a\operatorname{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{b\operatorname{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \right.}{f}$$
$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.12, size = 99, normalized size = 1.19

$$\frac{\sqrt{-b}\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)-\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[a]*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e
 + f*x)]]]) + Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b
*Cos[2*(e + f*x)]]])/f
```

fricas [B] time = 0.65, size = 1158, normalized size = 13.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^
6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 2*sqrt(a)*log(2*((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b
)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8
*(4*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 16
0*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*
a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16
*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2
```

```
 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/f, 1/4*(sqrt(b)*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f
, 1/4*(2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)
)) + sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f
*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*
x + e))))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Warning, replacing 0 by ` u`, a substitution variable should
perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable sho
uld perhaps be purged.Warning, replacing 0 by ` u`, a substitution variable
 should perhaps be purged.Warning, replacing 0 by ` u`, a substitution vari
able should perhaps be purged.Warning, replacing 0 by ` u`, a substitution
variable should perhaps be purged.Warning, replacing 0 by ` u`, a substitut
ion variable should perhaps be purged.Warning, replacing 0 by ` u`, a subst
itution variable should perhaps be purged.Warning, replacing 0 by ` u`, a s
ubstitution variable should perhaps be purged.Warning, replacing 0 by ` u`,
 a substitution variable should perhaps be purged.Warning, integration of a
bs or sign assumes constant sign by intervals (correct if the argument is r
eal):Check [abs(t_nostep)]Warning, need to choose a branch for the root of
a polynomial with parameters. This might be wrong.Non regular value [0] was
 discarded and replaced randomly by 0=[63]Warning, need to choose a branch
for the root of a polynomial with parameters. This might be wrong.Non regul
ar value [0] was discarded and replaced randomly by 0=[-14]Warning, need to
 choose a branch for the root of a polynomial with parameters. This might b
e wrong.Non regular value [0] was discarded and replaced randomly by 0=[-42
]Warning, need to choose a branch for the root of a polynomial with paramet
ers. This might be wrong.Non regular value [0] was discarded and replaced r
andomly by 0=[9]Warning, need to choose a branch for the root of a polynomi
al with parameters. This might be wrong.Non regular value [0] was discarded
 and replaced randomly by 0=[65]Warning, need to choose a branch for the ro
ot of a polynomial with parameters. This might be wrong.Non regular value [
0] was discarded and replaced randomly by 0=[-56]Evaluation time: 0.7index.
cc index_m operator + Error: Bad Argument Value
```

maple [B] time = 2.13, size = 174, normalized size = 2.10

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(\sqrt{b}\ \arctan\left(\frac{-2b\left(\cos^2\left(fx+e\right)\right)+a+b}{2\sqrt{b}\ \sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}}\right)-\sqrt{a}\ \ln\left(\frac{-\left(a-b\right)\left(\cos^2\left(f\cdots\right.\right.}{\cdots}\right)\right)}{2\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(1/2)*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-a^(1/2)*ln
((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-
a-b)/(-1+cos(f*x+e)^2)))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.45, size = 132, normalized size = 1.59

$$-\frac{2\,\sqrt{b}\,\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{ab+b^2}}\right)+\sqrt{a}\,\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)-\sqrt{a}\,\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{\cos\left(fx+e\right)+1}\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(a)*log(b - sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e)
- 1)) - sqrt(a)*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x +
 e) + 1) + a/(cos(f*x + e) + 1)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\csc\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x), x)
```

3.125 $\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=84

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

```
[Out] -1/2*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)
-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 378, 377, 206}

$$-\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2\sqrt{a}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*Sqrt[a]*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{(1-x^2)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\,\text{Subst}\Big(}{}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{2f} - \frac{(a+b)\,\text{Subst}\Big(}{}$$

$$= -\frac{(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2\sqrt{a}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\cot(e}{2f}$$

Mathematica [A] time = 0.29, size = 100, normalized size = 1.19

$$\frac{-2(a+b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{4\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*Sqrt[a]*f)
```

fricas [A] time = 0.54, size = 338, normalized size = 4.02

$$\left[\frac{4\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a\cos\left(fx+e\right)+\left((a+b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+2\right.}{}\right.}{8\left(af\cos\left(fx+e\right)^2-af\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + ((a + b)*cos(f*x +
 e)^2 - a - b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a*f*cos(f*x + e)^2 - a*f), 1/4*(((a +
b)*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a
 + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 +
 a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(a
*f*cos(f*x + e)^2 - a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Unable to divide, perhaps due to roun
ding error%%%{16,[4,4]%%%}+%%%{%%%{32,[1]%%%},[4,3]%%%}+%%%{%%%{16,[2]%%%},
[4,2]%%%}+%%%{%%%{-32,[1]%%%},[2,4]%%%}+%%%{%%%{-64,[2]%%%},[2,3]%%%}+%%%{%
%%{-32,[3]%%%},[2,2]%%%}+%%%{%%%{16,[2]%%%},[0,4]%%%}+%%%{%%%{32,[3]%%%},[0
,3]%%%}+%%%{%%%{16,[4]%%%},[0,2]%%%} / %%%{%%%{1,[1]%%%},[4,0]%%%}+%%%{%%%{
-2,[2]%%%},[2,0]%%%}+%%%{%%%{1,[3]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 1.79, size = 227, normalized size = 2.70

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(a\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}+a+b}{\sin\left(fx+e\right)^2}\right)\left(\sin^2\left(f\right.\right.}{4\sqrt{a}\sin\left(fx+e\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(a*ln(((a-b)*cos(f*x+e)^2+2*a^
(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x
+e)^2+b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2+2*a^(1/2)*(cos(f*x+e)^2*(a+b*sin(f
*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+fx\right)^2+a}}{\sin\left(e+fx\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\csc^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**3, x)
```

3.126 $\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4af}-\frac{(3a-b)\cot(}{}$$

```
[Out] -1/8*(3*a-b)*(a+b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a
^(3/2)/f-1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/a/f-1/8*(3*
a-b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.13, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8a^{3/2}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4af}-\frac{(3a-b)\cot(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((3*a - b)*(a + b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(8*a^(3/2)*f) - ((3*a - b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e +
f*x]*Csc[e + f*x])/(8*a*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]
*Csc[e + f*x]^3)/(4*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-bx^2}}{\left(1-x^2\right)^3}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4af} - \frac{(3a-b)\,\text{Subst}\Big(}{}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8af} - \frac{\big(a+b-b}{}$$

$$= -\frac{(3a-b)\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{8af} - \frac{\big(a+b-b}{}$$

$$= -\frac{(3a-b)(a+b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8a^{3/2}f} - \frac{(3a-b)\sqrt{a+b-b\cos^2(e}{}$$

Mathematica [A] time = 0.51, size = 127, normalized size = 0.89

$$\frac{\left(-6a^2-4ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)-\sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\,\big(2a}{16a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((-6*a^2 - 4*a*b + 2*b^2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x
)]]*Cot[e + f*x]*Csc[e + f*x]*(3*a + b + 2*a*Csc[e + f*x]^2))/(16*a^(3/2)*f
)
```

fricas [A] time = 0.73, size = 520, normalized size = 3.64

$$\left[-\frac{\left(\left(3a^2+2ab-b^2\right)\cos\left(fx+e\right)^4-2\left(3a^2+2ab-b^2\right)\cos\left(fx+e\right)^2+3a^2+2ab-b^2\right)\sqrt{a}\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(((3*a^2 + 2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos
(f*x + e)^2 + 3*a^2 + 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f
*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e
```

```
)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((3*a^2 + a*b)*c
os(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f), 1/16*(((3*a^2 +
2*a*b - b^2)*cos(f*x + e)^4 - 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 3*a^
2 + 2*a*b - b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(
f*x + e))) + 2*((3*a^2 + a*b)*cos(f*x + e)^3 - (5*a^2 + a*b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x +
e)^2 + a^2*f)]
```

giac [B] time = 0.55, size = 962, normalized size = 6.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2 + (7*a + 2*b)/a) + 8*(3*a^2
 + 2*a*b - b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)/sqrt(-a))/(sqrt(-a)*a) - 4*(3*a^(5/2) + 2*a^(3/2)*b - sqrt(a)*b^2)*log(ab
s(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqr
t(a)*b))/a^2 + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*
a^2 + 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 2*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*a^3 - 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))*a^2*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b
^2 - 3*a^(7/2))/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 -
a)^2*a))/f
```

maple [B] time = 2.20, size = 379, normalized size = 2.65

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(3a^3\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}+a+b}{\sin\left(fx+e\right)^2}\right)\left(\mathrm{si}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^3*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+2*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4*a^2-ln(((a-b)*cos(f*x+e)^2+
2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*b^2
```

```
*sin(f*x+e)^4*a+6*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(5
/2)+2*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^2*a^(3/2)+4*(cos
(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(5/2))/sin(f*x+e)^4/a^(5/2)/cos(f*x+e
)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin \left(fx+e\right)^{2}+a} \csc \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \sin \left(e+f x\right)^{2}+a}}{\sin \left(e+f x\right)^{5}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^5, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b \sin^{2}\left(e+fx\right)} \csc^{5}\left(e+fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**5, x)
```

3.127 $\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=259

$$-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a-2b)(a\cdots}{\cdots}$$

```
[Out] -1/15*(a+4*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/5*cos(f*
x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(2*a^2-3*a*b-8*b^2)*Ellip
ticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+
e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a-2*b)*(a+b)*EllipticF
(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2
/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.32, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 478, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2-3ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a-2b)(a\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
- (Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((2*a^2
- 3*a*b - 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e + f
*x]^2)/a]) + (2*a*(a - 2*b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[S
in[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b^2*
f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 478

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*(m + n*(p + q) + 1)), x] - Dist[e^n/(b*(m + n*(p + q) +
1)), Int[(e*x)^(m - n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[a*c*(m - n +
1) + (a*d*(m - n + 1) - n*q*(b*c - a*d))*x^n, x], x], x] /; FreeQ[{a, b, c,
 d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && GtQ[m - n
+ 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{\sqrt{1-x^2}}dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\text{se}\right.}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)}{}$$

$$= -\frac{(a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx)}{}$$

Mathematica [A] time = 1.43, size = 199, normalized size = 0.77

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(8a^2-4b(4a+7b)\cos(2(e+fx))+48ab+3b^2\cos(4(e+fx))+25b^2\right)+32a\left(a^2-ab-\right.}{240b^2f\sqrt{2a-b\cos(2(e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 - 3*a*b - 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 - a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e
 + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 + 48*a*b + 25*b^
2 - 4*b*(4*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e + f
*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

maple [A] time = 1.55, size = 413, normalized size = 1.59

$$\frac{3b^3\left(\sin^7\left(fx+e\right)\right)+4a\,b^2\left(\sin^5\left(fx+e\right)\right)+b^3\left(\sin^5\left(fx+e\right)\right)+2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\;\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(3*b^3*sin(f*x+e)^7+4*a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+2*(cos(f*x+e
)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a^3-2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin
(f*x+e),(-1/a*b)^(1/2))*b-4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(
1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*
sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(
f*x+e),(-1/a*b)^(1/2))*a*b^2+a^2*b*sin(f*x+e)^3-4*b^3*sin(f*x+e)^3-a^2*b*si
n(f*x+e)-4*a*b^2*sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.128 $\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{a+b\sin^2(}{3bf\sqrt{\frac{b\sin}{}}}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(a+2*b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2
)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(
a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$-\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{a+b\sin^2(}{3bf\sqrt{\frac{b\sin}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((a + 2*b)*
EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int\frac{a+(a+2b)\sin^2(}{\sqrt{a+b\sin^2(e+}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a(a+b))\int\frac{1}{\sqrt{a+b\sin^2(e+}}}{3b}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left((a+2b)\sqrt{a+b\sin^2(e}\right.}{3b\sqrt{1}}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{1+}}$$

Mathematica [A] time = 0.83, size = 159, normalized size = 1.00

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+2\sqrt{2}\,a(a+2b)\sqrt{}}{6\sqrt{2}\,bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

maple [A] time = 1.45, size = 266, normalized size = 1.67

$$-\frac{-b^2\left(\sin^5\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2+a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
[Out] -1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-2*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+a*b*sin(f*x+e))/b/cos(f*x+
e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+fx\right)^2\,\sqrt{b\sin\left(e+fx\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\sin^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sin(e + f*x)**2, x)
```

3.129 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 0.87, size = 71, normalized size = 1.39

$$\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\;\text{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.130 $\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right| -}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))*s
ec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2
/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 475, 21, 423, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right| -}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Ellipt
icF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/
a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
```

```
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^2\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{S}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+\right.}{f\sqrt{}}$$

Mathematica [A] time = 0.61, size = 137, normalized size = 0.79

$$\frac{-\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)+2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))\cdot}{a}}}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*(a + b)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*f*Sqrt[2*a + b
- b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

maple [A] time = 1.32, size = 156, normalized size = 0.90

$$\frac{\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\left(\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+\text{EllipticF}\left(\sin\left(fx+e\right)\right.\right.}{\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin
(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+fx\right)^2+a}}{\sin\left(e+fx\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\csc^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**2, x)
```

3.131 $\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=234

$$-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(a+b)\sqrt{\cos^2(e+}}{}$$

```
[Out] -1/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+2/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(
cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3188, 475, 583, 524, 426, 424, 421, 419}

$$-\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(a+b)\sqrt{\cos^2(e+}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x
]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((2*a + b)*Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (2*(a + b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

```
]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x^4\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\text{se}\right.}{}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx}{3f}$$

$$= -\frac{(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot(e+fx)\csc^2(e+fx}{3f}$$

Mathematica [A] time = 3.19, size = 188, normalized size = 0.80

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(4\left(2a^2+4ab+b^2\right)\cos(2(e+fx))-(2a+b)(8a+b\cos(4(e+fx))+3b)\right)}{2\sqrt{2}} + 4a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((4*(2*a^2 + 4*a*b + b^2)*Cos[2*(e + f*x)] - (2*a + b)*(8*a + 3*b + b*Cos[
4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) - 2*a*(2*a + b)*Sqr
t[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 4*a*(a + b
)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a*f
*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

maple [A] time = 1.49, size = 342, normalized size = 1.46

$$\frac{2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\ \mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)+2b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*sin(f*x+e)^3-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6+b^2*
sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-2*a*b*sin
(f*x+e)^2-a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a}\,\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/sin(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e+fx\right)}\,\csc^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*csc(e + f*x)**4, x)
```

3.132 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=169

$$\frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6bf}+\frac{(a-5b)\cos(e+fx)\left(a-b\right.}{24bf}$$

```
[Out] 1/16*(a-5*b)*(a+b)^2*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/
b^(3/2)/f+1/24*(a-5*b)*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/b/f-1/6*cos(f*
x+e)*(a+b-b*cos(f*x+e)^2)^(5/2)/b/f+1/16*(a-5*b)*(a+b)*cos(f*x+e)*(a+b-b*co
s(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.15, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 388, 195, 217, 203}

$$\frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16b^{3/2}f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6bf}+\frac{(a-5b)\cos(e+fx)\left(a-b\right.}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - 5*b)*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f
*x]^2]])/(16*b^(3/2)*f) + ((a - 5*b)*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Co
s[e + f*x]^2])/(16*b*f) + ((a - 5*b)*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2
)^(3/2))/(24*b*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{5/2}}{6bf} + \frac{(a-5b)\,\text{Subst}\left(\int\left(a+b\right.\right.}{6}$$

$$= \frac{(a-5b)\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{24bf} - \frac{\cos(e+fx)\left(a+b\right.}{6}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf} + \frac{(a-5b)\cos(e}{}$$

$$= \frac{(a-5b)(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{16bf} + \frac{(a-5b)\cos(e}{}$$

$$= \frac{(a-5b)(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{(a-5b)(a+b)\cos(e+f}{16}$$

Mathematica [A] time = 0.78, size = 152, normalized size = 0.90

$$\frac{\frac{(a+b)^2(5b-a)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{(-b)^{3/2}} - \frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(3a^2-b(7a+9b)\cos(2(e+fx))+29ab+b^2\cos(4(e+\right.}{3\sqrt{2}b}}{16f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/3*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(3*a^2 + 29*a*b + 23
*b^2 - b*(7*a + 9*b)*Cos[2*(e + f*x)] + b^2*Cos[4*(e + f*x)]))/(Sqrt[2]*b)
+ ((a + b)^2*(-a + 5*b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b -
b*Cos[2*(e + f*x)]]])/(-b)^(3/2))/(16*f)
```

fricas [A] time = 1.95, size = 579, normalized size = 3.43

$$\left[\frac{3\left(a^3-3a^2b-9ab^2-5b^3\right)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2ab^3+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(-b)*log(128*b^4*cos(f*x +
e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos
(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2
```

```
 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqr
t(-b)) - 8*(8*b^3*cos(f*x + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*
(a^2*b + 12*a*b^2 + 11*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/
(b^2*f), -1/192*(3*(a^3 - 3*a^2*b - 9*a*b^2 - 5*b^3)*sqrt(b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)
*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(8*b^3*cos(f*x
 + e)^5 - 2*(7*a*b^2 + 13*b^3)*cos(f*x + e)^3 + 3*(a^2*b + 12*a*b^2 + 11*b^
3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
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i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to ch
eck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2
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2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check s
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2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign:
(2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
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x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to chec
k sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)U
nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
n: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable
 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)-2*((192*b^5*f^12/2304/b^4/f^8*cos(f*x+exp(1))/f*cos(f*x+exp(1))/f+(
-624*b^5*f^10-336*b^4*f^10*a)/2304/b^4/f^8)*cos(f*x+exp(1))/f*cos(f*x+exp(1
))/f+(792*b^5*f^8+864*b^4*f^8*a+72*b^3*f^8*a^2)/2304/b^4/f^8)*cos(f*x+exp(1
))/f*sqrt(a-b*f^2*(-cos(f*x+exp(1))/f)^2+b)+2*(a^3-3*a^2*b-9*a*b^2-5*b^3)/3
2/b/sqrt(-b)/abs(f)*ln(abs(sqrt(a-b*f^2*(-cos(f*x+exp(1))/f)^2+b)+sqrt(-b*f
^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.82, size = 446, normalized size = 2.64

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(16\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b^{\frac{7}{2}}\left(\cos^4\left(fx+e\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(16*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)*b^(7/2)*cos(f*x+e)^4-4*b^(5/2)*(-b*cos(f*x+e)^4+(a+b)*cos
(f*x+e)^2)^(1/2)*(13*b+7*a)*cos(f*x+e)^2+66*b^(7/2)*(-b*cos(f*x+e)^4+(a+b)*
cos(f*x+e)^2)^(1/2)+72*a*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(5/2)
+6*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(3/2)+3*arctan(1/2*(-2*
b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))*a^3
*b-9*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2))*a^2*b^2-27*b^3*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2
)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-15*b^4*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/c
os(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.45, size = 248, normalized size = 1.47

$$\frac{\frac{3\,(a+b)^2 a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}} + \frac{3\,(a+b)^2 \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - \frac{18\,(a+b)a \arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}} - 18\,(a+b)\sqrt{b}\arcsin\left(\frac{b\cos(fx+e)}{\sqrt{(a+b)b}}\right) - 12\Big(}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(a + b)^2*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + 3*(a +
 b)^2*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*a*arcsin(
b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) - 18*(a + b)*sqrt(b)*arcsin(b*cos(f
*x + e)/sqrt((a + b)*b)) - 12*(-b*cos(f*x + e)^2 + a + b)^(3/2)*cos(f*x + e
) - 18*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*x + e) - 8*(-b*cos(f*x
 + e)^2 + a + b)^(5/2)*cos(f*x + e)/b + 2*(-b*cos(f*x + e)^2 + a + b)^(3/2)
*(a + b)*cos(f*x + e)/b + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2*cos(f
*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.133 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=114

$$-\frac{3(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{a-b\cdots}}\right.}{8\sqrt{b}f}$$

```
[Out] -1/4*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)/f-3/8*(a+b)^2*arctan(cos(f*x+e)*
b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)-3/8*(a+b)*cos(f*x+e)*(a+b-b*c
os(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 195, 217, 203}

$$-\frac{3(a+b)\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{8f}-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cdots}{\sqrt{a-b\cdots}}\right.}{8\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]
)/(8*Sqrt[b]*f) - (3*(a + b)*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(
8*f) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(3/2))/(4*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \left(a+b-bx^2\right)^{3/2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{3/2}}{4f} - \frac{(3(a+b))\,\text{Subst}\left(\int \sqrt{a+\ldots}\right.}{4\ldots}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)\left(a+b-\ldots\right.}{4f\ldots}$$

$$= -\frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{8f} - \frac{\cos(e+fx)\left(a+b-\ldots\right.}{4f\ldots}$$

$$= -\frac{3(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{b}f} - \frac{3(a+b)\cos(e+fx)\sqrt{a+b-b\ldots}}{8f\ldots}$$

Mathematica [A] time = 0.42, size = 113, normalized size = 0.99

$$-\frac{\frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\,(5a-b\cos(2(e+fx))+4b)}{\sqrt{2}}+\frac{3(a+b)^2\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*((Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(5*a + 4*b - b*Cos[2
*(e + f*x)]))/Sqrt[2] + (3*(a + b)^2*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/f
```

fricas [A] time = 0.82, size = 495, normalized size = 4.34

$$\left[-\frac{3\left(a^2+2ab+b^2\right)\sqrt{-b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2ab^3+b^4\right)\cos\left(\ldots\right.\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/64*(3*(a^2 + 2*a*b + b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*
b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*
x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a
*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) - 8*(2*
b^2*cos(f*x + e)^3 - 5*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
 + b))/(b*f), 1/32*(3*(a^2 + 2*a*b + b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x
 + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x +
e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*(2*b^2*cos(f*x + e)^3 - 5
*(a*b + b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
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to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*p
i/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*
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nable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)
>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sig
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 to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)-2*(-4*b^3*f^6/32/b^2/f^4*cos(f*x+exp(1))/f*cos(f*x+exp(1))/f+(10*b^
3*f^4+10*b^2*f^4*a)/32/b^2/f^4)*cos(f*x+exp(1))/f*sqrt(a-b*f^2*(-cos(f*x+ex
p(1))/f)^2+b)+2*(-3*a^2-6*a*b-3*b^2)/16/sqrt(-b)/abs(f)*ln(abs(sqrt(a-b*f^2
*(-cos(f*x+exp(1))/f)^2+b)+sqrt(-b*f^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.68, size = 309, normalized size = 2.71

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\,\left(4\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b^{\frac{3}{2}}\left(\cos^2\left(fx+e\right)\right)\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(4*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)*b^(3/2)*cos(f*x+e)^2-10*b^(3/2)*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)+3*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2))*a^2+6*b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/
b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+3*b^2*arctan(1/2*(-2*b*
cos(f*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-10*a*
(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^(1/2))/b^(1/2)/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.43, size = 106, normalized size = 0.93

$$-\frac{\frac{3\,(a+b)a\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}+3\,(a+b)\sqrt{b}\,\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)+2\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}\cos\left(fx+e\right)+3\sqrt{-b\cos}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*(a + b)*a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/sqrt(b) + 3*(a + b
)*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt((a + b)*b)) + 2*(-b*cos(f*x + e)^2 + a
 + b)^(3/2)*cos(f*x + e) + 3*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*cos(f*
x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.134 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}$$

```
[Out] -a^(3/2)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(3*a+
b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*b^(1/2)/f-1/2*b*co
s(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3186, 416, 523, 217, 203, 377, 206}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{b\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2f}-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*
x]^2]])/(2*f) - (a^(3/2)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[
e + f*x]^2]])/f - (b*Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=-\frac{\text{Subst}\left(\int\frac{\left(a+b-bx^2\right)^{3/2}}{1-x^2}\,dx,x,\cos(e+fx)\right)}{f}$$

$$=-\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}+\frac{\text{Subst}\left(\int\frac{-(a+b)(2a+b)+b(3a+b}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.}{2f}$$

$$=-\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{\left(1-x^2\right)\sqrt{a+b-bx^2}}\right.}{f}$$

$$=-\frac{b\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2f}-\frac{a^2\,\text{Subst}\left(\int\frac{1}{1-ax^2}\,dx,x,\frac{}{\sqrt{a+}}\right.}{f}$$

$$=-\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2f}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right.}{f}$$

Mathematica [A] time = 0.92, size = 141, normalized size = 1.16

$$-\frac{4a^{3/2}\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)+\sqrt{2}\,b\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}-2\sqrt{-b}\,(3a+b)\log\left(\sqrt{2a-b}\right.}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(4*a^(3/2)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos
[2*(e + f*x)]]] + Sqrt[2]*b*Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
 - 2*Sqrt[-b]*(3*a + b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b -
b*Cos[2*(e + f*x)]]])/f
```

fricas [B] time = 0.88, size = 1282, normalized size = 10.51

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (3*a + b)*sqrt(-
b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2
```

```
*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
+ b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*c
os(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3
)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)) - 4*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(
f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x +
e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2
+ 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/16*(8*sqrt(-a
)*a*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) - 8*sqrt(-b
*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) + (3*a + b)*sqrt(-b)*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 +
b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b
 + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 2
4*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3
- (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-b)))/f, 1/8*((3*a + b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b
*cos(f*x + e) + 2*a^(3/2)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*
a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos
(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(co
s(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/f, 1/8*(4*sqrt(-a)*a*arctan(-1/2*((a
 - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b
*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (3*a + b)*sqrt(b)*arctan(1/4
*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 +
b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) - 4*sqrt(-b*co
s(f*x + e)^2 + a + b)*b*cos(f*x + e))/f]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Error: Bad Argument Type
```

maple [B] time = 2.08, size = 255, normalized size = 2.09

$$\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(b^{\frac{3}{2}}\arctan\left(\frac{-2b\left(\cos^2(fx+e)\right)+a+b}{2\sqrt{b}\,\sqrt{-b\left(\cos^4(fx+e)\right)+(a+b)\left(\cos^2(fx+e)\right)}}\right)-2a^{\frac{3}{2}}\ln\left(\frac{-(a-b)\left(\cos^2\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(b^(3/2)*arctan(1/2*(-2*b*cos(f
*x+e)^2+a+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*a^(3/2)*
ln((-(a-b)*cos(f*x+e)^2-2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2
)-a-b)/(-1+cos(f*x+e)^2))+3*b^(1/2)*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^
(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))-2*b*(-b*cos(f*x+e)^4+(a+b
)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.48, size = 179, normalized size = 1.47

$$-\frac{3\,a\sqrt{b}\,\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{ab+b^2}}\right)+b^{\frac{3}{2}}\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{ab+b^2}}\right)+\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)+a^{\frac{3}{2}}\log\left(b-\frac{\sqrt{-}}{}\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*sqrt(b)*arcsin(b*cos(f*x + e)/sqrt(a*b + b^2)) + b^(3/2)*arcsin(b
*cos(f*x + e)/sqrt(a*b + b^2)) + sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x
+ e) + a^(3/2)*log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e
) - 1) - a/(cos(f*x + e) - 1)) - a^(3/2)*log(-b + sqrt(-b*cos(f*x + e)^2 +
a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1)))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.135 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e}{2f}$$

```
[Out] -b^(3/2)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f-1/2*(a+3*b
)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))*a^(1/2)/f-1/2*a*co
t(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.15, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3186, 413, 523, 217, 203, 377, 206}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2f}-\frac{a\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((b^(3/2)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/f
) - (Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e
+ f*x]^2]])/(2*f) - (a*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e +
f*x])/(2*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx=-\frac{\text{Subst}\left(\int\frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^2}dx,x,\cos(e+fx)\right)}{f}$$

$$=-\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f}+\frac{\text{Subst}\left(\int\frac{-(a+b}{(1-x}\right.}{}$$

$$=-\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f}-\frac{b^2\,\text{Subst}\left(\int\frac{}{\sqrt{a}}\right.}{}$$

$$=-\frac{a\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2f}-\frac{b^2\,\text{Subst}\left(\int\frac{}{1+}\right.}{}$$

$$=-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}-\frac{\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx}}\right.}{2f}$$

Mathematica [A] time = 1.22, size = 147, normalized size = 1.15

$$-\frac{4(-b)^{3/2}\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)+2\sqrt{a}\,(a+3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)+}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/4*(2*Sqrt[a]*(a + 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*a*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Co
t[e + f*x]*Csc[e + f*x] + 4*(-b)^(3/2)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] +
Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/f
```

fricas [B] time = 0.94, size = 1449, normalized size = 11.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(f*x + e)^2
- b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6
```

```
 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 -
8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + ((a + 3*b)*cos(f*x + e)^2 - a
- 3*b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b
 - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^
4 - 2*cos(f*x + e)^2 + 1)))/(f*cos(f*x + e)^2 - f), 1/8*(2*((a + 3*b)*cos(f
*x + e)^2 - a - 3*b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*
cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) + (b*cos(
f*x + e)^2 - b)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos
(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b
+ 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*
x + e)^2 - 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*
(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/(f*cos(f*x + e)^2 -
f), 1/8*(2*(b*cos(f*x + e)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4
- 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (
a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*a
*cos(f*x + e) + ((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(a)*log(2*((a^2 -
6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((
a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(
f*cos(f*x + e)^2 - f), 1/4*(((a + 3*b)*cos(f*x + e)^2 - a - 3*b)*sqrt(-a)*a
rctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) + (b*cos(f*x + e
)^2 - b)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*c
os(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*co
s(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e))/(f*cos(f*x
 + e)^2 - f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (4*pi/t_nostep/2)>(-4*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Error: Bad Argument Type
```

maple [B] time = 2.36, size = 287, normalized size = 2.24

$$\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(2b^{\frac{3}{2}}\arctan\left(\frac{2b\left(\sin^2\left(fx+e\right)\right)+a-b}{2\sqrt{b}\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}}\right)\left(\sin^2\left(fx+e\right)\right)-a^{\frac{3}{2}}\ln\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*arctan(1/2/b^(1/2)*(
2*b*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))*sin(f*x+e)^2
```

```
-a^(3/2)*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)
^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^2-3*a^(1/2)*b*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^2-2*a*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+e)^2/cos(f
*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \csc\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.136 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=128

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)\cot(e+f\cdots}{}$$

```
[Out] -1/4*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3/f-3/8*(a+b)^2*arcta
nh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)-3/8*(a+b)*cot(f
*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 378, 377, 206}

$$-\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{8\sqrt{a}f}-\frac{\cot(e+fx)\csc^3(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}{4f}-\frac{3(a+b)\cot(e+f\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]
])/(8*Sqrt[a]*f) - (3*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*C
sc[e + f*x])/(8*f) - ((a + b - b*Cos[e + f*x]^2)^(3/2)*Cot[e + f*x]*Csc[e +
 f*x]^3)/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2} dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^3} dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{4f} - \frac{(3(a+b))\,\text{Su}}{}$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f} - \frac{\left(a+b-\right.}{}$$

$$= -\frac{3(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{8f} - \frac{\left(a+b-\right.}{}$$

$$= -\frac{3(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{8\sqrt{a}f} - \frac{3(a+b)\sqrt{a+b-b\cos^2(e+}}{8}$$

Mathematica [A] time = 0.72, size = 114, normalized size = 0.89

$$-\frac{\frac{6(a+b)^2\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{a}} + \sqrt{2}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(2a\csc^2(e+fx)+3a+5\right.}{16f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/16*((6*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b
*Cos[2*(e + f*x)]]])/Sqrt[a] + Sqrt[2]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*C
ot[e + f*x]*Csc[e + f*x]*(3*a + 5*b + 2*a*Csc[e + f*x]^2))/f
```

fricas [A] time = 0.74, size = 484, normalized size = 3.78

$$\left[\frac{3\left(\left(a^2+2ab+b^2\right)\cos\left(fx+e\right)^4-2\left(a^2+2ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2ab+b^2\right)\sqrt{a}\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos\left(fx+\right.\right.}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*((a^2 + 2*a*b + b^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*
x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x +
e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 +
 (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*
b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) + 4*((3*a^2 + 5*a*b)*cos(
f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/16*(3*((a^2 + 2*a*b + b
^2)*cos(f*x + e)^4 - 2*(a^2 + 2*a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b
^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))) +
```

```
2*((3*a^2 + 5*a*b)*cos(f*x + e)^3 - 5*(a^2 + a*b)*cos(f*x + e))*sqrt(-b*cos
(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

giac [B] time = 0.70, size = 960, normalized size = 7.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a)*(a*tan(1/2*f*x + 1/2*e)^2 + (7*a^2 + 10*a*b)/a) + 2
4*(a^2 + 2*a*b + b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))/sqrt(-a))/sqrt(-a) - 12*(a^(5/2) + 2*a^(3/2)*b + sqrt(a)*b^2)*log(a
bs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sq
rt(a)*b))/a + 4*(4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 10*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 8*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*a^3 - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))*a^2*b - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*
b^2 - 3*a^(7/2) - 4*a^(5/2)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))^2 - a)^2)/f
```

maple [B] time = 1.95, size = 376, normalized size = 2.94

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(3a^2\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}+a+b}{\sin\left(fx+e\right)^2}\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(3*a^2*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*si
n(f*x+e)^4+6*a*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*co
s(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^4+3*b^2*ln(((a-b)*cos(f*x+e
)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)
*sin(f*x+e)^4+6*a^(3/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^
2+10*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*a^(1/2)*sin(f*x+e)^2+4*a^(3/
2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/a^(1/2)/sin(f*x+e)^4/cos(f*x+e)
/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+fx\right)^2+a\right)^{3/2}}{\sin\left(e+fx\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.137 $\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=197

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\cot(e+fx)\csc^5(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6af}-\frac{(5a-b)\cot}{}$$

```
[Out] -1/16*(5*a-b)*(a+b)^2*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2)
)/a^(3/2)/f-1/24*(5*a-b)*(a+b-b*cos(f*x+e)^2)^(3/2)*cot(f*x+e)*csc(f*x+e)^3
/a/f-1/6*(a+b-b*cos(f*x+e)^2)^(5/2)*cot(f*x+e)*csc(f*x+e)^5/a/f-1/16*(5*a-b
)*(a+b)*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.17, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 382, 378, 377, 206}

$$-\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{16a^{3/2}f}-\frac{\cot(e+fx)\csc^5(e+fx)\left(a-b\cos^2(e+fx)+b\right)^{5/2}}{6af}-\frac{(5a-b)\cot}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((5*a - b)*(a + b)^2*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e +
 f*x]^2]])/(16*a^(3/2)*f) - ((5*a - b)*(a + b)*Sqrt[a + b - b*Cos[e + f*x]^
2]*Cot[e + f*x]*Csc[e + f*x])/(16*a*f) - ((5*a - b)*(a + b - b*Cos[e + f*x]
^2)^(3/2)*Cot[e + f*x]*Csc[e + f*x]^3)/(24*a*f) - ((a + b - b*Cos[e + f*x]^
2)^(5/2)*Cot[e + f*x]*Csc[e + f*x]^5)/(6*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^{3/2}}{\left(1-x^2\right)^4}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(a+b-b\cos^2(e+fx)\right)^{5/2}\cot(e+fx)\csc^5(e+fx)}{6af} - \frac{(5a-b)\,\text{Subs}}{}$$

$$= -\frac{(5a-b)\left(a+b-b\cos^2(e+fx)\right)^{3/2}\cot(e+fx)\csc^3(e+fx)}{24af} - \frac{\left(a+\right.}{}$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{16af} - \frac{(5}{}$$

$$= -\frac{(5a-b)(a+b)\sqrt{a+b-b\cos^2(e+fx)}\cot(e+fx)\csc(e+fx)}{16af} - \frac{(5}{}$$

$$= -\frac{(5a-b)(a+b)^2\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{16a^{3/2}f} - \frac{(5a-b)(a+b)\sqrt{a+b}}{}$$

Mathematica [A] time = 1.18, size = 161, normalized size = 0.82

$$\frac{-\sqrt{2}\sqrt{a}\csc^2(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\left(\left(15a^2+22ab+3b^2\right)\cos(e+fx)+2a\cot(e+fx)\csc(e+fx)\left(\right.\right.}{96a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-6*(5*a - b)*(a + b)^2*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]*Csc[e + f*x]^2*((15*a^2 + 22*a*b + 3*b^2)*Cos[e + f*x] + 2*a*Cot[e + f*x]
*Csc[e + f*x]*(5*a + 7*b + 4*a*Csc[e + f*x]^2)))/(96*a^(3/2)*f)
```

fricas [A] time = 1.78, size = 752, normalized size = 3.82

$$\left[-\frac{3\left(\left(5a^3+9a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^6-3\left(5a^3+9a^2b+3ab^2-b^3\right)\cos\left(fx+e\right)^4-5a^3-9a^2b-3ab^2-\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/192*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^6 - 3*(5*a^3 + 9
*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3 - 9*a^2*b - 3*a*b^2 + b^3 +
3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6
*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a
 - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4
*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b + 3*a
*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(-b*
cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4 + 3
*a^2*f*cos(f*x + e)^2 - a^2*f), 1/96*(3*((5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*
cos(f*x + e)^6 - 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x + e)^4 - 5*a^3
 - 9*a^2*b - 3*a*b^2 + b^3 + 3*(5*a^3 + 9*a^2*b + 3*a*b^2 - b^3)*cos(f*x +
e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e)))
+ 2*((15*a^3 + 22*a^2*b + 3*a*b^2)*cos(f*x + e)^5 - 2*(20*a^3 + 29*a^2*b +
3*a*b^2)*cos(f*x + e)^3 + 3*(11*a^3 + 12*a^2*b + a*b^2)*cos(f*x + e))*sqrt(
-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^6 - 3*a^2*f*cos(f*x + e)^4
+ 3*a^2*f*cos(f*x + e)^2 - a^2*f)]
```

giac [B] time = 1.14, size = 1708, normalized size = 8.67

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/384*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*((a*tan(1/2*f*x + 1/2*e)^2 + (8*a^3 + 7*a^2*b)/a^2
)*tan(1/2*f*x + 1/2*e)^2 + (37*a^3 + 51*a^2*b + 6*a*b^2)/a^2) + 24*(5*a^3 +
 9*a^2*b + 3*a*b^2 - b^3)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a) - 12*(5*a^(7/2) + 9*a^(5/2)*b + 3*a^(3/2
)*b^2 - sqrt(a)*b^3)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^2 + 2*(45*(sqrt(a)*tan(1/2*f*x + 1/2*
e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a))^5*a^3 + 132*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^5*a^2*b + 108*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^5*a*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5
*b^3 + 63*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) +
120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b + 48*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 50*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 - 156*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b - 96*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 78*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^2*a^(9/2) - 108*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
```

```
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^2*a^(7/2)*b + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))*a^5 + 72*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*b + 36
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b^2 - 12*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^3 + 31*a^(11/2) + 36*a^(
9/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^3*a))
/f
```

maple [B] time = 2.52, size = 565, normalized size = 2.87

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(15a^4\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}+a+b}{\sin\left(fx+e\right)^2}\right)\left(\sin\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/96*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(15*a^4*ln(((a-b)*cos(f*x+e)^
2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*s
in(f*x+e)^6+27*a^3*b*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b
)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6+9*b^2*ln(((a-b)*cos(f
*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e
)^2)*sin(f*x+e)^6*a^2-3*b^3*ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)
^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e)^6*a+30*a^(7/2)*(
cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4+44*b*(cos(f*x+e)^2*(a+b
*sin(f*x+e)^2))^(1/2)*sin(f*x+e)^4*a^(5/2)+6*b^2*(cos(f*x+e)^2*(a+b*sin(f*x
+e)^2))^(1/2)*sin(f*x+e)^4*a^(3/2)+20*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)
^2))^(1/2)*sin(f*x+e)^2+28*b*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*sin(f*
x+e)^2*a^(5/2)+16*a^(7/2)*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/sin(f*x+
e)^6/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\sin\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^7, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.138 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=325

$$-\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e}{35b^2f\sqrt{a+}}$$

```
[Out] -1/35*(a^2+11*a*b+8*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
-2/35*(4*a+3*b)*cos(f*x+e)*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*co
s(f*x+e)*sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a+2*b)*(a^2-4*a*b-4*
b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+
b*sin(f*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2*a^2-
5*a*b-8*b^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(
1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 477, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(a^2+11ab+8b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2-5ab-8b^2\right)\sqrt{\cos^2(e+fx)}\sec(e}{35b^2f\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^2 + 11*a*b + 8*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(35*b*f) - (2*(4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]^3*Sqrt[a + b*Sin[e
+ f*x]^2])/(35*f) - (b*Cos[e + f*x]*Sin[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^
2])/(7*f) - (2*(a + 2*b)*(a^2 - 4*a*b - 4*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipti
cE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(
35*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 - 5*a*b - 8*b^
2)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 477

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(d*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)
^(q - 1))/(b*e*(m + n*(p + q) + 1)), x] + Dist[1/(b*(m + n*(p + q) + 1)), I
nt[(e*x)^m*(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*((c*b - a*d)*(m + 1) +
c*b*n*(p + q)) + (d*(c*b - a*d)*(m + 1) + d*n*(q - 1)*(b*c - a*d) + c*b*d*n
*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m, p}, x] && NeQ[b*c - a
*d, 0] && IGtQ[n, 0] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q
, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\sqrt{1-x^2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\cos(e+fx)\sin^5(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{\left(\sqrt{\cos^2(e+fx)}\right. \text{s}}{}$$

$$= -\frac{2(4a+3b)\cos(e+fx)\sin^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos(e}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(}{}$$

$$= -\frac{\left(a^2+11ab+8b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} - \frac{2(}{}$$

Mathematica [**A**] time = 2.65, size = 249, normalized size = 0.77

$$\frac{\sqrt{2}\,b\sin(2(e+fx))\left(-32a^3+b\left(144a^2+480ab+299b^2\right)\cos(2(e+fx))-496a^2b-2b^2(26a+27b)\cos(4(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-128*a*(a^3 - 2*a^2*b - 12*a*b^2 - 8*b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x
)])/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 - 3*a^2*b - 13*a*b^2 - 8*b^
3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*b*(-32*a^3 - 496*a^2*b - 684*a*b^2 - 250*b^3 + b*(144*a^2 + 480*a*b + 2
99*b^2)*Cos[2*(e + f*x)] - 2*b^2*(26*a + 27*b)*Cos[4*(e + f*x)] + 5*b^3*Cos
[6*(e + f*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*
x)]])
```

fricas [**F**] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^6-(a+3\,b)\cos\left(fx+e\right)^4+(2\,a+3\,b)\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - (a + 3*b)*cos(f*x + e)^4 + (2*a + 3*b)*cos(f*
x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

maple [B] time = 1.61, size = 602, normalized size = 1.85

$$\frac{5b^{4}\left(\sin^{9}\left(fx+e\right)\right)+13a\,b^{3}\left(\sin^{7}\left(fx+e\right)\right)+b^{4}\left(\sin^{7}\left(fx+e\right)\right)+9a^{2}b^{2}\left(\sin^{5}\left(fx+e\right)\right)+4a\,b^{3}\left(\sin^{5}\left(fx+e\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*b^4*sin(f*x+e)^9+13*a*b^3*sin(f*x+e)^7+b^4*sin(f*x+e)^7+9*a^2*b^2*s
in(f*x+e)^5+4*a*b^3*sin(f*x+e)^5+2*b^4*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*
((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4-3*(co
s(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a^3*b-13*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4
+4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^3*b+24*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+16*(cos(f*x+e)^2)^(1/2)*((a+b*s
in(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3+a^3*b*sin(
f*x+e)^3+2*a^2*b^2*sin(f*x+e)^3-9*a*b^3*sin(f*x+e)^3-8*b^4*sin(f*x+e)^3-a^3
*b*sin(f*x+e)-11*a^2*b^2*sin(f*x+e)-8*a*b^3*sin(f*x+e))/b^2/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sin\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\sin\left(e+f\,x\right)}^4\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.139 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=218

$$\frac{(3a^2+13ab+8b^2)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a+$$

```
[Out] -1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f-1/15*(3*a+4*b)*cos(f*
x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/15*(3*a^2+13*a*b+8*b^2)*(cos(f
*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e
)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/15*a*(a+b)*(3*a+4*b)*(cos(f*x+e
)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/
a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{(3a^2+13ab+8b^2)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f}-\frac{(3a+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((3*a + 4*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f)
- (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*f) + ((3*a^2
+ 13*a*b + 8*b^2)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(1
5*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(a + b)*(3*a + 4*b)*EllipticF[e
+ f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5f} + \frac{1}{5}\int \sqrt{a+b\sin^2(}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{\cos(e+f}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{\cos(e+f}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{\cos(e+f}{}$$

$$= -\frac{(3a+4b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{\cos(e+f}{}$$

Mathematica [A] time = 1.38, size = 201, normalized size = 0.92

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(48a^2-4b(9a+7b)\cos(2(e+fx))+68ab+3b^2\cos(4(e+fx))+25b^2\right)-16a\left(3a^2+7ab+\right.}{240bf\sqrt{2a-b\cos(2(e-}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (16*a*(3*a^2 + 13*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] - 16*a*(3*a^2 + 7*a*b + 4*b^2)*Sqrt[(2*a + b - b*Cos[
2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(48*a^2 + 68*a*b +
25*b^2 - 4*b*(9*a + 7*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(
e + f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-(a+2\,b)\cos\left(fx+e\right)^2+a+b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - (a + 2*b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(
f*x + e)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

maple [A] time = 1.41, size = 429, normalized size = 1.97

$$-\frac{-3b^3\left(\sin^7\left(fx+e\right)\right)-9a\,b^2\left(\sin^5\left(fx+e\right)\right)-b^3\left(\sin^5\left(fx+e\right)\right)+3\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\ \text{Ellip}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5-b^3*sin(f*x+e)^5+3*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a^3+7*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(s
in(f*x+e),(-1/a*b)^(1/2))*b+4*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+
b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+5*a*b^2*sin(f*x+e)^3+4
*b^3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)+4*a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^2\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.140 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin\cdots}}{3f\sqrt{\frac{b\sin\cdots}{\cdots}}}$$

```
[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.375$, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin\cdots}}{3f\sqrt{\frac{b\sin\cdots}{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\sin}{\sqrt{a+b\sin^2(e+f}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e+}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+fx)}\right.}{3\sqrt{1+\frac{b\sin^2(}{a}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+}}{3f\sqrt{1+\frac{b\sin^2(e+fx}{a}}}$$

Mathematica [A] time = 0.80, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}\,a(2a+b)\sqrt{}}{6\sqrt{2}\,f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.44, size = 266, normalized size = 1.73

$$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+\ldots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*a
*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.141 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=181

$$-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right|-}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -a*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-(a-b)*EllipticE(sin(f*x+e),(-b/a)^
(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(co
s(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3188, 474, 524, 426, 424, 421, 419}

$$-\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right|-}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - ((a - b)*Sqrt[Cos[e + f*
x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
 + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*Sqrt[Cos[e + f*
x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{x^2\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right.}{f}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\,\mathrm{se}\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left((-a+b)\sqrt{\cos^2(e+fx)}\,\mathrm{se}\right.}{}$$

$$= -\frac{a\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\mathrm{sin}\right.}{}$$

Mathematica [A] time = 1.37, size = 141, normalized size = 0.78

$$-\frac{a\left(\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)-2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+2(a-b)\sqrt{\frac{2a-b\cos\right.}{}}}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/2*(a*(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] + 2*(a - b)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*(a + b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]))/(f*Sqr
t[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\csc\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*
x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

maple [A] time = 1.40, size = 174, normalized size = 0.96

$$\frac{a\left(\sin\left(fx+e\right)\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\left(\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+\text{EllipticF}\left(\sin\left(fx+\right.\right.\right.\right.}{\sin\left(fx+e\right)\cos}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] a*(sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b
)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)
^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\sin\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.142 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)(2a+3b)\sqrt{c}}{}$$

```
[Out] -2/3*(a+2*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*a*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/f-2/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*
x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(
f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)
^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3188, 474, 583, 524, 426, 424, 421, 419}

$$-\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)(2a+3b)\sqrt{c}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (a*Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (2*(a + 2*b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*(2*a +
3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{x^4\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{a\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(\sqrt{\cos^2(e+fx)}\ \text{s}\right.}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+}{}$$

$$= -\frac{2(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{a\cot(e+fx)\csc^2(e+}{}$$

Mathematica [A] time = 4.51, size = 201, normalized size = 0.85

$$\frac{2\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{\cot(e+fx)\csc^2(e+fx)\left(2\left(2a^2+7ab+4b^2\right)\cos(2(e+fx))-8a^2-b(a+2b)\cos(4(e\right.}{\sqrt{2}}}{6f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((-8*a^2 - 13*a*b - 6*b^2 + 2*(2*a^2 + 7*a*b + 4*b^2)*Cos[2*(e + f*x)] - b
*(a + 2*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a
+ 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] +
2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[
e + f*x, -(b/a)])/(6*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*
x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

maple [A] time = 1.73, size = 408, normalized size = 1.73

$$2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\ \mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)+5b\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\ \sqrt{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+5*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3+3*b^2*(c
os(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))*sin(f*x+e)^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-4*(cos(f*x+e)^2)^(1/2
)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin
(f*x+e)^3+2*a*b*sin(f*x+e)^6+4*b^2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4+3*a*b*si
n(f*x+e)^4-4*b^2*sin(f*x+e)^4-a^2*sin(f*x+e)^2-5*a*b*sin(f*x+e)^2-a^2)/sin(
f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\frac{\left(b\sin\left(e+fx\right)^2+a\right)^{3/2}}{\sin\left(e+fx\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.143 $\int \left(a+b\sin^2(c+dx)\right)^{5/2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}\,E\left(c+dx\left|-\frac{b}{a}\right.\right)}{15d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}-\frac{4b(2a}{}$$

```
[Out] -1/5*b*cos(d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(3/2)/d-4/15*b*(2*a+b)*cos(
d*x+c)*sin(d*x+c)*(a+b*sin(d*x+c)^2)^(1/2)/d+1/15*(23*a^2+23*a*b+8*b^2)*(co
s(d*x+c)^2)^(1/2)/cos(d*x+c)*EllipticE(sin(d*x+c),(-b/a)^(1/2))*(a+b*sin(d*
x+c)^2)^(1/2)/d/(1+b*sin(d*x+c)^2/a)^(1/2)-4/15*a*(a+b)*(2*a+b)*(cos(d*x+c)
^2)^(1/2)/cos(d*x+c)*EllipticF(sin(d*x+c),(-b/a)^(1/2))*(1+b*sin(d*x+c)^2/a
)^(1/2)/d/(a+b*sin(d*x+c)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3180, 3170, 3172, 3178, 3177, 3183, 3182}

$$\frac{\left(23a^2+23ab+8b^2\right)\sqrt{a+b\sin^2(c+dx)}\,E\left(c+dx\left|-\frac{b}{a}\right.\right)}{15d\sqrt{\frac{b\sin^2(c+dx)}{a}+1}}-\frac{b\sin(c+dx)\cos(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d}-\frac{4b(2a}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (-4*b*(2*a + b)*Cos[c + d*x]*Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^2])/(15*d
) - (b*Cos[c + d*x]*Sin[c + d*x]*(a + b*Sin[c + d*x]^2)^(3/2))/(5*d) + ((23
*a^2 + 23*a*b + 8*b^2)*EllipticE[c + d*x, -(b/a)]*Sqrt[a + b*Sin[c + d*x]^2
])/(15*d*Sqrt[1 + (b*Sin[c + d*x]^2)/a]) - (4*a*(a + b)*(2*a + b)*EllipticF
[c + d*x, -(b/a)]*Sqrt[1 + (b*Sin[c + d*x]^2)/a])/(15*d*Sqrt[a + b*Sin[c +
d*x]^2])
```

Rule 3170

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[(B*Cos[e + f*x]*Sin[e + f*x]*(a + b*Si
n[e + f*x]^2)^p)/(2*f*(p + 1)), x] + Dist[1/(2*(p + 1)), Int[(a + b*Sin[e +
 f*x]^2)^(p - 1)*Simp[a*B + 2*a*A*(p + 1) + (2*A*b*(p + 1) + B*(b + 2*a*p +
 2*b*p))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && GtQ[
p, 0]
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^{5/2} dx = -\frac{b\cos(c+dx)\sin(c+dx)\left(a+b\sin^2(c+dx)\right)^{3/2}}{5d} + \frac{1}{5}\int \sqrt{a+b\sin^2(c+dx)}\ (\ldots$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin(c\ldots}{\ldots}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin(c\ldots}{\ldots}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin(c\ldots}{\ldots}$$

$$= -\frac{4b(2a+b)\cos(c+dx)\sin(c+dx)\sqrt{a+b\sin^2(c+dx)}}{15d} - \frac{b\cos(c+dx)\sin(c\ldots}{\ldots}$$

Mathematica [A] time = 1.42, size = 194, normalized size = 0.92

$$\frac{-\sqrt{2}\,b\sin(2(c+dx))\left(88a^2-28b(2a+b)\cos(2(c+dx))+88ab+3b^2\cos(4(c+dx))+25b^2\right)-64a\left(2a^2+3a\ldots\right.}{240d\sqrt{2a-b\cos(2(\ldots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^(5/2),x]
```

```
[Out] (16*a*(23*a^2 + 23*a*b + 8*b^2)*Sqrt[(2*a + b - b*Cos[2*(c + d*x)])/a]*Elli
pticE[c + d*x, -(b/a)] - 64*a*(2*a^2 + 3*a*b + b^2)*Sqrt[(2*a + b - b*Cos[2
```

```
*(c + d*x)])/a]*EllipticF[c + d*x, -(b/a)] - Sqrt[2]*b*(88*a^2 + 88*a*b + 2
5*b^2 - 28*b*(2*a + b)*Cos[2*(c + d*x)] + 3*b^2*Cos[4*(c + d*x)])*Sin[2*(c
+ d*x)])/(240*d*Sqrt[2*a + b - b*Cos[2*(c + d*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\cos(dx+c)^4 - 2\left(ab+b^2\right)\cos(dx+c)^2 + a^2 + 2\,ab + b^2\right)\sqrt{-b\cos(dx+c)^2 + a + b}\,, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b +
 b^2)*sqrt(-b*cos(d*x + c)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin(dx+c)^2 + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

maple [A] time = 1.62, size = 437, normalized size = 2.08

$$-\frac{b^3\sin(dx+c)\left(\cos^6(dx+c)\right)}{5} + \frac{\left(14a\,b^2+10b^3\right)\left(\cos^4(dx+c)\right)\sin(dx+c)}{15} + \frac{\left(-11a^2b-18a\,b^2-7b^3\right)\left(\cos^2(dx+c)\right)\sin(dx+c)}{15} - \frac{8\sqrt{\frac{\cos(2dx+2c)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\mathrm{co}}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^2)^(5/2),x)
```

```
[Out] (-1/5*b^3*sin(d*x+c)*cos(d*x+c)^6+1/15*(14*a*b^2+10*b^3)*cos(d*x+c)^4*sin(d
*x+c)+1/15*(-11*a^2*b-18*a*b^2-7*b^3)*cos(d*x+c)^2*sin(d*x+c)-8/15*(cos(d*x
+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b
)^(1/2))*a^3-4/5*a^2*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)
*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b-4/15*a*(cos(d*x+c)^2)^(1/2)*(-b/a*c
os(d*x+c)^2+(a+b)/a)^(1/2)*EllipticF(sin(d*x+c),(-1/a*b)^(1/2))*b^2+23/15*(
cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),
(-1/a*b)^(1/2))*a^3+23/15*(cos(d*x+c)^2)^(1/2)*(-b/a*cos(d*x+c)^2+(a+b)/a)^
(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a^2*b+8/15*(cos(d*x+c)^2)^(1/2)*
(-b/a*cos(d*x+c)^2+(a+b)/a)^(1/2)*EllipticE(sin(d*x+c),(-1/a*b)^(1/2))*a*b^
2)/cos(d*x+c)/(a+b*sin(d*x+c)^2)^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin(dx+c)^2 + a\right)^{\frac{5}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\left(b\sin(c+d\,x)^2 + a\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^2)^(5/2),x)
```

```
[Out] int((a + b*sin(c + d*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.144 $\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=83

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2bf}$$

```
[Out] 1/2*(a-b)*arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f-1
/2*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.10, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 388, 217, 203}

$$\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2b^{3/2}f}-\frac{\cos(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a - b)*ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(2*
b^(3/2)*f) - (Cos[e + f*x]*Sqrt[a + b - b*Cos[e + f*x]^2])/(2*b*f)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+$$

$$= -\frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf} + \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx}{\sqrt{a+b-b\cos^2(}}$$

$$= \frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\cos(e+fx)\sqrt{a+b-b\cos^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.28, size = 105, normalized size = 1.27

$$\frac{(a-b)\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{2\sqrt{-b}\,bf} - \frac{\cos(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{2\sqrt{2}\,bf}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*(Cos[e + f*x]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/(Sqrt[2]*b*f) + ((a
- b)*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
])/(2*Sqrt[-b]*b*f)
```

fricas [B] time = 0.57, size = 438, normalized size = 5.28

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,b\cos\left(fx+e\right)-(a-b)\sqrt{-b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*cos(f*x + e) - (a - b)*sqrt(-b)
*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b
^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos
(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*
cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(
f*x + e)^2 + a + b)*sqrt(-b)))/(b^2*f), -1/8*((a - b)*sqrt(b)*arctan(1/4*(8
*b^2*cos(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3
)*cos(f*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e))) + 4*sqrt(-b*cos(f
*x + e)^2 + a + b)*b*cos(f*x + e))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
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heck sign: (2*pi/x/2)>(-2*pi/x/2)-1/2/b*cos(f*x+exp(1))/f*sqrt(b*(1-(cos(f*
x+exp(1))/f*f)^2)+a)+2*(a-b)/4/b/sqrt(-b)/abs(f)*ln(abs(sqrt(b*(1-(cos(f*x+
exp(1))/f*f)^2)+a)+sqrt(-b*f^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.48, size = 186, normalized size = 2.24

$$-\frac{\sqrt{\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)}\left(2b^{\frac{3}{2}}\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^{2}\left(fx+e\right)\right)}-b^{2}\arctan\left(\frac{\cdots}{2\sqrt{b}\cdots}\right.\right.}{4b^{\frac{5}{2}}\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+\cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(2*b^(3/2)*(-b*cos(f*x+e)^4+(a
+b)*cos(f*x+e)^2)^(1/2)-b^2*arctan(1/2*(-2*b*cos(f*x+e)^2+a+b)/b^(1/2)/(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2))+b*a*arctan(1/2*(-2*b*cos(f*x+e)^2+a
+b)/b^(1/2)/(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)))/b^(5/2)/cos(f*x+e)
/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.43, size = 75, normalized size = 0.90

$$\frac{\frac{a\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}-\frac{\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}}-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cos\left(fx+e\right)}{b}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) - arcsin(b*cos(f*x +
e)/sqrt((a + b)*b))/sqrt(b) - sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/
b)/f
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.145 $\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=41

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3186, 217, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[b]*f)
)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f}\\
&= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{b}f}
\end{aligned}$$

Mathematica [A] time = 0.11, size = 53, normalized size = 1.29

$$-\frac{\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]]/(
Sqrt[-b]*f))
```

fricas [B] time = 0.52, size = 370, normalized size = 9.02

$$\left[-\frac{\sqrt{-b}\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+b^4\right)\cos\left(fx+e\right)^6+160\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4+a^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^
6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 +
 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2
*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e)
)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b))/(b*f), 1/4*arctan(1/4*(8*b^2*co
s(f*x + e)^4 - 8*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f
*x + e)^3 + (a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)))/(sqrt(b)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
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heck sign: (2*pi/x/2)>(-2*pi/x/2)-1/sqrt(-b)/abs(f)*ln(abs(sqrt(a-b*f^2*(-c
os(f*x+exp(1))/f)^2+b)+sqrt(-b*f^2)*cos(f*x+exp(1))/f))
```

maple [B] time = 1.18, size = 99, normalized size = 2.41

$$\frac{\sqrt{\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)}\,\arctan\left(\frac{2b\left(\sin^{2}\left(fx+e\right)\right)+a-b}{2\sqrt{b}\,\sqrt{\left(\cos^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)}}\right)}{2\sqrt{b}\,\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/b^(1/2)*arctan(1/2/b^(1/2)*(2*b
*sin(f*x+e)^2+a-b)/(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2))/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.44, size = 24, normalized size = 0.59

$$-\frac{\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/(sqrt(b)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\sin\left(e+f\,x\right)}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sin(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.146 $\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx$

Optimal. Leaf size=41

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/f/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 41, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 377, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(Sqrt[a]*f
))
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\csc(e+fx)}{\sqrt{a+b\sin^2(e+fx)}} dx &= -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}} dx, x, \cos(e+fx)\right)}{f} \\
&= -\frac{\text{Subst}\left(\int \frac{1}{1-ax^2} dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{f} \\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{\sqrt{a}f}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 48, normalized size = 1.17

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
]/(Sqrt[a]*f))
```

fricas [B] time = 0.51, size = 219, normalized size = 5.34

$$\left[\frac{\log\left(\frac{2\left(\left(a^2-6ab+b^2\right)\cos\left(fx+e\right)^4+2\left(3a^2+2ab-b^2\right)\cos\left(fx+e\right)^2-4\left((a-b)\cos\left(fx+e\right)^3+(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}+a^2+2ab+b}{\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1}\right.}{4\sqrt{a}f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*co
s(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(
f*x + e)^2 + 1))/(sqrt(a)*f), 1/2*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e
)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 -
 (a^2 + a*b)*cos(f*x + e)))/(a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-81,22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[53,42]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Eva
luation time: 0.41Error: Bad Argument Type
```

maple [B] time = 1.39, size = 112, normalized size = 2.73

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\;\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}+a+b}{\sin\left(fx+e\right)^2}\right)}{2\sqrt{a}\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)/a^(1/2)*ln(((a-b)*cos(f*x+e)^2
+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)/co
s(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.44, size = 109, normalized size = 2.66

$$-\frac{\frac{\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)}{\sqrt{a}}-\frac{\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)+1}+\frac{a}{\cos\left(fx+e\right)+1}\right)}{\sqrt{a}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) -
a/(cos(f*x + e) - 1))/sqrt(a) - log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1))/sqrt(a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sin\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.147 $\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=89

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2af}$$

```
[Out] -1/2*(a-b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/f
-1/2*cot(f*x+e)*csc(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 382, 377, 206}

$$-\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{3/2}f}-\frac{\cot(e+fx)\csc(e+fx)\sqrt{a-b\cos^2(e+fx)+b}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((a - b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])/(
2*a^(3/2)*f) - (Sqrt[a + b - b*Cos[e + f*x]^2]*Cot[e + f*x]*Csc[e + f*x])/(
2*a*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\right.}{2af}$$

$$= -\frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+fx)}{2af} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{}{\sqrt{a}}\right.}{2af}$$

$$= -\frac{(a-b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{3/2}f} - \frac{\sqrt{a+b-b\cos^2(e+fx)}\,\cot(e+fx)\csc(e+f}{2af}$$

Mathematica [A] time = 0.31, size = 102, normalized size = 1.15

$$\frac{-2(a-b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) - \sqrt{2}\sqrt{a}\cot(e+fx)\csc(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{4a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a - b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] - Sqrt[2]*Sqrt[a]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*
x]*Csc[e + f*x])/(4*a^(3/2)*f)
```

fricas [A] time = 0.56, size = 347, normalized size = 3.90

$$\left[\frac{4\sqrt{-b\cos(fx+e)^2+a+b}\,a\cos(fx+e) - \left((a-b)\cos(fx+e)^2-a+b\right)\sqrt{a}\log\left(\frac{2\left((a^2-6ab+b^2)\cos(fx+e)^4+2(3a^2\right.}{}\right.}{8\left(a^2f\cos(fx+e)^2-a^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e) - ((a - b)*cos(f*x +
 e)^2 - a + b)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2
 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*
x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f
*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/(a^2*f*cos(f*x + e)^2 - a^2*f), 1/4*(((
a - b)*cos(f*x + e)^2 - a + b)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2
 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a
^2 + a*b)*cos(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a*cos(f*x + e)
)/(a^2*f*cos(f*x + e)^2 - a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-81,22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[53,42]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/
2)>(-4*pi/t_nostep/2)Evaluation time: 0.5Unable to divide, perhaps due to r
ounding error%%%{1,[4,0]%%%}+%%%{%%%{-2,[1]%%%},[2,0]%%%}+%%%{%%%{1,[2]%%%}
,[0,0]%%%} / %%%{%%%{1,[1]%%%},[4,0]%%%}+%%%{%%%{-2,[2]%%%},[2,0]%%%}+%%%{%
%%{1,[3]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [B] time = 2.10, size = 231, normalized size = 2.60

$$-\frac{\sqrt{\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)}\left(\ln\left(\frac{(a-b)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{a}\,\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)+a+b}}{\sin\left(fx+e\right)^2}\right)\left(\sin^2\left(\right.\right.\right.}{4\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4*(cos(f*x+e)^2*(a+b*sin(f*x+e)^2))^(1/2)*(ln(((a-b)*cos(f*x+e)^2+2*a^(1
/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+a+b)/sin(f*x+e)^2)*sin(f*x+e
)^2*a^2-ln(((a-b)*cos(f*x+e)^2+2*a^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)+a+b)/sin(f*x+e)^2)*b*sin(f*x+e)^2*a+2*a^(3/2)*(cos(f*x+e)^2*(a+b*s
in(f*x+e)^2))^(1/2))/sin(f*x+e)^2/a^(5/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+fx\right)^3\,\sqrt{b\sin\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.148 $\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=206

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a-b)*EllipticE
(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b
^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3188, 479, 524, 426, 424, 421, 419}

$$\frac{a(2a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a - b
)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*
(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[
e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 479

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q) + 1)), x] - Dist[e^(2*n)/(b*d
*(m + n*(p + q) + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp
[a*c*(m - 2*n + 1) + (a*d*(m + n*(q - 1) + 1) + b*c*(m + n*(p - 1) + 1))*x^
n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b*c - a*d, 0] && IG
tQ[n, 0] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subs}}{3}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{3}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3b^2}$$

Mathematica [A] time = 0.90, size = 163, normalized size = 0.79

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)+2\sqrt{2}\,a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-4\sqrt{2}\,a(a-b)}{6\sqrt{2}\,b^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]
*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e +
f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.44, size = 268, normalized size = 1.30

$$\frac{b^2\left(\sin^5\left(fx+e\right)\right)+2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2-a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\cdots}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*(cos(f*x+e)^2)^(1
/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2
*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-a*b*sin(f*x+e))/b^2/cos(
f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.149 $\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=111

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)-a*(cos(f*x+e)^2)^(1/2)/cos(
f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a
+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3172, 3178, 3177, 3183, 3182}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Si
n[e + f*x]^2)/a]) - (a*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
```

```
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{b} - \frac{a\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{b}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{b\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{\left(a\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\right)\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{b\sqrt{a+b\sin^2(e+fx)}}$$

$$= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} - \frac{aF\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{bf\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.22, size = 78, normalized size = 0.70

$$\frac{\sqrt{2a-b\cos(2(e+fx))+b}\left(E\left(e+fx\middle|-\frac{b}{a}\right)-F\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*(EllipticE[e + f*x, -(b/a)] - EllipticF
[e + f*x, -(b/a)]))/(b*f*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right)}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b*cos(f*x +
e)^2 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.24, size = 93, normalized size = 0.84

$$-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\left(\operatorname{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\operatorname{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)\right)}{b\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)/b*(EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/cos(f*x+e)/(a+b*sin
(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sin(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.150 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\ &= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [C] time = 0.32, size = 60, normalized size = 1.18

$$\frac{\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)-a-b}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sin\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.151 $\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=177

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+
e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 480, 12, 493, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*Elliptic
F[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
)/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{bx^2}{\sqrt{1-x^2}\,\sqrt{\cdots}}\cdots\right)}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{x\cdots}{\sqrt{1-x^2}\cdots}\cdots\right)}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{\cdots}}\cdots\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+f\cdots}\right.}{af\sqrt{1+\frac{b\sin^2\cdots}{\cdots}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(\cdots}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.64, size = 138, normalized size = 0.78

$$\frac{-\sqrt{2}\cot(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(\right.}{2af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-(Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*Sqrt[(2*a + b -
b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(2*a*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\csc\left(fx+e\right)^{2}}{b\cos\left(fx+e\right)^{2}-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.53, size = 140, normalized size = 0.79

$$\frac{\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,a\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-\mathrm{EllipticE}\left(\sin\left(fx+\right.\right.\right.}{a\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))-EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+b*cos
(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/a/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(
1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.152 $\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=244

$$-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -2/3*(a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f-2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b*si
n(f*x+e)^2/a)^(1/2)+1/3*(2*a-b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+
e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(
1/2)
```

Rubi [A] time = 0.27, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 480, 583, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e + f
*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (2*(a - b)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a - b)*S
qrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sq
rt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 480

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q
 + 1))/(a*c*e*(m + 1)), x] - Dist[1/(a*c*e^n*(m + 1)), Int[(e*x)^(m + n)*(a
 + b*x^n)^p*(c + d*x^n)^q*Simp[(b*c + a*d)*(m + n + 1) + n*(b*c*p + a*d*q)
+ b*d*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p, q
}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b
, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^4\sqrt{1-x^2}\,\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{S}}{}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3af}$$

$$= -\frac{2(a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin}}{3af}$$

Mathematica [A] time = 3.98, size = 195, normalized size = 0.80

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(2\left(2a^2+ab-2b^2\right)\cos(2(e+fx))-8a^2+b(b-a)\cos(4(e+fx))-ab+3b^2\right)}{\sqrt{2}} + 2a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\right|-}{6a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((-8*a^2 - a*b + 3*b^2 + 2*(2*a^2 + a*b - 2*b^2)*Cos[2*(e + f*x)] + b*(-a
+ b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] - 4*a*(a - b)*S
qrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2*a*(2*a
 - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6
*a^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.48, size = 354, normalized size = 1.45

$$2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\,\operatorname{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)-b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\ldots}{\ldots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+
e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3-b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)
^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-2*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2*sin(f*x+e)^3+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+2*a*b*sin(f*x+e)^6-2*b^
2*sin(f*x+e)^6+2*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4+2*b^2*sin(f*x+e)^4-a^2
*sin(f*x+e)^2+a*b*sin(f*x+e)^2-a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*
x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(csc(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.153 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{a\cos(e+fx)}{bf(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}f}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(3/2)/f+a*cos(f*x+
e)/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3186, 385, 217, 203}

$$\frac{a\cos(e+fx)}{bf(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(3/2)*f)
) + (a*Cos[e + f*x])/(b*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] && !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{bf}$$

$$= \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1+bx^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{bf}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{3/2}f} + \frac{a\cos(e+fx)}{b(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A] time = 0.42, size = 96, normalized size = 1.22

$$\frac{\frac{\sqrt{2}ab\cos(e+fx)}{(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} + \sqrt{-b}\log\left(\sqrt{2a-b\cos(2(e+fx))+b} + \sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((Sqrt[2]*a*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]) +
Sqrt[-b]*Log[Sqrt[2]*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*
x)]]])/(b^2*f)
```

fricas [B] time = 0.64, size = 564, normalized size = 7.14

$$\left[-\frac{8\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,ab\cos\left(fx+e\right) + \left(\left(ab+b^2\right)\cos\left(fx+e\right)^2 - a^2 - 2ab - b^2\right)\sqrt{-b}\log\left(128b^4\cos\left(\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(8*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) + ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256
*(a*b^3 + b^4)*cos(f*x + e)^6 + 160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^
4 + a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a
*b^3 + b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*co
s(f*x + e)^5 + 10*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b +
 3*a*b^2 + b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)))/((
a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b^3 + b^4)*f), -1/4*(4*sqrt(
-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 -
 a^2 - 2*a*b - b^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*a*b^
2 + b^3)*cos(f*x + e))))/((a*b^3 + b^4)*f*cos(f*x + e)^2 - (a^2*b^2 + 2*a*b
^3 + b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (
2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x
/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/
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heck sign: (2*pi/x/2)>(-2*pi/x/2)-4*a/(-4*b^2-4*b*a)*cos(f*x+exp(1))/f*sqrt
(b*(1-(cos(f*x+exp(1))/f*f)^2)+a)/(b*(1-(cos(f*x+exp(1))/f*f)^2)+a)-1/b/sqr
t(-b)/abs(f)*ln(abs(sqrt(b*(1-(cos(f*x+exp(1))/f*f)^2)+a)+sqrt(-b*f^2)*cos(
f*x+exp(1))/f))
```

maple [B] time = 2.38, size = 156, normalized size = 1.97

$$\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(\sin^{2}\left(fx+e\right)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}}\right)}{2b^{\frac{3}{2}}}+\frac{a\left(\cos^{2}\left(fx+e\right)\right)}{b(a+b)\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(f\right.\right.}}\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(3/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))+a/b*cos(f
```

```
*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/cos(f*x+e)/(a+b*si
n(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.58, size = 81, normalized size = 1.03

$$-\frac{\frac{\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{3}{2}}}+\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)}-\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(3/2) + cos(f*x + e)/(sqrt(-b*co
s(f*x + e)^2 + a + b)*(a + b)) - cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a +
 b)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.154 $\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=34

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

```
[Out] -cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 34, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3186, 191}

$$-\frac{\cos(e+fx)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] -(Cos[e + f*x]/((a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}\\ &= -\frac{\cos(e+fx)}{(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.12, size = 41, normalized size = 1.21

$$-\frac{\sqrt{2}\cos(e+fx)}{f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] -((Sqrt[2]*Cos[e + f*x])/((a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]))
```

fricas [A] time = 0.44, size = 57, normalized size = 1.68

$$\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)}{\left(ab+b^2\right)f\cos\left(fx+e\right)^2-\left(a^2+2\,ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)/((a*b + b^2)*f*cos(f*x + e)^2
- (a^2 + 2*a*b + b^2)*f)
```

giac [A] time = 0.58, size = 53, normalized size = 1.56

$$\frac{\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}\,\cos\left(fx+e\right)}{\left(\left(\cos\left(fx+e\right)^{2}-1\right)b-a\right)(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sqrt(-(cos(f*x + e)^2 - 1)*b + a)*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a
)*(a + b)*f)
```

maple [A] time = 0.88, size = 31, normalized size = 0.91

$$-\frac{\cos\left(fx+e\right)}{(a+b)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -cos(f*x+e)/(a+b)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.39, size = 32, normalized size = 0.94

$$-\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*f)
```

mupad [B] time = 15.18, size = 119, normalized size = 3.50

$$-\frac{\sqrt{2}\,\sqrt{2\,a+b-b\,\cos\left(2\,e+2\,f\,x\right)}\,\left(4\,a\,\cos\left(e+f\,x\right)+b\,\cos\left(e+f\,x\right)-b\,\cos\left(3\,e+3\,f\,x\right)\right)}{f\,\left(a+b\right)\,\left(8\,a\,b+8\,a^2+3\,b^2-4\,b^2\,\cos\left(2\,e+2\,f\,x\right)+b^2\,\cos\left(4\,e+4\,f\,x\right)-8\,a\,b\,\cos\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] -(2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*cos(e + f*x) + b*cos(e
+ f*x) - b*cos(3*e + 3*f*x)))/(f*(a + b)*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*cos
(2*e + 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.155 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=79

$$\frac{b\cos(e+fx)}{af(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(3/2)/f+b*cos(f*x
+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3186, 382, 377, 206}

$$\frac{b\cos(e+fx)}{af(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(3/2)*f
)) + (b*Cos[e + f*x])/(a*(a + b)*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+b-bx^2)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-bx^2}}\,dx, x, \cos(e+fx)\right)}{af}$$

$$= \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-ax^2}\,dx, x, \frac{\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{af}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{3/2}f} + \frac{b\cos(e+fx)}{a(a+b)f\sqrt{a+b-b\cos^2(e+fx)}}$$

Mathematica [A] time = 0.35, size = 93, normalized size = 1.18

$$\frac{\frac{\sqrt{2}\sqrt{a}\,b\cos(e+fx)}{(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} - \tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
] + (Sqrt[2]*Sqrt[a]*b*Cos[e + f*x])/((a + b)*Sqrt[2*a + b - b*Cos[2*(e + f
*x)]]))/(a^(3/2)*f)
```

fricas [B] time = 0.57, size = 422, normalized size = 5.34

$$\left[-\frac{4\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,ab\cos\left(fx+e\right) - \left(\left(ab+b^2\right)\cos\left(fx+e\right)^2 - a^2 - 2\,ab - b^2\right)\sqrt{a}\,\log\left(\frac{2\left(\left(a^2-6\,ab+b^2\right)c\right.}{\cdots}\right.}{4\left(\left(a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2 - \left(a^4\cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(4*sqrt(-b*cos(f*x + e)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*
x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)
^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 +
2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)))/((a^3*b + a^2*b^2)*f
*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f), -1/2*(2*sqrt(-b*cos(f*x + e
)^2 + a + b)*a*b*cos(f*x + e) - ((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x + e))))
/((a^3*b + a^2*b^2)*f*cos(f*x + e)^2 - (a^4 + 2*a^3*b + a^2*b^2)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-81,22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[53,42]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)War
ning, replacing 0 by ` u`, a substitution variable should perhaps be purged
.Warning, replacing 0 by ` u`, a substitution variable should perhaps be pu
rged.Warning, replacing 0 by ` u`, a substitution variable should perhaps b
e purged.Evaluation time: 0.62Error: Bad Argument Type
```

maple [B] time = 2.38, size = 165, normalized size = 2.09

$$\frac{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(\frac{b\left(\cos^2\left(fx+e\right)\right)}{a(a+b)\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+(-a+b)\left(\sin^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}{\sin^2\left(fx+e\right)}\right)}{2a^{\frac{3}{2}}}\right)}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/a*b*cos(f*x+e)^2/(a+b)/(-(-b*s
in(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/2/a^(3/2)*ln((2*a+(-a+b)*sin(f*x+e)^2+
2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2))/cos(f*x
+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.49, size = 165, normalized size = 2.09

$$\frac{\frac{2\,b^2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a^2b+\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,ab^2}-\frac{\log\left(b-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)-1}-\frac{a}{\cos\left(fx+e\right)-1}\right)}{a^{\frac{3}{2}}}+\frac{\log\left(-b+\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}}{\cos\left(fx+e\right)+1}+\frac{a}{\cos\left(fx+e\right)+1}\right)}{a^{\frac{3}{2}}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b + sqrt(-b*co
s(f*x + e)^2 + a + b)*a*b^2) - log(b - sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1))/a^(3/2) + log(-b + sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) + 1) + a/(cos(f*x + e) + 1))/a
^(3/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(e + fx\right)}{\left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.156 $\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{b(a+3b)\cos(e+fx)}{2a^2f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b\cos^2(e+fx)+b}}$$

```
[Out] -1/2*(a-3*b)*arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2)
/f-1/2*b*(a+3*b)*cos(f*x+e)/a^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(1/2)-1/2*cot(
f*x+e)*csc(f*x+e)/a/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3186, 414, 527, 12, 377, 206}

$$-\frac{b(a+3b)\cos(e+fx)}{2a^2f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}-\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{2a^{5/2}f}-\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a - 3*b)*ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]])
/(2*a^(5/2)*f) - (b*(a + 3*b)*Cos[e + f*x])/(2*a^2*(a + b)*f*Sqrt[a + b - b
*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x])/(2*a*f*Sqrt[a + b - b*Cos[e
 + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{a-b-2bx^2}{\left(1-x^2\right)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{2af}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int\right.}{}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(a-3b)\,\text{S}}{}$$

$$= -\frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc(e+fx)}{2af\sqrt{a+b-b\cos^2(e+fx)}} - \frac{(a-3b)\,\text{S}}{}$$

$$= -\frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{2a^{5/2}f} - \frac{b(a+3b)\cos(e+fx)}{2a^2(a+b)f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\text{c}}{2af}$$

Mathematica [A] time = 0.70, size = 134, normalized size = 1.00

$$\frac{\frac{\cot(e+fx)\csc(e+fx)\left(-2a^2+b(a+3b)\cos(2(e+fx))-3ab-3b^2\right)}{\sqrt{2}\,a^2(a+b)\sqrt{2a-b\cos(2(e+fx))+b}} - \frac{(a-3b)\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{5/2}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(((a - 3*b)*ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2
*(e + f*x)]]])/a^(5/2)) + ((-2*a^2 - 3*a*b - 3*b^2 + b*(a + 3*b)*Cos[2*(e +
 f*x)])*Cot[e + f*x]*Csc[e + f*x])/(Sqrt[2]*a^2*(a + b)*Sqrt[2*a + b - b*Co
s[2*(e + f*x)]]))/(2*f)
```

fricas [B] time = 0.85, size = 634, normalized size = 4.73

$$\left[-\frac{\left(\left(a^2b-2\,ab^2-3\,b^3\right)\cos\left(fx+e\right)^4+a^3-a^2b-5\,ab^2-3\,b^3-\left(a^3-7\,ab^2-6\,b^3\right)\cos\left(fx+e\right)^2\right)\sqrt{a}\,\log\left(\frac{2\left(\cdots\right.}{\cdots}\right.}{8\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 -
3*b^3 - (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*((a^2 - 6*a*b
 + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2)*cos(f*x + e)^2 + 4*((a - b
)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)) - 4*((a
^2*b + 3*a*b^2)*cos(f*x + e)^3 - (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b))/((a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 +
 3*a^4*b + 2*a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/4*
(((a^2*b - 2*a*b^2 - 3*b^3)*cos(f*x + e)^4 + a^3 - a^2*b - 5*a*b^2 - 3*b^3
- (a^3 - 7*a*b^2 - 6*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos
(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x
+ e)^3 - (a^2 + a*b)*cos(f*x + e))) + 2*((a^2*b + 3*a*b^2)*cos(f*x + e)^3 -
 (a^3 + 2*a^2*b + 3*a*b^2)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/(
(a^4*b + a^3*b^2)*f*cos(f*x + e)^4 - (a^5 + 3*a^4*b + 2*a^3*b^2)*f*cos(f*x
+ e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

giac [B] time = 0.86, size = 520, normalized size = 3.88

$$\frac{\frac{\left(\frac{\left(a^5b+a^4b^2\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^6b+a^5b^2}+\frac{2\left(a^5b+3\,a^4b^2+6\,a^3b^3\right)}{a^6b+a^5b^2}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\frac{a^5b+a^4b^2-8\,a^3b^3}{a^6b+a^5b^2}}{\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}}-\frac{2\left(a^{\frac{3}{2}}-3\,\sqrt{a}\,b\right)\log\left(\left|-\left(\sqrt{a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\cdots\right.}\right.\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/8*((((a^5*b + a^4*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^6*b + a^5*b^2) + 2*(a^5*
b + 3*a^4*b^2 + 6*a^3*b^3)/(a^6*b + a^5*b^2))*tan(1/2*f*x + 1/2*e)^2 + (a^5
*b + a^4*b^2 - 8*a^3*b^3)/(a^6*b + a^5*b^2))/sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - 2*(a^(3/2)
 - 3*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))*a - a^(3/2) - 2*sqrt(a)*b))/a^3 + 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^2*sqrt(a) - a^(3/2))*a^2))/f
```

maple [B] time = 2.80, size = 274, normalized size = 2.04

$$\frac{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(-\frac{b^2\left(\cos^2\left(fx+e\right)\right)}{a^2(a+b)\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}+\frac{3b\ln\left(\frac{2a+(-a+b)\left(\sin^2\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-\cdots}}{\sin\left(fx\cdots\right.}\right.}{4a^{\frac{5}{2}}}\cdots\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\mathrm{s}\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(-b^2/a^2*cos(f*x+e)^2/(a+b)/(-(-
b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)+3/4/a^(5/2)*b*ln((2*a+(-a+b)*sin(f*x+
e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)-1/2
/a^2/sin(f*x+e)^2*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/4/a^(3/2)*ln(
(2*a+(-a+b)*sin(f*x+e)^2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2
))/sin(f*x+e)^2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^3}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)^3\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.157 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{a(8a-b)\sqrt{\co}}{}$$

```
[Out] a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(4*a+b)*co
s(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b^2/(a+b)/f-1/3*(8*a^2+3*a*b-2
*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a
+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(8*a-b)
*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*si
n(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 470, 582, 524, 426, 424, 421, 419}

$$-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(4a+b)\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) -
((4*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a
+ b)*f) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^3*(a + b
)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(8*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^
2)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2(3a+(}{\sqrt{1-x^2}}\right.}{b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e}}{3b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(4a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e}}{3b^2(a+b)f}$$

Mathematica [A] time = 1.31, size = 197, normalized size = 0.72

$$\frac{b\sin(2(e+fx))\left(-8a^2+b(a+b)\cos(2(e+fx))-3ab-b^2\right)+2\sqrt{2}\,a\left(8a^2+7ab-b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e\right.}{6\sqrt{2}\,b^3f(a+b)\sqrt{2a-b\cos(2(e+fx))+}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*(8*a^2 + 3*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a
]*EllipticE[e + f*x, -(b/a)] + 2*Sqrt[2]*a*(8*a^2 + 7*a*b - b^2)*Sqrt[(2*a
+ b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-8*a^2 - 3*a*b
 - b^2 + b*(a + b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^3*(a +
b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^6-3\cos\left(fx+e\right)^4+3\cos\left(fx+e\right)^2-1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-
b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^
2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.93, size = 405, normalized size = 1.48

$$\frac{a\,b^{2}\left(\sin^{5}\left(fx+e\right)\right)+b^{3}\left(\sin^{5}\left(fx+e\right)\right)+8\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{3}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(a*b^2*sin(f*x+e)^5+b^3*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f
*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*a^2*(cos(f*x+e
)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*b^2-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*
x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+2*(cos(f*x+e)^2
)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a
*b^2+4*a^2*b*sin(f*x+e)^3-b^3*sin(f*x+e)^3-4*a^2*b*sin(f*x+e)-a*b^2*sin(f*x
+e))/b^3/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^6}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.158 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)}{b^2f(a+}$$

```
[Out] a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2)/b^2/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*a*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3188, 470, 524, 426, 424, 421, 419}

$$-\frac{2a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\sec(e+fx)}{b^2f(a+}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(b^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]
^2)/a]) - (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{a+(-2}{\sqrt{1-x^2}}\right.}{b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{\sqrt{1-}}\right.}{b^2 f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((-2a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b}\right.}{b^2(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{b(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\right|}{b^2(a+b)f\sqrt{1+}}$$

Mathematica [A] time = 0.69, size = 136, normalized size = 0.67

$$\frac{a\left(-4(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+2(2a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}b\sin(2(e+fx)\right.}{2b^2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*(2*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x
, -(b/a)] + Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*b^2*(a + b)*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 + a
 + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^
2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.67, size = 241, normalized size = 1.19

$$-\frac{a\left(2a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\,\mathrm{E}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -a*(2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+
e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))-(cos(f*x+e)^2)^(1/2)*((
a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b+b*sin(f*x
+e)^3-b*sin(f*x+e))/b^2/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^{4}}{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.159 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=153

$$-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2
)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)/b/
(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Elliptic
F(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2
)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{\sin(e+fx)\cos(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cos[e + f*x]*Sin[e + f*x])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])) - (El
lipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(b*(a + b)*f*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
```

```
f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{\int \frac{a-a\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}+\frac{\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{b}-\frac{\int \sqrt{a+b\sin^2(e+fx)}}{b(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{b(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}-\frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{b(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}+\frac{F\left(e+\right.}{b}$$

Mathematica [A] time = 0.45, size = 138, normalized size = 0.90

$$\frac{\sqrt{2}(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-b\sin(2(e+fx))}{\sqrt{2}bf(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/
a)]) + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] - b*Sin[2*(e + f*x)])/(Sqrt[2]*b*(a + b)*f*Sqrt[2*a + b - b*C
os[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right)}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b^2*cos(f*x
 + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.52, size = 191, normalized size = 1.25

$$\frac{a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\mathrm{Elliptic}}{b\left(a+b\right)\cos\left(f\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(
-1/a*b)^(1/2))+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))+b*sin(f*x+e)^3-b*sin(f*x+e))/b/(
a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^2}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.160 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.15, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

maple [A] time = 1.74, size = 103, normalized size = 1.02

$$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b(\cos^2(fx+e))}{a}+\frac{a+b}{a}}\;a\,\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)b}{a\left(a+b\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] ((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.161 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f(a+b)}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\mathrm{si}\right.}{a^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*cot(f*x+e)*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)/f-(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec
(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/(a+b)/f/(1+b*sin(
f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)
^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 472, 583, 524, 426, 424, 421, 419}

$$-\frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f(a+b)}-\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\mathrm{si}\right.}{a^2f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (b*Cot[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((a + 2*b)*Cot[
e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*(a + b)*f) - ((a + 2*b)*Sqrt[Cos[
e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b
*Sin[e + f*x]^2])/(a^2*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\,(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a-}{x^2\sqrt{1-x}}\right.}{a(a+b)f}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{(}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{(}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} + \frac{(}{}$$

$$= \frac{b\cot(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2(a+b)f} - \frac{(a}{}$$

Mathematica [A] time = 1.28, size = 170, normalized size = 0.72

$$\frac{\cot(e+fx)\left(-2a^2+b(a+2b)\cos(2(e+fx))-3ab-2b^2\right)+\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{}}{\sqrt{2}\,a^2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-2*a^2 - 3*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*(e + f*x)])*Cot[e + f*x] - Sqr
t[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Elliptic
F[e + f*x, -(b/a)])/(Sqrt[2]*a^2*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)
]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.80, size = 199, normalized size = 0.85

$$\frac{\sin\left(fx+e\right)\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin\left(fx+e\right.\right.\right.}{a^2\sin\left(fx+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*b)+(a*b+2*b^2)*cos(f*x+e)^4+(-a^2-2*a*b-2*b^2)*cos(f*x+e)^2)/a^2/sin(f*x+e
)/(a+b)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.162 $\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=137

$$-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{5/2}f}+\frac{a(3a+5b)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{a\sin^2(e+fx)\cos(e+fx)}{3bf(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -arctan(cos(f*x+e)*b^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*a*cos(
f*x+e)*sin(f*x+e)^2/b/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*a*(3*a+5*b)*co
s(f*x+e)/b^2/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3186, 413, 385, 217, 203}

$$\frac{a(3a+5b)\cos(e+fx)}{3b^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{b^{5/2}f}+\frac{a\sin^2(e+fx)\cos(e+fx)}{3bf(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(b^(5/2)*f)
) + (a*(3*a + 5*b)*Cos[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e +
f*x]^2]) + (a*Cos[e + f*x]*Sin[e + f*x]^2)/(3*b*(a + b)*f*(a + b - b*Cos[e
+ f*x]^2)^(3/2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a-3b+3(a+b)x^2}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx\right.}{3b(a+b)f}$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} -$$

$$= \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(e+fx)\sin^2(e+fx)}{3b(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} -$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{b^{5/2}f} + \frac{a(3a+5b)\cos(e+fx)}{3b^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} + \frac{a\cos(}{3b(a+b)f\,(}$$

Mathematica [A] time = 0.78, size = 133, normalized size = 0.97

$$\frac{\frac{2\sqrt{2}\,a\cos(e+fx)\left(3a^2-b(2a+3b)\cos(2(e+fx))+7ab+3b^2\right)}{(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}} - \frac{3\log\left(\sqrt{2a-b\cos(2(e+fx))+b}+\sqrt{2}\sqrt{-b}\cos(e+fx)\right)}{\sqrt{-b}}}{3b^2f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*Sqrt[2]*a*Cos[e + f*x]*(3*a^2 + 7*a*b + 3*b^2 - b*(2*a + 3*b)*Cos[2*(e
+ f*x)]))/((a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)) - (3*Log[Sqrt[2]
*Sqrt[-b]*Cos[e + f*x] + Sqrt[2*a + b - b*Cos[2*(e + f*x)]]])/Sqrt[-b])/(3*
b^2*f)
```

fricas [B] time = 1.21, size = 885, normalized size = 6.46

$$\left[-\frac{3\left(\left(a^2b^2+2\,ab^3+b^4\right)\cos\left(fx+e\right)^4+a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4-2\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)\cos\left(fx\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2
*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2
```

```
)*sqrt(-b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + b^4)*cos(f*x + e)^6 +
160*(a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*b^2 +
4*a*b^3 + b^4 - 32*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2 + 8*(
16*b^3*cos(f*x + e)^7 - 24*(a*b^2 + b^3)*cos(f*x + e)^5 + 10*(a^2*b + 2*a*b
^2 + b^3)*cos(f*x + e)^3 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(f*x + e))*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)) + 8*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*
x + e)^3 - 3*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x +
e)^2 + a + b))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3
*a^2*b^5 + 3*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b
^5 + 4*a*b^6 + b^7)*f), 1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 +
 a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3
 + b^4)*cos(f*x + e)^2)*sqrt(b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b +
 b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(b)/(2*b^3*cos(f*x + e)^5 - 3*(a*b^2 + b^3)*cos(f*x + e)^3 + (a^2*b + 2*
a*b^2 + b^3)*cos(f*x + e))) - 4*(2*(2*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3
*(a^3*b + 3*a^2*b^2 + 2*a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b
))/((a^2*b^5 + 2*a*b^6 + b^7)*f*cos(f*x + e)^4 - 2*(a^3*b^4 + 3*a^2*b^5 + 3
*a*b^6 + b^7)*f*cos(f*x + e)^2 + (a^4*b^3 + 4*a^3*b^4 + 6*a^2*b^5 + 4*a*b^6
 + b^7)*f)]
```

giac [B] time = 0.95, size = 338, normalized size = 2.47

$$-\frac{\frac{\left(\left(\frac{(3a^3b^8+5a^2b^9)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^2b^{10}+2ab^{11}+b^{12}}+\frac{3(a^3b^8+7a^2b^9+8ab^{10})}{a^2b^{10}+2ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\frac{3(a^3b^8+7a^2b^9+8ab^{10})}{a^2b^{10}+2ab^{11}+b^{12}}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\frac{3a^3b^8+5a^2b^9}{a^2b^{10}+2ab^{11}+b^{12}}}{\left(a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a\right)^{\frac{3}{2}}}-\frac{6\arctan\left(-\right.}{}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((3*a^3*b^8 + 5*a^2*b^9)*tan(1/2*f*x + 1/2*e)^2/(a^2*b^10 + 2*a*b^1
1 + b^12) + 3*(a^3*b^8 + 7*a^2*b^9 + 8*a*b^10)/(a^2*b^10 + 2*a*b^11 + b^12)
)*tan(1/2*f*x + 1/2*e)^2 - 3*(a^3*b^8 + 7*a^2*b^9 + 8*a*b^10)/(a^2*b^10 + 2
*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 - (3*a^3*b^8 + 5*a^2*b^9)/(a^2*b^10
 + 2*a*b^11 + b^12))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 6*arctan(-1/2*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + sqrt(a))/sqrt(b))/b^(5/2))/f
```

maple [A] time = 3.28, size = 243, normalized size = 1.77

$$\frac{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}\left(\frac{\arctan\left(\frac{\sqrt{b}\left(\sin^2\left(fx+e\right)-\frac{-a+b}{2b}\right)}{\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2\left(fx+e\right)\right)}}\right)}{2b^{\frac{5}{2}}}+\frac{2a\left(\cos^2\left(fx+e\right)\right)}{b^2(a+b)\sqrt{-\left(-b\left(\sin^2\left(fx+e\right)\right)-a\right)\left(\cos^2(\right.}}\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/2/b^(5/2)*arctan(b^(1/2)*(sin(
f*x+e)^2-1/2*(-a+b)/b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))+2*a/b^2*c
os(f*x+e)^2/(a+b)/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/3*a^2/b^2*(2*
b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2
)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.46, size = 283, normalized size = 2.07

$$-\frac{\left(\frac{3\cos\left(fx+e\right)^{2}}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}-\frac{2a}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b^{2}}-\frac{2}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}\right)\cos\left(fx+e\right)+\frac{3\arcsin\left(\frac{b\cos\left(fx+e\right)}{\sqrt{(a+b)b}}\right)}{b^{\frac{5}{2}}}+\frac{2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*cos(f*x + e)^2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) - 2*a/((-b*co
s(f*x + e)^2 + a + b)^(3/2)*b^2) - 2/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b))
*cos(f*x + e) + 3*arcsin(b*cos(f*x + e)/sqrt((a + b)*b))/b^(5/2) + 2*cos(f*
x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x + e)/((-b*cos(
f*x + e)^2 + a + b)^(3/2)*(a + b)) - 3*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2
 + a + b)*b^2) + 2*a*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*
b^2) - 2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*b) + 4*cos(f*x + e
)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin\left(e+f\,x\right)^5}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.163 $\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=81

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\sin^2(e+fx)\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)^2/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*x
+e)/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3186, 378, 191}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\sin^2(e+fx)\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*Cos[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2]) - (Cos[e +
 f*x]*Sin[e + f*x]^2)/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sin^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} - \frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+fx)\right)}{3(a+b)f}$$

$$= -\frac{2\cos(e+fx)}{3(a+b)^2 f\sqrt{a+b-b\cos^2(e+fx)}} - \frac{\cos(e+fx)\sin^2(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.32, size = 64, normalized size = 0.79

$$\frac{\sqrt{2}\cos(e+fx)((a+3b)\cos(2(e+fx))-5a-3b)}{3f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sqrt[2]*Cos[e + f*x]*(-5*a - 3*b + (a + 3*b)*Cos[2*(e + f*x)]))/(3*(a + b)
^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [A] time = 0.66, size = 137, normalized size = 1.69

$$\frac{\left((a+3\,b)\cos\left(fx+e\right)^3 - 3\,(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{3\left(\left(a^2b^2+2\,ab^3+b^4\right)f\cos\left(fx+e\right)^4 - 2\left(a^3b+3\,a^2b^2+3\,ab^3+b^4\right)f\cos\left(fx+e\right)^2 + \left(a^4+4\,a^3b+6\,a^2b^2+4\,ab\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a + 3*b)*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e
)^2 + a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2
*b^2 + 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b
^3 + b^4)*f)
```

giac [B] time = 0.79, size = 149, normalized size = 1.84

$$-\frac{\sqrt{-\left(\cos\left(fx+e\right)^2-1\right)b+a}\left(\frac{3\left(abf^2+b^2f^2\right)}{a^2bf^2+2\,ab^2f^2+b^3f^2} - \frac{\left(abf^4+3\,b^2f^4\right)\cos\left(fx+e\right)^2}{\left(a^2bf^2+2\,ab^2f^2+b^3f^2\right)f^2}\right)\cos\left(fx+e\right)}{3\left(\left(\cos\left(fx+e\right)^2-1\right)b-a\right)^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*sqrt(-(cos(f*x + e)^2 - 1)*b + a)*(3*(a*b*f^2 + b^2*f^2)/(a^2*b*f^2 +
2*a*b^2*f^2 + b^3*f^2) - (a*b*f^4 + 3*b^2*f^4)*cos(f*x + e)^2/((a^2*b*f^2 +
 2*a*b^2*f^2 + b^3*f^2)*f^2))*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a)^2*
f)
```

maple [A] time = 1.31, size = 64, normalized size = 0.79

$$-\frac{\left(a\left(\sin^2\left(fx+e\right)\right)+3b\left(\sin^2\left(fx+e\right)\right)+2a\right)\cos\left(fx+e\right)}{3\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a^2+2ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(a*sin(f*x+e)^2+3*b*sin(f*x+e)^2+2*a)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3
/2)/(a^2+2*a*b+b^2)/f
```

maxima [A] time = 0.35, size = 121, normalized size = 1.49

$$-\frac{\frac{2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}+\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)}-\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}b}+\frac{\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)b}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)) - cos(f*x + e)/((-b*cos(f*
x + e)^2 + a + b)^(3/2)*b) + cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*
(a + b)*b))/f
```

mupad [B] time = 20.87, size = 176, normalized size = 2.17

$$\frac{2\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(a+3\,b-10\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+a\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-6\,b\,\mathrm{e}^{e\,2\mathrm{i}+}\right.}{3\,f\,{\left(a+b\right)}^2\,{\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a + 3*b - 10*a*exp(e*2i + f*
x*2i) + a*exp(e*4i + f*x*4i) - 6*b*exp(e*2i + f*x*2i) + 3*b*exp(e*4i + f*x*
4i)))/(3*f*(a + b)^2*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) +
 b*exp(e*4i + f*x*4i))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.164 $\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -1/3*cos(f*x+e)/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)-2/3*cos(f*x+e)/(a+b)^2/f
/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 192, 191}

$$-\frac{2\cos(e+fx)}{3f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\cos(e+fx)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -Cos[e + f*x]/(3*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) - (2*Cos[e + f
*x])/(3*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sin(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx &= -\frac{\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{5/2}}\,dx,x,\cos(e+fx)\right)}{f}\\
&= -\frac{\cos(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}-\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+b-bx^2\right)^{3/2}}\,dx,x,\cos(e+fx)\right)}{3(a+b)f}\\
&= -\frac{\cos(e+fx)}{3(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}}-\frac{2\cos(e+fx)}{3(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 60, normalized size = 0.82

$$\frac{2\sqrt{2}\cos(e+fx)(-3a+b\cos(2(e+fx))-2b)}{3f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*Sqrt[2]*Cos[e + f*x]*(-3*a - 2*b + b*Cos[2*(e + f*x)]))/(3*(a + b)^2*f*(
2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [B] time = 0.60, size = 134, normalized size = 1.84

$$\frac{\left(2\,b\cos\left(fx+e\right)^{3}-3\,(a+b)\cos\left(fx+e\right)\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{3\left(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{4}-2\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)f\cos\left(fx+e\right)^{2}+\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*b*cos(f*x + e)^3 - 3*(a + b)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 +
a + b)/((a^2*b^2 + 2*a*b^3 + b^4)*f*cos(f*x + e)^4 - 2*(a^3*b + 3*a^2*b^2 +
 3*a*b^3 + b^4)*f*cos(f*x + e)^2 + (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*f)
```

giac [B] time = 0.70, size = 137, normalized size = 1.88

$$\frac{\left(\frac{2\,b^{2}f^{2}\cos\left(fx+e\right)^{2}}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}-\frac{3\left(abf^{2}+b^{2}f^{2}\right)}{a^{2}bf^{2}+2\,ab^{2}f^{2}+b^{3}f^{2}}\right)\sqrt{-\left(\cos\left(fx+e\right)^{2}-1\right)b+a}\,\cos\left(fx+e\right)}{3\left(\left(\cos\left(fx+e\right)^{2}-1\right)b-a\right)^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(2*b^2*f^2*cos(f*x + e)^2/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2) - 3*(a*b*
f^2 + b^2*f^2)/(a^2*b*f^2 + 2*a*b^2*f^2 + b^3*f^2))*sqrt(-(cos(f*x + e)^2 -
 1)*b + a)*cos(f*x + e)/(((cos(f*x + e)^2 - 1)*b - a)^2*f)
```

maple [A] time = 1.31, size = 55, normalized size = 0.75

$$-\frac{\left(2b\left(\sin^{2}\left(fx+e\right)\right)+3a+b\right)\cos\left(fx+e\right)}{3\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\left(a^{2}+2ab+b^{2}\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2)/(a^2+2*a*
b+b^2)/f
```

maxima [A] time = 0.37, size = 63, normalized size = 0.86

$$-\frac{\frac{2\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,(a+b)^{2}}+\frac{\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^{2}+a+b\right)^{\frac{3}{2}}(a+b)}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)^2) + cos(f*x
+ e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*(a + b)))/f
```

mupad [B] time = 20.80, size = 159, normalized size = 2.18

$$\frac{4\,\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\sqrt{a+b\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(b-6\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-4\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}{3\,f\,{\left(a+b\right)}^2\,{\left(b-4\,a\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-2\,b\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+b\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) + 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b - 6*a*exp(e*2i + f*x*2i) -
 4*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i)))/(3*f*(a + b)^2*(b - 4*a*ex
p(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.165 $\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=129

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{b(5a+3b)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}+\frac{b\cos(e+fx)}{3af(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

```
[Out] -arctanh(cos(f*x+e)*a^(1/2)/(a+b-b*cos(f*x+e)^2)^(1/2))/a^(5/2)/f+1/3*b*cos
(f*x+e)/a/(a+b)/f/(a+b-b*cos(f*x+e)^2)^(3/2)+1/3*b*(5*a+3*b)*cos(f*x+e)/a^2
/(a+b)^2/f/(a+b-b*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 129, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3186, 414, 527, 12, 377, 206}

$$\frac{b(5a+3b)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a-b\cos^2(e+fx)+b}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a-b\cos^2(e+fx)+b}}\right)}{a^{5/2}f}+\frac{b\cos(e+fx)}{3af(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[a]*Cos[e + f*x])/Sqrt[a + b - b*Cos[e + f*x]^2]]/(a^(5/2)*f
)) + (b*Cos[e + f*x])/(3*a*(a + b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2)) + (b
*(5*a + 3*b)*Cos[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b - b*Cos[e + f*x]^2
])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\csc(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+b-bx^2\right)^{5/2}}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-3a-b-2bx^2}{(1-x^2)\left(a+b-bx^2\right)^{3/2}}\,dx, x, \cos(e+\right.}{3a(a+b)f}$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} -$$

$$= \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5a+3b)\cos(e+fx)}{3a^2(a+b)^2f\sqrt{a+b-b\cos^2(e+fx)}} -$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a}\cos(e+fx)}{\sqrt{a+b-b\cos^2(e+fx)}}\right)}{a^{5/2}f} + \frac{b\cos(e+fx)}{3a(a+b)f\left(a+b-b\cos^2(e+fx)\right)^{3/2}} + \frac{b(5}{3a^2(a+b}$$

Mathematica [A] time = 0.63, size = 127, normalized size = 0.98

$$\frac{\frac{\sqrt{2}b\cos(e+fx)\left(12a^2-b(5a+3b)\cos(2(e+fx))+13ab+3b^2\right)}{3a^2(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}} - \frac{\tanh^{-1}\left(\frac{\sqrt{2}\sqrt{a}\cos(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{a^{5/2}}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(ArcTanh[(Sqrt[2]*Sqrt[a]*Cos[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]
]]/a^(5/2)) + (Sqrt[2]*b*Cos[e + f*x]*(12*a^2 + 13*a*b + 3*b^2 - b*(5*a + 3
*b)*Cos[2*(e + f*x)]))/(3*a^2*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2
)))/f
```

fricas [B] time = 0.90, size = 752, normalized size = 5.83

$$\left[\frac{3\left(\left(a^{2}b^{2}+2\,ab^{3}+b^{4}\right)\cos\left(fx+e\right)^{4}+a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}-2\left(a^{3}b+3\,a^{2}b^{2}+3\,ab^{3}+b^{4}\right)\cos\left(fx \cdots\right.\right.}{12\left(\left(a^{5}b \cdots\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(f*x + e)^2)
*sqrt(a)*log(2*((a^2 - 6*a*b + b^2)*cos(f*x + e)^4 + 2*(3*a^2 + 2*a*b - b^2
)*cos(f*x + e)^2 - 4*((a - b)*cos(f*x + e)^3 + (a + b)*cos(f*x + e))*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a) + a^2 + 2*a*b + b^2)/(cos(f*x + e)^4 - 2*
cos(f*x + e)^2 + 1)) - 4*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b
 + 3*a^2*b^2 + a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*
b^2 + 2*a^4*b^3 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*
b^3 + a^3*b^4)*f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 +
a^3*b^4)*f), 1/6*(3*((a^2*b^2 + 2*a*b^3 + b^4)*cos(f*x + e)^4 + a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4 - 2*(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*cos(
f*x + e)^2)*sqrt(-a)*arctan(-1/2*((a - b)*cos(f*x + e)^2 + a + b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/(a*b*cos(f*x + e)^3 - (a^2 + a*b)*cos(f*x +
 e))) - 2*((5*a^2*b^2 + 3*a*b^3)*cos(f*x + e)^3 - 3*(2*a^3*b + 3*a^2*b^2 +
a*b^3)*cos(f*x + e))*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 2*a^4*b^3
 + a^3*b^4)*f*cos(f*x + e)^4 - 2*(a^6*b + 3*a^5*b^2 + 3*a^4*b^3 + a^3*b^4)*
f*cos(f*x + e)^2 + (a^7 + 4*a^6*b + 6*a^5*b^2 + 4*a^4*b^3 + a^3*b^4)*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[-81,22]Warning, need to choose a branch for the root of a p
olynomial with parameters. This might be wrong.The choice was done assuming
 [a,b]=[53,42]Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)War
ning, replacing 0 by ` u`, a substitution variable should perhaps be purged
.Warning, replacing 0 by ` u`, a substitution variable should perhaps be pu
rged.Warning, replacing 0 by ` u`, a substitution variable should perhaps b
e purged.Evaluation time: 0.66Error: Bad Argument Type
```

maple [B] time = 3.71, size = 249, normalized size = 1.93

$$\frac{\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}\left(\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a^{2}(a+b)\sqrt{-\left(-b\left(\sin^{2}\left(fx+e\right)\right)-a\right)\left(\cos^{2}\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+(-a+b)\left(\sin^{2}\left(fx+e\right)\right)+2\sqrt{a}\sqrt{-(\cdots}}{\sin\left(fx \cdots\right.}\right.}{2a^{\frac{5}{2}} \cdots}\right.}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+ \cdots\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] (-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)*(1/a^2*b*cos(f*x+e)^2/(a+b)/(-(-b
*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2)-1/2/a^(5/2)*ln((2*a+(-a+b)*sin(f*x+e)^
2+2*a^(1/2)*(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)^(1/2))/sin(f*x+e)^2)+1/3/a*
b*(2*b*sin(f*x+e)^2+3*a+b)*cos(f*x+e)^2/(-(-b*sin(f*x+e)^2-a)*cos(f*x+e)^2)
^(1/2)/(a+b*sin(f*x+e)^2)/(a^2+2*a*b+b^2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1
/2)/f
```

maxima [B] time = 0.53, size = 306, normalized size = 2.37

$$\frac{\frac{4\,b^3\cos\left(fx+e\right)}{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a^3b^2+2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a^2b^3+\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,ab^4}+\frac{2\,b^2\cos\left(fx+e\right)}{\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}a^2b+\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}}ab^2}+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(4*b^3*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2 + a + b)*a^3*b^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*a^2*b^3 + sqrt(-b*cos(f*x + e)^2 + a + b)*a*b^4)
+ 2*b^2*cos(f*x + e)/((-b*cos(f*x + e)^2 + a + b)^(3/2)*a^2*b + (-b*cos(f*x
 + e)^2 + a + b)^(3/2)*a*b^2) + 6*b^2*cos(f*x + e)/(sqrt(-b*cos(f*x + e)^2
+ a + b)*a^3*b + sqrt(-b*cos(f*x + e)^2 + a + b)*a^2*b^2) - 3*log(b - sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x + e) - 1) - a/(cos(f*x + e) - 1
))/a^(5/2) + 3*log(-b + sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a)/(cos(f*x +
e) + 1) + a/(cos(f*x + e) + 1))/a^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.166 $\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=285

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{a(8a+9b)\sqrt{c\cdots}}{\cdots}$$

```
[Out] 1/3*a*cos(f*x+e)*sin(f*x+e)^3/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*a*(2*a
+3*b)*cos(f*x+e)*sin(f*x+e)/b^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(8*a
^2+13*a*b+3*b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^
2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/b^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-
1/3*a*(8*a+9*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^3/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 285, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 470, 578, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^3f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(2a+3b)\sin\cdots}{3b^2f(a+b)^2\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x]^3)/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*a*(2*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b^2*(a + b)^2*f*Sqrt[a +
 b*Sin[e + f*x]^2]) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*Ellipt
icE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/
(3*b^3*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (a*(8*a + 9*b)*Sqrt[Co
s[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])/(3*b^3*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2(}{\sqrt{}}\right.}{3b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(\sqrt{}}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a(}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{((-}{}$$

$$= \frac{a\cos(e+fx)\sin^3(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2a(2a+3b)\cos(e+fx)\sin(e+fx)}{3b^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a}{}$$

Mathematica [A] time = 2.14, size = 192, normalized size = 0.67

$$-\frac{a\left(\sqrt{2}b\sin(2(e+fx))\left(-8a^2+b(5a+7b)\cos(2(e+fx))-17ab-7b^2\right)+2a\left(8a^2+17ab+9b^2\right)\left(\frac{2a-b\cos(2(e+}{a}\right.\right.}{6b^3f(a+b)^2(2a-b\cos(2(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(a*(-2*a*(8*a^2 + 13*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(
3/2)*EllipticE[e + f*x, -(b/a)] + 2*a*(8*a^2 + 17*a*b + 9*b^2)*((2*a + b -
b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(-8*a^2
 - 17*a*b - 7*b^2 + b*(5*a + 7*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(b^3
*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(fx+e\right)^6-3\cos\left(fx+e\right)^4+3\cos\left(fx+e\right)^2-1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^6 - 3*cos(f*x + e)^4 + 3*cos(f*x + e)^2 - 1)*sqrt(-b
*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)
```

```
^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)
^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 1.88, size = 698, normalized size = 2.45

$$-\frac{\left(\left(5a\,b^{2}+7b^{3}\right)\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(-4a^{2}b-11a\,b^{2}-7b^{3}\right)\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{-\frac{b\left(\cos^{2}\left(fx\right.\right.}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((5*a*b^2+7*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-11*a*b^2-7*b^3)*cos
(f*x+e)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)
*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+17*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b+9*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(
f*x+e),(-1/a*b)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-3*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+25*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b+26*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2+9*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3
-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3-21*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-16*(cos(f*x+e)^2)^(1/2)*(
-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2
-3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*b^3)*a/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^3/cos(f*x+e)/
f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^6}{\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.167 $\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=269

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e\ldots}{3b^2\ldots}$$

```
[Out] 1/3*a*cos(f*x+e)*sin(f*x+e)/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a+2*b)*
cos(f*x+e)*sin(f*x+e)/b/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a+2*b)*Elli
pticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x
+e)^2)^(1/2)/b^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a+3*b)*Ellipti
cF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)
^2/a)^(1/2)/b^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 269, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3188, 470, 527, 524, 426, 424, 421, 419}

$$\frac{(2a+3b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3b^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e\ldots}{3b^2\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (a*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a + 2*b)*Cos[e + f*x]*Sin[e + f*x])/(3*b*(a + b)^2*f*Sqrt[a + b*Sin[e
 + f*x]^2]) - (2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*(a + b)^2*f*Sq
rt[1 + (b*Sin[e + f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
/(3*b^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{a+(-}{\sqrt{1-x^2}}\right.}{3b(a+b)f}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+2\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+2\right.}{}$$

$$= \frac{a\cos(e+fx)\sin(e+fx)}{3b(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a+2b)\cos(e+fx)\sin(e+fx)}{3b(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a+2b}{}$$

Mathematica [A] time = 1.71, size = 182, normalized size = 0.68

$$-\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(-a^2+b(a+2b)\cos(2(e+fx))-4ab-2b^2\right)-a\left(2a^2+5ab+3b^2\right)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2} F}{3b^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*(2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e
+ f*x, -(b/a)] - a*(2*a^2 + 5*a*b + 3*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/
a)^(3/2)*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(-a^2 - 4*a*b - 2*b^2 + b*(
a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(b^2*(a + b)^2*f*(2*a + b - b*
Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*sqrt(-b*cos(f*x + e)^2 +
a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b
 - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 1.79, size = 623, normalized size = 2.32

$$\frac{\left(2a\,b^2+4b^3\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(-a^2b-5a\,b^2-4b^3\right)\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{\frac{\cos\left(2fx+2e\right)}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*a*b^2+4*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b-5*a*b^2-4*b^3)*cos(f*x
+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*b*(
2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e
)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+8*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^
2+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*b^3-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)
^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-4*
(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/b^2/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\sin\left(e+f\,x\right)}^4}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(sin(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.168 $\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=221

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\cdots}{\cdots}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(a-b)*cos(f
*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(a-b)*(cos(f*x+e)
^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^
(1/2)/a/b/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/(a+
b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3173, 3172, 3178, 3177, 3183, 3182}

$$-\frac{(a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{\sin(e+fx)\cos(e+fx)}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\cdots}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - (
(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]
^2]) - ((a - b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a
*b*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (EllipticF[e + f*x, -(b/a)
]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]
)
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \frac{\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\int \frac{a+a\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \frac{2a^2-a(a\cdots}{\sqrt{a+b\cdots}}}{3a^2\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)\int\cdots}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((a-b)\sqrt{\cdots}\right.}{\cdots}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(a-b)\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)E\left(\cdots\right.}{3a\cdots}$$

Mathematica [A] time = 1.53, size = 174, normalized size = 0.79

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(4a^2+b(b-a)\cos(2(e+fx))+ab-b^2\right)+2a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)\cdots}{6abf(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x,
 -(b/a)] + 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF
[e + f*x, -(b/a)] - Sqrt[2]*b*(4*a^2 + a*b - b^2 + b*(-a + b)*Cos[2*(e + f*
x)])*Sin[2*(e + f*x)])/(6*a*b*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3
/2))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\left(\cos\left(fx+e\right)^2-1\right)}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*(cos(f*x + e)^2 - 1)/(b^3*cos(f*x
+ e)^6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3
*(a^2*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.75, size = 483, normalized size = 2.19

$$\frac{\left(ab^2-b^3\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(-2a^2b-ab^2+b^3\right)\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((a*b^2-b^3)*sin(f*x+e)*cos(f*x+e)^4+(-2*a^2*b-a*b^2+b^3)*cos(f*x+e)^2*
sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
b)*cos(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(
3/2)/(a+b)^2/a/b/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(sin(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Timed out
```

3.169 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+f}{3af(a+b)\left(a+b\sin^2(e+\right.}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+f}{3af(a+b)\left(a+b\sin^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a(3a\cdots}{\cdots}}{\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{\cdots}{\sqrt{a+b\cdots}}}{3a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2a\cdots\right.}{\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+\cdots}{\cdots}$$

Mathematica [A] time = 1.38, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\cdots\right.}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

maple [B] time = 1.93, size = 547, normalized size = 2.45

$$-\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}b\left(\sin^{2}\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.170 $\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=322

$$\frac{2b(3a+2b)\cot(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(3a+4b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right|}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*b*cot(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(3*a+2*b)*cot(f*x
+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a^2+13*a*b+8*b^2)*cot(f*x
+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)^2/f-1/3*(3*a^2+13*a*b+8*b^2)*Ellipti
cE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2)/a^3/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(3*a+4*b)*EllipticF(
sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/
a)^(1/2)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.41, antiderivative size = 322, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3188, 472, 579, 583, 524, 426, 424, 421, 419}

$$-\frac{\left(3a^2+13ab+8b^2\right)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)^2}-\frac{\left(3a^2+13ab+8b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\,\mathrm{s}}}{3a^3f(a+b)^2\sqrt{\frac{b\sin^2(e+}{a}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (b*Cot[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*b*(3*a +
 2*b)*Cot[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^
2 + 13*a*b + 8*b^2)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f) - ((3*a^2 + 13*a*b + 8*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)
^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((3*a + 4*b)*Sqrt[Cos[e + f*x]^2]*El
lipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]
^2)/a])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 472

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(b*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n
)^(q + 1))/(a*e*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p + 1
)), Int[(e*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*b*(m + 1) + n*(b*c
 - a*d)*(p + 1) + d*b*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a,
b, c, d, e, m, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && I
ntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{1-x^2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{}{x^2}\right.}{3a(a+b)f}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\text{co}\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2\right.}{}$$

$$= \frac{b\cot(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(3a+2b)\cot(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(3a^2\right.}{}$$

Mathematica [A] time = 2.36, size = 214, normalized size = 0.66

$$\frac{4a^2\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(\left(3a^2+7ab+4b^2\right)F\left(e+fx\middle|-\frac{b}{a}\right)-\left(3a^2+13ab+8b^2\right)E\left(e+fx\middle|-\frac{b}{a}\right)\right)-2\sqrt{2}\left(2ab\right.}{12a^3f(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (4*a^2*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(-((3*a^2 + 13*a*b + 8*b^2)
*EllipticE[e + f*x, -(b/a)]) + (3*a^2 + 7*a*b + 4*b^2)*EllipticF[e + f*x, -
(b/a)]) - 2*Sqrt[2]*(3*(a + b)^2*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f
*x] + 2*a*b^2*(a + b)*Sin[2*(e + f*x)] + b^2*(7*a + 5*b)*(2*a + b - b*Cos[2
*(e + f*x)])*Sin[2*(e + f*x)]))/(12*a^3*(a + b)^2*f*(2*a + b - b*Cos[2*(e +
 f*x)])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\csc\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*csc(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 2.04, size = 527, normalized size = 1.64

$$-\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,ab\left(3\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}+7\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\cdots}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(3*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*
b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*a^2-13*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b-8*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)*cos(f*x+e)^2+(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3
+10*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+11*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*a*b^2+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-3*EllipticE(si
n(f*x+e),(-1/a*b)^(1/2))*a^3-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-
21*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-8*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*b^3)*sin(f*x+e)+(-3*a^2*b^2-13*a*b^3-8*b^4)*cos(f*x+e)^6+(6*a^3*b
+26*a^2*b^2+38*a*b^3+16*b^4)*cos(f*x+e)^4+(-3*a^4-12*a^3*b-26*a^2*b^2-25*a*
b^3-8*b^4)*cos(f*x+e)^2)/(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/sin(f*x+e)/a^3/co
s(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin\left(e+f\,x\right)^{2}\,\left(b\,\sin\left(e+f\,x\right)^{2}+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(csc(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.171 $\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=122

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos\right.}{f}$$

```
[Out] -d*AppellF1(1/2,1/2-1/2*m,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x
+e)*(a+b-b*cos(f*x+e)^2)^p*(d*sin(f*x+e))^(-1+m)*(sin(f*x+e)^2)^(1/2-1/2*m)
/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3189, 430, 429}

$$-\frac{d\cos(e+fx)\sin^2(e+fx)^{\frac{1-m}{2}}(d\sin(e+fx))^{m-1}\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(
a + b)]*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*(d*Sin[e + f*x])^(-1 + m)
*(Sin[e + f*x]^2)^((1 - m)/2))/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3189

```
Int[((d_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[
(ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Sin[e + f*x])^(2*FracPart[(m - 1)/2]))/
(f*(Sin[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)
/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, d,
 e, f, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\sin(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx = -\frac{\left(d(d\sin(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\sin^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\mathrm{Subst}\left(\int \left(1-x^2\right)^{\frac{1}{2}}\right.}{f}$$

$$= -\frac{\left(d\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}(d\sin(e+fx))^{2}\right.}{}$$

$$= -\frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b\right.}{}$$

Mathematica [A] time = 0.47, size = 113, normalized size = 0.93

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)(d\sin(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{1}{2},-p;\frac{m+3}{2};\sin^2(e\right.}{f(m+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + m)/2, 1/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^
2)/a)]*Sqrt[Cos[e + f*x]^2]*(d*Sin[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan
[e + f*x])/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\sin\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*sin(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

maple [F] time = 2.65, size = 0, normalized size = 0.00

$$\int \left(d\sin\left(fx+e\right)\right)^m\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\sin\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*sin(f*x + e))^m, x)
```

mupad **[F]**   time = 0.00, size = -1, normalized size = -0.01

$$\int \left(d\,\sin\left(e+f\,x\right)\right)^m\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((d*sin(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

sympy **[F(-1)]**   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*sin(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.172 $\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=220

$$-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] (3*a-2*b*(2+p))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15)
-(3*a^2-4*a*b*(1+p)+4*b^2*(p^2+3*p+2))*cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hy
pergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b^2/f/(4*p^2+16*p+15)/((1-b*c
os(f*x+e)^2/(a+b))^p)-cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)*sin(f*x+e)^2/b/
f/(5+2*p)
```

Rubi [A] time = 0.22, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3186, 416, 388, 246, 245}

$$-\frac{\left(3a^2-4ab(p+1)+4b^2\left(p^2+3p+2\right)\right)\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] ((3*a - 2*b*(2 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b^2*
f*(3 + 2*p)*(5 + 2*p)) - ((3*a^2 - 4*a*b*(1 + p) + 4*b^2*(2 + 3*p + p^2))*C
os[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2, (b
*Cos[e + f*x]^2)/(a + b)])/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 - (b*Cos[e + f*x]^
2)/(a + b))^p) - (Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p)*Sin[e + f
*x]^2)/(b*f*(5 + 2*p))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
```

```
0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)^2\left(a+b-bx^2\right)^p\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}\sin^2(e+fx)}{bf(5+2p)} + \frac{\text{Subst}\left(\int \left(a\right.\right.}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e+fx}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\cos(e+fx}{}$$

$$= \frac{(3a-2b(2+p))\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)} - \frac{\left(3a^2-4ab\right.}{}$$

Mathematica [C] time = 0.55, size = 98, normalized size = 0.45

$$\frac{\sin^5(e+fx)\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(3;\frac{1}{2},-p;4;\sin^2(e+fx),-\frac{b\sin^2(}{a}\right.}{6f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3, 1/2, -p, 4, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(6*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\,\cos\left(fx+e\right)^2+1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

maple [F] time = 4.56, size = 0, normalized size = 0.00

$$\int \left(\sin^5\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \sin\left(e+f\,x\right)^5\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.173 $\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=131

$$\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{bf(2p+3)}-\frac{\cos(e+fx)\left(a\right.}{}$$

```
[Out] -cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1+p)/b/f/(3+2*p)+(a-2*b*(1+p))*cos(f*x+e)
*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2/(a+b))/b/f
/(3+2*p)/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.11, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3186, 388, 246, 245}

$$\frac{(a-2b(p+1))\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{bf(2p+3)}-\frac{\cos(e+fx)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((a
- 2*b*(1 + p))*Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[
1/2, -p, 3/2, (b*Cos[e + f*x]^2)/(a + b)])/(b*f*(3 + 2*p)*(1 - (b*Cos[e + f
*x]^2)/(a + b))^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+b-bx^2\right)^p\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\,\text{Subst}\left(\right.}{}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left((a-2b(1+p))\left(a+b\right.\right.}{}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{(a-2b(1+p))\cos(e+}{}$$

Mathematica [C] time = 0.35, size = 98, normalized size = 0.75

$$\frac{\sin^3(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(2;\frac{1}{2},-p;3;\sin^2(e+fx),-\frac{b\sin}{}\right.}{4f}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[2, 1/2, -p, 3, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[Cos[
e + f*x]^2]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(4*f*((a
+ b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

maple [F] time = 8.26, size = 0, normalized size = 0.00

$$\int \left(\sin^3\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F]   time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^3 \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^3, x)
```

mupad [F]   time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^3\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^p \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)]   time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.174 $\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=74

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -cos(f*x+e)*(a+b-b*cos(f*x+e)^2)^p*hypergeom([1/2, -p],[3/2],b*cos(f*x+e)^2
/(a+b))/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.05, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3186, 246, 245}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int\left(a+b-bx^2\right)^p\,dx,x,\cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int\left(1-\frac{bx^2}{a+b}\right)^p \ldots\right.}{f}$$

$$= -\frac{\cos(e+fx)\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\,{}_2F_1\left(\frac{1}{2},-p \ldots\right.}{f}$$

Mathematica [A] time = 0.20, size = 74, normalized size = 1.00

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] -((Cos[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p*Hypergeometric2F1[1/2, -p, 3/2
, (b*Cos[e + f*x]^2)/(a + b)])/(f*(1 - (b*Cos[e + f*x]^2)/(a + b))^p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sin\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sin(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

maple [F] time = 2.60, size = 0, normalized size = 0.00

$$\int\sin\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sin\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.175 $\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -AppellF1(1/2,1,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 1, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^p}{1-x^2}\,dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{1-x^2}\,dx\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos\right.}{f}$$

Mathematica [F] time = 4.84, size = 0, normalized size = 0.00

$$\int \csc(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

maple [F] time = 2.43, size = 0, normalized size = 0.00

$$\int \csc\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e), x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+fx\right)^2+a\right)^p}{\sin\left(e+fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x), x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.176 $\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -AppellF1(1/2,2,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.08, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 2, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{\left(a+b-bx^2\right)^p}{\left(1-x^2\right)^2}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{\left(1-x^2\right)^2}dx, x\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\cos^2(\right.}{f}$$

Mathematica [F] time = 92.38, size = 0, normalized size = 0.00

$$\int \csc^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

maple [F] time = 2.21, size = 0, normalized size = 0.00

$$\int \left(\csc^3\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.177 $\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=83

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

```
[Out] -AppellF1(1/2,3,-p,3/2,cos(f*x+e)^2,b*cos(f*x+e)^2/(a+b))*cos(f*x+e)*(a+b-b
*cos(f*x+e)^2)^p/f/((1-b*cos(f*x+e)^2/(a+b))^p)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3186, 430, 429}

$$-\frac{\cos(e+fx)\left(a-b\cos^2(e+fx)+b\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[1/2, 3, -p, 3/2, Cos[e + f*x]^2, (b*Cos[e + f*x]^2)/(a + b)]*Co
s[e + f*x]*(a + b - b*Cos[e + f*x]^2)^p)/(f*(1 - (b*Cos[e + f*x]^2)/(a + b)
)^p))
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3186

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - b*ff^2*x^2)^p, x], x, Cos[e +
f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = -\frac{\text{Subst}\left(\int \frac{(a+b-bx^2)^p}{(1-x^2)^3}dx, x, \cos(e+fx)\right)}{f}$$

$$= -\frac{\left(\left(a+b-b\cos^2(e+fx)\right)^p\left(1-\frac{b\cos^2(e+fx)}{a+b}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1-\frac{bx^2}{a+b}\right)^p}{(1-x^2)^3} d\right.}{f}$$

$$= -\frac{F_1\left(\frac{1}{2};3,-p;\frac{3}{2};\cos^2(e+fx),\frac{b\cos^2(e+fx)}{a+b}\right)\cos(e+fx)\left(a+b-b\,\text{co}\right.}{f}$$

Mathematica [F] time = 100.93, size = 0, normalized size = 0.00

$$\int \csc^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

maple [F] time = 1.33, size = 0, normalized size = 0.00

$$\int \left(\csc^5\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.178 $\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\right.}{5f}$$

```
[Out] 1/5*AppellF1(5/2,1/2,-p,7/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)^4*(a
+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^
p)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3188, 511, 510}

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\frac{x^4\left(a+bx^2\right)^p}{\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{S}}{f}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\sin^4(e+fx)}{5f}$$

Mathematica [A] time = 0.54, size = 102, normalized size = 1.01

$$\frac{\sin^4(e+fx)\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{a+b\sin^2(e+fx)}{a}\right)^{-p}F_1\left(\frac{5}{2};\frac{1}{2},-p;\frac{7}{2};\sin^2(e+fx),-\frac{b\sin^2(}{a}\right.}{5f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[5/2, 1/2, -p, 7/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*Sin[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(5*f*
((a + b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(\cos\left(fx+e\right)^4-2\cos\left(fx+e\right)^2+1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((cos(f*x + e)^4 - 2*cos(f*x + e)^2 + 1)*(-b*cos(f*x + e)^2 + a + b
)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

maple [F] time = 4.42, size = 0, normalized size = 0.00

$$\int\left(\sin^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^4\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int(sin(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.179 $\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=99

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)}{3f}$$

```
[Out] 1/3*AppellF1(3/2,2+p,-p,5/2,-tan(f*x+e)^2,-(a+b)*tan(f*x+e)^2/a)*(sec(f*x+e
)^2)^p*(a+b*sin(f*x+e)^2)^p*tan(f*x+e)^3/f/((1+(a+b)*tan(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.17, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3174, 511, 510}

$$\frac{\tan^3(e+fx)\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(\frac{(a+b)\tan^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};p+2,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[3/2, 2 + p, -p, 5/2, -Tan[e + f*x]^2, -(((a + b)*Tan[e + f*x]^2)/
a)]*(Sec[e + f*x]^2)^p*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x]^3)/(3*f*(1 + (
(a + b)*Tan[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3174

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, D
ist[(ff*(a + b*Sin[e + f*x]^2)^p*(Sec[e + f*x]^2)^p)/(f*(a + (a + b)*Tan[e
+ f*x]^2)^p), Subst[Int[((a + (a + b)*ff^2*x^2)^p*(A + (A + B)*ff^2*x^2))/(
1 + ff^2*x^2)^(p + 2), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, A
, B}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(a+(a+b)\tan^2(e+fx)\right)^{-p}\right)\mathrm{S}}{f}$$

$$= \frac{\left(\sec^2(e+fx)^p\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{(a+b)\tan^2(e+fx)}{a}\right)^{-p}\right)\mathrm{Subst}\big(}{f}$$

$$= \frac{F_1\left(\frac{3}{2};2+p,-p;\frac{5}{2};-\tan^2(e+fx),-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sec^2(e+fx)^p\,(a}{3f}$$

Mathematica [B] time = 0.69, size = 240, normalized size = 2.42

$$-\frac{2^{-p-2}\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}\sqrt{\frac{b\cos^2(e+fx)}{a+b}}\,(2a-b\cos(2(e+fx))+b)^{p+1}\left(2a(p+2)F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((2^(-2 - p)*Sqrt[(b*Cos[e + f*x]^2)/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)
])^(1 + p)*(2*a*(2 + p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2
*(e + f*x)])/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)] - (1 + p)*A
ppellF1[2 + p, 1/2, 1/2, 3 + p, (2*a + b - b*Cos[2*(e + f*x)])/(2*(a + b)),
 (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*(2*a + b - b*Cos[2*(e + f*x)]))*Csc[
2*(e + f*x)]*Sqrt[-((b*Sin[e + f*x]^2)/a)])/(b^2*f*(1 + p)*(2 + p)))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\left(\cos\left(fx+e\right)^2-1\right)\left(-b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral(-(cos(f*x + e)^2 - 1)*(-b*cos(f*x + e)^2 + a + b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

maple [F] time = 5.34, size = 0, normalized size = 0.00

$$\int \left(\sin^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p \sin\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sin(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \sin\left(e+f\,x\right)^2\,\left(b\sin\left(e+f\,x\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(sin(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.180 $\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),\right.}{f}$$

```
[Out] -AppellF1(-1/2,1/2,-p,1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)*sec(f*
x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cos^2(e+fx)}\,\csc(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, 1/2, -p, 1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(
1 + (b*Sin[e + f*x]^2)/a)^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{x^2\sqrt{1-x^2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{S}}{f}$$

$$= -\frac{F_1\left(-\frac{1}{2};\frac{1}{2},-p;\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\csc(e+fx}{f}$$

Mathematica [F] time = 4.86, size = 0, normalized size = 0.00

$$\int \csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

maple [F] time = 1.69, size = 0, normalized size = 0.00

$$\int \left(\csc^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.181 $\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),\right.}{3f}$$

```
[Out] -1/3*AppellF1(-3/2,1/2,-p,-1/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*csc(f*x+e)^3
*sec(f*x+e)*(a+b*sin(f*x+e)^2)^p*(cos(f*x+e)^2)^(1/2)/f/((1+b*sin(f*x+e)^2/
a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3188, 511, 510}

$$-\frac{\sqrt{\cos^2(e+fx)}\csc^3(e+fx)\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, 1/2, -p, -1/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sq
rt[Cos[e + f*x]^2]*Csc[e + f*x]^3*Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(3
*f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3188

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x^4\sqrt{1-x^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-}\right.}{f}$$

$$= -\frac{F_1\left(-\frac{3}{2};\frac{1}{2},-p;-\frac{1}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\csc^3(}{3f}$$

Mathematica [F] time = 6.60, size = 0, normalized size = 0.00

$$\int \csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Csc[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\csc\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*csc(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

maple [F] time = 1.19, size = 0, normalized size = 0.00

$$\int \left(\csc^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\csc\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*csc(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\sin\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^2)^p/sin(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.182 $\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=335

$$\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+}{\sqrt{a^{2/3}+\sqrt[3]{-1}}}\right.}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

```
[Out] 3/8*x/b+a*cos(d*x+c)/b^2/d-3/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)*sin
(d*x+c)^3/b/d-2/3*a^(5/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2
/3)-b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*a^(5/3
)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3
)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1)^(2/
3)*a^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-
(-1)^(2/3)*b^(2/3))^(1/2))/b^(7/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.70, antiderivative size = 335, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2638, 2635, 8, 2660, 618, 204}

$$\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\,\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+}{\sqrt{a^{2/3}+\sqrt[3]{-1}}}\right.}{3b^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*x)/(8*b) + (2*(-1)^(2/3)*a^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Ta
n[(c + d*x)/2])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)
^(2/3)*b^(2/3)]*b^(7/3)*d) - (2*a^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(7/3)*d) +
(2*(-1)^(1/3)*a^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)
]*b^(7/3)*d) + (a*Cos[c + d*x])/(b^2*d) - (3*Cos[c + d*x]*Sin[c + d*x])/(8*
b*d) - (Cos[c + d*x]*Sin[c + d*x]^3)/(4*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
```

```
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a\sin(c+dx)}{b^2}+\frac{\sin^4(c+dx)}{b}+\frac{a^2\sin(c+dx)}{b^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= -\frac{a\int \sin(c+dx)\,dx}{b^2}+\frac{a^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b^2}+\frac{\int \sin^4(c+dx)\,dx}{b}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}+\frac{a^2\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}-\frac{}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{}\right.}\right.}{}$$

$$= \frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}-\frac{a^{5/3}\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,\mathrm{s}}}{3b^{7/3}}$$

$$= \frac{3x}{8b}+\frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}-\frac{\left(2a^{5/3}\right)\mathrm{S}}{}$$

$$= \frac{3x}{8b}+\frac{a\cos(c+dx)}{b^2d}-\frac{3\cos(c+dx)\sin(c+dx)}{8bd}-\frac{\cos(c+dx)\sin^3(c+dx)}{4bd}+\frac{\left(4a^{5/3}\right)\mathrm{S}}{}$$

$$= \frac{3x}{8b}+\frac{2(-1)^{2/3}a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{7/3}d}-\frac{2a^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{7/3}d}+$$

Mathematica [C] time = 0.52, size = 219, normalized size = 0.65

$$\frac{-32a^2\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan}{\#1^4b-2\#1^2b-4i\#1a+b}\right.}{96b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^7/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (96*a*Cos[c + d*x] - 32*a^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I
)*b*#1^4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^2 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1
^2 + b*#1^4) & ] + 3*b*(12*(c + d*x) - 8*Sin[2*(c + d*x)] + Sin[4*(c + d*x)
]))/(96*b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{7}}{b\sin\left(dx+c\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^7/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.56, size = 366, normalized size = 1.09

$$\frac{3\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{2\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{11\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4db\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}+\frac{6\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a}{d\,b^{2}\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{4}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^7+2/d/b^2/(1+tan(1/2*
d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^6*a+11/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*
tan(1/2*d*x+1/2*c)^5+6/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^
4*a-11/4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^3+6/d/b^2/(1+tan
(1/2*d*x+1/2*c)^2)^4*tan(1/2*d*x+1/2*c)^2*a-3/4/d/b/(1+tan(1/2*d*x+1/2*c)^2
)^4*tan(1/2*d*x+1/2*c)+2/d/b^2/(1+tan(1/2*d*x+1/2*c)^2)^4*a+3/4/d/b*arctan(
tan(1/2*d*x+1/2*c))+2/3/d*a^2/b^2*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_
R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 15.24, size = 1978, normalized size = 5.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((150994944*a^12*b^3*sin(c/2 + (d*x)/2) - 56623104*a^13*b^2*cos(c
/2 + (d*x)/2) - 12582912*a^15*cos(c/2 + (d*x)/2) + 679477248*root(729*a^2*b
^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)*a^11*b^5*sin(c/2 +
(d*x)/2) + 679477248*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^
4 + a^10, z, k)^2*a^9*b^8*cos(c/2 + (d*x)/2) - 42467328*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^2*a^11*b^6*cos(c/2 + (d*
x)/2) - 402653184*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 +
 a^10, z, k)^2*a^13*b^4*cos(c/2 + (d*x)/2) + 4586471424*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^8*b^10*cos(c/2 + (d*
x)/2) - 503316480*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 +
 a^10, z, k)^3*a^12*b^6*cos(c/2 + (d*x)/2) + 1911029760*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^7*b^12*cos(c/2 + (d*
x)/2) + 1774190592*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4
+ a^10, z, k)^4*a^9*b^10*cos(c/2 + (d*x)/2) - 301989888*root(729*a^2*b^14*z
^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^11*b^8*cos(c/2 + (d*
x)/2) - 18345885696*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4
 + a^10, z, k)^5*a^4*b^16*cos(c/2 + (d*x)/2) + 17199267840*root(729*a^2*b^1
4*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a^6*b^14*cos(c/2 +
(d*x)/2) + 32614907904*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*
z^4 + a^10, z, k)^5*a^8*b^12*cos(c/2 + (d*x)/2) + 9172942848*root(729*a^2*b
^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^6*a^5*b^16*cos(c/2
+ (d*x)/2) + 4416602112*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10
*z^4 + a^10, z, k)^6*a^7*b^14*cos(c/2 + (d*x)/2) - 130459631616*root(729*a^
2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^7*a^4*b^18*cos(c
/2 + (d*x)/2) + 122305904640*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4
*b^10*z^4 + a^10, z, k)^7*a^6*b^16*cos(c/2 + (d*x)/2) + 1613758464*root(729
*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^2*a^10*b^7*si
n(c/2 + (d*x)/2) + 1073741824*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^
4*b^10*z^4 + a^10, z, k)^2*a^12*b^5*sin(c/2 + (d*x)/2) - 4076863488*root(72
9*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^7*b^11*s
in(c/2 + (d*x)/2) + 2420637696*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a
^4*b^10*z^4 + a^10, z, k)^3*a^9*b^9*sin(c/2 + (d*x)/2) + 4831838208*root(72
9*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^3*a^11*b^7*s
in(c/2 + (d*x)/2) + 2293235712*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a
^4*b^10*z^4 + a^10, z, k)^4*a^8*b^11*sin(c/2 + (d*x)/2) + 11475615744*root(
729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^4*a^10*b^9
*sin(c/2 + (d*x)/2) - 2293235712*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243
*a^4*b^10*z^4 + a^10, z, k)^5*a^5*b^15*sin(c/2 + (d*x)/2) - 27844411392*roo
t(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^5*a^7*b^
13*sin(c/2 + (d*x)/2) + 25367150592*root(729*a^2*b^14*z^6 - 729*b^16*z^6 +
243*a^4*b^10*z^4 + a^10, z, k)^5*a^9*b^11*sin(c/2 + (d*x)/2) + 16307453952*
root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^6*a^8
*b^13*sin(c/2 + (d*x)/2) - 40768634880*root(729*a^2*b^14*z^6 - 729*b^16*z^6
 + 243*a^4*b^10*z^4 + a^10, z, k)^7*a^5*b^17*sin(c/2 + (d*x)/2) + 326149079
04*root(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)^7*
a^7*b^15*sin(c/2 + (d*x)/2) + 33554432*root(729*a^2*b^14*z^6 - 729*b^16*z^6
 + 243*a^4*b^10*z^4 + a^10, z, k)*a^15*b*sin(c/2 + (d*x)/2) - 70778880*root
(729*a^2*b^14*z^6 - 729*b^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k)*a^12*b^4*
cos(c/2 + (d*x)/2))/(b^9*cos(c/2 + (d*x)/2)))*root(729*a^2*b^14*z^6 - 729*b
^16*z^6 + 243*a^4*b^10*z^4 + a^10, z, k), k, 1, 6)/d + (log((cos(c/2 + (d*x
)/2)*1i + sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2))*3i)/(8*b*d) - (log((cos(c
/2 + (d*x)/2)*1i - sin(c/2 + (d*x)/2))/cos(c/2 + (d*x)/2))*3i)/(8*b*d) - si
n(2*c + 2*d*x)/(4*b*d) + sin(4*c + 4*d*x)/(32*b*d) + (a*cos(c + d*x))/(b^2*
d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.183 $\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=273

$$-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\sin(c$$

```
[Out] 1/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-2/3*a*arctan((b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(5/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3
*a*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2
/3)+b^(2/3))^(1/2))/b^(5/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*a*arct
anh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^
(2/3))^(1/2))/b^(5/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.57, antiderivative size = 273, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3220, 2635, 8, 2660, 618, 204, 206}

$$-\frac{2a\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{5/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2a\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{5/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}-\sin(c$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] x/(2*b) - (2*a*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b
^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3) - (
-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])
/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(5/3)*d) + (2*a*ArcTanh[(b^(1/3
) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]
])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(5/3)*d) - (Cos[c + d*x]*Sin[c +
 d*x])/(2*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin^2(c+dx)}{b} - \frac{a\sin^2(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sin^2(c+dx)\,dx}{b} - \frac{a\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\int 1\,dx}{2b} - \frac{a\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin\right.}\right.}{b}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{a\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{5/3}} - \frac{a\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{5/3}}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{(2a)\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right.}{3b^{5/3}d}$$

$$= \frac{x}{2b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{(4a)\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\right.}{3b^{5/3}d}$$

$$= \frac{x}{2b} - \frac{2a\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{5/3}d} + \frac{2a\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{5/3}d} + \frac{2a\tanh}{3}$$

Mathematica [C] time = 0.28, size = 255, normalized size = 0.93

$$\frac{-2ia\mathrm{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2\right.}{}\right.}{12}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (6*(c + d*x) - (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 -
(4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] - 3*Sin[2*(c + d*x)])/(12*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^5}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.57, size = 163, normalized size = 0.60

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\frac{\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}-\frac{4a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+\right.}\right.}{3db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/d/b/(1+tan(1/2*d*x+1/2*c)^2)^2*tan(1/2*d*x+1/2*c)^3-1/d/b/(1+tan(1/2*d*x+
1/2*c)^2)^2*tan(1/2*d*x+1/2*c)+1/d/b*arctan(tan(1/2*d*x+1/2*c))-4/3/d*a/b*s
um(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf
(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.47, size = 1962, normalized size = 7.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

```
[Out] tan(c/2 + (d*x)/2)^3/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(c/2 + (d*x
)/2)^4) - tan(c/2 + (d*x)/2)/(b*d + 2*b*d*tan(c/2 + (d*x)/2)^2 + b*d*tan(c/
2 + (d*x)/2)^4) + symsum(log((134217728*a^9*b^2 - 16777216*a^11 - 402653184
*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 +
a^6, z, k)*a^8*b^4 + 50331648*a^10*b*tan(c/2 + (d*x)/2) - 2415919104*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^2*a^7*b^6 + 914358272*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^
8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^2*a^9*b^4 + 7247757312*root(729*a^2*b^1
0*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^3*a^6*
b^8 - 478150656*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27
*a^4*b^4*z^2 + a^6, z, k)^3*a^8*b^6 + 10871635968*root(729*a^2*b^10*z^6 - 7
29*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^5*b^10 - 21
214789632*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b
^4*z^2 + a^6, z, k)^4*a^7*b^8 - 301989888*root(729*a^2*b^10*z^6 - 729*b^12*
z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^9*b^6 - 32614907904
*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 +
a^6, z, k)^5*a^4*b^12 + 59567505408*root(729*a^2*b^10*z^6 - 729*b^12*z^6 +
243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^5*a^6*b^10 + 4529848320*root(
729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z
, k)^5*a^8*b^8 + 55717134336*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2
*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6*a^5*b^12 - 42127589376*root(729*a^
2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6
*a^7*b^10 - 130459631616*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8
*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^7*a^4*b^14 + 122305904640*root(729*a^2*b
^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^7*a^
6*b^12 - 452984832*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 -
 27*a^4*b^4*z^2 + a^6, z, k)*a^9*b^3*tan(c/2 + (d*x)/2) + 1509949440*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^2*a^8*b^5*tan(c/2 + (d*x)/2) + 201326592*root(729*a^2*b^10*z^6 - 729*b^
12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^2*a^10*b^3*tan(c/2 +
 (d*x)/2) - 2717908992*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z
^4 - 27*a^4*b^4*z^2 + a^6, z, k)^3*a^7*b^7*tan(c/2 + (d*x)/2) - 2717908992*
root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a
^6, z, k)^3*a^9*b^5*tan(c/2 + (d*x)/2) + 4076863488*root(729*a^2*b^10*z^6 -
 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^6*b^9*tan
(c/2 + (d*x)/2) + 6039797760*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2
*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^4*a^8*b^7*tan(c/2 + (d*x)/2) - 40768
63488*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z
^2 + a^6, z, k)^5*a^5*b^11*tan(c/2 + (d*x)/2) - 679477248*root(729*a^2*b^10
*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^5*a^7*b
^9*tan(c/2 + (d*x)/2) + 16307453952*root(729*a^2*b^10*z^6 - 729*b^12*z^6 +
243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)^6*a^6*b^11*tan(c/2 + (d*x)/2)
 - 40768634880*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*
a^4*b^4*z^2 + a^6, z, k)^7*a^5*b^13*tan(c/2 + (d*x)/2) + 32614907904*root(7
29*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z,
 k)^7*a^7*b^11*tan(c/2 + (d*x)/2) + 33554432*root(729*a^2*b^10*z^6 - 729*b^
12*z^6 + 243*a^2*b^8*z^4 - 27*a^4*b^4*z^2 + a^6, z, k)*a^11*b*tan(c/2 + (d*
x)/2))/b^5)*root(729*a^2*b^10*z^6 - 729*b^12*z^6 + 243*a^2*b^8*z^4 - 27*a^4
*b^4*z^2 + a^6, z, k), k, 1, 6)/d - (log(tan(c/2 + (d*x)/2) - 1i)*1i)/(2*b*
d) + (log(tan(c/2 + (d*x)/2) + 1i)*1i)/(2*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.184 $\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=259

$$-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\cdots}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right.}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] x/b-2/3*a^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3
))^(1/2))/b/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*a^(1/3)*arctan(((-1)^(2/3)*b^(1/3
)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b/d/(a^(2
/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*a^(1/3)*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(
2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b/d/(a
^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.46, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3220, 3213, 2660, 618, 204}

$$-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3bd\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\cdots}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right.}{3bd\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] x/b - (2*a^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) -
 b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b*d) - (2*a^(1/3)*ArcTan[((-1)^(2/3)
*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(
3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b*d) + (2*a^(1/3)*ArcTan[((-1)^(1/3)*(
b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
```

```
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{b} - \frac{a}{b\left(a+b\sin^3(c+dx)\right)}\right) dx$$

$$= \frac{x}{b} - \frac{a\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= \frac{x}{b} - \frac{a\int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} - \frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin\right.}}{b}$$

$$= \frac{x}{b} + \frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b} + \frac{\sqrt[3]{a}\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}}}{3b}$$

$$= \frac{x}{b} + \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd} + \frac{\left(2\sqrt[3]{a}\right)\text{Subst}\left(\int \right.}{}$$

$$= \frac{x}{b} - \frac{\left(4\sqrt[3]{a}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd} - \frac{\left(4\sqrt[3]{a}\right)}{}$$

$$= \frac{x}{b} + \frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,bd} - \frac{2\sqrt[3]{a}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,bd} - \frac{2\sqrt[3]{a}}{}$$

Mathematica [C] time = 0.18, size = 140, normalized size = 0.54

$$\frac{2ia\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]+3c+3}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (3*c + 3*d*x + (2*I)*a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#
1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/
(3*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^3}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.47, size = 106, normalized size = 0.41

$$\frac{2\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db}-\frac{a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 2/d/b*arctan(tan(1/2*d*x+1/2*c))-1/3/d*a/b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R
^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_
Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{-8\,ab\int\frac{}{b^3\cos(6\,dx+6\,c)^2+9\,b^3\cos(4\,dx+4\,c)^2+64\,a^2b\cos(3\,dx+3\,c)^2+9\,b^3\cos(2\,dx+2\,c)^2+b^3\sin(6\,dx+6\,c)^2+9\,b^3\sin(4\,dx+4\,c)^2+64\,a^2b\sin(3\,dx+3\,c}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] (8*a*b*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)*sin(6*d*x +
6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6*c)*sin(3*d*x
 + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*x + 3*c)^2 -
3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c) - b)*sin(3*d*
x + 3*c))/(b^3*cos(6*d*x + 6*c)^2 + 9*b^3*cos(4*d*x + 4*c)^2 + 64*a^2*b*cos
(3*d*x + 3*c)^2 + 9*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(6*d*x + 6*c)^2 + 9*b^3
*sin(4*d*x + 4*c)^2 + 64*a^2*b*sin(3*d*x + 3*c)^2 - 48*a*b^2*cos(3*d*x + 3*
c)*sin(2*d*x + 2*c) + 9*b^3*sin(2*d*x + 2*c)^2 - 6*b^3*cos(2*d*x + 2*c) + b
^3 - 2*(3*b^3*cos(4*d*x + 4*c) - 3*b^3*cos(2*d*x + 2*c) - 8*a*b^2*sin(3*d*x
 + 3*c) + b^3)*cos(6*d*x + 6*c) - 6*(3*b^3*cos(2*d*x + 2*c) + 8*a*b^2*sin(3
*d*x + 3*c) - b^3)*cos(4*d*x + 4*c) - 2*(8*a*b^2*cos(3*d*x + 3*c) + 3*b^3*s
in(4*d*x + 4*c) - 3*b^3*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(8*a*b^2*cos
(3*d*x + 3*c) - 3*b^3*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) + 16*(3*a*b^2*cos(
2*d*x + 2*c) - a*b^2)*sin(3*d*x + 3*c)), x) + x)/b
```

mupad [B] time = 14.90, size = 1672, normalized size = 6.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(134217728*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 +
 27*a^2*b^2*z^2 + a^2, z, k)*a^7*tan(c/2 + (d*x)/2) - 268435456*root(729*a^
2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^2*a
^7*b - 1073741824*a^6*tan(c/2 + (d*x)/2) + 4831838208*root(729*a^2*b^6*z^6
- 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^2*a^5*b^3 + 3
3722204160*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^
2*z^2 + a^2, z, k)^3*a^6*b^3 + 15703474176*root(729*a^2*b^6*z^6 - 729*b^8*z
^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^4*a^5*b^5 - 4831838208*r
oot(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2,
 z, k)^4*a^7*b^3 - 130459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^
2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*a^4*b^7 + 154014842880*root(729*a
^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*
a^6*b^5 + 35332816896*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4
+ 27*a^2*b^2*z^2 + a^2, z, k)^6*a^5*b^7 - 21743271936*root(729*a^2*b^6*z^6
- 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^7*b^5 - 1
30459631616*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b
^2*z^2 + a^2, z, k)^7*a^4*b^9 + 122305904640*root(729*a^2*b^6*z^6 - 729*b^8
*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^6*b^7 + 2013265920
*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^
2, z, k)*a^6*b - 3221225472*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^
4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)*a^5*b^2*tan(c/2 + (d*x)/2) - 1858915532
8*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a
^2, z, k)^2*a^6*b^2*tan(c/2 + (d*x)/2) - 17716740096*root(729*a^2*b^6*z^6 -
 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^3*a^5*b^4*tan(
c/2 + (d*x)/2) + 2818572288*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^
4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^3*a^7*b^2*tan(c/2 + (d*x)/2) + 86973087
744*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 +
 a^2, z, k)^4*a^4*b^6*tan(c/2 + (d*x)/2) - 88181047296*root(729*a^2*b^6*z^6
 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^4*a^6*b^4*ta
n(c/2 + (d*x)/2) - 30802968576*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2
*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^5*a^5*b^6*tan(c/2 + (d*x)/2) + 18119
393280*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^
2 + a^2, z, k)^5*a^7*b^4*tan(c/2 + (d*x)/2) + 86973087744*root(729*a^2*b^6*
z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^4*b^8
*tan(c/2 + (d*x)/2) - 70665633792*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*
a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^6*a^6*b^6*tan(c/2 + (d*x)/2) - 40
768634880*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2
*z^2 + a^2, z, k)^7*a^5*b^8*tan(c/2 + (d*x)/2) + 32614907904*root(729*a^2*b
^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^4 + 27*a^2*b^2*z^2 + a^2, z, k)^7*a^7*
b^6*tan(c/2 + (d*x)/2))*root(729*a^2*b^6*z^6 - 729*b^8*z^6 + 243*a^2*b^4*z^
4 + 27*a^2*b^2*z^2 + a^2, z, k), k, 1, 6)/d - (log(tan(c/2 + (d*x)/2) - 1i)
*1i)/(b*d) + (log(tan(c/2 + (d*x)/2) + 1i)*1i)/(b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.185 $\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=267

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

```
[Out] -2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a
^(1/3)/b^(1/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*(-1)^(1/3)*arctan(((-1)^(2/3)*
b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(
1/3)/b^(1/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)+2/3*(-1)^(2/3)*arctan(((-
1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2))/a^(1/3)/b^(1/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3220, 2660, 618, 204}

$$\frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt[3]{b}d\sqrt{a^{2/3}+\sqrt[3]{-1}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a
^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)
]*b^(1/3)*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
- b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) - b^(2/3)]*b^(1/3)*d) + (2*(-1)^(1/3)*
ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^
(1/3)*b^(2/3)]])/(3*a^(1/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(1/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
```

```
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \cdots\right)$$

$$= -\frac{\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} + \frac{\sqrt[3]{-1}\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3\sqrt[3]{a}\sqrt[3]{b}} - \frac{(-1)^{2/3}\int \frac{1}{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(\cdots}}{3\sqrt[3]{a}\sqrt[3]{b}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} + \frac{\left(2\sqrt[3]{-1}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2(-1)^{2/3}\cdots}\right.}{3\sqrt[3]{\cdots}}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3\sqrt[3]{a}\sqrt[3]{b}d} - \frac{\left(4\sqrt[3]{-1}\right)\mathrm{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{2(-1)^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\sqrt[3]{b}d} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt[3]{a}\sqrt{a^{2/3}-b^{2/3}}\sqrt[3]{b}d} + \frac{2\sqrt[3]{-1}\tan\cdots}{3\sqrt[3]{a}\cdots}$$

Mathematica [C] time = 0.18, size = 172, normalized size = 0.64

$$-\frac{\mathrm{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan^{-}\cdots}{\#1^4b-2\#1^2b-4i\#1a+b}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/3*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Co
s[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ]/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.53, size = 78, normalized size = 0.29

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 2/3/d*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

mupad [B] time = 16.12, size = 652, normalized size = 2.44

$$\sum_{k=1}^{6}\ln\left(-8192\,a^3\,b+\mathrm{root}\left(729\,a^4\,b^2\,d^6-729\,a^2\,b^4\,d^6+243\,a^2\,b^2\,d^4+1,d,k\right)^2\,a^3\,b^3\,294912+\mathrm{root}\left(729\,a^4\,b^2\,d^6\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4
+ 1, d, k)^2*a^3*b^3 - 8192*a^3*b + 1548288*root(729*a^4*b^2*d^6 - 729*a^2*
b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^4*b^3 + 1990656*root(729*a^4*b^2*d
^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^4*a^5*b^3 - 7962624*root(
729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^4*b^5 +
5971968*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)
^5*a^6*b^3 + 65536*a^2*b^2*tan(c/2 + (d*x)/2) + 196608*root(729*a^4*b^2*d^6
 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)*a^3*b^2*tan(c/2 + (d*x)/2)
+ 294912*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k
)^2*a^4*b^2*tan(c/2 + (d*x)/2) - 1769472*root(729*a^4*b^2*d^6 - 729*a^2*b^4
*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) + 221184*roo
t(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^3*a^5*b^2*
tan(c/2 + (d*x)/2) + 2654208*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a
^2*b^2*d^4 + 1, d, k)^4*a^4*b^4*tan(c/2 + (d*x)/2) - 1990656*root(729*a^4*b
^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k)^5*a^5*b^4*tan(c/2 + (
d*x)/2))*root(729*a^4*b^2*d^6 - 729*a^2*b^4*d^6 + 243*a^2*b^2*d^4 + 1, d, k
), k, 1, 6)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.186 $\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=264

$$-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}$$

```
[Out] -arctanh(cos(d*x+c))/a/d-2/3*b^(1/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/
2*c))/(a^(2/3)-b^(2/3))^(1/2))/a/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*b^(1/3)*arct
anh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(
2/3))^(1/2))/a/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*b^(1/3)*arctanh((b^
(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^
(1/2))/a/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.36, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3220, 3770, 2660, 618, 204, 206}

$$-\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3ad\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3ad\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}+\frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3ad\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*b^(1/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2
/3)]])/(3*a*Sqrt[a^(2/3) - b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (2*b
^(1/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^
(2/3)*a^(2/3)) + b^(2/3)]])/(3*a*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*d) +
 (2*b^(1/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc(c+dx)}{a} - \frac{b\sin^2(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right) dx$$

$$= \frac{\int \csc(c+dx)\,dx}{a} - \frac{b\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{b\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}(}\right.}{a}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\sqrt[3]{b}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a} - \frac{\sqrt[3]{b}\int \frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a} - \frac{\sqrt[3]{}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} - \frac{\left(2\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3ad} - \frac{(2}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\left(4\sqrt[3]{b}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b} + 2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+\right.\right.}{3ad}$$

$$= -\frac{2\sqrt[3]{b}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{2\sqrt[3]{b}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan}{\sqrt{-(-1)^{2/3}a^{2/3}}}\right.}{3a\sqrt{-(-1)^{2/3}a^{2/3}+b}}$$

Mathematica [C] time = 0.27, size = 264, normalized size = 1.00

$$-\frac{ib\text{RootSum}\left[i\#1^6b - 3i\#1^4b + 8\#1^3a + 3i\#1^2b - ib\&, \frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#1\,c\right.}{\#1}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/6*(6*Log[Cos[(c + d*x)/2]] - 6*Log[Sin[(c + d*x)/2]] + I*b*RootSum[(-I)*
b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 4*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*Log[1 - 2
```

```
*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) &
])/(a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.76, size = 98, normalized size = 0.37

$$\frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{4b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^3),x)
[Out] 1/a/d*ln(tan(1/2*d*x+1/2*c))-4/3/d/a*b*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_
R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+
a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 15.66, size = 1439, normalized size = 5.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^3)),x)
[Out] symsum(log(98304*b^5 + 1048576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4
*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)*b^6*tan(c/2 + (d*x)/2) - 98304*root(
729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z,
k)^2*a^2*b^5 + 5898240*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4
 + 27*a^2*b^2*z^2 - b^2, z, k)^3*a^3*b^5 - 7962624*root(729*a^6*b^2*z^6 - 7
29*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^4*a^4*b^5 - 6635
52*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 -
```

```
b^2, z, k)^4*a^6*b^3 - 5308416*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4
*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^5*b^5 + 10616832*root(729*a^6*b^
2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^7*b
^3 + 7962624*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*
b^2*z^2 - b^2, z, k)^6*a^6*b^5 - 9953280*root(729*a^6*b^2*z^6 - 729*a^8*z^6
 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^8*b^3 - 589824*root(72
9*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)
*a*b^5 - 24576*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^
2*b^2*z^2 - b^2, z, k)*a^2*b^4*tan(c/2 + (d*x)/2) - 3145728*root(729*a^6*b^
2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^2*a*b^6
*tan(c/2 + (d*x)/2) + 466944*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b
^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^2*a^3*b^4*tan(c/2 + (d*x)/2) - 1887436
8*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b
^2, z, k)^3*a^2*b^6*tan(c/2 + (d*x)/2) - 3981312*root(729*a^6*b^2*z^6 - 729
*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^3*a^4*b^4*tan(c/2
+ (d*x)/2) + 56623104*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4
+ 27*a^2*b^2*z^2 - b^2, z, k)^4*a^3*b^6*tan(c/2 + (d*x)/2) + 20791296*root(
729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z,
k)^4*a^5*b^4*tan(c/2 + (d*x)/2) + 84934656*root(729*a^6*b^2*z^6 - 729*a^8*z
^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^5*a^4*b^6*tan(c/2 + (d*x
)/2) - 78962688*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a
^2*b^2*z^2 - b^2, z, k)^5*a^6*b^4*tan(c/2 + (d*x)/2) - 254803968*root(729*a
^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*
a^5*b^6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^6*b^2*z^6 - 729*a^8*z^6 -
 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b^2, z, k)^6*a^7*b^4*tan(c/2 + (d*x)/2)
)*root(729*a^6*b^2*z^6 - 729*a^8*z^6 - 243*a^4*b^2*z^4 + 27*a^2*b^2*z^2 - b
^2, z, k), k, 1, 6)/d + log(tan(c/2 + (d*x)/2))/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.187 $\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=287

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/2*cot(d*x+c)*csc(d*x+c)/a/d-2/3*b*arctan((b^
(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/
3)-b^(2/3))^(1/2)-2/3*b*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*
c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/
3))^(1/2)+2/3*b*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1
/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(5/3)/d/(a^(2/3)-(-1)^(2/3)*b^
(2/3))^(1/2)
```

Rubi [A] time = 0.40, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 3768, 3770, 3213, 2660, 618, 204}

$$-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}+\frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*b*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])
/(3*a^(5/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*b*ArcTan[((-1)^(2/3)*b^(1/3) +
a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(5/3)*S
qrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) + (2*b*ArcTan[((-1)^(1/3)*(b^(1/3) + (
-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(
3*a^(5/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d) - ArcTanh[Cos[c + d*x]]/(2*
a*d) - (Cot[c + d*x]*Csc[c + d*x])/(2*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx &= \int \left(\frac{\csc^3(c+dx)}{a} - \frac{b}{a\left(a+b\sin^3(c+dx)\right)}\right) dx \\
&= \frac{\int \csc^3(c+dx)\,dx}{a} - \frac{b\int \frac{1}{a+b\sin^3(c+dx)}\,dx}{a} \\
&= -\frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\int \csc(c+dx)\,dx}{2a} - \frac{b\int\left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)} - \frac{}{3a^{2/3}\left(-\sqrt[3]{a}\right.}\right.}{} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{b\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{5/3}} + \frac{b\int \frac{}{-\sqrt[3]{a}+}}{} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} + \frac{(2b)\operatorname{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x\right.}{3a^{5/3}d} \\
&= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad} - \frac{\cot(c+dx)\csc(c+dx)}{2ad} - \frac{(4b)\operatorname{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -\right.}{3a^{5/3}d} \\
&= \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2b\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{5/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2b\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a}}\right.}{3a^{5/3}\sqrt{a^{2/3}+}}
\end{aligned}
$$

Mathematica [C] time = 0.40, size = 181, normalized size = 0.63

$$
\frac{-3\left(\csc^2\left(\frac{1}{2}(c+dx)\right)-\sec^2\left(\frac{1}{2}(c+dx)\right)-4\log\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right)\right)+16ib\operatorname{RootSum}\left[\#1\right.}{24ad}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((16*I)*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] - 3*(Csc[(c + d*x)/2]^2
+ 4*Log[Cos[(c + d*x)/2]] - 4*Log[Sin[(c + d*x)/2]] - Sec[(c + d*x)/2]^2))/
(24*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^3}{b\sin(dx+c)^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^3/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.80, size = 144, normalized size = 0.50

$$\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad} - \frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2} + \frac{\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2ad} - \frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\right.}{\_R^5a+2\_R^3a+4\_}\right.}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/8/a/d/tan(1/2*d*x+1/2*c)^2+1/2/a/d*ln(tan(1/
2*d*x+1/2*c))-1/3/d/a*b*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)
*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
+ 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 32*(a*b*d*
cos(4*d*x + 4*c)^2 + 4*a*b*d*cos(2*d*x + 2*c)^2 + a*b*d*sin(4*d*x + 4*c)^2
- 4*a*b*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*b*d*sin(2*d*x + 2*c)^2 -
4*a*b*d*cos(2*d*x + 2*c) + a*b*d - 2*(2*a*b*d*cos(2*d*x + 2*c) - a*b*d)*cos
(4*d*x + 4*c))*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)*sin(
6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6*c)*s
in(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*x + 3
*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c) - b)*
sin(3*d*x + 3*c))/(a*b^2*cos(6*d*x + 6*c)^2 + 9*a*b^2*cos(4*d*x + 4*c)^2 +
```

```
64*a^3*cos(3*d*x + 3*c)^2 + 9*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(6*d*x +
6*c)^2 + 9*a*b^2*sin(4*d*x + 4*c)^2 + 64*a^3*sin(3*d*x + 3*c)^2 - 48*a^2*b*
cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*a*b^2*sin(2*d*x + 2*c)^2 - 6*a*b^2*co
s(2*d*x + 2*c) + a*b^2 - 2*(3*a*b^2*cos(4*d*x + 4*c) - 3*a*b^2*cos(2*d*x +
2*c) - 8*a^2*b*sin(3*d*x + 3*c) + a*b^2)*cos(6*d*x + 6*c) - 6*(3*a*b^2*cos(
2*d*x + 2*c) + 8*a^2*b*sin(3*d*x + 3*c) - a*b^2)*cos(4*d*x + 4*c) - 2*(8*a^
2*b*cos(3*d*x + 3*c) + 3*a*b^2*sin(4*d*x + 4*c) - 3*a*b^2*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c) + 6*(8*a^2*b*cos(3*d*x + 3*c) - 3*a*b^2*sin(2*d*x + 2*c))
*sin(4*d*x + 4*c) + 16*(3*a^2*b*cos(2*d*x + 2*c) - a^2*b)*sin(3*d*x + 3*c))
, x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 -
4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^2 + 2*co
s(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - (2*
(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4*c)^2 - 4*cos(2*d*
x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 4*s
in(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^2 - 2*cos(d*x)*cos
(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) + 4*(sin(3*d*x
+ 3*c) + sin(d*x + c))*sin(4*d*x + 4*c) - 8*sin(3*d*x + 3*c)*sin(2*d*x + 2*
c) - 8*sin(2*d*x + 2*c)*sin(d*x + c) + 4*cos(d*x + c))/(a*d*cos(4*d*x + 4*c
)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*a*d*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*cos(2*d*x + 2*c)
+ a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))
```

mupad [B] time = 15.03, size = 1573, normalized size = 5.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log(-(65536*a*b^9 - 262144*b^10*tan(c/2 + (d*x)/2) - 131072*root(729
*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k
)*a^2*b^9 - 61440*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 -
27*a^4*b^4*z^2 - b^6, z, k)*a^4*b^7 + 860160*root(729*a^10*b^2*z^6 - 729*a^
12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^5*b^7 - 3244032*
root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b
^6, z, k)^3*a^6*b^7 - 1105920*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^
8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^3*a^8*b^5 + 3538944*root(729*a^10*b
^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^4*a^7
*b^7 + 3870720*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*
a^4*b^4*z^2 - b^6, z, k)^4*a^9*b^5 + 663552*root(729*a^10*b^2*z^6 - 729*a^1
2*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^5*a^10*b^5 - 4976640*
root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b
^6, z, k)^5*a^12*b^3 - 7962624*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a
^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^11*b^5 + 9953280*root(729*a^10
*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a
^13*b^3 + 24576*a^2*b^8*tan(c/2 + (d*x)/2) + 540672*root(729*a^10*b^2*z^6 -
 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)*a^3*b^8*tan(c
/2 + (d*x)/2) - 7077888*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*
z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^2*a^4*b^8*tan(c/2 + (d*x)/2) + 442368*roo
t(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6,
 z, k)^2*a^6*b^6*tan(c/2 + (d*x)/2) - 2359296*root(729*a^10*b^2*z^6 - 729*a
^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^3*a^5*b^8*tan(c/2 +
 (d*x)/2) + 7741440*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4
- 27*a^4*b^4*z^2 - b^6, z, k)^3*a^7*b^6*tan(c/2 + (d*x)/2) - 80953344*root(
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k)^4*a^8*b^6*tan(c/2 + (d*x)/2) + 1990656*root(729*a^10*b^2*z^6 - 729*a^1
2*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^4*a^10*b^4*tan(c/2 +
(d*x)/2) - 31850496*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4
- 27*a^4*b^4*z^2 - b^6, z, k)^5*a^9*b^6*tan(c/2 + (d*x)/2) + 26873856*root(
```

```
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k)^5*a^11*b^4*tan(c/2 + (d*x)/2) + 254803968*root(729*a^10*b^2*z^6 - 729*
a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^10*b^6*tan(c/2
 + (d*x)/2) - 252813312*root(729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*
z^4 - 27*a^4*b^4*z^2 - b^6, z, k)^6*a^12*b^4*tan(c/2 + (d*x)/2))/a^5)*root(
729*a^10*b^2*z^6 - 729*a^12*z^6 - 243*a^8*b^2*z^4 - 27*a^4*b^4*z^2 - b^6, z
, k), k, 1, 6)/d - cot(c/2 + (d*x)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)^2/(8*a
*d) + log(tan(c/2 + (d*x)/2))/(2*a*d)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

3.188 $\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=344

$$\frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

```
[Out] -3/8*arctanh(cos(d*x+c))/a/d+b*cot(d*x+c)/a^2/d-3/8*cot(d*x+c)*csc(d*x+c)/a
/d-1/4*cot(d*x+c)*csc(d*x+c)^3/a/d-2/3*b^(5/3)*arctan((b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-b^(2/3))^(1/2)+
2/3*(-1)^(1/3)*b^(5/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c
))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3
))^(1/2)+2/3*(-1)^(2/3)*b^(5/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*
d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(7/3)/d/(a^(2/3)-(-1)^(2/
3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3220, 3767, 8, 3768, 3770, 2660, 618, 204}

$$\frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\sqrt[3]{-1}\,b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{7/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*(-1)^(2/3)*b^(5/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - (-1)^(2/3)
*b^(2/3)]*d) - (2*b^(5/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*(-1)^(1/3)*
b^(5/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 + (-1)^(1/3)*b^(2/3)]])/(3*a^(7/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) -
 (3*ArcTanh[Cos[c + d*x]])/(8*a*d) + (b*Cot[c + d*x])/(a^2*d) - (3*Cot[c +
d*x]*Csc[c + d*x])/(8*a*d) - (Cot[c + d*x]*Csc[c + d*x]^3)/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc^2(c+dx)}{a^2}+\frac{\csc^5(c+dx)}{a}+\frac{b^2\sin(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^5(c+dx)\,dx}{a}-\frac{b\int \csc^2(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= -\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\int \csc^3(c+dx)\,dx}{4a}+\frac{b^2\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}\right.\ldots}{\ldots}$$

$$= \frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\csc^3(c+dx)}{4ad}+\frac{3\int \csc(c+\ldots}{8a}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\,\ldots}{4a\ldots}$$

$$= -\frac{3\tanh^{-1}(\cos(c+dx))}{8ad}+\frac{b\cot(c+dx)}{a^2d}-\frac{3\cot(c+dx)\csc(c+dx)}{8ad}-\frac{\cot(c+dx)\,\ldots}{4a\ldots}$$

$$= \frac{2(-1)^{2/3}b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}-\frac{2b^{5/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{7/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\sqrt[3]{\ldots}}{\ldots}$$

Mathematica [C] time = 2.02, size = 290, normalized size = 0.84

$$\frac{3\left(-a\csc^4\left(\frac{1}{2}(c+dx)\right)-6a\csc^2\left(\frac{1}{2}(c+dx)\right)+a\sec^4\left(\frac{1}{2}(c+dx)\right)+6a\sec^2\left(\frac{1}{2}(c+dx)\right)+24a\log\left(\sin\left(\frac{1}{2}(c+dx)\right.\right.\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-64*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6 & , (-2*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ] + 3*(32*b*Co
t[(c + d*x)/2] - 6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[C
os[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 6*a*Sec[(c + d*x)/2]^2 + a*
Sec[(c + d*x)/2]^4 - 32*b*Tan[(c + d*x)/2]))/(192*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^5}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^5/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.82, size = 217, normalized size = 0.63

$$\frac{\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)}{64da}+\frac{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)b}{2d\,a^2}-\frac{1}{64da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^4}-\frac{1}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^2}+\frac{3\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8ad}+\frac{}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/64/d/a*tan(1/2*d*x+1/2*c)^4+1/8/a/d*tan(1/2*d*x+1/2*c)^2-1/2/d/a^2*tan(1/
2*d*x+1/2*c)*b-1/64/d/a/tan(1/2*d*x+1/2*c)^4-1/8/a/d/tan(1/2*d*x+1/2*c)^2+3
/8/a/d*ln(tan(1/2*d*x+1/2*c))+1/2/d*b/a^2/tan(1/2*d*x+1/2*c)+2/3/d*b^2/a^2*
sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=
RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 14.60, size = 1560, normalized size = 4.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log((262144*b^14*tan(c/2 + (d*x)/2) - 3072*a^3*b^11 + 155648*root(72
9*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^4*b^11 - 3
93216*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)
^2*a^5*b^11 + 774144*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^
4 - b^10, z, k)^2*a^7*b^9 - 2064384*root(729*a^14*b^2*z^6 - 729*a^16*z^6 -
243*a^10*b^4*z^4 - b^10, z, k)^3*a^8*b^9 + 2073600*root(729*a^14*b^2*z^6 -
729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^10*b^7 - 9510912*root(729
*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^4*a^11*b^7 +
2737152*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
k)^4*a^13*b^5 + 10616832*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^
4*z^4 - b^10, z, k)^5*a^12*b^7 - 10285056*root(729*a^14*b^2*z^6 - 729*a^16*
z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^14*b^5 + 3732480*root(729*a^14*b^2
*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^16*b^3 + 7962624*r
oot(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^6*a^15
*b^5 - 9953280*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^
10, z, k)^6*a^17*b^3 + 98304*a^2*b^12*tan(c/2 + (d*x)/2) - 262144*root(729*
a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)*a^3*b^12*tan(c
/2 + (d*x)/2) + 165888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*
z^4 - b^10, z, k)*a^5*b^10*tan(c/2 + (d*x)/2) - 1327104*root(729*a^14*b^2*z
^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^2*a^6*b^10*tan(c/2 + (d*
x)/2) + 165888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^
10, z, k)^2*a^8*b^8*tan(c/2 + (d*x)/2) + 2359296*root(729*a^14*b^2*z^6 - 72
9*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^7*b^10*tan(c/2 + (d*x)/2) -
 7077888*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z,
 k)^3*a^9*b^8*tan(c/2 + (d*x)/2) + 82944*root(729*a^14*b^2*z^6 - 729*a^16*z
^6 - 243*a^10*b^4*z^4 - b^10, z, k)^3*a^11*b^6*tan(c/2 + (d*x)/2) + 8139571
2*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^4*a
^10*b^8*tan(c/2 + (d*x)/2) - 1714176*root(729*a^14*b^2*z^6 - 729*a^16*z^6 -
 243*a^10*b^4*z^4 - b^10, z, k)^4*a^12*b^6*tan(c/2 + (d*x)/2) + 27869184*ro
ot(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^5*a^13*
b^6*tan(c/2 + (d*x)/2) - 23141376*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 24
3*a^10*b^4*z^4 - b^10, z, k)^5*a^15*b^4*tan(c/2 + (d*x)/2) - 254803968*root
(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k)^6*a^14*b^
6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^14*b^2*z^6 - 729*a^16*z^6 - 243
*a^10*b^4*z^4 - b^10, z, k)^6*a^16*b^4*tan(c/2 + (d*x)/2))/a^9)*root(729*a^
14*b^2*z^6 - 729*a^16*z^6 - 243*a^10*b^4*z^4 - b^10, z, k), k, 1, 6)/d - co
t(c/2 + (d*x)/2)^2/(8*a*d) - cot(c/2 + (d*x)/2)^4/(64*a*d) + tan(c/2 + (d*x
)/2)^2/(8*a*d) + tan(c/2 + (d*x)/2)^4/(64*a*d) + (3*log(tan(c/2 + (d*x)/2))
)/(8*a*d) + (b*cot(c/2 + (d*x)/2))/(2*a^2*d) - (b*tan(c/2 + (d*x)/2))/(2*a^
2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.189 $\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=293

$$\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] -a*x/b^2-cos(d*x+c)/b/d+1/3*cos(d*x+c)^3/b/d+2/3*a^(4/3)*arctan((b^(1/3)+a^
(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^2/d/(a^(2/3)-b^(2/3))^
(1/2)+2/3*a^(4/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a
^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-
2/3*a^(4/3)*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c
))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(
1/2)
```

Rubi [A] time = 0.39, antiderivative size = 293, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 2633, 3213, 2660, 618, 204}

$$\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^2d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((a*x)/b^2) + (2*a^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^2*d) + (2*a^(4/3)*ArcTan[
((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b
^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^2*d) - (2*a^(4/3)*ArcTan[
((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) -
 (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*b^2*d) - Cos[c
 + d*x]/(b*d) + Cos[c + d*x]^3/(3*b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{a}{b^2}+\frac{\sin^3(c+dx)}{b}+\frac{a^2}{b^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= -\frac{ax}{b^2}+\frac{a^2\int\frac{1}{a+b\sin^3(c+dx)}\,dx}{b^2}+\frac{\int\sin^3(c+dx)\,dx}{b}$$

$$= -\frac{ax}{b^2}+\frac{a^2\int\left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{\ldots}\right.}\right.}{b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{a^{4/3}\int\frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^2}-\frac{a^{4/3}\int\frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{\ldots}}}{3b^2}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}-\frac{\left(2a^{4/3}\right)\text{Subst}\left(\int\frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}\ldots\right.\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{\cos(c+dx)}{bd}+\frac{\cos^3(c+dx)}{3bd}+\frac{\left(4a^{4/3}\right)\text{Subst}\left(\int\frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-\ldots\right.}{3b^2d}$$

$$= -\frac{ax}{b^2}-\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^2d}+\frac{2a^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^2d}+\frac{2a\ldots}{\ldots}$$

Mathematica [C] time = 0.28, size = 164, normalized size = 0.56

$$-\frac{8ia^2\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]+12a\ldots}{12b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/12*(12*a*c + 12*a*d*x + 9*b*Cos[c + d*x] - b*Cos[3*(c + d*x)] + (8*I)*a^
2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2
```

```
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/(b^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^6}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^6/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.58, size = 166, normalized size = 0.57

$$-\frac{4\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{4}{3db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,b^2}+\frac{a^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3\right.}\right.}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -4/d/b/(1+tan(1/2*d*x+1/2*c)^2)^3*tan(1/2*d*x+1/2*c)^2-4/3/d/b/(1+tan(1/2*d
*x+1/2*c)^2)^3-2/d/b^2*a*arctan(tan(1/2*d*x+1/2*c))+1/3/d*a^2/b^2*sum((_R^4
+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=Roo
tOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-96\,a^2b^2d\int\frac{}{b^4\cos(6\,dx+6\,c)^2+9\,b^4\cos(4\,dx+4\,c)^2+64\,a^2b^2\cos(3\,dx+3\,c)^2+9\,b^4\cos(2\,dx+2\,c)^2+b^4\sin(6\,dx+6\,c)^2+9\,b^4\sin(4\,dx+4\,c)^2+64\,a^2b^2\sin}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/12*(96*a^2*b^2*d*integrate(-(8*a*cos(3*d*x + 3*c)^2 - b*cos(3*d*x + 3*c)
*sin(6*d*x + 6*c) + 3*b*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) + b*cos(6*d*x + 6
*c)*sin(3*d*x + 3*c) - 3*b*cos(4*d*x + 4*c)*sin(3*d*x + 3*c) + 8*a*sin(3*d*
x + 3*c)^2 - 3*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + (3*b*cos(2*d*x + 2*c)
- b)*sin(3*d*x + 3*c))/(b^4*cos(6*d*x + 6*c)^2 + 9*b^4*cos(4*d*x + 4*c)^2 +
 64*a^2*b^2*cos(3*d*x + 3*c)^2 + 9*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(6*d*x +
 6*c)^2 + 9*b^4*sin(4*d*x + 4*c)^2 + 64*a^2*b^2*sin(3*d*x + 3*c)^2 - 48*a*b
^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 9*b^4*sin(2*d*x + 2*c)^2 - 6*b^4*cos
(2*d*x + 2*c) + b^4 - 2*(3*b^4*cos(4*d*x + 4*c) - 3*b^4*cos(2*d*x + 2*c) -
8*a*b^3*sin(3*d*x + 3*c) + b^4)*cos(6*d*x + 6*c) - 6*(3*b^4*cos(2*d*x + 2*c
) + 8*a*b^3*sin(3*d*x + 3*c) - b^4)*cos(4*d*x + 4*c) - 2*(8*a*b^3*cos(3*d*x
 + 3*c) + 3*b^4*sin(4*d*x + 4*c) - 3*b^4*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
```

```
 + 6*(8*a*b^3*cos(3*d*x + 3*c) - 3*b^4*sin(2*d*x + 2*c))*sin(4*d*x + 4*c) +
 16*(3*a*b^3*cos(2*d*x + 2*c) - a*b^3)*sin(3*d*x + 3*c)), x) + 12*a*d*x - b
*cos(3*d*x + 3*c) + 9*b*cos(d*x + c))/(b^2*d)
```

mupad [B] time = 14.58, size = 1800, normalized size = 6.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((1073741824*a^13*tan(c/2 + (d*x)/2) + 2013265920*root(729*a^2*b^
12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^12*
b^2 - 4831838208*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 2
7*a^6*b^4*z^2 + a^8, z, k)^2*a^10*b^5 + 268435456*root(729*a^2*b^12*z^6 - 7
29*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^2*a^12*b^3 + 33
722204160*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b
^4*z^2 + a^8, z, k)^3*a^10*b^6 - 15703474176*root(729*a^2*b^12*z^6 - 729*b^
14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^4*a^8*b^9 + 48318382
08*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2
+ a^8, z, k)^4*a^10*b^7 - 130459631616*root(729*a^2*b^12*z^6 - 729*b^14*z^6
 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^6*b^12 + 154014842880*
root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a
^8, z, k)^5*a^8*b^10 - 35332816896*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 2
43*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^6*a^6*b^13 + 21743271936*root(
729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z
, k)^6*a^8*b^11 - 130459631616*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a
^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^7*a^4*b^16 + 122305904640*root(729
*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k
)^7*a^6*b^14 - 3221225472*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^
8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)*a^11*b^3*tan(c/2 + (d*x)/2) + 185891553
28*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2
+ a^8, z, k)^2*a^11*b^4*tan(c/2 + (d*x)/2) - 17716740096*root(729*a^2*b^12*
z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^3*a^9*b^
7*tan(c/2 + (d*x)/2) + 2818572288*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 24
3*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^3*a^11*b^5*tan(c/2 + (d*x)/2) -
 86973087744*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^
6*b^4*z^2 + a^8, z, k)^4*a^7*b^10*tan(c/2 + (d*x)/2) + 88181047296*root(729
*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k
)^4*a^9*b^8*tan(c/2 + (d*x)/2) - 30802968576*root(729*a^2*b^12*z^6 - 729*b^
14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^7*b^11*tan(c/2 +
 (d*x)/2) + 18119393280*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*
z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^5*a^9*b^9*tan(c/2 + (d*x)/2) - 8697308774
4*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 +
 a^8, z, k)^6*a^5*b^14*tan(c/2 + (d*x)/2) + 70665633792*root(729*a^2*b^12*z
^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^6*a^7*b^1
2*tan(c/2 + (d*x)/2) - 40768634880*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 2
43*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)^7*a^5*b^15*tan(c/2 + (d*x)/2)
+ 32614907904*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a
^6*b^4*z^2 + a^8, z, k)^7*a^7*b^13*tan(c/2 + (d*x)/2) + 134217728*root(729*
a^2*b^12*z^6 - 729*b^14*z^6 + 243*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k)
*a^13*b*tan(c/2 + (d*x)/2))/b^7)*root(729*a^2*b^12*z^6 - 729*b^14*z^6 + 243
*a^4*b^8*z^4 + 27*a^6*b^4*z^2 + a^8, z, k), k, 1, 6)/d - (4*tan(c/2 + (d*x)
/2)^2)/(b*d + 3*b*d*tan(c/2 + (d*x)/2)^2 + 3*b*d*tan(c/2 + (d*x)/2)^4 + b*d
*tan(c/2 + (d*x)/2)^6) - 4/(3*(b*d + 3*b*d*tan(c/2 + (d*x)/2)^2 + 3*b*d*tan
(c/2 + (d*x)/2)^4 + b*d*tan(c/2 + (d*x)/2)^6)) + (a*log(tan(c/2 + (d*x)/2)
- 1i)*1i)/(b^2*d) - (a*log(tan(c/2 + (d*x)/2) + 1i)*1i)/(b^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^6(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**6/(a + b*sin(c + d*x)**3), x)
```

3.190 $\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a^{2/3}+\sqrt[3]{-}}}\right.}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2}}}$$

```
[Out] -cos(d*x+c)/b/d+2/3*a^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^(2
/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(
2/3)*a^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.42, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2638, 2660, 618, 204}

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a^{2/3}+\sqrt[3]{-}}}\right.}{3b^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*a^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(4/3)*d) - (2*(-1)^(1/3)
*a^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d)
- Cos[c + d*x]/(b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\sin(c+dx)}{b} - \frac{a\sin(c+dx)}{b\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \sin(c+dx)\,dx}{b} - \frac{a\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{b}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3\sqrt[3]{a}\sqrt[3]{b}\left(\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} + \frac{\left((\right.}{}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{\left(2a^{2/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{4/3}d} - \frac{\left(2\sqrt[3]{-1}\,a^{2/3}\right)}{}$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{\left(4a^{2/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{4/3}d} +$$

$$= -\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{4/3}d} - \frac{2\sqrt[3]{-}}{}$$

Mathematica [C] time = 0.26, size = 186, normalized size = 0.66

$$\frac{-3\cos(c+dx) + a\text{RootSum}\left[i\#1^6b - 3i\#1^4b + 8\#1^3a + 3i\#1^2b - ib\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+\right.}{\#1^4b-2\#1^2}\right.}{3bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cos[c + d*x] + a*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^
4 + I*b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2
*Cos[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
- I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*
#1^4) & ])/(3*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sin(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.56, size = 106, normalized size = 0.38

$$-\frac{2}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}-\frac{2a\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
[Out] -2/d/b/(1+tan(1/2*d*x+1/2*c)^2)-2/3/d*a/b*sum((_R^3+_R)/(_R^5*a+2*_R^3*a+4*
_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3
*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 15.07, size = 665, normalized size = 2.37

$$\sum_{k=1}^{6}\ln\left(8192\,a^8\,b^5-\mathrm{root}\left(729\,a^2\,b^8\,d^6-729\,b^{10}\,d^6+243\,a^2\,b^6\,d^4+a^4,d,k\right)^2\,a^6\,b^9\,294912+\mathrm{root}\left(729\,a^2\,b^8\,c\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
[Out] symsum(log(8192*a^8*b^5 - 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*
a^2*b^6*d^4 + a^4, d, k)^2*a^6*b^9 + 1548288*root(729*a^2*b^8*d^6 - 729*b^1
0*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^6*b^10 - 1990656*root(729*a^2*b^8*
d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^4*a^6*b^11 - 7962624*root
(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^4*b^14 +
 5971968*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)
^5*a^6*b^12 - 65536*a^7*b^6*tan(c/2 + (d*x)/2) + 196608*root(729*a^2*b^8*d^
6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)*a^7*b^7*tan(c/2 + (d*x)/2)
- 294912*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)
^2*a^7*b^8*tan(c/2 + (d*x)/2) - 1769472*root(729*a^2*b^8*d^6 - 729*b^10*d^6
 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^5*b^11*tan(c/2 + (d*x)/2) + 221184*root
```

```
(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^3*a^7*b^9*ta
n(c/2 + (d*x)/2) - 2654208*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^
6*d^4 + a^4, d, k)^4*a^5*b^12*tan(c/2 + (d*x)/2) - 1990656*root(729*a^2*b^8
*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k)^5*a^5*b^13*tan(c/2 + (d*
x)/2))*root(729*a^2*b^8*d^6 - 729*b^10*d^6 + 243*a^2*b^6*d^4 + a^4, d, k),
k, 1, 6)/d - 2/(b*d + b*d*tan(c/2 + (d*x)/2)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sin(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.191 $\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/((-1)^(1/3)*
a^(2/3)+b^(2/3))^(1/2)-2/3*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+
1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)
+b^(2/3))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3220, 2660, 618, 204, 206}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3b^{2/3}d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}\right)}{3b^{2/3}d\sqrt{\sqrt[3]{-1}a^{2/3}+b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1
/3)*Tan[(c + d*x)/2])/Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2/3)*d) - (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a
^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^
(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int\left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{}{3b^{2/3}\left((-1)^{2}\right.}\right.$$
$$=\frac{\int\frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int\frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}+\frac{\int\frac{1}{(-1)^{2/3}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{2/3}}$$
$$=\frac{2\,\text{Subst}\left(\int\frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}+\frac{2\,\text{Subst}\left(\int\frac{1}{-\sqrt[3]{-1}\sqrt[3]{a}+2\sqrt[3]{b}x-\sqrt[3]{-1}\sqrt[3]{a}\right.}{3b^{2/3}d}$$
$$=-\frac{4\,\text{Subst}\left(\int\frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^{2/3}d}-\frac{4\,\text{Subst}\left(\int\frac{}{-4\left((-1)^{2/3}a\right.}\right.}{}$$
$$=\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3\sqrt{a^{2/3}-b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}}\right)}{3\sqrt{-(-1)^{2/3}a^{2/3}+b^{2/3}}\,b^{2/3}d}-\frac{2\tanh^{-1}\left(\frac{\sqrt[3]{b}+(-1)^{2}}{\sqrt{\sqrt[3]{}}}\right.}{3\sqrt{\sqrt[3]{-1}\,a^{2/3}}}$$

Mathematica [C] time = 0.19, size = 231, normalized size = 0.96

$$\frac{i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#1\cos(\right.}{\#1^5b-}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((I/6)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
 , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ])/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.45, size = 76, normalized size = 0.32

$$\frac{4\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\_R^2\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 4/3/d*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R
=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

mupad [B] time = 15.65, size = 590, normalized size = 2.46

$$\sum_{k=1}^{6}\ln\left(-\frac{8192\,a^4\left(-729\,a^2\,b^3-81\,a^2\,b^2\,\mathrm{root}\left(d^6-27\,b^2\,d^4+243\,b^4\,d^2+729\,b^4\left(a^2-b^2\right),d,k\right)+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^4+324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a\,b^3\,\mathrm{root}\left(d^6-2\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a^4*(12*b*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(
a^2 - b^2), d, k)^4 + 324*b^4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4
*(a^2 - b^2), d, k) + 972*b^5 + 4*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729
*b^4*(a^2 - b^2), d, k)^5 - 729*a^2*b^3 - 72*b^2*root(d^6 - 27*b^2*d^4 + 24
3*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 - 216*b^3*root(d^6 - 27*b^2*d^4 +
243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 - 81*a^2*b^2*root(d^6 - 27*b^2*d
^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k) + 243*a*b^4*tan(c/2 + (d*x)/2
) + 3*a*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a
^2 - b^2), d, k)^4 + 162*a*b^2*tan(c/2 + (d*x)/2)*root(d^6 - 27*b^2*d^4 + 2
43*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^2 + 36*a*b*tan(c/2 + (d*x)/2)*root(
d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^3 + 324*a*tan(c
/2 + (d*x)/2)*b^3*root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2)
, d, k)))/root(d^6 - 27*b^2*d^4 + 243*b^4*d^2 + 729*b^4*(a^2 - b^2), d, k)^
5)*root(729*a^2*b^4*d^6 - 729*b^6*d^6 + 243*b^4*d^4 - 27*b^2*d^2 + 1, d, k)
, k, 1, 6)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
[Out] Integral(sin(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.192 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-(-1
)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
```

```
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{}{3a^{2/3}\left(-\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}$$

$$= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}-\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\right.}{3a^{2/3}d}$$

$$= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}+\frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}+\sqrt[3]{\right.}}\right.}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+}{\sqrt{a^{2/3}}}\right.}{3a^{2/3}\sqrt{a^{2/3}+}}$$

Mathematica [C] time = 0.13, size = 126, normalized size = 0.51

$$-\frac{2i\mathrm{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.53, size = 83, normalized size = 0.34

$$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin\left(dx+c\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

mupad [B] time = 15.88, size = 609, normalized size = 2.49

$$\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\left(-729\,a^5+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\,\left(a^2-b^2\right),d,k\right)+972\,a^3\,b^2+a^3\,b\,\mathrm{root}(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*d^
4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4*b*t
an(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(c/2 + (
d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
+ 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 7
29*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 + 27*a^2*d^4 +
243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*d^6 - 729*a^6*
d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin^3\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

3.193 $\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=281

$$-\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-\cdots}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2\cdots}}}\right.}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

```
[Out] -cot(d*x+c)/a/d+2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^
(2/3)-b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*(-1)^(1/3)*b^(2
/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(
2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3
)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(4/3)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 281, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3220, 3767, 8, 2660, 618, 204}

$$-\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-\cdots}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2\cdots}}}\right.}{3a^{4/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*b^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - (-1)^(2/3
)*b^(2/3)]*d) + (2*b^(2/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt
[a^(2/3) - b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*(-1)^(1/3)
*b^(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3
) + (-1)^(1/3)*b^(2/3)]])/(3*a^(4/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d)
- Cot[c + d*x]/(a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e, x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{\csc^2(c+dx)}{a} - \frac{b\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^2(c+dx)\,dx}{a} - \frac{b\int \frac{\sin(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a}$$

$$= -\frac{b\int \left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{\sqrt[3]{-1}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\right.}\right.}{a}$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{b^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{4/3}} - \frac{\left(\sqrt[3]{-1}\,b^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{4/3}} +$$

$$= -\frac{\cot(c+dx)}{ad} + \frac{\left(2b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{4/3}d} - \left(2\sqrt[3]{-1}\,b\right.$$

$$= -\frac{\cot(c+dx)}{ad} - \frac{\left(4b^{2/3}\right)\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, 2\sqrt[3]{b}+2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{4/3}d}$$

$$= -\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{4/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - 2$$

Mathematica [C] time = 0.31, size = 196, normalized size = 0.70

$$\frac{2b\,\mathrm{RootSum}\left[\#1^6 b - 3\#1^4 b - 8i\#1^3 a + 3\#1^2 b - b\&, \frac{-i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+i\log\left(\#1^2-2\#1\cos(c+dx)+1\right)+2\#1^2\tan^{-1}\left(\right.}{\#1^4 b-2\#1^2 b-4i\#1 a+b}\right.}{6ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-3*Cot[(c + d*x)/2] + 2*b*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4
+ b*#1^6 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2] + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4
) & ] + 3*Tan[(c + d*x)/2])/(6*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(csc(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.76, size = 119, normalized size = 0.42

$$\frac{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{1}{2ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{2b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^3+\_R\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}\right)}{3da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
[Out] 1/2/a/d*tan(1/2*d*x+1/2*c)-1/2/a/d/tan(1/2*d*x+1/2*c)-2/3/d/a*b*sum((_R^3+_
R)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6
*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 14.42, size = 697, normalized size = 2.48

$$\frac{\left(\sum_{k=1}^{6}\ln\left(8192\,a^7\,b^6-\mathrm{root}\left(729\,a^8\,b^2\,d^6-729\,a^{10}\,d^6-243\,a^6\,b^2\,d^4-b^4,d,k\right)^2\,a^9\,b^6\,294912+\mathrm{root}\left(729\,a^8\,b^2\,d^6\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
[Out] (symsum(log(8192*a^7*b^6 - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243
*a^6*b^2*d^4 - b^4, d, k)^2*a^9*b^6 + 1548288*root(729*a^8*b^2*d^6 - 729*a^
10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^11*b^5 - 1990656*root(729*a^8*b^2
*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^4*a^13*b^4 - 7962624*roo
t(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^13*b^5
+ 5971968*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
```

```
)^5*a^15*b^3 - 65536*a^6*b^7*tan(c/2 + (d*x)/2) + 196608*root(729*a^8*b^2*d
^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)*a^8*b^6*tan(c/2 + (d*x)/2)
 - 294912*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
)^2*a^10*b^5*tan(c/2 + (d*x)/2) - 1769472*root(729*a^8*b^2*d^6 - 729*a^10*d
^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^10*b^6*tan(c/2 + (d*x)/2) + 221184*ro
ot(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^3*a^12*b^4
*tan(c/2 + (d*x)/2) - 2654208*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6
*b^2*d^4 - b^4, d, k)^4*a^12*b^5*tan(c/2 + (d*x)/2) - 1990656*root(729*a^8*
b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k)^5*a^14*b^4*tan(c/2 +
(d*x)/2))*root(729*a^8*b^2*d^6 - 729*a^10*d^6 - 243*a^6*b^2*d^4 - b^4, d, k
), k, 1, 6) - 1/(2*a*tan(c/2 + (d*x)/2)) + tan(c/2 + (d*x)/2)/(2*a))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.194 $\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=296

$$\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\,\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{(-1}{}\right.}{3a^2d\sqrt{\sqrt[3]{}}}$$

```
[Out] b*arctanh(cos(d*x+c))/a^2/d-cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+2/3*b^(4/3)
*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^2/d
/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)+(-1)^(2/3)*a^(1/3)*ta
n(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/((-1)^(1/3)*a^(
2/3)+b^(2/3))^(1/2)-2/3*b^(4/3)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2
*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/a^2/d/(-(-1)^(2/3)*a^(2/3
)+b^(2/3))^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 296, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.304$, Rules used = {3220, 3770, 3767, 2660, 618, 204, 206}

$$\frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\,\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}\right)}{3a^2d\sqrt{b^{2/3}-(-1)^{2/3}a^{2/3}}}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}\right)}{3a^2d\sqrt{\sqrt[3]{-1}\,a^{2/3}+b^{2/3}}}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*b^(4/3)*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/
3)]])/(3*a^2*Sqrt[a^(2/3) - b^(2/3)]*d) + (b*ArcTanh[Cos[c + d*x]])/(a^2*d)
 - (2*b^(4/3)*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
-((-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*a^2*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2
/3)]*d) - (2*b^(4/3)*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]
)/Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*a^2*Sqrt[(-1)^(1/3)*a^(2/3) + b^(
2/3)]*d) - Cot[c + d*x]/(a*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{b\csc(c+dx)}{a^2}+\frac{\csc^4(c+dx)}{a}+\frac{b^2\sin^2(c+dx)}{a^2\left(a+b\sin^3(c+dx)\right)}\right)dx$$

$$= \frac{\int \csc^4(c+dx)\,dx}{a}-\frac{b\int \csc(c+dx)\,dx}{a^2}+\frac{b^2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}+\frac{b^2\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}+\ldots\right.}{a^2}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{b^{4/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^2}+\ldots$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\left(2b^{4/3}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+2\sqrt[3]{b}x+\ldots}\right.}{3a^2\ldots}$$

$$= \frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}-\frac{\left(4b^{4/3}\right)\text{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)\ldots}\right.}{\ldots}$$

$$= \frac{2b^{4/3}\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^2\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{b\tanh^{-1}(\cos(c+dx))}{a^2d}-\frac{2b^{4/3}\tanh^{-1}\left(\frac{\sqrt[3]{b}-\sqrt[3]{-1}\ldots}{\sqrt{-(-1)\ldots}}\right)}{3a^2\sqrt{-(-1)^{2/3}a^{2/3}\ldots}}$$

Mathematica [C] time = 2.15, size = 333, normalized size = 1.12

$$4ib^2\text{RootSum}\left[\#1^6b-3\#1^4b-8i\#1^3a+3\#1^2b-b\&,\frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#\ldots\right.}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-8*a*Cot[(c + d*x)/2] + 24*b*Log[Cos[(c + d*x)/2]] - 24*b*Log[Sin[(c + d*x
)/2]] + (4*I)*b^2*RootSum[-b + 3*b*#1^2 - (8*I)*a*#1^3 - 3*b*#1^4 + b*#1^6
& , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] - 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (2*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 -
2*b*#1^3 + b*#1^5) & ] + 8*a*Csc[c + d*x]^3*Sin[(c + d*x)/2]^4 - (a*Csc[(c
+ d*x)/2]^4*Sin[c + d*x])/2 + 8*a*Tan[(c + d*x)/2])/(24*a^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^4}{b\sin(dx+c)^3 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.79, size = 176, normalized size = 0.59

$$\frac{\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)}{24da}+\frac{3\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{1}{24da\tan\left(\frac{dx}{2}+\frac{c}{2}\right)^3}-\frac{3}{8ad\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}-\frac{b\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d\,a^2}+\frac{4b^2\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/24/d/a*tan(1/2*d*x+1/2*c)^3+3/8/a/d*tan(1/2*d*x+1/2*c)-1/24/d/a/tan(1/2*d
*x+1/2*c)^3-3/8/a/d/tan(1/2*d*x+1/2*c)-1/d/a^2*b*ln(tan(1/2*d*x+1/2*c))+4/3
/d*b^2/a^2*sum(_R^2/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_
R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 16.38, size = 1503, normalized size = 5.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] symsum(log((98304*b^11 + 589824*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*
a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)*a^2*b^10 - 98304*root(729*a^12*b^
2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^4*
b^9 - 5898240*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a
^4*b^6*z^2 - b^8, z, k)^3*a^6*b^8 - 7962624*root(729*a^12*b^2*z^6 - 729*a^1
4*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^4*a^8*b^7 - 663552*ro
ot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8
, z, k)^4*a^10*b^5 + 5308416*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8
*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^10*b^6 - 10616832*root(729*a^12*
b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^
12*b^4 + 7962624*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 2
7*a^4*b^6*z^2 - b^8, z, k)^6*a^12*b^5 - 9953280*root(729*a^12*b^2*z^6 - 729
*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^14*b^3 + 2457
6*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 -
 b^8, z, k)*a^3*b^9*tan(c/2 + (d*x)/2) - 3145728*root(729*a^12*b^2*z^6 - 72
9*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^3*b^10*tan(c
/2 + (d*x)/2) + 466944*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z
^4 + 27*a^4*b^6*z^2 - b^8, z, k)^2*a^5*b^8*tan(c/2 + (d*x)/2) + 18874368*ro
ot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8
, z, k)^3*a^5*b^9*tan(c/2 + (d*x)/2) + 3981312*root(729*a^12*b^2*z^6 - 729*
a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^3*a^7*b^7*tan(c/2
+ (d*x)/2) + 56623104*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^
4 + 27*a^4*b^6*z^2 - b^8, z, k)^4*a^7*b^8*tan(c/2 + (d*x)/2) + 20791296*roo
t(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8,
 z, k)^4*a^9*b^6*tan(c/2 + (d*x)/2) - 84934656*root(729*a^12*b^2*z^6 - 729*
a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^9*b^7*tan(c/2
+ (d*x)/2) + 78962688*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^
4 + 27*a^4*b^6*z^2 - b^8, z, k)^5*a^11*b^5*tan(c/2 + (d*x)/2) - 254803968*r
oot(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^
8, z, k)^6*a^11*b^6*tan(c/2 + (d*x)/2) + 252813312*root(729*a^12*b^2*z^6 -
729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)^6*a^13*b^4*tan
(c/2 + (d*x)/2) - 1048576*root(729*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^
4*z^4 + 27*a^4*b^6*z^2 - b^8, z, k)*a*b^11*tan(c/2 + (d*x)/2))/a^6)*root(72
9*a^12*b^2*z^6 - 729*a^14*z^6 - 243*a^8*b^4*z^4 + 27*a^4*b^6*z^2 - b^8, z,
k), k, 1, 6)/d - (a*((3*cot(c/2 + (d*x)/2))/8 - (3*tan(c/2 + (d*x)/2))/8 +
cot(c/2 + (d*x)/2)^3/24 - tan(c/2 + (d*x)/2)^3/24) + b*log(tan(c/2 + (d*x)/
2)))/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.195 $\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=177

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(a+b)\cos(c+dx)}{b^2d}+\frac{\cos^5(c+dx)}{5bd}-\frac{2\cos^3(c+dx)}{3bd}$$

```
[Out] (a+b)*cos(d*x+c)/b^2/d-2/3*cos(d*x+c)^3/b/d+1/5*cos(d*x+c)^5/b/d-1/2*a^(3/2
)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(9/4)/d/(a^(1/2)-b^(
1/2))^(1/2)-1/2*a^(3/2)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))
/b^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$-\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{(a+b)\cos(c+dx)}{b^2d}+\frac{\cos^5(c+dx)}{5bd}-\frac{2\cos^3(c+dx)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(a^(3/2)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(9/4)*d) - (a^(3/2)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(9/4)*d) + ((a + b)*Cos[
c + d*x])/(b^2*d) - (2*Cos[c + d*x]^3)/(3*b*d) + Cos[c + d*x]^5/(5*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{a+b}{b^2}+\frac{2x^2}{b}-\frac{x^4}{b}+\frac{a^2}{b^2\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d}-\frac{2\cos^3(c+dx)}{3bd}+\frac{\cos^5(c+dx)}{5bd}-\frac{a^2\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, \ldots\right.}{b^2d}$$

$$= \frac{(a+b)\cos(c+dx)}{b^2d}-\frac{2\cos^3(c+dx)}{3bd}+\frac{\cos^5(c+dx)}{5bd}+\frac{a^{3/2}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\ldots\right.}{2b^{3/2}d}$$

$$= -\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{9/4}d}-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{9/4}d}+\frac{(a+b)\cos(c+dx)}{b^2d}-\frac{2\cos\ldots}{\ldots}$$

Mathematica [C] time = 0.48, size = 228, normalized size = 1.29

$$\frac{\cos(c+dx)(120a-28b\cos(2(c+dx))+3b\cos(4(c+dx))+89b)+60ia^2\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+\ldots\right.}{12\ldots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Cos[c + d*x]*(120*a + 89*b - 28*b*Cos[2*(c + d*x)] + 3*b*Cos[4*(c + d*x)])
 + (60*I)*a^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1
^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^
4 + b*#1^6) & ])/(120*b^2*d)
```

fricas [B] time = 0.58, size = 872, normalized size = 4.93

$$\frac{12\,b\cos\left(dx+c\right)^5-15\,b^2d\sqrt{-\frac{\left(ab^4-b^5\right)\sqrt{\frac{a^7}{\left(a^2b^9-2\,ab^{10}+b^{11}\right)d^4}}\,d^2+a^3}{\left(ab^4-b^5\right)d^2}}\,\log\left(a^5\cos\left(dx+c\right)+\left(a^4b^2d-\left(ab^7-b^8\right)\sqrt{\frac{\ldots}{\left(a^2b^9-2\ldots\right.}}\right.\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/60*(12*b*cos(d*x + c)^5 - 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^
9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(d*x
+ c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4
))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^
```

```
2 + a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^
2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log(a^5*cos(
d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))
*d^2 - a^3)/((a*b^4 - b^5)*d^2))) + 15*b^2*d*sqrt(-((a*b^4 - b^5)*sqrt(a^7/
((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))*log(-a^5
*cos(d*x + c) + (a^4*b^2*d - (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 +
b^11)*d^4))*d^3)*sqrt(-((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b^11)
*d^4))*d^2 + a^3)/((a*b^4 - b^5)*d^2))) - 15*b^2*d*sqrt(((a*b^4 - b^5)*sqrt
(a^7/((a^2*b^9 - 2*a*b^10 + b^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))*log
(-a^5*cos(d*x + c) - (a^4*b^2*d + (a*b^7 - b^8)*sqrt(a^7/((a^2*b^9 - 2*a*b^
10 + b^11)*d^4))*d^3)*sqrt(((a*b^4 - b^5)*sqrt(a^7/((a^2*b^9 - 2*a*b^10 + b
^11)*d^4))*d^2 - a^3)/((a*b^4 - b^5)*d^2))) - 40*b*cos(d*x + c)^3 + 60*(a +
 b)*cos(d*x + c))/(b^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-42,-63]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[-2,-75]-2/d*(-15*((1-cos(c+
d*x))/(1+cos(c+d*x)))^4*a-60*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a-90*((1-cos
(c+d*x))/(1+cos(c+d*x)))^2*a-80*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b-60*(1-c
os(c+d*x))/(1+cos(c+d*x))*a-40*(1-cos(c+d*x))/(1+cos(c+d*x))*b-15*a-8*b)*1/
15/b^2/((1-cos(c+d*x))/(1+cos(c+d*x))+1)^5-2/d/b^2*2/d*((-2*a^4*b+12*a^3*b^
2-6*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+2*a^3*a*b-3*a^3*sqrt(a*b)*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))-10*a^2*b^3+12*a^2*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))
-12*a^2*b*a*b+6*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+2*a*b^3*sqrt
(a^2-a*b+sqrt(a*b)*(-a+b))+10*a*b^2*a*b+a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a
*b)*(-a+b)))*abs(a-b)/(24*a^4*b-96*a^3*b^2+112*a^2*b^3-32*a*b^4-8*b^5)*(ata
n(tan(c+d*x)/sqrt(-(-8*a+sqrt(8*a*8*a-4*(4*a-4*b)*4*a))/2/(4*a-4*b)))+pi*fl
oor((c+d*x)/pi+1/2))-(-2*a^4*b+12*a^3*b^2+6*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(a
-b))+2*a^3*a*b-3*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-10*a^2*b^3-12*
a^2*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-12*a^2*b*a*b+6*a^2*b*sqrt(a*b)*sqrt(a
^2-a*b+sqrt(a*b)*(a-b))-2*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+10*a*b^2*a*b+
a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)/(24*a^4*b-96*a^3*b^
2+112*a^2*b^3-32*a*b^4-8*b^5)*(atan(tan(c+d*x)/sqrt(-(-8*a-sqrt(8*a*8*a-4*(
4*a-4*b)*4*a))/2/(4*a-4*b)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [A] time = 0.37, size = 159, normalized size = 0.90

$$\frac{\cos^5(dx+c)}{5bd}-\frac{2\left(\cos^3(dx+c)\right)}{3bd}+\frac{a\cos(dx+c)}{b^2d}+\frac{\cos(dx+c)}{bd}-\frac{a^2\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2db\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{a^2\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2db\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/5*cos(d*x+c)^5/b/d-2/3*cos(d*x+c)^3/b/d+a*cos(d*x+c)/b^2/d+cos(d*x+c)/b/d
-1/2/d*a^2/b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((
a*b)^(1/2)+b)*b)^(1/2))-1/2/d*a^2/b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*a
rctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/240*(240*b^2*d*integrate(8*(4*a^2*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2
*(8*a^3 - 3*a^2*b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^3 - 3*a^2*b)*
cos(4*d*x + 4*c)*sin(3*d*x + 3*c) - (a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d
*x + 3*c))*cos(8*d*x + 8*c) + 4*(a^2*b*sin(5*d*x + 5*c) - a^2*b*sin(3*d*x +
 3*c))*cos(6*d*x + 6*c) - 2*(2*a^2*b*sin(2*d*x + 2*c) + (8*a^3 - 3*a^2*b)*s
in(4*d*x + 4*c))*cos(5*d*x + 5*c) + (a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d
*x + 3*c))*sin(8*d*x + 8*c) - 4*(a^2*b*cos(5*d*x + 5*c) - a^2*b*cos(3*d*x +
 3*c))*sin(6*d*x + 6*c) + (4*a^2*b*cos(2*d*x + 2*c) - a^2*b + 2*(8*a^3 - 3*
a^2*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*a^2*b*cos(2*d*x + 2*c) - a^2
*b)*sin(3*d*x + 3*c))/(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 +
 16*b^4*cos(2*d*x + 2*c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*
c)^2 + 16*b^4*sin(2*d*x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2
*b^2 - 48*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*
b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) - 2*(4*b^4*cos(6*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b
^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c)
- b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3
 - 3*b^4 - 4*(8*a*b^3 - 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^
4*sin(6*d*x + 6*c) + 2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x +
 4*c))*sin(8*d*x + 8*c) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*si
n(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + 3*b*cos(5*d*x + 5*c) - 25*b*cos(3*d
*x + 3*c) + 30*(8*a + 5*b)*cos(d*x + c))/(b^2*d)
```

mupad [B] time = 14.44, size = 1067, normalized size = 6.03

$$\frac{\cos(c+dx)^5}{5bd}-\frac{2\cos(c+dx)^3}{3bd}+\frac{\cos(c+dx)\left(\frac{a-b}{b^2}+\frac{2}{b}\right)}{d}+\frac{\mathrm{atan}\left(-\frac{a^4\cos(c+dx)\sqrt{-\frac{\sqrt{a^7b^9}}{16\left(ab^9-b^{10}\right)}-\frac{a^3b^5}{16\left(ab^9-b^{10}\right)}}\,8i}{\frac{2a^6b^7}{ab^9-b^{10}}+\frac{2a^3b^2\sqrt{a^7b^9}}{ab^9-b^{10}}}+\frac{a^4b^9\,\mathrm{co}}{\frac{2a^6b^{16}}{ab^9-b^{10}}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^4*b^9*cos(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b
^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^
15)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (2*a^4*b
^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*cos(c + d*x)*(- (a^7*b^9)^(1/2)
/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^7
)/(a*b^9 - b^10) + (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) + (a*b^4*cos
(c + d*x)*(- (a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b
^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15
)/(a*b^9 - b^10) + (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) - (2*a^4*b^1
0*(a^7*b^9)^(1/2))/(a*b^9 - b^10)))*(-((a^7*b^9)^(1/2) + a^3*b^5)/(16*(a*b^
9 - b^10)))^(1/2)*2i)/d - (atan((a^4*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b
^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*8i)/((2*a^6*b^7)/(a*b^9
- b^10) - (2*a^3*b^2*(a^7*b^9)^(1/2))/(a*b^9 - b^10)) - (a^4*b^9*cos(c + d*
x)*((a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3*b^5)/(16*(a*b^9 - b^10)))^(1
/2)*8i)/((2*a^6*b^16)/(a*b^9 - b^10) - (2*a^7*b^15)/(a*b^9 - b^10) - (2*a^3
*b^11*(a^7*b^9)^(1/2))/(a*b^9 - b^10) + (2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9
 - b^10)) + (a*b^4*cos(c + d*x)*((a^7*b^9)^(1/2)/(16*(a*b^9 - b^10)) - (a^3
*b^5)/(16*(a*b^9 - b^10)))^(1/2)*(a^7*b^9)^(1/2)*8i)/((2*a^6*b^16)/(a*b^9 -
```

```
b^10) - (2*a^7*b^15)/(a*b^9 - b^10) - (2*a^3*b^11*(a^7*b^9)^(1/2))/(a*b^9
- b^10) + (2*a^4*b^10*(a^7*b^9)^(1/2))/(a*b^9 - b^10)))*(((a^7*b^9)^(1/2) -
 a^3*b^5)/(16*(a*b^9 - b^10)))^(1/2)*2i)/d - (2*cos(c + d*x)^3)/(3*b*d) + c
os(c + d*x)^5/(5*b*d) + (cos(c + d*x)*((a - b)/b^2 + 2/b))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.196 $\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=148

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

```
[Out] cos(d*x+c)/b/d-1/3*cos(d*x+c)^3/b/d-1/2*a*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2
)-b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*a*arctanh(b^(1/4)*c
os(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1166, 205, 208}

$$-\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cos^3(c+dx)}{3bd}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(a*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[Sqrt[a]
 - Sqrt[b]]*b^(7/4)*d) + (a*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(7/4)*d) + Cos[c + d*x]/(b*d) - Cos[
c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{x^2}{b}+\frac{a-ax^2}{b\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)}{bd} - \frac{\cos^3(c+dx)}{3bd} - \frac{\text{Subst}\left(\int \frac{a-ax^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)}{bd} - \frac{\cos^3(c+dx)}{3bd} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2bd} + \frac{a\,\text{Subst}\left(\right.}{}$$

$$= -\frac{a\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{7/4}d} + \frac{a\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{7/4}d} + \frac{\cos(c+dx)}{bd} - \frac{\cos^3(c+dx)}{3bd}$$

Mathematica [C] time = 0.29, size = 310, normalized size = 2.09

$$-3ia\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-3i\#1^2\log\left(\#1\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (18*Cos[c + d*x] - 2*Cos[3*(c + d*x)] - (3*I)*a*RootSum[b - 4*b*#1^2 - 16*a
*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#
1^5 + b*#1^7) & ])/(24*b*d)
```

fricas [B] time = 0.56, size = 849, normalized size = 5.74

$$3bd\sqrt{-\frac{\left(ab^3-b^4\right)d^2\sqrt{\frac{a^5}{\left(a^2b^7-2ab^8+b^9\right)d^4}}+a^2}{\left(ab^3-b^4\right)d^2}}\log\left(a^3\cos(dx+c)+\left(a^2b^2d-\left(ab^5-b^6\right)d^3\sqrt{\frac{a^5}{\left(a^2b^7-2ab^8+b^9\right)d^4}}\right)\sqrt{-\frac{\left(ab^3-b^4\right)d^2}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5
```

```
- b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*
d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2)))
- 3*b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) -
 a^2)/((a*b^3 - b^4)*d^2))*log(a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6
)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 3*b*
d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)
/((a*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) + (a^2*b^2*d - (a*b^5 - b^6)*d^
3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + 3*b*d*s
qrt(((a*b^3 - b^4)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a
*b^3 - b^4)*d^2))*log(-a^3*cos(d*x + c) - (a^2*b^2*d + (a*b^5 - b^6)*d^3*sq
rt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^2)/((a*b^3 - b^4)*d^2))) - 4*cos(d*x +
c)^3 + 12*cos(d*x + c))/(b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[76,51]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[11,92]-2/d*(-6*(1-cos(c+d*x))
/(1+cos(c+d*x))-2)*1/3/b/((1-cos(c+d*x))/(1+cos(c+d*x))+1)^3+2/d*4*a/b*2/d*
(1/(8*b*2)*(c+d*x)+((2*a^4*b-8*a^3*b^2-2*a^3*a*b+3*a^3*sqrt(a*b)*sqrt(a^2-a
*b+sqrt(a*b)*(a-b))+10*a^2*b^3+8*a^2*b*a*b-12*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+
sqrt(a*b)*(a-b))-4*a*b^4-10*a*b^2*a*b+11*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(
a*b)*(a-b))+4*b^3*a*b+2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-
b)*b^2+(-4*a^4*b^2-3*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+8*a^3*b^3+9*a^3*b^
2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*a^3*b*a*b-4*a^2*b^4-5*a^2*b^3*sqrt(a^2-a*
b+sqrt(a*b)*(a-b))-8*a^2*b^2*a*b-a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*a*b^
3*a*b)*abs(a-b)*abs(b)+(2*a^4*b^3-4*a^3*b^4-2*a^3*b^2*a*b+3*a^3*b^2*sqrt(a*
b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2*a^2*b^5+4*a^2*b^3*a*b-6*a^2*b^3*sqrt(a*b
)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-2*a*b^4*a*b-a*b^4*sqrt(a*b)*sqrt(a^2-a*b+sq
rt(a*b)*(a-b)))*abs(a-b))/((96*a^6*b^2-480*a^5*b^3+832*a^4*b^4-576*a^3*b^5+
96*a^2*b^6+32*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(32*a*b+sqrt(32*a*b*32*
a*b+4*(-16*a*b+16*b^2)*16*a*b))/2/(-16*a*b+16*b^2)))+pi*floor((c+d*x)/pi+1/
2))-((2*a^4*b-8*a^3*b^2-2*a^3*a*b+3*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-
a+b))+10*a^2*b^3+8*a^2*b*a*b-12*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+
b))-4*a*b^4-10*a*b^2*a*b+11*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+
4*b^3*a*b+2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)*b^2+(-4*
a^4*b^2+3*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+8*a^3*b^3-9*a^3*b^2*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))+4*a^3*b*a*b-4*a^2*b^4+5*a^2*b^3*sqrt(a^2-a*b+sqrt(a*
b)*(-a+b))-8*a^2*b^2*a*b+a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+4*a*b^3*a*b)*
abs(a-b)*abs(b)+(2*a^4*b^3-4*a^3*b^4-2*a^3*b^2*a*b+3*a^3*b^2*sqrt(a*b)*sqrt
(a^2-a*b+sqrt(a*b)*(-a+b))+2*a^2*b^5+4*a^2*b^3*a*b-6*a^2*b^3*sqrt(a*b)*sqrt
(a^2-a*b+sqrt(a*b)*(-a+b))-2*a*b^4*a*b-a*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*
b)*(-a+b)))*abs(a-b))/((96*a^6*b^2-480*a^5*b^3+832*a^4*b^4-576*a^3*b^5+96*a
^2*b^6+32*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(32*a*b-sqrt(32*a*b*32*a*b+
4*(-16*a*b+16*b^2)*16*a*b))/2/(-16*a*b+16*b^2)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [A] time = 0.31, size = 115, normalized size = 0.78

$$-\frac{\cos^{3}\left(dx+c\right)}{3bd}+\frac{\cos\left(dx+c\right)}{bd}+\frac{a\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2db\sqrt{\left(\sqrt{ab}\,+b\right)b}}-\frac{a\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2db\sqrt{\left(\sqrt{ab}\,-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/3*cos(d*x+c)^3/b/d+cos(d*x+c)/b/d+1/2/d*a/b/(((a*b)^(1/2)+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d*a/b/(((a*b)^(1/2)-b)*b)
^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/12*(12*b*d*integrate(-2*(12*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*a*
b*cos(d*x + c)*sin(2*d*x + 2*c) + 4*a*b*cos(2*d*x + 2*c)*sin(d*x + c) - a*b
*sin(d*x + c) + (a*b*sin(7*d*x + 7*c) - 3*a*b*sin(5*d*x + 5*c) + 3*a*b*sin(
3*d*x + 3*c) - a*b*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*a*b*sin(6*d*x + 6*
c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(7*d*x +
 7*c) + 4*(3*a*b*sin(5*d*x + 5*c) - 3*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x +
c))*cos(6*d*x + 6*c) - 6*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*
x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a^2 - 3*a*b)*sin(3*d*x + 3*c) - (8*a^2
 - 3*a*b)*sin(d*x + c))*cos(4*d*x + 4*c) - (a*b*cos(7*d*x + 7*c) - 3*a*b*co
s(5*d*x + 5*c) + 3*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*sin(8*d*x + 8*c
) - (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a
*b)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*a*b*cos(5*d*x + 5*c) - 3*a*b*
cos(3*d*x + 3*c) + a*b*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*a*b*cos(2*d*x
+ 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*
(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)*cos(d*x + c))*sin(4*d*x
+ 4*c) - 3*(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x
+ 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(
8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*
b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*
c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3
*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3
*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x +
 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4
*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x
+ 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*
c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2
*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + c
os(3*d*x + 3*c) - 9*cos(d*x + c))/(b*d)
```

mupad [B] time = 14.27, size = 1119, normalized size = 7.56

$$\frac{\cos\left(c+d\,x\right)}{b\,d}-\frac{{\cos\left(c+d\,x\right)}^3}{3\,b\,d}+\frac{\mathrm{atan}\left(-\frac{a^3\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right)}}\,8\mathrm{i}}{\frac{2\,a^4}{b^2}+\frac{2\,a^4\,b^6}{a\,b^7-b^8}+\frac{2\,a^2\,b^2\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}}+\frac{a^3\,b^8\,\cos(c+d\,x)\,\sqrt{-\frac{\sqrt{a^5\,b^7}}{16\,\left(a\,b^7-b^8\right)}-\frac{a^2\,b^4}{16\,\left(a\,b^7-b^8\right.}}}{2\,a^4\,b^6-2\,a^5\,b^5+\frac{2\,a^4\,b^{14}}{a\,b^7-b^8}-\frac{2\,a^5\,b^{13}}{a\,b^7-b^8}+\frac{2\,a^2\,b^{10}\,\sqrt{a^5\,b^7}}{a\,b^7-b^8}-\frac{2}{}}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
```

```
[Out] cos(c + d*x)/(b*d) - cos(c + d*x)^3/(3*b*d) + (atan((a^3*b^8*cos(c + d*x)*(
- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*
8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 - b^8) - (2*a^5*b^13)/(a*b
^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 - b^8) - (2*a^3*b^9*(a^5*b^
7)^(1/2))/(a*b^7 - b^8)) - (a^3*cos(c + d*x)*(- (a^5*b^7)^(1/2)/(16*(a*b^7
- b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/((2*a^4)/b^2 + (2*a^4*b^6
)/(a*b^7 - b^8) + (2*a^2*b^2*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b^4*cos(c
 + d*x)*(- (a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)
))^(1/2)*(a^5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 -
 b^8) - (2*a^5*b^13)/(a*b^7 - b^8) + (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 -
b^8) - (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)))*(-((a^5*b^7)^(1/2) + a^2
*b^4)/(16*(a*b^7 - b^8)))^(1/2)*2i)/d - (atan((a^3*cos(c + d*x)*((a^5*b^7)^
(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*8i)/((2*a^4)
/b^2 + (2*a^4*b^6)/(a*b^7 - b^8) - (2*a^2*b^2*(a^5*b^7)^(1/2))/(a*b^7 - b^8
)) - (a^3*b^8*cos(c + d*x)*((a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/
(16*(a*b^7 - b^8)))^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7
- b^8) - (2*a^5*b^13)/(a*b^7 - b^8) - (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 -
 b^8) + (2*a^3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)) + (a*b^4*cos(c + d*x)*((
a^5*b^7)^(1/2)/(16*(a*b^7 - b^8)) - (a^2*b^4)/(16*(a*b^7 - b^8)))^(1/2)*(a^
5*b^7)^(1/2)*8i)/(2*a^4*b^6 - 2*a^5*b^5 + (2*a^4*b^14)/(a*b^7 - b^8) - (2*a
^5*b^13)/(a*b^7 - b^8) - (2*a^2*b^10*(a^5*b^7)^(1/2))/(a*b^7 - b^8) + (2*a^
3*b^9*(a^5*b^7)^(1/2))/(a*b^7 - b^8)))*(((a^5*b^7)^(1/2) - a^2*b^4)/(16*(a*
b^7 - b^8)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.197 $\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=138

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos(c+dx)}{bd}$$

```
[Out] cos(d*x+c)/b/d-1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*a^(1/
2)/b^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*a^(1/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1170, 1093, 205, 208}

$$-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\cos(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[a]*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*Sqrt[S
qrt[a] - Sqrt[b]]*b^(5/4)*d) - (Sqrt[a]*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(5/4)*d) + Cos[c + d*x]/
(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
```

```
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a}{b\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\cos(c+dx)}{bd} - \frac{a\,\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{bd}$$

$$= \frac{\cos(c+dx)}{bd} + \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2\sqrt{b}\,d} - \frac{\sqrt{a}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}\ldots}\right.}{2\ldots}$$

$$= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{5/4}d} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{5/4}d} + \frac{\cos(c+dx)}{bd}$$

Mathematica [C] time = 0.25, size = 198, normalized size = 1.43

$$\frac{2\cos(c+dx) + ia\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\#1\log\left(\#1^2-2\#1\ldots\right.}{\#\ldots}\right.}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (2*Cos[c + d*x] + I*a*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^
6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^3 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 -
 3*b*#1^4 + b*#1^6) & ])/(2*b*d)
```

fricas [B] time = 0.54, size = 815, normalized size = 5.91

$$-\frac{bd\sqrt{-\frac{\left(ab^2-b^3\right)d^2\sqrt{\frac{a^3}{\left(a^2b^5-2ab^6+b^7\right)d^4}}+a}{\left(ab^2-b^3\right)d^2}}\log\left(a^2\cos(dx+c)-\left(\left(ab^4-b^5\right)d^3\sqrt{\frac{a^3}{\left(a^2b^5-2ab^6+b^7\right)d^4}}-a^2bd\right)\sqrt{-\frac{\left(ab^2-b^3\right)d^2\ldots}{\ldots}}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a)/((a*b^2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(
a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sq
rt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) - b*d*sq
```

```
rt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^
2 - b^3)*d^2))*log(a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5
 - 2*a*b^6 + b^7)*d^4)) + a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b
^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))) - b*d*sqrt(-((a*b^2 -
b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2)
)*log(-a^2*cos(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 +
 b^7)*d^4)) - a^2*b*d)*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^
6 + b^7)*d^4)) + a)/((a*b^2 - b^3)*d^2))) + b*d*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-a^2*c
os(d*x + c) - ((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
+ a^2*b*d)*sqrt(((a*b^2 - b^3)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) - a)/((a*b^2 - b^3)*d^2))) - 4*cos(d*x + c))/(b*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-42,-63]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[-2,-75]2/d/b/((1-cos(c+d*x)
)/(1+cos(c+d*x))+1)-2/d/b*2/d*((-2*a^3*b+12*a^2*b^2-6*a^2*b*sqrt(a^2-a*b+sq
rt(a*b)*(-a+b))+2*a^2*a*b-3*a^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-10
*a*b^3+12*a*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-12*a*b*a*b+6*a*b*sqrt(a*b)*s
qrt(a^2-a*b+sqrt(a*b)*(-a+b))+2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+10*b^2*a
*b+b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(24*a^4*b-96*a^3*
b^2+112*a^2*b^3-32*a*b^4-8*b^5)*(atan(tan(c+d*x)/sqrt(-(-8*a+sqrt(8*a*8*a-4
*(4*a-4*b)*4*a))/2/(4*a-4*b)))+pi*floor((c+d*x)/pi+1/2))-(-2*a^3*b+12*a^2*b
^2+6*a^2*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2*a^2*a*b-3*a^2*sqrt(a*b)*sqrt(a^2
-a*b+sqrt(a*b)*(a-b))-10*a*b^3-12*a*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-12*a*
b*a*b+6*a*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-2*b^3*sqrt(a^2-a*b+sqrt
(a*b)*(a-b))+10*b^2*a*b+b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-
b)/(24*a^4*b-96*a^3*b^2+112*a^2*b^3-32*a*b^4-8*b^5)*(atan(tan(c+d*x)/sqrt(-
(-8*a-sqrt(8*a*8*a-4*(4*a-4*b)*4*a))/2/(4*a-4*b)))+pi*floor((c+d*x)/pi+1/2)
))
```

maple [A] time = 0.36, size = 103, normalized size = 0.75

$$\frac{\cos \left(d x+c\right)}{b d}-\frac{a \,\mathrm{arctanh}\left(\frac{\cos \left(d x+c\right) b}{\sqrt{\left(\sqrt{a b}+b\right) b}}\right)}{2 d \sqrt{a b}\, \sqrt{\left(\sqrt{a b}+b\right) b}}-\frac{a \arctan \left(\frac{\cos \left(d x+c\right) b}{\sqrt{\left(\sqrt{a b}-b\right) b}}\right)}{2 d \sqrt{a b}\, \sqrt{\left(\sqrt{a b}-b\right) b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] cos(d*x+c)/b/d-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(
1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (b*d*integrate(8*(4*a*b*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a^2 - 3*a*
b)*cos(3*d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c)*s
in(3*d*x + 3*c) - (a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(8*d*x +
 8*c) + 4*(a*b*sin(5*d*x + 5*c) - a*b*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) -
2*(2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*cos(5*d*x + 5
*c) + (a*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(a
*b*cos(5*d*x + 5*c) - a*b*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*a*b*cos(2
*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) -
(4*a*b*cos(2*d*x + 2*c) - a*b)*sin(3*d*x + 3*c))/(b^3*cos(8*d*x + 8*c)^2 +
16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c
)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d
*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(6
4*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x
+ 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(
4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*
d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos
(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b
^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c)
 + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + cos(d*x + c)
)/(b*d)
```

mupad [B] time = 14.26, size = 1001, normalized size = 7.25

$$\frac{\cos(c+dx)}{bd}-\frac{2\,\mathrm{atanh}\left(\frac{8a^2b^7\cos(c+dx)\sqrt{-\frac{\sqrt{a^3b^5}}{16\left(ab^5-b^6\right)}-\frac{ab^3}{16\left(ab^5-b^6\right)}}}{\frac{2a^3b^{11}}{ab^5-b^6}-\frac{2a^4b^{10}}{ab^5-b^6}+\frac{2a^2b^8\sqrt{a^3b^5}}{ab^5-b^6}-\frac{2a^3b^7\sqrt{a^3b^5}}{ab^5-b^6}}-\frac{8a^2b\cos(c+dx)\sqrt{-\frac{\sqrt{a^3b^5}}{16\left(ab^5-b^6\right)}-\frac{ab^3}{16\left(ab^5-b^6\right)}}}{\frac{2a^3b^5}{ab^5-b^6}+\frac{2a^2b^2\sqrt{a^3b^5}}{ab^5-b^6}}+\frac{8ab^4\cos(c+d\cdots}{\frac{2a^3b^{11}}{ab^5-b^6}-\cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

```
[Out] cos(c + d*x)/(b*d) - (2*atanh((8*a^2*b^7*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(1
6*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^11)/(a*b^5
- b^6) - (2*a^4*b^10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 -
b^6) - (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b*cos(c + d*x)*(
- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/(
(2*a^3*b^5)/(a*b^5 - b^6) + (2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) + (8
*a*b^4*cos(c + d*x)*(- (a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*
b^5 - b^6)))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (2*a^4*b^
10)/(a*b^5 - b^6) + (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) - (2*a^3*b^7*
(a^3*b^5)^(1/2))/(a*b^5 - b^6)))*(-((a^3*b^5)^(1/2) + a*b^3)/(16*(a*b^5 - b
^6)))^(1/2))/d + (2*atanh((8*a^2*b*cos(c + d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5
 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*b^5)/(a*b^5 - b^6) -
(2*a^2*b^2*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) - (8*a^2*b^7*cos(c + d*x)*((a^3*
b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)))^(1/2))/((2*a^3*
b^11)/(a*b^5 - b^6) - (2*a^4*b^10)/(a*b^5 - b^6) - (2*a^2*b^8*(a^3*b^5)^(1/
2))/(a*b^5 - b^6) + (2*a^3*b^7*(a^3*b^5)^(1/2))/(a*b^5 - b^6)) + (8*a*b^4*c
os(c + d*x)*((a^3*b^5)^(1/2)/(16*(a*b^5 - b^6)) - (a*b^3)/(16*(a*b^5 - b^6)
))^(1/2)*(a^3*b^5)^(1/2))/((2*a^3*b^11)/(a*b^5 - b^6) - (2*a^4*b^10)/(a*b^5
 - b^6) - (2*a^2*b^8*(a^3*b^5)^(1/2))/(a*b^5 - b^6) + (2*a^3*b^7*(a^3*b^5)^
(1/2))/(a*b^5 - b^6)))*(((a^3*b^5)^(1/2) - a*b^3)/(16*(a*b^5 - b^6)))^(1/2)
)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.198 $\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

```
[Out] -1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/(a^(1/2)-
b^(1/2))^(1/2)+1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/b^(3
/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3215, 1166, 205, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2b^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[Sqrt[a] - S
qrt[b]]*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]
]/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2d}$$
$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/4}d}$$

Mathematica [C] time = 0.17, size = 285, normalized size = 2.48

$$-\frac{i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-3i\#1^2\log\left(\#1^2-2\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((-1/8*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 &
 , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1
)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(
-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/d
```

fricas [B] time = 0.53, size = 703, normalized size = 6.11

$$\frac{1}{4}\sqrt{-\frac{\left(ab-b^2\right)d^2\sqrt{\frac{a}{\left(a^2b^3-2\,ab^4+b^5\right)d^4}}+1}{\left(ab-b^2\right)d^2}}\log\left(-\left(\left(ab^2-b^3\right)d^3\sqrt{\frac{a}{\left(a^2b^3-2\,ab^4+b^5\right)d^4}}-bd\right)\sqrt{-\frac{\left(ab-b^2\right)d^2\sqrt{\frac{a}{\left(a^2b^3\right.}}}{\left(ab-b\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a
*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d
^4)) - b*d)*sqrt(-((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
+ 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) - 1/4*sqrt(-((a*b - b^2)*d^2*sqrt(a
/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^
3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - b*d)*sqrt(-((a*b - b^2)*d^
2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1)/((a*b - b^2)*d^2)) - cos(d*x
 + c)) - 1/4*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))
- 1)/((a*b - b^2)*d^2))*log(-((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
+ b^5)*d^4)) + b*d)*sqrt(((a*b - b^2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)
*d^4)) - 1)/((a*b - b^2)*d^2)) + cos(d*x + c)) + 1/4*sqrt(((a*b - b^2)*d^2*
sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2))*log(-((a*b^
2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + b*d)*sqrt(((a*b - b^
2)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1)/((a*b - b^2)*d^2)) - co
s(d*x + c))
```

giac [B] time = 0.74, size = 166, normalized size = 1.44

$$\frac{\sqrt{-b^2-\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b+\sqrt{ab}\right)d|b|}+\frac{\sqrt{-b^2+\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(b-\sqrt{ab}\right)d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 + sqrt((a
- b)*b*d^4 + b^2*d^4))/(b*d^4))))/((b + sqrt(a*b))*d*abs(b)) + 1/2*sqrt(-b^
2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2 - sqrt((a - b)*b*d^4 +
 b^2*d^4))/(b*d^4))))/((b - sqrt(a*b))*d*abs(b))
```

maple [A] time = 0.24, size = 78, normalized size = 0.68

$$\frac{\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2d\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2d\sqrt{\left(\sqrt{ab}-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1
/2))-1/2/d/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin\left(dx+c\right)^3}{b\sin\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^3/(b*sin(d*x + c)^4 - a), x)
```

mupad [B] time = 0.51, size = 976, normalized size = 8.49

$$\frac{2\operatorname{atanh}\left(\frac{8\,a\,b^2\,\cos(c+d\,x)\sqrt{\frac{\sqrt{a\,b^3}}{16\left(a\,b^3-b^4\right)}-\frac{b^2}{16\left(a\,b^3-b^4\right)}}}{2\,a\,b+\frac{2\,a\,b^5}{a\,b^3-b^4}-\frac{2\,a\,b^3\,\sqrt{a\,b^3}}{a\,b^3-b^4}}-\frac{8\,a\,b^6\,\cos(c+d\,x)\sqrt{\frac{\sqrt{a\,b^3}}{16\left(a\,b^3-b^4\right)}-\frac{b^2}{16\left(a\,b^3-b^4\right)}}}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\frac{2\,a\,b^9}{a\,b^3-b^4}+\frac{2\,a^2\,b^6\,\sqrt{a\,b^3}}{a\,b^3-b^4}-\frac{2\,a\,b^7\,\sqrt{a\,b^3}}{a\,b^3-b^4}}+\frac{8\,a\,b^4\,\cos(c+d\,x)\,\sqrt{a}\cdots}{2\,a\,b^5-2\,a^2\,b^4-\frac{2\,a^2\,b^8}{a\,b^3-b^4}+\cdots}\cdots}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (2*atanh((8*a*b^2*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16*
(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/(a*b^3 - b^4) - (2*a*b^3*(a*b^3)^
(1/2))/(a*b^3 - b^4)) - (8*a*b^6*cos(c + d*x)*((a*b^3)^(1/2)/(16*(a*b^3 - b
^4)) - b^2/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a
*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4) + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 -
b^4) - (2*a*b^7*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c + d*x)*(a*b^
```

```
3)^(1/2)*((a*b^3)^(1/2)/(16*(a*b^3 - b^4)) - b^2/(16*(a*b^3 - b^4)))^(1/2))
/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4)
 + (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) - (2*a*b^7*(a*b^3)^(1/2))/(a*b^3
 - b^4)))*(-(b^2 - (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1/2))/d - (2*atanh((
8*a*b^6*cos(c + d*x)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 -
 b^4)))^(1/2))/(2*a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)
/(a*b^3 - b^4) - (2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*a*b^7*(a*b^3)
^(1/2))/(a*b^3 - b^4)) - (8*a*b^2*cos(c + d*x)*(- b^2/(16*(a*b^3 - b^4)) -
(a*b^3)^(1/2)/(16*(a*b^3 - b^4)))^(1/2))/(2*a*b + (2*a*b^5)/(a*b^3 - b^4) +
 (2*a*b^3*(a*b^3)^(1/2))/(a*b^3 - b^4)) + (8*a*b^4*cos(c + d*x)*(a*b^3)^(1/
2)*(- b^2/(16*(a*b^3 - b^4)) - (a*b^3)^(1/2)/(16*(a*b^3 - b^4)))^(1/2))/(2*
a*b^5 - 2*a^2*b^4 - (2*a^2*b^8)/(a*b^3 - b^4) + (2*a*b^9)/(a*b^3 - b^4) - (
2*a^2*b^6*(a*b^3)^(1/2))/(a*b^3 - b^4) + (2*a*b^7*(a*b^3)^(1/2))/(a*b^3 - b
^4)))*(-(b^2 + (a*b^3)^(1/2))/(16*(a*b^3 - b^4)))^(1/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.199 $\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] -1/2*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(1/4)/d/a^(1/2)/(
a^(1/2)-b^(1/2))^(1/2)-1/2*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/
2))/b^(1/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {3215, 1093, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt[4]{b}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqr
t[a] - Sqrt[b]]*b^(1/4)*d) - ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
Sqrt[b]]]/(2*Sqrt[a]*Sqrt[Sqrt[a] + Sqrt[b]]*b^(1/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2\sqrt{a}\,d} - \frac{\sqrt{b}\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c\right.}{2\sqrt{a}\,d}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}-\sqrt{b}}\,\sqrt[4]{b}\,d} - \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2\sqrt{a}\sqrt{\sqrt{a}+\sqrt{b}}\,\sqrt[4]{b}\,d}$$

Mathematica [C] time = 0.16, size = 183, normalized size = 1.46

$$\frac{i\text{RootSum}\left[\#1^8 b - 4\#1^6 b - 16\#1^4 a + 6\#1^4 b - 4\#1^2 b + b\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\#1^3\log\left(\#1^2\right.}{\#1^6 b-3\#1^4 b-8\#1^2 a+3\#1^2 b-}\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((I/2)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
(-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*#
1^6) & ])/d
```

fricas [B] time = 0.56, size = 703, normalized size = 5.62

$$-\frac{1}{4}\sqrt{-\frac{\left(a^2-ab\right)d^2\sqrt{\frac{1}{\left(a^3b-2a^2b^2+ab^3\right)d^4}}+1}{\left(a^2-ab\right)d^2}}\log\left(-\left(\left(a^2b-ab^2\right)d^3\sqrt{\frac{1}{\left(a^3b-2a^2b^2+ab^3\right)d^4}}-ad\right)\sqrt{-\frac{\left(a^2-ab\right)d^2}{\left(a\right.}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a
*b^3)*d^4)) - a*d)*sqrt(-((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) + 1/4*sqrt(-((a^2 - a*b)*d
^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b)*d^2))*log(-(
(a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - a*d)*sqrt(-
((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a^2 - a*b
)*d^2)) - cos(d*x + c)) + 1/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*
b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a
^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a^2 - a*b)*d^2)) + cos(d*x + c)) - 1
/4*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a
^2 - a*b)*d^2))*log(-((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^
3)*d^4)) + a*d)*sqrt(((a^2 - a*b)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d
^4)) - 1)/((a^2 - a*b)*d^2)) - cos(d*x + c))
```

giac [B] time = 0.76, size = 183, normalized size = 1.46

$$-\frac{\sqrt{ab}\sqrt{-b^2-\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2+\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab+\sqrt{ab}\,a\right)d|b|}+\frac{\sqrt{ab}\sqrt{-b^2+\sqrt{ab}\,b}\arctan\left(\frac{\cos(dx+c)}{d\sqrt{-\frac{bd^2-\sqrt{(a-b)bd^4+b^2d^4}}{bd^4}}}\right)}{2\left(ab-\sqrt{ab}\,a\right)d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d^2
 + sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b + sqrt(a*b)*a)*d*abs(b))
 + 1/2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(b*d
^2 - sqrt((a - b)*b*d^4 + b^2*d^4))/(b*d^4))))/((a*b - sqrt(a*b)*a)*d*abs(b
))
```

maple [A] time = 0.36, size = 90, normalized size = 0.72

$$-\frac{b\,\mathrm{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}\,+b\right)b}}-\frac{b\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}\,-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2*b/d/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)
^(1/2)+b)*b)^(1/2))-1/2*b/d/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(co
s(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sin(dx+c)}{b\sin(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)/(b*sin(d*x + c)^4 - a), x)
```

mupad [B] time = 15.11, size = 361, normalized size = 2.89

$$\frac{\ln\left(4\,a\,b^3\sqrt{\frac{1}{a\,b+\sqrt{a^3\,b}}}-4\,b^3\cos\left(c+d\,x\right)+\frac{4\,a\,b^4\cos(c+d\,x)}{a\,b+\sqrt{a^3\,b}}\right)\sqrt{-\frac{a\,b-\sqrt{a^3\,b}}{16\left(a^3\,b-a^2\,b^2\right)}}}{d}+\frac{\ln\left(4\,b^3\cos\left(c+d\,x\right)-4\,a\,b^3\sqrt{-\ldots}\right.}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (log(4*a*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - 4*b^3*cos(c + d*x) + (4*a*b^
4*cos(c + d*x))/(a*b + (a^3*b)^(1/2)))*(-(a*b - (a^3*b)^(1/2))/(16*(a^3*b -
 a^2*b^2)))^(1/2))/d + (log(4*b^3*cos(c + d*x) - 4*a*b^3*(1/(a*b - (a^3*b)^
(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b - (a^3*b)^(1/2)))*(-(a*b + (a^3
*b)^(1/2))/(16*(a^3*b - a^2*b^2)))^(1/2))/d - (log(4*b^3*cos(c + d*x) + 4*a
*b^3*(1/(a*b + (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b + (a^3*b
)^(1/2)))*(1/(a*b + (a^3*b)^(1/2)))^(1/2))/(4*d) - (log(4*b^3*cos(c + d*x)
```

```
+ 4*a*b^3*(1/(a*b - (a^3*b)^(1/2)))^(1/2) - (4*a*b^4*cos(c + d*x))/(a*b - (
a^3*b)^(1/2)))*(1/(a*b - (a^3*b)^(1/2)))^(1/2))/(4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.200 $\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

```
[Out] -arctanh(cos(d*x+c))/a/d-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(
1/2))^(1/2))/a/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*
x+c)/(a^(1/2)+b^(1/2))^(1/2))/a/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2ad\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\tanh^{-1}(\cos(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a*Sqrt
[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a*d) + (b^(1/4)*ArcTanh[(b^
(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a*Sqrt[Sqrt[a] + Sqrt[b]]*
d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-b+2bx^2-bx^4)}\,dx, x, \cos(c+dx)\right)}{d}$$
$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a(-1+x^2)}+\frac{b-bx^2}{a(a-b+2bx^2-bx^4)}\right)dx, x, \cos(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{ad} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{ad}$$
$$= -\frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{b\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\,dx, x, \cos(c+dx)\right)}{2ad} + \frac{b\,\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$
$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{ad} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [C] time = 0.26, size = 318, normalized size = 2.34

$$-\frac{ib\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-3i\#1^2\log\left(\#1^2\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/8*(8*Log[Cos[(c + d*x)/2]] - 8*Log[Sin[(c + d*x)/2]] + I*b*RootSum[b - 4
*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)] + I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*
b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a*d)
```

fricas [B] time = 0.65, size = 773, normalized size = 5.68

$$\frac{ad\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}+b}{(a^3-a^2b)d^2}}\log\left(b\cos(dx+c)-\left((a^4-a^3b)d^3\sqrt{\frac{b}{(a^5-2a^4b+a^3b^2)d^4}}-abd\right)\sqrt{-\frac{(a^3-a^2b)d^2\sqrt{\frac{}{(a^5-2a}}}{(a^3-a^2b)}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
 b)/((a^3 - a^2*b)*d^2))*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a
^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^
5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(((a^3 -
a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)
)*log(b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)
*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) - b)/((a^3 - a^2*b)*d^2))) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b/((a^5
 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) -
 ((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - a*b*d)*sqrt(-
((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + b)/((a^3 - a^2
*b)*d^2))) + a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*
d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-b*cos(d*x + c) - ((a^4 - a^3*b)*d^3*sq
rt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + a*b*d)*sqrt(((a^3 - a^2*b)*d^2*sqrt
(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2))) - 2*log(1/2*
cos(d*x + c) + 1/2) + 2*log(-1/2*cos(d*x + c) + 1/2))/(a*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[76,51]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[11,92]2/d*1/4/a*ln(abs(1-cos(
c+d*x))/abs(1+cos(c+d*x)))+2/d*4*b/a*2/d*(1/(8*b*2)*(c+d*x)+((2*a^4*b-8*a^3
*b^2-2*a^3*a*b+3*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+10*a^2*b^3+8*a
^2*b*a*b-12*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-4*a*b^4-10*a*b^2*
a*b+11*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*b^3*a*b+2*b^3*sqrt(a
*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)*b^2+(-4*a^4*b^2-3*a^4*b*sqrt(a^
2-a*b+sqrt(a*b)*(a-b))+8*a^3*b^3+9*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*
a^3*b*a*b-4*a^2*b^4-5*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-8*a^2*b^2*a*b-a
*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*a*b^3*a*b)*abs(a-b)*abs(b)+(2*a^4*b^3-
4*a^3*b^4-2*a^3*b^2*a*b+3*a^3*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2
*a^2*b^5+4*a^2*b^3*a*b-6*a^2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-2*
a*b^4*a*b-a*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b))/((96*a^6
*b^2-480*a^5*b^3+832*a^4*b^4-576*a^3*b^5+96*a^2*b^6+32*a*b^7)*abs(b))*(atan
(tan(c+d*x)/sqrt(-(32*a*b+sqrt(32*a*b*32*a*b+4*(-16*a*b+16*b^2)*16*a*b))/2/
(-16*a*b+16*b^2)))+pi*floor((c+d*x)/pi+1/2))-((2*a^4*b-8*a^3*b^2-2*a^3*a*b+
3*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+10*a^2*b^3+8*a^2*b*a*b-12*a^
2*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-4*a*b^4-10*a*b^2*a*b+11*a*b^2*
sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+4*b^3*a*b+2*b^3*sqrt(a*b)*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)*b^2+(-4*a^4*b^2+3*a^4*b*sqrt(a^2-a*b+sqrt(
a*b)*(-a+b))+8*a^3*b^3-9*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+4*a^3*b*a*b
-4*a^2*b^4+5*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-8*a^2*b^2*a*b+a*b^4*sqr
t(a^2-a*b+sqrt(a*b)*(-a+b))+4*a*b^3*a*b)*abs(a-b)*abs(b)+(2*a^4*b^3-4*a^3*b
^4-2*a^3*b^2*a*b+3*a^3*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+2*a^2*b
^5+4*a^2*b^3*a*b-6*a^2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-2*a*b^4
*a*b-a*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b))/((96*a^6*b^2
-480*a^5*b^3+832*a^4*b^4-576*a^3*b^5+96*a^2*b^6+32*a*b^7)*abs(b))*(atan(tan
(c+d*x)/sqrt(-(32*a*b-sqrt(32*a*b*32*a*b+4*(-16*a*b+16*b^2)*16*a*b))/2/(-16
*a*b+16*b^2)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [A] time = 0.49, size = 120, normalized size = 0.88

$$\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2ad}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{2da}+\frac{b\,\mathrm{arctanh}\left(\frac{\cos\left(dx+c\right)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{2da\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{b\arctan\left(\frac{\cos\left(dx+c\right)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{2da\sqrt{\left(\sqrt{ab}-b\right)b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/a/d*ln(cos(d*x+c)-1)-1/2/d/a*ln(1+cos(d*x+c))+1/2/d/a*b/(((a*b)^(1/2)+b
)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d/a*b/(((a*b
)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a*d*integrate(-2*(12*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*b^2*
cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^2*cos(2*d*x + 2*c)*sin(d*x + c) - b^2*s
in(d*x + c) + (b^2*sin(7*d*x + 7*c) - 3*b^2*sin(5*d*x + 5*c) + 3*b^2*sin(3*
d*x + 3*c) - b^2*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x + 6*c)
 + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7
*c) + 4*(3*b^2*sin(5*d*x + 5*c) - 3*b^2*sin(3*d*x + 3*c) + b^2*sin(d*x + c)
)*cos(6*d*x + 6*c) - 6*(2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x
+ 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b -
 3*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^2*cos(7*d*x + 7*c) - 3*b^2*cos(
5*d*x + 5*c) + 3*b^2*cos(3*d*x + 3*c) - b^2*cos(d*x + c))*sin(8*d*x + 8*c)
- (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2
)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^2*cos(5*d*x + 5*c) - 3*b^2*co
s(3*d*x + 3*c) + b^2*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4*b^2*cos(2*d*x +
2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) + 2*(3*(8
*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin(4*d*x +
4*c) - 3*(4*b^2*cos(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x
+ 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^
2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2
*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*
cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*
d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4
*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*
a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2
- 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin
(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x +
4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)
*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + log(cos(d*x)^2 + 2*cos(d*x)*cos(
c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - log(cos(d*x)^2
 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2
))/(a*d)
```

mupad [B] time = 15.36, size = 2031, normalized size = 14.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] - (atan((((((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^4
 - 192*a^3*b^5 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)
^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a
^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))
/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(
16*(a^4*b - a^5)))^(1/2)*1i + (((((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)
))^(1/2)*(192*a^3*b^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b
^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c
 + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a
^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2
*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i)/((((((a^2*b + (a^5*b)^(1/
2))/(16*(a^4*b - a^5)))^(1/2)*(256*a^4*b^4 - 192*a^3*b^5 + cos(c + d*x)*(76
8*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)
) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^
(1/2) + 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 6*b^
5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - (((((a
^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*(192*a^3*b^5 - 256*a^4*b^4
+ cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b + (a^5*b)^(1/2))/(16*(a^
4*b - a^5)))^(1/2)) - 144*a^2*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(1
6*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a
^5)))^(1/2) + 6*b^5*cos(c + d*x))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5
)))^(1/2)))*((a^2*b + (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*2i)/d - atan
h((32*b^4*cos(c + d*x))/(32*b^4 - (18*b^5)/a) - (18*b^5*cos(c + d*x))/(32*a
*b^4 - 18*b^5))/(a*d) - (atan(((6*b^5*cos(c + d*x) + (((256*a^4*b^4 - 192*a
^3*b^5 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/
(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/
2) - 144*a^2*b^5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))
^(1/2) + 12*a*b^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^
2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i + (6*b^5*cos(c + d*x) + (
((192*a^3*b^5 - 256*a^4*b^4 + cos(c + d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^
2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(1
6*(a^4*b - a^5)))^(1/2) - 144*a^2*b^5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2)
)/(16*(a^4*b - a^5)))^(1/2) - 12*a*b^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b
 - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2)*1i)/((6
*b^5*cos(c + d*x) + (((256*a^4*b^4 - 192*a^3*b^5 + cos(c + d*x)*(768*a^4*b^
5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*
b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 144*a^2*b^5*cos(c + d*x))*((
a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) + 12*a*b^5)*((a^2*b - (a^5
*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b -
 a^5)))^(1/2) - (6*b^5*cos(c + d*x) + (((192*a^3*b^5 - 256*a^4*b^4 + cos(c
+ d*x)*(768*a^4*b^5 - 512*a^5*b^4)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^
5)))^(1/2))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 144*a^2*b^
5*cos(c + d*x))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2) - 12*a*b
^5)*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5)))^(1/2))*((a^2*b - (a^5*b)^(
1/2))/(16*(a^4*b - a^5)))^(1/2)))*((a^2*b - (a^5*b)^(1/2))/(16*(a^4*b - a^5
)))^(1/2)*2i)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)/(a - b*sin(c + d*x)**4), x)
```

3.201 $\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(\cos(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c}{2ad}$$

```
[Out] -1/2*arctanh(cos(d*x+c))/a/d-1/4/a/d/(1-cos(d*x+c))+1/4/a/d/(1+cos(d*x+c))-
1/2*b^(3/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a
^(1/2)-b^(1/2))^(1/2)-1/2*b^(3/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/
2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1170, 207, 1093, 205, 208}

$$-\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(\cos(c+dx)+1)}-\frac{\tanh^{-1}(\cos(c}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(3/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^(3/2
)*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(2*a*d) - (b^(3/4)*Arc
Tanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^(3/2)*Sqrt[Sqrt[
a] + Sqrt[b]]*d) - 1/(4*a*d*(1 - Cos[c + d*x])) + 1/(4*a*d*(1 + Cos[c + d*x
]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a-b+2bx^2-bx^4\right)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(\frac{1}{4a(-1+x)^2}+\frac{1}{4a(1+x)^2}-\frac{1}{2a\left(-1+x^2\right)}+\frac{b}{a\left(a-b+2bx^2-bx^4\right)}\right) dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{2ad}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{1}{4ad(1-\cos(c+dx))}+\frac{1}{4ad(1+\cos(c+dx))}+\frac{b^{3/2}\,\text{Subst}}{}$$

$$= -\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\tanh^{-1}(\cos(c+dx))}{2ad}-\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^{3/2}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{}{4ad}$$

Mathematica [C] time = 0.34, size = 242, normalized size = 1.32

$$\frac{4ib\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\#1^3}{\#1^6b-3\#1^4b-8\#1^2a+}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-Csc[(c + d*x)/2]^2 - 4*Log[Cos[(c + d*x)/2]] + 4*Log[Sin[(c + d*x)/2]] +
(4*I)*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & ,
 (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + I*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - I*Log[1
- 2*Cos[c + d*x]*#1 + #1^2]*#1^3)/(-b - 8*a*#1^2 + 3*b*#1^2 - 3*b*#1^4 + b*
#1^6) & ] + Sec[(c + d*x)/2]^2)/(8*a*d)
```

fricas [B] time = 0.68, size = 924, normalized size = 5.02

$$-\frac{\left(ad\cos(dx+c)^2-ad\right)\sqrt{-\frac{\left(a^4-a^3b\right)d^2\sqrt{\frac{b^3}{\left(a^7-2a^6b+a^5b^2\right)d^4}}+b^2}{\left(a^4-a^3b\right)d^2}}\log\left(b^2\cos(dx+c)-\left(\left(a^5-a^4b\right)d^3\sqrt{\frac{b^3}{\left(a^7-2a^6b+a^5b^2\right)d^4}}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 -
2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(b^2*cos(d*x + c) -
 ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - a^2*b*d)*sq
rt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a
^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqr
t(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(b^2*
cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4
)) - b^2)/((a^4 - a^3*b)*d^2))) - (a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 -
a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*
d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b +
 a^5*b^2)*d^4)) - a^2*b*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*
b + a^5*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))) + (a*d*cos(d*x + c)^2 - a*d
)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/
((a^4 - a^3*b)*d^2))*log(-b^2*cos(d*x + c) - ((a^5 - a^4*b)*d^3*sqrt(b^3/((
a^7 - 2*a^6*b + a^5*b^2)*d^4)) + a^2*b*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^3/
((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))) + (cos(d*x +
c)^2 - 1)*log(1/2*cos(d*x + c) + 1/2) - (cos(d*x + c)^2 - 1)*log(-1/2*cos(d
*x + c) + 1/2) - 2*cos(d*x + c))/(a*d*cos(d*x + c)^2 - a*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-35,-31]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[87,-13]-2/d*(-(1-cos(c+d*x)
)/(1+cos(c+d*x))*1/16/a+(2*(1-cos(c+d*x))/(1+cos(c+d*x))+1)*1/16/a/(1-cos(c
+d*x))*(1+cos(c+d*x))-1/8/a*ln(abs(1-cos(c+d*x))/abs(1+cos(c+d*x))))-2/d/a*
2/d*((2*a^3*b-12*a^2*b^2-6*a^2*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-2*a^2*a*b+3*
a^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+10*a*b^3+12*a*b^2*sqrt(a^2-a*b+
sqrt(a*b)*(a-b))+12*a*b*a*b-6*a*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2
*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-10*b^2*a*b-b^2*sqrt(a*b)*sqrt(a^2-a*b+sq
rt(a*b)*(a-b)))*abs(a-b)/(24*a^5-96*a^4*b+112*a^3*b^2-32*a^2*b^3-8*a*b^4)*(
atan(tan(c+d*x)/sqrt(-(8*a+sqrt(8*a*8*a+4*(4*b-4*a)*4*a))/2/(4*b-4*a)))+pi*
floor((c+d*x)/pi+1/2))-(2*a^3*b-12*a^2*b^2+6*a^2*b*sqrt(a^2-a*b+sqrt(a*b)*(
-a+b))-2*a^2*a*b+3*a^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+10*a*b^3-12
*a*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+12*a*b*a*b-6*a*b*sqrt(a*b)*sqrt(a^2-a
*b+sqrt(a*b)*(-a+b))-2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-10*b^2*a*b-b^2*sq
rt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(24*a^5-96*a^4*b+112*a^3*b
^2-32*a^2*b^3-8*a*b^4)*(atan(tan(c+d*x)/sqrt(-(8*a-sqrt(8*a*8*a+4*(4*b-4*a)
*4*a))/2/(4*b-4*a)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [A] time = 0.57, size = 170, normalized size = 0.92

$$\frac{1}{4da\left(\cos\left(dx+c\right)-1\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{4ad}+\frac{1}{4ad\left(1+\cos\left(dx+c\right)\right)}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{4da}-\frac{b^{2}\operatorname{arctanh}\left(\frac{\cos\left(dx+c\right)b}{\sqrt{\left(\sqrt{ab}+b\right)}}\right.}{2da\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d/a/(cos(d*x+c)-1)+1/4/a/d*ln(cos(d*x+c)-1)+1/4/a/d/(1+cos(d*x+c))-1/4/
d/a*ln(1+cos(d*x+c))-1/2/d/a*b^2/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arct
anh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d/a*b^2/(a*b)^(1/2)/(((a*b)
^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(cos(3*d*x + 3*c) + cos(d*x + c))*cos(4*d*x + 4*c) - 4*(2*cos(2*d*x
+ 2*c) - 1)*cos(3*d*x + 3*c) - 8*cos(2*d*x + 2*c)*cos(d*x + c) + 4*(a*d*cos
(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 - 4*a*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d*cos(2
*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c))*inte
grate(8*(4*b^2*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + 2*(8*a*b - 3*b^2)*cos(3*
d*x + 3*c)*sin(4*d*x + 4*c) - 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(3*d*x
+ 3*c) - (b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) + 4
*(b^2*sin(5*d*x + 5*c) - b^2*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*(2*b^2*
sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) + (b^
2*cos(5*d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(8*d*x + 8*c) - 4*(b^2*cos(5*
d*x + 5*c) - b^2*cos(3*d*x + 3*c))*sin(6*d*x + 6*c) + (4*b^2*cos(2*d*x + 2*
c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(5*d*x + 5*c) - (4*b^2*co
s(2*d*x + 2*c) - b^2)*sin(3*d*x + 3*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^
2*cos(6*d*x + 6*c)^2 + 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)
^2 + 16*a*b^2*sin(6*d*x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*co
s(2*d*x + 2*c) + a*b^2 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b
^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8
*d*x + 8*c) + 8*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a
*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*
a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x +
8*c) + 16*(2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos
(4*d*x + 4*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log
(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c)
 + sin(c)^2) - (2*(2*cos(2*d*x + 2*c) - 1)*cos(4*d*x + 4*c) - cos(4*d*x + 4
*c)^2 - 4*cos(2*d*x + 2*c)^2 - sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(
2*d*x + 2*c) - 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) - 1)*log(cos(d*x)^
2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^
2) + 4*(sin(3*d*x + 3*c) + sin(d*x + c))*sin(4*d*x + 4*c) - 8*sin(3*d*x + 3
*c)*sin(2*d*x + 2*c) - 8*sin(2*d*x + 2*c)*sin(d*x + c) + 4*cos(d*x + c))/(a
*d*cos(4*d*x + 4*c)^2 + 4*a*d*cos(2*d*x + 2*c)^2 + a*d*sin(4*d*x + 4*c)^2 -
 4*a*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*a*d*sin(2*d*x + 2*c)^2 - 4*a*d
*cos(2*d*x + 2*c) + a*d - 2*(2*a*d*cos(2*d*x + 2*c) - a*d)*cos(4*d*x + 4*c)
)
```

mupad [B] time = 15.19, size = 2779, normalized size = 15.10

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(cos(c + d*x)*1i)*1i)/(d*(2*a - 2*a*cos(c + d*x)^2)) - cos(c + d*x)/(d
*(2*a - 2*a*cos(c + d*x)^2)) - (atan(cos(c + d*x)*1i)*cos(c + d*x)^2*1i)/(d
*(2*a - 2*a*cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(((a^7*b^3)^(1/2)
 + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(((a^7*b
^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x)*(
((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*cos(
c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^5*b
^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*12i
 - a^7*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(
1/2)*4i + a^7*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*
a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^
6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^
2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2
) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*(((a^
7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(2*b^3*(a^7*b^3)^
(1/2) + a^3*b^5 + a^5*b^3 - a*b^2*(a^7*b^3)^(1/2) + a^2*b*(a^7*b^3)^(1/2)))
*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2*a*
cos(c + d*x)^2)) + (a*atan((a^13*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)
/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(-((a^7*b^3)^(1/2)
- a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x)*(-((a^7*b^3
)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*cos(c + d*x)*
(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^5*b^4*cos(c
 + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*12i - a^7*
b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4
i + a^7*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))
^(3/2)*320i - a^8*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b
- 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/
(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2)
- a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*(-((a^7
*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(a^3*b^5 - 2*b^3*(
a^7*b^3)^(1/2) + a^5*b^3 + a*b^2*(a^7*b^3)^(1/2) - a^2*b*(a^7*b^3)^(1/2)))*
(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2*a*
cos(c + d*x)^2)) - (a*cos(c + d*x)^2*atan((a^13*cos(c + d*x)*(((a^7*b^3)^(1
/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(((a^
7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c + d*x
)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4*b^5*c
os(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*8i + a^
5*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
12i - a^7*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7)
)^(1/2)*4i + a^7*b^4*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b -
16*a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3*b^2)/(16
*a^6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(((a^7*b^3)^(1/2) + a^3
*b^2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(((a^7*b^3)^(
1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*cos(c + d*x)*((
(a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/(2*b^3*(a^7*b^
3)^(1/2) + a^3*b^5 + a^5*b^3 - a*b^2*(a^7*b^3)^(1/2) + a^2*b*(a^7*b^3)^(1/2
)))*(((a^7*b^3)^(1/2) + a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*4i)/(d*(2*a - 2
*a*cos(c + d*x)^2)) - (a*cos(c + d*x)^2*atan((a^13*cos(c + d*x)*(-((a^7*b^3
)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*2048i + a^10*b*cos(c + d*x)*(
-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*64i - a^12*b*cos(c
+ d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*7168i - a^4
*b^5*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
8i + a^5*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7)
)^(1/2)*12i - a^7*b^2*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b
- 16*a^7))^(1/2)*4i + a^7*b^4*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(1
6*a^6*b - 16*a^7))^(3/2)*320i - a^8*b^3*cos(c + d*x)*(-((a^7*b^3)^(1/2) - a
^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*576i + a^9*b^2*cos(c + d*x)*(-((a^7*b^3)
^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(3/2)*192i - a^10*b^3*cos(c + d*x)*(
-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*3072i + a^11*b^2*co
```

```
s(c + d*x)*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(5/2)*8192i)/
(a^3*b^5 - 2*b^3*(a^7*b^3)^(1/2) + a^5*b^3 + a*b^2*(a^7*b^3)^(1/2) - a^2*b*
(a^7*b^3)^(1/2)))*(-((a^7*b^3)^(1/2) - a^3*b^2)/(16*a^6*b - 16*a^7))^(1/2)*
4i)/(d*(2*a - 2*a*cos(c + d*x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**3/(a - b*sin(c + d*x)**4), x)
```

3.202 $\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=229

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{}{16a}$$

```
[Out] -1/8*(3*a+8*b)*arctanh(cos(d*x+c))/a^2/d-1/16/a/d/(1-cos(d*x+c))^2-3/16/a/d
/(1-cos(d*x+c))+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*x+c))-1/2*b^(5/
4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2/d/(a^(1/2)-b^(1/2
))^(1/2)+1/2*b^(5/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^
2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1170, 207, 1166, 205, 208}

$$-\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{}{16a}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(5/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] - Sqrt[b]]*d) - ((3*a + 8*b)*ArcTanh[Cos[c + d*x]])/(8*a^2*d) +
(b^(5/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sq
rt[Sqrt[a] + Sqrt[b]]*d) - 1/(16*a*d*(1 - Cos[c + d*x])^2) - 3/(16*a*d*(1 -
 Cos[c + d*x])) + 1/(16*a*d*(1 + Cos[c + d*x])^2) + 3/(16*a*d*(1 + Cos[c +
d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-b+2bx^2-bx^4\right)}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{8a(-1+x)^3}+\frac{3}{16a(-1+x)^2}+\frac{1}{8a(1+x)^3}+\frac{3}{16a(1+x)^2}+\frac{-3a-8b}{8a^2\left(-1+x^2\right)}-\frac{b^2\left(-1+x^2\right)}{a^2\left(a-b+2bx^2-b\right.}\right.\right.}{d}$$

$$= -\frac{1}{16ad(1-\cos(c+dx))^2}-\frac{3}{16ad(1-\cos(c+dx))}+\frac{1}{16ad(1+\cos(c+dx))^2}+\frac{}{16ad}$$

$$= -\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}-\frac{1}{16ad(1-\cos(c+dx))^2}-\frac{3}{16ad(1-\cos(c+dx))}$$

$$= -\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{(3a+8b)\tanh^{-1}(\cos(c+dx))}{8a^2d}+\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+\right.}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [C] time = 1.17, size = 409, normalized size = 1.79

$$-8ib^2\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{2\#1^6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-3i\#1^2\,\text{lo}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-6*a*Csc[(c + d*x)/2]^2 - a*Csc[(c + d*x)/2]^4 - 24*a*Log[Cos[(c + d*x)/2]
] - 64*b*Log[Cos[(c + d*x)/2]] + 24*a*Log[Sin[(c + d*x)/2]] + 64*b*Log[Sin[
(c + d*x)/2]] - (8*I)*b^2*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b
*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2] + 6*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
2 - (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 6*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^4 + (3*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ] + 6*
a*Sec[(c + d*x)/2]^2 + a*Sec[(c + d*x)/2]^4)/(64*a^2*d)
```

fricas [B] time = 0.77, size = 1089, normalized size = 4.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/16*(6*a*cos(d*x + c)^3 + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2
 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) + b^3)/((a^5 - a^4*b)*d^2))*log(b^4*cos(d*x + c) + (a^2*b^3*d - (a^7 - a^
6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(-((a^5 - a^4*b)*d^
2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))) -
4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*
b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2)
)*log(b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*
a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b +
a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 4*(a^2*d*cos(d*x + c)^4 - 2*a
^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^
8*b + a^7*b^2)*d^4)) + b^3)/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) + (a
^2*b^3*d - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqr
t(-((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^3)/((a^
5 - a^4*b)*d^2))) + 4*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*
d)*sqrt(((a^5 - a^4*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)
/((a^5 - a^4*b)*d^2))*log(-b^4*cos(d*x + c) - (a^2*b^3*d + (a^7 - a^6*b)*d^
3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)))*sqrt(((a^5 - a^4*b)*d^2*sqrt(b
^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - b^3)/((a^5 - a^4*b)*d^2))) - 10*a*cos
(d*x + c) - ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a + 8*b)*cos(d*x + c)^2 + 3*
a + 8*b)*log(1/2*cos(d*x + c) + 1/2) + ((3*a + 8*b)*cos(d*x + c)^4 - 2*(3*a
 + 8*b)*cos(d*x + c)^2 + 3*a + 8*b)*log(-1/2*cos(d*x + c) + 1/2))/(a^2*d*co
s(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[76,51]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[11,92]-2/d*((-32*((1-cos(c+d*
x))/(1+cos(c+d*x)))^2*a-256*(1-cos(c+d*x))/(1+cos(c+d*x))*a)*1/4096/a^2+(18
*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a+48*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b
+8*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)*1/128/a^2/((1-cos(c+d*x))/(1+cos(c+d*
x)))^2+(-3*a-8*b)*1/32/a^2*ln(abs(1-cos(c+d*x))/abs(1+cos(c+d*x))))+2/d*4*b
^2/a^2*2/d*(1/(8*b*2)*(c+d*x)+((2*a^4*b-8*a^3*b^2-2*a^3*a*b+3*a^3*sqrt(a*b)
*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+10*a^2*b^3+8*a^2*b*a*b-12*a^2*b*sqrt(a*b)*sq
rt(a^2-a*b+sqrt(a*b)*(a-b))-4*a*b^4-10*a*b^2*a*b+11*a*b^2*sqrt(a*b)*sqrt(a^
2-a*b+sqrt(a*b)*(a-b))+4*b^3*a*b+2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-
b)))*abs(a-b)*b^2+(-4*a^4*b^2-3*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+8*a^3*b
^3+9*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*a^3*b*a*b-4*a^2*b^4-5*a^2*b^3*
sqrt(a^2-a*b+sqrt(a*b)*(a-b))-8*a^2*b^2*a*b-a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a
-b))+4*a*b^3*a*b)*abs(a-b)*abs(b)+(2*a^4*b^3-4*a^3*b^4-2*a^3*b^2*a*b+3*a^3*
b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2*a^2*b^5+4*a^2*b^3*a*b-6*a^2*b
^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-2*a*b^4*a*b-a*b^4*sqrt(a*b)*sqrt
(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b))/((96*a^6*b^2-480*a^5*b^3+832*a^4*b^4-5
76*a^3*b^5+96*a^2*b^6+32*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(32*a*b+sqrt
```

```
(32*a*b*32*a*b+4*(-16*a*b+16*b^2)*16*a*b))/2/(-16*a*b+16*b^2)))+pi*floor((c
+d*x)/pi+1/2))-((2*a^4*b-8*a^3*b^2-2*a^3*a*b+3*a^3*sqrt(a*b)*sqrt(a^2-a*b+s
qrt(a*b)*(-a+b))+10*a^2*b^3+8*a^2*b*a*b-12*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqr
t(a*b)*(-a+b))-4*a*b^4-10*a*b^2*a*b+11*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*
b)*(-a+b))+4*b^3*a*b+2*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-
b)*b^2+(-4*a^4*b^2+3*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+8*a^3*b^3-9*a^3*b
^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+4*a^3*b*a*b-4*a^2*b^4+5*a^2*b^3*sqrt(a^2-
a*b+sqrt(a*b)*(-a+b))-8*a^2*b^2*a*b+a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+4*
a*b^3*a*b)*abs(a-b)*abs(b)+(2*a^4*b^3-4*a^3*b^4-2*a^3*b^2*a*b+3*a^3*b^2*sqr
t(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+2*a^2*b^5+4*a^2*b^3*a*b-6*a^2*b^3*sqr
t(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-2*a*b^4*a*b-a*b^4*sqrt(a*b)*sqrt(a^2-
a*b+sqrt(a*b)*(-a+b)))*abs(a-b))/((96*a^6*b^2-480*a^5*b^3+832*a^4*b^4-576*a
^3*b^5+96*a^2*b^6+32*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(32*a*b-sqrt(32*
a*b*32*a*b+4*(-16*a*b+16*b^2)*16*a*b))/2/(-16*a*b+16*b^2)))+pi*floor((c+d*x
)/pi+1/2)))
```

maple [A] time = 0.54, size = 232, normalized size = 1.01

$$-\frac{1}{16da\left(\cos\left(dx+c\right)-1\right)^{2}}+\frac{3}{16da\left(\cos\left(dx+c\right)-1\right)}+\frac{3\ln\left(\cos\left(dx+c\right)-1\right)}{16ad}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)b}{2d\,a^{2}}+\frac{}{16ad\left(1+c\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/16/d/a/(cos(d*x+c)-1)^2+3/16/d/a/(cos(d*x+c)-1)+3/16/a/d*ln(cos(d*x+c)-1
)+1/2/d/a^2*ln(cos(d*x+c)-1)*b+1/16/a/d/(1+cos(d*x+c))^2+3/16/a/d/(1+cos(d*
x+c))-3/16/d/a*ln(1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))*b+1/2/d*b^2/a^2/
(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1
/2/d*b^2/a^2/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)
*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/16*(48*a*cos(2*d*x + 2*c)*cos(d*x + c) - 176*a*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 48*a*sin(2*d*x + 2*c)*sin(d*x + c) - 4*(3*a*cos(7*d*x + 7*c) - 1
1*a*cos(5*d*x + 5*c) - 11*a*cos(3*d*x + 3*c) + 3*a*cos(d*x + c))*cos(8*d*x
+ 8*c) + 12*(4*a*cos(6*d*x + 6*c) - 6*a*cos(4*d*x + 4*c) + 4*a*cos(2*d*x +
2*c) - a)*cos(7*d*x + 7*c) - 16*(11*a*cos(5*d*x + 5*c) + 11*a*cos(3*d*x + 3
*c) - 3*a*cos(d*x + c))*cos(6*d*x + 6*c) + 44*(6*a*cos(4*d*x + 4*c) - 4*a*c
os(2*d*x + 2*c) + a)*cos(5*d*x + 5*c) + 24*(11*a*cos(3*d*x + 3*c) - 3*a*cos
(d*x + c))*cos(4*d*x + 4*c) - 44*(4*a*cos(2*d*x + 2*c) - a)*cos(3*d*x + 3*c
) - 12*a*cos(d*x + c) + 16*(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*cos(6*d*x +
 6*c)^2 + 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d
*sin(8*d*x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin(4*d*x + 4*
c)^2 - 48*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*sin(2*d*x + 2*
c)^2 - 8*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x + 6*c) - 6*a
^2*d*cos(4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c)
- 8*(6*a^2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x
 + 6*c) - 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 4*(2*a^2
*d*sin(6*d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d*sin(2*d*x + 2*c))
*sin(6*d*x + 6*c))*integrate(-2*(12*b^3*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) -
```

```
 4*b^3*cos(d*x + c)*sin(2*d*x + 2*c) + 4*b^3*cos(2*d*x + 2*c)*sin(d*x + c)
- b^3*sin(d*x + c) + (b^3*sin(7*d*x + 7*c) - 3*b^3*sin(5*d*x + 5*c) + 3*b^3
*sin(3*d*x + 3*c) - b^3*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*b^3*sin(6*d*x
 + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*cos(
7*d*x + 7*c) + 4*(3*b^3*sin(5*d*x + 5*c) - 3*b^3*sin(3*d*x + 3*c) + b^3*sin
(d*x + c))*cos(6*d*x + 6*c) - 6*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)
*sin(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(3*(8*a*b^2 - 3*b^3)*sin(3*d*x + 3*
c) - (8*a*b^2 - 3*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - (b^3*cos(7*d*x + 7*
c) - 3*b^3*cos(5*d*x + 5*c) + 3*b^3*cos(3*d*x + 3*c) - b^3*cos(d*x + c))*si
n(8*d*x + 8*c) - (4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2
*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(3*b^3*cos(5*d*x
+ 5*c) - 3*b^3*cos(3*d*x + 3*c) + b^3*cos(d*x + c))*sin(6*d*x + 6*c) + 3*(4
*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*sin(5*d
*x + 5*c) + 2*(3*(8*a*b^2 - 3*b^3)*cos(3*d*x + 3*c) - (8*a*b^2 - 3*b^3)*cos
(d*x + c))*sin(4*d*x + 4*c) - 3*(4*b^3*cos(2*d*x + 2*c) - b^3)*sin(3*d*x +
3*c))/(a^2*b^2*cos(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*
b^2*cos(2*d*x + 2*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x
+ 6*c)^2 + 16*a^2*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2
*b^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 4
8*a^3*b + 9*a^2*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2
*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x
 + 8*c) + 8*(4*a^2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b -
 3*a^2*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6
*c) + 2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*
sin(8*d*x + 8*c) + 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*s
in(4*d*x + 4*c))*sin(6*d*x + 6*c)), x) + ((3*a + 8*b)*cos(8*d*x + 8*c)^2 +
16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 16*
(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a +
 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a + 8*
b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2 -
2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3*a
 + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)*c
os(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2*c) + 3*a + 8*b)*cos(6*d*x + 6
*c) - 12*(4*(3*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(4*d*x + 4*c) - 8*
(3*a + 8*b)*cos(2*d*x + 2*c) - 4*(2*(3*a + 8*b)*sin(6*d*x + 6*c) - 3*(3*a +
 8*b)*sin(4*d*x + 4*c) + 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) -
 16*(3*(3*a + 8*b)*sin(4*d*x + 4*c) - 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c) + 3*a + 8*b)*log(cos(d*x)^2 + 2*cos(d*x)*cos(c) + cos(c)^2 + si
n(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) - ((3*a + 8*b)*cos(8*d*x + 8*c)^2
+ 16*(3*a + 8*b)*cos(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*cos(4*d*x + 4*c)^2 + 1
6*(3*a + 8*b)*cos(2*d*x + 2*c)^2 + (3*a + 8*b)*sin(8*d*x + 8*c)^2 + 16*(3*a
 + 8*b)*sin(6*d*x + 6*c)^2 + 36*(3*a + 8*b)*sin(4*d*x + 4*c)^2 - 48*(3*a +
8*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(3*a + 8*b)*sin(2*d*x + 2*c)^2
- 2*(4*(3*a + 8*b)*cos(6*d*x + 6*c) - 6*(3*a + 8*b)*cos(4*d*x + 4*c) + 4*(3
*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(8*d*x + 8*c) - 8*(6*(3*a + 8*b)
*cos(4*d*x + 4*c) - 4*(3*a + 8*b)*cos(2*d*x + 2*c) + 3*a + 8*b)*cos(6*d*x +
 6*c) - 12*(4*(3*a + 8*b)*cos(2*d*x + 2*c) - 3*a - 8*b)*cos(4*d*x + 4*c) -
8*(3*a + 8*b)*cos(2*d*x + 2*c) - 4*(2*(3*a + 8*b)*sin(6*d*x + 6*c) - 3*(3*a
 + 8*b)*sin(4*d*x + 4*c) + 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c)
 - 16*(3*(3*a + 8*b)*sin(4*d*x + 4*c) - 2*(3*a + 8*b)*sin(2*d*x + 2*c))*sin
(6*d*x + 6*c) + 3*a + 8*b)*log(cos(d*x)^2 - 2*cos(d*x)*cos(c) + cos(c)^2 +
sin(d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) - 4*(3*a*sin(7*d*x + 7*c) - 11*a
*sin(5*d*x + 5*c) - 11*a*sin(3*d*x + 3*c) + 3*a*sin(d*x + c))*sin(8*d*x + 8
*c) + 24*(2*a*sin(6*d*x + 6*c) - 3*a*sin(4*d*x + 4*c) + 2*a*sin(2*d*x + 2*c
))*sin(7*d*x + 7*c) - 16*(11*a*sin(5*d*x + 5*c) + 11*a*sin(3*d*x + 3*c) - 3
*a*sin(d*x + c))*sin(6*d*x + 6*c) + 88*(3*a*sin(4*d*x + 4*c) - 2*a*sin(2*d*
x + 2*c))*sin(5*d*x + 5*c) + 24*(11*a*sin(3*d*x + 3*c) - 3*a*sin(d*x + c))*
```

```
sin(4*d*x + 4*c))/(a^2*d*cos(8*d*x + 8*c)^2 + 16*a^2*d*cos(6*d*x + 6*c)^2 +
 36*a^2*d*cos(4*d*x + 4*c)^2 + 16*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(8*d*
x + 8*c)^2 + 16*a^2*d*sin(6*d*x + 6*c)^2 + 36*a^2*d*sin(4*d*x + 4*c)^2 - 48
*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*d*sin(2*d*x + 2*c)^2 - 8*
a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(4*a^2*d*cos(6*d*x + 6*c) - 6*a^2*d*cos(
4*d*x + 4*c) + 4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 8*(6*a^
2*d*cos(4*d*x + 4*c) - 4*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) -
 12*(4*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 4*(2*a^2*d*sin(6*
d*x + 6*c) - 3*a^2*d*sin(4*d*x + 4*c) + 2*a^2*d*sin(2*d*x + 2*c))*sin(8*d*x
 + 8*c) - 16*(3*a^2*d*sin(4*d*x + 4*c) - 2*a^2*d*sin(2*d*x + 2*c))*sin(6*d*
x + 6*c))
```

mupad [B] time = 15.52, size = 3692, normalized size = 16.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^
7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^
4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9
*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*
b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(1
6*(a^8*b - a^9)))^(1/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(
1/2) - (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)
^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)*1i - (((768*a^3*b^8 - 144*a^5*b
^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) +
(cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(
16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b
 - a^9)))^(1/2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))
/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(((a^9*b
^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 +
96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^
9)))^(1/2)*1i)/((((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 -
 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 81
92*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4
))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(
2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^
4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b -
 a^9)))^(1/2) - (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*((
(a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (((768*a^3*b^8 - 144
*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a
^5) + (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(((a^9*b^5)^(1/2) + a^4*
b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*
(a^8*b - a^9)))^(1/2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6
*b^5))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*((
(a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*
b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(((a^9*b^5)^(1/2) + a^4*b^3)/(16*(a^8*
b - a^9)))^(1/2) + (9*a*b^8 + 24*b^9)/(32*a^5)))*(((a^9*b^5)^(1/2) + a^4*b^
3)/(16*(a^8*b - a^9)))^(1/2)*2i)/d + (atan(((((768*a^3*b^8 - 144*a^5*b^6)/(
64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) - (cos(
c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(
a^8*b - a^9)))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b -
a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(1
6*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^
5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c + d*x)*(48*a*b^8 + 9
6*b^9 + 9*a^2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^
9)))^(1/2)*1i - (((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 -
 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) + (cos(c + d*x)*(12288*a^8*b^5 - 81
```

```
92*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^
4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c + d*x)
*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(-((a^9*b^5)^(1/2) -
 a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8
*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4)
)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)*1i)/((((768*a^3*b
^8 - 144*a^5*b^6)/(64*a^5) + (((10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^
4)/(64*a^5) - (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/
2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4
*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6
 + 144*a^6*b^5))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9))
)^(1/2))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) - (cos(c +
 d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b
^3)/(16*(a^8*b - a^9)))^(1/2) + (((768*a^3*b^8 - 144*a^5*b^6)/(64*a^5) + ((
(10240*a^8*b^5 - 12288*a^7*b^6 + 6144*a^9*b^4)/(64*a^5) + (cos(c + d*x)*(12
288*a^8*b^5 - 8192*a^9*b^4)*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)
))^(1/2))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2)
 - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4))*(-((
a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2))*(-((a^9*b^5)^(1/2) - a
^4*b^3)/(16*(a^8*b - a^9)))^(1/2) + (cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^
2*b^7))/(16*a^4))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a^9)))^(1/2) +
 (9*a*b^8 + 24*b^9)/(32*a^5)))*(-((a^9*b^5)^(1/2) - a^4*b^3)/(16*(a^8*b - a
^9)))^(1/2)*2i)/d - ((5*cos(c + d*x))/(8*a) - (3*cos(c + d*x)^3)/(8*a))/(d*
(cos(c + d*x)^4 - cos(c + d*x)^2 + sin(c + d*x)^2)) - (atan(((((cos(c + d*x
)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4) - (((12*a^3*b^8 - (9*a^5*b^6)/4
)/a^5 + ((3*a + 8*b)*((((160*a^8*b^5 - 192*a^7*b^6 + 96*a^9*b^4)/a^5 - (cos
(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(3*a + 8*b))/(256*a^6))*(3*a + 8*b
))/(16*a^2) + (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16
*a^4)))/(16*a^2))*(3*a + 8*b))/(16*a^2))*(3*a + 8*b)*1i)/(16*a^2) + (((cos(
c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2*b^7))/(16*a^4) + (((12*a^3*b^8 - (9*a^5
*b^6)/4)/a^5 + ((3*a + 8*b)*((((160*a^8*b^5 - 192*a^7*b^6 + 96*a^9*b^4)/a^5
 + (cos(c + d*x)*(12288*a^8*b^5 - 8192*a^9*b^4)*(3*a + 8*b))/(256*a^6))*(3*
a + 8*b))/(16*a^2) - (cos(c + d*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^
5))/(16*a^4)))/(16*a^2))*(3*a + 8*b))/(16*a^2))*(3*a + 8*b)*1i)/(16*a^2))/(
((9*a*b^8)/32 + (3*b^9)/4)/a^5 - (((cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9*a^2
*b^7))/(16*a^4) - (((12*a^3*b^8 - (9*a^5*b^6)/4)/a^5 + ((3*a + 8*b)*((((160
*a^8*b^5 - 192*a^7*b^6 + 96*a^9*b^4)/a^5 - (cos(c + d*x)*(12288*a^8*b^5 - 8
192*a^9*b^4)*(3*a + 8*b))/(256*a^6))*(3*a + 8*b))/(16*a^2) + (cos(c + d*x)*
(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4)))/(16*a^2))*(3*a + 8*b
))/(16*a^2))*(3*a + 8*b))/(16*a^2) + (((cos(c + d*x)*(48*a*b^8 + 96*b^9 + 9
*a^2*b^7))/(16*a^4) + (((12*a^3*b^8 - (9*a^5*b^6)/4)/a^5 + ((3*a + 8*b)*(((
(160*a^8*b^5 - 192*a^7*b^6 + 96*a^9*b^4)/a^5 + (cos(c + d*x)*(12288*a^8*b^5
 - 8192*a^9*b^4)*(3*a + 8*b))/(256*a^6))*(3*a + 8*b))/(16*a^2) - (cos(c + d
*x)*(2304*a^4*b^7 + 768*a^5*b^6 + 144*a^6*b^5))/(16*a^4)))/(16*a^2))*(3*a +
 8*b))/(16*a^2))*(3*a + 8*b))/(16*a^2)))*(3*a + 8*b)*1i)/(8*a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.203 $\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=184

$$\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x(a+b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}+\frac{5\sin(c+d\ldots}{8\ldots}$$

```
[Out] 5/8*x/b-(a+b)*x/b^2+5/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*sin(d*x+
c)/b/d+1/2*a^(5/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d
/(a^(1/2)-b^(1/2))^(1/2)+1/2*a^(5/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x
+c)/a^(1/4))/b^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3217, 1287, 199, 203, 1166, 205}

$$\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x(a+b)}{b^2}-\frac{\sin(c+dx)\cos^3(c+dx)}{4bd}+\frac{5\sin(c+d\ldots}{8\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (5*x)/(8*b) - ((a + b)*x)/b^2 + (a^(5/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Ta
n[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(5/4)*ArcTan[(
Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*
b^2*d) + (5*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*Sin[c + d*
x])/(4*b*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{\left(1+x^2\right)^3\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b\left(1+x^2\right)^3}+\frac{2}{b\left(1+x^2\right)^2}+\frac{-a-b}{b^2\left(1+x^2\right)}+\frac{a^2\left(1+x^2\right)}{b^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a^2\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2 d} - \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3}\,dx, x, \tan(c+dx)\right)}{bd} + \cdots$$

$$= -\frac{(a+b)x}{b^2} + \frac{\cos(c+dx)\sin(c+dx)}{bd} - \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\left(a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)\right)\text{S}\cdots}{\cdots}$$

$$= \frac{x}{b} - \frac{(a+b)x}{b^2} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2 d} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2 d} + \frac{5\cos\cdots}{\cdots}$$

$$= \frac{5x}{8b} - \frac{(a+b)x}{b^2} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2 d} + \frac{a^{5/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2 d} + \frac{5\,\text{co}\cdots}{\cdots}$$

Mathematica [A] time = 0.92, size = 172, normalized size = 0.93

$$-\frac{-\frac{16a^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{16a^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} + 4(8a+3b)(c+dx) - 8b\sin(2(c+dx)) + b\sin(4(c+\cdots}{32b^2 d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/32*(4*(8*a + 3*b)*(c + d*x) - (16*a^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Ta
n[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (16*a^(
```

```
3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 8*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(
b^2*d)
```

fricas [B] time = 0.68, size = 1311, normalized size = 7.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*cos(d*x + c)^2 - 1/4*a^3 - 1/4*
(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/(
(a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c)
- (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*
sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*
d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*sqrt(-((a*b^4 - b^5)*d^2*sqrt(a^
5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))*log(1/4*a^3*
cos(d*x + c)^2 - 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a
^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b
^2*d*cos(d*x + c)*sin(d*x + c) - (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a
*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^4 - b^5)*d^2*sqrt(
a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) + a^3)/((a*b^4 - b^5)*d^2))) - b^2*d*s
qrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/((a
*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2 + 1/4*a^3 - 1/4*(2*(a^2*b^3 -
 a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4)*d^2)*sqrt(a^5/((a^2*b^7 - 2*
a*b^8 + b^9)*d^4)) + 1/2*(a^2*b^2*d*cos(d*x + c)*sin(d*x + c) + (a*b^5 - b^
6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4))*cos(d*x + c)*sin(d*x + c))
*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - a^3)/(
(a*b^4 - b^5)*d^2))) + b^2*d*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 - 2
*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))*log(-1/4*a^3*cos(d*x + c)^2
 + 1/4*a^3 - 1/4*(2*(a^2*b^3 - a*b^4)*d^2*cos(d*x + c)^2 - (a^2*b^3 - a*b^4
)*d^2)*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^4)) - 1/2*(a^2*b^2*d*cos(d*x +
 c)*sin(d*x + c) + (a*b^5 - b^6)*d^3*sqrt(a^5/((a^2*b^7 - 2*a*b^8 + b^9)*d^
4))*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^4 - b^5)*d^2*sqrt(a^5/((a^2*b^7 -
 2*a*b^8 + b^9)*d^4)) - a^3)/((a*b^4 - b^5)*d^2))) + (8*a + 3*b)*d*x + (2*b
*cos(d*x + c)^3 - 5*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

giac [B] time = 1.11, size = 461, normalized size = 2.51

$$\frac{4\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^3-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{ab^2+\sqrt{a^2b^4-(ab^2-b^3)ab^2}}{ab^2-b^3}}}\right)\right)|-a+b|}{3a^4b^2-12a^3b^3+14a^2b^4-4ab^5-b^6}+\frac{4\left(3\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3 - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^2)*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 + sqrt(a^2*b^4 - (a*
b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^4*b^2 - 12*a^3*b^3 + 1
4*a^2*b^4 - 4*a*b^5 - b^6) + 4*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3 -
 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b - sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*
b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*
```

```
a^4*b^2 - 12*a^3*b^3 + 14*a^2*b^4 - 4*a*b^5 - b^6) - (d*x + c)*(8*a + 3*b)/
b^2 + (5*tan(d*x + c)^3 + 3*tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b))/d
```

maple [B] time = 0.35, size = 605, normalized size = 3.29

$$\frac{5\left(\tan^3(dx+c)\right)}{8db\left(\tan^2(dx+c)+1\right)^2}+\frac{3\tan(dx+c)}{8db\left(\tan^2(dx+c)+1\right)^2}-\frac{3\arctan(\tan(dx+c))}{8db}-\frac{\arctan(\tan(dx+c))\,a}{d\,b^2}+\frac{a^3\arctan}{2d\,b^2\,(a-b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 5/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3+3/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+
c)-3/8/d/b*arctan(tan(d*x+c))-1/d/b^2*arctan(tan(d*x+c))*a+1/2/d*a^3/b^2/(a
-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a
)*(a-b))^(1/2))-1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a^3/b/(a*b)
^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^
(1/2)+a)*(a-b))^(1/2))+1/2/d*a^3/b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a^2/b/(a-b)/(((a
*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))
^(1/2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((
-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/2)/(a-b)
/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-
b))^(1/2))-1/2/d*a^2/b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan
(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 16.87, size = 5022, normalized size = 27.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(
64*b^5) - (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(
1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16
*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d
*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5
 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)
))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a
^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 -
b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 +
 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*
b^8 - b^9)))^(1/2)*1i - (((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 +
 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*
(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12
288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(
1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b
```

^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i)/((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (63*a^8*b - 216*a^9 + 27*a^6*b^3 + 126*a^7*b^2)/(32*b^5)))*(-((a^5*b^9)^(1/2) + a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*2i)/d + (atan(((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i - (((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*1i)/((((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*b^4 - 768*a^8*b^3)/(64*b^5))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (((((2048*a^3*b^10 + 8192*a^4*b^9 - 22528*a^5*b^8 + 12288*a^6*b^7)/(

```
64*b^5) + (tan(c + d*x)*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1
/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*
b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2) + (tan(c + d*x
)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 +
 2304*a^7*b^4))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^
(1/2) - (144*a^3*b^8 + 624*a^4*b^7 + 112*a^5*b^6 - 1648*a^6*b^5 + 1536*a^7*
b^4 - 768*a^8*b^3)/(64*b^5))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)
))^(1/2) - (tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259
*a^6*b^3 + 71*a^7*b^2))/(16*b^4))*(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 -
 b^9)))^(1/2) + (63*a^8*b - 216*a^9 + 27*a^6*b^3 + 126*a^7*b^2)/(32*b^5)))*
(((a^5*b^9)^(1/2) - a^3*b^4)/(16*(a*b^8 - b^9)))^(1/2)*2i)/d + ((3*tan(c +
d*x))/(8*b) + (5*tan(c + d*x)^3)/(8*b))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)
^4 + 1)) + (atan((((a*8i + b*3i)*((tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a
^8*b + 93*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4) - ((((9*a^3*b^8)/4
+ (39*a^4*b^7)/4 + (7*a^5*b^6)/4 - (103*a^6*b^5)/4 + 24*a^7*b^4 - 12*a^8*b^
3)/b^5 + (((tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a
^5*b^6 + 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4) - (((32*a^3*b^10 + 128*a^4*
b^9 - 352*a^5*b^8 + 192*a^6*b^7)/b^5 - (tan(c + d*x)*(a*8i + b*3i)*(12288*a
^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i
 + b*3i))/(16*b^2))*(a*8i + b*3i))/(16*b^2))*(a*8i + b*3i))/(16*b^2))*1i)/(
16*b^2) + ((a*8i + b*3i)*((tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 9
3*a^5*b^4 + 259*a^6*b^3 + 71*a^7*b^2))/(16*b^4) + ((((9*a^3*b^8)/4 + (39*a^
4*b^7)/4 + (7*a^5*b^6)/4 - (103*a^6*b^5)/4 + 24*a^7*b^4 - 12*a^8*b^3)/b^5 -
 (((tan(c + d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 +
 2048*a^6*b^5 + 2304*a^7*b^4))/(16*b^4) + (((32*a^3*b^10 + 128*a^4*b^9 - 35
2*a^5*b^8 + 192*a^6*b^7)/b^5 + (tan(c + d*x)*(a*8i + b*3i)*(12288*a^2*b^11
- 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i + b*3i)
)/(16*b^2))*(a*8i + b*3i))/(16*b^2))*(a*8i + b*3i))/(16*b^2))*1i)/(16*b^2))
/(((63*a^8*b)/32 - (27*a^9)/4 + (27*a^6*b^3)/32 + (63*a^7*b^2)/16)/b^5 + ((
a*8i + b*3i)*((tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 +
259*a^6*b^3 + 71*a^7*b^2))/(16*b^4) - ((((9*a^3*b^8)/4 + (39*a^4*b^7)/4 + (
7*a^5*b^6)/4 - (103*a^6*b^5)/4 + 24*a^7*b^4 - 12*a^8*b^3)/b^5 + (((tan(c +
d*x)*(432*a^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^
5 + 2304*a^7*b^4))/(16*b^4) - (((32*a^3*b^10 + 128*a^4*b^9 - 352*a^5*b^8 +
192*a^6*b^7)/b^5 - (tan(c + d*x)*(a*8i + b*3i)*(12288*a^2*b^11 - 12288*a^3*
b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i + b*3i))/(16*b^2))*
(a*8i + b*3i))/(16*b^2))*(a*8i + b*3i))/(16*b^2)))/(16*b^2) - ((a*8i + b*3i
)*((tan(c + d*x)*(9*a^4*b^5 - 96*a^9 - 336*a^8*b + 93*a^5*b^4 + 259*a^6*b^3
 + 71*a^7*b^2))/(16*b^4) + ((((9*a^3*b^8)/4 + (39*a^4*b^7)/4 + (7*a^5*b^6)/
4 - (103*a^6*b^5)/4 + 24*a^7*b^4 - 12*a^8*b^3)/b^5 - (((tan(c + d*x)*(432*a
^2*b^9 + 1584*a^3*b^8 - 880*a^4*b^7 - 5488*a^5*b^6 + 2048*a^6*b^5 + 2304*a^
7*b^4))/(16*b^4) + (((32*a^3*b^10 + 128*a^4*b^9 - 352*a^5*b^8 + 192*a^6*b^7
)/b^5 + (tan(c + d*x)*(a*8i + b*3i)*(12288*a^2*b^11 - 12288*a^3*b^10 - 1228
8*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i + b*3i))/(16*b^2))*(a*8i + b*3
i))/(16*b^2))*(a*8i + b*3i))/(16*b^2)))/(16*b^2)))*(a*8i + b*3i)*1i)/(8*b^2
*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.204 $\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{x}{2b}$$

```
[Out] -1/2*x/b+1/2*cos(d*x+c)*sin(d*x+c)/b/d+1/2*a^(3/4)*arctan((a^(1/2)-b^(1/2))
^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*a^(3/4)*ar
ctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2)
)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3217, 1287, 199, 203, 1130, 205}

$$\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^{3/2}d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sin(c+dx)\cos(c+dx)}{2bd}-\frac{x}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -x/(2*b) + (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])
/(2*Sqrt[Sqrt[a] - Sqrt[b]]*b^(3/2)*d) - (a^(3/4)*ArcTan[(Sqrt[Sqrt[a] + Sq
rt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b^(3/2)*d) + (Cos
[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{(1+x^2)^2\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{b(1+x^2)^2}-\frac{1}{b(1+x^2)}+\frac{ax^2}{b\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{a\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{x}{b} + \frac{\cos(c+dx)\sin(c+dx)}{2bd} + \frac{\left(a\left(\sqrt{a}+\sqrt{b}\right)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c\right.}{2b^{3/2}d}$$

$$= -\frac{x}{2b} + \frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^{3/2}d} - \frac{a^{3/4}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^{3/2}d} + \frac{\cos(c+dx)\sin}{2bd}$$

Mathematica [A] time = 0.83, size = 157, normalized size = 1.01

$$\frac{-\frac{2a\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{2a\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-2\sqrt{b}(c+dx)+\sqrt{b}\sin(2(c+dx))}{4b^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*Sqrt[b]*(c + d*x) - (2*a*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt
[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (2*a*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]] + Sqrt[b]*Sin[2*(c + d*x)])/(4*b^(3/2)*d)
```

fricas [B] time = 0.64, size = 1275, normalized size = 8.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*x + c)^2 - 1/4*a^2 - 1/4*(2
*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a
^2*b^5 - 2*a*b^6 + b^7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 -
 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)
) + a^2)/((a*b^3 - b^4)*d^2))) - b*d*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))*log(1/4*a^2*cos(d*
x + c)^2 - 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2
 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^
5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c)
- a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a*b^3 - b^4)*d^2*sqrt(a^3/((a^
2*b^5 - 2*a*b^6 + b^7)*d^4)) + a^2)/((a*b^3 - b^4)*d^2))) + b*d*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1/4*(2*(a^2*b^2 - a*b^3)*d^
2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^
7)*d^4)) + 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4))
*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^
3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - a^2)/((a*b^3 - b^4
)*d^2))) - b*d*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))*log(-1/4*a^2*cos(d*x + c)^2 + 1/4*a^2 - 1
/4*(2*(a^2*b^2 - a*b^3)*d^2*cos(d*x + c)^2 - (a^2*b^2 - a*b^3)*d^2)*sqrt(a^
3/((a^2*b^5 - 2*a*b^6 + b^7)*d^4)) - 1/2*((a*b^4 - b^5)*d^3*sqrt(a^3/((a^2*
b^5 - 2*a*b^6 + b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)
*sin(d*x + c))*sqrt(((a*b^3 - b^4)*d^2*sqrt(a^3/((a^2*b^5 - 2*a*b^6 + b^7)*
d^4)) - a^2)/((a*b^3 - b^4)*d^2))) + 4*d*x - 4*cos(d*x + c)*sin(d*x + c))/(
b*d)
```

giac [B] time = 1.02, size = 695, normalized size = 4.48

$$-\frac{\frac{dx+c}{b} + \frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)b^2|-a+b|-\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^3b-\cdots\right.}{\left(3a^5b^2-15a^4b^3+26a^3b^4-18a^2b\cdots\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 -
 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3)*abs(-a +
 b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a
^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3
*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 6*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a*b^3)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*
b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3*a*b^6 + b^7)*abs(b)) - tan(d
*x + c)/((tan(d*x + c)^2 + 1)*b))/d
```

maple [B] time = 0.35, size = 551, normalized size = 3.55

$$\frac{\tan(dx+c)}{2db\left(\tan^2(dx+c)+1\right)}-\frac{\arctan(\tan(dx+c))}{2db}+\frac{a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2db\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{a^3\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2db\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/b*tan(d*x+c)/(tan(d*x+c)^2+1)-1/2/d/b*arctan(tan(d*x+c))+1/2/d*a^2/b/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))-1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/
2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a^2/b/(a-
b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))+1/2/d*a^3/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*
(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a
^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(4*b*d*integrate(-4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)^
2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(8*a^2 - 3*a*b)
*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(8*a^2 - 3*a*b)*sin(4*d*x +
4*c)^2 + 2*(8*a^2 - 7*a*b)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (a*b*cos(6*d
*x + 6*c) - 2*a*b*cos(4*d*x + 4*c) + a*b*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 + (8*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 7*a*b)*cos(4*d*x + 4*c))*cos(
6*d*x + 6*c) + 2*(a*b + (8*a^2 - 7*a*b)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c)
- (a*b*sin(6*d*x + 6*c) - 2*a*b*sin(4*d*x + 4*c) + a*b*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 7*a*b)*sin(4*d*x + 4*
c))*sin(6*d*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 +
 16*b^3*cos(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*
c)^2 + 16*b^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2
*b - 48*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)
*sin(4*d*x + 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) - 2*(4*b^3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 -
 3*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^
3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3
*b^3 - 4*(8*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*si
n(6*d*x + 6*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c
))*sin(8*d*x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*
d*x + 4*c))*sin(6*d*x + 6*c)), x) - 2*d*x + sin(2*d*x + 2*c))/(b*d)
```

mupad [B] time = 16.40, size = 1273, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4),x)
```

```
[Out] sin(2*c + 2*d*x)/(4*b*d) - (atan((a*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^
2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(((a^3*b^7)^(1/2)
- a^2*b^3)/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(((a^3*b^7)
^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c + d*x)*(((a
^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b^6*sin(c +
d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*24i + a^3*b^5*
sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i + a
^4*b^4*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)
*8i + a^2*b^8*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7)
)^(3/2)*448i + a^3*b^7*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6
- 16*b^7))^(3/2)*320i + a^2*b^10*sin(c + d*x)*(((a^3*b^7)^(1/2) - a^2*b^3)/
(16*a*b^6 - 16*b^7))^(5/2)*3072i)/(a^2*b^5*cos(c + d*x) + a^3*b^4*cos(c + d
*x) - a^4*b^3*cos(c + d*x) - a^2*cos(c + d*x)*(a^3*b^7)^(1/2) + 2*a*b*cos(c
 + d*x)*(a^3*b^7)^(1/2)))*(((a^3*b^7)^(1/2) - a^2*b^3)/(16*a*b^6 - 16*b^7))
^(1/2)*2i)/d - (atan((a*b^7*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*
a*b^6 - 16*b^7))^(1/2)*4i - b^12*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)
/(16*a*b^6 - 16*b^7))^(5/2)*3072i - b^10*sin(c + d*x)*(-((a^3*b^7)^(1/2) +
a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a*b^9*sin(c + d*x)*(-((a^3*b^7)^
(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3/2)*192i + a^2*b^6*sin(c + d*x)*(-(
(a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*24i + a^3*b^5*sin(c +
 d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*4i + a^4*b^4
*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(1/2)*8i +
 a^2*b^8*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^(3
/2)*448i + a^3*b^7*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 1
6*b^7))^(3/2)*320i + a^2*b^10*sin(c + d*x)*(-((a^3*b^7)^(1/2) + a^2*b^3)/(1
6*a*b^6 - 16*b^7))^(5/2)*3072i)/(a^2*b^5*cos(c + d*x) + a^3*b^4*cos(c + d*x
) - a^4*b^3*cos(c + d*x) + a^2*cos(c + d*x)*(a^3*b^7)^(1/2) - 2*a*b*cos(c +
 d*x)*(a^3*b^7)^(1/2)))*(-((a^3*b^7)^(1/2) + a^2*b^3)/(16*a*b^6 - 16*b^7))^
(1/2)*2i)/d - atan(sin(c + d*x)/cos(c + d*x))/(2*b*d)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.205 $\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

```
[Out] -x/b+1/2*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b/d/(a^
(1/2)-b^(1/2))^(1/2)+1/2*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/
a^(1/4))/b/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1287, 203, 1166, 205}

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2bd\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(
2*Sqrt[Sqrt[a] - Sqrt[b]]*b*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + Sqrt[b]]*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
```

```
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a(1+x^2)}{b\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{a\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}$$

$$= -\frac{x}{b} + \frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd} + \frac{\left(a\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\right)\text{Sub}\ldots}{}$$

$$= -\frac{x}{b} + \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,bd} + \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,bd}$$

Mathematica [A] time = 0.42, size = 143, normalized size = 1.13

$$\frac{\frac{\sqrt{a}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\sqrt{a}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}} - 2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + (Sqrt[a]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
 Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[a]*ArcTanh[((Sqrt[a]
- Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqr
t[b]])/(2*b*d)
```

fricas [B] time = 0.59, size = 1125, normalized size = 8.86

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a
)/((a*b^2 - b^3)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt
(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b
^5)*d^4)) + a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2
 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) - b*sqrt
(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + a)/((a*b^2 -
 b^3)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) - b*d*cos(d*x + c)*sin(d
```

```
*x + c))*sqrt(-((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) +
 a)/((a*b^2 - b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b
^2)*d^2)*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - 1/4) + b*sqrt(((a*b^2 -
b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*
log(-1/4*cos(d*x + c)^2 + 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4
 + b^5)*d^4))*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b^2 - b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2
- b^3)*d^2)) - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - b*sqrt(((a*b^2 - b^3)*d^2*sqr
t(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))*log(-1/4*cos
(d*x + c)^2 - 1/2*((a*b^2 - b^3)*d^3*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)
)*cos(d*x + c)*sin(d*x + c) + b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a*b^2 -
 b^3)*d^2*sqrt(a/((a^2*b^3 - 2*a*b^4 + b^5)*d^4)) - a)/((a*b^2 - b^3)*d^2))
 - 1/4*(2*(a*b - b^2)*d^2*cos(d*x + c)^2 - (a*b - b^2)*d^2)*sqrt(a/((a^2*b^
3 - 2*a*b^4 + b^5)*d^4)) + 1/4) - 8*x)/b
```

giac [B] time = 0.98, size = 912, normalized size = 7.18

$$-\frac{\frac{2(dx+c)}{b}+\frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}a^2-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}ab-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\sqrt{ab}b^2\right)b^2|-a+b|-\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}a^3b-9\sqrt{a^2\cdots}\right.\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 + 5*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a
*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3*
a*b^6 + b^7)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^3*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 + 5*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*b^4)*abs(-a + b)*abs(b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^2*b^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
 c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*
a*b))/(a*b - b^2))))/((3*a^5*b^2 - 15*a^4*b^3 + 26*a^3*b^4 - 18*a^2*b^5 + 3
*a*b^6 + b^7)*abs(b)))/d
```

maple [B] time = 0.28, size = 517, normalized size = 4.07

$$-\frac{\arctan\left(\tan\left(dx+c\right)\right)}{db}+\frac{a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2db\left(a-b\right)\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}-\frac{a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\sqrt{ab}\left(a-b\right)\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}+\frac{a^2\arctan\left(\cdots\right)}{2d\sqrt{ab}\left(a-b\right)\sqrt{\cdots}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/b*arctan(tan(d*x+c))+1/2/d*a^2/b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*a
rctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/
2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1
/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a^2/b/(a-b
)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a
-b))^(1/2))-1/2/d*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(
d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/
2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))
*b-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan
(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*(a-
b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 16.27, size = 2991, normalized size = 23.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

```
[Out] - atan((18*a^5*tan(c + d*x))/(18*a^5 - 50*a^4*b + 32*a^3*b^2) - (50*a^4*tan
(c + d*x))/(32*a^3*b - 50*a^4 + (18*a^5)/b) + (32*a^3*b*tan(c + d*x))/(32*a
^3*b - 50*a^4 + (18*a^5)/b))/(b*d) - (atan((((-(a*b^2 - (a*b^5)^(1/2))/(16*
(a*b^4 - b^5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)
*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*
b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 -
 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80
*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2
) + 12*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2) + tan(c + d*x)*(18*a^4
*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4
- b^5)))^(1/2)*1i + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((
(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b
^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16
*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b
^4)) + tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))
*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^
4 - 28*a^3*b^3 + 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 -
 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i)/(6*a^
3*b - 6*a^4 + ((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(((-(a*b
^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 4
48*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^
4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) +
 tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))*(-(a*
b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28
*a^3*b^3 - 24*a^4*b^2) + tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^
3*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - ((-(a*b^2 - (
a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(((-(a*b^2 - (a*b^5)^(1/2))/(16*(a*
b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 +
tan(c + d*x)*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b
^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(176*a^2*b^5
- 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a
```

```
*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2) + ta
n(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2))*(-(a*b^2 - (a*b^5)^
(1/2))/(16*(a*b^4 - b^5)))^(1/2)))*(-(a*b^2 - (a*b^5)^(1/2))/(16*(a*b^4 - b
^5)))^(1/2)*2i)/d - (atan(((tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20
*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d
*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*
b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b^5 - 64*a^2*b^6 - 448*a^4*b
^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)
))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2
 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12*a^5*b - 16*a^2*b^4 + 28*a^
3*b^3 - 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i
 + (tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (
a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a
^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 -
 b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b^4 - 192*a^5*b^3 + tan(c
 + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 -
768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a
*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^3*b^3 + 24*a^4*b^2))*(-(
a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*1i)/((tan(c + d*x)*(18*a^4
*b + 6*a^5 - 4*a^2*b^3 - 20*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4
 - b^5)))^(1/2)*((tan(c + d*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 14
4*a^5*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(320*a^3*b
^5 - 64*a^2*b^6 - 448*a^4*b^4 + 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^
5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^
5 + 768*a^5*b^4)))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) + 12
*a^5*b - 16*a^2*b^4 + 28*a^3*b^3 - 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(
16*(a*b^4 - b^5)))^(1/2) - (tan(c + d*x)*(18*a^4*b + 6*a^5 - 4*a^2*b^3 - 20
*a^3*b^2) + (-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*((tan(c + d
*x)*(176*a^2*b^5 - 400*a^3*b^4 + 80*a^4*b^3 + 144*a^5*b^2) + (-(a*b^2 + (a*
b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*(64*a^2*b^6 - 320*a^3*b^5 + 448*a^4*b
^4 - 192*a^5*b^3 + tan(c + d*x)*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)
))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)))*(-(a*b^2
 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) - 12*a^5*b + 16*a^2*b^4 - 28*a^
3*b^3 + 24*a^4*b^2))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2) +
6*a^3*b - 6*a^4))*(-(a*b^2 + (a*b^5)^(1/2))/(16*(a*b^4 - b^5)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.206 $\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(1/4)/d/b^(1/2)/(a
^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4)
)/a^(1/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3217, 1130, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\,\sqrt{b}\,d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a
^(1/4)]/(2*a^(1/4)*Sqrt[Sqrt[a] + Sqrt[b]]*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a}{2d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}-\sqrt{b}}\sqrt{b}\,d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt[4]{a}\sqrt{\sqrt{a}+\sqrt{b}}\sqrt{b}\,d}$$

Mathematica [A] time = 0.36, size = 137, normalized size = 1.10

$$-\frac{\tan^{-1}\left(\frac{(\sqrt{a}+\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2\sqrt{b}\,d\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{(\sqrt{a}-\sqrt{b})\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2\sqrt{b}\,d\sqrt{\sqrt{a}\sqrt{b}-a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/(
Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d) - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c +
 d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]/(2*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]*d)
```

fricas [B] time = 0.62, size = 1087, normalized size = 8.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/
((a*b - b^2)*d^2))*log(1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1
/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b
^3)*d^4)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 -
 (a^2 - a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) + 1/8*sq
rt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1)/((a*b -
 b^2)*d^2))*log(1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*
b - 2*a^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) - a*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4
)) + 1)/((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 -
 a*b)*d^2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1/4) - 1/8*sqrt(((a*
b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^
2))*log(-1/4*cos(d*x + c)^2 + 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a
^2*b^2 + a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/
((a*b - b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^
2)*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4) + 1/8*sqrt(((a*b - b^2)
*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b - b^2)*d^2))*log(
-1/4*cos(d*x + c)^2 - 1/2*((a^2*b - a*b^2)*d^3*sqrt(1/((a^3*b - 2*a^2*b^2 +
 a*b^3)*d^4))*cos(d*x + c)*sin(d*x + c) + a*d*cos(d*x + c)*sin(d*x + c))*sq
rt(((a*b - b^2)*d^2*sqrt(1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) - 1)/((a*b -
b^2)*d^2)) - 1/4*(2*(a^2 - a*b)*d^2*cos(d*x + c)^2 - (a^2 - a*b)*d^2)*sqrt(
1/((a^3*b - 2*a^2*b^2 + a*b^3)*d^4)) + 1/4)
```

giac [B] time = 0.99, size = 398, normalized size = 3.18

$$\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((
4*a + sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^4
*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b -
sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi +
 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a
- b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5))
/d
```

maple [B] time = 0.43, size = 492, normalized size = 3.94

$$\frac{a\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\,(a-b)\sqrt{\left(\sqrt{ab}\,-a\right)(a-b)}}-\frac{a^2\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\,(a-b)\sqrt{\left(\sqrt{ab}\,-a\right)(a-b)}}+\frac{a\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2d\,(a-b)\sqrt{\left(\sqrt{ab}\,+a\right)(a-b)}}+\frac{a^2}{2d\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a/(
a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)
*(a-b))^(1/2))+1/2/d*a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*
tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*
(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a
/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/((
(a*b)^(1/2)+a)*(a-b))^(1/2))*b
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\sin(dx+c)^2}{b\sin(dx+c)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(sin(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

mupad [B] time = 16.19, size = 443, normalized size = 3.54

$$\frac{\ln\left(ab-a^2-\frac{a\tan(c+dx)\,(a-b)\sqrt{-\frac{1}{ab+\sqrt{ab^3}}}\left(2ab^2+a\sqrt{ab^3}+b\sqrt{ab^3}\right)}{ab+\sqrt{ab^3}}\right)\sqrt{-\frac{1}{ab+\sqrt{ab^3}}}}{4d}-\frac{\ln\left(ab-a^2-\frac{a\tan(c+dx)\sqrt{-\frac{1}{ab-\sqrt{ab^3}}}}{ab}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (log(a*b - a^2 - (a*tan(c + d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*(
2*a*b^2 + a*(a*b^3)^(1/2) + b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*(-1/(a
*b + (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 - (a*tan(c + d*x)*(-1/(a
*b - (a*b^3)^(1/2)))^(1/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^3)^(
1/2)))/(a*b - (a*b^3)^(1/2)))*((a*b + (a*b^3)^(1/2))/(16*(a*b^3 - a^2*b^2))
)^(1/2))/d + (log(a*b - a^2 + (a*tan(c + d*x)*(-1/(a*b - (a*b^3)^(1/2)))^(1
/2)*(a - b)*(a*(a*b^3)^(1/2) - 2*a*b^2 + b*(a*b^3)^(1/2)))/(a*b - (a*b^3)^(
1/2)))*(-1/(a*b - (a*b^3)^(1/2)))^(1/2))/(4*d) - (log(a*b - a^2 + (a*tan(c
+ d*x)*(a - b)*(-1/(a*b + (a*b^3)^(1/2)))^(1/2)*(2*a*b^2 + a*(a*b^3)^(1/2)
+ b*(a*b^3)^(1/2)))/(a*b + (a*b^3)^(1/2)))*((a*b - (a*b^3)^(1/2))/(16*(a*b^
3 - a^2*b^2)))^(1/2))/d
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.207 $\int \frac{1}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

```
[Out] 1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b
^(1/2))^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4
)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3209, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\sqrt{\sqrt{a}+\sqrt{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(2*a^(3/4)*Sqrt[Sqrt
[a] - Sqrt[b]]*d) + ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/
(2*a^(3/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\,\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2d} + \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\,\sqrt{b}+(a}\right)}{2d}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}$$

Mathematica [A] time = 0.26, size = 128, normalized size = 1.11

$$\frac{\frac{\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}} - \frac{\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\,\sqrt{b}-a}}}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-1),x]
```

```
[Out] (ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]]/Sqrt[
a + Sqrt[a]*Sqrt[b]] - ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a +
 Sqrt[a]*Sqrt[b]]]/Sqrt[-a + Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*d)
```

fricas [B] time = 0.60, size = 1079, normalized size = 9.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a
^2 - a*b)*d^2))*log(1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((
a^5 - 2*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d
^4)) + 1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a
^3 - a^2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) - 1/8*sqr
t(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1)/((a^2 - a*
b)*d^2))*log(1/4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2
*a^4*b + a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a*b*d*cos(d*x + c)*sin(
d*x + c))*sqrt(-((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) +
1)/((a^2 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^
2*b)*d^2)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1/4*b) + 1/8*sqrt(((a^2
 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))
*log(-1/4*b*cos(d*x + c)^2 + 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b
+ a^3*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2
 - a*b)*d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2
)*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b) - 1/8*sqrt(((a^2 - a*b)*
d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*d^2))*log(-1/
4*b*cos(d*x + c)^2 - 1/2*((a^4 - a^3*b)*d^3*sqrt(b/((a^5 - 2*a^4*b + a^3*b^
2)*d^4))*cos(d*x + c)*sin(d*x + c) + a*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(
((a^2 - a*b)*d^2*sqrt(b/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) - 1)/((a^2 - a*b)*
```

```
d^2)) - 1/4*(2*(a^3 - a^2*b)*d^2*cos(d*x + c)^2 - (a^3 - a^2*b)*d^2)*sqrt(b
/((a^5 - 2*a^4*b + a^3*b^2)*d^4)) + 1/4*b)
```

giac [B] time = 0.42, size = 361, normalized size = 3.14

$$\frac{\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^2\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\tan(dx+c)}{\sqrt{\frac{4a+\sqrt{-16(a-b)a+16a^2}}{a-b}}}\right)\right)|a-b|}{3a^5-12a^4b+14a^3b^2-4a^2b^3-ab^4}+\frac{\left(3\sqrt{a^2-ab-\cdots}\right.}{\cdots}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*a*b - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^2)*(pi*floor((d*x
+ c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(-16*(a - b)*a + 16*
a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 - a*
b^4) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2 - 6*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^2)*(pi*floor((d*
x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a - sqrt(-16*(a - b)*a + 1
6*a^2))/(a - b))))*abs(a - b)/(3*a^5 - 12*a^4*b + 14*a^3*b^2 - 4*a^2*b^3 -
a*b^4))/d
```

maple [B] time = 0.36, size = 492, normalized size = 4.28

$$\frac{a\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{a\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)b}{2d\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{a\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)b}{2d\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}+\frac{\cdots}{2d\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))
^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b+1/2/d*a/(
a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a
*b)^(1/2)+a)*(a-b))^(1/2))*b+1/2/d*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*ta
n(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)
^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2
))*b^2-1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\sin\left(dx+c\right)^4-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(d*x + c)^4 - a), x)
```

mupad [B] time = 14.99, size = 671, normalized size = 5.83

$$\mathrm{atan}\left(\frac{a^3\tan(c+d\,x)\,\sqrt{-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}}\,4\mathrm{i}+a^5\tan(c+d\,x)\left(-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}\right)^{3/2}64\mathrm{i}+a^3\tan(c+d\,x)\left(-\frac{1}{16\,a^2+16\,\sqrt{a^3\,b}}\right)^{3/2}\sqrt{a^3\,b}\,64\mathrm{i}+a^2\,b\tan(c+d\,x)\,\sqrt{-\cdots}}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^3*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i + a^5*tan
(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*64i + a^3*tan(c + d*x)*(-1
/(16*a^2 + 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i + a^2*b*tan(c + d*x)*
(-1/(16*a^2 + 16*(a^3*b)^(1/2)))^(1/2)*4i - a^4*b*tan(c + d*x)*(-1/(16*a^2
+ 16*(a^3*b)^(1/2)))^(3/2)*64i + a*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1
/2)))^(1/2)*(a^3*b)^(1/2)*4i + b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1/2
)))^(1/2)*(a^3*b)^(1/2)*4i - a^2*b*tan(c + d*x)*(-1/(16*a^2 + 16*(a^3*b)^(1
/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b + (a^3*b)^(1/2)))*(-1/(16*a^2 + 16*(a^3
*b)^(1/2)))^(1/2)*2i)/d + (atan((a^3*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^
(1/2)))^(1/2)*4i + a^5*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*
64i - a^3*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)
*64i + a^2*b*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*4i - a^4*b
*tan(c + d*x)*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*64i - a*tan(c + d*x)*(
-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i - b*tan(c + d*x)*(-1
/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*(a^3*b)^(1/2)*4i + a^2*b*tan(c + d*x)*(
-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(3/2)*(a^3*b)^(1/2)*64i)/(a*b - (a^3*b)^(1/
2)))*(-1/(16*a^2 - 16*(a^3*b)^(1/2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.208 $\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=139

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1
/2)/a^(5/4)/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*ta
n(d*x+c)/a^(1/4))*b^(1/2)/a^(5/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 139, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(5/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(5/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a
*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{x^2\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{bx^2}{a\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= -\frac{\cot(c+dx)}{ad}+\frac{\left(\left(1-\frac{\sqrt{a}}{\sqrt{b}}\right)b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2ad}+\frac{\left(\left(1+\frac{\sqrt{a}}{\sqrt{b}}\right)\right.}{}$$
$$= \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{5/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\cot(c+dx)}{ad}$$

Mathematica [A] time = 1.08, size = 143, normalized size = 1.03

$$-\frac{\frac{\sqrt{b}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{\sqrt{b}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+2\cot(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/2*((Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*S
qrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])
*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + 2*
Cot[c + d*x])/(a*d)
```

fricas [B] time = 0.62, size = 1229, normalized size = 8.84

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)
) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x + c)^2 - 1/4*b^2 - 1/4*(2*(
a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5
*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d*cos(d*x + c)*sin(d*x + c))*
sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a
^3 - a^2*b)*d^2)))*sin(d*x + c) - a*d*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a
^7 - 2*a^6*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2))*log(1/4*b^2*cos(d*x
 + c)^2 - 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)
*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sq
rt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) - a^2*b*d
*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4)) + b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + a*d*sqrt(((a^
3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b
```

```
)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 - 1/4*(2*(a^4 - a^3*b)*d^2*co
s(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))
 + 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4))*cos(d*x
 + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3 - a^2*b)
*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) - b)/((a^3 - a^2*b)*d^2)))*s
in(d*x + c) - a*d*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^
2)*d^4)) - b)/((a^3 - a^2*b)*d^2))*log(-1/4*b^2*cos(d*x + c)^2 + 1/4*b^2 -
1/4*(2*(a^4 - a^3*b)*d^2*cos(d*x + c)^2 - (a^4 - a^3*b)*d^2)*sqrt(b^3/((a^7
 - 2*a^6*b + a^5*b^2)*d^4)) - 1/2*((a^5 - a^4*b)*d^3*sqrt(b^3/((a^7 - 2*a^6
*b + a^5*b^2)*d^4))*cos(d*x + c)*sin(d*x + c) + a^2*b*d*cos(d*x + c)*sin(d*
x + c))*sqrt(((a^3 - a^2*b)*d^2*sqrt(b^3/((a^7 - 2*a^6*b + a^5*b^2)*d^4)) -
 b)/((a^3 - a^2*b)*d^2)))*sin(d*x + c) + 8*cos(d*x + c))/(a*d*sin(d*x + c))
```

giac [B] time = 1.16, size = 672, normalized size = 4.83

$$-\frac{\left(\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2b-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab^2-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^3\right)a^2|a-b|-\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^5-6\sqrt{a^2-ab\cdots}\right.\right.}{\left(3a^8-15a^7b+26a^6b^2-18a^5b^3+3a^4b^4+a^3b^5\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b - s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2)*abs(a - b))*(pi*floor
((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^2 - a*
b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^
4*b^4 + a^3*b^5)*abs(a)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b
)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^4*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2)*
abs(a - b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 -
 sqrt(a^4 - (a^2 - a*b)*a^2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^
2 - 18*a^5*b^3 + 3*a^4*b^4 + a^3*b^5)*abs(a)) + 2/(a*tan(d*x + c)))/d
```

maple [B] time = 0.49, size = 518, normalized size = 3.73

$$-\frac{1}{d a \tan\left(dx+c\right)}+\frac{b \operatorname{arctanh}\left(\frac{\left(-a+b\right)\tan\left(dx+c\right)}{\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}\right)}{2d\left(a-b\right)\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}-\frac{a \operatorname{arctanh}\left(\frac{\left(-a+b\right)\tan\left(dx+c\right)}{\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}\right)b}{2d\sqrt{ab}\,\left(a-b\right)\sqrt{\left(\sqrt{ab}-a\right)\left(a-b\right)}}+\frac{b \arctan\left(\frac{\left(a-b\right)\tan\left(d\cdots\right.}{\sqrt{\left(\sqrt{ab}+a\right)\cdots}}\right.}{2d\left(a-b\right)\sqrt{\left(\sqrt{ab}+a\cdots\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/a/tan(d*x+c)+1/2/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*
b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^
(1/2))*b+1/2/d*b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c
)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/
d/a*b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(
```

```
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2-1/2/d/a*b
^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2
)+a)*(a-b))^(1/2))-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*ar
ctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] ((a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2*a*d*cos(2*d*x + 2*c)
+ a*d)*integrate(-4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 +
4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 - 4*(8*a*b - 3*b^2)*cos
(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 4*(8*a*b - 3*b^2)*sin(4*d*x + 4*c)
^2 + 2*(8*a*b - 7*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x +
 6*c) - 2*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (
8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 7*b^2)*cos(4*d*x + 4*c))*cos(6*d*
x + 6*c) + 2*(b^2 + (8*a*b - 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b
^2*sin(6*d*x + 6*c) - 2*b^2*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*
d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a*b - 7*b^2)*sin(4*d*x + 4*c))*
sin(6*d*x + 6*c))/(a*b^2*cos(8*d*x + 8*c)^2 + 16*a*b^2*cos(6*d*x + 6*c)^2 +
 16*a*b^2*cos(2*d*x + 2*c)^2 + a*b^2*sin(8*d*x + 8*c)^2 + 16*a*b^2*sin(6*d*
x + 6*c)^2 + 16*a*b^2*sin(2*d*x + 2*c)^2 - 8*a*b^2*cos(2*d*x + 2*c) + a*b^2
 + 4*(64*a^3 - 48*a^2*b + 9*a*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^3 - 48*a^2*
b + 9*a*b^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c)*s
in(2*d*x + 2*c) - 2*(4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) -
a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a*b
^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(6
*d*x + 6*c) - 4*(8*a^2*b - 3*a*b^2 - 4*(8*a^2*b - 3*a*b^2)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 4*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c)
 + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*(2*a*b^2*sin
(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c)), x)
 - 2*sin(2*d*x + 2*c))/(a*d*cos(2*d*x + 2*c)^2 + a*d*sin(2*d*x + 2*c)^2 - 2
*a*d*cos(2*d*x + 2*c) + a*d)
```

mupad [B] time = 14.51, size = 371, normalized size = 2.67

$$\frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+d\,x)\left(4\,a^4\,b^4-4\,a^6\,b^2\right)-\frac{\tan(c+d\,x)\left(\sqrt{a^5\,b^3}+a^3\,b\right)\left(64\,a^9\,b-128\,a^8\,b^2+64\,a^7\,b^3\right)}{16\left(a^5\,b-a^6\right)}\right)\sqrt{\frac{\sqrt{a^5\,b^3}+a^3\,b}{16\left(a^5\,b-a^6\right)}}}{2\,a^3\,b^4-2\,a^4\,b^3}\right)\sqrt{\frac{\sqrt{a^5\,b^3}+a^3\,b}{16\left(a^5\,b-a^6\right)}}}{d}+\frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(\cdots\right.}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (2*atanh((2*(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) - (tan(c + d*x)*((a^5*b^3
)^(1/2) + a^3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)))
*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^4*b
^3))*(((a^5*b^3)^(1/2) + a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d + (2*atanh((2*
(tan(c + d*x)*(4*a^4*b^4 - 4*a^6*b^2) + (tan(c + d*x)*((a^5*b^3)^(1/2) - a^
3*b)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2))/(16*(a^5*b - a^6)))*(-((a^5*b^3
)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/(2*a^3*b^4 - 2*a^4*b^3))*(-((a^
5*b^3)^(1/2) - a^3*b)/(16*(a^5*b - a^6)))^(1/2))/d - cot(c + d*x)/(a*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**2/(a - b*sin(c + d*x)**4), x)
```

3.209 $\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=149

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-1/3*cot(d*x+c)^3/a/d+1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*t
an(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b*arctan((a^(1/2)+
b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{\cot^3(c+dx)}{3ad}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(7/4)*Sqrt[
Sqrt[a] - Sqrt[b]]*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(2*a^(7/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - Cot[c + d*x]/(a*d) - Cot[c
+ d*x]^3/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{x^4\left(a+2ax^2+(a-b)x^4\right)}\,dx,x,\tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{1}{ax^2}+\frac{b\left(1+x^2\right)}{a\left(a+2ax^2+(a-b)x^4\right)}\right)dx,x,\tan(c+dx)\right)}{d}$$
$$= -\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{b\,\text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a-b)x^4}\,dx,x,\tan(c+dx)\right)}{ad}$$
$$= -\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+dx)}{3ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx,x,\tan(c\right.}{2a^{3/2}d}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{7/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{\cot(c+dx)}{ad}-\frac{\cot^3(c+}{3ad}$$

Mathematica [A] time = 1.56, size = 165, normalized size = 1.11

$$\frac{\frac{3b\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{3b\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}-4\sqrt{a}\cot(c+dx)-2\sqrt{a}\cot(c+dx)\csc^2(c+dx)}{6a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*b*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])
/Sqrt[a + Sqrt[a]*Sqrt[b]] - (3*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x]
)/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] - 4*Sqrt[a]*Cot[c
 + d*x] - 2*Sqrt[a]*Cot[c + d*x]*Csc[c + d*x]^2)/(6*a^(3/2)*d)
```

fricas [B] time = 0.61, size = 1365, normalized size = 9.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/24*(3*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9
 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x
 + c)^2 - 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b -
a^4*b^2)*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*co
s(d*x + c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a
^9 - 2*a^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3
*(a*d*cos(d*x + c)^2 - a*d)*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8
*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2))*log(1/4*b^4*cos(d*x + c)^2
- 1/4*b^4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)
*d^2)*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x +
c)*sin(d*x + c) - (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4
))*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a
^8*b + a^7*b^2)*d^4)) + b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) - 3*(a*d*co
```

```
s(d*x + c)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*
b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^
4 - 1/4*(2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sq
rt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)) + 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d
*x + c) + (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d
*x + c)*sin(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^
7*b^2)*d^4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 3*(a*d*cos(d*x + c
)^2 - a*d)*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4)
) - b^2)/((a^4 - a^3*b)*d^2))*log(-1/4*b^4*cos(d*x + c)^2 + 1/4*b^4 - 1/4*(
2*(a^5*b - a^4*b^2)*d^2*cos(d*x + c)^2 - (a^5*b - a^4*b^2)*d^2)*sqrt(b^5/((
a^9 - 2*a^8*b + a^7*b^2)*d^4)) - 1/2*(a^2*b^3*d*cos(d*x + c)*sin(d*x + c) +
 (a^7 - a^6*b)*d^3*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^4))*cos(d*x + c)*s
in(d*x + c))*sqrt(((a^4 - a^3*b)*d^2*sqrt(b^5/((a^9 - 2*a^8*b + a^7*b^2)*d^
4)) - b^2)/((a^4 - a^3*b)*d^2)))*sin(d*x + c) + 16*cos(d*x + c)^3 - 24*cos(
d*x + c))/((a*d*cos(d*x + c)^2 - a*d)*sin(d*x + c))
```

giac [B] time = 1.04, size = 938, normalized size = 6.30

$$-\frac{3\left(\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2b-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab^2-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^3\right)a^2|a-b|-\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^4b-9\sqrt{a^2-ab+\sqrt{ab}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/6*(3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*a^4*b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 5*sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4)
*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - sqrt(a^2 - a*b
 + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b))*(pi*floor((d*x + c)/pi
 + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 + sqrt(a^4 - (a^2 - a*b)*a^2))/(a^2
 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^4 + a^3*b
^5)*abs(a)) - 3*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6
*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*sqrt(a*b)*b^3)*a^2*abs(a - b) + (3*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*a^4*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^2 + 5*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*a*b^4)*abs(a - b)*abs(a) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*a^4*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3)*abs(a - b))*(pi*floor((d*x
 + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2 - sqrt(a^4 - (a^2 - a*b)*a^
2))/(a^2 - a*b))))/((3*a^8 - 15*a^7*b + 26*a^6*b^2 - 18*a^5*b^3 + 3*a^4*b^4
 + a^3*b^5)*abs(a)) + 2*(3*tan(d*x + c)^2 + 1)/(a*tan(d*x + c)^3))/d
```

maple [B] time = 0.53, size = 542, normalized size = 3.64

$$-\frac{1}{3da\tan(dx+c)^3}-\frac{1}{da\tan(dx+c)}+\frac{b\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)b^2}{2d\sqrt{ab}\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{a}{2d\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/3/d/a/tan(d*x+c)^3-1/d/a/tan(d*x+c)+1/2/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/(a*b
)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))*b^2+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2+1/2/d*b
/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+
a)*(a-b))^(1/2))-1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-
a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d/a*b^3/(a*b)^(1/2)/(a-b
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*
(a-b))^(1/2))-1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/a*b^2/(a-b)/(((
a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^
(1/2))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 15.55, size = 1670, normalized size = 11.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan((((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4
- 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(
a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) - tan(c + d*x)*
(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(
1/2)*1i - ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b
^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)/(1
6*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) + tan(c + d*
x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8))
)^(1/2)*1i)/(((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^
5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)
/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) - tan(c +
 d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^
8)))^(1/2) + ((((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^
5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(((a^7*b^5)^(1/2) + a^4*b^2)
/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 128*a^8*b^2)) + tan(c +
 d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*b - a^
8)))^(1/2) - 2*a^2*b^5 + 2*a^3*b^4))*(((a^7*b^5)^(1/2) + a^4*b^2)/(16*(a^7*
b - a^8)))^(1/2)*2i)/d + (atan((((-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b -
 a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d*x)*(-((a^7*
b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^3 - 12
8*a^8*b^2)) - tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5)^(1/2) - a^
4*b^2)/(16*(a^7*b - a^8)))^(1/2)*1i - ((-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a
^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c + d*x)*(-
((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*a^7*b^
3 - 128*a^8*b^2)) + tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5)^(1/2
) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*1i)/(((-((a^7*b^5)^(1/2) - a^4*b^2)/
(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 + tan(c + d
*x)*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b + 64*
a^7*b^3 - 128*a^8*b^2)) - tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*b^5
)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2) + ((-((a^7*b^5)^(1/2) - a^4*b^
```

```
2)/(16*(a^7*b - a^8)))^(1/2)*(16*a^5*b^4 - 32*a^6*b^3 + 16*a^7*b^2 - tan(c
+ d*x)*(-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*(64*a^9*b +
64*a^7*b^3 - 128*a^8*b^2)) + tan(c + d*x)*(4*a^3*b^5 - 4*a^5*b^3))*(-((a^7*
b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2) - 2*a^2*b^5 + 2*a^3*b^4))*(
-((a^7*b^5)^(1/2) - a^4*b^2)/(16*(a^7*b - a^8)))^(1/2)*2i)/d - (1/(3*a) + t
an(c + d*x)^2/a)/(d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Integral(csc(c + d*x)**4/(a - b*sin(c + d*x)**4), x)
```

3.210 $\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx}{3ad}$$

```
[Out] -(a+b)*cot(d*x+c)/a^2/d-2/3*cot(d*x+c)^3/a/d-1/5*cot(d*x+c)^5/a/d+1/2*b^(3/
2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(9/4)/d/(a^(1/2)-b^
(1/2))^(1/2)-1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))
/a^(9/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1130, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^5(c+dx)}{5ad}-\frac{2\cot^3(c+dx}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(9/4)
*Sqrt[Sqrt[a] - Sqrt[b]]*d) - (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[
c + d*x])/a^(1/4)])/(2*a^(9/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c
+ d*x])/(a^2*d) - (2*Cot[c + d*x]^3)/(3*a*d) - Cot[c + d*x]^5/(5*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1130

```
Int[((d_.)*(x_))^(m_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Wi
th[{q = Rt[b^2 - 4*a*c, 2]}, Dist[(d^2*(b/q + 1))/2, Int[(d*x)^(m - 2)/(b/2
 + q/2 + c*x^2), x], x] - Dist[(d^2*(b/q - 1))/2, Int[(d*x)^(m - 2)/(b/2 -
q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && G
eQ[m, 2]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
```

```
IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^6\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^6}+\frac{2}{ax^4}+\frac{a+b}{a^2x^2}+\frac{b^2x^2}{a^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{b^2\,\text{Subst}\left(\int \frac{x^2}{a+2ax^2+(a-b)x^4}\,dx, x\right.}{a^2d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{2\cot^3(c+dx)}{3ad}-\frac{\cot^5(c+dx)}{5ad}+\frac{\left(\left(\sqrt{a}+\sqrt{b}\right)b^{3/2}\right)\text{Subst}\left(\int \frac{}{a+}\right.}{2a}$$

$$= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{9/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{(a+b)\cot(c+dx)}{a^2d}-$$

Mathematica [A] time = 4.67, size = 174, normalized size = 0.98

$$-\frac{\frac{15b^{3/2}\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{15b^{3/2}\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}+2\cot(c+dx)\left(3a\csc^4(c+dx)+4a\csc^2(c+dx)+8a+\right.}{30a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/30*((15*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[
a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + (15*b^(3/2)*ArcTanh[((Sqrt[a] - S
qrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b
]] + 2*Cot[c + d*x]*(8*a + 15*b + 4*a*Csc[c + d*x]^2 + 3*a*Csc[c + d*x]^4))
/(a^2*d)
```

fricas [B] time = 0.64, size = 1477, normalized size = 8.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/120*(8*(8*a + 15*b)*cos(d*x + c)^5 - 80*(2*a + 3*b)*cos(d*x + c)^3 - 15*
(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(-((a^5 - a^4*b
)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2
))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(
d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d
^4)) + 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) - (a^8 - a^7*b)*sqrt(b^7/((
a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^5
 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 + b^3)/((a^5 - a^
4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^
2 + a^2*d)*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))
```

```
*d^2 + b^3)/((a^5 - a^4*b)*d^2))*log(1/4*b^5*cos(d*x + c)^2 - 1/4*b^5 - 1/4
*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/
((a^11 - 2*a^10*b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x +
c) - (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x
+ c)*sin(d*x + c))*sqrt(-((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^
2)*d^4))*d^2 + b^3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 15*(a^2*d*cos(d*x
+ c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^1
1 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2))*log(-1/4*b^5*
cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5*b^2)*d^2*cos(d*x + c)^2 - (a
^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4)) + 1/2*(a^3
*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*b)*sqrt(b^7/((a^11 - 2*a^10*b
 + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5 - a^4*b)*sqrt(b
^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5 - a^4*b)*d^2)))*sin(
d*x + c) - 15*(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sqrt(
((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^3)/((a^5
 - a^4*b)*d^2))*log(-1/4*b^5*cos(d*x + c)^2 + 1/4*b^5 - 1/4*(2*(a^6*b - a^5
*b^2)*d^2*cos(d*x + c)^2 - (a^6*b - a^5*b^2)*d^2)*sqrt(b^7/((a^11 - 2*a^10*
b + a^9*b^2)*d^4)) - 1/2*(a^3*b^3*d*cos(d*x + c)*sin(d*x + c) + (a^8 - a^7*
b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^5 - a^4*b)*sqrt(b^7/((a^11 - 2*a^10*b + a^9*b^2)*d^4))*d^2 - b^
3)/((a^5 - a^4*b)*d^2)))*sin(d*x + c) + 120*(a + b)*cos(d*x + c))/((a^2*d*c
os(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)*sin(d*x + c))
```

giac [B] time = 0.99, size = 471, normalized size = 2.65

$$\frac{15\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2b-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab^2-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/30*(15*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/
sqrt((a^3 + sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a
^7 - 12*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 15*(3*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*fl
oor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - sqrt(a^6 - (a^3 -
 a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5*b^2 -
4*a^4*b^3 - a^3*b^4) - 2*(15*a*tan(d*x + c)^4 + 15*b*tan(d*x + c)^4 + 10*a*
tan(d*x + c)^2 + 3*a)/(a^2*tan(d*x + c)^5))/d
```

maple [B] time = 0.53, size = 585, normalized size = 3.29

$$-\frac{1}{5da\tan(dx+c)^5}-\frac{1}{da\tan(dx+c)}-\frac{b}{d\,a^2\tan(dx+c)}-\frac{2}{3da\tan(dx+c)^3}+\frac{b^2\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2da\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/5/d/a/tan(d*x+c)^5-1/d/a/tan(d*x+c)-1/d/a^2/tan(d*x+c)*b-2/3/d/a/tan(d*x
+c)^3+1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*
```

```
x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2
+1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/((
(a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b)
)^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2-1/2/d*b^
3/a^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)
^(1/2)-a)*(a-b))^(1/2))+1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b^3
/a^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))-1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^
(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/15*(300*b*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + 10*(3*b*sin(8*d*x + 8*c) -
12*b*sin(6*d*x + 6*c) + 2*(8*a + 9*b)*sin(4*d*x + 4*c) - 4*(2*a + 3*b)*sin(
2*d*x + 2*c))*cos(10*d*x + 10*c) + 50*(6*b*sin(6*d*x + 6*c) - 4*(4*a + 3*b)
*sin(4*d*x + 4*c) + (8*a + 9*b)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 200*((
8*a + 3*b)*sin(4*d*x + 4*c) - (4*a + 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c
) + 15*(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*x + 8*c)^2 + 100*a^2*
d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 + 25*a^2*d*cos(2*d*x +
2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d*x + 8*c)^2 + 100*a^2
*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 - 100*a^2*d*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2 - 10*a^2*d*cos(2*d*x
+ 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*d*cos(6*d*x + 6*c) +
10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(10*d*x +
10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4*d*x + 4*c) + 5*a^2*d
*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a^2*d*cos(4*d*x + 4*c)
 - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c) - 20*(5*a^2*d*cos(2*d
*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*d*x + 8*c) - 2*a^2*d*
sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(1
0*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d*sin(4*d*x + 4*c) + a
^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d*sin(4*d*x + 4*c) - a
^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-4*(4*b^3*cos(6*d*x + 6*
c)^2 + 4*b^3*cos(2*d*x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*
x + 2*c)^2 - b^3*cos(2*d*x + 2*c) - 4*(8*a*b^2 - 3*b^3)*cos(4*d*x + 4*c)^2
- 4*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(8*a*b^2 - 7*b^3)*sin(4*d*x +
4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) - 2*b^3*cos(4*d*x + 4*c) + b^
3*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8
*a*b^2 - 7*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(b^3 + (8*a*b^2 - 7*
b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*sin(6*d*x + 6*c) - 2*b^3*sin
(4*d*x + 4*c) + b^3*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x
 + 2*c) + (8*a*b^2 - 7*b^3)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2*co
s(8*d*x + 8*c)^2 + 16*a^2*b^2*cos(6*d*x + 6*c)^2 + 16*a^2*b^2*cos(2*d*x + 2
*c)^2 + a^2*b^2*sin(8*d*x + 8*c)^2 + 16*a^2*b^2*sin(6*d*x + 6*c)^2 + 16*a^2
*b^2*sin(2*d*x + 2*c)^2 - 8*a^2*b^2*cos(2*d*x + 2*c) + a^2*b^2 + 4*(64*a^4
- 48*a^3*b + 9*a^2*b^2)*cos(4*d*x + 4*c)^2 + 4*(64*a^4 - 48*a^3*b + 9*a^2*b
^2)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) - 2*(4*a^2*b^2*cos(6*d*x + 6*c) + 4*a^2*b^2*cos(2*d*x + 2*c) - a^2
*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*a^
2*b^2*cos(2*d*x + 2*c) - a^2*b^2 + 2*(8*a^3*b - 3*a^2*b^2)*cos(4*d*x + 4*c)
)*cos(6*d*x + 6*c) - 4*(8*a^3*b - 3*a^2*b^2 - 4*(8*a^3*b - 3*a^2*b^2)*cos(2
*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*a^2*b^2*sin(6*d*x + 6*c) + 2*a^2*b^2*s
in(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c)
+ 16*(2*a^2*b^2*sin(2*d*x + 2*c) + (8*a^3*b - 3*a^2*b^2)*sin(4*d*x + 4*c))*
```

```
sin(6*d*x + 6*c)), x) - 2*(15*b*cos(8*d*x + 8*c) - 60*b*cos(6*d*x + 6*c) +
10*(8*a + 9*b)*cos(4*d*x + 4*c) - 20*(2*a + 3*b)*cos(2*d*x + 2*c) + 8*a + 1
5*b)*sin(10*d*x + 10*c) - 10*(30*b*cos(6*d*x + 6*c) - 20*(4*a + 3*b)*cos(4*
d*x + 4*c) + 5*(8*a + 9*b)*cos(2*d*x + 2*c) - 8*a - 12*b)*sin(8*d*x + 8*c)
- 20*(10*(8*a + 3*b)*cos(4*d*x + 4*c) - 10*(4*a + 3*b)*cos(2*d*x + 2*c) + 8
*a + 9*b)*sin(6*d*x + 6*c) - 60*(5*b*cos(2*d*x + 2*c) - 2*b)*sin(4*d*x + 4*
c) - 30*b*sin(2*d*x + 2*c))/(a^2*d*cos(10*d*x + 10*c)^2 + 25*a^2*d*cos(8*d*
x + 8*c)^2 + 100*a^2*d*cos(6*d*x + 6*c)^2 + 100*a^2*d*cos(4*d*x + 4*c)^2 +
25*a^2*d*cos(2*d*x + 2*c)^2 + a^2*d*sin(10*d*x + 10*c)^2 + 25*a^2*d*sin(8*d
*x + 8*c)^2 + 100*a^2*d*sin(6*d*x + 6*c)^2 + 100*a^2*d*sin(4*d*x + 4*c)^2 -
 100*a^2*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*a^2*d*sin(2*d*x + 2*c)^2
- 10*a^2*d*cos(2*d*x + 2*c) + a^2*d - 2*(5*a^2*d*cos(8*d*x + 8*c) - 10*a^2*
d*cos(6*d*x + 6*c) + 10*a^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) +
 a^2*d)*cos(10*d*x + 10*c) - 10*(10*a^2*d*cos(6*d*x + 6*c) - 10*a^2*d*cos(4
*d*x + 4*c) + 5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(8*d*x + 8*c) - 20*(10*a
^2*d*cos(4*d*x + 4*c) - 5*a^2*d*cos(2*d*x + 2*c) + a^2*d)*cos(6*d*x + 6*c)
- 20*(5*a^2*d*cos(2*d*x + 2*c) - a^2*d)*cos(4*d*x + 4*c) - 10*(a^2*d*sin(8*
d*x + 8*c) - 2*a^2*d*sin(6*d*x + 6*c) + 2*a^2*d*sin(4*d*x + 4*c) - a^2*d*si
n(2*d*x + 2*c))*sin(10*d*x + 10*c) - 50*(2*a^2*d*sin(6*d*x + 6*c) - 2*a^2*d
*sin(4*d*x + 4*c) + a^2*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 100*(2*a^2*d
*sin(4*d*x + 4*c) - a^2*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 15.26, size = 416, normalized size = 2.34

$$\frac{2\,\mathrm{atanh}\left(\frac{2\left(\tan(c+d\,x)\left(4\,a^7\,b^6-4\,a^9\,b^4\right)-\frac{\tan(c+d\,x)\left(\sqrt{a^9\,b^7}+a^5\,b^3\right)\left(64\,a^{14}\,b-128\,a^{13}\,b^2+64\,a^{12}\,b^3\right)}{16\left(a^9\,b-a^{10}\right)}\right)\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\left(a^9\,b-a^{10}\right)}}}{2\,a^5\,b^7-2\,a^6\,b^6}\right)\sqrt{\frac{\sqrt{a^9\,b^7}+a^5\,b^3}{16\left(a^9\,b-a^{10}\right)}}}{d}+\frac{2\,\mathrm{atanh}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) - (tan(c + d*x)*((a^9*b^7
)^(1/2) + a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9*b - a
^10)))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*a^5*b^7
- 2*a^6*b^6))*(((a^9*b^7)^(1/2) + a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/d +
(2*atanh((2*(tan(c + d*x)*(4*a^7*b^6 - 4*a^9*b^4) + (tan(c + d*x)*((a^9*b^7
)^(1/2) - a^5*b^3)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2))/(16*(a^9*b - a
^10)))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/(2*a^5*b^7
 - 2*a^6*b^6))*(-((a^9*b^7)^(1/2) - a^5*b^3)/(16*(a^9*b - a^10)))^(1/2))/d
- (1/(5*a) + (2*tan(c + d*x)^2)/(3*a) + (tan(c + d*x)^4*(a + b))/a^2)/(d*ta
n(c + d*x)^5)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.211 $\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=197

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^7(c+d}{7ad}$$

```
[Out] -(a+b)*cot(d*x+c)/a^2/d-1/3*(3*a+b)*cot(d*x+c)^3/a^2/d-3/5*cot(d*x+c)^5/a/d
-1/7*cot(d*x+c)^7/a/d+1/2*b^2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(
1/4))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(1/2)+1/2*b^2*arctan((a^(1/2)+b^(1/2))^(
1/2)*tan(d*x+c)/a^(1/4))/a^(11/4)/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1287, 1166, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}d\sqrt{\sqrt{a}+\sqrt{b}}}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^7(c+d}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(11/4)*Sq
rt[Sqrt[a] - Sqrt[b]]*d) + (b^2*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x
])/a^(1/4)])/(2*a^(11/4)*Sqrt[Sqrt[a] + Sqrt[b]]*d) - ((a + b)*Cot[c + d*x]
)/(a^2*d) - ((3*a + b)*Cot[c + d*x]^3)/(3*a^2*d) - (3*Cot[c + d*x]^5)/(5*a*
d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
```

```
IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{x^8\left(a+2ax^2+(a-b)x^4\right)}\,dx,x,\tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^8}+\frac{3}{ax^6}+\frac{3a+b}{a^2x^4}+\frac{a+b}{a^2x^2}+\frac{b^2\left(1+x^2\right)}{a^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx,x,\tan(c+dx)\right)}{d}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{b^2\,\mathrm{S}}{}$$

$$= -\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{(3a+b)\cot^3(c+dx)}{3a^2d}-\frac{3\cot^5(c+dx)}{5ad}-\frac{\cot^7(c+dx)}{7ad}+\frac{((\sqrt{}}{}$$

$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}-\sqrt{b}}\,d}+\frac{b^2\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{11/4}\sqrt{\sqrt{a}+\sqrt{b}}\,d}-\frac{(a+b)\cot(c+dx)}{a^2d}-$$

Mathematica [A] time = 6.32, size = 277, normalized size = 1.41

$$\frac{b^2\tan^{-1}\left(\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan(c+dx)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{2a^{5/2}d\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{b^2\tanh^{-1}\left(\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan(c+dx)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{2a^{5/2}d\sqrt{\sqrt{a}\sqrt{b}-a}}+\frac{\csc^3(c+dx)(-24a\cos(c+dx)-35b\cos(c+dx)}{105a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^2*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*Sqrt[b]]
*Sqrt[b])])/(2*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*d) - (b^2*ArcTanh[((Sqrt[a
]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b])])/(2*a^(5
/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*d) - (2*(24*a*Cos[c + d*x] + 35*b*Cos[c + d*
x])*Csc[c + d*x])/(105*a^2*d) + ((-24*a*Cos[c + d*x] - 35*b*Cos[c + d*x])*C
sc[c + d*x]^3)/(105*a^2*d) - (6*Cot[c + d*x]*Csc[c + d*x]^4)/(35*a*d) - (Co
t[c + d*x]*Csc[c + d*x]^6)/(7*a*d)
```

fricas [B] time = 0.65, size = 1585, normalized size = 8.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/840*(16*(24*a + 35*b)*cos(d*x + c)^7 - 56*(24*a + 35*b)*cos(d*x + c)^5 +
 560*(3*a + 4*b)*cos(d*x + c)^3 + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d
*x + c)^4 + 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt
(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*
b^7*cos(d*x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^
2 - (a^7*b^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) +
 1/2*(a^3*b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a
```

```
^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b
)*d^2)))*sin(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4
+ 3*a^2*d*cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 + (a^6 - a^5*b)*sqrt(b^9/((a^1
3 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(1/4*b^7*cos(d*
x + c)^2 - 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b
^2 - a^6*b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*
b^5*d*cos(d*x + c)*sin(d*x + c) - (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b
 + a^11*b^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 + (a^6 - a^5*b
)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*s
in(d*x + c) - 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*
cos(d*x + c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12
*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2
+ 1/4*b^7 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*
b^3)*d^2)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) + 1/2*(a^3*b^5*d*cos
(d*x + c)*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b
^2)*d^4))*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^
9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x +
c) + 105*(a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a^2*d*cos(d*x +
 c)^2 - a^2*d)*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11
*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2))*log(-1/4*b^7*cos(d*x + c)^2 + 1/4*b^7
 - 1/4*(2*(a^7*b^2 - a^6*b^3)*d^2*cos(d*x + c)^2 - (a^7*b^2 - a^6*b^3)*d^2)
*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4)) - 1/2*(a^3*b^5*d*cos(d*x + c)
*sin(d*x + c) + (a^10 - a^9*b)*sqrt(b^9/((a^13 - 2*a^12*b + a^11*b^2)*d^4))
*d^3*cos(d*x + c)*sin(d*x + c))*sqrt(-(b^4 - (a^6 - a^5*b)*sqrt(b^9/((a^13
- 2*a^12*b + a^11*b^2)*d^4))*d^2)/((a^6 - a^5*b)*d^2)))*sin(d*x + c) - 840*
(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^6 - 3*a^2*d*cos(d*x + c)^4 + 3*a
^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))
```

giac [B] time = 1.02, size = 467, normalized size = 2.37

$$\frac{105\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^3-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan(dx+c)}{\sqrt{\frac{a^3+\sqrt{a^6-(a^3-a^2b)a^3}}{a^3-a^2b}}}\right)\right)|a-b|}{3a^7-12a^6b+14a^5b^2-4a^4b^3-a^3b^4}+\frac{105\left(3\sqrt{a\cdots\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/210*(105*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 6*sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^4)*(pi
*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 + sqrt(a^6 - (a^
3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12*a^6*b + 14*a^5*b^2
 - 4*a^4*b^3 - a^3*b^4) + 105*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^
2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a
^3 - sqrt(a^6 - (a^3 - a^2*b)*a^3))/(a^3 - a^2*b))))*abs(a - b)/(3*a^7 - 12
*a^6*b + 14*a^5*b^2 - 4*a^4*b^3 - a^3*b^4) - 2*(105*a*tan(d*x + c)^6 + 105*
b*tan(d*x + c)^6 + 105*a*tan(d*x + c)^4 + 35*b*tan(d*x + c)^4 + 63*a*tan(d*
x + c)^2 + 15*a)/(a^2*tan(d*x + c)^7))/d
```

maple [B] time = 0.55, size = 624, normalized size = 3.17

$$-\frac{1}{7da\tan(dx+c)^7}-\frac{1}{da\tan(dx+c)}-\frac{b}{d\,a^2\tan(dx+c)}-\frac{1}{da\tan(dx+c)^3}-\frac{b}{3d\,a^2\tan(dx+c)^3}-\frac{3}{5da\tan(dx+c)^5}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/7/d/a/tan(d*x+c)^7-1/d/a/tan(d*x+c)-1/d/a^2/tan(d*x+c)*b-1/d/a/tan(d*x+c
)^3-1/3/d/a^2/tan(d*x+c)^3*b-3/5/d/a/tan(d*x+c)^5+1/2/d/a*b^2/(a-b)/(((a*b)
^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1
/2))-1/2/d/a*b^3/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-
a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d/a*b^3/(a*b)^(1/2)/(a-b
)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a
-b))^(1/2))+1/2/d/a*b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b^3/a^2/(a-b)/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2
/d*b^4/a^2/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*t
an(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b^4/a^2/(a*b)^(1/2)/(a-b)/((
(a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))
^(1/2))-1/2/d*b^3/a^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 17.08, size = 1704, normalized size = 8.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^8*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan((((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b
^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)
/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - ta
n(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11
*b - a^12)))^(1/2)*1i - ((((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12))
)^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(((a^11*b^9)
^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 12
8*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^11*b^9)^(1/2) + a
^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i)/(((((a^11*b^9)^(1/2) + a^6*b^4)/(16
*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c +
d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b +
 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(((a^
11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) + ((((a^11*b^9)^(1/2)
+ a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*
b^3 - tan(c + d*x)*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2
)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a
^8*b^5))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) - 2*a^4*
b^8 + 2*a^5*b^7))*(((a^11*b^9)^(1/2) + a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)
*2i)/d + (atan((((-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)
*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 + tan(c + d*x)*(-((a^11*b^9)^(1/2)
 - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13
*b^2)) - tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^
4)/(16*(a^11*b - a^12)))^(1/2)*1i - ((-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^
11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)
*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(64*a^14*b + 64
*a^12*b^3 - 128*a^13*b^2)) + tan(c + d*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11
*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*1i)/(((-((a^11*b^9)^(1/2
```

```
) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*a^10*b^4 + 16*a^1
1*b^3 + tan(c + d*x)*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(
1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) - tan(c + d*x)*(4*a^6*b^7 -
4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2) + ((
-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a^12)))^(1/2)*(16*a^9*b^5 - 32*
a^10*b^4 + 16*a^11*b^3 - tan(c + d*x)*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a
^11*b - a^12)))^(1/2)*(64*a^14*b + 64*a^12*b^3 - 128*a^13*b^2)) + tan(c + d
*x)*(4*a^6*b^7 - 4*a^8*b^5))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(a^11*b - a
^12)))^(1/2) - 2*a^4*b^8 + 2*a^5*b^7))*(-((a^11*b^9)^(1/2) - a^6*b^4)/(16*(
a^11*b - a^12)))^(1/2)*2i)/d - (1/(7*a) + (3*tan(c + d*x)^2)/(5*a) + (tan(c
 + d*x)^6*(a + b))/a^2 + (tan(c + d*x)^4*(3*a + b))/(3*a^2))/(d*tan(c + d*x
)^7)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.212 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cos^4(c+d\right.}$$

```
[Out] -cos(d*x+c)/b^2/d-1/4*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*
b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/
2))^(1/2))*a^(1/2)*(5*a^(1/2)-6*b^(1/2))/b^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)+
1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*a^(1/2)*(5*a^(1/2)+
6*b^(1/2))/b^(9/4)/d/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.48, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1676, 1166, 205, 208}

$$-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{4b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{a}\left(5\sqrt{a}+\right.}{8b^9}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (Sqrt[a]*(5*Sqrt[a] - 6*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
 - Sqrt[b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(9/4)*d) + (Sqrt[a]*(5*Sqrt[a]
 + 6*Sqrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(9/4)*d) - Cos[c + d*x]/(b^2*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(4*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
- b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
```

```
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1676

```
Int[(Pq_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandInte
grand[Pq/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a\left(a+\frac{a^2}{b}-4b\right)-2a(7\ldots}{a-b+2b\ldots}\right.}{8\ldots}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int\left(-\frac{8a(a-b)}{b}+\frac{2(a^{\ldots}}{b(\ldots}\right.\right.}{8\ldots}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int\ldots\right.}{\ldots}$$

$$= -\frac{\cos(c+dx)}{b^2d} - \frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(\sqrt{a}\left(5\sqrt{a}\ldots\right.\right.}{\ldots}$$

$$= \frac{\sqrt{a}\left(5\sqrt{a}-6\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{9/4}d} + \frac{\sqrt{a}\left(5\sqrt{a}+6\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{9/4}d} - \ldots$$

Mathematica [C] time = 1.19, size = 486, normalized size = 2.06

$$-\frac{ia\,\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+40\#1^4a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-54\#1^4b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+20i\#1^2a\log\left(\#1^2-\ldots\right.}{\ldots}\right.}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*(32*Cos[c + d*x] + (32*a*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))
/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (I*a*R
ootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1
^2] - 40*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 54*b*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^2 - (27*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 40*a*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 54*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^4 - (20*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (27*I)*
b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c +
 d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) -
8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(b^2*d)
```

fricas [B] time = 0.90, size = 2649, normalized size = 11.22

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(16*(a*b - b^2)*cos(d*x + c)^5 - 4*(7*a*b - 8*b^2)*cos(d*x + c)^3 + (
(a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^
2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt
((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*
b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^1
5)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 -
b^7)*d^2))*log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x
 + c) + (2*(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sq
rt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^
6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b
^15)*d^4)) - (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqr
t(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b +
7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4
*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^
2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b
^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^
3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 -
 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5
*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) -
15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*
log((625*a^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*
(2*a^4*b^7 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7
 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a
^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4))
+ (125*a^5*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4
 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2
 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a
^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b
^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) - ((a*b^3 - b^4)*d*cos(d*
x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*
sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b
 + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*
a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47
*a^2*b + 36*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a
^5 - 2625*a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7
 - 9*a^3*b^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^
6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 +
15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - (125*a^5
```

```
*b^2 - 520*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(-((a^3*b^4 - 3*a^2*
b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a
^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 +
 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + 15*a^3 - 47*a^2*b + 36*a*b^2)/((a^3
*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4
- 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^
3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^
5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 -
 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 3
6*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-(625*a^5 - 2625*
a^4*b + 3684*a^3*b^2 - 1728*a^2*b^3)*cos(d*x + c) + (2*(2*a^4*b^7 - 9*a^3*b
^8 + 15*a^2*b^9 - 11*a*b^10 + 3*b^11)*d^3*sqrt((625*a^7 - 3450*a^6*b + 7161
*a^5*b^2 - 6624*a^4*b^3 + 2304*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^1
1 - 20*a^3*b^12 + 15*a^2*b^13 - 6*a*b^14 + b^15)*d^4)) + (125*a^5*b^2 - 520
*a^4*b^3 + 723*a^3*b^4 - 336*a^2*b^5)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2*sqrt((625*a^7 - 3450*a^6*b + 7161*a^5*b^2 - 6624*a^4*b^3 + 23
04*a^3*b^4)/((a^6*b^9 - 6*a^5*b^10 + 15*a^4*b^11 - 20*a^3*b^12 + 15*a^2*b^1
3 - 6*a*b^14 + b^15)*d^4)) - 15*a^3 + 47*a^2*b - 36*a*b^2)/((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2))) - 4*(5*a^2 - 7*a*b + 4*b^2)*cos(d*x + c))/((a
*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (a^2*b^2
- 2*a*b^3 + b^4)*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-54,3]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[65,-69]2/d*(5*((1-cos(c+d*x))
/(1+cos(c+d*x)))^4*a^2-6*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a*b+20*((1-cos(c
+d*x))/(1+cos(c+d*x)))^3*a^2-26*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a*b+30*((
1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^2-94*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a*
b+64*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b^2+20*(1-cos(c+d*x))/(1+cos(c+d*x))
*a^2-14*(1-cos(c+d*x))/(1+cos(c+d*x))*a*b+5*a^2-4*a*b)/(-4*a*b^2+4*b^3)/(((
1-cos(c+d*x))/(1+cos(c+d*x)))^5*a+5*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+10*
((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*b+
10*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^2
*b+5*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)+2/d/(-4*a*b^2+4*b^3)*2/d*(-a/4*(c+d
*x)+(-10*a^4*b+72*a^3*b^2+33*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+10*a^3*a*b
-15*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-118*a^2*b^3-102*a^2*b^2*sqr
t(a^2-a*b+sqrt(a*b)*(a-b))-72*a^2*b*a*b+42*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqr
t(a*b)*(a-b))+56*a*b^4+61*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+118*a*b^2*a*b
-19*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+12*b^4*sqrt(a^2-a*b+sqrt(
a*b)*(a-b))-56*b^3*a*b-4*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a
-b)/(24*a^4*b-96*a^3*b^2+112*a^2*b^3-32*a*b^4-8*b^5)*(atan(tan(c+d*x)/sqrt(
-(8*a+sqrt(8*a*8*a+4*(-4*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor((c+d*x)/pi+1/
2))-(-10*a^4*b+72*a^3*b^2-33*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+10*a^3*a*
b-15*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-118*a^2*b^3+102*a^2*b^2*s
qrt(a^2-a*b+sqrt(a*b)*(-a+b))-72*a^2*b*a*b+42*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+
sqrt(a*b)*(-a+b))+56*a*b^4-61*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+118*a*b^
2*a*b-19*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-12*b^4*sqrt(a^2-a*b
+sqrt(a*b)*(-a+b))-56*b^3*a*b-4*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)
))*abs(a-b)/(24*a^4*b-96*a^3*b^2+112*a^2*b^3-32*a*b^4-8*b^5)*(atan(tan(c+d*
x)/sqrt(-(8*a-sqrt(8*a*8*a+4*(-4*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor((c+d*
x)/pi+1/2)))
```

maple [B] time = 0.34, size = 482, normalized size = 2.04

$$-\frac{\cos\left(dx+c\right)}{b^{2}d}-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{4db\left(b\left(\cos^{4}\left(dx+c\right)\right)-2b\left(\cos^{2}\left(dx+c\right)\right)-a+b\right)\left(a-b\right)}+\frac{a^{2}\cos\left(dx+\right.}{4d\,b^{2}\left(b\left(\cos^{4}\left(dx+c\right)\right)-2b\left(\cos^{2}\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -cos(d*x+c)/b^2/d-1/4/d*a/b/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos
(d*x+c)^3+1/4/d*a^2/b^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x
+c)+1/4/d*a/b/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+1/8/d*
a/b/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b
)^(1/2))-3/4/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+5/8/d*a^2/b/(a-b)/(a*b)^(1/2)/(((a*b)^(1/
2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d*a/b/(a
-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2)
)-3/4/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(
((a*b)^(1/2)-b)*b)^(1/2))+5/8/d*a^2/b/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)
^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c)*cos(d*x + c) - 4*(2*a*b^2 - 3*b^3)*
sin(3*d*x + 3*c)*sin(2*d*x + 2*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c)*sin(
d*x + c) - ((a*b^2 - b^3)*cos(9*d*x + 9*c) - 4*(a*b^2 - b^3)*cos(7*d*x + 7*
c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(5*d*x + 5*c) - 4*(a*b^2 - b^3)*cos(
3*d*x + 3*c) + (a*b^2 - b^3)*cos(d*x + c))*cos(10*d*x + 10*c) - (a*b^2 - b^
3 - (2*a*b^2 - 3*b^3)*cos(8*d*x + 8*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(
6*d*x + 6*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 -
3*b^3)*cos(2*d*x + 2*c))*cos(9*d*x + 9*c) - (4*(2*a*b^2 - 3*b^3)*cos(7*d*x
+ 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*cos(5*d*x + 5*c) + 4*(2*a*b^2 - 3*
b^3)*cos(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*cos(d*x + c))*cos(8*d*x + 8*c) +
4*(a*b^2 - b^3 - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(6*d*x + 6*c) - (20*a^2*b
 - 17*a*b^2 + 2*b^3)*cos(4*d*x + 4*c) - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))
*cos(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(5*d*x +
 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a
*b^2 + 2*b^3)*cos(d*x + c))*cos(6*d*x + 6*c) + 2*(8*a^2*b - 11*a*b^2 + 3*b^
3 - (160*a^3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*cos(4*d*x + 4*c) - (16*a^2*b -
 30*a*b^2 + 9*b^3)*cos(2*d*x + 2*c))*cos(5*d*x + 5*c) - (4*(20*a^2*b - 17*a
*b^2 + 2*b^3)*cos(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*cos(d*x + c)
)*cos(4*d*x + 4*c) + 4*(a*b^2 - b^3 - (2*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*c
os(3*d*x + 3*c) - (a*b^2 - b^3)*cos(d*x + c) + 2*((a*b^4 - b^5)*d*cos(9*d*x
 + 9*c)^2 + 16*(a*b^4 - b^5)*d*cos(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2
*b^3 + 57*a*b^4 - 9*b^5)*d*cos(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*cos(3*d*
x + 3*c)^2 - 8*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)*cos(d*x + c) + (a*b^4 - b^5
)*d*cos(d*x + c)^2 + (a*b^4 - b^5)*d*sin(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*
d*sin(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*si
n(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*sin(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*
d*sin(3*d*x + 3*c)*sin(d*x + c) + (a*b^4 - b^5)*d*sin(d*x + c)^2 - 2*(4*(a*
b^4 - b^5)*d*cos(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*
x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c
))*cos(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c
```

```
) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) - (a*b^4 - b^5)*d*cos(d*x + c))*cos(
7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(3*d*x + 3*c) - (8*
a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(d*x + c))*cos(5*d*x + 5*c) - 2*(4*(a*b^4
- b^5)*d*sin(7*d*x + 7*c) + 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x +
5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*s
in(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) +
4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*b^4 - b^5)*d*sin(d*x + c))*sin(7*d*
x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(3*d*x + 3*c) - (8*a^2*
b^3 - 11*a*b^4 + 3*b^5)*d*sin(d*x + c))*sin(5*d*x + 5*c))*integrate(-1/2*(4
*a*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*a*b^2*cos(2*d*x + 2*c)*sin(d*x + c
) + a*b^2*sin(d*x + c) + 4*(20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c)*sin(2*d*x
 + 2*c) - (a*b^2*sin(7*d*x + 7*c) - a*b^2*sin(d*x + c) + (20*a^2*b - 27*a*b
^2)*sin(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))*cos(8*d*x +
8*c) - 2*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b -
3*a*b^2)*sin(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(a*b^2*sin(d*x + c) - (20*a
^2*b - 27*a*b^2)*sin(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*sin(3*d*x + 3*c))
*cos(6*d*x + 6*c) - 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*sin(4*d*x + 4*c) +
2*(20*a^2*b - 27*a*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*((160*a^3 -
276*a^2*b + 81*a*b^2)*sin(3*d*x + 3*c) + (8*a^2*b - 3*a*b^2)*sin(d*x + c))*
cos(4*d*x + 4*c) + (a*b^2*cos(7*d*x + 7*c) - a*b^2*cos(d*x + c) + (20*a^2*b
 - 27*a*b^2)*cos(5*d*x + 5*c) - (20*a^2*b - 27*a*b^2)*cos(3*d*x + 3*c))*sin
(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*a*b^2*cos(2*d*x + 2*c) - a*b^
2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(a*b^2*cos
(d*x + c) - (20*a^2*b - 27*a*b^2)*cos(5*d*x + 5*c) + (20*a^2*b - 27*a*b^2)*
cos(3*d*x + 3*c))*sin(6*d*x + 6*c) - (20*a^2*b - 27*a*b^2 - 2*(160*a^3 - 27
6*a^2*b + 81*a*b^2)*cos(4*d*x + 4*c) - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x +
2*c))*sin(5*d*x + 5*c) + 2*((160*a^3 - 276*a^2*b + 81*a*b^2)*cos(3*d*x + 3*
c) + (8*a^2*b - 3*a*b^2)*cos(d*x + c))*sin(4*d*x + 4*c) + (20*a^2*b - 27*a*
b^2 - 4*(20*a^2*b - 27*a*b^2)*cos(2*d*x + 2*c))*sin(3*d*x + 3*c))/(a*b^4 -
b^5 + (a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a*b^4 - b^5)*cos(6*d*x + 6*c)^
2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16
*(a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a*b^4 - b^5)*sin(8*d*x + 8*c)^2 + 16*(
a*b^4 - b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 -
9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 16*(a*b^4 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a*b^4 - b^5
 - 4*(a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(
4*d*x + 4*c) - 4*(a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^
4 - b^5 - 2*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a*b^4 - b^
5)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^3 - 11*a*b^4 + 3*b^5 - 4
*(8*a^2*b^3 - 11*a*b^4 + 3*b^5)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b
^4 - b^5)*cos(2*d*x + 2*c) - 4*(2*(a*b^4 - b^5)*sin(6*d*x + 6*c) + (8*a^2*b
^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2*(a*b^4 - b^5)*sin(2*d*x + 2*c))
*sin(8*d*x + 8*c) + 16*((8*a^2*b^3 - 11*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2
*(a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - ((a*b^2 - b^3)*sin
(9*d*x + 9*c) - 4*(a*b^2 - b^3)*sin(7*d*x + 7*c) - 2*(8*a^2*b - 11*a*b^2 +
3*b^3)*sin(5*d*x + 5*c) - 4*(a*b^2 - b^3)*sin(3*d*x + 3*c) + (a*b^2 - b^3)*
sin(d*x + c))*sin(10*d*x + 10*c) + ((2*a*b^2 - 3*b^3)*sin(8*d*x + 8*c) + (2
0*a^2*b - 17*a*b^2 + 2*b^3)*sin(6*d*x + 6*c) + (20*a^2*b - 17*a*b^2 + 2*b^3
)*sin(4*d*x + 4*c) + (2*a*b^2 - 3*b^3)*sin(2*d*x + 2*c))*sin(9*d*x + 9*c) -
 (4*(2*a*b^2 - 3*b^3)*sin(7*d*x + 7*c) + 2*(16*a^2*b - 30*a*b^2 + 9*b^3)*si
n(5*d*x + 5*c) + 4*(2*a*b^2 - 3*b^3)*sin(3*d*x + 3*c) - (2*a*b^2 - 3*b^3)*s
in(d*x + c))*sin(8*d*x + 8*c) - 4*((20*a^2*b - 17*a*b^2 + 2*b^3)*sin(6*d*x
+ 6*c) + (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(4*d*x + 4*c) + (2*a*b^2 - 3*b^3)
*sin(2*d*x + 2*c))*sin(7*d*x + 7*c) - (2*(160*a^3 - 196*a^2*b + 67*a*b^2 -
6*b^3)*sin(5*d*x + 5*c) + 4*(20*a^2*b - 17*a*b^2 + 2*b^3)*sin(3*d*x + 3*c)
- (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(d*x + c))*sin(6*d*x + 6*c) - 2*((160*a^
3 - 196*a^2*b + 67*a*b^2 - 6*b^3)*sin(4*d*x + 4*c) + (16*a^2*b - 30*a*b^2 +
 9*b^3)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - (4*(20*a^2*b - 17*a*b^2 + 2*b^
3)*sin(3*d*x + 3*c) - (20*a^2*b - 17*a*b^2 + 2*b^3)*sin(d*x + c))*sin(4*d*x
```

```
 + 4*c))/((a*b^4 - b^5)*d*cos(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*cos(7*d*x
 + 7*c)^2 + 4*(64*a^3*b^2 - 112*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*cos(5*d*x + 5
*c)^2 + 16*(a*b^4 - b^5)*d*cos(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*d*cos(3*d*x
 + 3*c)*cos(d*x + c) + (a*b^4 - b^5)*d*cos(d*x + c)^2 + (a*b^4 - b^5)*d*sin
(9*d*x + 9*c)^2 + 16*(a*b^4 - b^5)*d*sin(7*d*x + 7*c)^2 + 4*(64*a^3*b^2 - 1
12*a^2*b^3 + 57*a*b^4 - 9*b^5)*d*sin(5*d*x + 5*c)^2 + 16*(a*b^4 - b^5)*d*si
n(3*d*x + 3*c)^2 - 8*(a*b^4 - b^5)*d*sin(3*d*x + 3*c)*sin(d*x + c) + (a*b^4
 - b^5)*d*sin(d*x + c)^2 - 2*(4*(a*b^4 - b^5)*d*cos(7*d*x + 7*c) + 2*(8*a^2
*b^3 - 11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x +
 3*c) - (a*b^4 - b^5)*d*cos(d*x + c))*cos(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 -
11*a*b^4 + 3*b^5)*d*cos(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*cos(3*d*x + 3*c) -
 (a*b^4 - b^5)*d*cos(d*x + c))*cos(7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^
4 + 3*b^5)*d*cos(3*d*x + 3*c) - (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*cos(d*x +
c))*cos(5*d*x + 5*c) - 2*(4*(a*b^4 - b^5)*d*sin(7*d*x + 7*c) + 2*(8*a^2*b^3
 - 11*a*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c
) - (a*b^4 - b^5)*d*sin(d*x + c))*sin(9*d*x + 9*c) + 8*(2*(8*a^2*b^3 - 11*a
*b^4 + 3*b^5)*d*sin(5*d*x + 5*c) + 4*(a*b^4 - b^5)*d*sin(3*d*x + 3*c) - (a*
b^4 - b^5)*d*sin(d*x + c))*sin(7*d*x + 7*c) + 4*(4*(8*a^2*b^3 - 11*a*b^4 +
3*b^5)*d*sin(3*d*x + 3*c) - (8*a^2*b^3 - 11*a*b^4 + 3*b^5)*d*sin(d*x + c))*
sin(5*d*x + 5*c))
```

mupad [B] time = 16.00, size = 3941, normalized size = 16.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - cos(c + d*x)/(b^2*d) - ((cos(c + d*x)*(a*b + a^2))/(4*(a - b)) - (a*b*cos
(c + d*x)^3)/(4*(a - b)))/(d*(a*b^2 - b^3 + 2*b^3*cos(c + d*x)^2 - b^3*cos(
c + d*x)^4)) - (atan(((((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b
^5 - 2*a*b^4 + a^2*b^3)) - (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*
(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/
2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*
a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/
2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(
a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (co
s(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*(
(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 -
15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a
^3*b^9)))^(1/2)*1i - (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^
5 - 2*a*b^4 + a^2*b^3)) + (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(
a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2
))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a
^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2
) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a
^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) - (cos
(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((
25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 1
5*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^
3*b^9)))^(1/2)*1i)/((36*a^3*b - 25*a^4)/(32*(b^5 - 2*a*b^4 + a^2*b^3)) + ((
(1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2*b^3))
 - (cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b
^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^1
2 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3*b^5))/
(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1
/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*
a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (cos(c + d*x)*(25*a^4 - 59*
a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2)
+ 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3
```

```
*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (((179
2*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2*b^3)) + (
cos(c + d*x)*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 +
 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 -
3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3*b^5))/(4*(
a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2)
- 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^
11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 59*a^3*
b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*((25*a^2*(a^3*b^9)^(1/2) + 48
*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^5 - 69*a*b*(a^3*b^9
)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)))*((25*a^2*(
a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) - 36*a*b^7 + 47*a^2*b^6 - 15*a^3*b^
5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))
^(1/2)*2i)/d - (atan(((((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b
^5 - 2*a*b^4 + a^2*b^3)) - (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2
*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1
/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512
*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)^(
1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b
*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (
cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))
*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6
 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10
+ a^3*b^9)))^(1/2)*1i - (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*
(b^5 - 2*a*b^4 + a^2*b^3)) + (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b
^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^
(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 5
12*a^2*b^6 + 256*a^3*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)
^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a
*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) -
 (cos(c + d*x)*(25*a^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)
))*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b
^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^1
0 + a^3*b^9)))^(1/2)*1i)/((36*a^3*b - 25*a^4)/(32*(b^5 - 2*a*b^4 + a^2*b^3)
) + (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2
*b^3)) - (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) +
 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^1
1 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3
*b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3
*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/
(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) + (cos(c + d*x)*(25*a
^4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^
9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69
*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)
 + (((1792*a^2*b^6 - 3072*a^3*b^5 + 1280*a^4*b^4)/(64*(b^5 - 2*a*b^4 + a^2*
b^3)) + (cos(c + d*x)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) +
36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11
 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2)*(256*a*b^7 - 512*a^2*b^6 + 256*a^3*
b^5))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*
b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(
256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2) - (cos(c + d*x)*(25*a^
4 - 59*a^3*b + 36*a^2*b^2))/(4*(a^2*b - 2*a*b^2 + b^3)))*(-(25*a^2*(a^3*b^9
)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^6 + 15*a^3*b^5 - 69*
a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10 + a^3*b^9)))^(1/2))
)*(-(25*a^2*(a^3*b^9)^(1/2) + 48*b^2*(a^3*b^9)^(1/2) + 36*a*b^7 - 47*a^2*b^
6 + 15*a^3*b^5 - 69*a*b*(a^3*b^9)^(1/2))/(256*(3*a*b^11 - b^12 - 3*a^2*b^10
 + a^3*b^9)))^(1/2)*2i)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.213 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(2-\cos^2(c+d\right.}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2\right.}$$

```
[Out] -1/4*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*
x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/2)-4
*b^(1/2))/b^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*cos(d*x+c)/
(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+4*b^(1/2))/b^(7/4)/d/(a^(1/2)+b^(1/2))^
(3/2)
```

Rubi [A] time = 0.33, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{a\cos(c+dx)\left(2-\cos^2(c+d\right.}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((3*Sqrt[a] - 4*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(7/4)*d) - ((3*Sqrt[a] + 4*Sqrt[b])*Ar
cTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*(Sqrt[a] + Sqrt[b
])^(3/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*(a - b)*b*d*
(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=-\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^2}\,dx,x,\cos(c+dx)\right)}{d}$$

$$=-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{4a(a-2b)-2a(3a-}{a-b+2bx^2-b}\right.}{8a(a}$$

$$=-\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)}-\frac{\left(3a-\sqrt{a}\sqrt{b}-4b\right)\text{Sul}}{}$$

$$=\frac{\left(3\sqrt{a}-4\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{\left(3\sqrt{a}+4\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{7/4}d}-\frac{}{4(a}$$

Mathematica [C] time = 0.61, size = 565, normalized size = 2.69

$$\frac{\frac{16a(\cos(3(c+dx))-5\cos(c+dx))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b}-i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{-6\#1^6a\tan^{-1}\left(\frac{\sin(c+d}{\cos(c+dx)}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*
x)] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
 4*b*#1^6 + b*#1^8 & , (6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 8*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (3*I)*a*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2] + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 10*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^2 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*b*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^4 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (5*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^4 - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 8*b*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (3*I)*a*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^6 - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
 - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(32*(a - b)*b*d)
```

fricas [B] time = 0.84, size = 2507, normalized size = 11.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^
4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b
^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b
^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3 - 3*a^2*b
^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 384*b^3)*co
s(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*b^9)*d^3*
sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(-((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(
a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3 -
 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1
392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 +
 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log((81*a^3 - 405*a^2*b + 680*a*b^2 - 38
4*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*a*b^8 + 5*
b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4
)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d)*sqrt(((a
^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^3
*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^
3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b - 16*b^2)/((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)
^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((
a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b + 1273*a^
3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a
^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b + 16*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^2*b + 680*
a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2*b^7 - 18*
a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 +
 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11
- 6*a*b^12 + b^13)*d^4)) - 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 - 48*b^5)*d
)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 3*a^2 - 15*a*b +
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - ((a*b^2 - b^3)*d*c
os(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*
d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 - 522*a^4*b
 + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*
b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 + 15*a*b -
 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(-(81*a^3 - 405*a^
2*b + 680*a*b^2 - 384*b^3)*cos(d*x + c) + ((3*a^4*b^5 - 14*a^3*b^6 + 24*a^2
*b^7 - 18*a*b^8 + 5*b^9)*d^3*sqrt((81*a^5 - 522*a^4*b + 1273*a^3*b^2 - 1392
*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 2*(9*a^3*b^2 - 47*a^2*b^3 + 82*a*b^4 -
 48*b^5)*d)*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((81*a^5 -
522*a^4*b + 1273*a^3*b^2 - 1392*a^2*b^3 + 576*a*b^4)/((a^6*b^7 - 6*a^5*b^8
+ 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) - 3*a^2 +
```

```
 15*a*b - 16*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))) - 8*a*cos(d
*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2
 - (a^2*b - 2*a*b^2 + b^3)*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[54,-78]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-23,24]2/d*(((1-cos(c+d*x))/
(1+cos(c+d*x)))^3*a-3*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a+8*((1-cos(c+d*x))
/(1+cos(c+d*x)))^2*b-5*(1-cos(c+d*x))/(1+cos(c+d*x))*a-a)/(4*a*b-4*b^2)/(((
1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a+6*(
(1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+4
*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)+2/d/(4*a*b-4*b^2)*2/d*(-(3*a-4*b)/(2*b*
2)*(c+d*x)+((-6*a^5*b+30*a^4*b^2+6*a^4*a*b-9*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqr
t(a*b)*(a-b))-58*a^3*b^3-30*a^3*b*a*b+45*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(
a*b)*(a-b))+50*a^2*b^4+58*a^2*b^2*a*b-75*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqr
t(a*b)*(a-b))-16*a*b^5-50*a*b^3*a*b+39*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*
b)*(a-b))+16*b^4*a*b+8*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b
)*b^2+(12*a^5*b^2+9*a^5*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-32*a^4*b^3-33*a^4*b
^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-12*a^4*b*a*b+28*a^3*b^4+33*a^3*b^3*sqrt(a^
2-a*b+sqrt(a*b)*(a-b))+32*a^3*b^2*a*b-8*a^2*b^5-7*a^2*b^4*sqrt(a^2-a*b+sqrt
(a*b)*(a-b))-28*a^2*b^3*a*b-2*a*b^5*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+8*a*b^4*a
*b)*abs(a-b)*abs(b)+(-8*a^5*b^3+28*a^4*b^4+8*a^4*b^2*a*b-12*a^4*b^2*sqrt(a*
b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-32*a^3*b^5-28*a^3*b^3*a*b+42*a^3*b^3*sqrt(
a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+12*a^2*b^6+32*a^2*b^4*a*b-32*a^2*b^4*sqr
t(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-12*a*b^5*a*b-6*a*b^5*sqrt(a*b)*sqrt(a^
2-a*b+sqrt(a*b)*(a-b)))*abs(a-b))/((24*a^6*b^2-120*a^5*b^3+208*a^4*b^4-144*
a^3*b^5+24*a^2*b^6+8*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(8*a*b+sqrt(8*a*
b*8*a*b+4*(-4*a*b+4*b^2)*4*a*b))/2/(-4*a*b+4*b^2)))+pi*floor((c+d*x)/pi+1/2
))-((-6*a^5*b+30*a^4*b^2+6*a^4*a*b-9*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(
-a+b))-58*a^3*b^3-30*a^3*b*a*b+45*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-
a+b))+50*a^2*b^4+58*a^2*b^2*a*b-75*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)
*(-a+b))-16*a*b^5-50*a*b^3*a*b+39*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-
a+b))+16*b^4*a*b+8*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)*b
^2+(12*a^5*b^2-9*a^5*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-32*a^4*b^3+33*a^4*b^2
*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-12*a^4*b*a*b+28*a^3*b^4-33*a^3*b^3*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))+32*a^3*b^2*a*b-8*a^2*b^5+7*a^2*b^4*sqrt(a^2-a*b+sqrt
(a*b)*(-a+b))-28*a^2*b^3*a*b+2*a*b^5*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+8*a*b^4
*a*b)*abs(a-b)*abs(b)+(-8*a^5*b^3+28*a^4*b^4+8*a^4*b^2*a*b-12*a^4*b^2*sqrt(
a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-32*a^3*b^5-28*a^3*b^3*a*b+42*a^3*b^3*sq
rt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+12*a^2*b^6+32*a^2*b^4*a*b-32*a^2*b^4
*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-12*a*b^5*a*b-6*a*b^5*sqrt(a*b)*sq
rt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b))/((24*a^6*b^2-120*a^5*b^3+208*a^4*b^
4-144*a^3*b^5+24*a^2*b^6+8*a*b^7)*abs(b))*(atan(tan(c+d*x)/sqrt(-(8*a*b-sqr
t(8*a*b*8*a*b+4*(-4*a*b+4*b^2)*4*a*b))/2/(-4*a*b+4*b^2)))+pi*floor((c+d*x)/
pi+1/2)))
```

maple [B] time = 0.32, size = 394, normalized size = 1.88

$$-\frac{a\left(\cos^{3}\left(dx+c\right)\right)}{4db\left(b\left(\cos^{4}\left(dx+c\right)\right)-2b\left(\cos^{2}\left(dx+c\right)\right)-a+b\right)\left(a-b\right)}+\frac{a\cos\left(dx+c\right)}{2db\left(b\left(\cos^{4}\left(dx+c\right)\right)-2b\left(\cos^{2}\left(dx+c\right)\right)-a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d*a/b/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/2/d*a
/b/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)-3/8/d*a/b/(a-b)/(
((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/
2/d/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b
)^(1/2))-1/8/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+3/8/d*a/b/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)
*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/2/d/(a-b)/(((a*b)^(1/2)-b
)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/8/d*a/(a-b)/(a*
b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^
(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b*sin(3*d*x + 3*c)*sin(2*d*
x + 2*c) + 4*a*b*sin(2*d*x + 2*c)*sin(d*x + c) - a*b*cos(d*x + c) - (a*b*co
s(7*d*x + 7*c) - 5*a*b*cos(5*d*x + 5*c) - 5*a*b*cos(3*d*x + 3*c) + a*b*cos(
d*x + c))*cos(8*d*x + 8*c) + (4*a*b*cos(6*d*x + 6*c) + 4*a*b*cos(2*d*x + 2*
c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))*cos(7*d*x + 7*c) - 4*(5*a*b*
cos(5*d*x + 5*c) + 5*a*b*cos(3*d*x + 3*c) - a*b*cos(d*x + c))*cos(6*d*x + 6
*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b + 2*(8*a^2 - 3*a*b)*cos(4*d*x + 4*c))
*cos(5*d*x + 5*c) - 2*(5*(8*a^2 - 3*a*b)*cos(3*d*x + 3*c) - (8*a^2 - 3*a*b)
*cos(d*x + c))*cos(4*d*x + 4*c) - 5*(4*a*b*cos(2*d*x + 2*c) - a*b)*cos(3*d*
x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6
*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x +
 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*
b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2
*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3
- b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x +
4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*
c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^
4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2
 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)
*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11
*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*si
n(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*
(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/2*(4*(5*a*
b - 12*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) - 4*(3*a*b - 4*b^2)*cos(d*x +
 c)*sin(2*d*x + 2*c) + 4*(3*a*b - 4*b^2)*cos(2*d*x + 2*c)*sin(d*x + c) + ((
3*a*b - 4*b^2)*sin(7*d*x + 7*c) - (5*a*b - 12*b^2)*sin(5*d*x + 5*c) + (5*a*
b - 12*b^2)*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))*cos(8*d*x + 8*
c) + 2*(2*(3*a*b - 4*b^2)*sin(6*d*x + 6*c) + (24*a^2 - 41*a*b + 12*b^2)*sin
(4*d*x + 4*c) + 2*(3*a*b - 4*b^2)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((
5*a*b - 12*b^2)*sin(5*d*x + 5*c) - (5*a*b - 12*b^2)*sin(3*d*x + 3*c) + (3*a
*b - 4*b^2)*sin(d*x + c))*cos(6*d*x + 6*c) - 2*((40*a^2 - 111*a*b + 36*b^2)
*sin(4*d*x + 4*c) + 2*(5*a*b - 12*b^2)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) -
 2*((40*a^2 - 111*a*b + 36*b^2)*sin(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^
2)*sin(d*x + c))*cos(4*d*x + 4*c) - ((3*a*b - 4*b^2)*cos(7*d*x + 7*c) - (5*
a*b - 12*b^2)*cos(5*d*x + 5*c) + (5*a*b - 12*b^2)*cos(3*d*x + 3*c) - (3*a*b
 - 4*b^2)*cos(d*x + c))*sin(8*d*x + 8*c) + (3*a*b - 4*b^2 - 4*(3*a*b - 4*b^
```

```
2)*cos(6*d*x + 6*c) - 2*(24*a^2 - 41*a*b + 12*b^2)*cos(4*d*x + 4*c) - 4*(3*
a*b - 4*b^2)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) - 4*((5*a*b - 12*b^2)*cos(5
*d*x + 5*c) - (5*a*b - 12*b^2)*cos(3*d*x + 3*c) + (3*a*b - 4*b^2)*cos(d*x +
 c))*sin(6*d*x + 6*c) - (5*a*b - 12*b^2 - 2*(40*a^2 - 111*a*b + 36*b^2)*cos
(4*d*x + 4*c) - 4*(5*a*b - 12*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*(
(40*a^2 - 111*a*b + 36*b^2)*cos(3*d*x + 3*c) - (24*a^2 - 41*a*b + 12*b^2)*c
os(d*x + c))*sin(4*d*x + 4*c) + (5*a*b - 12*b^2 - 4*(5*a*b - 12*b^2)*cos(2*
d*x + 2*c))*sin(3*d*x + 3*c) - (3*a*b - 4*b^2)*sin(d*x + c))/(a*b^3 - b^4 +
 (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos(6*d*x + 6*c)^2 + 4
*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x + 4*c)^2 + 16*(a*b^3
 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8*c)^2 + 16*(a*b^3 -
 b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*si
n(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c)*sin(2
*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a*b^3 - b^4 - 4*(a*b
^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4
*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^3 - b^4 -
 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*
d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4 - 4*(8*a^2*b
^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^3 - b^4)
*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(8*d*
x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 -
 b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (a*b*sin(7*d*x + 7*c) - 5*a
*b*sin(5*d*x + 5*c) - 5*a*b*sin(3*d*x + 3*c) + a*b*sin(d*x + c))*sin(8*d*x
+ 8*c) + 2*(2*a*b*sin(6*d*x + 6*c) + 2*a*b*sin(2*d*x + 2*c) + (8*a^2 - 3*a*
b)*sin(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(5*a*b*sin(5*d*x + 5*c) + 5*a*b*s
in(3*d*x + 3*c) - a*b*sin(d*x + c))*sin(6*d*x + 6*c) - 10*(2*a*b*sin(2*d*x
+ 2*c) + (8*a^2 - 3*a*b)*sin(4*d*x + 4*c))*sin(5*d*x + 5*c) - 2*(5*(8*a^2 -
 3*a*b)*sin(3*d*x + 3*c) - (8*a^2 - 3*a*b)*sin(d*x + c))*sin(4*d*x + 4*c))/
((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2
 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*
(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 1
6*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3
 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*
b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(
6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b
^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*
a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*
x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 +
 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x +
4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^
4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c
) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)
*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 15.87, size = 3612, normalized size = 17.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] ((a*cos(c + d*x)^3)/(4*b*(a - b)) - (a*cos(c + d*x))/(2*b*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((1024*a*b^6 - 1536*
a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*(-(9*
a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 -
29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(
256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(
a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*
```

```
b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos
(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*
(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a
*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*1i -
(((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2))
+ (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 +
16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8
 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*
b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^
6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^
3*b^7)))^(1/2) - (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a
*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b
^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a
^3*b^7)))^(1/2)*1i)/((((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 -
 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7
)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b
^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3
*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/
2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 -
b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b +
9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1
/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 -
 b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (((1024*a*b^6 - 1536*a^2*b^5 + 512*a
^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) + (cos(c + d*x)*(-(9*a^2*(a*b^7)^(1/
2) + 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^
(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512
*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) +
24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2)
)/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a
*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*(-(9*a^2*(a*b^7)^(1/2) +
 24*b^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2
))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (64*a*b^2 - 84*a^2
*b + 27*a^3)/(32*(b^4 - 2*a*b^3 + a^2*b^2))))*(-(9*a^2*(a*b^7)^(1/2) + 24*b
^2*(a*b^7)^(1/2) - 15*a*b^5 + 16*b^6 + 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(2
56*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*2i)/d - (atan(((((1024*a*
b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c +
 d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*
a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)
))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*
((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^
4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/
2) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*
((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^
4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/
2)*1i - (((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^
2*b^2)) + (cos(c + d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a
*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*
a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2
 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 -
 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^
8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2
 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 -
 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^
8 + a^3*b^7)))^(1/2)*1i)/((((1024*a*b^6 - 1536*a^2*b^5 + 512*a^3*b^4)/(64*(
b^4 - 2*a*b^3 + a^2*b^2)) - (cos(c + d*x)*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a
*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3
*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256
*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^
(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9
```

```
 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (cos(c + d*x)*(16*a*b^2 - 23*a^2*b
 + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) + 24*b^2*(a*b^7)^
(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256*(3*a*b^9
 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) + (((1024*a*b^6 - 1536*a^2*b^5 + 512
*a^3*b^4)/(64*(b^4 - 2*a*b^3 + a^2*b^2)) + (cos(c + d*x)*((9*a^2*(a*b^7)^(1
/2) + 24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)
^(1/2))/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*(256*a*b^6 - 51
2*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) +
24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2)
)/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (cos(c + d*x)*(16*a
*b^2 - 23*a^2*b + 9*a^3))/(4*(a^2 - 2*a*b + b^2)))*((9*a^2*(a*b^7)^(1/2) +
24*b^2*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2)
)/(256*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2) - (64*a*b^2 - 84*a^2*
b + 27*a^3)/(32*(b^4 - 2*a*b^3 + a^2*b^2))))*((9*a^2*(a*b^7)^(1/2) + 24*b^2
*(a*b^7)^(1/2) + 15*a*b^5 - 16*b^6 - 3*a^2*b^4 - 29*a*b*(a*b^7)^(1/2))/(256
*(3*a*b^9 - b^10 - 3*a^2*b^8 + a^3*b^7)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.214 $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=217

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+\right.}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c\right.}$$

```
[Out] -1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)+1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(a^(1/2)-2
*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cos
(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1/2))/b^(5/4)/d/a^(1/2)/(a^(
1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.26, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1205, 1166, 205, 208}

$$\frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+dx)+\right.}{4bd(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]
]])/(8*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(3/2)*b^(5/4)*d) + ((Sqrt[a] + 2*Sqrt[b]
)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*Sqrt[a]*(Sqrt
[a] + Sqrt[b])^(3/2)*b^(5/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x]^2))
/(4*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2a(a-3b)+2abx^2}{a-b+2bx^2-bx^4}\right.}{8a(a-b}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} - \frac{\left(\sqrt{a}-2\sqrt{b}\right)\text{Subst}\left(\int\right.}{8\sqrt{a}\,\left(\right.}$$

$$= \frac{\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{5/4}d} + \frac{\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{5/4}d} - \frac{}{4(a-b}$$

Mathematica [C] time = 0.66, size = 469, normalized size = 2.16

$$-\frac{\frac{32\cos(c+dx)(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6b\tan^{-1}\left(\frac{\sin(c+}{\cos(c+dx}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 8*a*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^2 + 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (4*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (11*I)*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 + 8*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
4 - 22*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (4*I)*a*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^4 + (11*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#
1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1
^7) & ])/((a - b)*b*d)
```

fricas [B] time = 0.82, size = 2507, normalized size = 11.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^
3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3
 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64
*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*
b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4
 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(
a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*
b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 +
15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 -
 24*a*b^4)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4
 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^
5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b -
4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) + ((a*b^2 - b^3)*d
*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3
)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^
3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 -
20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/(
(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log((a^3 - 9*a^2*b + 28*a*b
^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^
3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*
b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (
a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 +
 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b
^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^
10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 -
 a*b^5)*d^2))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*
x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) + a^2 - a*b - 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b^3)*cos(d*x + c) - (2*(a^4*b^5 -
3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a
*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9
 - 6*a^2*b^10 + a*b^11)*d^4)) - (a^4*b - 8*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)
*d)*sqrt(((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*
b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20
*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + a^2 - a*b - 4*b^2)/((a
^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))) - ((a*b^2 - b^3)*d*cos(d*x +
 c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(
-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a
^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8
 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 -
 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d^2))*log(-(a^3 - 9*a^2*b + 28*a*b^2 - 32*b
^3)*cos(d*x + c) - (2*(a^4*b^5 - 3*a^3*b^6 + 3*a^2*b^7 - a*b^8)*d^3*sqrt((a
^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7*b^5 - 6*a^6*b^6 + 15*
a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^11)*d^4)) + (a^4*b - 8
*a^3*b^2 + 23*a^2*b^3 - 24*a*b^4)*d)*sqrt(-((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^
4 - a*b^5)*d^2*sqrt((a^4 - 10*a^3*b + 41*a^2*b^2 - 80*a*b^3 + 64*b^4)/((a^7
*b^5 - 6*a^6*b^6 + 15*a^5*b^7 - 20*a^4*b^8 + 15*a^3*b^9 - 6*a^2*b^10 + a*b^
11)*d^4)) - a^2 + a*b + 4*b^2)/((a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d
^2))) - 4*(a + b)*cos(d*x + c))/((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2
- b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-57,-84]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[27,61]2/d*(((1-cos(c+d*x))/
(1+cos(c+d*x)))^3*a-2*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*b+3*((1-cos(c+d*x))
/(1+cos(c+d*x)))^2*a-8*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+3*(1-cos(c+d*x))
/(1+cos(c+d*x))*a+2*(1-cos(c+d*x))/(1+cos(c+d*x))*b+a)/(-4*a*b+4*b^2)/(((1-
cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a+6*((1
-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+4*(
1-cos(c+d*x))/(1+cos(c+d*x))*a+a)+2/d/(-4*a*b+4*b^2)*2/d*(-1/4*(c+d*x)+(-2*
a^4*b+16*a^3*b^2+9*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+2*a^3*a*b-3*a^3*sqrt
(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-30*a^2*b^3-30*a^2*b^2*sqrt(a^2-a*b+sqrt
(a*b)*(a-b))-16*a^2*b*a*b+6*a^2*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+1
6*a*b^4+21*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+30*a*b^2*a*b+a*b^2*sqrt(a*b)
*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+4*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-16*b^3*a
*b)*abs(a-b)/(24*a^5*b-96*a^4*b^2+112*a^3*b^3-32*a^2*b^4-8*a*b^5)*(atan(tan
(c+d*x)/sqrt(-(8*a+sqrt(8*a*8*a+4*(-4*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor(
(c+d*x)/pi+1/2))-(-2*a^4*b+16*a^3*b^2-9*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)
)+2*a^3*a*b-3*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-30*a^2*b^3+30*a^
2*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-16*a^2*b*a*b+6*a^2*b*sqrt(a*b)*sqrt(a^
2-a*b+sqrt(a*b)*(-a+b))+16*a*b^4-21*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+30
*a*b^2*a*b+a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-4*b^4*sqrt(a^2-a*
b+sqrt(a*b)*(-a+b))-16*b^3*a*b)*abs(a-b)/(24*a^5*b-96*a^4*b^2+112*a^3*b^3-3
2*a^2*b^4-8*a*b^5)*(atan(tan(c+d*x)/sqrt(-(8*a-sqrt(8*a*8*a+4*(-4*a+4*b)*4*
a))/2/(-4*a+4*b)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [B] time = 0.30, size = 440, normalized size = 2.03

$$-\frac{\cos^3(dx+c)}{4d\left(b\left(\cos^4(dx+c)\right)-2b\left(\cos^2(dx+c)\right)-a+b\right)(a-b)}+\frac{a\cos(dx+c)}{4db\left(b\left(\cos^4(dx+c)\right)-2b\left(\cos^2(dx+c)\right)-a+b\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/4/d*a/b/(
b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+1/4/d/(b*cos(d*x+c)^4
-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+1/8/d/(a-b)/(((a*b)^(1/2)+b)*b)^(1/
2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/4/d/(a-b)/(a*b)^(1/2)/
(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b
+1/8/d*a/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(
((a*b)^(1/2)+b)*b)^(1/2))-1/8/d/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(
d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/4/d/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)-b
)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+1/8/d*a/(a-b)/(
a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x +
2*c))*cos(3*d*x + 3*c) + 2*((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)
*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^
4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3
*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d
*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d
- 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*
cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) +
 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4
*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b
^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (
8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d
*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate
(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(d*x
 + c) + 4*(4*a*b - 11*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d*x
+ c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) + (4*a*b - 11*b^2)*sin(5*d*
x + 5*c) - (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^2*s
in(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c)
)*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) - (4*a*b - 11*b^2)*sin(5*d*x + 5*c
) + (4*a*b - 11*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) - 2*((32*a^2 - 100*
a*b + 33*b^2)*sin(4*d*x + 4*c) + 2*(4*a*b - 11*b^2)*sin(2*d*x + 2*c))*cos(5
*d*x + 5*c) - 2*((32*a^2 - 100*a*b + 33*b^2)*sin(3*d*x + 3*c) + (8*a*b - 3*
b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x +
 c) + (4*a*b - 11*b^2)*cos(5*d*x + 5*c) - (4*a*b - 11*b^2)*cos(3*d*x + 3*c)
)*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^2
 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x +
c) - (4*a*b - 11*b^2)*cos(5*d*x + 5*c) + (4*a*b - 11*b^2)*cos(3*d*x + 3*c))
*sin(6*d*x + 6*c) - (4*a*b - 11*b^2 - 2*(32*a^2 - 100*a*b + 33*b^2)*cos(4*d
*x + 4*c) - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((32*
a^2 - 100*a*b + 33*b^2)*cos(3*d*x + 3*c) + (8*a*b - 3*b^2)*cos(d*x + c))*si
n(4*d*x + 4*c) + (4*a*b - 11*b^2 - 4*(4*a*b - 11*b^2)*cos(2*d*x + 2*c))*sin
(3*d*x + 3*c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3
- b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*c
os(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin
(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a
^2*b^2 + 57*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 +
3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c
)^2 + 2*(a*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11
*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*
d*x + 8*c) - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x +
4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 -
11*a*b^3 + 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d
*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^
4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x)
```

```
- (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c)
 - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x +
6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d*x
 + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b + b
^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(4*
d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((32*a^
2 - 4*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*sin(4*d
*x + 4*c))/((a*b^3 - b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d
*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4
*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x +
 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2
 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^
4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c
)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 -
b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*
c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c)
 + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)
*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 -
 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*c
os(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a
*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a
*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 16.63, size = 3839, normalized size = 17.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (cos(c + d*x)^3/(4*(a - b)) - (cos(c + d*x)*(a + b))/(4*b*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((768*a*b^4 - 1024*a
^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a^2*(a^3*
b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a
^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(25
6*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*
b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a
^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (
cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^
3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*
(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*1
i - (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) + (
cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2
*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*
a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a^2
*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 -
2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*b^5 - a
^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4
*b^6 - a^5*b^5)))^(1/2)*1i)/((((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64
*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b
^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*
b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 2
56*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b
^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*
b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (cos(c + d*x)*(a^2*b - 3*a
*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3
*b^5)^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^
```

```
2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) - (a - 4*b)/(32*(a^2 - 2*a
*b + b^2)) + (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b +
b^2)) + (cos(c + d*x)*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*
b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7
+ 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*
(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*a*
b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7
+ 3*a^4*b^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(
4*(a^2 - 2*a*b + b^2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) - 4*
a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^
7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)))*(-(a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)
^(1/2) - 4*a*b^5 - a^2*b^4 + a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8
 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*2i)/d - (atan(((((768*a*b^4 - 1
024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a^2*(
a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*
b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(a
^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b
*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
+ (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(
a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*
b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)
*1i - (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)) +
 (cos(c + d*x)*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^
2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*
b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2
*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^2
*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*b
^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2 -
2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 + a^
2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^4*
b^6 - a^5*b^5)))^(1/2)*1i)/((((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*
(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5
)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^
8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256
*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)
^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8
 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) + (cos(c + d*x)*(a^2*b - 3*a*b^
2 + 4*b^3))/(4*(a^2 - 2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5
)^(1/2) + 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^
8 - 3*a^3*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2) - (a - 4*b)/(32*(a^2 - 2*a*b +
 b^2)) + (((768*a*b^4 - 1024*a^2*b^3 + 256*a^3*b^2)/(64*(a^2 - 2*a*b + b^2)
) + (cos(c + d*x)*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 +
 a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a
^4*b^6 - a^5*b^5)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2
- 2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 +
a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a^
4*b^6 - a^5*b^5)))^(1/2) - (cos(c + d*x)*(a^2*b - 3*a*b^2 + 4*b^3))/(4*(a^2
 - 2*a*b + b^2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) + 4*a*b^5 +
 a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3*b^7 + 3*a
^4*b^6 - a^5*b^5)))^(1/2)))*((a^2*(a^3*b^5)^(1/2) + 8*b^2*(a^3*b^5)^(1/2) +
 4*a*b^5 + a^2*b^4 - a^3*b^3 - 5*a*b*(a^3*b^5)^(1/2))/(256*(a^2*b^8 - 3*a^3
*b^7 + 3*a^4*b^6 - a^5*b^5)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.215 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=186

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

```
[Out] -1/4*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)
^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/b^(3/4)/d/a^(1/2
)/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^
(1/2))/b^(3/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.18, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\,b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a]
 - Sqrt[b])^(3/2)*b^(3/4)*d) + ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a]
+ Sqrt[b]]]/(8*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/4)*d) - (Cos[c + d*x]
*(2 - Cos[c + d*x]^2))/(4*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c +
 d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
```

```
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{-4ab+2abx^2}{a-b+2bx^2-bx^4}\,dx \ldots\right)}{8a(a-b)\ldots}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}+b-bx^2}\ldots\right)}{8\sqrt{a}\left(\sqrt{a}-\ldots\right)}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/4}d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/4}d} - \frac{\cos(c+dx)\ldots}{4(a-b)d\left(a-b+2b\,\text{c}\ldots\right)}$$

Mathematica [C] time = 0.34, size = 345, normalized size = 1.85

$$\frac{16(\cos(3(c+dx))-5\cos(c+dx))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{2\#1^6\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1\ldots}\right)}{\ldots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)]) - I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4
*b*#1^6 + b*#1^8 & , (-2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + I*Log[1
 - 2*Cos[c + d*x]*#1 + #1^2] + 14*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*
#1^2 - (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 14*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 + (7*I)*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4
+ 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])
/(32*(a - b)*d)
```

fricas [B] time = 0.69, size = 2049, normalized size = 11.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*cos(d*x + c)^3 - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*c
os(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^
3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^
5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^5*b^
2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3
 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^
4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)
*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b - 3*a^3*b^2
 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2
)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^
5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4
*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log((a + 3*b)*cos(d*x + c) - ((a^
5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7
*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9
)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b
^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*
a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2))) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b -
b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 +
3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/(
(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x + c) -
 ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4)) - 2*(a^2*b + 3*a*b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5
 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 3*a + b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(
a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^
4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a -
 b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-(a + 3*b)*cos(d*x +
 c) - ((a^5*b^2 - 2*a^4*b^3 + 2*a^2*b^5 - a*b^6)*d^3*sqrt((a^2 + 6*a*b + 9*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
^8 + a*b^9)*d^4)) + 2*(a^2*b + 3*a*b^2)*d)*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2
*b^3 - a*b^4)*d^2*sqrt((a^2 + 6*a*b + 9*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5
*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) - 3*a - b)/((a^4*
b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))) - 8*cos(d*x + c))/((a*b - b^2)*d*
cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

giac [B] time = 1.04, size = 605, normalized size = 3.25

$$-\frac{\frac{\cos(dx+c)^3}{d} - \frac{2\cos(dx+c)}{d}}{4\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 - a + b\right)(a-b)} + \frac{\left(\left(a^2b - 2\,ab^2 + b^3\right)\sqrt{ab}\,\sqrt{-b^2 + \sqrt{ab}\,b}\,d^4 - 2\left(a^2b - ab^2\right)\sqrt{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(cos(d*x + c)^3/d - 2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x
 + c)^2 - a + b)*(a - b)) + 1/8*((a^2*b - 2*a*b^2 + b^3)*sqrt(a*b)*sqrt(-b^
2 + sqrt(a*b)*b)*d^4 - 2*(a^2*b - a*b^2)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(-
a*d^2 + b*d^2) + (a*d^2 - b*d^2)^2*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*a)*ar
ctan(cos(d*x + c)/(d*sqrt(-(a*b*d^2 - b^2*d^2 + sqrt((a*b*d^2 - b^2*d^2)^2
```

```
+ (a*b*d^4 - b^2*d^4)*(a^2 - 2*a*b + b^2)))/(a*b*d^4 - b^2*d^4))))/((a^4*b
- 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^3*abs(-a*d^2 + b*d^2)*abs(b)) - 1/8*((a^
2*b - 2*a*b^2 + b^3)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 + 2*(a^2*b - a*
b^2)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(-a*d^2 + b*d^2) + (a*d^2 - b*d^2)^2*s
qrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*a)*arctan(cos(d*x + c)/(d*sqrt(-(a*b*d^2
- b^2*d^2 - sqrt((a*b*d^2 - b^2*d^2)^2 + (a*b*d^4 - b^2*d^4)*(a^2 - 2*a*b +
 b^2)))/(a*b*d^4 - b^2*d^4))))/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^3
*abs(-a*d^2 + b*d^2)*abs(b))
```

maple [A] time = 0.26, size = 213, normalized size = 1.15

$$\frac{\sqrt{ab}\,\cos(dx+c)}{8da\left(\sqrt{ab}+b\right)\left(-b\left(\cos^2(dx+c)\right)+\sqrt{ab}+b\right)}+\frac{\sqrt{ab}\,\operatorname{arctanh}\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}+b\right)b}}\right)}{8da\left(\sqrt{ab}+b\right)\sqrt{\left(\sqrt{ab}+b\right)b}}-\frac{\sqrt{ab}\,\cos(dx+}{8da\left(\sqrt{ab}-b\right)\left(b\left(\cos^2(dx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] 1/8/d*(a*b)^(1/2)/a*cos(d*x+c)/((a*b)^(1/2)+b)/(-b*cos(d*x+c)^2+(a*b)^(1/2)
+b)+1/8/d*(a*b)^(1/2)/a/((a*b)^(1/2)+b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(c
os(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d*(a*b)^(1/2)/a*cos(d*x+c)/((a*b
)^(1/2)-b)/(b*cos(d*x+c)^2-b+(a*b)^(1/2))-1/8/d*(a*b)^(1/2)/a/((a*b)^(1/2)-
b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 15.60, size = 3060, normalized size = 16.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (cos(c + d*x)^3/(4*(a - b)) - cos(c + d*x)/(2*(a - b)))/(d*(a - b + 2*b*cos
(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((512*a*b^4 - 512*a^2*b^3)/(64*(
a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/
2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^
(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a
*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 -
 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^3))/(4
*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 +
3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*1i - ((
(512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*((a*(a^3
*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^
3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4
))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b
^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) -
(cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) +
 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4
*b^4 - a^5*b^3)))^(1/2)*1i)/(b/(32*(a^2 - 2*a*b + b^2)) + (((512*a*b^4 - 51
2*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*((a*(a^3*b^3)^(1/2) + 3
```

```
*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b
^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*
a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)
/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(
a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^
(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)
))^(1/2) + (((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c +
d*x)*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a
^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5
+ 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3
)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^
3)))^(1/2) - (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*((a*(a^3
*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^
3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)))*((a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^
(1/2) + a*b^3 + 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)
))^(1/2)*2i)/d - (atan(((((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)
) - (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^
2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6
- 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2)
 + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a
^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b +
b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256
*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*1i - (((512*a*b^4 - 51
2*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) +
3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*
b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2
*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^
2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) - (cos(c + d*x)
*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^
3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b
^3)))^(1/2)*1i)/(b/(32*(a^2 - 2*a*b + b^2)) + (((512*a*b^4 - 512*a^2*b^3)/(
64*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3
)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^
3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^3*b^4))/(4*(a^2 - 2*a*b + b^2))
)*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2
*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2) + (cos(c + d*x)*(a*b^2 + b^
3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a
*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)
+ (((512*a*b^4 - 512*a^2*b^3)/(64*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(-(a
*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 -
 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1/2)*(256*a*b^6 - 512*a^2*b^5 + 256*a^
3*b^4))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2)
 - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1
/2) - (cos(c + d*x)*(a*b^2 + b^3))/(4*(a^2 - 2*a*b + b^2)))*(-(a*(a^3*b^3)^
(1/2) + 3*b*(a^3*b^3)^(1/2) - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5
+ 3*a^4*b^4 - a^5*b^3)))^(1/2)))*(-(a*(a^3*b^3)^(1/2) + 3*b*(a^3*b^3)^(1/2)
 - a*b^3 - 3*a^2*b^2)/(256*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)))^(1
/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.216 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=221

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+\right.}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\right.}$$

```
[Out] -1/4*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)-1/8*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a^(1/2)
-2*b^(1/2))/a^(3/2)/b^(1/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctanh(b^(1/4)*c
os(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(3*a^(1/2)+2*b^(1/2))/a^(3/2)/b^(1/4)/d/
(a^(1/2)+b^(1/2))^(3/2)
```

Rubi [A] time = 0.27, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.227, Rules used = {3215, 1092, 1166, 205, 208}

$$-\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\cos(c+dx)\left(a-b\cos^2(c+\right.}{4ad(a-b)\left(a-b\cos^4(c+dx)+2b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -((3*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(1/4)*d) - ((3*Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(3/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*b^(1/4)*d) - (Cos[c + d*x]*(a + b - b*Cos[c + d*x
]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-2(4(a-}{a-b+2bx^2}\right.}{8a}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} + \frac{\left(\left(3\sqrt{a}-2\sqrt{b}\right)\sqrt{b}\right)\text{Subs}}{8a^{3/}}$$

$$= -\frac{\left(3\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} - \frac{\left(3\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt[4]{b}\,d} - \frac{}{4a(a-}$$

Mathematica [C] time = 0.45, size = 469, normalized size = 2.12

$$-\frac{\frac{32\cos(c+dx)(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + i\text{RootSum}\left[\#1^8 b - 4\#1^6 b - 16\#1^4 a + 6\#1^4 b - 4\#1^2 b + b\&, \frac{2\#1^6 b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/32*((32*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 +
 6*b*#1^4 - 4*b*#1^6 + b*#1^8 & , (-2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)] + I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 24*a*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^
2 - (12*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b*Log[1 - 2*Cos
[c + d*x]*#1 + #1^2]*#1^2 - 24*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#
1^4 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (12*I)*a*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - I*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*
#1^7) & ])/(a*(a - b)*d)
```

fricas [B] time = 0.81, size = 2269, normalized size = 10.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*b*cos(d*x + c)^3 - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b -
a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b
+ 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 1
5*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log((8
1*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3
 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
(27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*
b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4
*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) + ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b +
 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^
2 - a^3*b^3)*d^2))*log((81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*
b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b
+ 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^
4*b^6 + a^3*b^7)*d^4)) + (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*
a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b -
6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4
)) - 15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)))
 + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 -
 (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^
2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-(81*a^2 - 81*a*b + 20*b^2)
*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b^3 - 5*a^4*b^4 + a^3*b^5)*
d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a
^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - (27*a^4 - 24*a^3*b + 5*a
^2*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 -
90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5
 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 15*a^2 - 15*a*b + 4*b^2)/((a^6 - 3*a^5*b +
3*a^4*b^2 - a^3*b^3)*d^2))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b
- a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b
 + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8
*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) -
15*a^2 + 15*a*b - 4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-
(81*a^2 - 81*a*b + 20*b^2)*cos(d*x + c) + (2*(2*a^7*b - 7*a^6*b^2 + 9*a^5*b
^3 - 5*a^4*b^4 + a^3*b^5)*d^3*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a
^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))
+ (27*a^4 - 24*a^3*b + 5*a^2*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3
*b^3)*d^2*sqrt((81*a^2 - 90*a*b + 25*b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3
- 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - 15*a^2 + 15*a*b -
4*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))) - 4*(a + b)*cos(d*x +
c))/((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2
- (a^3 - 2*a^2*b + a*b^2)*d)
```

giac [B] time = 0.89, size = 693, normalized size = 3.14

$$-\frac{\frac{b\cos(dx+c)^3}{d} - \frac{a\cos(dx+c)}{d} - \frac{b\cos(dx+c)}{d}}{4\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 - a + b\right)\left(a^2 - ab\right)} + \frac{\left(\left(3\,a^4b - 8\,a^3b^2 + 7\,a^2b^3 - 2\,ab^4\right)\sqrt{-b^2 + \sqrt{ab}\,b}\,d^4 - \right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/4*(b*cos(d*x + c)^3/d - a*cos(d*x + c)/d - b*cos(d*x + c)/d)/((b*cos(d*x
 + c)^4 - 2*b*cos(d*x + c)^2 - a + b)*(a^2 - a*b)) + 1/8*((3*a^4*b - 8*a^3*
b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^4 - (3*a^2 - 4*a*b +
b^2)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(-a^2*d^2 + a*b*d^2) + (a^2*
d^2 - a*b*d^2)^2*sqrt(-b^2 + sqrt(a*b)*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(
a^2*b*d^2 - a*b^2*d^2 + sqrt((a^2*b*d^2 - a*b^2*d^2)^2 + (a^2*b*d^4 - a*b^2
*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a*b^2*d^4))))/((a^4 - 3*a^3*b
+ 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d^2 + a*b*d^2)*abs(b)) - 1/8*((
3*a^4*b - 8*a^3*b^2 + 7*a^2*b^3 - 2*a*b^4)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 + (
3*a^2 - 4*a*b + b^2)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(-a^2*d^2 +
a*b*d^2) + (a^2*d^2 - a*b*d^2)^2*sqrt(-b^2 - sqrt(a*b)*b)*b)*arctan(cos(d*x
 + c)/(d*sqrt(-(a^2*b*d^2 - a*b^2*d^2 - sqrt((a^2*b*d^2 - a*b^2*d^2)^2 + (a
^2*b*d^4 - a*b^2*d^4)*(a^3 - 2*a^2*b + a*b^2)))/(a^2*b*d^4 - a*b^2*d^4))))/
((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*sqrt(a*b)*d^3*abs(-a^2*d^2 + a*b*d^2)*
abs(b))
```

maple [B] time = 0.52, size = 488, normalized size = 2.21

$$-\frac{\cos(dx+c)}{8da(a-b)\left(\cos^2(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}-\frac{\cos(dx+c)}{8d\sqrt{ab}\,(a-b)\left(\cos^2(dx+c)+\frac{\sqrt{ab}}{b}-1\right)}-\frac{b\arctan\left(\frac{\cos(dx+c)b}{\sqrt{\left(\sqrt{ab}-b\right)b}}\right)}{8da(a-b)\sqrt{\left(\sqrt{ab}-b\right)b}}-\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/8/d/a/(a-b)*cos(d*x+c)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)-1/8/d/(a*b)^(1/2)/
(a-b)*cos(d*x+c)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)-1/8/d*b/a/(a-b)/(((a*b)^(1/
2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-3/8/d/(a-b)/(
a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b
)^(1/2))*b+1/4/d*b^2/(a*b)^(1/2)/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(c
os(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/8/d/a/(a-b)*cos(d*x+c)/(cos(d*x+c)
^2-1-(a*b)^(1/2)/b)+1/8/d/(a*b)^(1/2)/(a-b)*cos(d*x+c)/(cos(d*x+c)^2-1-(a*b
)^(1/2)/b)+1/8/d*b/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(
((a*b)^(1/2)+b)*b)^(1/2))-3/8/d/(a-b)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)
*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b+1/4/d*b^2/(a*b)^(1/2)/a/
(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1
/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*b^2*cos(2*d*x + 2*c)*cos(d*x + c) + 4*b^2*sin(2*d*x + 2*c)*sin(d*x +
 c) - b^2*cos(d*x + c) - 4*(4*a*b + b^2)*sin(3*d*x + 3*c)*sin(2*d*x + 2*c)
- (b^2*cos(7*d*x + 7*c) + b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c)
 - (4*a*b + b^2)*cos(3*d*x + 3*c))*cos(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*
c) + 4*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*cos
(7*d*x + 7*c) + 4*(b^2*cos(d*x + c) - (4*a*b + b^2)*cos(5*d*x + 5*c) - (4*a
*b + b^2)*cos(3*d*x + 3*c))*cos(6*d*x + 6*c) + (4*a*b + b^2 - 2*(32*a^2 - 4
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 4*(4*a*b + b^2)*cos(2*d*x + 2*c))*cos(5*d*
x + 5*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*c
os(d*x + c))*cos(4*d*x + 4*c) + (4*a*b + b^2 - 4*(4*a*b + b^2)*cos(2*d*x +
2*c))*cos(3*d*x + 3*c) + 2*((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^
```

```
2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 -
9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 +
 (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x
+ 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)
^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2
*d*x + 2*c) + (a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*
c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a
*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*co
s(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11
*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d
)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b -
 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x
 + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*
x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integr
ate(-1/2*(4*b^2*cos(d*x + c)*sin(2*d*x + 2*c) - 4*b^2*cos(2*d*x + 2*c)*sin(
d*x + c) - 4*(12*a*b - 5*b^2)*cos(3*d*x + 3*c)*sin(2*d*x + 2*c) + b^2*sin(d
*x + c) - (b^2*sin(7*d*x + 7*c) - b^2*sin(d*x + c) - (12*a*b - 5*b^2)*sin(5
*d*x + 5*c) + (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(8*d*x + 8*c) - 2*(2*b^
2*sin(6*d*x + 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4
*c))*cos(7*d*x + 7*c) - 4*(b^2*sin(d*x + c) + (12*a*b - 5*b^2)*sin(5*d*x +
5*c) - (12*a*b - 5*b^2)*sin(3*d*x + 3*c))*cos(6*d*x + 6*c) + 2*((96*a^2 - 7
6*a*b + 15*b^2)*sin(4*d*x + 4*c) + 2*(12*a*b - 5*b^2)*sin(2*d*x + 2*c))*cos
(5*d*x + 5*c) + 2*((96*a^2 - 76*a*b + 15*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3
*b^2)*sin(d*x + c))*cos(4*d*x + 4*c) + (b^2*cos(7*d*x + 7*c) - b^2*cos(d*x
+ c) - (12*a*b - 5*b^2)*cos(5*d*x + 5*c) + (12*a*b - 5*b^2)*cos(3*d*x + 3*c
))*sin(8*d*x + 8*c) + (4*b^2*cos(6*d*x + 6*c) + 4*b^2*cos(2*d*x + 2*c) - b^
2 + 2*(8*a*b - 3*b^2)*cos(4*d*x + 4*c))*sin(7*d*x + 7*c) + 4*(b^2*cos(d*x +
 c) + (12*a*b - 5*b^2)*cos(5*d*x + 5*c) - (12*a*b - 5*b^2)*cos(3*d*x + 3*c)
)*sin(6*d*x + 6*c) + (12*a*b - 5*b^2 - 2*(96*a^2 - 76*a*b + 15*b^2)*cos(4*d
*x + 4*c) - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((96*
a^2 - 76*a*b + 15*b^2)*cos(3*d*x + 3*c) - (8*a*b - 3*b^2)*cos(d*x + c))*sin
(4*d*x + 4*c) - (12*a*b - 5*b^2 - 4*(12*a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(
3*d*x + 3*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*c)^2 + 16*
(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 -
 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c)^2 + (a
^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)^
2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c)^2 + 16*(
8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2
*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^2 - a*b^3)
*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4
*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - a*b^3
- 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - a*b^3)
*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3 - 4
*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a
^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d*x + 6*c)
+ (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*si
n(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) - (b^2*sin(7*d*x + 7*c) + b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*
c) - (4*a*b + b^2)*sin(3*d*x + 3*c))*sin(8*d*x + 8*c) + 2*(2*b^2*sin(6*d*x
+ 6*c) + 2*b^2*sin(2*d*x + 2*c) + (8*a*b - 3*b^2)*sin(4*d*x + 4*c))*sin(7*d
*x + 7*c) + 4*(b^2*sin(d*x + c) - (4*a*b + b^2)*sin(5*d*x + 5*c) - (4*a*b +
 b^2)*sin(3*d*x + 3*c))*sin(6*d*x + 6*c) - 2*((32*a^2 - 4*a*b - 3*b^2)*sin(
4*d*x + 4*c) + 2*(4*a*b + b^2)*sin(2*d*x + 2*c))*sin(5*d*x + 5*c) - 2*((32*
a^2 - 4*a*b - 3*b^2)*sin(3*d*x + 3*c) - (8*a*b - 3*b^2)*sin(d*x + c))*sin(4
*d*x + 4*c))/((a^2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)
*d*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos
```

```
(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*
b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(
64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3
*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^
2 - a*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) +
(a^2*b^2 - a*b^3)*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*
b - 11*a^2*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*
d*x + 2*c) - (a^2*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2
*b^2 + 3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)
 - (a^2*b^2 - a*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*cos(2*d*x + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x +
4*c) - 4*(2*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(
a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 16.79, size = 3507, normalized size = 15.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] ((b*cos(c + d*x)^3)/(4*a*(a - b)) - (cos(c + d*x)*(a + b))/(4*a*(a - b)))/(
d*(a - b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) + (atan(((((256*a^3*b^5
- 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) - (cos(c + d*x
)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2)
 + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^
7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*
a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
 a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b^5 - 11*a*b^4
 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/
2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*
a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b
^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b
^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a
^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)
)/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9
*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a
^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2
*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4
*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/
2)*1i)/((9*a*b^3 - 4*b^4)/(32*(a^5 - 2*a^4*b + a^3*b^2)) + (((256*a^3*b^5 -
 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) - (cos(c + d*x)
*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2)
+ 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7
*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a
*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b^5 - 11*a*b^4
+ 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2
) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a
^7*b^3 - 3*a^8*b^2)))^(1/2) + (((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/
(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 +
 256*a^5*b^4)*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b
^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4
*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^
(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b
^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3
*b + a^2*b^2)))*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3
*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)))
```

```
*(-(15*a^5*b - 9*a*(a^9*b)^(1/2) + 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b
^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i)/d + (atan(((
((256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2))
- (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a
*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-
(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)
/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*
b^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9
*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b
- a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((256*a^3*b^5 - 1024*a^4*b
^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b
^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*
b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^
8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1
/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3
*a^7*b^3 - 3*a^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3
))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^
9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*
a^8*b^2)))^(1/2)*1i)/((9*a*b^3 - 4*b^4)/(32*(a^5 - 2*a^4*b + a^3*b^2)) + ((
(256*a^3*b^5 - 1024*a^4*b^4 + 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) -
 (cos(c + d*x)*(256*a^3*b^6 - 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a*
(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a
^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(
15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/
(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (cos(c + d*x)*(4*b
^5 - 11*a*b^4 + 9*a^2*b^3))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*
a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b -
 a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2) + (((256*a^3*b^5 - 1024*a^4*b^4 +
 768*a^5*b^3)/(64*(a^5 - 2*a^4*b + a^3*b^2)) + (cos(c + d*x)*(256*a^3*b^6 -
 512*a^4*b^5 + 256*a^5*b^4)*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(
1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^
2)))^(1/2))/(4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1/2)
- 5*b*(a^9*b)^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7
*b^3 - 3*a^8*b^2)))^(1/2) - (cos(c + d*x)*(4*b^5 - 11*a*b^4 + 9*a^2*b^3))/(
4*(a^4 - 2*a^3*b + a^2*b^2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)
^(1/2) + 4*a^3*b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*
b^2)))^(1/2)))*(-(15*a^5*b + 9*a*(a^9*b)^(1/2) - 5*b*(a^9*b)^(1/2) + 4*a^3*
b^3 - 15*a^4*b^2)/(256*(a^9*b - a^6*b^4 + 3*a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i
)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.217 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=325

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}-\tanh^{-1}$$

```
[Out] -arctanh(cos(d*x+c))/a^2/d-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-b
+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/8*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^
(1/2)-b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*arctanh
(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(3/2)/d/(a^(1/2)+b^(1/2))^(3
/2)-1/2*b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^2/d/(a
^(1/2)-b^(1/2))^(1/2)+1/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/
2))^(1/2))/a^2/d/(a^(1/2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 325, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.318, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2d\sqrt{\sqrt{a}+\sqrt{b}}}+\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{8a^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\tanh^{-1}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -(b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(3/2
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqr
t[Sqrt[a] - Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c +
d*x]]/(a^2*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(8*a^(3/2)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*
Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(2*a^2*Sqrt[Sqrt[a] + Sqrt[b]]*d) -
 (b*Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a*(a - b)*d*(a - b + 2*b*Cos[c +
d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
```

```
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^2(-1+x^2)}+\frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^2}+\frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)}\right)dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^2d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^2d} -$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^2d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)} +$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}+\sqrt{b}}\,d} -$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^2\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^2d} +$$

Mathematica [C] time = 0.86, size = 600, normalized size = 1.85

$$-\frac{ib\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{10\#1^6a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-8\#1^6b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-38\#1^4a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+24\#1^4b\tan^{-1}\left(\frac{\text{si}}{\text{cos}}\right.}{}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((16*a*b*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(8*a - 3*b + 4*b*Co
s[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - 32*Log[Cos[(c + d*x)/2]] + 32*Log[S
in[(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#
1^6 + b*#1^8 & , (-10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] - (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 38*a*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^2 - 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 - (19*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (12*I)*b*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 38*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^4 + 24*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (19*I)*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (12*I)*b*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^4 + 10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 8*b*ArcT
an[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (5*I)*a*Log[1 - 2*Cos[c + d*x]*
#1 + #1^2]*#1^6 + (4*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1)
- 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b))/(32*a^2*d)
```

fricas [B] time = 1.26, size = 2711, normalized size = 8.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/16*(4*a*b*cos(d*x + c)^3 - 8*a*b*cos(d*x + c) + ((a^3*b - a^2*b^2)*d*cos
(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b
^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1
450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^1
1*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b -
47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^
3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*
b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2
 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b - 137*a^4*
b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)
/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a
^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
 a^4*b^3)*d^2))) - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2
)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*
b + 3*a^5*b^2 - a^4*b^3)*d^2))*log((625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 -
128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a
^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*
b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5
 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)
*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3
*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 -
```

```
20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^
2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) - ((a^3*b - a^2*b
^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*
b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*
a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b
 + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 35
*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*lo
g(-(625*a^3*b - 1125*a^2*b^2 + 664*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10
 - 18*a^9*b + 24*a^8*b^2 - 14*a^7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 14
50*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 2*(75*a^5*b
- 137*a^4*b^2 + 82*a^3*b^3 - 16*a^2*b^4)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b
^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4
 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a
^8*b^5 + a^7*b^6)*d^4)) + 35*a^2*b - 47*a*b^2 + 16*b^3)/((a^7 - 3*a^6*b + 3
*a^5*b^2 - a^4*b^3)*d^2))) + ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b
 - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*
a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*
b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 1
5*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*b + 47*a*b^2 - 16*b^3)/((a^
7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))*log(-(625*a^3*b - 1125*a^2*b^2 + 6
64*a*b^3 - 128*b^4)*cos(d*x + c) + ((5*a^10 - 18*a^9*b + 24*a^8*b^2 - 14*a^
7*b^3 + 3*a^6*b^4)*d^3*sqrt((625*a^4*b - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*
a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4
- 6*a^8*b^5 + a^7*b^6)*d^4)) + 2*(75*a^5*b - 137*a^4*b^2 + 82*a^3*b^3 - 16*
a^2*b^4)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*sqrt((625*a^4*b
 - 1450*a^3*b^2 + 1241*a^2*b^3 - 464*a*b^4 + 64*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 35*a^2*
b + 47*a*b^2 - 16*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2))) + 8*((
a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^
2)*log(1/2*cos(d*x + c) + 1/2) - 8*((a*b - b^2)*cos(d*x + c)^4 - 2*(a*b - b
^2)*cos(d*x + c)^2 - a^2 + 2*a*b - b^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^3
*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^
4 - 2*a^3*b + a^2*b^2)*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-89,-82]Warning, nee
d to choose a branch for the root of a polynomial with parameters. This mig
ht be wrong.The choice was done assuming [a,b]=[91,55]2/d*((((1-cos(c+d*x))
/(1+cos(c+d*x)))^3*a*b-3*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a*b+8*((1-cos(c+
d*x))/(1+cos(c+d*x)))^2*b^2-5*(1-cos(c+d*x))/(1+cos(c+d*x))*a*b-a*b)/(4*a^3
-4*a^2*b)/(((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d
*x)))^3*a+6*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c
+d*x)))^2*b+4*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)+1/4/a^2*ln(abs(1-cos(c+d*x
))/abs(1+cos(c+d*x))))+2/d/(4*a^3-4*a^2*b)*2/d*((-4*b+5*a)/4*(c+d*x)+(-2*a^
4*b-15*a^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-8*a^3*b^2+48*a^3*b*sqrt(a^2-a*b+sq
rt(a*b)*(a-b))+2*a^3*a*b+27*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+26*
a^2*b^3-31*a^2*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+8*a^2*b*a*b-78*a^2*b*sqrt(
a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-16*a*b^4-6*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*
(a-b))-26*a*b^2*a*b+39*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+16*b^3
*a*b+8*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)/(24*a^5-96*a^4
*b+112*a^3*b^2-32*a^2*b^3-8*a*b^4)*(atan(tan(c+d*x)/sqrt(-(8*a+sqrt(8*a*8*a
```

```
+4*(4*b-4*a)*4*a))/2/(4*b-4*a)))+pi*floor((c+d*x)/pi+1/2))-(-2*a^4*b+15*a^4
*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-8*a^3*b^2-48*a^3*b*sqrt(a^2-a*b+sqrt(a*b)*(
-a+b))+2*a^3*a*b+27*a^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+26*a^2*b^3
+31*a^2*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+8*a^2*b*a*b-78*a^2*b*sqrt(a*b)*s
qrt(a^2-a*b+sqrt(a*b)*(-a+b))-16*a*b^4+6*a*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b
))-26*a*b^2*a*b+39*a*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+16*b^3*a*
b+8*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(24*a^5-96*a^4*b
+112*a^3*b^2-32*a^2*b^3-8*a*b^4)*(atan(tan(c+d*x)/sqrt(-(8*a-sqrt(8*a*8*a+4
*(4*b-4*a)*4*a))/2/(4*b-4*a)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [A] time = 0.53, size = 450, normalized size = 1.38

$$\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2d\,a^2}-\frac{\ln\left(1+\cos\left(dx+c\right)\right)}{2d\,a^2}-\frac{b\left(\cos^3\left(dx+c\right)\right)}{4da\left(b\left(\cos^4\left(dx+c\right)\right)-2b\left(\cos^2\left(dx+c\right)\right)-a+b\right)\left(a-b\right)}+\frac{}{2da\left(b\left(c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] 1/2/d/a^2*ln(cos(d*x+c)-1)-1/2/d/a^2*ln(1+cos(d*x+c))-1/4/d/a*b/(b*cos(d*x+
c)^4-2*b*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)^3+1/2/d/a*b/(b*cos(d*x+c)^4-2*b
*cos(d*x+c)^2-a+b)/(a-b)*cos(d*x+c)+5/8/d*b/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/
2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/2/d/a^2*b^2/(a-b)/(((a
*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/8/d
*b^2/(a*b)^(1/2)/a/(a-b)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((
a*b)^(1/2)+b)*b)^(1/2))-5/8/d*b/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(co
s(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/2/d/a^2*b^2/(a-b)/(((a*b)^(1/2)-b)*
b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/8/d*b^2/(a*b)^(1/
2)/a/(a-b)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b
)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(4*a*b^2*cos(2*d*x + 2*c)*cos(d*x + c) - 20*a*b^2*sin(3*d*x + 3*c)*sin(
2*d*x + 2*c) + 4*a*b^2*sin(2*d*x + 2*c)*sin(d*x + c) - a*b^2*cos(d*x + c) -
 (a*b^2*cos(7*d*x + 7*c) - 5*a*b^2*cos(5*d*x + 5*c) - 5*a*b^2*cos(3*d*x + 3
*c) + a*b^2*cos(d*x + c))*cos(8*d*x + 8*c) + (4*a*b^2*cos(6*d*x + 6*c) + 4*
a*b^2*cos(2*d*x + 2*c) - a*b^2 + 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*co
s(7*d*x + 7*c) - 4*(5*a*b^2*cos(5*d*x + 5*c) + 5*a*b^2*cos(3*d*x + 3*c) - a
*b^2*cos(d*x + c))*cos(6*d*x + 6*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2 +
 2*(8*a^2*b - 3*a*b^2)*cos(4*d*x + 4*c))*cos(5*d*x + 5*c) - 2*(5*(8*a^2*b -
 3*a*b^2)*cos(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*cos(d*x + c))*cos(4*d*x +
4*c) - 5*(4*a*b^2*cos(2*d*x + 2*c) - a*b^2)*cos(3*d*x + 3*c) - 2*((a^3*b^2
- a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c)
^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(8*
d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 11
2*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a
^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2
*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^
3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^4
*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*c
os(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^4*b -
```

```
11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2
*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^4*b - 11*
a^3*b^2 + 3*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3
)*d)*cos(4*d*x + 4*c) - 4*(2*(a^3*b^2 - a^2*b^3)*d*sin(6*d*x + 6*c) + (8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)
*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))*integrate(-1/2*(4*(19*a*b^2 - 12*b^3)*cos(3*d*x + 3*c)*sin(2*d*x + 2
*c) - 4*(5*a*b^2 - 4*b^3)*cos(d*x + c)*sin(2*d*x + 2*c) + 4*(5*a*b^2 - 4*b^
3)*cos(2*d*x + 2*c)*sin(d*x + c) + ((5*a*b^2 - 4*b^3)*sin(7*d*x + 7*c) - (1
9*a*b^2 - 12*b^3)*sin(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) -
 (5*a*b^2 - 4*b^3)*sin(d*x + c))*cos(8*d*x + 8*c) + 2*(2*(5*a*b^2 - 4*b^3)*
sin(6*d*x + 6*c) + (40*a^2*b - 47*a*b^2 + 12*b^3)*sin(4*d*x + 4*c) + 2*(5*a
*b^2 - 4*b^3)*sin(2*d*x + 2*c))*cos(7*d*x + 7*c) + 4*((19*a*b^2 - 12*b^3)*s
in(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*sin(3*d*x + 3*c) + (5*a*b^2 - 4*b^3)*
sin(d*x + c))*cos(6*d*x + 6*c) - 2*((152*a^2*b - 153*a*b^2 + 36*b^3)*sin(4*
d*x + 4*c) + 2*(19*a*b^2 - 12*b^3)*sin(2*d*x + 2*c))*cos(5*d*x + 5*c) - 2*(
(152*a^2*b - 153*a*b^2 + 36*b^3)*sin(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 +
12*b^3)*sin(d*x + c))*cos(4*d*x + 4*c) - ((5*a*b^2 - 4*b^3)*cos(7*d*x + 7*c
) - (19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) + (19*a*b^2 - 12*b^3)*cos(3*d*x +
3*c) - (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(8*d*x + 8*c) + (5*a*b^2 - 4*b^3
- 4*(5*a*b^2 - 4*b^3)*cos(6*d*x + 6*c) - 2*(40*a^2*b - 47*a*b^2 + 12*b^3)*c
os(4*d*x + 4*c) - 4*(5*a*b^2 - 4*b^3)*cos(2*d*x + 2*c))*sin(7*d*x + 7*c) -
4*((19*a*b^2 - 12*b^3)*cos(5*d*x + 5*c) - (19*a*b^2 - 12*b^3)*cos(3*d*x + 3
*c) + (5*a*b^2 - 4*b^3)*cos(d*x + c))*sin(6*d*x + 6*c) - (19*a*b^2 - 12*b^3
 - 2*(152*a^2*b - 153*a*b^2 + 36*b^3)*cos(4*d*x + 4*c) - 4*(19*a*b^2 - 12*b
^3)*cos(2*d*x + 2*c))*sin(5*d*x + 5*c) + 2*((152*a^2*b - 153*a*b^2 + 36*b^3
)*cos(3*d*x + 3*c) - (40*a^2*b - 47*a*b^2 + 12*b^3)*cos(d*x + c))*sin(4*d*x
 + 4*c) + (19*a*b^2 - 12*b^3 - 4*(19*a*b^2 - 12*b^3)*cos(2*d*x + 2*c))*sin(
3*d*x + 3*c) - (5*a*b^2 - 4*b^3)*sin(d*x + c))/(a^3*b^2 - a^2*b^3 + (a^3*b^
2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)^2
 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 16*
(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x + 8*
c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b +
57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a
^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*d*
x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)
- 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*
b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4*b
 - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2*d
*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*a^
4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b
^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)
+ (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^
3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b
^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c)), x) - (a*b^2 - b^3 + (a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b
^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*
x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x
+ 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57
*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^
2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)
*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*
(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 -
 b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 -
4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^
2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b
- 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*si
```

```
n(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b
^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(cos(d*x)^2 + 2*cos(d*x)*c
os(c) + cos(c)^2 + sin(d*x)^2 - 2*sin(d*x)*sin(c) + sin(c)^2) + (a*b^2 - b^
3 + (a*b^2 - b^3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2
+ 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2
- b^3)*cos(2*d*x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 -
b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d
*x + 4*c)^2 + 16*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 16*(a*b^2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^
3)*cos(6*d*x + 6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(
a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2
*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))
*cos(6*d*x + 6*c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 +
 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*
c) - 4*(2*(a*b^2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin
(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8
*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c))*log(cos(d*x)^2 - 2*cos(d*x)*cos(c) + cos(c)^2 + sin(
d*x)^2 + 2*sin(d*x)*sin(c) + sin(c)^2) - (a*b^2*sin(7*d*x + 7*c) - 5*a*b^2*
sin(5*d*x + 5*c) - 5*a*b^2*sin(3*d*x + 3*c) + a*b^2*sin(d*x + c))*sin(8*d*x
 + 8*c) + 2*(2*a*b^2*sin(6*d*x + 6*c) + 2*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b
 - 3*a*b^2)*sin(4*d*x + 4*c))*sin(7*d*x + 7*c) - 4*(5*a*b^2*sin(5*d*x + 5*c
) + 5*a*b^2*sin(3*d*x + 3*c) - a*b^2*sin(d*x + c))*sin(6*d*x + 6*c) - 10*(2
*a*b^2*sin(2*d*x + 2*c) + (8*a^2*b - 3*a*b^2)*sin(4*d*x + 4*c))*sin(5*d*x +
 5*c) - 2*(5*(8*a^2*b - 3*a*b^2)*sin(3*d*x + 3*c) - (8*a^2*b - 3*a*b^2)*sin
(d*x + c))*sin(4*d*x + 4*c))/((a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 16
*(a^3*b^2 - a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*
b^2 - 9*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x
+ 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3
)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*d*
sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 8*(a^3*
b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(4*(a^3*b^2 -
 a^2*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4
*d*x + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3
)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(4*d*x
 + 4*c) + 4*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) - (a^3*b^2 - a^2*b^3)*d)
*cos(6*d*x + 6*c) + 4*(4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*cos(2*d*x + 2
*c) - (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^3*b^
2 - a^2*b^3)*d*sin(6*d*x + 6*c) + (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(
4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*d*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a
^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 17.55, size = 7491, normalized size = 23.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^2),x)
```

```
[Out] ((b*cos(c + d*x)^3)/(4*a*(a - b)) - (b*cos(c + d*x))/(2*a*(a - b)))/(d*(a -
 b + 2*b*cos(c + d*x)^2 - b*cos(c + d*x)^4)) - (atan(((((3072*a^3*b^7 - 109
44*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152*a^7*b
^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*a^6*b
+ a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) +
35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b -
 a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6 + 2293
76*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a
^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29
```

```
*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) +
(cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(128*(a^6 -
2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*
b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 +
 a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2)
 + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*
b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(768*b^7 - 2048*a*b
^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2)
 + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b
)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*1i - (((3072*
a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (
((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a
^7 - 2*a^6*b + a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^
9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(25
6*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a
^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2))
)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47
*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^
2)))^(1/2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))
/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1
/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^
10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*
(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/
(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(c + d*x)*(768*b
^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*
(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 -
29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*
1i)/((((3072*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b +
a^5*b^2)) - (((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10
*b^4)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2
) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*
b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b
^7 - 262144*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*
b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16
*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b
^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29
312*a^6*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b
^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2
))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(
1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a
^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c
+ d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2
)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 -
47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*
b^2)))^(1/2) - (125*a*b^5 - 80*b^6)/(128*(a^7 - 2*a^6*b + a^5*b^2)) + (((30
72*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2))
- (((49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256
*(a^7 - 2*a^6*b + a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b)^(1/2) + 8*b^2*
(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/
(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 26214
4*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^
2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 -
 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9
*b^2)))^(1/2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^
5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)
^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3
*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*((25*a^2*(a^9*b)^(1/2) + 8*b
^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2
))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(c + d*x)*(76
```

8*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*((25*a
^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 - 47*a^5*b^2
 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/
2)))*((25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) + 35*a^6*b + 16*a^4*b^3 -
 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9
*b^2)))^(1/2)*2i)/d - (atan(((((3072*a^3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5
)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152*a^7*b^7 - 155648*a^8*b^6 + 172
032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (cos(c + d*
x)*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 +
47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*
b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11
b^4))/(128(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^
9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(25
6*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(18432*a^
4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-
(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^
5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2))
)^(1/2))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*
b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 -
3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(
128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/
2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^1
0*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*1i - (((3072*a^3*b^7 - 10944*a^4*
b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152*a^7*b^7 - 15
5648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*a^6*b + a^5*b
^2)) + (cos(c + d*x)*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6
*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11
+ a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6 + 229376*a^1
0*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)
^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*
(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(
c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(128*(a^6 - 2*a^5
*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b -
16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8
*b^3 - 3*a^9*b^2)))^(1/2))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) -
35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b -
 a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(c + d*x)*(768*b^7 - 2048*a*b^6
+ 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) +
 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^
(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*1i)/((((3072*a^
3*b^7 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((
49152*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7
 - 2*a^6*b + a^5*b^2)) - (cos(c + d*x)*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9
*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256
*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^
9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))
*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47
*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^
2)))^(1/2) + (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))
/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(
1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a
^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^
2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2)
)/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) + (cos(c + d*x)*(768
*b^7 - 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a
^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2
 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/
2) - (125*a*b^5 - 80*b^6)/(128*(a^7 - 2*a^6*b + a^5*b^2)) + (((3072*a^3*b^7
 - 10944*a^4*b^6 + 9776*a^5*b^5)/(256*(a^7 - 2*a^6*b + a^5*b^2)) - (((49152

```
*a^7*b^7 - 155648*a^8*b^6 + 172032*a^9*b^5 - 65536*a^10*b^4)/(256*(a^7 - 2*
a^6*b + a^5*b^2)) + (cos(c + d*x)*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(
1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a
^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)*(98304*a^8*b^7 - 262144*a^9*b^6
 + 229376*a^10*b^5 - 65536*a^11*b^4))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(2
5*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*
b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^
(1/2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(128
*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2)
- 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b
 - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2))*(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^
9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(25
6*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2) - (cos(c + d*x)*(768*b^7
- 2048*a*b^6 + 1425*a^2*b^5))/(128*(a^6 - 2*a^5*b + a^4*b^2)))*(-(25*a^2*(a
^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*a^5*b^2 - 29
*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2)))^(1/2)))*
(-(25*a^2*(a^9*b)^(1/2) + 8*b^2*(a^9*b)^(1/2) - 35*a^6*b - 16*a^4*b^3 + 47*
a^5*b^2 - 29*a*b*(a^9*b)^(1/2))/(256*(3*a^10*b - a^11 + a^8*b^3 - 3*a^9*b^2
)))^(1/2)*2i)/d - (atan((((((((192*a^7*b^7 - 608*a^8*b^6 + 672*a^9*b^5 - 25
6*a^10*b^4)/(2*(a^7 - 2*a^6*b + a^5*b^2)) - (cos(c + d*x)*(98304*a^8*b^7 -
262144*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(512*a^2*(a^6 - 2*a^5*b
 + a^4*b^2)))/(2*a^2) + (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 2931
2*a^6*b^5))/(256*(a^6 - 2*a^5*b + a^4*b^2)))/(2*a^2) - (12*a^3*b^7 - (171*a
^4*b^6)/4 + (611*a^5*b^5)/16)/(2*(a^7 - 2*a^6*b + a^5*b^2)))*1i)/(2*a^2) -
(cos(c + d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^2*b^5)*1i)/(256*(a^6 - 2*a^5*b
 + a^4*b^2)))/a^2 - ((((((192*a^7*b^7 - 608*a^8*b^6 + 672*a^9*b^5 - 256*a^1
0*b^4)/(2*(a^7 - 2*a^6*b + a^5*b^2)) + (cos(c + d*x)*(98304*a^8*b^7 - 26214
4*a^9*b^6 + 229376*a^10*b^5 - 65536*a^11*b^4))/(512*a^2*(a^6 - 2*a^5*b + a^
4*b^2)))/(2*a^2) - (cos(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6
*b^5))/(256*(a^6 - 2*a^5*b + a^4*b^2)))/(2*a^2) - (12*a^3*b^7 - (171*a^4*b^
6)/4 + (611*a^5*b^5)/16)/(2*(a^7 - 2*a^6*b + a^5*b^2)))*1i)/(2*a^2) + (cos(
c + d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^2*b^5)*1i)/(256*(a^6 - 2*a^5*b + a^
4*b^2)))/a^2)/(((125*a*b^5)/128 - (5*b^6)/8)/(a^7 - 2*a^6*b + a^5*b^2) + ((
(((192*a^7*b^7 - 608*a^8*b^6 + 672*a^9*b^5 - 256*a^10*b^4)/(2*(a^7 - 2*a^6*
b + a^5*b^2)) - (cos(c + d*x)*(98304*a^8*b^7 - 262144*a^9*b^6 + 229376*a^10
*b^5 - 65536*a^11*b^4))/(512*a^2*(a^6 - 2*a^5*b + a^4*b^2)))/(2*a^2) + (cos
(c + d*x)*(18432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(256*(a^6 - 2*a^
5*b + a^4*b^2)))/(2*a^2) - (12*a^3*b^7 - (171*a^4*b^6)/4 + (611*a^5*b^5)/16
)/(2*(a^7 - 2*a^6*b + a^5*b^2)))/(2*a^2) - (cos(c + d*x)*(768*b^7 - 2048*a*
b^6 + 1425*a^2*b^5))/(256*(a^6 - 2*a^5*b + a^4*b^2)))/a^2 + (((((192*a^7*b^
7 - 608*a^8*b^6 + 672*a^9*b^5 - 256*a^10*b^4)/(2*(a^7 - 2*a^6*b + a^5*b^2))
 + (cos(c + d*x)*(98304*a^8*b^7 - 262144*a^9*b^6 + 229376*a^10*b^5 - 65536*
a^11*b^4))/(512*a^2*(a^6 - 2*a^5*b + a^4*b^2)))/(2*a^2) - (cos(c + d*x)*(18
432*a^4*b^7 - 45440*a^5*b^6 + 29312*a^6*b^5))/(256*(a^6 - 2*a^5*b + a^4*b^2
)))/(2*a^2) - (12*a^3*b^7 - (171*a^4*b^6)/4 + (611*a^5*b^5)/16)/(2*(a^7 - 2
*a^6*b + a^5*b^2)))/(2*a^2) + (cos(c + d*x)*(768*b^7 - 2048*a*b^6 + 1425*a^
2*b^5))/(256*(a^6 - 2*a^5*b + a^4*b^2)))/a^2))*1i)/(a^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.218 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=320

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{}}\right.}{2b^2d\sqrt{\sqrt{a}+}}$$

```
[Out] x/b^2+1/8*a^(1/4)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^(3/2
)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*a^(1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(
d*x+c)/a^(1/4))/b^(3/2)/d/(a^(1/2)+b^(1/2))^(3/2)-1/2*a^(1/4)*arctan((a^(1/
2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)-b^(1/2))^(1/2)-1/2*a^(
1/4)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/b^2/d/(a^(1/2)+b^(1
/2))^(1/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*tan(d*x+c)^5/b/d/(a+2*a*tan(d*x+c)^
2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.45, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 9, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3217, 1313, 1275, 12, 1122, 1166, 205, 1287, 203}

$$\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2b^2d\sqrt{\sqrt{a}-\sqrt{b}}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}}{\sqrt[4]{}}\right.}{2b^2d\sqrt{\sqrt{a}+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] x/b^2 - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2
*Sqrt[Sqrt[a] - Sqrt[b]]*b^2*d) + (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*
Tan[c + d*x])/a^(1/4)])/(8*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) - (a^(1/4)*
ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*Sqrt[Sqrt[a] + S
qrt[b]]*b^2*d) - (a^(1/4)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*(Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*
d) + Tan[c + d*x]^5/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4
))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
```

```
x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1287

```
Int[(((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)^(q_.))/((a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[((f*x)^m*(d + e*x^2)^q)/(a
 + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e, f, m}, x] && NeQ[b^2 - 4
*a*c, 0] && IntegerQ[q] && IntegerQ[m]
```

Rule 1313

```
Int[(((f_.)*(x_))^(m_.)*((a_.) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_))/((d_.)
+ (e_.)*(x_)^2), x_Symbol] :> -Dist[f^4/(c*d^2 - b*d*e + a*e^2), Int[(f*x)^
(m - 4)*(a*d + (b*d - a*e)*x^2)*(a + b*x^2 + c*x^4)^p, x], x] + Dist[(d^2*f
^4)/(c*d^2 - b*d*e + a*e^2), Int[((f*x)^(m - 4)*(a + b*x^2 + c*x^4)^(p + 1)
)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0
] && LtQ[p, -1] && GtQ[m, 2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{x^4(a+ax^2)}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{bd} - \frac{\text{Subst}\left(\int \frac{x^4}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)}\,dx, x,\right.}{bd}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2abx^4}{a+2ax^2+(a-b)x^4}\,dx, x, \text{ta}\right.}{8ab^2d}$$

$$= \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{b^2d}$$

$$= \frac{x}{b^2} - \frac{\tan(c+dx)}{4(a-b)bd} + \frac{\tan^5(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\left(a\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\right.}{}$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}+\sqrt{b}}\,b^2d} - \frac{\tan(c+dx)}{4(a-b)bd} +$$

$$= \frac{x}{b^2} - \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2\sqrt{\sqrt{a}-\sqrt{b}}\,b^2d} + \frac{\sqrt[4]{a}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d} - \frac{\sqrt[4]{a}\tan^{-1}\left(\sqrt{\right.}{2\sqrt{\sqrt{a}}$$

Mathematica [A] time = 5.02, size = 262, normalized size = 0.82

$$\frac{-\frac{\sqrt{a}\left(4\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{2ab(\sin(4(c+dx))-6\sin(2(c+dx)))}{(a-b)(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} + \frac{\sqrt{a}\left(4\sqrt{a}-5\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} + 8}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (8*(c + d*x) - (Sqrt[a]*(4*Sqrt[a] + 5*Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])
*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sq
rt[a]*Sqrt[b]]) + (Sqrt[a]*(4*Sqrt[a] - 5*Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt
[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-
a + Sqrt[a]*Sqrt[b]]) + (2*a*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/((
a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])))/(8*b^2*d)
```

fricas [B] time = 1.33, size = 3544, normalized size = 11.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(32*(a*b - b^2)*d*x*cos(d*x + c)^4 - 64*(a*b - b^2)*d*x*cos(d*x + c)^2
 - 32*(a^2 - 2*a*b + b^2)*d*x + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3
- b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*
```

```
a^2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450
*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15
*a^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^
4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2
- 625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)
^2 + 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqr
t((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7
- 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4
))*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*
b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b
^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)
/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12
+ b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b
^6 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6
+ 25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*
a*b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3
 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
1 - 6*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 -
b^4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(-((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a
^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a
^2*b^11 - 6*a*b^12 + b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4
- 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(32*a^3 - 166*a^2*b + 1125/4*a*b^2 -
625/4*b^3 - 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2
 - 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt(
(64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 -
6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))
*cos(d*x + c)*sin(d*x + c) - (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^
5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7
)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/(
(a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 +
b^13)*d^4)) + 16*a^3 - 47*a^2*b + 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6
 - b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 +
25*b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*
b^6 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 +
 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11
- 6*a*b^12 + b^13)*d^4))) + ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^
4)*d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b
^5 + 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*
b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*
b^11 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3
*a^2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 62
5/4*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 +
 1/2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((6
4*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*
a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*c
os(d*x + c)*sin(d*x + c) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)
*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d
^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^
6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^1
3)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 -
b^7)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*
b^7)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6
 + 25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 62
5*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6
*a*b^12 + b^13)*d^4))) - ((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*
d*cos(d*x + c)^2 - (a^2*b^2 - 2*a*b^3 + b^4)*d)*sqrt(((a^3*b^4 - 3*a^2*b^5
+ 3*a*b^6 - b^7)*d^2*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3
 + 625*a*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^1
```

```
1 - 6*a*b^12 + b^13)*d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^
2*b^5 + 3*a*b^6 - b^7)*d^2))*log(-32*a^3 + 166*a^2*b - 1125/4*a*b^2 + 625/4
*b^3 + 1/4*(128*a^3 - 664*a^2*b + 1125*a*b^2 - 625*b^3)*cos(d*x + c)^2 - 1/
2*(2*(2*a^4*b^5 - 9*a^3*b^6 + 15*a^2*b^7 - 11*a*b^8 + 3*b^9)*d^3*sqrt((64*a
^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b^7 - 6*a^5
*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*d^4))*cos(
d*x + c)*sin(d*x + c) + (24*a^3*b^2 - 127*a^2*b^3 + 220*a*b^4 - 125*b^5)*d*
cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7)*d^2*
sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a*b^4)/((a^6*b
^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*b^12 + b^13)*
d^4)) - 16*a^3 + 47*a^2*b - 35*a*b^2)/((a^3*b^4 - 3*a^2*b^5 + 3*a*b^6 - b^7
)*d^2)) + 1/4*(2*(16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 + 25*b^7
)*d^2*cos(d*x + c)^2 - (16*a^4*b^3 - 73*a^3*b^4 + 123*a^2*b^5 - 91*a*b^6 +
25*b^7)*d^2)*sqrt((64*a^5 - 464*a^4*b + 1241*a^3*b^2 - 1450*a^2*b^3 + 625*a
*b^4)/((a^6*b^7 - 6*a^5*b^8 + 15*a^4*b^9 - 20*a^3*b^10 + 15*a^2*b^11 - 6*a*
b^12 + b^13)*d^4))) - 8*(a*b*cos(d*x + c)^3 - 2*a*b*cos(d*x + c))*sin(d*x +
 c))/((a*b^3 - b^4)*d*cos(d*x + c)^4 - 2*(a*b^3 - b^4)*d*cos(d*x + c)^2 - (
a^2*b^2 - 2*a*b^3 + b^4)*d)
```

giac [B] time = 1.17, size = 1563, normalized size = 4.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 21*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 16*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^3)*(a*b^2 - b^3)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^5*b^2 - 63*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^3 + 116*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^3*b^4 - 86*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^
2*b^5 + 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*b^7)*abs(-a*b^2 + b^3)*abs(-a + b) - (9*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^4 - 51*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*sqrt(a*b)*a^4*b^5 + 102*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^6 - 82*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 17*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arc
tan(tan(d*x + c)/sqrt((a^2*b^2 - a*b^3 + sqrt((a^2*b^2 - a*b^3)^2 - (a^2*b^
2 - a*b^3)*(a^2*b^2 - 2*a*b^3 + b^4)))/(a^2*b^2 - 2*a*b^3 + b^4))))/((3*a^7
*b^4 - 21*a^6*b^5 + 59*a^5*b^6 - 85*a^4*b^7 + 65*a^3*b^8 - 23*a^2*b^9 + a*b
^10 + b^11)*abs(-a*b^2 + b^3)) - (2*(6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^3 - 21*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 16
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b^2 - b^3)^2*abs(-a + b) + (12*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*a^5*b^2 - 63*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*a^4*b^3 + 116*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - 86*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a^2*b^5 + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
a*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^7)*abs(-a*b^2 + b^3)*abs(-a
 + b) - (9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^4 - 51*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^5 + 102*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 82*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^7 + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^8
+ 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^9)*abs(-a + b))*(pi*flo
or((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b^2 - a*b^3 - sqrt((
a^2*b^2 - a*b^3)^2 - (a^2*b^2 - a*b^3)*(a^2*b^2 - 2*a*b^3 + b^4)))/(a^2*b^2
 - 2*a*b^3 + b^4))))/((3*a^7*b^4 - 21*a^6*b^5 + 59*a^5*b^6 - 85*a^4*b^7 + 6
5*a^3*b^8 - 23*a^2*b^9 + a*b^10 + b^11)*abs(-a*b^2 + b^3)) - 2*(2*a*tan(d*x
```

```
 + c)^3 + a*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d
*x + c)^2 + a)*(a*b - b^2)) + 8*(d*x + c)/b^2)/d
```

maple [B] time = 0.31, size = 644, normalized size = 2.01

$$\frac{\arctan(\tan(dx+c))}{d\,b^2} - \frac{a\left(\tan^3(dx+c)\right)}{2db\left(\left(\tan^4(dx+c)\right)a - \left(\tan^4(dx+c)\right)b + 2a\left(\tan^2(dx+c)\right) + a\right)(a-b)} - \frac{}{4db\left(\left(\tan^4(\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] 1/d/b^2*arctan(tan(d*x+c))-1/2/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan
(d*x+c)^2+a)/(a-b)*tan(d*x+c)^3-1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*
a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)-1/2/d*a^2/b^2/(a-b)/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/4/d*a/
b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/
2)-a)*(a-b))^(1/2))+3/8/d*a^2/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-5/8/d*a/(a*b)
^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b
)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a^2/b^2/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/4/d*a/b/(a-b)/(((a
*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(
1/2))-3/8/d*a^2/b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a
-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+5/8/d*a/(a*b)^(1/2)/(a-b)/(((
a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^
(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 17.60, size = 7640, normalized size = 23.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (atan(((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 30
72*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208
896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan(c
+ d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5
 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a
^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*
a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
 - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^1
0 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2*b^8 - 84
864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 -
 b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^
5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 +
a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 +
 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*
```

a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i - (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i)/((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c + d*x)*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (656*a^5 - 2049*a^4*b + 1600*a^3*b^2)/(128*(a*b^5 - b^6))))*((8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) - 35*a*b^6 + 47*a^2*b^5 - 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256

```
*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*2i)/d + (atan(((((5120*a^2
*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(
a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 1679
36*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) - (tan(c + d*x)*(-(8*a^2*(a
*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 2
9*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(
98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a
*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47
*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2
*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54
912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*(-(8*a^2
*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4
- 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2
))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 +
16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*
b^8)))^(1/2) + (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a
^4*b^2))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
 + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^1
0 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*1i - (((5120*a^2*b^7 - 17664*a^3*b^
6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((
20480*a^2*b^11 - 110592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*
a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c + d*x)*(-(8*a^2*(a*b^9)^(1/2) + 25*b^
2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2)
)/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 19
6608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*(-(8*
a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b
^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(
1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864
*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25
*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1
/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*(-(8*a^2*(a*b^9)
^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b
*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan
(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^
4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^
2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^
9 + a^3*b^8)))^(1/2)*1i)/((((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 -
 16320*a^5*b^4 + 3072*a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 11
0592*a^3*b^10 + 208896*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^
5 - b^6)) - (tan(c + d*x)*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 3
5*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 -
b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196
608*a^5*b^9 - 98304*a^6*b^8))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) +
 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)
^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x
)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^
6*b^4))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
+ 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10
 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*
b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(25
6*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(768*a^6
+ 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*(-(8*a^2
*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4
- 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2
) + (((5120*a^2*b^7 - 17664*a^3*b^6 + 26688*a^4*b^5 - 16320*a^5*b^4 + 3072*
a^6*b^3)/(256*(a*b^5 - b^6)) + (((20480*a^2*b^11 - 110592*a^3*b^10 + 208896
*a^4*b^9 - 167936*a^5*b^8 + 49152*a^6*b^7)/(256*(a*b^5 - b^6)) + (tan(c + d
*x)*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 +
 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3
```

```
*b^8)))^(1/2)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^
6*b^8))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2)
+ 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10
 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) + (tan(c + d*x)*(21376*a^2*b^8 - 848
64*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4))/(128*(a*b^4 -
b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^
5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 +
a^3*b^8)))^(1/2))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(a*b^9)^(1/2) + 35*a*b^6
- 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(256*(3*a*b^10 - b^11 - 3
*a^2*b^9 + a^3*b^8)))^(1/2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a
^3*b^3 - 5295*a^4*b^2))/(128*(a*b^4 - b^5)))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^
2*(a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2)
)/(256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2) - (656*a^5 - 2049*a^
4*b + 1600*a^3*b^2)/(128*(a*b^5 - b^6))))*(-(8*a^2*(a*b^9)^(1/2) + 25*b^2*(
a*b^9)^(1/2) + 35*a*b^6 - 47*a^2*b^5 + 16*a^3*b^4 - 29*a*b*(a*b^9)^(1/2))/(
256*(3*a*b^10 - b^11 - 3*a^2*b^9 + a^3*b^8)))^(1/2)*2i)/d - atan(((((((((8
0*a^2*b^11 - 432*a^3*b^10 + 816*a^4*b^9 - 656*a^5*b^8 + 192*a^6*b^7)*1i)/(2
*(a*b^5 - b^6)) - (tan(c + d*x)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*
a^5*b^9 - 98304*a^6*b^8))/(512*b^2*(a*b^4 - b^5)))*1i)/(2*b^2) + (tan(c + d
*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*
a^6*b^4)*1i)/(256*(a*b^4 - b^5)))*1i)/(2*b^2) + ((20*a^2*b^7 - 69*a^3*b^6 +
 (417*a^4*b^5)/4 - (255*a^5*b^4)/4 + 12*a^6*b^3)*1i)/(2*(a*b^5 - b^6)))/(2*
b^2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))
/(256*(a*b^4 - b^5)))/b^2 - (((((((80*a^2*b^11 - 432*a^3*b^10 + 816*a^4*b^
9 - 656*a^5*b^8 + 192*a^6*b^7)*1i)/(2*(a*b^5 - b^6)) + (tan(c + d*x)*(98304
*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(512*b^2*(a*
b^4 - b^5)))*1i)/(2*b^2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 5
4912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4)*1i)/(256*(a*b^4 - b^5)))*1i)/
(2*b^2) + ((20*a^2*b^7 - 69*a^3*b^6 + (417*a^4*b^5)/4 - (255*a^5*b^4)/4 + 1
2*a^6*b^3)*1i)/(2*(a*b^5 - b^6)))/(2*b^2) + (tan(c + d*x)*(768*a^6 + 800*a^
2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2))/(256*(a*b^4 - b^5)))/b^2)/(((((((((8
0*a^2*b^11 - 432*a^3*b^10 + 816*a^4*b^9 - 656*a^5*b^8 + 192*a^6*b^7)*1i)/(2
*(a*b^5 - b^6)) - (tan(c + d*x)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*
a^5*b^9 - 98304*a^6*b^8))/(512*b^2*(a*b^4 - b^5)))*1i)/(2*b^2) + (tan(c + d
*x)*(21376*a^2*b^8 - 84864*a^3*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*
a^6*b^4)*1i)/(256*(a*b^4 - b^5)))*1i)/(2*b^2) + ((20*a^2*b^7 - 69*a^3*b^6 +
 (417*a^4*b^5)/4 - (255*a^5*b^4)/4 + 12*a^6*b^3)*1i)/(2*(a*b^5 - b^6)))*1i)
/(2*b^2) - (tan(c + d*x)*(768*a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b
^2)*1i)/(256*(a*b^4 - b^5)))/b^2 + ((((((((80*a^2*b^11 - 432*a^3*b^10 + 81
6*a^4*b^9 - 656*a^5*b^8 + 192*a^6*b^7)*1i)/(2*(a*b^5 - b^6)) + (tan(c + d*x
)*(98304*a^2*b^12 - 196608*a^3*b^11 + 196608*a^5*b^9 - 98304*a^6*b^8))/(512
*b^2*(a*b^4 - b^5)))*1i)/(2*b^2) - (tan(c + d*x)*(21376*a^2*b^8 - 84864*a^3
*b^7 + 54912*a^4*b^6 + 20864*a^5*b^5 - 18432*a^6*b^4)*1i)/(256*(a*b^4 - b^5
)))*1i)/(2*b^2) + ((20*a^2*b^7 - 69*a^3*b^6 + (417*a^4*b^5)/4 - (255*a^5*b^
4)/4 + 12*a^6*b^3)*1i)/(2*(a*b^5 - b^6)))*1i)/(2*b^2) + (tan(c + d*x)*(768*
a^6 + 800*a^2*b^4 + 4832*a^3*b^3 - 5295*a^4*b^2)*1i)/(256*(a*b^4 - b^5)))/b
^2 - ((41*a^5)/8 - (2049*a^4*b)/128 + (25*a^3*b^2)/2)/(a*b^5 - b^6)))/(b^2*
d) - ((a*tan(c + d*x)^3)/(2*b*(a - b)) + (a*tan(c + d*x))/(4*b*(a - b)))/(d
*(a + 2*a*tan(c + d*x)^2 + tan(c + d*x)^4*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.219 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=233

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{}{4bd\left((a-b\right.}$$

```
[Out] -1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-3*b^(1/2
))/a^(1/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))^(
1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+3*b^(1/2))/a^(1/4)/b^(3/2)/d/(a^(1/2)+b
^(1/2))^(3/2)-1/4*tan(d*x+c)/(a-b)/b/d+1/4*sec(d*x+c)^2*tan(d*x+c)^3/b/d/(a
+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.35, antiderivative size = 233, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1120, 1279, 1166, 205}

$$-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\,b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)}{4bd(a-b)}+\frac{}{4bd\left((a-b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -((2*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(8*a^(1/4)*(Sqrt[a] - Sqrt[b])^(3/2)*b^(3/2)*d) + ((2*Sqrt[a] + 3*Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(1/4)*(
Sqrt[a] + Sqrt[b])^(3/2)*b^(3/2)*d) - Tan[c + d*x]/(4*(a - b)*b*d) + (Sec[c
 + d*x]^2*Tan[c + d*x]^3)/(4*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx=\frac{\text{Subst}\left(\int \frac{x^6}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx,x,\tan(c+dx)\right)}{d}$$

$$=\frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}-\frac{\text{Subst}\left(\int \frac{x^2\left(6a+2ax^2\right)}{a+2ax^2+(a-b)x^4}\,dx,x,\tan\right.}{8abd}$$

$$=-\frac{\tan(c+dx)}{4(a-b)bd}+\frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}+\frac{\text{Subst}\left(\int \frac{2a^2-2a}{a+2ax^2}\right.}{8}$$

$$=-\frac{\tan(c+dx)}{4(a-b)bd}+\frac{\sec^2(c+dx)\tan^3(c+dx)}{4bd\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)}-\frac{\left(a-\frac{2\sqrt{a}(a-2b)}{\sqrt{b}}-\right.}{}$$

$$=-\frac{\left(2\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8\sqrt[4]{a}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}b^{3/2}d}+\frac{\left(2\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right.}{8\sqrt[4]{a}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}b^{3/2}d}$$

Mathematica [A] time = 2.66, size = 238, normalized size = 1.02

$$\frac{\frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}+2a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{4b\sin(2(c+dx))(-2a+b\cos(2(c+dx))-b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b}-\frac{\sqrt{b}\left(\sqrt{a}\sqrt{b}-2a+3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{8b^2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (((2*a + Sqrt[a]*Sqrt[b] - 3*b)*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - (Sqrt[b]*(-2
*a + Sqrt[a]*Sqrt[b] + 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt
[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (4*b*(-2*a - b + b*Co
s[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos
[4*(c + d*x)]))/(8*(a - b)*b^2*d)
```

fricas [B] time = 1.24, size = 3135, normalized size = 13.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 -
 (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^
2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a
^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2)/(
(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2 - 81*a*b + 81*b
^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^
4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/
((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 +
a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)
*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 2
0*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15*a*b + 15*b^2
)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b
^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*
a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 8
1*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2
*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*(a*b^2 - b^3)*d*
cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(-((a^3*b^3 - 3*a^2*b^4 + 3
*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 1
5*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 - 15
*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*log(1/4*(20*a^2
 - 81*a*b + 81*b^2)*cos(d*x + c)^2 - 5*a^2 + 81/4*a*b - 81/4*b^2 - 1/2*((a^
5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*sqrt((25*a^2 - 9
0*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^
7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(5*a^3*b - 19*a^
2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^3*b^3 - 3*a^2*b^4
+ 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4)) + 4*a^2 -
 15*a*b + 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2)) + 1/4*(2*(4*
a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*cos(d*x + c)^2
- (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2)*sqrt((25*
a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15
*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + ((a*b^2 - b^3)*d*cos(d*x + c)^4 - 2*
(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*sqrt(((a^3*b^3
- 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3
- 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4
)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2))*
log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 - 81/4*a*b + 81/
4*b^2 + 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6 + 3*a*b^7)*d^3*
sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4
*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*
(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^3*b
^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b
^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*
d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2
)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)*d^2*
cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4 + 9*a*b^5)
*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 -
20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) - ((a*b^2 - b^3)*d*cos(
d*x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)*
sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b + 81*
b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b
```

```
^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a*b^
5 - b^6)*d^2))*log(-1/4*(20*a^2 - 81*a*b + 81*b^2)*cos(d*x + c)^2 + 5*a^2 -
 81/4*a*b + 81/4*b^2 - 1/2*((a^5*b^3 - 6*a^4*b^4 + 12*a^3*b^5 - 10*a^2*b^6
+ 3*a*b^7)*d^3*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a
^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))*cos(d*x + c)*si
n(d*x + c) - 2*(5*a^3*b - 19*a^2*b^2 + 18*a*b^3)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(((a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d^2*sqrt((25*a^2 - 90*a*b +
81*b^2)/((a^7*b^3 - 6*a^6*b^4 + 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^
2*b^8 + a*b^9)*d^4)) - 4*a^2 + 15*a*b - 15*b^2)/((a^3*b^3 - 3*a^2*b^4 + 3*a
*b^5 - b^6)*d^2)) + 1/4*(2*(4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2*b^4
+ 9*a*b^5)*d^2*cos(d*x + c)^2 - (4*a^5*b - 21*a^4*b^2 + 39*a^3*b^3 - 31*a^2
*b^4 + 9*a*b^5)*d^2)*sqrt((25*a^2 - 90*a*b + 81*b^2)/((a^7*b^3 - 6*a^6*b^4
+ 15*a^5*b^5 - 20*a^4*b^6 + 15*a^3*b^7 - 6*a^2*b^8 + a*b^9)*d^4))) + 8*(b*c
os(d*x + c)^3 - (a + b)*cos(d*x + c))*sin(d*x + c))/((a*b^2 - b^3)*d*cos(d*
x + c)^4 - 2*(a*b^2 - b^3)*d*cos(d*x + c)^2 - (a^2*b - 2*a*b^2 + b^3)*d)
```

giac [B] time = 1.20, size = 1481, normalized size = 6.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*(((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 17*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
3)*(a*b - b^2)^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b
 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^2 + 14*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^3*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^4 - s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^5)*abs(-a*b + b^2)*abs(-a + b) - 2*(
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b - 18*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 + 38*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^4*b^3 - 32*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^4 + 7*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^5 + 2*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^6)*abs(-a + b))*(pi*floor((d*x +
 c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2*b - a*b^2 + sqrt((a^2*b - a*b
^2)^2 - (a^2*b - a*b^2)*(a^2*b - 2*a*b^2 + b^3)))/(a^2*b - 2*a*b^2 + b^3)))
)/((3*a^8*b^2 - 21*a^7*b^3 + 59*a^6*b^4 - 85*a^5*b^5 + 65*a^4*b^6 - 23*a^3*
b^7 + a^2*b^8 + a*b^9)*abs(-a*b + b^2)) - ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
*b + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 3*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a*b - b^2)^2*abs(-a + b) - (3*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*a^4*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^3 - 4*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a^2*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b
^5)*abs(-a*b + b^2)*abs(-a + b) - 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^6*b - 18*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2
+ 38*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 - 32*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 7*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^2*b^5 + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*a*b^6)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sq
rt((a^2*b - a*b^2 - sqrt((a^2*b - a*b^2)^2 - (a^2*b - a*b^2)*(a^2*b - 2*a*b
^2 + b^3)))/(a^2*b - 2*a*b^2 + b^3))))/((3*a^8*b^2 - 21*a^7*b^3 + 59*a^6*b^
4 - 85*a^5*b^5 + 65*a^4*b^6 - 23*a^3*b^7 + a^2*b^8 + a*b^9)*abs(-a*b + b^2)
) - 2*(a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x + c))/((a*tan(d*x +
c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)*(a*b - b^2)))/d
```

maple [B] time = 0.32, size = 674, normalized size = 2.89

$$-\frac{a\left(\tan^{3}(dx+c)\right)}{4db\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)b+2a\left(\tan^{2}(dx+c)\right)+a\right)(a-b)}-\frac{\tan}{4d\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x
+c)^3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*
x+c)^3-1/4/d*a/b/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*t
an(d*x+c)-1/8/d*a/b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(
d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/4/d*a^2/b/(
a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/8/d*a/
b/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)
+a)*(a-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/4/d*a^2/b/(a*b)^(1/2)/(a-b)/(((a*
b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1
/2))+1/2/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*t
an(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*((a*b^3
- b^4)*d*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64
*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 -
 b^4)*d*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3
 - b^4)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4
)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*
c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^
4)*d*cos(2*d*x + 2*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x +
6*c) + 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4
)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2
- 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c)
 - (a*b^3 - b^4)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*
d*cos(2*d*x + 2*c) - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4
*(2*(a*b^3 - b^4)*d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin
(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(
(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2
*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - 3*b^2)*cos(6*d*x + 6*
c)^2 + 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - 3*b^2)*cos(2*d*x
+ 2*c)^2 + 4*(2*a*b - 3*b^2)*sin(6*d*x + 6*c)^2 + 12*(8*a*b - 3*b^2)*sin(4*
d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 21*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c)^2 - (6*b^2*cos(4*d*x + 4*c) + (2*a*
b - 3*b^2)*cos(6*d*x + 6*c) + (2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x +
 8*c) - (2*a*b - 3*b^2 - 2*(16*a^2 - 30*a*b + 21*b^2)*cos(4*d*x + 4*c) - 8*
```

```
(2*a*b - 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(3*b^2 - (16*a^2 - 3
0*a*b + 21*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a*b - 3*b^2)*cos(2*
d*x + 2*c) - (6*b^2*sin(4*d*x + 4*c) + (2*a*b - 3*b^2)*sin(6*d*x + 6*c) + (
2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((16*a^2 - 30*a*b + 2
1*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c))/(a*b^3 - b^4 + (a*b^3 - b^4)*cos(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*cos
(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*cos(4*d*x +
 4*c)^2 + 16*(a*b^3 - b^4)*cos(2*d*x + 2*c)^2 + (a*b^3 - b^4)*sin(8*d*x + 8
*c)^2 + 16*(a*b^3 - b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 5
7*a*b^3 - 9*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^3 - b^4)*sin(2*d*x + 2*c)^2 + 2*(a
*b^3 - b^4 - 4*(a*b^3 - b^4)*cos(6*d*x + 6*c) - 2*(8*a^2*b^2 - 11*a*b^3 + 3
*b^4)*cos(4*d*x + 4*c) - 4*(a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
 - 8*(a*b^3 - b^4 - 2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(4*d*x + 4*c) - 4*(
a*b^3 - b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^2*b^2 - 11*a*b^3 +
 3*b^4 - 4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c
) - 8*(a*b^3 - b^4)*cos(2*d*x + 2*c) - 4*(2*(a*b^3 - b^4)*sin(6*d*x + 6*c)
+ (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*sin(2*d
*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*b^3 + 3*b^4)*sin(4*d*x
+ 4*c) + 2*(a*b^3 - b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (b^2 + (
2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4*d*x + 4*c) - (2*a*b +
 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b + 3*b^2 - 2*(16*a^2 + 2
*a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(2*d*x + 2*c))*sin(6*d*
x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(2*d*x + 2*c))*si
n(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a*b^3 - b^4)*d*cos(8*d*x
 + 8*c)^2 + 16*(a*b^3 - b^4)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b
^2 + 57*a*b^3 - 9*b^4)*d*cos(4*d*x + 4*c)^2 + 16*(a*b^3 - b^4)*d*cos(2*d*x
+ 2*c)^2 + (a*b^3 - b^4)*d*sin(8*d*x + 8*c)^2 + 16*(a*b^3 - b^4)*d*sin(6*d*
x + 6*c)^2 + 4*(64*a^3*b - 112*a^2*b^2 + 57*a*b^3 - 9*b^4)*d*sin(4*d*x + 4*
c)^2 + 16*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c
) + 16*(a*b^3 - b^4)*d*sin(2*d*x + 2*c)^2 - 8*(a*b^3 - b^4)*d*cos(2*d*x + 2
*c) + (a*b^3 - b^4)*d - 2*(4*(a*b^3 - b^4)*d*cos(6*d*x + 6*c) + 2*(8*a^2*b^
2 - 11*a*b^3 + 3*b^4)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*
c) - (a*b^3 - b^4)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^2*b^2 - 11*a*b^3 + 3*b^4
)*d*cos(4*d*x + 4*c) + 4*(a*b^3 - b^4)*d*cos(2*d*x + 2*c) - (a*b^3 - b^4)*d
)*cos(6*d*x + 6*c) + 4*(4*(8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*cos(2*d*x + 2*c)
 - (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d)*cos(4*d*x + 4*c) - 4*(2*(a*b^3 - b^4)*
d*sin(6*d*x + 6*c) + (8*a^2*b^2 - 11*a*b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*
(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b^2 - 11*a*
b^3 + 3*b^4)*d*sin(4*d*x + 4*c) + 2*(a*b^3 - b^4)*d*sin(2*d*x + 2*c))*sin(6
*d*x + 6*c))
```

mupad [B] time = 16.54, size = 3400, normalized size = 14.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] (atan(((((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (ta
n(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4
+ 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^
2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a
*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256
*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3
 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b
^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2
*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i - (((256*a^2*b^5 - 512*a^3*b^4 + 256
*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2)
 + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*
```

a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a
^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1
/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6
)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*
b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^
4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i)/(
(27*a*b^2 - 21*a^2*b + 4*a^3)/(32*(a*b^3 - b^4)) + (((256*a^2*b^5 - 512*a^3
*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*((15*a*b^5 - 5*a*(a*
b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^
2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b
^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*
(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7
 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^
2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) -
 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(
1/2) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (ta
n(c + d*x)*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4
+ 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^
2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a
*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256
*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) - (tan(c + d*x)*(9*a*b^3
 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 - 5*a*(a*b
^9)^(1/2) + 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2
*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)))*((15*a*b^5 - 5*a*(a*b^9)^(1/2) + 9*b*
(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7
 - a^4*b^6)))^(1/2)*2i)/d + (atan(((((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b
^3)/(64*(a*b^3 - b^4)) - (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b
*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^
7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3
))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) -
15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1
/2) + (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 -
b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*
a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*1i - (((256
*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((
15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/
(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a
^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a
*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a
^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b +
 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9
b(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*
b^7 - a^4*b^6)))^(1/2)*1i)/((27*a*b^2 - 21*a^2*b + 4*a^3)/(32*(a*b^3 - b^4)
) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b^3)/(64*(a*b^3 - b^4)) - (tan(c
 + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4
*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*(256*a^2*b
^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3))/(4*(a*b^2 - b^3)))*((15*a*b^
5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a
*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (tan(c + d*x)*(9*a*b^3 -
15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)
^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^
8 + 3*a^3*b^7 - a^4*b^6)))^(1/2) + (((256*a^2*b^5 - 512*a^3*b^4 + 256*a^4*b
^3)/(64*(a*b^3 - b^4)) + (tan(c + d*x)*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b
*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^
7 - a^4*b^6)))^(1/2)*(256*a^2*b^6 - 768*a^3*b^5 + 768*a^4*b^4 - 256*a^5*b^3
))/(4*(a*b^2 - b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) -
15*a^2*b^4 + 4*a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1
/2) - (tan(c + d*x)*(9*a*b^3 - 15*a^3*b + 4*a^4 + 10*a^2*b^2))/(4*(a*b^2 -
b^3)))*((15*a*b^5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*

```
a^3*b^3)/(256*(a*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)))*((15*a*b^
5 + 5*a*(a*b^9)^(1/2) - 9*b*(a*b^9)^(1/2) - 15*a^2*b^4 + 4*a^3*b^3)/(256*(a
*b^9 - 3*a^2*b^8 + 3*a^3*b^7 - a^4*b^6)))^(1/2)*2i)/d - ((a*tan(c + d*x))/(
4*(a*b - b^2)) + (tan(c + d*x)^3*(a + b))/(4*(a*b - b^2)))/(d*(a + 2*a*tan(
c + d*x)^2 + tan(c + d*x)^4*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.220 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=195

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan^5(c+dx)}{4ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c}{4ad(}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b
^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4)
)/a^(3/4)/d/b^(1/2)/(a^(1/2)+b^(1/2))^(3/2)-1/4*tan(d*x+c)/a/(a-b)/d+1/4*ta
n(d*x+c)^5/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.23, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3217, 1275, 12, 1122, 1166, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan^5(c+dx)}{4ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{\tan(c}{4ad(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)]/(8*a^(3/4)*(Sqrt[a]
- Sqrt[b])^(3/2)*Sqrt[b]*d) - ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])
/a^(1/4)]/(8*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - Tan[c + d*x]/(4
*a*(a - b)*d) + Tan[c + d*x]^5/(4*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1122

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> Simp[(d^3*(d*x)^(m - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(m + 4*p + 1)),
 x] - Dist[d^4/(c*(m + 4*p + 1)), Int[(d*x)^(m - 4)*Simp[a*(m - 3) + b*(m +
 2*p - 1)*x^2, x]*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x
] && NeQ[b^2 - 4*a*c, 0] && GtQ[m, 3] && NeQ[m + 4*p + 1, 0] && IntegerQ[2*
p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1+x^2)}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int -\frac{2bx^4}{a+2ax^2+(a-b)x^4}\,dx, x, \text{t}\right.}{8abd}$$

$$= \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{x^4}{a+2ax^2+(a-b)x^4}\,dx, x, \text{tan}\right.}{4ad}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{a+}{a+2ax^2}\right.}{}$$

$$= -\frac{\tan(c+dx)}{4a(a-b)d} + \frac{\tan^5(c+dx)}{4ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(2\sqrt{a}-\frac{a+b}{\sqrt{b}}\right)\text{Su}}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\tan(c+dx)}{4a(a-b)d} + \frac{}{4ad\left(a+2a\right.}$$

Mathematica [A] time = 4.41, size = 225, normalized size = 1.15

$$-\frac{\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{2(\sin(4(c+dx))-6\sin(2(c+dx)))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + \frac{\left(\sqrt{a}+\sqrt{b}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}}{8d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] -1/8*(((Sqrt[a] - Sqrt[b])*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a
 + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) + ((Sqrt[
a] + Sqrt[b])*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*
Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (2*(-6*Sin[2*(c +
 d*x)] + Sin[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c
+ d*x)]))/((a - b)*d)
```

fricas [B] time = 0.86, size = 2796, normalized size = 14.34

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^
2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt
((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a
^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*
b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5
*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*
b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos
(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x +
 c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b
 + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*
b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^
2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^
5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b
- 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d
^4)) - 3/4*a - 1/4*b) - ((a*b - b^2)*d*cos(d*x + c)^4 - 2*(a*b - b^2)*d*cos
(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(1/4*(3*a + b)*cos(d*x + c)^2 - 1/2*
(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)
/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a
^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3 + 4*a^2*b + a*b^2)*d*cos(d
*x + c)*sin(d*x + c))*sqrt(-((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + a + 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^
2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos
(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*
b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4
*b^6 + a^3*b^7)*d^4)) - 3/4*a - 1/4*b) + ((a*b - b^2)*d*cos(d*x + c)^4 - 2*
(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(((a^4*b - 3*a^3*
b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2
 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/4*(3*a + b)*cos(
d*x + c)^2 + 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d^3*sqrt((9*a
^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^
5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3 + 4*a^2*b
 + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a - 3*b)/((a^4*b -
3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sq
rt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15
*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) - ((a*b - b^2)*d*cos
(d*x + c)^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)*sqrt(
((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a
^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b
```

```
^7)*d^4)) - a - 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2))*log(-1/
4*(3*a + b)*cos(d*x + c)^2 - 1/2*(2*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d^3*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 - 20*a^6
*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) +
(3*a^3 + 4*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4*b - 3*a^3
*b^2 + 3*a^2*b^3 - a*b^4)*d^2*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^
2 + 15*a^7*b^3 - 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) - a -
 3*b)/((a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d^2)) - 1/4*(2*(a^5 - 3*a^4*
b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 -
a^2*b^3)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/((a^9*b - 6*a^8*b^2 + 15*a^7*b^3 -
 20*a^6*b^4 + 15*a^5*b^5 - 6*a^4*b^6 + a^3*b^7)*d^4)) + 3/4*a + 1/4*b) + 8*
(cos(d*x + c)^3 - 2*cos(d*x + c))*sin(d*x + c))/((a*b - b^2)*d*cos(d*x + c)
^4 - 2*(a*b - b^2)*d*cos(d*x + c)^2 - (a^2 - 2*a*b + b^2)*d)
```

giac [B] time = 1.06, size = 1264, normalized size = 6.48

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] 1/8*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^
2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*
b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2 + (3*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^5)
*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^2
 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - 2*a*
b + b^2))))/(3*a^8*b - 21*a^7*b^2 + 59*a^6*b^3 - 85*a^5*b^4 + 65*a^4*b^5 -
23*a^3*b^6 + a^2*b^7 + a*b^8) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*b^3)*(a - b)^2 - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 12*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^2*b^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*
b + sqrt(a*b)*(a - b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^2 - a*b - sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 -
 2*a*b + b^2)))/(a^2 - 2*a*b + b^2))))/(3*a^8*b - 21*a^7*b^2 + 59*a^6*b^3 -
 85*a^5*b^4 + 65*a^4*b^5 - 23*a^3*b^6 + a^2*b^7 + a*b^8) - 2*(2*tan(d*x + c
)^3 + tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c
)^2 + a)*(a - b)))/d
```

maple [B] time = 0.27, size = 478, normalized size = 2.45

$$-\frac{\tan^{3}(dx+c)}{2d\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)b+2a\left(\tan^{2}(dx+c)\right)+a\right)(a-b)}-\frac{\tan(dx}{4d\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)^
3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)
-1/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan
(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/8/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2
)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*
b+1/4/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*
b)^(1/2)-a)*(a-b))^(1/2))+1/8/d*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/8/d/(a*b)^(
1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1
/2)+a)*(a-b))^(1/2))*b+1/4/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 15.99, size = 2980, normalized size = 15.28

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (atan(((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d*
x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3
*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 +
768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^
2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*b^3
)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4
+ 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i - (((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)
/(32*(a - b)) + (tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a
^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*
(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(
a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a
^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (tan(c + d*x)*(6*a*b + a^2 + b^2))/
(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)
/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i)/((((128*a*b^3
 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) - (tan(c + d*x)*((3*a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*
a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4
*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a
^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c +
 d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)
^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2
)))^(1/2) + (((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) + (tan(c +
 d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(
a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4
 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*
b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6
*b^2)))^(1/2) + (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*((3*a*(a^3*
b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b
^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - 1/(16*(a - b))))*((3*a*(a^3*b^3)^(1/2)
```

```
+ b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*
b^3 - a^6*b^2)))^(1/2)*2i)/d - (atan(((((128*a*b^3 + 128*a^3*b - 256*a^2*b^
2)/(32*(a - b)) - (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2)
- a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/
2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*(-(3
*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 -
 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^
2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2
*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*1i - (((128*
a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b)) + (tan(c + d*x)*(-(3*a*(a^3*b
^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^
4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 7
68*a^4*b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*
b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (
tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(
a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 -
 a^6*b^2)))^(1/2)*1i)/((((128*a*b^3 + 128*a^3*b - 256*a^2*b^2)/(32*(a - b))
 - (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2
*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*(256*a^5*b -
 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*
a^5*b^3 - a^6*b^2)))^(1/2) - (tan(c + d*x)*(6*a*b + a^2 + b^2))/(4*(a - b))
)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3
*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (((128*a*b^3 + 128*a^3*b
- 256*a^2*b^2)/(32*(a - b)) + (tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3
*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^
6*b^2)))^(1/2)*(256*a^5*b - 256*a^2*b^4 + 768*a^3*b^3 - 768*a^4*b^2))/(4*(a
 - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(25
6*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2) + (tan(c + d*x)*(6*a*
b + a^2 + b^2))/(4*(a - b)))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a
^3*b - 3*a^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)
- 1/(16*(a - b))))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a
^2*b^2)/(256*(a^3*b^5 - 3*a^4*b^4 + 3*a^5*b^3 - a^6*b^2)))^(1/2)*2i)/d - (t
an(c + d*x)^3/(2*(a - b)) + tan(c + d*x)/(4*(a - b)))/(d*(a + 2*a*tan(c + d
*x)^2 + tan(c + d*x)^4*(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.221 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=219

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)\left((a+b)\tan\right.}{4ad(a-b)\left((a-b)\tan^4(c+d\right.}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))/
a^(5/4)/d/(a^(1/2)-b^(1/2))^(3/2)/b^(1/2)-1/8*arctan((a^(1/2)+b^(1/2))^(1/2
)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(5/4)/d/b^(1/2)/(a^(1/2)+b^(1/2
))^(3/2)-1/4*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x+c)^
2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.30, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3217, 1333, 1166, 205}

$$\frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{\tan(c+dx)\left((a+b)\tan\right.}{4ad(a-b)\left((a-b)\tan^4(c+d\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4
)])/(8*a^(5/4)*(Sqrt[a] - Sqrt[b])^(3/2)*Sqrt[b]*d) - ((2*Sqrt[a] + Sqrt[b]
)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(5/4)*(Sqrt[
a] + Sqrt[b])^(3/2)*Sqrt[b]*d) - (Tan[c + d*x]*(a + (a + b)*Tan[c + d*x]^2)
)/(4*a*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
```

```
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)^2}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b}{a-b}-\frac{2a(3a-b)bx^2}{a-b}}{a+2ax^2+(a-b)x^4}\right.}{8a^2bc}$$

$$= -\frac{\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(2a+\sqrt{a}\sqrt{b}-b\right)\text{Subs}}{8a}$$

$$= \frac{\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\sqrt{b}\,d} - \frac{\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\sqrt{b}\,d} -$$

Mathematica [A] time = 2.14, size = 255, normalized size = 1.16

$$\frac{-\frac{\sqrt{a}\left(-\sqrt{a}\sqrt{b}+2a-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{4\sqrt{a}\sin(2(c+dx))(2a-b\cos(2(c+dx))+b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} - \frac{\sqrt{a}\left(\sqrt{a}\sqrt{b}+2a-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-((Sqrt[a]*(2*a - Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
 d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt[b])) - (
Sqrt[a]*(2*a + Sqrt[a]*Sqrt[b] - b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*
x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[-a + Sqrt[a]*Sqrt[b]]*Sqrt[b]) - (4*
Sqrt[a]*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*C
os[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(8*a^(3/2)*(a - b)*d)
```

fricas [B] time = 1.18, size = 3445, normalized size = 15.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] 1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^
2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*
```

```
b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b -
6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^
4)) + 4*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*l
og(8*a^3 - 7*a^2*b + 9/4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2
 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^
4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a
^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5
*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos
(d*x + c)*sin(d*x + c))*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^
2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*
b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4
*a^2 + a*b - b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2
*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 -
 (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 -
80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3
- 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) - ((a^2*b - a*b^2)*
d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*
b^2)*d)*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 -
 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3
 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2
)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2))*log(8*a^3 - 7*a^2*b + 9/
4*a*b^2 - 1/4*b^3 - 1/4*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2
- 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((
64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15
*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c
)*sin(d*x + c) - 2*(8*a^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))
*sqrt(-((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3
*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a
^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) + 4*a^2 + a*b - b^2)/((a^5
*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15
*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15
*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 -
 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b
^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) + ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a
^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b -
3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 -
 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b
^5 - 6*a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 +
3*a^3*b^3 - a^2*b^4)*d^2))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4
*(32*a^3 - 28*a^2*b + 9*a*b^2 - b^3)*cos(d*x + c)^2 + 1/2*((3*a^8*b - 10*a^
7*b^2 + 12*a^6*b^3 - 6*a^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*
a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 +
 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a
^5 - 5*a^4*b + a^3*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b
^2 + 3*a^3*b^3 - a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b
^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*
a^6*b^6 + a^5*b^7)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b
^3 - a^2*b^4)*d^2)) + 1/4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2*cos(d*x + c)^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a
^3*b^4)*d^2)*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4))) - ((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x
+ c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 -
a^2*b^4)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*
b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7
)*d^4)) - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2
))*log(-8*a^3 + 7*a^2*b - 9/4*a*b^2 + 1/4*b^3 + 1/4*(32*a^3 - 28*a^2*b + 9*
a*b^2 - b^3)*cos(d*x + c)^2 - 1/2*((3*a^8*b - 10*a^7*b^2 + 12*a^6*b^3 - 6*a
^5*b^4 + a^4*b^5)*d^3*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4
```

```
)/((a^11*b - 6*a^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6
+ a^5*b^7)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(8*a^5 - 5*a^4*b + a^3*b^2)*
d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4
)*d^2*sqrt((64*a^4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a
^10*b^2 + 15*a^9*b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))
 - 4*a^2 - a*b + b^2)/((a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d^2)) + 1/
4*(2*(4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2*cos(d*x + c)
^2 - (4*a^7 - 13*a^6*b + 15*a^5*b^2 - 7*a^4*b^3 + a^3*b^4)*d^2)*sqrt((64*a^
4 - 80*a^3*b + 41*a^2*b^2 - 10*a*b^3 + b^4)/((a^11*b - 6*a^10*b^2 + 15*a^9*
b^3 - 20*a^8*b^4 + 15*a^7*b^5 - 6*a^6*b^6 + a^5*b^7)*d^4))) - 8*(b*cos(d*x
+ c)^3 - (a + b)*cos(d*x + c))*sin(d*x + c))/((a^2*b - a*b^2)*d*cos(d*x + c
)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)
```

giac [B] time = 1.12, size = 1407, normalized size = 6.42

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^4)*(a^2 - a*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6
*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*a^4*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^4 -
 sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^5)*abs(-a^2 + a*b)*abs(-a + b) -
 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8 - 12*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^6*b^2 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5
*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^4)*abs(-a + b))*
(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3 - a^2*b + sqr
t((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^2*b + a*b^2)))/(a^3 - 2*a^2*b
+ a*b^2))))/((3*a^10*b - 21*a^9*b^2 + 59*a^8*b^3 - 85*a^7*b^4 + 65*a^6*b^5
- 23*a^5*b^6 + a^4*b^7 + a^3*b^8)*abs(-a^2 + a*b)) - ((9*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 21*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^2 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 +
 sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^2 - a*b)^2*abs(-a +
b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b - 12*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a^5*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^3 -
4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^2*b^5)*abs(-a^2 + a*b)*abs(-a + b) - 2*(3*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*a^8 - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^7*b + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^2 - 4*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^3 - sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^4*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2
) + arctan(tan(d*x + c)/sqrt((a^3 - a^2*b - sqrt((a^3 - a^2*b)^2 - (a^3 - a
^2*b)*(a^3 - 2*a^2*b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10*b - 21*
a^9*b^2 + 59*a^8*b^3 - 85*a^7*b^4 + 65*a^6*b^5 - 23*a^5*b^6 + a^4*b^7 + a^3
*b^8)*abs(-a^2 + a*b)) + 2*(a*tan(d*x + c)^3 + b*tan(d*x + c)^3 + a*tan(d*x
 + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)*(a^2
 - a*b)))/d
```

maple [B] time = 0.45, size = 534, normalized size = 2.44

$$-\frac{\tan^{3}(dx+c)}{4d\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right)b+2a\left(\tan^{2}(dx+c)\right)+a\right)(a-b)}-\frac{\left(\tan^{3}(d\right.}{4d\left(\left(\tan^{4}(dx+c)\right)a-\left(\tan^{4}(dx+c)\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d*x+c)^
3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)/a*tan(d*x+
c)^3*b-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)*tan(d
*x+c)+3/8/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))-1/8/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*a
rctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-1/4/d*a/(a*b)^(1/
2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))+3/8/d/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)
*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/8/d/a/(a-b)/(((a*b)^(1/2)+a)*(
a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+1/4/d*
a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(
((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((2*a*b
 - b^2)*sin(6*d*x + 6*c) - (8*a*b - 3*b^2)*sin(4*d*x + 4*c) - (2*a*b + 3*b^
2)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*((16*a^2 + 2*a*b - 3*b^2)*sin(4*d
*x + 4*c) + 4*(2*a*b + b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^2*b^
2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c)^2
 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 + 16*
(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b +
57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a
*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(2*d*
x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)*d -
 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 + 3*a*
b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2
 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos
(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b^3)*
d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x + 2
*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*b^2
- a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*
a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(2*a*b - b^2)*cos(6*d*x + 6
*c)^2 - 4*(32*a^2 - 20*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(2*a*b - b^2)*co
s(2*d*x + 2*c)^2 + 4*(2*a*b - b^2)*sin(6*d*x + 6*c)^2 - 4*(32*a^2 - 20*a*b
+ 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c)*
sin(2*d*x + 2*c) + 4*(2*a*b - b^2)*sin(2*d*x + 2*c)^2 - ((2*a*b - b^2)*cos(
6*d*x + 6*c) - 2*(4*a*b - b^2)*cos(4*d*x + 4*c) + (2*a*b - b^2)*cos(2*d*x +
 2*c))*cos(8*d*x + 8*c) - (2*a*b - b^2 - 2*(16*a^2 - 30*a*b + 7*b^2)*cos(4*
d*x + 4*c) - 8*(2*a*b - b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(4*a*b
- b^2 + (16*a^2 - 30*a*b + 7*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (2*a
*b - b^2)*cos(2*d*x + 2*c) - ((2*a*b - b^2)*sin(6*d*x + 6*c) - 2*(4*a*b - b
^2)*sin(4*d*x + 4*c) + (2*a*b - b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2
*((16*a^2 - 30*a*b + 7*b^2)*sin(4*d*x + 4*c) + 4*(2*a*b - b^2)*sin(2*d*x +
2*c))*sin(6*d*x + 6*c))/(a^2*b^2 - a*b^3 + (a^2*b^2 - a*b^3)*cos(8*d*x + 8*
c)^2 + 16*(a^2*b^2 - a*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57
*a^2*b^2 - 9*a*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^2 - a*b^3)*cos(2*d*x + 2
*c)^2 + (a^2*b^2 - a*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*sin(6*d
```

```
*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*sin(4*d*x + 4*c
)^2 + 16*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - a*b^3 - 4*(a^2*b^
2 - a*b^3)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x
+ 4*c) - 4*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^
2 - a*b^3 - 2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(4*d*x + 4*c) - 4*(a^2*b^
2 - a*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 11*a^2*b^2 + 3
*a*b^3 - 4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4
*c) - 8*(a^2*b^2 - a*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^2 - a*b^3)*sin(6*d
*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2
- a*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b - 11*a^2*b^2 + 3
*a*b^3)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (b^2 + (2*a*b - b^2)*cos(6*d*x + 6*c) - (8*a*b - 3*b^2)*cos(4
*d*x + 4*c) - (2*a*b + 3*b^2)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (2*a*b +
 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2)*cos(4*d*x + 4*c) - 8*(2*a*b + b^2)*cos(
2*d*x + 2*c))*sin(6*d*x + 6*c) + (8*a*b - 3*b^2 - 2*(16*a^2 + 2*a*b - 3*b^2
)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (2*a*b - b^2)*sin(2*d*x + 2*c))/((a^
2*b^2 - a*b^3)*d*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*
c)^2 + 4*(64*a^4 - 112*a^3*b + 57*a^2*b^2 - 9*a*b^3)*d*cos(4*d*x + 4*c)^2 +
 16*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c)^2 + (a^2*b^2 - a*b^3)*d*sin(8*d*x
+ 8*c)^2 + 16*(a^2*b^2 - a*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^4 - 112*a^3*
b + 57*a^2*b^2 - 9*a*b^3)*d*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b - 11*a^2*b^2 +
 3*a*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^2 - a*b^3)*d*sin(
2*d*x + 2*c)^2 - 8*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) + (a^2*b^2 - a*b^3)
*d - 2*(4*(a^2*b^2 - a*b^3)*d*cos(6*d*x + 6*c) + 2*(8*a^3*b - 11*a^2*b^2 +
3*a*b^3)*d*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2
*b^2 - a*b^3)*d)*cos(8*d*x + 8*c) + 8*(2*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d
*cos(4*d*x + 4*c) + 4*(a^2*b^2 - a*b^3)*d*cos(2*d*x + 2*c) - (a^2*b^2 - a*b
^3)*d)*cos(6*d*x + 6*c) + 4*(4*(8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*cos(2*d*x
 + 2*c) - (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d)*cos(4*d*x + 4*c) - 4*(2*(a^2*
b^2 - a*b^3)*d*sin(6*d*x + 6*c) + (8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*
d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*
((8*a^3*b - 11*a^2*b^2 + 3*a*b^3)*d*sin(4*d*x + 4*c) + 2*(a^2*b^2 - a*b^3)*
d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 17.35, size = 3842, normalized size = 17.54

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (tan(c + d*x)/(4*(a - b)) + (tan(c + d*x)^3*(a + b))/(4*a*(a - b)))/(d*(a
 + 2*a*tan(c + d*x)^2 + tan(c + d*x)^4*(a - b))) - (atan(((((256*a^5*b + 25
6*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*(-(8*a^2*(a^5*b
^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*
b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a
^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*
(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b
*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)
- (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^
2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a
*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2
)*1i - (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) + (tan(
c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3
 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 -
 a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4
*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a
^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7
*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/
```

(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*1i)/((((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (12*a^2 - 7*a*b + b^2)/(32*(a^2*b - a^3)) + (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) + (tan(c + d*x)*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)))*(-(8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) - 4*a^5*b + a^3*b^3 - a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*2i)/d - (atan(((((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*1i - (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) + (tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*1i)/((((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) - (tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a^3 + b^3))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) - (12*a^2 - 7*a*b + b^2)/(32*(a^2*b - a^3)) + (((256*a^5*b + 256*a^3*b^3 - 512*a^4*b^2)/(64*(a^2*b - a^3)) + (tan(c + d*x)*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*(256*a^6*b - 256*a^3*b^4 + 768*a^4*b^3 - 768*a^5*b^2))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2) + (tan(c + d*x)*(9*a^2*b - 6*a*b^2 + 4*a

```
^3 + b^3))/(4*(a*b - a^2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^3)^(1/2) +
 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b^5 - 3*a^6
*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)))*((8*a^2*(a^5*b^3)^(1/2) + b^2*(a^5*b^
3)^(1/2) + 4*a^5*b - a^3*b^3 + a^4*b^2 - 5*a*b*(a^5*b^3)^(1/2))/(256*(a^5*b
^5 - 3*a^6*b^4 + 3*a^7*b^3 - a^8*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.222 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=210

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left(2\right.}{4ad(a-b)\left((a-b)\tan^4(c\right.}$$

```
[Out] 1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a^(1/2)-3*b^(1/2)
)/a^(7/4)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(
d*x+c)/a^(1/4))*(4*a^(1/2)+3*b^(1/2))/a^(7/4)/d/(a^(1/2)+b^(1/2))^(3/2)-1/4
*b*tan(d*x+c)*(1+2*tan(d*x+c)^2)/a/(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*
x+c)^4)
```

Rubi [A] time = 0.26, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3209, 1205, 1166, 205}

$$\frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)\left(2\right.}{4ad(a-b)\left((a-b)\tan^4(c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] ((4*Sqrt[a] - 3*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(8*a^(7/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + ((4*Sqrt[a] + 3*Sqrt[b])*Ar
cTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^(7/4)*(Sqrt[a] +
 Sqrt[b])^(3/2)*d) - (b*Tan[c + d*x]*(1 + 2*Tan[c + d*x]^2))/(4*a*(a - b)*d
*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
```

```
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-\frac{2a(4a-3b)b}{a-b}-\frac{4a(2a-}{a-}}{a+2ax^2+(a-b)}\right.}{8a^2}$$

$$= -\frac{b\tan(c+dx)\left(1+2\tan^2(c+dx)\right)}{4a(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(4a-\sqrt{a}\sqrt{b}-3b\right)\text{Sub}}{8}$$

$$= \frac{\left(4\sqrt{a}-3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\left(4\sqrt{a}+3\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}$$

Mathematica [A] time = 2.87, size = 230, normalized size = 1.10

$$\frac{\frac{\left(-\sqrt{a}\sqrt{b}+4a-3b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{2\sqrt{a}\,b(\sin(4(c+dx))-6\sin(2(c+dx)))}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b}-\frac{\left(\sqrt{a}\sqrt{b}+4a-3b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{8a^{3/2}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-2),x]
```

```
[Out] (((4*a - Sqrt[a]*Sqrt[b] - 3*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] - ((4*a + Sqrt[a]*Sqrt
[b] - 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqr
t[b]]])/Sqrt[-a + Sqrt[a]*Sqrt[b]] + (2*Sqrt[a]*b*(-6*Sin[2*(c + d*x)] + Si
n[4*(c + d*x)]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]))/(
8*a^(3/2)*(a - b)*d)
```

fricas [B] time = 1.20, size = 3477, normalized size = 16.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)
^2 - (a^3 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3
)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/(
```

(a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7
*b^6)*d^4)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^
3)*d^2))*log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3
*b - 680*a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*
a^9*b + 9*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c
) - (120*a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b
 - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15
*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2
- 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16
*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 -
 (16*a^8 - 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a
^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b
+ 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((
a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3
 - 2*a^2*b + a*b^2)*d)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqr
t((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6
*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4
)) + 16*a^2 - 15*a*b + 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*
log(96*a^3*b - 170*a^2*b^2 + 405/4*a*b^3 - 81/4*b^4 - 1/4*(384*a^3*b - 680*
a^2*b^2 + 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9
*a^8*b^2 - 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a
^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3
+ 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (120*
a^5*b - 217*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(-((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a
^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2
 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) + 16*a^2 - 15*a*b
+ 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57
*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8
- 57*a^7*b + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 13
92*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11
*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) + ((a^2*b - a
*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*
b + a*b^2)*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4
*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b +
15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^
2 + 15*a*b - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^
3*b + 170*a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 +
 405*a*b^3 - 81*b^4)*cos(d*x + c)^2 + 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2
- 5*a^7*b^3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 -
522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*
b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (120*a^5*b - 2
17*a^4*b^2 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((
a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1
273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10
*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/(
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57*a^7*b + 7
5*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8 - 57*a^7*b
 + 75*a^6*b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2
 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*
a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) - ((a^2*b - a*b^2)*d*co
s(d*x + c)^4 - 2*(a^2*b - a*b^2)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)
*d)*sqrt(((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*
a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^
2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b
 - 3*b^2)/((a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^2))*log(-96*a^3*b + 170*

```
a^2*b^2 - 405/4*a*b^3 + 81/4*b^4 + 1/4*(384*a^3*b - 680*a^2*b^2 + 405*a*b^3
 - 81*b^4)*cos(d*x + c)^2 - 1/2*(2*(2*a^10 - 7*a^9*b + 9*a^8*b^2 - 5*a^7*b^
3 + a^6*b^4)*d^3*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^3 - 522*a*b^4
+ 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*a^9*b^4 - 6*a^
8*b^5 + a^7*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (120*a^5*b - 217*a^4*b^2
 + 132*a^3*b^3 - 27*a^2*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^6 - 3*a^
5*b + 3*a^4*b^2 - a^3*b^3)*d^2*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^2*b^
3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 + 15*
a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4)) - 16*a^2 + 15*a*b - 3*b^2)/((a^6 - 3*a
^5*b + 3*a^4*b^2 - a^3*b^3)*d^2)) + 1/4*(2*(16*a^8 - 57*a^7*b + 75*a^6*b^2
- 43*a^5*b^3 + 9*a^4*b^4)*d^2*cos(d*x + c)^2 - (16*a^8 - 57*a^7*b + 75*a^6*
b^2 - 43*a^5*b^3 + 9*a^4*b^4)*d^2)*sqrt((576*a^4*b - 1392*a^3*b^2 + 1273*a^
2*b^3 - 522*a*b^4 + 81*b^5)/((a^13 - 6*a^12*b + 15*a^11*b^2 - 20*a^10*b^3 +
 15*a^9*b^4 - 6*a^8*b^5 + a^7*b^6)*d^4))) + 8*(b*cos(d*x + c)^3 - 2*b*cos(d
*x + c))*sin(d*x + c))/((a^2*b - a*b^2)*d*cos(d*x + c)^4 - 2*(a^2*b - a*b^2
)*d*cos(d*x + c)^2 - (a^3 - 2*a^2*b + a*b^2)*d)
```

giac [B] time = 0.44, size = 1506, normalized size = 7.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*((2*(6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
3)*(a^2 - a*b)^2*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6
- 57*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b + 92*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a^4*b^2 - 58*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^3 + 8*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^4 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^5)*abs(-a^2 + a*b)*abs(-a + b) - (15*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^7 - 69*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
a^6*b + 106*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 - 62*sqrt
(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 + 7*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^2*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a^3 - a^2*b + sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^
2*b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10 - 21*a^9*b + 59*a^8*b^2
- 85*a^7*b^3 + 65*a^6*b^4 - 23*a^5*b^5 + a^4*b^6 + a^3*b^7)*abs(-a^2 + a*b)
) - (2*(6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 15*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^2 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)
*(a^2 - a*b)^2*abs(-a + b) + (12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6 -
57*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b + 92*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^4*b^2 - 58*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^3 + 8*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^4 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a*b^5)*abs(-a^2 + a*b)*abs(-a + b) - (15*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^7 - 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
6*b + 106*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^2 - 62*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^3 + 7*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a^3*b^4 + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^2*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x
+ c)/sqrt((a^3 - a^2*b - sqrt((a^3 - a^2*b)^2 - (a^3 - a^2*b)*(a^3 - 2*a^2*
b + a*b^2)))/(a^3 - 2*a^2*b + a*b^2))))/((3*a^10 - 21*a^9*b + 59*a^8*b^2 -
85*a^7*b^3 + 65*a^6*b^4 - 23*a^5*b^5 + a^4*b^6 + a^3*b^7)*abs(-a^2 + a*b))
+ 2*(2*b*tan(d*x + c)^3 + b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x +
c)^4 + 2*a*tan(d*x + c)^2 + a)*(a^2 - a*b)))/d
```

maple [B] time = 0.40, size = 618, normalized size = 2.94

$$-\frac{\left(\tan^3(dx+c)\right)b}{2d\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx+c)\right)b+2a\left(\tan^2(dx+c)\right)+a\right)(a-b)a}-\frac{b\,\mathrm{t}}{4d\left(\left(\tan^4(dx+c)\right)a-\left(\tan^4(dx\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/2/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)/(a-b)/a*tan(d*x+c
)^3*b-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)*b/a/(a-b)*ta
n(d*x+c)+1/2/d/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c
)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/4/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2
)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-5/8/d/(a*b)^(1
/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(
1/2)-a)*(a-b))^(1/2))*b+3/8/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^2+1/2/d/(a-
b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))-1/4/d/a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d
*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b+5/8/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2
)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-
3/8/d/a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 16.52, size = 3675, normalized size = 17.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4)^2,x)
```

```
[Out] - (atan(((((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3
*b - a^4)) - (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 1
5*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 +
 a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*
a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) -
 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10
 + a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^
4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(
1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b
- a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i - (((512*a^6*b - 384*a^3*b^4 + 128
0*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*((24*a^2*(a^7*
b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^
7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b
- 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^
7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(
a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c
+ d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a
```

```
^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29
*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i)
/((32*a^2*b - 34*a*b^2 + 9*b^3)/(16*(a^3*b - a^4)) + (((512*a^6*b - 384*a^3
*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)*((24
*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 -
29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(
256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*((
24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2
- 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)
 - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3
)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^
4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))
^(1/2) + (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3
*b - a^4)) + (tan(c + d*x)*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) - 1
5*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 +
 a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*
a^6*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) -
 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10
 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^
4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(
1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b
- a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)))*((24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7
*b)^(1/2) - 15*a^5*b + 16*a^6 + 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a
^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i)/d - (atan(((((512*a^6*b - 38
4*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)
*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*
b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(
1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3
)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a
^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2))
)^(1/2) - (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*
b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^
6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^
8*b^2)))^(1/2)*1i - (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^
2)/(32*(a^3*b - a^4)) + (tan(c + d*x)*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*
b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^
9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^
5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a
^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3
*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26
*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9
*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/
(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*1i)/((32*a^2*b - 34*a*b
^2 + 9*b^3)/(16*(a^3*b - a^4)) + (((512*a^6*b - 384*a^3*b^4 + 1280*a^4*b^3
- 1408*a^5*b^2)/(32*(a^3*b - a^4)) - (tan(c + d*x)*(-(24*a^2*(a^7*b)^(1/2)
+ 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2
))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2)*(256*a^7*b - 256*a^4
*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/
2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(
1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) - (tan(c + d*x)*(
16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7
*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a
^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8*b^2)))^(1/2) + (((512*a
^6*b - 384*a^3*b^4 + 1280*a^4*b^3 - 1408*a^5*b^2)/(32*(a^3*b - a^4)) + (tan
(c + d*x)*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*a^6
 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*a^8
*b^2)))^(1/2)*(256*a^7*b - 256*a^4*b^4 + 768*a^5*b^3 - 768*a^6*b^2))/(4*(a^
2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*b - 16*
a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b^3 - 3*
```

```
a^8*b^2)))^(1/2) + (tan(c + d*x)*(16*a^3*b - 26*a*b^3 + 9*b^4 + 9*a^2*b^2))
/(4*(a^2*b - a^3)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 15*a^5*
b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 + a^7*b
^3 - 3*a^8*b^2)))^(1/2)))*(-(24*a^2*(a^7*b)^(1/2) + 9*b^2*(a^7*b)^(1/2) + 1
5*a^5*b - 16*a^6 - 3*a^4*b^2 - 29*a*b*(a^7*b)^(1/2))/(256*(3*a^9*b - a^10 +
 a^7*b^3 - 3*a^8*b^2)))^(1/2)*2i)/d - ((b*tan(c + d*x)^3)/(2*a*(a - b)) + (
b*tan(c + d*x))/(4*a*(a - b)))/(d*(a + 2*a*tan(c + d*x)^2 + tan(c + d*x)^4*
(a - b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**2,x)
```

```
[Out] Timed out
```

3.223 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx$

Optimal. Leaf size=236

$$\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)}{4a^2d(a-b)\left((a-b)\mathrm{ta}\right.}$$

```
[Out] -cot(d*x+c)/a^2/d+1/8*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(6
*a^(1/2)-5*b^(1/2))*b^(1/2)/a^(9/4)/d/(a^(1/2)-b^(1/2))^(3/2)-1/8*arctan((a
^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(6*a^(1/2)+5*b^(1/2))/a^(
9/4)/d/(a^(1/2)+b^(1/2))^(3/2)-1/4*b*tan(d*x+c)*(a+(a+b)*tan(d*x+c)^2)/a^2/
(a-b)/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.53, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1334, 1664, 1166, 205}

$$\frac{\sqrt{b}\left(6\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt{b}\left(6\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}-\frac{b\tan(c+dx)}{4a^2d(a-b)\left((a-b)\mathrm{ta}\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] ((6*Sqrt[a] - 5*Sqrt[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*
x])/a^(1/4)])/(8*a^(9/4)*(Sqrt[a] - Sqrt[b])^(3/2)*d) - ((6*Sqrt[a] + 5*Sqr
t[b])*Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(8*a^
(9/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) - Cot[c + d*x]/(a^2*d) - (b*Tan[c + d*x]
*(a + (a + b)*Tan[c + d*x]^2))/(4*a^2*(a - b)*d*(a + 2*a*Tan[c + d*x]^2 + (
a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
```

```
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{x^2\left(a+2ax^2+(a-b)x^4\right)^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-8ab-\frac{2a(8a-\cdots}{a-\cdots}}{x^2(a+\cdots}\right.}{\cdots}$$

$$= -\frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{\text{Subst}\left(\int\left(-\frac{8b}{x^2}+\frac{2\cdots}{(a-\cdots}\right.\right.}{\cdots}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} - \frac{b\,\text{Sub}\cdots}{\cdots}$$

$$= -\frac{\cot(c+dx)}{a^2d} - \frac{b\tan(c+dx)\left(a+(a+b)\tan^2(c+dx)\right)}{4a^2(a-b)d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)} + \frac{\left(\left(7a\cdots\right.\right.}{\cdots}$$

$$= \frac{\left(6\sqrt{a}-5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{8a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} - \frac{\left(6\sqrt{a}+5\sqrt{b}\right)\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{\cdots}}}{\cdots}\right)}{8a^{9/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d}\cdots$$

Mathematica [A] time = 2.28, size = 274, normalized size = 1.16

$$\frac{-\frac{\left(6a\sqrt{b}+5\sqrt{a}b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{4\sqrt{a}b\sin(2(c+dx))(2a-b\cos(2(c+dx))+b)}{(a-b)(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} - \frac{\left(6a\sqrt{b}-5\sqrt{a}b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\left(\sqrt{a}-\sqrt{b}\right)\sqrt{\sqrt{a}\sqrt{b}-a}} - \cdots}{8a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^2,x]
```

```
[Out] (-(((6*a*Sqrt[b] + 5*Sqrt[a]*b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/S
qrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])*Sqrt[a + Sqrt[a]*Sqrt[b]]))
```

```
 - ((6*a*Sqrt[b] - 5*Sqrt[a]*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/
Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] - Sqrt[b])*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) - 8*Sqrt[a]*Cot[c + d*x] - (4*Sqrt[a]*b*(2*a + b - b*Cos[2*(c + d*x)])*S
in[2*(c + d*x)])/((a - b)*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c +
d*x)])))/(8*a^(5/2)*d)
```

fricas [B] time = 1.49, size = 3648, normalized size = 15.46

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="fricas")
```

```
[Out] -1/32*(8*(4*a*b - 5*b^2)*cos(d*x + c)^5 - 8*(7*a*b - 10*b^2)*cos(d*x + c)^3
 - ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)
^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b
^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625
*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*
b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^
5*b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4
*b^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c
)^2 + 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*s
qrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a
^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9
*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^
4*b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*
a^5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3
450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^1
1*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 -
 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^
7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b
+ 183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^
3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^
2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) +
 ((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2
 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(-((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3
)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b
^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^
5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*
b^2 - a^4*b^3)*d^2))*log(432*a^3*b^2 - 921*a^2*b^3 + 2625/4*a*b^4 - 625/4*b
^5 - 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^
2 - 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqr
t((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^1
5 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b
^6)*d^4))*cos(d*x + c)*sin(d*x + c) - 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*
b^3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^7 - 3*a^6*b + 3*a^
5*b^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 345
0*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*
b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)) + 36*a^2*b - 47*a*b^2 + 15*b^3)/((a^7 - 3
*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*
b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b +
183*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*
b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2
- 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) - (
(a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 -
 (a^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d
^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)
/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 +
 a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2
 - a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 - 2625/4*a*b^4 + 625/4*b^5
```

```
 + 1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2
+ 1/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt(
(2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15
- 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6
)*d^4))*cos(d*x + c)*sin(d*x + c) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^
3 - 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b
^2 - a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a
*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4
 - 6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^
6*b + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2
 - 111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183
*a^7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4
 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 2
0*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) + ((a^
3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a
^4 - 2*a^3*b + a^2*b^2)*d)*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2 - a^4*b^3)*d^2*
sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((
a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^
9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b + 3*a^5*b^2 -
a^4*b^3)*d^2))*log(-432*a^3*b^2 + 921*a^2*b^3 - 2625/4*a*b^4 + 625/4*b^5 +
1/4*(1728*a^3*b^2 - 3684*a^2*b^3 + 2625*a*b^4 - 625*b^5)*cos(d*x + c)^2 - 1
/2*((7*a^11 - 26*a^10*b + 36*a^9*b^2 - 22*a^8*b^3 + 5*a^7*b^4)*d^3*sqrt((23
04*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6
*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d
^4))*cos(d*x + c)*sin(d*x + c) + 2*(144*a^6*b - 303*a^5*b^2 + 213*a^4*b^3 -
 50*a^3*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^7 - 3*a^6*b + 3*a^5*b^2
- a^4*b^3)*d^2*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 + 7161*a^2*b^5 - 3450*a*b^
6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a^12*b^3 + 15*a^11*b^4 -
6*a^10*b^5 + a^9*b^6)*d^4)) - 36*a^2*b + 47*a*b^2 - 15*b^3)/((a^7 - 3*a^6*b
 + 3*a^5*b^2 - a^4*b^3)*d^2)) + 1/4*(2*(36*a^9 - 133*a^8*b + 183*a^7*b^2 -
111*a^6*b^3 + 25*a^5*b^4)*d^2*cos(d*x + c)^2 - (36*a^9 - 133*a^8*b + 183*a^
7*b^2 - 111*a^6*b^3 + 25*a^5*b^4)*d^2)*sqrt((2304*a^4*b^3 - 6624*a^3*b^4 +
7161*a^2*b^5 - 3450*a*b^6 + 625*b^7)/((a^15 - 6*a^14*b + 15*a^13*b^2 - 20*a
^12*b^3 + 15*a^11*b^4 - 6*a^10*b^5 + a^9*b^6)*d^4)))*sin(d*x + c) - 8*(4*a^
2 - 7*a*b + 5*b^2)*cos(d*x + c))/(((a^3*b - a^2*b^2)*d*cos(d*x + c)^4 - 2*(
a^3*b - a^2*b^2)*d*cos(d*x + c)^2 - (a^4 - 2*a^3*b + a^2*b^2)*d)*sin(d*x +
c))
```

giac [B] time = 1.13, size = 1545, normalized size = 6.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="giac")
```

```
[Out] -1/8*(((21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*b^4)*(a^3 - a^2*b)^2*abs(-a + b) - (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b
))*a^7*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^2 + 14*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^5*b^3 - 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4
*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^5)*abs(-a^3 + a^2*b)*abs(-
a + b) - 2*(9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 42*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 66*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^8*b^2 - 40*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqr
t(a*b)*a^7*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 +
2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5)*abs(-a + b))*(pi*f
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - a^3*b + sqrt((a^
4 - a^3*b)^2 - (a^4 - a^3*b)*(a^4 - 2*a^3*b + a^2*b^2)))/(a^4 - 2*a^3*b + a
^2*b^2))))/((3*a^12 - 21*a^11*b + 59*a^10*b^2 - 85*a^9*b^3 + 65*a^8*b^4 - 2
```

```
3*a^7*b^5 + a^6*b^6 + a^5*b^7)*abs(-a^3 + a^2*b)) - ((21*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 57*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*a^2*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3
+ 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3 - a^2*b)^2*abs(
-a + b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b - 12*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*a^6*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b
^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^4 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*a^3*b^5)*abs(-a^3 + a^2*b)*abs(-a + b) - 2*(9*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 42*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^9*b + 66*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^
2 - 40*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^3 + 5*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 2*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^5*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^4 - a^3*b - sqrt((a^4 - a^3*b)^2 - (a^4 - a^3*b)*
(a^4 - 2*a^3*b + a^2*b^2)))/(a^4 - 2*a^3*b + a^2*b^2))))/((3*a^12 - 21*a^11
*b + 59*a^10*b^2 - 85*a^9*b^3 + 65*a^8*b^4 - 23*a^7*b^5 + a^6*b^6 + a^5*b^7
)*abs(-a^3 + a^2*b)) + 2*(4*a^2*tan(d*x + c)^4 - 7*a*b*tan(d*x + c)^4 + 5*b
^2*tan(d*x + c)^4 + 8*a^2*tan(d*x + c)^2 - 7*a*b*tan(d*x + c)^2 + 4*a^2 - 4
*a*b)/((a*tan(d*x + c)^5 - b*tan(d*x + c)^5 + 2*a*tan(d*x + c)^3 + a*tan(d*
x + c))*(a^3 - a^2*b)))/d
```

maple [B] time = 0.53, size = 708, normalized size = 3.00

$$-\frac{1}{d\,a^{2}\tan\left(dx+c\right)}-\frac{\left(\tan^{3}\left(dx+c\right)\right)b}{4d\left(\left(\tan^{4}\left(dx+c\right)\right)a-\left(\tan^{4}\left(dx+c\right)\right)b+2a\left(\tan^{2}\left(dx+c\right)\right)+a\right)\left(a-b\right)a}-\frac{}{4d\,a^{2}\left(\left(\tan^{4}\left(dx+\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x)
```

```
[Out] -1/d/a^2/tan(d*x+c)-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a
)/(a-b)/a*tan(d*x+c)^3*b-1/4/d*b^2/a^2/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*t
an(d*x+c)^2+a)/(a-b)*tan(d*x+c)^3-1/4/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*
tan(d*x+c)^2+a)*b/a/(a-b)*tan(d*x+c)+7/8/d/a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^
(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b-5/8/d*b^2/
a^2/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(
1/2)-a)*(a-b))^(1/2))-3/4/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b+1/2/d/a/(a*b)^(
1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))*b^2+7/8/d/a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arct
an((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-5/8/d*b^2/a^2/(a-b)/((
(a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))
^(1/2))+3/4/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b-1/2/d/a/(a*b)^(1/2)/(a-b)/(((a*
b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1
/2))*b^2
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((
6*a*b^2 - 5*b^3)*sin(8*d*x + 8*c) - 2*(13*a*b^2 - 10*b^3)*sin(6*d*x + 6*c)
- 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*sin(4*d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*
sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + (2*(48*a^2*b - 5*a*b^2 - 25*b^3)*sin
```

(6*d*x + 6*c) + 2*(112*a^2*b - 165*a*b^2 + 50*b^3)*sin(4*d*x + 4*c) + 5*(8*a*b^2 - 15*b^3)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 2*(2*(256*a^3 - 432*a^2*b + 210*a*b^2 - 25*b^3)*sin(4*d*x + 4*c) + (112*a^2*b - 165*a*b^2 + 50*b^3)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*((a^3*b^2 - a^2*b^3)*d*cos(10*d*x + 10*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 10*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(5*(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(10*d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(4*d*x + 4*c) - 2*(5*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*sin(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-(4*(6*a*b^2 - 5*b^3)*cos(6*d*x + 6*c)^2 - 4*(64*a^2*b - 64*a*b^2 + 15*b^3)*cos(4*d*x + 4*c)^2 + 4*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(6*a*b^2 - 5*b^3)*sin(6*d*x + 6*c)^2 - 4*(64*a^2*b - 64*a*b^2 + 15*b^3)*sin(4*d*x + 4*c)^2 + 2*(48*a^2*b - 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(6*a*b^2 - 5*b^3)*sin(2*d*x + 2*c)^2 - ((6*a*b^2 - 5*b^3)*cos(6*d*x + 6*c) - 2*(8*a*b^2 - 5*b^3)*cos(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (6*a*b^2 - 5*b^3 - 2*(48*a^2*b - 90*a*b^2 + 35*b^3)*cos(4*d*x + 4*c) - 8*(6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(8*a*b^2 - 5*b^3 + (48*a^2*b - 90*a*b^2 + 35*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (6*a*b^2 - 5*b^3)*cos(2*d*x + 2*c) - ((6*a*b^2 - 5*b^3)*sin(6*d*x + 6*c) - 2*(8*a*b^2 - 5*b^3)*sin(4*d*x + 4*c) + (6*a*b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((48*a^2*b - 90*a*b^2 + 35*b^3)*sin(4*d*x + 4*c) + 4*(6*a*b^2 - 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^3*b^2 - a^2*b^3 + (a^3*b^2 - a^2*b^3)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - a^2*b^3)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 112*a^4*b + 57*a^3*b^2 - 9*a^2*b^3)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - a^2*b^3 - 4*(a^3*b^2 - a^2*b^3)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - a^2*b^3 - 2*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3 - 4*(8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - a^2*b^3)*cos(2*d*x + 2*c) - 4*(2*(a^3*b^2 - a^2*b^3)*sin(6*d*x + 6*c) + (8*a^4*b - 11*a^3*b^2 + 3*a^2*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 11*a^3*b^2 + 3*a^2

```
*b^3)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - a^2*b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) - (4*a*b^2 - 5*b^3 + (6*a*b^2 - 5*b^3)*cos(8*d*x + 8*c) - 2*(13
*a*b^2 - 10*b^3)*cos(6*d*x + 6*c) - 2*(32*a^2*b - 47*a*b^2 + 15*b^3)*cos(4*
d*x + 4*c) - 2*(7*a*b^2 - 10*b^3)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + (1
4*a*b^2 - 20*b^3 - 2*(48*a^2*b - 5*a*b^2 - 25*b^3)*cos(6*d*x + 6*c) - 2*(11
2*a^2*b - 165*a*b^2 + 50*b^3)*cos(4*d*x + 4*c) - 5*(8*a*b^2 - 15*b^3)*cos(2
*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(32*a^2*b - 47*a*b^2 + 15*b^3 - 2*(256*a^
3 - 432*a^2*b + 210*a*b^2 - 25*b^3)*cos(4*d*x + 4*c) - (112*a^2*b - 165*a*b
^2 + 50*b^3)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + 2*(13*a*b^2 - 10*b^3 - (4
8*a^2*b - 5*a*b^2 - 25*b^3)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (6*a*b^2 -
 5*b^3)*sin(2*d*x + 2*c))/((a^3*b^2 - a^2*b^3)*d*cos(10*d*x + 10*c)^2 + 25*
(a^3*b^2 - a^2*b^3)*d*cos(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*
b^2 - 25*a^2*b^3)*d*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^
2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x +
 2*c)^2 + (a^3*b^2 - a^2*b^3)*d*sin(10*d*x + 10*c)^2 + 25*(a^3*b^2 - a^2*b^
3)*d*sin(8*d*x + 8*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)
*d*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d
*sin(4*d*x + 4*c)^2 + 20*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 25*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c)^2 - 10*(a^
3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d - 2*(5*(a^3*b^2
 - a^2*b^3)*d*cos(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos
(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5
*(a^3*b^2 - a^2*b^3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(10*d*x
 + 10*c) + 10*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(6*d*x + 6*c) - 2*
(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^
3)*d*cos(2*d*x + 2*c) + (a^3*b^2 - a^2*b^3)*d)*cos(8*d*x + 8*c) - 4*(2*(64*
a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^2*b^3)*d*cos(4*d*x + 4*c) + 5*(8*a^4*b
 - 13*a^3*b^2 + 5*a^2*b^3)*d*cos(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a
^2*b^3)*d)*cos(6*d*x + 6*c) + 4*(5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*cos
(2*d*x + 2*c) - (8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d)*cos(4*d*x + 4*c) - 2*
(5*(a^3*b^2 - a^2*b^3)*d*sin(8*d*x + 8*c) + 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2
*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x
 + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 10
*(2*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(6*d*x + 6*c) - 2*(8*a^4*b - 13
*a^3*b^2 + 5*a^2*b^3)*d*sin(4*d*x + 4*c) - 5*(a^3*b^2 - a^2*b^3)*d*sin(2*d*
x + 2*c))*sin(8*d*x + 8*c) - 4*(2*(64*a^5 - 144*a^4*b + 105*a^3*b^2 - 25*a^
2*b^3)*d*sin(4*d*x + 4*c) + 5*(8*a^4*b - 13*a^3*b^2 + 5*a^2*b^3)*d*sin(2*d*
x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 18.29, size = 4411, normalized size = 18.69

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^2),x)
```

```
[Out] (atan((((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*
a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*
b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6
- 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + tan(c
+ d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a
^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b
^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 -
 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3
- 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^
9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3
+ 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a
^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a
^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i - ((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^
2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(
```

```
1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8
- 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576
*a^15*b^3 + 4096*a^16*b^2 - tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2
*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1
/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b -
65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 22
93760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*
a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5
+ 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/
2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(
a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i)/(
((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3
 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3
*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920
*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + tan(c + d*x)*
(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3
+ 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*
a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 137625
6*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 45875
2*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 -
 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*
a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) - 36*a^7*b -
15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a
^9*b^3 - 3*a^10*b^2)))^(1/2) + ((-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3
)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256
*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^
11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3
+ 4096*a^16*b^2 - tan(c + d*x)*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)
^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*
(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12
*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16
*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*a^7*b^9 -
39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^
12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^
2*(a^9*b^3)^(1/2) - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(
1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2) - 4000*a^5*b^9
+ 27360*a^6*b^8 - 77504*a^7*b^7 + 116416*a^8*b^6 - 97824*a^9*b^5 + 43616*a^
10*b^4 - 8064*a^11*b^3))*(-(48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2)
 - 36*a^7*b - 15*a^5*b^3 + 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^1
1*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*2i)/d - (1/a + (tan(c + d*x)^4*(
4*a^2 - 7*a*b + 5*b^2))/(4*a^2*(a - b)) + (tan(c + d*x)^2*(8*a - 7*b))/(4*a
*(a - b)))/(d*(a*tan(c + d*x) + 2*a*tan(c + d*x)^3 + tan(c + d*x)^5*(a - b)
)) + (atan(((((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b +
 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 +
a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*
b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + ta
n(c + d*x)*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 1
5*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^
9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^
7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b
^3 - 458752*a^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952
*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b
^3 + 9216*a^14*b^2))*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36
*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b -
 a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i - (((48*a^2*(a^9*b^3)^(1/2) + 25*b
^2*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^
(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8
 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 2457
6*a^15*b^3 + 4096*a^16*b^2 - tan(c + d*x)*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2
```

```
*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1
/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b -
65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 22
93760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*
a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5
+ 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^14*b^2))*((48*a^2*(a^9*b^3)^(1/2
) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a
^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*1i)/((
((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 -
 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a
^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^7 + 61440*a^12*b^6 - 81920*a
^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 4096*a^16*b^2 + tan(c + d*x)*((
48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 - 4
7*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^1
0*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 + 458752*a^13*b^7 - 1376256*a
^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 + 1376256*a^17*b^3 - 458752*a
^18*b^2)) + tan(c + d*x)*(6400*a^7*b^9 - 39424*a^8*b^8 + 93952*a^9*b^7 - 10
0352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4 - 36608*a^13*b^3 + 9216*a^1
4*b^2))*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^7*b + 15*a
^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^12 + a^9*b
^3 - 3*a^10*b^2)))^(1/2) + (((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/
2) + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a
^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(4096*a^10*b^8 - 24576*a^11*b^
7 + 61440*a^12*b^6 - 81920*a^13*b^5 + 61440*a^14*b^4 - 24576*a^15*b^3 + 409
6*a^16*b^2 - tan(c + d*x)*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2)
 + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^1
1*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*(65536*a^19*b - 65536*a^12*b^8 +
 458752*a^13*b^7 - 1376256*a^14*b^6 + 2293760*a^15*b^5 - 2293760*a^16*b^4 +
 1376256*a^17*b^3 - 458752*a^18*b^2)) - tan(c + d*x)*(6400*a^7*b^9 - 39424*
a^8*b^8 + 93952*a^9*b^7 - 100352*a^10*b^6 + 26368*a^11*b^5 + 40448*a^12*b^4
 - 36608*a^13*b^3 + 9216*a^14*b^2))*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*
b^3)^(1/2) + 36*a^7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(
256*(3*a^11*b - a^12 + a^9*b^3 - 3*a^10*b^2)))^(1/2) - 4000*a^5*b^9 + 27360
*a^6*b^8 - 77504*a^7*b^7 + 116416*a^8*b^6 - 97824*a^9*b^5 + 43616*a^10*b^4
- 8064*a^11*b^3))*((48*a^2*(a^9*b^3)^(1/2) + 25*b^2*(a^9*b^3)^(1/2) + 36*a^
7*b + 15*a^5*b^3 - 47*a^6*b^2 - 69*a*b*(a^9*b^3)^(1/2))/(256*(3*a^11*b - a^
12 + a^9*b^3 - 3*a^10*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**2,x)
[Out] Timed out
```

3.224 $\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=315

$$\frac{\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{\left(-14\sqrt{a}\sqrt{b}+5a+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{9/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\ (14$$

```
[Out] -1/8*a*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b^2/d/(a-b+2*b*cos(d*x+c)^2-b*
cos(d*x+c)^4)^2+1/32*cos(d*x+c)*(9*a^2-11*a*b-10*b^2-2*(2*a-5*b)*b*cos(d*x+
c)^2)/(a-b)^2/b^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^(1/
4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a+12*b-14*a^(1/2)*b^(1/2))/b^(9/4
)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/2)-1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2
)+b^(1/2))^(1/2))*(5*a+12*b+14*a^(1/2)*b^(1/2))/b^(9/4)/d/a^(1/2)/(a^(1/2)+
b^(1/2))^(5/2)
```

Rubi [A] time = 0.57, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1678, 1166, 205, 208}

$$\frac{\cos(c+dx)\left(9a^2-2b(2a-5b)\cos^2(c+dx)-11ab-10b^2\right)}{32b^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{a\cos(c+dx)\left(a-b\cos^2(c+dx)+b\right)}{8b^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((5*a - 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt
[a] - Sqrt[b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(9/4)*d) - ((5*a +
 14*Sqrt[a]*Sqrt[b] + 12*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + S
qrt[b]]])/(64*Sqrt[a]*(Sqrt[a] + Sqrt[b])^(5/2)*b^(9/4)*d) - (a*Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*(a - b)*b^2*d*(a - b + 2*b*Cos[c + d*x]^2
- b*Cos[c + d*x]^4)^2) + (Cos[c + d*x]*(9*a^2 - 11*a*b - 10*b^2 - 2*(2*a -
5*b)*b*Cos[c + d*x]^2))/(32*(a - b)^2*b^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b
*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^9(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^4}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{\frac{2a\left(a^2+ab-8b^2\right)}{b}-2a(\cdots}{\left(a-b+\cdots\right.}\right.}{\cdots}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2-11ab\cdots\right.}{32(a-b)^2b^2d\left(a-b+\cdots\right.}$$

$$= -\frac{a\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)b^2d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(9a^2-11ab\cdots\right.}{32(a-b)^2b^2d\left(a-b+\cdots\right.}$$

$$= -\frac{\left(5a-14\sqrt{a}\sqrt{b}+12b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{9/4}d} - \frac{\left(5a+14\sqrt{a}\sqrt{b}+12b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\,c\cdots}{\sqrt{\cdots}}\right.}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{9/4}d}$$

Mathematica [C] time = 1.59, size = 785, normalized size = 2.49

$$i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{4\#1^6ab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-10\#1^6b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+20\#1^4a^2\cdots}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^9/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-9*a^2 + 13*a*b + 5*b^2 + (2*a - 5*b)*b*Cos[2*(c + d*x)
]))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b
)*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)])^2 + I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#
1^4 - 4*b*#1^6 + b*#1^8 & , (-4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
] + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + (2*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2] - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 56*a*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 - 78*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (28*I)*a*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (39*I)*b^2*Log[1 - 2*Cos[c + d*x]*#
1 + #1^2]*#1^2 + 20*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 56*
a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 78*b^2*ArcTan[Sin[c + d
*x]/(Cos[c + d*x] - #1)]*#1^4 - (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2
]*#1^4 + (28*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (39*I)*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^6 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - (2*
I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (5*I)*b^2*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7)
& ])/(128*(a - b)^2*b^2*d)
```

fricas [B] time = 1.54, size = 4640, normalized size = 14.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(8*(2*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 12*(3*a^2*b - a*b^2 - 10*b^3)*c
os(d*x + c)^5 + 24*(3*a^2*b - 2*a*b^2 - 5*b^3)*cos(d*x + c)^3 + ((a^2*b^4 -
 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x +
c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*
b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 +
6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*
b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8
- a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^
8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*
b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 -
a*b^9)*d^2))*log((625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96
336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8
 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 1
2*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 +
 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b
^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 -
 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a
*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 +
 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 1
55*a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^
```

3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640 *a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3* b^7 + 5*a^2*b^8 - a*b^9)*d^2))) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c) ^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^ 6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)* sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4 - (a^6*b^4 - 5*a ^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6 700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^ 5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5 *b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*log((625*a^6 - 52 50*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55 296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b ^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)) - (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^ 4 - (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2 *sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b ^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^ 9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4))) /((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))) - ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^ 6)*d*cos(d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155* a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b ^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 11 7532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a* b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 1 0*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*log(-(625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57 820*a^3*b^3 + 96336*a^2*b^4 - 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8 *b^7 - 6*a^7*b^8 + 27*a^6*b^9 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640* a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)) + (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^ 4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4 + (a^6*b^4 - 5*a^5*b^5 + 10 *a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432* a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^1 1 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^1 6 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a ^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))) + ((a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^6 - 2*(a^3* b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4*(a^3*b^3 - 3*a^2*b^

```
4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*b^3 + 6*a^2*b^4 - 4*
a*b^5 + b^6)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 - 76*a*b^3 - 144*b^4
- (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2*s
qrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b^4
 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^9
- 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14 +
 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4)))/(
(a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b^8 - a*b^9)*d^2))*l
og(-(625*a^6 - 5250*a^5*b + 22509*a^4*b^2 - 57820*a^3*b^3 + 96336*a^2*b^4 -
 98304*a*b^5 + 55296*b^6)*cos(d*x + c) - ((a^8*b^7 - 6*a^7*b^8 + 27*a^6*b^9
 - 80*a^5*b^10 + 135*a^4*b^11 - 126*a^3*b^12 + 61*a^2*b^13 - 12*a*b^14)*d^3
*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b^3 + 269641*a^4*b
^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 147456*b^8)/((a^11*b^
9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^13 - 252*a^6*b^14
 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18 + a*b^19)*d^4))
- (125*a^7*b^2 - 1045*a^6*b^3 + 4305*a^5*b^4 - 10583*a^4*b^5 + 16798*a^3*b^
6 - 16320*a^2*b^7 + 8448*a*b^8)*d)*sqrt((15*a^4 - 94*a^3*b + 155*a^2*b^2 -
76*a*b^3 - 144*b^4 - (a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2
*b^8 - a*b^9)*d^2*sqrt((625*a^8 - 6700*a^7*b + 35406*a^6*b^2 - 117532*a^5*b
^3 + 269641*a^4*b^4 - 437952*a^3*b^5 + 498432*a^2*b^6 - 368640*a*b^7 + 1474
56*b^8)/((a^11*b^9 - 10*a^10*b^10 + 45*a^9*b^11 - 120*a^8*b^12 + 210*a^7*b^
13 - 252*a^6*b^14 + 210*a^5*b^15 - 120*a^4*b^16 + 45*a^3*b^17 - 10*a^2*b^18
 + a*b^19)*d^4)))/((a^6*b^4 - 5*a^5*b^5 + 10*a^4*b^6 - 10*a^3*b^7 + 5*a^2*b
^8 - a*b^9)*d^2))) + 20*(a^3 - 4*a^2*b + a*b^2 + 2*b^3)*cos(d*x + c))/((a^2
*b^4 - 2*a*b^5 + b^6)*d*cos(d*x + c)^8 - 4*(a^2*b^4 - 2*a*b^5 + b^6)*d*cos(
d*x + c)^6 - 2*(a^3*b^3 - 5*a^2*b^4 + 7*a*b^5 - 3*b^6)*d*cos(d*x + c)^4 + 4
*(a^3*b^3 - 3*a^2*b^4 + 3*a*b^5 - b^6)*d*cos(d*x + c)^2 + (a^4*b^2 - 4*a^3*
b^3 + 6*a^2*b^4 - 4*a*b^5 + b^6)*d)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[-82,8]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[-37,16]-2/d*(5*((1-cos(c+d*x)
)/(1+cos(c+d*x)))^7*a^3-11*((1-cos(c+d*x))/(1+cos(c+d*x)))^7*a^2*b+12*((1-c
os(c+d*x))/(1+cos(c+d*x)))^7*a*b^2+35*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^3
-85*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^2*b+104*((1-cos(c+d*x))/(1+cos(c+d*
x)))^6*a*b^2+105*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a^3-407*((1-cos(c+d*x))/
(1+cos(c+d*x)))^5*a^2*b+652*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a*b^2-320*((1
-cos(c+d*x))/(1+cos(c+d*x)))^5*b^3+175*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^
3-865*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^2*b+1696*((1-cos(c+d*x))/(1+cos(c
+d*x)))^4*a*b^2-1408*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*b^3+175*((1-cos(c+d*
x))/(1+cos(c+d*x)))^3*a^3-849*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a^2*b+756*(
(1-cos(c+d*x))/(1+cos(c+d*x)))^3*a*b^2+320*((1-cos(c+d*x))/(1+cos(c+d*x)))^
3*b^3+105*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^3-383*((1-cos(c+d*x))/(1+cos(
c+d*x)))^2*a^2*b+248*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a*b^2+35*(1-cos(c+d*
x))/(1+cos(c+d*x))*a^3-77*(1-cos(c+d*x))/(1+cos(c+d*x))*a^2*b-12*(1-cos(c+d
*x))/(1+cos(c+d*x))*a*b^2+5*a^3-11*a^2*b)/(-32*a^2*b^2+64*a*b^3-32*b^4)/(((
1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a+6*(
(1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+4
*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)^2-2/d/(32*a^2*b^2-64*a*b^3+32*b^4)*2/d*
(-(-2*a+5*b)/2*(c+d*x)+(10*a^5*b-82*a^4*b^2-42*a^4*b*sqrt(a^2-a*b+sqrt(a*b)
*(a-b))-10*a^4*a*b+15*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+190*a^3*b
```

```
^3+180*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+82*a^3*b*a*b-39*a^3*b*sqrt(a*b
)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-198*a^2*b^4-250*a^2*b^3*sqrt(a^2-a*b+sqrt(a
*b)*(a-b))-190*a^2*b^2*a*b-11*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b
))+80*a*b^5+112*a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+198*a*b^3*a*b+51*a*b^3*
sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+24*b^5*sqrt(a^2-a*b+sqrt(a*b)*(a-b)
)-80*b^4*a*b+8*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)/(24*a^
5*b-96*a^4*b^2+112*a^3*b^3-32*a^2*b^4-8*a*b^5)*(atan(tan(c+d*x)/sqrt(-(8*a+
sqrt(8*a*8*a+4*(-4*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor((c+d*x)/pi+1/2))-(1
0*a^5*b-82*a^4*b^2+42*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-10*a^4*a*b+15*a^
4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+190*a^3*b^3-180*a^3*b^2*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))+82*a^3*b*a*b-39*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*
b)*(-a+b))-198*a^2*b^4+250*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-190*a^2*b
^2*a*b-11*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+80*a*b^5-112*a*b
^4*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+198*a*b^3*a*b+51*a*b^3*sqrt(a*b)*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))-24*b^5*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-80*b^4*a*b+8*b
^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(24*a^5*b-96*a^4*b^2+
112*a^3*b^3-32*a^2*b^4-8*a*b^5)*(atan(tan(c+d*x)/sqrt(-(8*a-sqrt(8*a*8*a+4*
(-4*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [**B**] time = 0.42, size = 1164, normalized size = 3.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 1/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7*
a-5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)
^7*b-9/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*a*b+b^2)*cos(d
*x+c)^5*a^2+3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*
cos(d*x+c)^5*a+15/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b
+b^2)*cos(d*x+c)^5+9/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*
a*b+b^2)*cos(d*x+c)^3*a^2-3/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^
2-2*a*b+b^2)*cos(d*x+c)^3*a-15/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2
*b/(a^2-2*a*b+b^2)*cos(d*x+c)^3+5/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b
)^2/b^2/(a-b)*cos(d*x+c)*a^2-15/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^
2/b/(a-b)*cos(d*x+c)*a-5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)
*cos(d*x+c)-1/16/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(
d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a+5/32/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+
b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-5/64/d/(a^2-2*a
*b+b^2)/b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b
)^(1/2)+b)*b)^(1/2))*a^2+11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+
b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a-3/16/d/(a^2-2
*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a
*b)^(1/2)+b)*b)^(1/2))+1/16/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*a
rctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-5/32/d/(a^2-2*a*b+b^2)/(((a
*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-5/64/d
/(a^2-2*a*b+b^2)/b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*
b/(((a*b)^(1/2)-b)*b)^(1/2))*a^2+11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b
)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-3/16/d
/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*
b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^9/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 19.29, size = 6675, normalized size = 21.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^9/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] ((cos(c + d*x)^7*(2*a - 5*b))/(16*(a^2 - 2*a*b + b^2)) + (3*cos(c + d*x)^5*
(a*b - 3*a^2 + 10*b^2))/(32*b*(a^2 - 2*a*b + b^2)) - (5*cos(c + d*x)*(3*a*b
 - a^2 + 2*b^2))/(32*b^2*(a - b)) - (3*cos(c + d*x)^3*(2*a*b - 3*a^2 + 5*b^
2))/(16*b*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2
) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x
)^8)) + (atan(((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a
^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4
*b^3)) - (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2)
- 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*
b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2)
)/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 -
a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b
^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))
*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b
^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) -
480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5
*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (co
s(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a
^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) +
 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^
6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) -
134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10
*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*1i - (((180224*a*b^8 - 483328*a^2
*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b
^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^
(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94
*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(
1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^
12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b
^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4
+ b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b
^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5
+ 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*
b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a
^6*b^10 - a^7*b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 +
 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^
2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a
^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2
) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14
 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*1
i)/((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40
960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) - (c
os(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9
 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^
9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a
^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^
(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*
a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*
(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^
3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(
a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 +
```

```
 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (cos(c + d*x)*
(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*
b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a
^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*
b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(
a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 +
 5*a^6*b^10 - a^7*b^9)))^(1/2) + (((180224*a*b^8 - 483328*a^2*b^7 + 466944*
a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5
 - 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^
4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*
a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3
*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^
11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3
*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*
b^3 - 4*a^3*b^2)))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 14
4*a*b^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*
(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(1
6384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*
b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161
*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*(-(25*a^4
*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b^9 - 76*a^2*b^8 + 155*a
^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*
(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13
+ 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) - (668*a*b^2 -
277*a^2*b + 50*a^3 - 720*b^3)/(8192*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4
+ a^4*b^3))))*(-(25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) - 144*a*b
^9 - 76*a^2*b^8 + 155*a^3*b^7 - 94*a^4*b^6 + 15*a^5*b^5 + 349*a^2*b^2*(a^3*
b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*
(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9))
)^(1/2)*2i)/d + (atan(((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 -
204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b
^4 + a^4*b^3)) - (cos(c + d*x)*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)
^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 3
49*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9
)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*
b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 6553
6*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3
*b^2)))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76
*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1
/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^
14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)
 + (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(
256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*((25*a^4*(a^3*b^9)^(1
/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a
^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/
2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12
 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*1i - (((180224*a*b^8 - 48332
8*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 -
4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*((25*a^4*(a^3*b
^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7
+ 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^
9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^
4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a
^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*
b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^
3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b
^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a
^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 +
5*a^6*b^10 - a^7*b^9)))^(1/2) - (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^
```

```
3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3
*b^2)))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76
*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1
/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^
14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2)
*1i)/((((180224*a*b^8 - 483328*a^2*b^7 + 466944*a^3*b^6 - 204800*a^4*b^5 +
40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^4*b^3)) -
(cos(c + d*x)*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^
9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b
^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(
a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))
^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b^7 - 65536*a^4*b^6 + 16384
*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*((25*a^4*
(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^
3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(
a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 +
 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) + (cos(c + d*x)*
(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^2*b^2))/(256*(a^4*b - 4*a*
b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^
3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b
^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a
^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 +
5*a^6*b^10 - a^7*b^9)))^(1/2) + (((180224*a*b^8 - 483328*a^2*b^7 + 466944*a
^3*b^6 - 204800*a^4*b^5 + 40960*a^5*b^4)/(16384*(b^7 - 4*a*b^6 + 6*a^2*b^5
- 4*a^3*b^4 + a^4*b^3)) + (cos(c + d*x)*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*
(a^3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^
5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b
*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11
 + 5*a^6*b^10 - a^7*b^9)))^(1/2)*(16384*a*b^9 - 65536*a^2*b^8 + 98304*a^3*b
^7 - 65536*a^4*b^6 + 16384*a^5*b^5))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^
3 - 4*a^3*b^2)))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a
*b^9 + 76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^
3*b^9)^(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(1638
4*(a^2*b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9
)))^(1/2) - (cos(c + d*x)*(25*a^4 - 94*a^3*b - 164*a*b^3 + 144*b^4 + 161*a^
2*b^2))/(256*(a^4*b - 4*a*b^4 + b^5 + 6*a^2*b^3 - 4*a^3*b^2)))*((25*a^4*(a^
3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 + 76*a^2*b^8 - 155*a^3*b
^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^(1/2) - 480*a*b^3*(a^3
*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*b^14 - 5*a^3*b^13 + 10
*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/2) - (668*a*b^2 - 277*
a^2*b + 50*a^3 - 720*b^3)/(8192*(b^7 - 4*a*b^6 + 6*a^2*b^5 - 4*a^3*b^4 + a^
4*b^3))))*((25*a^4*(a^3*b^9)^(1/2) + 384*b^4*(a^3*b^9)^(1/2) + 144*a*b^9 +
76*a^2*b^8 - 155*a^3*b^7 + 94*a^4*b^6 - 15*a^5*b^5 + 349*a^2*b^2*(a^3*b^9)^
(1/2) - 480*a*b^3*(a^3*b^9)^(1/2) - 134*a^3*b*(a^3*b^9)^(1/2))/(16384*(a^2*
b^14 - 5*a^3*b^13 + 10*a^4*b^12 - 10*a^5*b^11 + 5*a^6*b^10 - a^7*b^9)))^(1/
2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**9/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.225 $\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=290

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx\right.}{32bd(a-b)^2\left(a-b\cos^4(c+dx)+2b\,c\right.}$$

```
[Out] -1/8*a*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*
x+c)^4)^2+1/32*cos(d*x+c)*(5*a-17*b-3*(a-3*b)*cos(d*x+c)^2)/(a-b)^2/b/d/(a-
b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+3/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-
b^(1/2))^(1/2))*(a^(1/2)-2*b^(1/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)-b^(1/2))^(5/
2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(a^(1/2)+2*b^(1
/2))/b^(7/4)/d/a^(1/2)/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A] time = 0.43, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\,b^{7/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(-3(a-3b)\cos^2(c+dx\right.}{32bd(a-b)^2\left(a-b\cos^4(c+dx)+2b\,c\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*(Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[
b]]])/(64*Sqrt[a]*(Sqrt[a] - Sqrt[b])^(5/2)*b^(7/4)*d) - (3*(Sqrt[a] + 2*Sq
rt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*Sqrt[a]
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(7/4)*d) - (a*Cos[c + d*x]*(2 - Cos[c + d*x]^2
))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) + (Cos
[c + d*x]*(5*a - 17*b - 3*(a - 3*b)*Cos[c + d*x]^2))/(32*(a - b)^2*b*d*(a -
 b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^7(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{4a(a-4b)-2a(3a \cdots}{\left(a-b+2bx^2-\cdots\right.}\cdots\right.}{16a(\cdots}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(5a - \cdots\right.}{32(a-b)^2bd\left(a-b+\cdots\right.}$$

$$= -\frac{a\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(5a - \cdots\right.}{32(a-b)^2bd\left(a-b+\cdots\right.}$$

$$= \frac{3\left(\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{3\left(\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64\sqrt{a}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{7/4}d} - \frac{\cdots}{8(a\cdots}$$

Mathematica [C] time = 1.15, size = 630, normalized size = 2.17

$$-3i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{-2\#1^6a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+6\#1^6b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+6\#1^4a\cdots}{\cdots}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^7/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(-7*a + 25*b + 3*(a - 3*b)*Cos[2*(c + d*x)]))/(8*a - 3*b
 + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) + (512*a*(a - b)*(-5*Cos[c +
d*x] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c +
 d*x)])^2 - (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 +
b*#1^8 & , (2*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 6*b*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (3*I
)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*
x] - #1)]*#1^2 + 34*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (3*I)
*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (17*I)*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^2 + 6*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 34*b
*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (3*I)*a*Log[1 - 2*Cos[c +
d*x]*#1 + #1^2]*#1^4 + (17*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 2*
a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + 6*b*ArcTan[Sin[c + d*x]/(
Cos[c + d*x] - #1)]*#1^6 + I*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - (3*
I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3
 - 3*b*#1^5 + b*#1^7) & ])/(256*(a - b)^2*b*d)
```

fricas [B] time = 1.13, size = 4185, normalized size = 14.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/128*(12*(a*b - 3*b^2)*cos(d*x + c)^7 - 4*(11*a*b - 35*b^2)*cos(d*x + c)^5
 + 4*(a^2 + 18*a*b - 43*b^2)*cos(d*x + c)^3 + 3*((a^2*b^3 - 2*a*b^4 + b^5)*
d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^
2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3
+ 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^
4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b
^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^
4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^
10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^1
5 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a^6*
b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27
*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b
^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*
b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a
^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a
^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^
3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 -
 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^
3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^
9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^1
4 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 +
 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8
)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a
*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*
cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 +
 (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*
b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b
 + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7
- 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 2
10*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^
3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^
```

3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) - 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + a^3 - 10*a^2*b + 21*a*b^2 + 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) + 3*((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-27*(a^4 - 10*a^3*b + 29*a^2*b^2 - 4*a*b^3 - 64*b^4)*cos(d*x + c) + 27*((a^8*b^5 - 8*a^7*b^6 + 23*a^6*b^7 - 30*a^5*b^8 + 15*a^4*b^9 + 4*a^3*b^10 - 7*a^2*b^11 + 2*a*b^12)*d^3*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) + (a^5*b^2 - 11*a^4*b^3 + 35*a^3*b^4 - 9*a^2*b^5 - 80*a*b^6)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((a^6 - 12*a^5*b + 46*a^4*b^2 - 28*a^3*b^3 - 167*a^2*b^4 + 160*a*b^5 + 256*b^6)/((a^11*b^7 - 10*a^10*b^8 + 45*a^9*b^9 - 120*a^8*b^10 + 210*a^7*b^11 - 252*a^6*b^12 + 210*a^5*b^13 - 120*a^4*b^14 + 45*a^3*b^15 - 10*a^2*b^16 + a*b^17)*d^4)) - a^3 + 10*a^2*b - 21*a*b^2 - 4*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))) - 4*(3*a^2 + 14*a*b - 17*b^2)*cos(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x

```
+ c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[61,-66]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[30,-29]-2/d*(-3*((1-cos(c+d*
x))/(1+cos(c+d*x)))^7*a^2*b+3*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^3-30*((1-
cos(c+d*x))/(1+cos(c+d*x)))^6*a^2*b+16*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a^
3-111*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a^2*b+80*((1-cos(c+d*x))/(1+cos(c+d
*x)))^5*a*b^2+35*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^3-26*((1-cos(c+d*x))/(
1+cos(c+d*x)))^4*a^2*b-64*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a*b^2+256*((1-c
os(c+d*x))/(1+cos(c+d*x)))^4*b^3+40*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a^3+9
5*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a^2*b-336*((1-cos(c+d*x))/(1+cos(c+d*x)
))^3*a*b^2+25*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^3+54*((1-cos(c+d*x))/(1+c
os(c+d*x)))^2*a^2*b-64*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a*b^2+8*(1-cos(c+d
*x))/(1+cos(c+d*x))*a^3+19*(1-cos(c+d*x))/(1+cos(c+d*x))*a^2*b+a^3+2*a^2*b)
/(16*a^3*b-32*a^2*b^2+16*a*b^3)/(((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-
cos(c+d*x))/(1+cos(c+d*x)))^3*a+6*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((
1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+4*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)^2-2/
d/(16*a^2*b-32*a*b^2+16*b^3)*2/d*(-(-3*a+9*b)/(4*b*2)*(c+d*x)+((6*a^5*b-42*
a^4*b^2-6*a^4*a*b+9*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+114*a^3*b^3
+42*a^3*b*a*b-63*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-126*a^2*b^4-
114*a^2*b^2*a*b+159*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+48*a*b^
5+126*a*b^3*a*b-129*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-48*b^4*a*
b-24*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)*b^2+(-12*a^5*b^2
-9*a^5*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+60*a^4*b^3+54*a^4*b^2*sqrt(a^2-a*b+s
qrt(a*b)*(a-b))+12*a^4*b*a*b-132*a^3*b^4-132*a^3*b^3*sqrt(a^2-a*b+sqrt(a*b)
*(a-b))-60*a^3*b^2*a*b+132*a^2*b^5+150*a^2*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b)
)+132*a^2*b^3*a*b-48*a*b^6-51*a*b^5*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-132*a*b^4
*a*b-12*b^6*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+48*b^5*a*b)*abs(a-b)*abs(b)+(6*a^
5*b^3-18*a^4*b^4-6*a^4*b^2*a*b+9*a^4*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(
a-b))-6*a^3*b^5+18*a^3*b^3*a*b-27*a^3*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*
(a-b))+42*a^2*b^6+6*a^2*b^4*a*b-21*a^2*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)
*(a-b))-24*a*b^7-42*a*b^5*a*b+75*a*b^5*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-
b))+24*b^6*a*b+12*b^6*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b))/((
48*a^6*b^2-240*a^5*b^3+416*a^4*b^4-288*a^3*b^5+48*a^2*b^6+16*a*b^7)*abs(b))
*(atan(tan(c+d*x)/sqrt(-(16*a*b+sqrt(16*a*b*16*a*b+4*(-8*a*b+8*b^2)*8*a*b))
/2/(-8*a*b+8*b^2)))+pi*floor((c+d*x)/pi+1/2))-((6*a^5*b-42*a^4*b^2-6*a^4*a*
b+9*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+114*a^3*b^3+42*a^3*b*a*b-6
3*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-126*a^2*b^4-114*a^2*b^2*a*
b+159*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+48*a*b^5+126*a*b^3*a
*b-129*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-48*b^4*a*b-24*b^4*sqr
t(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)*b^2+(-12*a^5*b^2+9*a^5*b*sq
rt(a^2-a*b+sqrt(a*b)*(-a+b))+60*a^4*b^3-54*a^4*b^2*sqrt(a^2-a*b+sqrt(a*b)*(
-a+b))+12*a^4*b*a*b-132*a^3*b^4+132*a^3*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-
60*a^3*b^2*a*b+132*a^2*b^5-150*a^2*b^4*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+132*a
^2*b^3*a*b-48*a*b^6+51*a*b^5*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-132*a*b^4*a*b+1
2*b^6*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+48*b^5*a*b)*abs(a-b)*abs(b)+(6*a^5*b^3
-18*a^4*b^4-6*a^4*b^2*a*b+9*a^4*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)
)-6*a^3*b^5+18*a^3*b^3*a*b-27*a^3*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+
b))+42*a^2*b^6+6*a^2*b^4*a*b-21*a^2*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-
a+b))-24*a*b^7-42*a*b^5*a*b+75*a*b^5*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b
```

```
))+24*b^6*a*b+12*b^6*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b))/((
48*a^6*b^2-240*a^5*b^3+416*a^4*b^4-288*a^3*b^5+48*a^2*b^6+16*a*b^7)*abs(b))
*(atan(tan(c+d*x)/sqrt(-(16*a*b-sqrt(16*a*b*16*a*b+4*(-8*a*b+8*b^2)*8*a*b))
/2/(-8*a*b+8*b^2)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [B] time = 0.36, size = 814, normalized size = 2.81

$$\frac{3\left(\cos^{7}(dx+c)\right)a}{32d\left(b\left(\cos^{4}(dx+c)\right)-2b\left(\cos^{2}(dx+c)\right)-a+b\right)^{2}\left(a^{2}-2ab+b^{2}\right)}-\frac{9\left(\cos^{7}(dx+c)\right.}{32d\left(b\left(\cos^{4}(dx+c)\right)-2b\left(\cos^{2}(dx+c\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7
*a-9/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c
)^7*b-11/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d
*x+c)^5*a+35/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b+b^2)
*cos(d*x+c)^5+1/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/b/(a^2-2*a*b+b
^2)*cos(d*x+c)^3*a^2+9/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*
a*b+b^2)*cos(d*x+c)^3*a-43/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(
a^2-2*a*b+b^2)*cos(d*x+c)^3-3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/
b/(a-b)*cos(d*x+c)*a-17/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)*
cos(d*x+c)-3/64/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d
*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*a+9/64/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b
)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-3/32/d/(a^2-2*a*
b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)
^(1/2)+b)*b)^(1/2))+3/64/d/(a^2-2*a*b+b^2)/b/(((a*b)^(1/2)-b)*b)^(1/2)*arct
an(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*a-9/64/d/(a^2-2*a*b+b^2)/(((a*b)
^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-3/32/d/(a
^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(
((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^7/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 19.62, size = 5824, normalized size = 20.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^7/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] ((3*cos(c + d*x)^7*(a - 3*b))/(32*(a^2 - 2*a*b + b^2)) - (cos(c + d*x)^5*(1
1*a - 35*b))/(32*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)^3*(18*a*b + a^2 - 43*
b^2))/(32*b*(a - b)^2) - (cos(c + d*x)*(3*a + 17*b))/(32*b*(a - b)))/(d*(a^
2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c + d*x)^4*(2*a*b -
6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) + (atan(((((3*(81920*a
*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*
b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c + d*x)*((9*(a^3*(a^3*b^7)^
(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^
4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 -
```

5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384
*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(2
56*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) +
 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a
b^2(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b
^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (cos(c + d*
x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 +
b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*
b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b
*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9
 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*1i - (((3*(81920*a*b^7 - 180224*a^2*b^6 + 1
14688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b
^3 + a^4*b^2)) + (cos(c + d*x)*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^
(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/
2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10
- 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 +
98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b
^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) +
 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*
a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^
5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) - (cos(c + d*x)*(81*a*b^2 - 54*a^2*b +
 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a
^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3
*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384
*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))
^(1/2)*1i)/((((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4
b^4))/(32768(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c +
 d*x)*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*
b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1
/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8
- a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4
*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(
(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 1
0*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(
16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b
^7)))^(1/2) + (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b
^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*
(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 +
 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (((3*(81920*a*b^
7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5
 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) + (cos(c + d*x)*((9*(a^3*(a^3*b^7)^(1/
2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 +
 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a
^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*
b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*
(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16
b^3(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^
2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11
 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) - (cos(c + d*x)*
(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4
 + 6*a^2*b^2)))*((9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) + 4*a*b^7
 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a
^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 +
5*a^6*b^8 - a^7*b^7)))^(1/2) + (3*(9*a^2 - 63*a*b + 108*b^2))/(16384*(b^6 -
 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2))))*((9*(a^3*(a^3*b^7)^(1/2) + 1
6*b^3*(a^3*b^7)^(1/2) + 4*a*b^7 + 21*a^2*b^6 - 10*a^3*b^5 + a^4*b^4 + 5*a*b
^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^1
1 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*2i)/d + (atan((

```
(((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(3276
8*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) - (cos(c + d*x)*(-(9*(
a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^
3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(1638
4*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7))
)^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 1638
4*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(a^3*(a
^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5
- a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2
*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2
) + (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3
*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b
^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)
^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b
^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*1i - (((3*(81920*a*b^7 - 18
0224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a
^2*b^4 - 4*a^3*b^3 + a^4*b^2)) + (cos(c + d*x)*(-(9*(a^3*(a^3*b^7)^(1/2) +
16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*
b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^
11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 -
 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3
*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a
^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 1
0*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) - (cos(c + d*x)*(81*
a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6
*a^2*b^2)))*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 -
21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*
b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a
^6*b^8 - a^7*b^7)))^(1/2)*1i)/((((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*
a^3*b^5 - 16384*a^4*b^4))/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a
^4*b^2)) - (cos(c + d*x)*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2)
 - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) -
6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*
a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304
*a^3*b^6 - 65536*a^4*b^5 + 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 +
b^4 + 6*a^2*b^2)))*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a
*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*
b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^
9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a
^3 + 36*b^3))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(a^3*
(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^
5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a
^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1
/2) + (((3*(81920*a*b^7 - 180224*a^2*b^6 + 114688*a^3*b^5 - 16384*a^4*b^4))
/(32768*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2)) + (cos(c + d*x)*
(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 +
 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))
/(16384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7
*b^7)))^(1/2)*(16384*a*b^8 - 65536*a^2*b^7 + 98304*a^3*b^6 - 65536*a^4*b^5
+ 16384*a^5*b^4))/(256*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(
a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^
3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(1638
4*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7))
)^(1/2) - (cos(c + d*x)*(81*a*b^2 - 54*a^2*b + 9*a^3 + 36*b^3))/(256*(a^4 -
 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*
(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^
3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(16384*(a^2*b^12 - 5*a^3*b^11 + 10
*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^7)))^(1/2) + (3*(9*a^2 - 63*a*b
```

```
+ 108*b^2))/(16384*(b^6 - 4*a*b^5 + 6*a^2*b^4 - 4*a^3*b^3 + a^4*b^2))))*(-(
9*(a^3*(a^3*b^7)^(1/2) + 16*b^3*(a^3*b^7)^(1/2) - 4*a*b^7 - 21*a^2*b^6 + 10
*a^3*b^5 - a^4*b^4 + 5*a*b^2*(a^3*b^7)^(1/2) - 6*a^2*b*(a^3*b^7)^(1/2)))/(1
6384*(a^2*b^12 - 5*a^3*b^11 + 10*a^4*b^10 - 10*a^5*b^9 + 5*a^6*b^8 - a^7*b^
7)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**7/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.226 $\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+3a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\cos(c+dx)\left(a^2+2\right.}{32abd(a-b)^2\left(a-\right.}$$

```
[Out] -1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/(a-b)/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)^2+1/32*cos(d*x+c)*(a^2-11*a*b-2*b^2+2*b*(2*a+b)*cos(d*x+c)^2)/a/(
a-b)^2/b/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)+1/64*arctan(b^(1/4)*cos(d*
x+c)/(a^(1/2)-b^(1/2))^(1/2))*(3*a+4*b-10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/
d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))
^(1/2))*(3*a+4*b+10*a^(1/2)*b^(1/2))/a^(3/2)/b^(5/4)/d/(a^(1/2)+b^(1/2))^(5
/2)
```

Rubi [A] time = 0.47, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3215, 1205, 1178, 1166, 205, 208}

$$\frac{\cos(c+dx)\left(a^2+2b(2a+b)\cos^2(c+dx)-11ab-2b^2\right)}{32abd(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}+\frac{\left(-10\sqrt{a}\sqrt{b}+3a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{5/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((3*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(5/4)*d) + ((3*a + 1
0*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(5/4)*d) - (Cos[c + d*x]*(a
+ b - b*Cos[c + d*x]^2))/(8*(a - b)*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos
[c + d*x]^4)^2) + (Cos[c + d*x]*(a^2 - 11*a*b - 2*b^2 + 2*b*(2*a + b)*Cos[c
 + d*x]^2))/(32*a*(a - b)^2*b*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x
]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2a(a-7b)+10abx^2}{\left(a-b+2bx^2-bx^4\right)^2}\right.}{16a(a-b}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11ab-\right.}{32a(a-b)^2bd\left(a-b+2\right.}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8(a-b)bd\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\cos(c+dx)\left(a^2-11ab-\right.}{32a(a-b)^2bd\left(a-b+2\right.}$$

$$= \frac{\left(3a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{5/4}d} + \frac{\left(3a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c\right.}{\sqrt{\sqrt{a}+}}}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{5/4}d}$$

Mathematica [C] time = 1.40, size = 786, normalized size = 2.51

$$i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&,\frac{-4\#1^6ab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-2\#1^6b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-12\#1^4a^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+64\#1^4ab\tan^{-}}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(a^2 - 9*a*b - b^2 + b*(2*a + b)*Cos[2*(c + d*x)]))/(a*(8
*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) - (512*(a - b)*Cos[c
 + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)]
+ b*Cos[4*(c + d*x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 -
4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + 12*a^2*ArcTan[Si
n[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 64*a*b*ArcTan[Sin[c + d*x]/(Cos[c +
d*x] - #1)]*#1^2 + 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (
6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (32*I)*a*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2
 - 12*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 64*a*b*ArcTan[Sin
[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)]*#1^4 + (6*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (32*I)
*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + (5*I)*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^6 + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6
)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/a)/(128*(a - b)^
2*b*d)
```

fricas [B] time = 1.47, size = 4524, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(8*(2*a*b^2 + b^3)*cos(d*x + c)^7 + 4*(a^2*b - 19*a*b^2 - 6*b^3)*cos
(d*x + c)^5 - 8*(5*a^2*b - 14*a*b^2 - 3*b^3)*cos(d*x + c)^3 + ((a^3*b^3 - 2
*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(
d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4
 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b -
4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229
*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*
b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 -
53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^
12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*
b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^
2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((8
1*a^5 - 1458*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)
*cos(d*x + c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^
6*b^8 + 114*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5
*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^
6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 25
2*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*
b^15)*d^4)) - (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*
a^3*b^5 - 160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3
 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^
```

3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 1043
61*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7
 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12
+ 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a
^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 +
 a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6
 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*
b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2
+ 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 +
 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^
4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*
b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 4
5*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120
*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*
b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log((81*a^5 - 14
58*a^4*b + 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x +
 c) + ((a^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 11
4*a^5*b^9 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814
*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*
b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10
 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)
) + (27*a^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 -
160*a^2*b^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4
- (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2
*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4
 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^1
0*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b
^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 1
0*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))) - ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*
cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*
b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^
3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^
3 - 4*a^2*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3
 - 16*b^4 + (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^
3*b^7)*d^2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 1043
61*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7
 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12
+ 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a
^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log(-(81*a^5 - 1458*a^4*b
+ 9389*a^3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a
^10*b^4 + 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9
 - 35*a^4*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2
- 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*
a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^
7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) - (27*a
^7*b - 411*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b
^6)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 + (a^8*b^
2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81
*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*
a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 2
10*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*
a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5
 + 5*a^4*b^6 - a^3*b^7)*d^2))) + ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x +
 c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a
^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3
*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2
*b^4 + a*b^5)*d)*sqrt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4
 - (a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^
2*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^

```
4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^
10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*
b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 -
10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2))*log(-(81*a^5 - 1458*a^4*b + 9389*a^
3*b^2 - 24972*a^2*b^3 + 10896*a*b^4 - 1280*b^5)*cos(d*x + c) + ((a^10*b^4 +
 10*a^9*b^5 - 69*a^8*b^6 + 160*a^7*b^7 - 185*a^6*b^8 + 114*a^5*b^9 - 35*a^4
*b^10 + 4*a^3*b^11)*d^3*sqrt((81*a^6 - 1548*a^5*b + 12814*a^4*b^2 - 53212*a
^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6400*b^6)/((a^13*b^5 - 10*a^12*b^6
+ 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^9 - 252*a^8*b^10 + 210*a^7*b^11 -
120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14 + a^3*b^15)*d^4)) + (27*a^7*b - 41
1*a^6*b^2 + 2383*a^5*b^3 - 5529*a^4*b^4 + 1962*a^3*b^5 - 160*a^2*b^6)*d)*sq
rt((15*a^4 - 30*a^3*b - 229*a^2*b^2 + 116*a*b^3 - 16*b^4 - (a^8*b^2 - 5*a^7
*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*b^6 - a^3*b^7)*d^2*sqrt((81*a^6 - 15
48*a^5*b + 12814*a^4*b^2 - 53212*a^3*b^3 + 104361*a^2*b^4 - 48160*a*b^5 + 6
400*b^6)/((a^13*b^5 - 10*a^12*b^6 + 45*a^11*b^7 - 120*a^10*b^8 + 210*a^9*b^
9 - 252*a^8*b^10 + 210*a^7*b^11 - 120*a^6*b^12 + 45*a^5*b^13 - 10*a^4*b^14
+ a^3*b^15)*d^4)))/((a^8*b^2 - 5*a^7*b^3 + 10*a^6*b^4 - 10*a^5*b^5 + 5*a^4*
b^6 - a^3*b^7)*d^2))) + 4*(3*a^3 + 12*a^2*b - 13*a*b^2 - 2*b^3)*cos(d*x + c
))/((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4
 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*
d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x +
c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[40,31]Warning, need
to choose a branch for the root of a polynomial with parameters. This might
 be wrong.The choice was done assuming [a,b]=[7,-19]-2/d*(-3*((1-cos(c+d*x)
)/(1+cos(c+d*x)))^7*a^3+13*((1-cos(c+d*x))/(1+cos(c+d*x)))^7*a^2*b-4*((1-co
s(c+d*x))/(1+cos(c+d*x)))^7*a*b^2-21*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^3+
99*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^2*b-24*((1-cos(c+d*x))/(1+cos(c+d*x)
))^6*a*b^2-63*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a^3+225*((1-cos(c+d*x))/(1+
cos(c+d*x)))^5*a^2*b-68*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a*b^2-64*((1-cos(
c+d*x))/(1+cos(c+d*x)))^5*b^3-105*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^3+183
*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^2*b-96*((1-cos(c+d*x))/(1+cos(c+d*x)))
^4*a*b^2-384*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*b^3-105*((1-cos(c+d*x))/(1+c
os(c+d*x)))^3*a^3-9*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a^2*b+452*((1-cos(c+d
*x))/(1+cos(c+d*x)))^3*a*b^2+64*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*b^3-63*((
1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^3-87*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^
2*b+120*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a*b^2-21*(1-cos(c+d*x))/(1+cos(c+
d*x))*a^3-37*(1-cos(c+d*x))/(1+cos(c+d*x))*a^2*b+4*(1-cos(c+d*x))/(1+cos(c+
d*x))*a*b^2-3*a^3-3*a^2*b)/(-32*a^3*b+64*a^2*b^2-32*a*b^3)/(((1-cos(c+d*x))
/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a+6*((1-cos(c+d*x)
)/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*b+4*(1-cos(c+d*x
))/(1+cos(c+d*x))*a+a)^2-2/d/(32*a^3*b-64*a^2*b^2+32*a*b^3)*2/d*(-(2*a+b)/2
*(c+d*x)+(-6*a^5*b+62*a^4*b^2+30*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+6*a^4*
a*b-9*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-178*a^3*b^3-132*a^3*b^2*s
qrt(a^2-a*b+sqrt(a*b)*(a-b))-62*a^3*b*a*b+33*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+s
qrt(a*b)*(a-b))+170*a^2*b^4+158*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+178*a
^2*b^2*a*b-51*a^2*b^2*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-48*a*b^5-24*a
*b^4*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-170*a*b^3*a*b+43*a*b^3*sqrt(a*b)*sqrt(a^
2-a*b+sqrt(a*b)*(a-b))-8*b^5*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+48*b^4*a*b+8*b^4
*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)/(24*a^5*b-96*a^4*b^2+112
```

```
*a^3*b^3-32*a^2*b^4-8*a*b^5)*(atan(tan(c+d*x)/sqrt(-(8*a+sqrt(8*a*8*a+4*(-4
*a+4*b)*4*a))/2/(-4*a+4*b)))+pi*floor((c+d*x)/pi+1/2))-(-6*a^5*b+62*a^4*b^2
-30*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+6*a^4*a*b-9*a^4*sqrt(a*b)*sqrt(a^2
-a*b+sqrt(a*b)*(-a+b))-178*a^3*b^3+132*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b
))-62*a^3*b*a*b+33*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+170*a^2*b
^4-158*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+178*a^2*b^2*a*b-51*a^2*b^2*sq
rt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-48*a*b^5+24*a*b^4*sqrt(a^2-a*b+sqrt(
a*b)*(-a+b))-170*a*b^3*a*b+43*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)
)+8*b^5*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+48*b^4*a*b+8*b^4*sqrt(a*b)*sqrt(a^2-
a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(24*a^5*b-96*a^4*b^2+112*a^3*b^3-32*a^2*b^4
-8*a*b^5)*(atan(tan(c+d*x)/sqrt(-(8*a-sqrt(8*a*8*a+4*(-4*a+4*b)*4*a))/2/(-4
*a+4*b)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [B] time = 0.39, size = 1167, normalized size = 3.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -1/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7
*b-1/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b^2/a/(a^2-2*a*b+b^2)*cos
(d*x+c)^7-1/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*co
s(d*x+c)^5*a+19/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b+b
^2)*cos(d*x+c)^5+3/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2*a*
b+b^2)*cos(d*x+c)^5*b^2+5/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2
-2*a*b+b^2)*cos(d*x+c)^3*a-7/8/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/
(a^2-2*a*b+b^2)*cos(d*x+c)^3-3/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2
/a/(a^2-2*a*b+b^2)*cos(d*x+c)^3*b^2-3/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2
-a+b)^2/b/(a-b)*cos(d*x+c)*a-15/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^
2/(a-b)*cos(d*x+c)-1/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a-b)/a*b
*cos(d*x+c)+1/16/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*
x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/32/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)
*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b+3/64/d/(a^2-2*a
*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^
(1/2)+b)*b)^(1/2))*a-13/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)
*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/16/d/a/(a^2-2*a
*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^
(1/2)+b)*b)^(1/2))*b^2-1/16/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arc
tan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/32/d/a/(a^2-2*a*b+b^2)/(((a*b
)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+3/64/d
/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/
(((a*b)^(1/2)-b)*b)^(1/2))*a-13/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^
(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/16/d/a/(
a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/((
(a*b)^(1/2)-b)*b)^(1/2))*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 20.15, size = 6362, normalized size = 20.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((cos(c + d*x)^3*(14*a*b - 5*a^2 + 3*b^2))/(16*a*(a - b)^2) - (cos(c + d*
x)^5*(19*a*b - a^2 + 6*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (b*cos(c + d*x)^7
*(2*a + b))/(16*a*(a^2 - 2*a*b + b^2)) + (cos(c + d*x)*(15*a*b + 3*a^2 + 2*
b^2))/(32*a*b*(a - b)))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b
^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d
*x)^8)) - (atan(((((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 - 18841
6*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*
a^5*b^2)) - (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2)
+ 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*
b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^
9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8
 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^
6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) -
 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^
4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(163
84*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)
))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a
^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*
(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^
5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9
*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^1
0*b^6 - a^11*b^5)))^(1/2)*1i - (((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a
^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 -
4*a^4*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a
^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^
7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b
^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*
(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*
b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^
9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b
^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^
5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b
^6 - a^11*b^5)))^(1/2) - (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*
a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^
2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^
4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) +
 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a
^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*1i)/((((16384*a^3*b^6 - 172032*a^4*
b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*
b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d*x)*((80*b^3*(a^9*b^5)^(1
/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a
^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))
/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11
*b^5)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^
5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2))
)*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b
^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86
*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*
b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 +
 16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^
3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^
9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*
a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (((16384*a^3*b^6 -
172032*a^4*b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a
^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((80*b^3*(
a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5
*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*
```

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b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10
*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 6
5536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 +
6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*b^7
- 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*b^5)
^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^
8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) - (cos(c + d*x)*(9*a^4*b -
100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b
^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1
/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 30
1*a*b^2*(a^9*b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^
7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (44*a*b^
2 + 143*a^2*b - 18*a^3 - 16*b^3)/(8192*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
 + 6*a^5*b^2))))*((80*b^3*(a^9*b^5)^(1/2) - 9*a^3*(a^9*b^5)^(1/2) + 16*a^3*
b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 - 301*a*b^2*(a^9*
b^5)^(1/2) + 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^
8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*2i)/d - (atan(((((16384
*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2
)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d*x)
*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^
6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*
a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b
^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*
a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4
*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) +
16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^
2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9
 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (cos(c + d*x)*
(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^
5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(
a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a
^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*
b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)
*1i - (((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 - 188416*a^6*b^3 +
 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) +
 (cos(c + d*x)*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^
7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^
5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*
b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*b^8 - 65536*a^
4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b
 + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9
*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*
b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^1
0 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) -
(cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 62*a^3*b^2))/(2
56*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9*b^5)^(1
/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*
a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2)
)/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^1
1*b^5)))^(1/2)*1i)/((((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a^5*b^4 - 18
8416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 +
 6*a^5*b^2)) - (cos(c + d*x)*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/
2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301
*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7
*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*(16384*a^3*
b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(256*
(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9*b^5)^(1/2)
 - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6
*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(
```

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16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b
^5)))^(1/2) + (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*a^2*b^3 - 6
2*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^
3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229
*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(
a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*
a^10*b^6 - a^11*b^5)))^(1/2) + (((16384*a^3*b^6 - 172032*a^4*b^5 + 319488*a
^5*b^4 - 188416*a^6*b^3 + 24576*a^7*b^2)/(16384*(a^7 - 4*a^6*b + a^3*b^4 -
4*a^4*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a
^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^
7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b
^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2)*
(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*
b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((9*a^3*(a^9
*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a^5*b
^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^9*b^
5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^10*b
^6 - a^11*b^5)))^(1/2) - (cos(c + d*x)*(9*a^4*b - 100*a*b^4 + 16*b^5 + 209*
a^2*b^3 - 62*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^
2)))*((9*a^3*(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^
4*b^6 + 229*a^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) -
 86*a^2*b*(a^9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a
^9*b^7 + 5*a^10*b^6 - a^11*b^5)))^(1/2) + (44*a*b^2 + 143*a^2*b - 18*a^3 -
16*b^3)/(8192*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))))*((9*a^3*
(a^9*b^5)^(1/2) - 80*b^3*(a^9*b^5)^(1/2) + 16*a^3*b^7 - 116*a^4*b^6 + 229*a
^5*b^5 + 30*a^6*b^4 - 15*a^7*b^3 + 301*a*b^2*(a^9*b^5)^(1/2) - 86*a^2*b*(a^
9*b^5)^(1/2))/(16384*(a^6*b^10 - 5*a^7*b^9 + 10*a^8*b^8 - 10*a^9*b^7 + 5*a^
10*b^6 - a^11*b^5)))^(1/2)*2i)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a-b*sin(d*x+c)**4)**3,x)
[Out] Timed out
```

3.227 $\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=288

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left(-\left((5a+b)\cos^2(c+d\right.\right.}{32ad(a-b)^2\left(a-b\cos^4(c+dx)+2b\right.}$$

```
[Out] -1/8*cos(d*x+c)*(2-cos(d*x+c)^2)/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)
^4)^2-1/32*cos(d*x+c)*(11*a+b-(5*a+b)*cos(d*x+c)^2)/a/(a-b)^2/d/(a-b+2*b*co
s(d*x+c)^2-b*cos(d*x+c)^4)-1/64*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))
^(1/2))*(5*a^(1/2)-2*b^(1/2))/a^(3/2)/b^(3/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/6
4*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*(5*a^(1/2)+2*b^(1/2))
/a^(3/2)/b^(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)
```

Rubi [A] time = 0.50, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3215, 1178, 1166, 205, 208}

$$-\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}b^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left(-(5a+b)\cos^2(c+d\right.}{32ad(a-b)^2\left(a-b\cos^4(c+dx)+2b\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt
[b]]])/(64*a^(3/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/4)*d) + ((5*Sqrt[a] + 2*S
qrt[b])*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(64*a^(3/2
)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/4)*d) - (Cos[c + d*x]*(2 - Cos[c + d*x]^2)
)/(8*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (Cos[c
+ d*x]*(11*a + b - (5*a + b)*Cos[c + d*x]^2))/(32*a*(a - b)^2*d*(a - b + 2*
b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
```

```
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{-12ab+10abx^2}{\left(a-b+2bx^2-bx^4\right)}\right.}{16a(a-}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left(11a+\right.}{32a(a-b)^2d\left(a-b+2\right.}$$

$$= -\frac{\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left(11a+\right.}{32a(a-b)^2d\left(a-b+2\right.}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/4}d} + \frac{\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/4}d} - \frac{}{8(}$$

Mathematica [C] time = 1.12, size = 631, normalized size = 2.19

$$i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{-10\#1^6a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-2\#1^6b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+94\#1^4a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-10\#1^4b\tan^{-1}\left(\right.}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*Cos[c + d*x]*(-17*a - b + (5*a + b)*Cos[2*(c + d*x)]))/(a*(8*a - 3*b +
 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*(a - b)*(-5*Cos[c + d*x
] + Cos[3*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*
x)])^2 + (I*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6 + b*#1^8
 & , (10*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)] - (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b*L
og[1 - 2*Cos[c + d*x]*#1 + #1^2] - 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
 #1)]*#1^2 + 10*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (47*I)*a*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1
```

```
+ #1^2]*#1^2 + 94*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 10*b*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (47*I)*a*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^4 + (5*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 10*a*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 2*b*ArcTan[Sin[c + d*x]/(Co
s[c + d*x] - #1)]*#1^6 + (5*I)*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + I
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 -
 3*b*#1^5 + b*#1^7) & ])/a)/(256*(a - b)^2*d)
```

fricas [B] time = 1.19, size = 4050, normalized size = 14.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(5*a*b + b^2)*cos(d*x + c)^7 - 12*(7*a*b + b^2)*cos(d*x + c)^5 -
12*(3*a^2 - 10*a*b - b^2)*cos(d*x + c)^3 + ((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d
*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4
*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b
^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a
^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*
a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a
*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 21
0*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4
*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b -
5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625*a
^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*a^
9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2*a
^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*sqr
t(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2
*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^1
3*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^
8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)
) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^4
)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(
a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^
3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 +
5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780
*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a
^4*b^12 + a^3*b^13)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b
- 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log((625
*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 - 16*
a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8 + 2
*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 12
25*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a
^3*b^13)*d^4)) + (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d)*s
qrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^
2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/((a^
13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b
^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4
)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^
3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) - ((a^3*b^2 - 2*a^2*b^3 + a*b^
4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*
```

```
(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a
^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 -
 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4
+ 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 107
80*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6
+ 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10
*a^4*b^12 + a^3*b^13)*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*
b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-(
625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2 -
16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b^8
+ 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 +
 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*
b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12
+ a^3*b^13)*d^4)) - (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4)*d
)*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6
)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4)/
((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a
^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)
*d^4)) + 105*a^3 + 70*a^2*b - 35*a*b^2 + 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*a^
6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) + ((a^3*b^2 - 2*a^2*b^3 +
a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6
- 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b -
 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b
^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b
^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 -
10780*a*b^3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b
^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 -
 10*a^4*b^12 + a^3*b^13)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a
^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log
(-(625*a^3 + 3750*a^2*b - 1491*a*b^2 + 140*b^3)*cos(d*x + c) + ((5*a^10*b^2
 - 16*a^9*b^3 + 3*a^8*b^4 + 50*a^7*b^5 - 85*a^6*b^6 + 60*a^5*b^7 - 19*a^4*b
^8 + 2*a^3*b^9)*d^3*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^
3 + 1225*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a
^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^
12 + a^3*b^13)*d^4)) + (325*a^5*b + 1977*a^4*b^2 - 609*a^3*b^3 + 35*a^2*b^4
)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b
^6)*d^2*sqrt((625*a^4 + 7700*a^3*b + 21966*a^2*b^2 - 10780*a*b^3 + 1225*b^4
)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252
*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^1
3)*d^4)) - 105*a^3 - 70*a^2*b + 35*a*b^2 - 4*b^3)/((a^8*b - 5*a^7*b^2 + 10*
a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))) + 4*(19*a^2 - 18*a*b - b
^2)*cos(d*x + c))/((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*
b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^
3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d
*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)
```

giac [B] time = 1.86, size = 1076, normalized size = 3.74

$$-\frac{\frac{5\,ab\cos(dx+c)^7}{d} + \frac{b^2\cos(dx+c)^7}{d} - \frac{21\,ab\cos(dx+c)^5}{d} - \frac{3\,b^2\cos(dx+c)^5}{d} - \frac{9\,a^2\cos(dx+c)^3}{d} + \frac{30\,ab\cos(dx+c)^3}{d} + \frac{3\,b^2\cos(dx+c)^3}{d} + \frac{19\,a^2}{}}{32\left(b\cos\left(dx+c\right)^4 - 2\,b\cos\left(dx+c\right)^2 - a + b\right)^2\left(a^3 - 2\,a^2b + ab^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/32*(5*a*b*cos(d*x + c)^7/d + b^2*cos(d*x + c)^7/d - 21*a*b*cos(d*x + c)^
5/d - 3*b^2*cos(d*x + c)^5/d - 9*a^2*cos(d*x + c)^3/d + 30*a*b*cos(d*x + c)
```

```
^3/d + 3*b^2*cos(d*x + c)^3/d + 19*a^2*cos(d*x + c)/d - 18*a*b*cos(d*x + c)
/d - b^2*cos(d*x + c)/d)/((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - a + b)^2
*(a^3 - 2*a^2*b + a*b^2)) - 1/64*(2*(4*a^6*b - 17*a^5*b^2 + 28*a^4*b^3 - 22
*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*d^4 + (13*
a^4*b - 27*a^3*b^2 + 15*a^2*b^3 - a*b^4)*sqrt(-b^2 + sqrt(a*b)*b)*d^2*abs(a
^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2)^2*s
qrt(a*b)*sqrt(-b^2 + sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(d*sqrt(-(
a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2 - sqrt((a^3*b*d^2 - 2*a^2*b^2*d^2 + a
*b^3*d^2)^2 + (a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a^3*b + 3*a^
2*b^2 - a*b^3)))/(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4))))/((a^7*b - 5*a^6
*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^3*abs(a^3*d^2 - 2*a
^2*b*d^2 + a*b^2*d^2)*abs(b)) + 1/64*(2*(4*a^6*b - 17*a^5*b^2 + 28*a^4*b^3
- 22*a^3*b^4 + 8*a^2*b^5 - a*b^6)*sqrt(-b^2 - sqrt(a*b)*b)*d^4 - (13*a^3 -
27*a^2*b + 15*a*b^2 - b^3)*sqrt(a*b)*sqrt(-b^2 - sqrt(a*b)*b)*d^2*abs(a^3*d
^2 - 2*a^2*b*d^2 + a*b^2*d^2) + (a^3*d^2 - 2*a^2*b*d^2 + a*b^2*d^2)^2*sqrt(
-b^2 - sqrt(a*b)*b)*(5*a + b))*arctan(cos(d*x + c)/(d*sqrt(-(a^3*b*d^2 - 2*
a^2*b^2*d^2 + a*b^3*d^2 + sqrt((a^3*b*d^2 - 2*a^2*b^2*d^2 + a*b^3*d^2)^2 +
(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)
))/(a^3*b*d^4 - 2*a^2*b^2*d^4 + a*b^3*d^4))))/((a^6 - 5*a^5*b + 10*a^4*b^2
- 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*sqrt(a*b)*d^3*abs(a^3*d^2 - 2*a^2*b*d^2 +
 a*b^2*d^2)*abs(b))
```

maple [B] time = 0.52, size = 1153, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -5/64/d/b/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-1/6
4/d/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-1/8/d/(
a*b)^(1/2)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+1/
32/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d
*x+c)^3-7/64/d/b/(a*b)^(1/2)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x
+c)-5/64/d/b/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)+1/32/d/(a*
b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-5/64/d/(a^2-2*
a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^
(1/2))-1/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)
*b/(((a*b)^(1/2)-b)*b)^(1/2))*b-1/8/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)
^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+1/32/d/a/
(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(
((a*b)^(1/2)-b)*b)^(1/2))*b^2-5/64/d/b/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^
2-2*a*b+b^2)*cos(d*x+c)^3-1/64/d/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*
a*b+b^2)*cos(d*x+c)^3+1/8/d/(a*b)^(1/2)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a
^2-2*a*b+b^2)*cos(d*x+c)^3-1/32/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/
2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+7/64/d/b/(a*b)^(1/2)/(cos(d*x+c)^2-1-(
a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)-5/64/d/b/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^
2/(a-b)*cos(d*x+c)-1/32/d/(a*b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a
-b)*cos(d*x+c)+5/64/d/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos
(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))+1/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)
+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b-1/8/d/(a^2-2
*a*b+b^2)*b/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a
*b)^(1/2)+b)*b)^(1/2))+1/32/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b
)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b^2
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 19.28, size = 5566, normalized size = 19.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - (atan(((((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*a^6*b^
3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c + d*x
)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^
4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5
*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(16384*
a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4))/(
256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^
(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*
a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6
 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (cos(c + d*x)*(4*b^5 - 31*
a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3
 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b
^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(1638
4*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))
^(1/2)*1i - (((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*a^6
*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c +
d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4
*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8
- 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(163
84*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b^4)
)/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^
3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 1
05*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*
b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) - (cos(c + d*x)*(4*b^5 -
31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*
b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^
3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(1
6384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3
)))^(1/2)*1i)/((((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*
a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) - (cos(c
 + d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*
a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b
^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(
16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b
^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9
*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3
+ 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a
^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (cos(c + d*x)*(4*b^5
 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a
^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4
*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))
/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*
b^3)))^(1/2) + (((16384*a^3*b^6 - 245760*a^4*b^5 + 442368*a^5*b^4 - 212992*
a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)) + (cos(c
 + d*x)*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*
a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b
^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*(
16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*a^6*b^5 + 16384*a^7*b
^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9
```

```
*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3
+ 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a
^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) - (cos(c + d*x)*(4*b^5
 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a
^3*b^3 + 6*a^4*b^2)))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(1/2) + 4
*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3)^(1/2))
/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*
b^3)))^(1/2) + (5*a*b^2 + 125*a^2*b - 4*b^3)/(16384*(a^7 - 4*a^6*b + a^3*b^
4 - 4*a^4*b^3 + 6*a^5*b^2))))*((35*b^2*(a^9*b^3)^(1/2) - 25*a^2*(a^9*b^3)^(
1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 - 154*a*b*(a^9*b^3
)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4
 - a^11*b^3)))^(1/2)*2i)/d - (atan(((((16384*a^3*b^6 - 245760*a^4*b^5 + 442
368*a^5*b^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 +
 6*a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1
/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)
^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4
- a^11*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 65536*
a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4
*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*
a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b
^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) +
 (cos(c + d*x)*(4*b^5 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 - 4*
a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35*b^
2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154
*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5
 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*1i - (((16384*a^3*b^6 - 245760*a^4*b^5 +
442368*a^5*b^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^
3 + 6*a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)
^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b
^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b
^4 - a^11*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 - 655
36*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*
a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 -
35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^
6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2
) - (cos(c + d*x)*(4*b^5 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^6 -
 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35
*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 +
154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*
b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*1i)/((((16384*a^3*b^6 - 245760*a^4*b^5
 + 442368*a^5*b^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4
*b^3 + 6*a^5*b^2)) - (cos(c + d*x)*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b
^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^
9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^1
0*b^4 - a^11*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 -
65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 +
 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5
 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)^(1/2))/(16384*
(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(
1/2) + (cos(c + d*x)*(4*b^5 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^
6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) -
 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2
 + 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a
^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (((16384*a^3*b^6 - 245760*a^4*b^5
 + 442368*a^5*b^4 - 212992*a^6*b^3)/(32768*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4
*b^3 + 6*a^5*b^2)) + (cos(c + d*x)*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b
^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^
9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^1
0*b^4 - a^11*b^3)))^(1/2)*(16384*a^3*b^8 - 65536*a^4*b^7 + 98304*a^5*b^6 -
```

```
65536*a^6*b^5 + 16384*a^7*b^4))/(256*(a^6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 +
 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) - 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5
 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2 + 154*a*b*(a^9*b^3)^(1/2))/(16384*
(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(
1/2) - (cos(c + d*x)*(4*b^5 - 31*a*b^4 + 74*a^2*b^3 + 25*a^3*b^2))/(256*(a^
6 - 4*a^5*b + a^2*b^4 - 4*a^3*b^3 + 6*a^4*b^2)))*((25*a^2*(a^9*b^3)^(1/2) -
 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105*a^6*b^2
 + 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^6 - 10*a
^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2) + (5*a*b^2 + 125*a^2*b - 4*b^3)/(16
384*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))))*((25*a^2*(a^9*b^3)
^(1/2) - 35*b^2*(a^9*b^3)^(1/2) + 4*a^3*b^5 - 35*a^4*b^4 + 70*a^5*b^3 + 105
*a^6*b^2 + 154*a*b*(a^9*b^3)^(1/2))/(16384*(a^6*b^8 - 5*a^7*b^7 + 10*a^8*b^
6 - 10*a^9*b^5 + 5*a^10*b^4 - a^11*b^3)))^(1/2)*2i)/d - ((3*cos(c + d*x)^3*
(10*a*b - 3*a^2 + b^2))/(32*a*(a - b)^2) + (cos(c + d*x)*(19*a + b))/(32*a*
(a - b)) - (3*b*cos(c + d*x)^5*(7*a + b))/(32*a*(a^2 - 2*a*b + b^2)) + (b*c
os(c + d*x)^7*(5*a + b))/(32*a*(a^2 - 2*a*b + b^2)))/(d*(a^2 - 2*a*b + b^2
+ cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*c
os(c + d*x)^6 + b^2*cos(c + d*x)^8))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.228 $\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$-\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(10\sqrt{a}\sqrt{b}+7a+4b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\cos(c+dx)\left((7a-\right.}{32a^2d(a-b)^2\left(a-\right.}$$

```
[Out] -1/8*cos(d*x+c)*(a+b-b*cos(d*x+c)^2)/a/(a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(
d*x+c)^4)^2-1/32*cos(d*x+c)*((7*a-3*b)*(a+2*b)-6*(2*a-b)*b*cos(d*x+c)^2)/a^
2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-3/64*arctan(b^(1/4)*cos(d
*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(7*a+4*b-10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)
/d/(a^(1/2)-b^(1/2))^(5/2)-3/64*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2)
)^(1/2))*(7*a+4*b+10*a^(1/2)*b^(1/2))/a^(5/2)/b^(1/4)/d/(a^(1/2)+b^(1/2))^(
5/2)
```

Rubi [A] time = 0.46, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3215, 1092, 1178, 1166, 205, 208}

$$-\frac{\cos(c+dx)\left((7a-3b)(a+2b)-6b(2a-b)\cos^2(c+dx)\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{3\left(-10\sqrt{a}\sqrt{b}+7a+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\sqrt[4]{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(10\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (-3*(7*a - 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqr
t[a] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(1/4)*d) - (3*(7*
a + 10*Sqrt[a]*Sqrt[b] + 4*b)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] +
 Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(1/4)*d) - (Cos[c + d*x
]*(a + b - b*Cos[c + d*x]^2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 -
b*Cos[c + d*x]^4)^2) - (Cos[c + d*x]*((7*a - 3*b)*(a + 2*b) - 6*(2*a - b)*b
*Cos[c + d*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c
 + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1092

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> -Simp[(x*(b^2 -
 2*a*c + b*c*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(b^2 - 2*a*c + 2*(p + 1)*(b^2
 - 4*a*c) + b*c*(4*p + 7)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ
[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int \frac{2(a-b)b+4b^2-4}{(a-b}\right.}{}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left((7a-3b\right.}{32a^2(a-b)^2d\left(a-b\right.}$$

$$= -\frac{\cos(c+dx)\left(a+b-b\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{\cos(c+dx)\left((7a-3b\right.}{32a^2(a-b)^2d\left(a-b\right.}$$

$$= -\frac{3\left(7a-10\sqrt{a}\sqrt{b}+4b\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt[4]{b}\,d} - \frac{3\left(7a+10\sqrt{a}\sqrt{b}+4b\right)\tanh^{-1}\left(\right.}{64a^{5/2}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt[4]{}}$$

Mathematica [C] time = 1.28, size = 784, normalized size = 2.50

$$3i\text{RootSum}\left[\#1^8b-4\#1^6b-16\#1^4a+6\#1^4b-4\#1^2b+b\&, \frac{-4\#1^6ab\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2\#1^6b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+28\#1^4}{}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-32*Cos[c + d*x]*(7*a^2 + 5*a*b - 3*b^2 + 3*b*(-2*a + b)*Cos[2*(c + d*x)]
))/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) - (512*a*(a - b)
*Cos[c + d*x]*(2*a + b - b*Cos[2*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c +
d*x)] + b*Cos[4*(c + d*x)])^2 + (3*I)*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8 & , (4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)] - 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - (2*I)*a*b*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2] + I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - 28*a^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 24*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^2 - 10*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^2 + (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (5*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^2 + 28*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 24*a*b*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 10*b^2*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1^4 - (14*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1
^4 + (12*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - (5*I)*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1^4 - 4*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^6 + 2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 + (2*I)*a*b
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
 #1^2]*#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(128*
a^2*(a - b)^2*d)
```

fricas [B] time = 1.41, size = 4160, normalized size = 13.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(24*(2*a*b^2 - b^3)*cos(d*x + c)^7 - 4*(7*a^2*b + 35*a*b^2 - 18*b^3)
*cos(d*x + c)^5 - 8*(a^2*b - 22*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 3*((a^4*b^2
 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4
)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*
x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (
a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^
3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a
^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^
2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b
^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 -
 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
+ 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 -
 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^
10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*
d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^
15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b
^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4))
 - (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-
(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b +
10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^
3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13
*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b
^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*
b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 +
a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c
)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a
^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b +
 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b
^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*
b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3
+ 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b
^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 +
```

a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) + (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) - 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) - (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) + 3*((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-27*(2401*a^4 - 4802*a^3*b + 4189*a^2*b^2 - 1788*a*b^3 + 336*b^4)*cos(d*x + c) - 27*((11*a^12*b - 66*a^11*b^2 + 169*a^10*b^3 - 240*a^9*b^4 + 205*a^8*b^5 - 106*a^7*b^6 + 31*a^6*b^7 - 4*a^5*b^8)*d^3*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)) + (343*a^7 - 623*a^6*b + 515*a^5*b^2 - 213*a^4*b^3 + 42*a^3*b^4)*d)*sqrt(-(105*a^4 - 210*a^3*b + 189*a^2*b^2 - 84*a*b^3 + 16*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((2401*a^4 - 5292*a^3*b + 4974*a^2*b^2 - 2268*a*b^3 + 441*b^4)/((a^15*b - 10*a^14*b^2 + 45*a^13*b^3 - 120*a^12*b^4 + 210*a^11*b^5 - 252*a^10*b^6 + 210*a^9*b^7 - 120*a^8*b^8 + 45*a^7*b^9 - 10*a^6*b^10 + a^5*b^11)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))) + 4*(11*a^3 + 4*a^2*b - 21*a*b^2 + 6*b^3)*cos(d*x + c))/((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*co

```
s(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)
^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 -
4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)
```

giac [B] time = 1.75, size = 766, normalized size = 2.45

$$-\frac{3\left(7\,a^{3}-5\,a^{2}b+2\,ab^{2}-\left(11\,a^{2}-11\,ab+4\,b^{2}\right)\sqrt{ab}\right)\sqrt{-b^{2}-\sqrt{ab}\,b}\,\arctan\left(\frac{\cos}{d\sqrt{-\frac{a^{4}bd^{2}-2\,a^{3}b^{2}d^{2}+a^{2}b^{3}d^{2}+\sqrt{\left(a^{4}bd^{2}-2\,a^{3}b^{2}d^{2}+a^{2}}}{a^{4}bd^{4}-}}}\right.}{64\left(a^{6}-3\,a^{5}b+3\,a^{4}b^{2}-a^{3}b^{3}\right)d|b|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -3/64*(7*a^3 - 5*a^2*b + 2*a*b^2 - (11*a^2 - 11*a*b + 4*b^2)*sqrt(a*b))*sqr
t(-b^2 - sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(a^4*b*d^2 - 2*a^3*b^2*d
^2 + a^2*b^3*d^2 + sqrt((a^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2)^2 + (a^4*
b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))
)/(a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4))))/((a^6 - 3*a^5*b + 3*a^4*b^2
- a^3*b^3)*d*abs(b)) - 3/64*(7*a^3 - 5*a^2*b + 2*a*b^2 + (11*a^2 - 11*a*b +
 4*b^2)*sqrt(a*b))*sqrt(-b^2 + sqrt(a*b)*b)*arctan(cos(d*x + c)/(d*sqrt(-(a
^4*b*d^2 - 2*a^3*b^2*d^2 + a^2*b^3*d^2 - sqrt((a^4*b*d^2 - 2*a^3*b^2*d^2 +
a^2*b^3*d^2)^2 + (a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4)*(a^5 - 3*a^4*b +
 3*a^3*b^2 - a^2*b^3)))/(a^4*b*d^4 - 2*a^3*b^2*d^4 + a^2*b^3*d^4))))/((a^6
- 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d*abs(b)) - 1/32*(12*a*b^2*cos(d*x + c)^7/
d - 6*b^3*cos(d*x + c)^7/d - 7*a^2*b*cos(d*x + c)^5/d - 35*a*b^2*cos(d*x +
c)^5/d + 18*b^3*cos(d*x + c)^5/d - 2*a^2*b*cos(d*x + c)^3/d + 44*a*b^2*cos(
d*x + c)^3/d - 18*b^3*cos(d*x + c)^3/d + 11*a^3*cos(d*x + c)/d + 4*a^2*b*co
s(d*x + c)/d - 21*a*b^2*cos(d*x + c)/d + 6*b^3*cos(d*x + c)/d)/((b*cos(d*x
+ c)^4 - 2*b*cos(d*x + c)^2 - a + b)^2*(a^4 - 2*a^3*b + a^2*b^2))
```

maple [B] time = 0.64, size = 1281, normalized size = 4.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -3/16/d/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+3/3
2/d*b/a^2/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-9/6
4/d/(a*b)^(1/2)/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*cos(d*x+c)
^3+3/64/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a^2-2*a*b+b^2)*
cos(d*x+c)^3-11/64/d/b/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)+
3/32/d/a^2/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-5/64/d/(a*b)^(
1/2)/a/(cos(d*x+c)^2+(a*b)^(1/2)/b-1)^2/(a-b)*cos(d*x+c)-3/16/d/a/(a^2-2*a*
b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1
/2))*b+3/32/d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(
d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-21/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/2)/(
((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))+27/
64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x
+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b^2-3/16/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*a*b+
b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2
))-3/16/d/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+3
/32/d*b/a^2/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3+9
/64/d/(a*b)^(1/2)/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2)*cos(d*x+
c)^3-3/64/d*b/(a*b)^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a^2-2*a*b+b^2
)*cos(d*x+c)^3-11/64/d/b/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c
```

```
)+3/32/d/a^2/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+5/64/d/(a*b)
^(1/2)/a/(cos(d*x+c)^2-1-(a*b)^(1/2)/b)^2/(a-b)*cos(d*x+c)+3/16/d/a/(a^2-2*
a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)
^(1/2))*b-3/32/d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(
cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-21/64/d/(a^2-2*a*b+b^2)*b/(a*b)^(1/
2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2)
)+27/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(c
os(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b^2-3/16/d*b^3/a^2/(a*b)^(1/2)/(a^2-
2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*
b)^(1/2))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 18.91, size = 5753, normalized size = 18.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] (atan(((((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^
4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^
2)) - (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 10
5*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^1
5*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13
*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 -
 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3
+ 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9
*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^
(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3
- 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 -
 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*
a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b -
 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2
)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*
a^14*b^2)))^(1/2)*1i - (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5
 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^
7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^
15*b)^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b
^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^
12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 +
 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*
b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)
^(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 -
54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^
4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b^6
+ 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4
- 4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/
2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a
*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 +
10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*1i)/((3*(684*a*b^5 - 144*b^6 - 1233*a^2*b
^4 + 882*a^3*b^3))/(8192*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)
) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^4 +
```

```
57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2))
 - (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a
^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b
)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^
3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 - 65
536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6
*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b
 - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/
2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5
*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 - 99
0*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*a^6
*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 105*a^9*b - 16
*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))
/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5*a^1
4*b^2)))^(1/2) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155
648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3
 + 6*a^8*b^2)) + (cos(c + d*x)*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^
(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 5
4*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4
 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304
*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 -
4*a^5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2)
 - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b
*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10
*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089
*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^
5*b^3 + 6*a^6*b^2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) - 1
05*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 54*a*b*(a^
15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^1
3*b^3 - 5*a^14*b^2)))^(1/2)))*((9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^
(1/2) - 105*a^9*b - 16*a^5*b^5 + 84*a^6*b^4 - 189*a^7*b^3 + 210*a^8*b^2 - 5
4*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4
 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*2i)/d - ((cos(c + d*x)*(15*a*b + 11*a^
2 - 6*b^2))/(32*a^2*(a - b)) - (cos(c + d*x)^3*(a^2*b - 22*a*b^2 + 9*b^3))/
(16*a^2*(a - b)^2) + (3*b*cos(c + d*x)^7*(2*a*b - b^2))/(16*a^2*(a^2 - 2*a*
b + b^2)) - (b*cos(c + d*x)^5*(35*a*b + 7*a^2 - 18*b^2))/(32*a^2*(a^2 - 2*a
*b + b^2)))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c
+ d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) + (a
tan(((((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b^4
+ 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b^2)
) - (cos(c + d*x)*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) + 105
*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a^15
*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*
b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6 -
65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 +
 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) + 105*a^9
*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a^15*b)^
(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3
- 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^5 -
 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 + 6*
a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) + 105*a^9*b
+ 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a^15*b)^(1/
2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3 - 5
*a^14*b^2)))^(1/2)*1i - (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^
5 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a
^7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(
a^15*b)^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8
*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*
a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7
```

```
 + 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15
*b)^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2
 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12
*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b
^6 + 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b
^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)
^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 -
54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^
4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*1i)/((3*(684*a*b^5 - 144*b^6 - 1233*a
^2*b^4 + 882*a^3*b^3))/(8192*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*
b^2)) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^5 - 155648*a^8*b
^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a^7*b^3 + 6*a^8*b
^2)) - (cos(c + d*x)*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) +
105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a
^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^
13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7 + 98304*a^7*b^6
 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^
3 + 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) + 105*
a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a^15*
b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b
^3 - 5*a^14*b^2)))^(1/2) + (cos(c + d*x)*(144*b^7 - 612*a*b^6 + 1089*a^2*b^
5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b^4 - 4*a^5*b^3 +
 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)^(1/2) + 105*a^9
*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 - 54*a*b*(a^15*b)^
(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^4 + 10*a^13*b^3
- 5*a^14*b^2)))^(1/2) + (((3*(16384*a^5*b^7 - 73728*a^6*b^6 + 155648*a^7*b^
5 - 155648*a^8*b^4 + 57344*a^9*b^3))/(16384*(a^10 - 4*a^9*b + a^6*b^4 - 4*a
^7*b^3 + 6*a^8*b^2)) + (cos(c + d*x)*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(
a^15*b)^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8
*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*
a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*(16384*a^5*b^8 - 65536*a^6*b^7
 + 98304*a^7*b^6 - 65536*a^8*b^5 + 16384*a^9*b^4))/(256*(a^8 - 4*a^7*b + a^
4*b^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15
*b)^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2
 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12
*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2) - (cos(c + d*x)*(144*b^7 - 612*a*b
^6 + 1089*a^2*b^5 - 990*a^3*b^4 + 441*a^4*b^3))/(256*(a^8 - 4*a^7*b + a^4*b
^4 - 4*a^5*b^3 + 6*a^6*b^2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^2*(a^15*b)
^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*a^8*b^2 -
54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 - 10*a^12*b^
4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)))*(-(9*(49*a^2*(a^15*b)^(1/2) + 21*b^
2*(a^15*b)^(1/2) + 105*a^9*b + 16*a^5*b^5 - 84*a^6*b^4 + 189*a^7*b^3 - 210*
a^8*b^2 - 54*a*b*(a^15*b)^(1/2)))/(16384*(a^15*b - a^10*b^6 + 5*a^11*b^5 -
10*a^12*b^4 + 10*a^13*b^3 - 5*a^14*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
[Out] Timed out
```

3.229 $\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=617

$$-\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}-\frac{\sqrt[4]{b}\left(5\sqrt{a}-2\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\left(5\sqrt{a}+2\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{64a^{5/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\sqrt[4]{b}\,\mathrm{t}}{8a}$$

```
[Out] -arctanh(cos(d*x+c))/a^3/d-1/8*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a/(a-b)/d/(a-b
+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)^2-1/4*b*cos(d*x+c)*(2-cos(d*x+c)^2)/a^2/(
a-b)/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/32*b*cos(d*x+c)*(11*a+b-(5*a
+b)*cos(d*x+c)^2)/a^2/(a-b)^2/d/(a-b+2*b*cos(d*x+c)^2-b*cos(d*x+c)^4)-1/64*
b^(1/4)*arctan(b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))*(5*a^(1/2)-2*b^(
1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(5/2)-1/8*b^(1/4)*arctan(b^(1/4)*cos(d*x+
c)/(a^(1/2)-b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)-b^(1/2))^(3/2)+1/8*b^(1/4)*a
rctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/
2))^(3/2)+1/64*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))*
(5*a^(1/2)+2*b^(1/2))/a^(5/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/2*b^(1/4)*arctan(
b^(1/4)*cos(d*x+c)/(a^(1/2)-b^(1/2))^(1/2))/a^3/d/(a^(1/2)-b^(1/2))^(1/2)+1
/2*b^(1/4)*arctanh(b^(1/4)*cos(d*x+c)/(a^(1/2)+b^(1/2))^(1/2))/a^3/d/(a^(1/
2)+b^(1/2))^(1/2)
```

Rubi [A] time = 0.84, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 7, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.318$, Rules used = {3215, 1238, 207, 1178, 1166, 205, 208}

$$-\frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{4a^2d(a-b)\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}-\frac{b\cos(c+dx)\left(-(5a+b)\cos^2(c+dx)+11a+b\right)}{32a^2d(a-b)^2\left(a-b\cos^4(c+dx)+2b\cos^2(c+dx)-b\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((5*Sqrt[a] - 2*Sqrt[b])*b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a
] - Sqrt[b]]])/(64*a^(5/2)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (b^(1/4)*ArcTan[(
b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b]]])/(8*a^(5/2)*(Sqrt[a] - Sqrt[
b])^(3/2)*d) - (b^(1/4)*ArcTan[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] - Sqrt[b
]]])/(2*a^3*Sqrt[Sqrt[a] - Sqrt[b]]*d) - ArcTanh[Cos[c + d*x]]/(a^3*d) + (b
^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(8*a^(5/2)*
(Sqrt[a] + Sqrt[b])^(3/2)*d) + (b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt
[Sqrt[a] + Sqrt[b]]])/(2*a^3*Sqrt[Sqrt[a] + Sqrt[b]]*d) + ((5*Sqrt[a] + 2*S
qrt[b])*b^(1/4)*ArcTanh[(b^(1/4)*Cos[c + d*x])/Sqrt[Sqrt[a] + Sqrt[b]]])/(6
4*a^(5/2)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b*Cos[c + d*x]*(2 - Cos[c + d*x]^
2))/(8*a*(a - b)*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)^2) - (b*
Cos[c + d*x]*(2 - Cos[c + d*x]^2))/(4*a^2*(a - b)*d*(a - b + 2*b*Cos[c + d*
x]^2 - b*Cos[c + d*x]^4)) - (b*Cos[c + d*x]*(11*a + b - (5*a + b)*Cos[c + d
*x]^2))/(32*a^2*(a - b)^2*d*(a - b + 2*b*Cos[c + d*x]^2 - b*Cos[c + d*x]^4)
)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1178

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symb
ol] :> Simp[(x*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2)*(a + b*x^2 +
 c*x^4)^(p + 1))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2
 - 4*a*c)), Int[Simp[(2*p + 3)*d*b^2 - a*b*e - 2*a*c*d*(4*p + 5) + (4*p + 7
)*(d*b - 2*a*e)*c*x^2, x]*(a + b*x^2 + c*x^4)^(p + 1), x], x] /; FreeQ[{a,
b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] &&
LtQ[p, -1] && IntegerQ[2*p]
```

Rule 1238

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q*(a + b*x^2 + c*x^4)^p, x], x]
 /; FreeQ[{a, b, c, d, e, p, q}, x] && NeQ[b^2 - 4*a*c, 0] && ((IntegerQ[p]
 && IntegerQ[q]) || IGtQ[p, 0] || IGtQ[q, 0])
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a-b+2bx^2-bx^4\right)^3}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \left(-\frac{1}{a^3(-1+x^2)} + \frac{b-bx^2}{a\left(a-b+2bx^2-bx^4\right)^3} + \frac{b-bx^2}{a^2\left(a-b+2bx^2-bx^4\right)^2} + \frac{b-bx^2}{a^3\left(a-b+2bx^2-bx^4\right)}\right) dx, x\right.}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \cos(c+dx)\right)}{a^3 d} - \frac{\text{Subst}\left(\int \frac{b-bx^2}{a-b+2bx^2-bx^4}\,dx, x, \cos(c+dx)\right)}{a^3 d} - \frac{\text{Su}}{}$$

$$= -\frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} - \frac{b\cos(c+dx)\left(2-\cos^2(c+dx)\right)}{8a(a-b)d\left(a-b+2b\cos^2(c+dx)-b\cos^4(c+dx)\right)^2} - \frac{}{4}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}+\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}+\sqrt{b}}\,d} - \frac{}{8a(}$$

$$= -\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{2a^3\sqrt{\sqrt{a}-\sqrt{b}}\,d} - \frac{\tanh^{-1}(\cos(c+dx))}{a^3 d} + \frac{\sqrt[4]{b}\,\text{t}}{8a^5}$$

$$= -\frac{\left(5\sqrt{a}-2\sqrt{b}\right)\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{64a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{5/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt{\sqrt{a}-\sqrt{b}}}\right)}{8a^{5/2}\left(\sqrt{a}-\sqrt{b}\right)^{3/2} d} - \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}}{\sqrt{}}\right.}{2a^3\sqrt{\sqrt{a}-}}$$

Mathematica [C] time = 4.28, size = 920, normalized size = 1.49

$$\frac{\frac{512b(\cos(3(c+dx))-5\cos(c+dx))a^2}{(a-b)(-8a+3b-4b\cos(2(c+dx))+b\cos(4(c+dx)))^2} + \frac{32b\cos(c+dx)(-41a+23b+(13a-7b)\cos(2(c+dx)))a}{(a-b)^2(8a-3b+4b\cos(2(c+dx))-b\cos(4(c+dx)))} - 256\log\left(\cos\left(\frac{1}{2}(c+dx)\right)\right) + 25}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((32*a*b*Cos[c + d*x]*(-41*a + 23*b + (13*a - 7*b)*Cos[2*(c + d*x)]))/((a -
 b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (512*a^2*b
*(-5*Cos[c + d*x] + Cos[3*(c + d*x)]))/((a - b)*(-8*a + 3*b - 4*b*Cos[2*(c
+ d*x)] + b*Cos[4*(c + d*x)])^2) - 256*Log[Cos[(c + d*x)/2]] + 256*Log[Sin[
(c + d*x)/2]] - (I*b*RootSum[b - 4*b*#1^2 - 16*a*#1^4 + 6*b*#1^4 - 4*b*#1^6
 + b*#1^8 & , (-90*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 142*a*b*A
rcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c
 + d*x] - #1)] + (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (71*I)*a*b*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2] + 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 506*a*b*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + 192*b^2*ArcTan[Sin[c + d*x]/(C
os[c + d*x] - #1)]*#1^2 - (199*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^
2 + (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (96*I)*b^2*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - 398*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^4 + 506*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - 192*
```

```
b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + (199*I)*a^2*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - (253*I)*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^4 + (96*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 + 90*a^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 - 142*a*b*ArcTan[Sin[c + d*x]/(Cos[c
+ d*x] - #1)]*#1^6 + 64*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^6 -
 (45*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^6 + (71*I)*a*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^6 - (32*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*
#1^6)/(-(b*#1) - 8*a*#1^3 + 3*b*#1^3 - 3*b*#1^5 + b*#1^7) & ])/(a - b)^2)/(
256*a^3*d)
```

fricas [B] time = 3.46, size = 5020, normalized size = 8.14

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/128*(4*(13*a^2*b^2 - 7*a*b^3)*cos(d*x + c)^7 - 4*(53*a^2*b^2 - 29*a*b^3)
*cos(d*x + c)^5 - 4*(17*a^3*b - 78*a^2*b^2 + 37*a*b^3)*cos(d*x + c)^3 - ((a
^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a
^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*
cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)
^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b
 - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b
+ 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b -
19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 -
 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 -
10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*
a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a
^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log((
4100625*a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b^3 - 20354340*a^3*b^4 + 10
504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(d*x + c) - ((45*a^16 - 280
*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12*b^4 - 540*a^11*b^5 + 165*
a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 3997108
6*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 740147
2*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 -
120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 +
45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) - (123525*a^9*b - 450359*a^8*b
^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5*b^5 - 95104*a^4*b^6 + 122
88*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^
4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6
*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679
452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 15360
00*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 21
0*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*
a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x +
c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^
5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*
a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^
3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b
^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^
6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 4967
9452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536
000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 2
10*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10
*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 +
 5*a^7*b^4 - a^6*b^5)*d^2))*log((4100625*a^6*b - 14762250*a^5*b^2 + 2322794
9*a^4*b^3 - 20354340*a^3*b^4 + 10504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^
7)*cos(d*x + c) - ((45*a^16 - 280*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 9
```

95*a^12*b^4 - 540*a^11*b^5 + 165*a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a
^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^
4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((
a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5
 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4
)) + (123525*a^9*b - 450359*a^8*b^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327
152*a^5*b^5 - 95104*a^4*b^6 + 12288*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^
3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b
^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a
^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*
a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
+ 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 -
 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^
10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))) + ((a^5*b^2 -
2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d
*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x +
 c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7
 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(3465*a^4*b - 9306*a
^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 + (a^11 - 5*a^10*b + 10*a^9*
b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*
a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992
*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b
 + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6
- 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a
^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(-(4100625*
a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b^3 - 20354340*a^3*b^4 + 10504896*a
^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(d*x + c) - ((45*a^16 - 280*a^15*b
+ 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12*b^4 - 540*a^11*b^5 + 165*a^10*b^6
 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^
3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^
7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18
*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*
b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) - (123525*a^9*b - 450359*a^8*b^2 + 715
183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5*b^5 - 95104*a^4*b^6 + 12288*a^3*b
^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 1024
*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^
2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*
b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8
 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b
^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9
 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^
4 - a^6*b^5)*d^2))) - ((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4
*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 +
7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3
- a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*
b^4)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 + 10045*a^2*b^3 - 5084*a*b^4 + 102
4*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d
^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5
*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^
8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*
b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^
9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b
^4 - a^6*b^5)*d^2))*log(-(4100625*a^6*b - 14762250*a^5*b^2 + 23227949*a^4*b
^3 - 20354340*a^3*b^4 + 10504896*a^2*b^5 - 3044864*a*b^6 + 393216*b^7)*cos(
d*x + c) - ((45*a^16 - 280*a^15*b + 747*a^14*b^2 - 1110*a^13*b^3 + 995*a^12
*b^4 - 540*a^11*b^5 + 165*a^10*b^6 - 22*a^9*b^7)*d^3*sqrt((4100625*a^8*b -
19010700*a^7*b^2 + 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 -
 21320992*a^3*b^6 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 -
10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*

```
a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)) + (1
23525*a^9*b - 450359*a^8*b^2 + 715183*a^7*b^3 - 630957*a^6*b^4 + 327152*a^5
*b^5 - 95104*a^4*b^6 + 12288*a^3*b^7)*d)*sqrt(-(3465*a^4*b - 9306*a^3*b^2 +
 10045*a^2*b^3 - 5084*a*b^4 + 1024*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10
*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((4100625*a^8*b - 19010700*a^7*b^2
+ 39971086*a^6*b^3 - 49679452*a^5*b^4 + 39947241*a^4*b^5 - 21320992*a^3*b^6
 + 7401472*a^2*b^7 - 1536000*a*b^8 + 147456*b^9)/((a^21 - 10*a^20*b + 45*a^
19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^
14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^11 - 5*a^10*b +
10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))) + 20*(7*a^3*b - 10*a^
2*b^2 + 3*a*b^3)*cos(d*x + c) + 64*((a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^
8 - 4*(a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^6 - 2*(a^3*b - 5*a^2*b^2 + 7*a
*b^3 - 3*b^4)*cos(d*x + c)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 +
4*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)*cos(d*x + c)^2)*log(1/2*cos(d*x + c)
+ 1/2) - 64*((a^2*b^2 - 2*a*b^3 + b^4)*cos(d*x + c)^8 - 4*(a^2*b^2 - 2*a*b^
3 + b^4)*cos(d*x + c)^6 - 2*(a^3*b - 5*a^2*b^2 + 7*a*b^3 - 3*b^4)*cos(d*x +
 c)^4 + a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4 + 4*(a^3*b - 3*a^2*b^2 +
3*a*b^3 - b^4)*cos(d*x + c)^2)*log(-1/2*cos(d*x + c) + 1/2))/((a^5*b^2 - 2*
a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*c
os(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c
)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 -
 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Warni
ng, need to choose a branch for the root of a polynomial with parameters. T
his might be wrong.The choice was done assuming [a,b]=[53,-89]Warning, need
 to choose a branch for the root of a polynomial with parameters. This migh
t be wrong.The choice was done assuming [a,b]=[-57,87]Precision problem cho
osing root in common_EXT, current precision 14-2/d*((-8*((1-cos(c+d*x))/(1+
cos(c+d*x)))^7*a^3*b+5*((1-cos(c+d*x))/(1+cos(c+d*x)))^7*a^2*b^2-5*((1-cos(
c+d*x))/(1+cos(c+d*x)))^6*a^3*b-86*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a^2*b^
2+64*((1-cos(c+d*x))/(1+cos(c+d*x)))^6*a*b^3+104*((1-cos(c+d*x))/(1+cos(c+d
*x)))^5*a^3*b-327*((1-cos(c+d*x))/(1+cos(c+d*x)))^5*a^2*b^2+208*((1-cos(c+d
*x))/(1+cos(c+d*x)))^5*a*b^3+315*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^3*b-10
74*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a^2*b^2+1728*((1-cos(c+d*x))/(1+cos(c+
d*x)))^4*a*b^3-768*((1-cos(c+d*x))/(1+cos(c+d*x)))^4*b^4+400*((1-cos(c+d*x)
)/(1+cos(c+d*x)))^3*a^3*b-1161*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a^2*b^2+56
0*((1-cos(c+d*x))/(1+cos(c+d*x)))^3*a*b^3+257*((1-cos(c+d*x))/(1+cos(c+d*x)
))^2*a^3*b-370*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a^2*b^2+128*((1-cos(c+d*x)
)/(1+cos(c+d*x)))^2*a*b^3+80*(1-cos(c+d*x))/(1+cos(c+d*x))*a^3*b-53*(1-cos(
c+d*x))/(1+cos(c+d*x))*a^2*b^2+9*a^3*b-6*a^2*b^2)/(16*a^5-32*a^4*b+16*a^3*b
^2)/(((1-cos(c+d*x))/(1+cos(c+d*x)))^4*a+4*((1-cos(c+d*x))/(1+cos(c+d*x)))^
3*a+6*((1-cos(c+d*x))/(1+cos(c+d*x)))^2*a-16*((1-cos(c+d*x))/(1+cos(c+d*x))
)^2*b+4*(1-cos(c+d*x))/(1+cos(c+d*x))*a+a)^2-1/4/a^3*ln(abs(1-cos(c+d*x))/a
bs(1+cos(c+d*x))))-2/d/(16*a^5-32*a^4*b+16*a^3*b^2)*2/d*((-32*b^2+71*b*a-45
*a^2)/8*(c+d*x)+(32*a^5*b+135*a^5*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-32*a^4*b^2-
579*a^4*b*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-32*a^4*a*b-222*a^4*sqrt(a*b)*sqrt(a
^2-a*b+sqrt(a*b)*(a-b))-184*a^3*b^3+729*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(a-b
))+32*a^3*b*a*b+840*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+312*a^2*b
^4-209*a^2*b^3*sqrt(a^2-a*b+sqrt(a*b)*(a-b))+184*a^2*b^2*a*b-910*a^2*b^2*sq
rt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))-128*a*b^5-52*a*b^4*sqrt(a^2-a*b+sqrt(
a*b)*(a-b))-312*a*b^3*a*b+252*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b))
+128*b^4*a*b+64*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(a-b)))*abs(a-b)/(48*a
```

```
^5-192*a^4*b+224*a^3*b^2-64*a^2*b^3-16*a*b^4)*(atan(tan(c+d*x)/sqrt(-(16*a+
sqrt(16*a*16*a+4*(8*b-8*a)*8*a))/2/(8*b-8*a)))+pi*floor((c+d*x)/pi+1/2))-(3
2*a^5*b-135*a^5*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))-32*a^4*b^2+579*a^4*b*sqrt(a^
2-a*b+sqrt(a*b)*(-a+b))-32*a^4*a*b-222*a^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)
*(-a+b))-184*a^3*b^3-729*a^3*b^2*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+32*a^3*b*a*
b+840*a^3*b*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+312*a^2*b^4+209*a^2*b^
3*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+184*a^2*b^2*a*b-910*a^2*b^2*sqrt(a*b)*sqrt
(a^2-a*b+sqrt(a*b)*(-a+b))-128*a*b^5+52*a*b^4*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)
)-312*a*b^3*a*b+252*a*b^3*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b))+128*b^4*
a*b+64*b^4*sqrt(a*b)*sqrt(a^2-a*b+sqrt(a*b)*(-a+b)))*abs(a-b)/(48*a^5-192*a
^4*b+224*a^3*b^2-64*a^2*b^3-16*a*b^4)*(atan(tan(c+d*x)/sqrt(-(16*a-sqrt(16*
a*16*a+4*(8*b-8*a)*8*a))/2/(8*b-8*a)))+pi*floor((c+d*x)/pi+1/2)))
```

maple [**B**] time = 0.61, size = 1139, normalized size = 1.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 1/2/d/a^3*ln(cos(d*x+c)-1)-1/2/d/a^3*ln(1+cos(d*x+c))-13/32/d/(b*cos(d*x+c)
^4-2*b*cos(d*x+c)^2-a+b)^2*b^2/a/(a^2-2*a*b+b^2)*cos(d*x+c)^7+7/32/d/a^2*b^
3/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^7+53/3
2/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2*a*b+b^2)*cos(d*x+c)^5*
b^2-29/32/d/a^2*b^3/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/(a^2-2*a*b+b^2)
*cos(d*x+c)^5+17/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2*b/(a^2-2*a*b+
b^2)*cos(d*x+c)^3-39/16/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-a+b)^2/a/(a^2-2*
a*b+b^2)*cos(d*x+c)^3*b^2+37/32/d/a^2*b^3/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^2-
a+b)^2/(a^2-2*a*b+b^2)*cos(d*x+c)^3-35/32/d/(b*cos(d*x+c)^4-2*b*cos(d*x+c)^
2-a+b)^2/(a-b)/a*b*cos(d*x+c)+15/32/d/a^2*b^2/(b*cos(d*x+c)^4-2*b*cos(d*x+c
)^2-a+b)^2/(a-b)*cos(d*x+c)+45/64/d/a/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(
1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*b-71/64/d*b^2/a^2/(a^2
-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)
*b)^(1/2))+1/2/d/a^3*b^3/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(
cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-1/4/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)
/(((a*b)^(1/2)+b)*b)^(1/2)*arctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))*
b^2+5/32/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*a*b+b^2)/(((a*b)^(1/2)+b)*b)^(1/2)*ar
ctanh(cos(d*x+c)*b/(((a*b)^(1/2)+b)*b)^(1/2))-45/64/d/a/(a^2-2*a*b+b^2)/(((
a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))*b+71/
64/d*b^2/a^2/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/
(((a*b)^(1/2)-b)*b)^(1/2))-1/2/d/a^3*b^3/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-b)*b
)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))-1/4/d/a/(a^2-2*a*b+b
^2)/(a*b)^(1/2)/(((a*b)^(1/2)-b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)
-b)*b)^(1/2))*b^2+5/32/d*b^3/a^2/(a*b)^(1/2)/(a^2-2*a*b+b^2)/(((a*b)^(1/2)-
b)*b)^(1/2)*arctan(cos(d*x+c)*b/(((a*b)^(1/2)-b)*b)^(1/2))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**B**] time = 20.83, size = 12247, normalized size = 19.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a - b*sin(c + d*x)^4)^3),x)

[Out] - ((5*cos(c + d*x)*(7*a*b - 3*b^2))/(32*a^2*(a - b)) - (cos(c + d*x)^3*(17*
a^2*b - 78*a*b^2 + 37*b^3))/(32*a^2*(a - b)^2) - (cos(c + d*x)^5*(53*a*b^2
- 29*b^3))/(32*a^2*(a - b)^2) + (b*cos(c + d*x)^7*(13*a*b - 7*b^2))/(32*a^2
*(a - b)^2))/(d*(a^2 - 2*a*b + b^2 + cos(c + d*x)^2*(4*a*b - 4*b^2) - cos(c
 + d*x)^4*(2*a*b - 6*b^2) - 4*b^2*cos(c + d*x)^6 + b^2*cos(c + d*x)^8)) - (
atan(((((((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1
158*a^15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 +
6*a^12*b^2)) - (cos(c + d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 34
89660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a
^17*b^4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)
))/(2*a^3) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 59274854
4*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^1
1*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6
*b^8)/64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5
)/8192)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*1i)/(2*
a^3) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 230
76232*a^3*b^6 + 8247825*a^4*b^5)*1i)/(1048576*(a^12 - 4*a^11*b + a^8*b^4 -
4*a^9*b^3 + 6*a^10*b^2)))/a^3 - ((((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^
13*b^7 - 2154*a^14*b^6 + 1158*a^15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b
+ a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*x)*(402653184*a^12*b^9
- 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395
008*a^16*b^5 - 268435456*a^17*b^4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4
 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (cos(c + d*x)*(75497472*a^6*b^9 - 33
7215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^
5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3)
 - (12*a^5*b^9 - (4311*a^6*b^8)/64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b
^6)/8192 + (546059*a^9*b^5)/8192)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b
^3 + 6*a^12*b^2)))*1i)/(2*a^3) + (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^
8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5)*1i)/(1048576*(a^
12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/a^3)/((((((192*a^11*b^9
 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1158*a^15*b^5 - 256*a^16*
b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c +
d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221
225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(2097152*a^3*(
a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) + (cos(c + d*
x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^
9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^
3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6*b^8)/64 + (307961*a^7*b
^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5)/8192)/(2*(a^14 - 4*a^1
3*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))/(2*a^3) - (cos(c + d*x)*(314572
8*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*
b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/a^3 +
 (((((192*a^11*b^9 - 990*a^12*b^8 + 2050*a^13*b^7 - 2154*a^14*b^6 + 1158*a^
15*b^5 - 256*a^16*b^4)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12
*b^2)) + (cos(c + d*x)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 34896609
28*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^
4))/(2097152*a^3*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*
a^3) - (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*
b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(1048576*(a^12 - 4*a^11*b +
a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))/(2*a^3) - (12*a^5*b^9 - (4311*a^6*b^8)/
64 + (307961*a^7*b^7)/2048 - (1290253*a^8*b^6)/8192 + (546059*a^9*b^5)/8192
)/(2*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))/(2*a^3) + (co
s(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*
b^6 + 8247825*a^4*b^5))/(1048576*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6
*a^10*b^2)))/a^3 + ((90009*a*b^7)/32768 - (81*b^8)/128 - (271845*a^2*b^6)/6
5536 + (1184625*a^3*b^5)/524288)/(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 +
 6*a^12*b^2)))*1i)/(a^3*d) - (atan((((((201326592*a^11*b^9 - 1038090240*a^
12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 -
```

268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c + d*x)*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 69895552*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*1i - (((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*x)*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 69895552*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(

-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6
*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1
/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^
16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*1i
)/((((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 22
58632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^1
4 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c + d*x)*(-(2025
a^4(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 +
 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) -
4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b -
 a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(4026531
84*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b
^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b +
a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a
^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 +
9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429
*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 +
10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(75497472*a^6*b^9 - 3372
15488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5)
)/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^
4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 50
84*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 469
4*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^
17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912
*a^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 69895
552*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2
)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 102
4*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*
b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*
(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/
2) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076
232*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*
b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3
465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2
000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b
)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10
*a^15*b^2)))^(1/2) + (((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 214958
0800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*
b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (co
s(c + d*x)*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*
b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3
*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/
(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2
)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 -
 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(
a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^
(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4
- 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^
13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b
^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*x)*(75497
472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 1
63684352*a^10*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10
b^2)))(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b -
 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a
^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16
384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))
^(1/2) - (12582912*a^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 1651523
84*a^8*b^6 + 69895552*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^1
1*b^3 + 6*a^12*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) -

3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (1440144*a*b^7 - 331776*b^8 - 2174760*a^2*b^6 + 1184625*a^3*b^5)/(524288*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2))))*(-(2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) - 3465*a^10*b - 1024*a^6*b^5 + 5084*a^7*b^4 - 10045*a^8*b^3 + 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*2i)/d - (atan((((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c + d*x)*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 69895552*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*1i - (((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*x)*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^1

3*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(1638
4*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(
1/2) - (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8 + 592748544*a^8*
b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(524288*(a^12 - 4*a^11*b + a
^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^1
3*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 93
06*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a
^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10
*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a^5*b^9 - 70631424*a^6*b^8 + 1
57676032*a^7*b^7 - 165152384*a^8*b^6 + 69895552*a^9*b^5)/(1048576*(a^14 - 4
*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*((2025*a^4*(a^13*b)^(1/2) +
384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045
*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(
1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*
a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(3145728*b^9
- 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 8247825*a^4*b^5))/
(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(
a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*
a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a
^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17
+ a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*1i)/((((((2013
26592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 2258632704*a^1
4*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^14 - 4*a^13*b
+ a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) - (cos(c + d*x)*((2025*a^4*(a^13*b)
^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4
+ 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a
^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*
b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(402653184*a^12*b^9 -
1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b^6 + 14763950
08*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a
^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) +
3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 -
2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13
b)^(1/2))/(16384(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 -
10*a^15*b^2)))^(1/2) + (cos(c + d*x)*(75497472*a^6*b^9 - 337215488*a^7*b^8
+ 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))/(524288*(a^1
2 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2
) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10
045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b
)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 -
5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a^5*b^9 - 7063
1424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 69895552*a^9*b^5)/(1
048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)))*((2025*a^4*
(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084
*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*
a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17
+ a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*
x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232*a^3*b^6 + 82
47825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2
)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024
*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b
)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(
5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2
) + (((((201326592*a^11*b^9 - 1038090240*a^12*b^8 + 2149580800*a^13*b^7 - 2
258632704*a^14*b^6 + 1214251008*a^15*b^5 - 268435456*a^16*b^4)/(1048576*(a^
14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2)) + (cos(c + d*x)*((2025
a^4(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 -
5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) -
4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b -

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 a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*(4026531
84*a^12*b^9 - 1879048192*a^13*b^8 + 3489660928*a^14*b^7 - 3221225472*a^15*b
^6 + 1476395008*a^16*b^5 - 268435456*a^17*b^4))/(524288*(a^12 - 4*a^11*b +
a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^
13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9
306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*
a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 1
0*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (cos(c + d*x)*(75497472*a^6*b^9 - 33721
5488*a^7*b^8 + 592748544*a^8*b^7 - 489406464*a^9*b^6 + 163684352*a^10*b^5))
/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3 + 6*a^10*b^2)))*((2025*a^4*
(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b^5 - 5084
*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2) - 4694*
a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16*b - a^17
 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2) - (12582912*a
^5*b^9 - 70631424*a^6*b^8 + 157676032*a^7*b^7 - 165152384*a^8*b^6 + 6989555
2*a^9*b^5)/(1048576*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2))
)*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a
^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^
(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*
a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)
+ (cos(c + d*x)*(3145728*b^9 - 14417920*a*b^8 + 26453264*a^2*b^7 - 23076232
*a^3*b^6 + 8247825*a^4*b^5))/(524288*(a^12 - 4*a^11*b + a^8*b^4 - 4*a^9*b^3
 + 6*a^10*b^2)))*((2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*
a^10*b + 1024*a^6*b^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*
a*b^3*(a^13*b)^(1/2) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1
/2))/(16384*(5*a^16*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^1
5*b^2)))^(1/2) + (1440144*a*b^7 - 331776*b^8 - 2174760*a^2*b^6 + 1184625*a^
3*b^5)/(524288*(a^14 - 4*a^13*b + a^10*b^4 - 4*a^11*b^3 + 6*a^12*b^2))))*((
2025*a^4*(a^13*b)^(1/2) + 384*b^4*(a^13*b)^(1/2) + 3465*a^10*b + 1024*a^6*b
^5 - 5084*a^7*b^4 + 10045*a^8*b^3 - 9306*a^9*b^2 - 2000*a*b^3*(a^13*b)^(1/2
) - 4694*a^3*b*(a^13*b)^(1/2) + 4429*a^2*b^2*(a^13*b)^(1/2))/(16384*(5*a^16
*b - a^17 + a^12*b^5 - 5*a^13*b^4 + 10*a^14*b^3 - 10*a^15*b^2)))^(1/2)*2i)/
d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.230 $\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{32abd(a-b)}+\frac{\cdots}{8ad\left((a-\cdots\right.}$$

```
[Out] -1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-5*b^(1/
2))/a^(3/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/2))
^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+5*b^(1/2))/a^(3/4)/b^(3/2)/d/(a^(1/2)
+b^(1/2))^(5/2)-1/32*(a+5*b)*tan(d*x+c)/a/(a-b)^2/b/d+1/32*tan(d*x+c)^3/a/(
a-b)/b/d+1/8*tan(d*x+c)^9/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/3
2*sec(d*x+c)^2*tan(d*x+c)^5/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.53, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.292, Rules used = {3217, 1275, 12, 1120, 1279, 1166, 205}

$$-\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^9(c\cdots}{8ad\left((a-b)\tan^4(c+dx)\cdots\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((2*Sqrt[a] - 5*Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(3/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((2*Sqrt[a] + 5*S
qrt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(3/4)
*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - ((a + 5*b)*Tan[c + d*x])/(32*a*(a -
 b)^2*b*d) + Tan[c + d*x]^3/(32*a*(a - b)*b*d) + Tan[c + d*x]^9/(8*a*d*(a +
 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Sec[c + d*x]^2*Tan[c +
d*x]^5)/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1120

```
Int[((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
 :> -Simp[(d^3*(d*x)^(m - 3)*(2*a + b*x^2)*(a + b*x^2 + c*x^4)^(p + 1))/(2*
(p + 1)*(b^2 - 4*a*c)), x] + Dist[d^4/(2*(p + 1)*(b^2 - 4*a*c)), Int[(d*x)^
(m - 4)*(2*a*(m - 3) + b*(m + 4*p + 3)*x^2)*(a + b*x^2 + c*x^4)^(p + 1), x]
, x] /; FreeQ[{a, b, c, d}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && GtQ[
m, 3] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1275

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_.), x_Symbol] :> Simp[(f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p + 1
)*(b*d - 2*a*e - (b*e - 2*c*d)*x^2))/(2*(p + 1)*(b^2 - 4*a*c)), x] - Dist[f
^2/(2*(p + 1)*(b^2 - 4*a*c)), Int[(f*x)^(m - 2)*(a + b*x^2 + c*x^4)^(p + 1)
*Simp[(m - 1)*(b*d - 2*a*e) - (4*p + 4 + m + 1)*(b*e - 2*c*d)*x^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] &&
GtQ[m, 1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1279

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(e*f*(f*x)^(m - 1)*(a + b*x^2 + c*x^4)^(p +
1))/(c*(m + 4*p + 3)), x] - Dist[f^2/(c*(m + 4*p + 3)), Int[(f*x)^(m - 2)*(
a + b*x^2 + c*x^4)^p*Simp[a*e*(m - 1) + (b*e*(m + 2*p + 1) - c*d*(m + 4*p +
 3))*x^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[m, 1] && NeQ[m + 4*p + 3, 0] && IntegerQ[2*p] && (IntegerQ[p] ||
 IntegerQ[m])
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^8(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^8(1+x^2)}{(a+2ax^2+(a-b)x^4)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} + \frac{\text{Subst}\left(\int -\frac{2bx^8}{(a+2ax^2+(a-b)x^4)^2}\,d\right.}{16abd}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{x^8}{(a+2ax^2+(a-b)x^4)^2}\,dx\right.}{8ad}$$

$$= \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\sec^2(c+dx)\tan}{32abd\left(a+2a\tan^2(c+dx)\right.}$$

$$= \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{}{32abd\left(a+\right.}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4\right.}$$

$$= -\frac{(a+5b)\tan(c+dx)}{32a(a-b)^2bd} + \frac{\tan^3(c+dx)}{32a(a-b)bd} + \frac{\tan^9(c+dx)}{8ad\left(a+2a\tan^2(c+dx)+(a-b)\tan^4\right.}$$

$$= -\frac{\left(2\sqrt{a}-5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(2\sqrt{a}+5\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+\right.}{\sqrt[4]{a}}}{64a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}b^{3/2}d}$$

Mathematica [A] time = 3.98, size = 331, normalized size = 1.04

$$\frac{\frac{\left(2a^{3/2}\sqrt{b}-8\sqrt{a}\,b^{3/2}+ab+5b^2\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\,\sqrt{b}+a}} + \frac{8b\sin(2(c+dx))((5b-2a)\cos(2(c+dx))+5a-14b)}{8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b} + \frac{64ab(a-b)(\sin(4(c+dx))-6\sin(2(}{(-8a-4b\cos(2(c+dx))+b\cos(4(c+}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^8/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((2*a^(3/2)*Sqrt[b] + a*b - 8*Sqrt[a]*b^(3/2) + 5*b^2)*ArcTan[((Sqrt[a] +
Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a
]*Sqrt[b]]) + ((2*Sqrt[a] - 5*Sqrt[b])*(Sqrt[a] + Sqrt[b])^2*Sqrt[b]*ArcTan
h[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*
Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (8*b*(5*a - 14*b + (-2*a + 5*b)*Cos[2*(c + d*
x)])*Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x
)]) + (64*a*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b
 - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*b^2*d)
```

fricas [B] time = 1.90, size = 5219, normalized size = 16.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4
+ b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d
*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4
*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(((a^6*b^3 - 5*a^5*b^4 +
 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^
3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a
^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*
b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*
a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^
7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a*b^2 + 625/4*b^3 - 1/4
*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*x + c)^2 + 1/2*((a^9*b
^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*
a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 62
5*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7
- 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^
3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b - 623*a^4*b^2 + 1161*a^
3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^6*b^
3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225
*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*
a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7
*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) - 4*a^3 +
 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3
*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^
3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^
2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a
^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 77
00*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 +
 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*
a^4*b^12 + a^3*b^13)*d^4))) - ((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 -
 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*
a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d
*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt((
(a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqr
t((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^
3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 +
210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) -
4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 -
 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(35*a^3 - 1491/4*a^2*b + 1875/2*a
*b^2 + 625/4*b^3 - 1/4*(140*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*
x + c)^2 - 1/2*((a^9*b^3 - 18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*
b^7 - 66*a^4*b^8 + 13*a^3*b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2
*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a
^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b
^11 - 10*a^4*b^12 + a^3*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (70*a^5*b -
 623*a^4*b^2 + 1161*a^3*b^3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 -
 a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625
*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 -
 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3
*b^13)*d^4)) - 4*a^3 + 35*a^2*b - 70*a*b^2 - 105*b^3)/((a^6*b^3 - 5*a^5*b^4
 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 4
5*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*
b^7)*d^2*cos(d*x + c)^2 - (4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4
 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*
b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^1
1*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^
```

10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4))) + ((a^2*b^3 - 2*a*b^4 + b
^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^
3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*
b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*
a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a
^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b
^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a
^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^
12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2 + 105*b^3)/((a^6*b^3 - 5
*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2))*log(-35*a^3 +
1491/4*a^2*b - 1875/2*a*b^2 - 625/4*b^3 + 1/4*(140*a^3 - 1491*a^2*b + 3750
*a*b^2 + 625*b^3)*cos(d*x + c)^2 + 1/2*((a^9*b^3 - 18*a^8*b^4 + 75*a^7*b^5
- 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*a^3*b^9)*d^3*sqrt((1225*a^4 -
10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b
^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 -
120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4))*cos(d*x + c)*si
n(d*x + c) + (70*a^5*b - 623*a^4*b^2 + 1161*a^3*b^3 + 995*a^2*b^4 + 125*a*b
^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 -
10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b + 21966*a
^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120
*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5
*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2 + 105*b
^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^
2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^
5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^2 - (4*a^8*b - 45*a^7*b^2 +
165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2)*sq
rt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b
^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 +
210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)))
- ((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)
*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c
)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*d*cos(d*x + c)^2 + (a^4*b - 4
*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)*sqrt(-((a^6*b^3 - 5*a^5*b^4 + 10*a
^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^4 - 10780*a^3*b +
21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b
^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10
+ 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35*a^2*b + 70*a*b^2
+ 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a
*b^8)*d^2))*log(-35*a^3 + 1491/4*a^2*b - 1875/2*a*b^2 - 625/4*b^3 + 1/4*(14
0*a^3 - 1491*a^2*b + 3750*a*b^2 + 625*b^3)*cos(d*x + c)^2 - 1/2*((a^9*b^3 -
18*a^8*b^4 + 75*a^7*b^5 - 140*a^6*b^6 + 135*a^5*b^7 - 66*a^4*b^8 + 13*a^3*
b^9)*d^3*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^
4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 25
2*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^
13)*d^4))*cos(d*x + c)*sin(d*x + c) + (70*a^5*b - 623*a^4*b^2 + 1161*a^3*b^
3 + 995*a^2*b^4 + 125*a*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^6*b^3 -
5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^6 + 5*a^2*b^7 - a*b^8)*d^2*sqrt((1225*a^
4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^1
2*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 210*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^
9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4*b^12 + a^3*b^13)*d^4)) + 4*a^3 - 35
*a^2*b + 70*a*b^2 + 105*b^3)/((a^6*b^3 - 5*a^5*b^4 + 10*a^4*b^5 - 10*a^3*b^
6 + 5*a^2*b^7 - a*b^8)*d^2)) + 1/4*(2*(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 -
290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*b^6 + 25*a^2*b^7)*d^2*cos(d*x + c)^2 -
(4*a^8*b - 45*a^7*b^2 + 165*a^6*b^3 - 290*a^5*b^4 + 270*a^4*b^5 - 129*a^3*
b^6 + 25*a^2*b^7)*d^2)*sqrt((1225*a^4 - 10780*a^3*b + 21966*a^2*b^2 + 7700*
a*b^3 + 625*b^4)/((a^13*b^3 - 10*a^12*b^4 + 45*a^11*b^5 - 120*a^10*b^6 + 21
0*a^9*b^7 - 252*a^8*b^8 + 210*a^7*b^9 - 120*a^6*b^10 + 45*a^5*b^11 - 10*a^4
*b^12 + a^3*b^13)*d^4))) - 8*(2*(2*a*b - 5*b^2)*cos(d*x + c)^7 - 3*(5*a*b -
13*b^2)*cos(d*x + c)^5 + 24*(a*b - 2*b^2)*cos(d*x + c)^3 - (a^2 + 18*a*b -

```
19*b^2)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x +
c)^8 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*d*cos(d*x + c)^6 - 2*(a^3*b^2 - 5*a^2*b
^3 + 7*a*b^4 - 3*b^5)*d*cos(d*x + c)^4 + 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 -
b^5)*d*cos(d*x + c)^2 + (a^4*b - 4*a^3*b^2 + 6*a^2*b^3 - 4*a*b^4 + b^5)*d)
```

giac [B] time = 2.16, size = 1989, normalized size = 6.24

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 45*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 77*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a*b^2 + 13*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
b^3)*(a^2*b - 2*a*b^2 + b^3)^2*abs(-a + b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^6*b - 49*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^3 + 112*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^4 - 87*sqrt(a^2 - a*b + sqrt(a*b)*(a -
b))*a^2*b^5 + 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^6 + 5*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*b^7)*abs(a^2*b - 2*a*b^2 + b^3)*abs(-a + b) - (6*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b - 63*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^7*b^2 + 229*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^6*b^3 - 367*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5
*b^4 + 233*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^5 + 27*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 89*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*s
qrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^9)*abs(-
a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^3*b - 2
*a^2*b^2 + a*b^3 + sqrt((a^3*b - 2*a^2*b^2 + a*b^3)^2 - (a^3*b - 2*a^2*b^2
+ a*b^3)*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)))/(a^3*b - 3*a^2*b^2 + 3*a*b^3
 - b^4))))/((3*a^10*b^2 - 27*a^9*b^3 + 104*a^8*b^4 - 224*a^7*b^5 + 294*a^6*
b^6 - 238*a^5*b^7 + 112*a^4*b^8 - 24*a^3*b^9 - a^2*b^10 + a*b^11)*abs(a^2*b
 - 2*a*b^2 + b^3)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3
- 45*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 77*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 13*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*b^3)*(a^2*b - 2*a*b^2 + b^3)^2*abs(-a + b) - (3*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*a^6*b - 49*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*
b^3 + 112*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^4 - 87*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a^2*b^5 + 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^6
+ 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^7)*abs(a^2*b - 2*a*b^2 + b^3)*abs
(-a + b) - (6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b - 63*sqrt
(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^2 + 229*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*sqrt(a*b)*a^6*b^3 - 367*sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^5*b^4 + 233*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4
*b^5 + 27*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^6 - 89*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^7 + 19*sqrt(a^2 - a*b - sqrt(
a*b)*(a - b))*sqrt(a*b)*a*b^8 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*b^9)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/
sqrt((a^3*b - 2*a^2*b^2 + a*b^3 - sqrt((a^3*b - 2*a^2*b^2 + a*b^3)^2 - (a^3
*b - 2*a^2*b^2 + a*b^3)*(a^3*b - 3*a^2*b^2 + 3*a*b^3 - b^4)))/(a^3*b - 3*a^
2*b^2 + 3*a*b^3 - b^4))))/((3*a^10*b^2 - 27*a^9*b^3 + 104*a^8*b^4 - 224*a^7
*b^5 + 294*a^6*b^6 - 238*a^5*b^7 + 112*a^4*b^8 - 24*a^3*b^9 - a^2*b^10 + a*
b^11)*abs(a^2*b - 2*a*b^2 + b^3)) - 2*(a^2*tan(d*x + c)^7 + 18*a*b*tan(d*x
+ c)^7 - 19*b^2*tan(d*x + c)^7 + 3*a^2*tan(d*x + c)^5 + 30*a*b*tan(d*x + c)
^5 - 9*b^2*tan(d*x + c)^5 + 3*a^2*tan(d*x + c)^3 + 21*a*b*tan(d*x + c)^3 +
a^2*tan(d*x + c) + 5*a*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^
4 + 2*a*tan(d*x + c)^2 + a)^2*(a^2*b - 2*a*b^2 + b^3)))/d
```

maple [B] time = 0.38, size = 1634, normalized size = 5.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x)

[Out] -1/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/b*tan(d*x+c)^7*a-19/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x+c)^7-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a^2-15/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a+9/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)^3-21/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)-5/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)-1/64/d/b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+7/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/32/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^3-11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/16/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/64/d/b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+7/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/32/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3+11/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-1/16/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-13/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+5/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-13/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-5/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sin(d*x+c)^8/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")

[Out] -1/8*(4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) + ((a*b^3 - 4*b^4)*sin(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*sin(12*d*x + 12*c) + 3*(48*a^2*b^2 - 73*a*b^3 + 20*b^4)*sin(10*d*x + 10*c) + (256*a^3*b - 832*a^2*b^2 + 550*a*b^3 - 175*b^4)*sin(8*d*x + 8*c) + (112*a^2*b^2 - 533*a*b^3 + 220*b^4)*sin(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 + 141*b^4)*sin(4*d*x + 4*c) - (17*a*b^3 - 44*b^4)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) + 2*(2*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*sin(12*d*x + 12*c) - 8*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(10*d*x + 10*c) - 3*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(8*d*x + 8*c) - 16*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(6*d*x + 6*c) + 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(4*d*x + 4*c) + 24*(3*a*b^3 - 8*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(10*d*x + 10*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(8*d*x + 8*c) - 2*(128*a^3* b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(6*d*x + 6*c) + 4*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*sin(4*d*x + 4*c) - 2*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(8*d*x + 8*c) + 8*(256*a^3*b + 2400*a^2*b^2 - 2379*a*b^3 + 560*b^4)*sin(6*d*x + 6*c) - 2*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*sin(4*d*x + 4*c) + 16*(32*a^2*b^2 - 151*a*b^3 + 62*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3 - 2800*b^4)*sin(6*d*x + 6*c) - (6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^4)*sin(4*d*x + 4*c) + 3*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 280*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*sin(4*d*x + 4*c) + 4*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^2*b^5 - 2*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c)^2 - 16*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) + (a^2*b^5 - 2*a*b^6 + b^7)*d - 2*(8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(6*d*x + 6*c) - 4*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6

- 245*b^7)*d*cos(4*d*x + 4*c) - 8*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(2*d*x + 2*c) + (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c) + (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(8*d*x + 8*c) + 4*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(6*d*x + 6*c) - (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(1/4*(4*(a*b - 4*b^2)*cos(6*d*x + 6*c)^2 + 36*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b - 4*b^2)*cos(2*d*x + 2*c)^2 + 4*(a*b - 4*b^2)*sin(6*d*x + 6*c)^2 + 36*(8*a*b - 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a^2 - 35*a*b + 48*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b - 4*b^2)*sin(2*d*x + 2*c)^2 - (18*b^2*cos(4*d*x + 4*c) + (a*b - 4*b^2)*cos(6*d*x + 6*c) + (a*b - 4*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b - 4*b^2 - 2*(8*a^2 - 35*a*b + 48*b^2)*cos(4*d*x + 4*c) - 8*(a*b - 4*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(9*b^2 - (8*a^2 - 35*a*b + 48*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b - 4*b^2)*cos(2*d*x + 2*c) - (18*b^2*sin(4*d*x + 4*c) + (a*b - 4*b^2)*sin(6*d*x + 6*c) + (a*b - 4*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2 - 35*a*b + 48*b^2)*sin(4*d*x + 4*c) + 4*(a*b - 4*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^2*b^3 - 2*a*b^4 + b^5 + (a^2*b^3 - 2*a*b^4 + b^5)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^4*b - 176*a^3*b^2 + 169*a^2*b^3 - 66*a*b^4 + 9*b^5)*cos(4*d*x + 4*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c)^2 + (a^2*b^3 - 2*a*b^4 + b^5)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^3 - 2*a*b^4 + b^5)*sin(6*d*x + 6*c)^2 + 4*(64*a^4*b - 176*a^3*b^2 + 169*a^2*b^3 - 66*a*b^4 + 9*b^5)*sin(4*d*x + 4*c)^2 + 16*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^2*b^3 - 2*a*b^4 + b^5)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^3 - 2*a*b^4 + b^5 - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(6*d*x + 6*c) - 2*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*cos(4*d*x + 4*c) - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^3 -

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2*a*b^4 + b^5 - 2*(8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*cos(4*d*x +
4*c) - 4*(a^2*b^3 - 2*a*b^4 + b^5)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(
8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5 - 4*(8*a^3*b^2 - 19*a^2*b^3 + 14*
a*b^4 - 3*b^5)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^2*b^3 - 2*a*b^4 +
b^5)*cos(2*d*x + 2*c) - 4*(2*(a^2*b^3 - 2*a*b^4 + b^5)*sin(6*d*x + 6*c) + (
8*a^3*b^2 - 19*a^2*b^3 + 14*a*b^4 - 3*b^5)*sin(4*d*x + 4*c) + 2*(a^2*b^3 -
2*a*b^4 + b^5)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^3*b^2 - 19*a^2
*b^3 + 14*a*b^4 - 3*b^5)*sin(4*d*x + 4*c) + 2*(a^2*b^3 - 2*a*b^4 + b^5)*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (2*a*b^3 - 5*b^4 + (a*b^3 - 4*b^4)*c
os(14*d*x + 14*c) - (32*a^2*b^2 - 58*a*b^3 - b^4)*cos(12*d*x + 12*c) + 3*(4
8*a^2*b^2 - 73*a*b^3 + 20*b^4)*cos(10*d*x + 10*c) + (256*a^3*b - 832*a^2*b^
2 + 550*a*b^3 - 175*b^4)*cos(8*d*x + 8*c) + (112*a^2*b^2 - 533*a*b^3 + 220*
b^4)*cos(6*d*x + 6*c) - (32*a^2*b^2 - 158*a*b^3 + 141*b^4)*cos(4*d*x + 4*c)
 - (17*a*b^3 - 44*b^4)*cos(2*d*x + 2*c))*sin(16*d*x + 16*c) + (17*a*b^3 - 4
4*b^4 - 4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(12*d*x + 12*c) + 16*(80*a^2
*b^2 - 145*a*b^3 + 44*b^4)*cos(10*d*x + 10*c) + 6*(384*a^3*b - 1312*a^2*b^2
 + 873*a*b^3 - 280*b^4)*cos(8*d*x + 8*c) + 32*(32*a^2*b^2 - 151*a*b^3 + 62*
b^4)*cos(6*d*x + 6*c) - 4*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*cos(4*d*x + 4*
c) - 48*(3*a*b^3 - 8*b^4)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c) + (32*a^2*b^
2 - 158*a*b^3 + 141*b^4 + 4*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4
)*cos(10*d*x + 10*c) - 2*(6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3 - 2485*b^
4)*cos(8*d*x + 8*c) - 4*(128*a^3*b + 2744*a^2*b^2 - 4711*a*b^3 + 1554*b^4)*
cos(6*d*x + 6*c) + 8*(400*a^2*b^2 - 918*a*b^3 + 497*b^4)*cos(4*d*x + 4*c) -
 4*(72*a^2*b^2 - 355*a*b^3 + 310*b^4)*cos(2*d*x + 2*c))*sin(12*d*x + 12*c)
- (112*a^2*b^2 - 533*a*b^3 + 220*b^4 - 2*(2048*a^4 + 18560*a^3*b - 24752*a^
2*b^2 + 13175*a*b^3 - 2800*b^4)*cos(8*d*x + 8*c) - 16*(256*a^3*b + 2400*a^2
*b^2 - 2379*a*b^3 + 560*b^4)*cos(6*d*x + 6*c) + 4*(128*a^3*b + 2744*a^2*b^2
 - 4711*a*b^3 + 1554*b^4)*cos(4*d*x + 4*c) - 32*(32*a^2*b^2 - 151*a*b^3 + 6
2*b^4)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - (256*a^3*b - 832*a^2*b^2 + 55
0*a*b^3 - 175*b^4 - 2*(2048*a^4 + 18560*a^3*b - 24752*a^2*b^2 + 13175*a*b^3
 - 2800*b^4)*cos(6*d*x + 6*c) + 2*(6400*a^3*b - 13888*a^2*b^2 + 8566*a*b^3
- 2485*b^4)*cos(4*d*x + 4*c) - 6*(384*a^3*b - 1312*a^2*b^2 + 873*a*b^3 - 28
0*b^4)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) - (144*a^2*b^2 - 219*a*b^3 + 60*b
^4 - 4*(128*a^3*b - 456*a^2*b^2 + 1233*a*b^3 - 434*b^4)*cos(4*d*x + 4*c) -
16*(80*a^2*b^2 - 145*a*b^3 + 44*b^4)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (
32*a^2*b^2 - 58*a*b^3 - b^4 - 4*(72*a^2*b^2 - 155*a*b^3 + 26*b^4)*cos(2*d*x
 + 2*c))*sin(4*d*x + 4*c) - (a*b^3 - 4*b^4)*sin(2*d*x + 2*c))/((a^2*b^5 - 2
*a*b^6 + b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(1
4*d*x + 14*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 +
49*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^
5 - 322*a*b^6 + 49*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5
*b^2 + 83712*a^4*b^3 - 67648*a^3*b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^
7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322
*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*
a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^2*b^5 - 2*a*b^6
+ b^7)*d*cos(2*d*x + 2*c)^2 + (a^2*b^5 - 2*a*b^6 + b^7)*d*sin(16*d*x + 16*c
)^2 + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^4*b^3
- 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*sin(12*d*x + 12*c)^2 +
64*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(10*
d*x + 10*c)^2 + 4*(16384*a^6*b - 57344*a^5*b^2 + 83712*a^4*b^3 - 67648*a^3*
b^4 + 32841*a^2*b^5 - 9170*a*b^6 + 1225*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256
*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*sin(6*d*x + 6*
c)^2 + 16*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*s
in(4*d*x + 4*c)^2 + 64*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*
c)^2 - 16*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) + (a^2*b^5 - 2*a*b^6
 + b^7)*d - 2*(8*(a^2*b^5 - 2*a*b^6 + b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^3*
b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^3*b^4 -
 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^4*b^3 - 352
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*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^3
*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^3*b^4 - 2
3*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5 - 2*a*b^6 + b
^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(16*d*x + 16*c) +
16*(4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(12*d*x + 12*c) - 8*
(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(10*d*x + 10*c) - 2*(128*
a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*cos(8*d*x + 8*c
) - 8*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(6*d*x + 6*c) + 4*(
8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(4*d*x + 4*c) + 8*(a^2*b^5
- 2*a*b^6 + b^7)*d*cos(2*d*x + 2*c) - (a^2*b^5 - 2*a*b^6 + b^7)*d)*cos(14*d
*x + 14*c) - 8*(8*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49
*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4
- 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*cos(8*d*x + 8*c) + 8*(128*a^4*b^3
- 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(6
4*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a*b^6 + 49*b^7)*d*cos(4*d*x + 4
*c) - 8*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (8
*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(20
48*a^5*b^2 - 6528*a^4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*
b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*
a*b^6 + 49*b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2
*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^3*b^4 - 39*a^2*b^5
+ 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) + (16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^
6 - 7*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^5*b^2 - 6528*a^4*b^3 + 8144
*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*cos(6*d*x + 6*c) - 4*(102
4*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b
^7)*d*cos(4*d*x + 4*c) - 8*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a
*b^6 + 35*b^7)*d*cos(2*d*x + 2*c) + (128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^
5 - 166*a*b^6 + 35*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^4*b^3 - 424*a^3*
b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^3*b^4
- 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (16*a^3*b^4 - 39*a^2*
b^5 + 30*a*b^6 - 7*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^3*b^4 - 23*a^2*b^5
+ 22*a*b^6 - 7*b^7)*d*cos(2*d*x + 2*c) - (8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6
 - 7*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^2*b^5 - 2*a*b^6 + b^7)*d*sin(14*d*x
 + 14*c) + 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*
c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) -
(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x
+ 8*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c)
+ 2*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2
*b^5 - 2*a*b^6 + b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^3
*b^4 - 23*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^3*b^4
- 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(10*d*x + 10*c) - (128*a^4*b^3 - 352*
a^3*b^4 + 355*a^2*b^5 - 166*a*b^6 + 35*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^3*
b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^3*b^4 - 23
*a^2*b^5 + 22*a*b^6 - 7*b^7)*d*sin(4*d*x + 4*c) + 4*(a^2*b^5 - 2*a*b^6 + b^
7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^4*b^3 - 424*a^3*b^
4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(10*d*x + 10*c) + (1024*a^5*b^2
- 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 - 245*b^7)*d*sin(
8*d*x + 8*c) + 4*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*
b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^4*b^3 - 240*a^3*b^4 + 337*a^2*b^5 - 210*a
*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^3*b^4 - 23*a^2*b^5 + 22*a*b^6 -
7*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^5*b^2 - 6528*a^
4*b^3 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(8*d*x + 8
*c) + 4*(256*a^4*b^3 - 736*a^3*b^4 + 753*a^2*b^5 - 322*a*b^6 + 49*b^7)*d*si
n(6*d*x + 6*c) - 2*(128*a^4*b^3 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 4
9*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*
d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^5*b^2 - 6528*a^4*b^3
 + 8144*a^3*b^4 - 5141*a^2*b^5 + 1722*a*b^6 - 245*b^7)*d*sin(6*d*x + 6*c) -
 (1024*a^5*b^2 - 3712*a^4*b^3 + 5304*a^3*b^4 - 3813*a^2*b^5 + 1442*a*b^6 -
245*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^4*b^3 - 352*a^3*b^4 + 355*a^2*b^5 -
```

```
166*a*b^6 + 35*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^4*b^3
 - 424*a^3*b^4 + 513*a^2*b^5 - 266*a*b^6 + 49*b^7)*d*sin(4*d*x + 4*c) + 2*(
16*a^3*b^4 - 39*a^2*b^5 + 30*a*b^6 - 7*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x +
 6*c))
```

mupad [B] time = 19.65, size = 5508, normalized size = 17.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^8/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] (atan(((((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 1
6384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (tan(c + d*x)
*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b
^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5
*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a
^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 -
16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*
b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4
 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*
a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c + d*x)*(259*a*
b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*
b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^
2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(163
84*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6))
)^(1/2)*1i - (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b
^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) + (tan(c
+ d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70
*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^
11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(1
6384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*
b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2
*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a
^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10
 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) - (tan(c + d*x)*(
259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 -
3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) +
105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2)
)/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8
*b^6)))^(1/2)*1i)/((4*a^2 - 77*a*b + 325*b^2)/(16384*(3*a*b^5 - b^6 - 3*a^2
*b^4 + a^3*b^3)) + (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768
*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (
tan(c + d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^
6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(
a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1
/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 8192
0*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((
25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5
 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^
4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c +
d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 -
b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1
/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)
^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7
 - a^8*b^6)))^(1/2) + (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32
768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3))
+ (tan(c + d*x)*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2
*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(1638
4*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))
```

^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) - (tan(c + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)))*((25*b^2*(a^3*b^9)^(1/2) - 35*a^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 + 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*2i)/d - ((tan(c + d*x)^7*(a + 19*b))/(32*(a*b - b^2)) + (3*tan(c + d*x)^3*(7*a*b + a^2))/(32*(a^2*b - 2*a*b^2 + b^3)) + (a*tan(c + d*x)*(a + 5*b))/(32*(a^2*b - 2*a*b^2 + b^3)) + (3*tan(c + d*x)^5*(10*a*b + a^2 - 3*b^2))/(32*(a - b)*(a*b - b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*tan(c + d*x)^2)) + (atan(((((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (tan(c + d*x)*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*1i - (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) + (tan(c + d*x)*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) - (tan(c + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*1i)/((4*a^2 - 77*a*b + 325*b^2)/(16384*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) + (((229376*a^2*b^6 - 81920*a*b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6 - 3*a^2*b^4 + a^3*b^3)) - (tan(c + d*x)*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (tan(c + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 35*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^

```
5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9
 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2) + (((229376*a^2*b^6 - 81920*a*
b^7 - 196608*a^3*b^5 + 32768*a^4*b^4 + 16384*a^5*b^3)/(32768*(3*a*b^5 - b^6
 - 3*a^2*b^4 + a^3*b^3)) + (tan(c + d*x)*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*
(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a
*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8
 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*(16384*a^2*b^8 - 81920*a^3*b^7 + 163840*a^4
*b^6 - 163840*a^5*b^5 + 81920*a^6*b^4 - 16384*a^7*b^3))/(256*(3*a*b^4 - b^5
 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)^(1/2) - 25*b^2*(a^3*b^9)^(1/2)
 + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^3 - 154*a*b*(a^3*b^9)^(1
/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 10*a^6*b^8 + 5*a^7*b^7 -
a^8*b^6)))^(1/2) - (tan(c + d*x)*(259*a*b^3 - 35*a^3*b + 4*a^4 + 25*b^4 + 3
5*a^2*b^2))/(256*(3*a*b^4 - b^5 - 3*a^2*b^3 + a^3*b^2)))*((35*a^2*(a^3*b^9)
^(1/2) - 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4
*a^5*b^3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*
b^9 - 10*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)))*((35*a^2*(a^3*b^9)^(1/2)
- 25*b^2*(a^3*b^9)^(1/2) + 105*a^2*b^6 + 70*a^3*b^5 - 35*a^4*b^4 + 4*a^5*b^
3 - 154*a*b*(a^3*b^9)^(1/2))/(16384*(a^3*b^11 - 5*a^4*b^10 + 10*a^5*b^9 - 1
0*a^6*b^8 + 5*a^7*b^7 - a^8*b^6)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**8/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.231 $\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=343

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\ \tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ \tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+}{32abd(}$$

```
[Out] -1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b-10*a^(1/2
)*b^(1/2))/a^(5/4)/b^(3/2)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b
^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(4*a+3*b+10*a^(1/2)*b^(1/2))/a^(5/4)/b^(3
/2)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b^2)*tan
(d*x+c)^2)/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*tan(d*x
+c)*(2*a*(a^2-a*b-8*b^2)/(a-b)^3+(2*a^2+15*a*b+3*b^2)*tan(d*x+c)^2/(a-b)^2)
/a/b/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.77, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\left(-10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\ \tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\ \tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}b^{3/2}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+}{32abd(}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -((4*a - 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(5/4)*(Sqrt[a] - Sqrt[b])^(5/2)*b^(3/2)*d) + ((4*a +
10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(64*a^(5/4)*(Sqrt[a] + Sqrt[b])^(5/2)*b^(3/2)*d) - (Tan[c + d*x]*(a*
(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Ta
n[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(a^2 - a*b
- 8*b^2))/(a - b)^3 + ((2*a^2 + 15*a*b + 3*b^2)*Tan[c + d*x]^2)/(a - b)^2))
/(32*a*b*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^6(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^6\left(1+x^2\right)^2}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^3b(a+3b)}{(a-b)^3}+\frac{2a^2b\cdots}{\cdots}}{\cdots}\right.}{\cdots}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(a^2-ab-8b\cdots\right)}{(a-b)^3}\cdots\right.}{32abd\left(a+2a\tan^2(c+\cdots\right.}$$

$$= -\frac{\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a\left(a^2-ab-8b\cdots\right)}{(a-b)^3}\cdots\right.}{32abd\left(a+2a\tan^2(c+\cdots\right.}$$

$$= -\frac{\left(4a-10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{5/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}b^{3/2}d} + \frac{\left(4a+10\sqrt{a}\sqrt{b}+3b\right)\tan^{-1}\left(\cdots\right.}{64a^{5/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\cdots}$$

Mathematica [A] time = 3.76, size = 350, normalized size = 1.02

$$\frac{\frac{4b\sin(2(c+dx))\left(4a^2+3b(a+b)\cos(2(c+dx))-19ab-3b^2\right)}{a(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)}+\frac{\sqrt{b}\left(10\sqrt{a}\sqrt{b}+4a+3b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{128b(a-b)\sin(2(c+dx}{(-8a-4b\cos(2(c+dx}}{64b^2d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^6/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*Sqrt[b]*(4*a + 10*Sqrt[a]*Sqrt[b] + 3*b)*ArcTan[((S
qrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a*Sqrt[a + Sqr
t[a]*Sqrt[b]]) + ((Sqrt[a] + Sqrt[b])^2*Sqrt[b]*(4*a - 10*Sqrt[a]*Sqrt[b] +
 3*b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]
])/(a*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + (4*b*(4*a^2 - 19*a*b - 3*b^2 + 3*b*(a +
 b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)
] - b*Cos[4*(c + d*x)])) - (128*(a - b)*b*(2*a + b - b*Cos[2*(c + d*x)])*Si
n[2*(c + d*x)])/(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)
/(64*(a - b)^2*b^2*d)
```

fricas [B] time = 2.85, size = 5961, normalized size = 17.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2
*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b
^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*
x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16
*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 +
 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*
a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^
6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 2
52*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*
b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 -
a^2*b^8)*d^2))*log(320*a^5 - 2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 7
29/2*a*b^4 - 81/4*b^5 - 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*
a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 + 1/2*((2*a^11*b^3 - 27*a^10*
b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 -
3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b
^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^1
3*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*
b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c
) + (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 +
9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2
*b^2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 +
 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 5
3212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^
4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9
- 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5
*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(1
6*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*
b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^
2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9
*a^3*b^8)*d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^
3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13
*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b
```

^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - ((a^3*b^3 - 2*a^2*b^4
+ a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^
6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4
*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2
+ 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^
2 + 30*a*b^3 - 15*b^4 + (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*
a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 5321
2*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 +
45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 1
20*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^
6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2))*log(320*a^5 -
2724*a^4*b + 6243*a^3*b^2 - 9389/4*a^2*b^3 + 729/2*a*b^4 - 81/4*b^5 - 1/4*(
1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*a^2*b^3 + 1458*a*b^4 - 81*b^5
)*cos(d*x + c)^2 - 1/2*((2*a^11*b^3 - 27*a^10*b^4 + 108*a^9*b^5 - 205*a^8*b
^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^
6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b
^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^
11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b
^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) + (320*a^7*b - 2404*a^6*b^2
+ 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 + (a
^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqr
t((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4
- 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^
6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11
- 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^
4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*
a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8
)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4
+ 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6
- 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^
5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^1
1*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^
12 + a^5*b^13)*d^4))) + ((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4
*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 +
7*a^2*b^4 - 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4
- a*b^5)*d*cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*
b^5)*d)*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*
b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((
6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1
548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 +
210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 1
0*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b
^6 + 5*a^3*b^7 - a^2*b^8)*d^2))*log(-320*a^5 + 2724*a^4*b - 6243*a^3*b^2 +
9389/4*a^2*b^3 - 729/2*a*b^4 + 81/4*b^5 + 1/4*(1280*a^5 - 10896*a^4*b + 249
72*a^3*b^2 - 9389*a^2*b^3 + 1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 + 1/2*((2*a
^11*b^3 - 27*a^10*b^4 + 108*a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b
^8 + 32*a^5*b^9 - 3*a^4*b^10)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4
*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10
*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*
a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*
x + c)*sin(d*x + c) - (320*a^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b
^4 + 49*a^3*b^5 + 9*a^2*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 1
16*a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*
b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b +
104361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^1
5*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*
b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^
4)))/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)

```
*d^2)) - 1/4*(2*(16*a^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*
a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a
^10*b - 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6
 + 121*a^4*b^7 - 9*a^3*b^8)*d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*
b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*
a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a
^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - ((a^
3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^
5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 - 3*a*b^5)*d*cos(d
*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*cos(d*x + c)^2 +
(a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)*sqrt(-(16*a^4 - 116*
a^3*b + 229*a^2*b^2 + 30*a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5
 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104
361*a^4*b^2 - 53212*a^3*b^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b
^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8
 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))
)/((a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^
2))*log(-320*a^5 + 2724*a^4*b - 6243*a^3*b^2 + 9389/4*a^2*b^3 - 729/2*a*b^4
 + 81/4*b^5 + 1/4*(1280*a^5 - 10896*a^4*b + 24972*a^3*b^2 - 9389*a^2*b^3 +
1458*a*b^4 - 81*b^5)*cos(d*x + c)^2 - 1/2*((2*a^11*b^3 - 27*a^10*b^4 + 108*
a^9*b^5 - 205*a^8*b^6 + 210*a^7*b^7 - 117*a^6*b^8 + 32*a^5*b^9 - 3*a^4*b^10
)*d^3*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814
*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 12
0*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*
a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))*cos(d*x + c)*sin(d*x + c) - (320*a
^7*b - 2404*a^6*b^2 + 4779*a^5*b^3 - 1025*a^4*b^4 + 49*a^3*b^5 + 9*a^2*b^6)
*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(16*a^4 - 116*a^3*b + 229*a^2*b^2 + 30*
a*b^3 - 15*b^4 - (a^7*b^3 - 5*a^6*b^4 + 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7
 - a^2*b^8)*d^2*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b
^3 + 12814*a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^1
3*b^5 - 120*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*
b^10 + 45*a^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4)))/((a^7*b^3 - 5*a^6*b^4 +
 10*a^5*b^5 - 10*a^4*b^6 + 5*a^3*b^7 - a^2*b^8)*d^2)) - 1/4*(2*(16*a^10*b -
 156*a^9*b^2 + 549*a^8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*
a^4*b^7 - 9*a^3*b^8)*d^2*cos(d*x + c)^2 - (16*a^10*b - 156*a^9*b^2 + 549*a^
8*b^3 - 965*a^7*b^4 + 930*a^6*b^5 - 486*a^5*b^6 + 121*a^4*b^7 - 9*a^3*b^8)*
d^2)*sqrt((6400*a^6 - 48160*a^5*b + 104361*a^4*b^2 - 53212*a^3*b^3 + 12814*
a^2*b^4 - 1548*a*b^5 + 81*b^6)/((a^15*b^3 - 10*a^14*b^4 + 45*a^13*b^5 - 120
*a^12*b^6 + 210*a^11*b^7 - 252*a^10*b^8 + 210*a^9*b^9 - 120*a^8*b^10 + 45*a
^7*b^11 - 10*a^6*b^12 + a^5*b^13)*d^4))) - 8*(3*(a*b^2 + b^3)*cos(d*x + c)^
7 + (2*a^2*b - 17*a*b^2 - 9*b^3)*cos(d*x + c)^5 - (11*a^2*b - 26*a*b^2 - 9*
b^3)*cos(d*x + c)^3 + (2*a^3 + 13*a^2*b - 12*a*b^2 - 3*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^3*b^3 - 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^8 - 4*(a^3*b^3 -
 2*a^2*b^4 + a*b^5)*d*cos(d*x + c)^6 - 2*(a^4*b^2 - 5*a^3*b^3 + 7*a^2*b^4 -
 3*a*b^5)*d*cos(d*x + c)^4 + 4*(a^4*b^2 - 3*a^3*b^3 + 3*a^2*b^4 - a*b^5)*d*
cos(d*x + c)^2 + (a^5*b - 4*a^4*b^2 + 6*a^3*b^3 - 4*a^2*b^4 + a*b^5)*d)
```

giac [B] time = 2.24, size = 2231, normalized size = 6.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*(((6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 - 63*sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b + 109*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*
a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3*b - 2*a^2
*b^2 + a*b^3)^2*abs(-a + b) + 2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^8*
b - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b^2 - 4*sqrt(a^2 - a*b + sqrt
```

```
(a*b)*(a - b))*a^6*b^3 + 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^4 - 3
3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^5 + 7*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^3*b^6 + 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^7)*abs(a^3
*b - 2*a^2*b^2 + a*b^3)*abs(-a + b) - (12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^11*b - 117*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^1
0*b^2 + 431*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b^3 - 773*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^4 + 703*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^5 - 279*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^6*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*
b^7 + 17*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^8 + sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^9)*abs(-a + b))*(pi*floor((d*x +
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4*b - 2*a^3*b^2 + a^2*b^3 + sqrt
((a^4*b - 2*a^3*b^2 + a^2*b^3)^2 - (a^4*b - 2*a^3*b^2 + a^2*b^3)*(a^4*b - 3
*a^3*b^2 + 3*a^2*b^3 - a*b^4)))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4))))/
((3*a^12*b^2 - 27*a^11*b^3 + 104*a^10*b^4 - 224*a^9*b^5 + 294*a^8*b^6 - 238
*a^7*b^7 + 112*a^6*b^8 - 24*a^5*b^9 - a^4*b^10 + a^3*b^11)*abs(a^3*b - 2*a^
2*b^2 + a*b^3)) - ((6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 - 6
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b + 109*sqrt(a^2 - a*b
- sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a
^3*b - 2*a^2*b^2 + a*b^3)^2*abs(-a + b) - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a^8*b - 9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^7*b^2 - 4*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*a^6*b^3 + 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*a^5*b^4 - 33*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^5 + 7*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*a^3*b^6 + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*
b^7)*abs(a^3*b - 2*a^2*b^2 + a*b^3)*abs(-a + b) - (12*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^11*b - 117*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*s
qrt(a*b)*a^10*b^2 + 431*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b
^3 - 773*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^4 + 703*sqrt(a
^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^5 - 279*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^6*b^6 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^5*b^7 + 17*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^8
+ sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^9)*abs(-a + b))*(pi*f
loor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4*b - 2*a^3*b^2 + a^
2*b^3 - sqrt((a^4*b - 2*a^3*b^2 + a^2*b^3)^2 - (a^4*b - 2*a^3*b^2 + a^2*b^3
)*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)))/(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
- a*b^4))))/((3*a^12*b^2 - 27*a^11*b^3 + 104*a^10*b^4 - 224*a^9*b^5 + 294*a
^8*b^6 - 238*a^7*b^7 + 112*a^6*b^8 - 24*a^5*b^9 - a^4*b^10 + a^3*b^11)*abs(
a^3*b - 2*a^2*b^2 + a*b^3)) - 2*(2*a^3*tan(d*x + c)^7 + 13*a^2*b*tan(d*x +
c)^7 - 12*a*b^2*tan(d*x + c)^7 - 3*b^3*tan(d*x + c)^7 + 6*a^3*tan(d*x + c)^
5 + 28*a^2*b*tan(d*x + c)^5 - 10*a*b^2*tan(d*x + c)^5 + 6*a^3*tan(d*x + c)^
3 + 19*a^2*b*tan(d*x + c)^3 - a*b^2*tan(d*x + c)^3 + 2*a^3*tan(d*x + c) + 4
*a^2*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x +
c)^2 + a)^2*(a^3*b - 2*a^2*b^2 + a*b^3)))/d
```

maple [B] time = 0.37, size = 1909, normalized size = 5.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -1/16/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3-7/32/d*b/(a^2-2*a
*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(
d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/16/d/b/(a^2-2*a*b+b^2)/(a*b)^(1/2)/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))*a^3+7/32/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^
(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/
2))-15/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(
```

```
d*x+c)^7+19/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^
(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+1/64/d*b^2
/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-
a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/32/d/b/(a^2-2*a*b+b^2)/(a-
b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(
a-b))^(1/2))*a^2-7/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2
/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a-1/32/d/b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2
)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*
a^2-19/64/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-3/32/d/(tan(d
*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+1/32/
d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*ta
n(d*x+c)^3+3/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)
*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/64/d/a*b^2/(a^2
-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((
a*b)^(1/2)+a)*(a-b))^(1/2))-1/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/((
(a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))
^(1/2))+5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-
2*a*b+b^2)*tan(d*x+c)^5-19/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+
c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)^3-1/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)-1/16/d/(tan(d*x+c)^4*
a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/b*tan(d*x+c)^7*a-3/16/d/(tan(d
*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)
^5*a^2-3/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a^2/b/(a
^2-2*a*b+b^2)*tan(d*x+c)^3-1/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*
x+c)^2+a)^2*a^2/b/(a^2-2*a*b+b^2)*tan(d*x+c)+19/64/d/(a^2-2*a*b+b^2)*a/(a-b
)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*
(a-b))^(1/2))+19/64/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)
*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-5/16/d*b/(a^2-2*a*b
+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)
^(1/2)-a)*(a-b))^(1/2))-5/16/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-
b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^6/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/16*(4*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c) + ((4*a^2*b^3 - 13*a*b^4 + 3*b^5)*sin(14*d*x + 14*c) - 3*(8*a
^2*b^3 - 33*a*b^4 + 7*b^5)*sin(12*d*x + 12*c) + (64*a^3*b^2 + 68*a^2*b^3 -
225*a*b^4 + 63*b^5)*sin(10*d*x + 10*c) - 3*(128*a^3*b^2 + 32*a^2*b^3 - 61*a
*b^4 + 35*b^5)*sin(8*d*x + 8*c) - (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4 - 105
*b^5)*sin(6*d*x + 6*c) + 3*(40*a^2*b^3 - 29*a*b^4 - 21*b^5)*sin(4*d*x + 4*c
) - (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(16*d*x + 16*c) +
2*(2*(32*a^3*b^2 - 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*sin(12*d*x + 12*c) - 8*(
64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(10*d*x + 10*c) - (512*a^4*
b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*sin(8*d*x + 8*c) + 16
*(172*a^2*b^3 - 37*a*b^4 - 21*b^5)*sin(6*d*x + 6*c) + 2*(32*a^3*b^2 - 372*a
^2*b^3 + 289*a*b^4 + 105*b^5)*sin(4*d*x + 4*c) + 8*(4*a^2*b^3 - 25*a*b^4 -
9*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(512*a^4*b - 672*a^3*b^2
 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(10*d*x + 10*c) - 3*(3072*a^4*b -
6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*sin(8*d*x + 8*c) - 2*(51
2*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(6*d*x + 6
*c) + 12*(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5)*sin(4*d*x + 4*c)
- 2*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*sin(2*d*x + 2*c))*cos(
12*d*x + 12*c) - 2*((8192*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3
 - 2079*a*b^4 - 735*b^5)*sin(8*d*x + 8*c) + 8*(1024*a^4*b + 3712*a^3*b^2 -
```

3692*a^2*b^3 + 483*a*b^4 + 147*b^5)*sin(6*d*x + 6*c) - 2*(512*a^4*b + 3936*
a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*sin(4*d*x + 4*c) - 16*(172*a
^2*b^3 - 37*a*b^4 - 21*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((8192
*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*
sin(6*d*x + 6*c) - 3*(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4
- 245*b^5)*sin(4*d*x + 4*c) + (512*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11
*a*b^4 - 315*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*((512*a^4*b - 672*
a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b
^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 1
6*((a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a
^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337
*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
- 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(10*d*x + 10*c)
^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841
*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^
3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c)^
2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*
cos(4*d*x + 4*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c)^2
+ (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^5 - 2*a^
2*b^6 + a*b^7)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*
a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^3
- 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c)^
2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a^5*b^3 - 67648*a^4*b^4 + 32841*
a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^3
- 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c)^2
+ 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*s
in(4*d*x + 4*c)^2 + 64*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*si
n(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*
d*x + 2*c)^2 - 16*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) + (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d - 2*(8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x
+ 14*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x +
12*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x +
10*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7
)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d
*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos
(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b
^5 - 2*a^2*b^6 + a*b^7)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^
5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^3 - 352*a^4*b
^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b
^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 -
23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2
*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d)*cos(14*
d*x + 14*c) - 8*(8*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 +
49*a*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4
*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*cos(8*d*x + 8*c) + 8*(128
*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x
+ 6*c) - 4*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7
)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*
cos(2*d*x + 2*c) + (8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d)*cos(1
2*d*x + 12*c) + 16*(2*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^
3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(8*d*x + 8*c) + 8*(256*a^5*b^3 - 736
*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c) - 4*(12
8*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x
+ 4*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x +
2*c) + (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d)*cos(10*d*x + 10*
c) + 4*(8*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722
*a^2*b^6 - 245*a*b^7)*d*cos(6*d*x + 6*c) - 4*(1024*a^6*b^2 - 3712*a^5*b^3 +
5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*cos(4*d*x + 4*c)

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 - 8*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*c
os(2*d*x + 2*c) + (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 +
35*a*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*
b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) + 8*(16*a^4*b^4 - 39*a^3*b
^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (16*a^4*b^4 - 39*a^3*b^5 +
30*a^2*b^6 - 7*a*b^7)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^4*b^4 - 23*a^3*b^5 +
22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (8*a^4*b^4 - 23*a^3*b^5 + 22*a^2
*b^6 - 7*a*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*
sin(14*d*x + 14*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*si
n(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin
(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 +
35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*
a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^
7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c))
*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)
*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*
d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b
^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6
 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7
*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x +
2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 -
 266*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c) + (1024*a^6*b^2 - 3712*a^5*b^
3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(8*d*x + 8
*c) + 4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*
d*sin(6*d*x + 6*c) - 2*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^
6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 -
 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32*((2048*a^6*b^2 - 6528
*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*sin(8*
d*x + 8*c) + 4*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*
a*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 26
6*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(16*a^4*b^4 - 39*a^3*b^5 + 30*
a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^6
*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^
7)*d*sin(6*d*x + 6*c) - (1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*
a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(4*d*x + 4*c) - 2*(128*a^5*b^3 - 3
52*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) - 64*((128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 +
 49*a*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7
*a*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/8*(4*(4*a^2*b -
13*a*b^2 + 3*b^3)*cos(6*d*x + 6*c)^2 + 12*(56*a^2*b - 29*a*b^2 + 3*b^3)*cos
(4*d*x + 4*c)^2 + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c)^2 + 4*(4*
a^2*b - 13*a*b^2 + 3*b^3)*sin(6*d*x + 6*c)^2 + 12*(56*a^2*b - 29*a*b^2 + 3*
b^3)*sin(4*d*x + 4*c)^2 + 2*(32*a^3 - 116*a^2*b + 147*a*b^2 - 21*b^3)*sin(4
*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2
*c)^2 - ((4*a^2*b - 13*a*b^2 + 3*b^3)*cos(6*d*x + 6*c) + 6*(7*a*b^2 - b^3)*
cos(4*d*x + 4*c) + (4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c))*cos(8*d*x
 + 8*c) - (4*a^2*b - 13*a*b^2 + 3*b^3 - 2*(32*a^3 - 116*a^2*b + 147*a*b^2 -
 21*b^3)*cos(4*d*x + 4*c) - 8*(4*a^2*b - 13*a*b^2 + 3*b^3)*cos(2*d*x + 2*c)
)*cos(6*d*x + 6*c) - 2*(21*a*b^2 - 3*b^3 - (32*a^3 - 116*a^2*b + 147*a*b^2
- 21*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (4*a^2*b - 13*a*b^2 + 3*b^3)
*cos(2*d*x + 2*c) - ((4*a^2*b - 13*a*b^2 + 3*b^3)*sin(6*d*x + 6*c) + 6*(7*a
*b^2 - b^3)*sin(4*d*x + 4*c) + (4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2*c
))*sin(8*d*x + 8*c) + 2*((32*a^3 - 116*a^2*b + 147*a*b^2 - 21*b^3)*sin(4*d*
x + 4*c) + 4*(4*a^2*b - 13*a*b^2 + 3*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c
))/(a^3*b^3 - 2*a^2*b^4 + a*b^5 + (a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(8*d*x +
 8*c)^2 + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^5*b
 - 176*a^4*b^2 + 169*a^3*b^3 - 66*a^2*b^4 + 9*a*b^5)*cos(4*d*x + 4*c)^2 + 1
6*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c)^2 + (a^3*b^3 - 2*a^2*b^4 +
 a*b^5)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(6*d*x + 6
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*c)^2 + 4*(64*a^5*b - 176*a^4*b^2 + 169*a^3*b^3 - 66*a^2*b^4 + 9*a*b^5)*sin
(4*d*x + 4*c)^2 + 16*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*d*x +
2*c)^2 + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5 - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*c
os(6*d*x + 6*c) - 2*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*cos(4*d
*x + 4*c) - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8
*c) - 8*(a^3*b^3 - 2*a^2*b^4 + a*b^5 - 2*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b
^4 - 3*a*b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x
+ 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5
 - 4*(8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3*a*b^5)*cos(2*d*x + 2*c))*cos(
4*d*x + 4*c) - 8*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*cos(2*d*x + 2*c) - 4*(2*(a^3
*b^3 - 2*a^2*b^4 + a*b^5)*sin(6*d*x + 6*c) + (8*a^4*b^2 - 19*a^3*b^3 + 14*a
^2*b^4 - 3*a*b^5)*sin(4*d*x + 4*c) + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*
d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b^2 - 19*a^3*b^3 + 14*a^2*b^4 - 3
*a*b^5)*sin(4*d*x + 4*c) + 2*(a^3*b^3 - 2*a^2*b^4 + a*b^5)*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c)), x) + (3*a*b^4 + 3*b^5 - (4*a^2*b^3 - 13*a*b^4 + 3*b^5)
*cos(14*d*x + 14*c) + 3*(8*a^2*b^3 - 33*a*b^4 + 7*b^5)*cos(12*d*x + 12*c) -
 (64*a^3*b^2 + 68*a^2*b^3 - 225*a*b^4 + 63*b^5)*cos(10*d*x + 10*c) + 3*(128
*a^3*b^2 + 32*a^2*b^3 - 61*a*b^4 + 35*b^5)*cos(8*d*x + 8*c) + (64*a^3*b^2 +
 452*a^2*b^3 - 9*a*b^4 - 105*b^5)*cos(6*d*x + 6*c) - 3*(40*a^2*b^3 - 29*a*b
^4 - 21*b^5)*cos(4*d*x + 4*c) + (4*a^2*b^3 - 37*a*b^4 - 21*b^5)*cos(2*d*x +
 2*c))*sin(16*d*x + 16*c) + (4*a^2*b^3 - 37*a*b^4 - 21*b^5 - 4*(32*a^3*b^2
- 84*a^2*b^3 - 83*a*b^4 + 21*b^5)*cos(12*d*x + 12*c) + 16*(64*a^3*b^2 - 84*
a^2*b^3 - 43*a*b^4 + 21*b^5)*cos(10*d*x + 10*c) + 2*(512*a^4*b - 3584*a^3*b
^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*cos(8*d*x + 8*c) - 32*(172*a^2*b^3
- 37*a*b^4 - 21*b^5)*cos(6*d*x + 6*c) - 4*(32*a^3*b^2 - 372*a^2*b^3 + 289*a
*b^4 + 105*b^5)*cos(4*d*x + 4*c) - 16*(4*a^2*b^3 - 25*a*b^4 - 9*b^5)*cos(2*
d*x + 2*c))*sin(14*d*x + 14*c) - (120*a^2*b^3 - 87*a*b^4 - 63*b^5 - 4*(512*
a^4*b - 672*a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*cos(10*d*x + 10*c)
 + 6*(3072*a^4*b - 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*cos(8
*d*x + 8*c) + 4*(512*a^4*b + 3936*a^3*b^2 - 6740*a^2*b^3 + 1281*a*b^4 + 441
*b^5)*cos(6*d*x + 6*c) - 24*(192*a^3*b^2 - 416*a^2*b^3 + 161*a*b^4 + 49*b^5
)*cos(4*d*x + 4*c) + 4*(32*a^3*b^2 - 372*a^2*b^3 + 289*a*b^4 + 105*b^5)*cos
(2*d*x + 2*c))*sin(12*d*x + 12*c) + (64*a^3*b^2 + 452*a^2*b^3 - 9*a*b^4 - 1
05*b^5 + 2*(8192*a^5 + 27136*a^4*b - 37696*a^3*b^2 + 17644*a^2*b^3 - 2079*a
*b^4 - 735*b^5)*cos(8*d*x + 8*c) + 16*(1024*a^4*b + 3712*a^3*b^2 - 3692*a^2
*b^3 + 483*a*b^4 + 147*b^5)*cos(6*d*x + 6*c) - 4*(512*a^4*b + 3936*a^3*b^2
- 6740*a^2*b^3 + 1281*a*b^4 + 441*b^5)*cos(4*d*x + 4*c) - 32*(172*a^2*b^3 -
 37*a*b^4 - 21*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + (384*a^3*b^2 + 9
6*a^2*b^3 - 183*a*b^4 + 105*b^5 + 2*(8192*a^5 + 27136*a^4*b - 37696*a^3*b^2
 + 17644*a^2*b^3 - 2079*a*b^4 - 735*b^5)*cos(6*d*x + 6*c) - 6*(3072*a^4*b -
 6272*a^3*b^2 + 2920*a^2*b^3 - 413*a*b^4 - 245*b^5)*cos(4*d*x + 4*c) + 2*(5
12*a^4*b - 3584*a^3*b^2 + 1388*a^2*b^3 - 11*a*b^4 - 315*b^5)*cos(2*d*x + 2*
c))*sin(8*d*x + 8*c) - (64*a^3*b^2 + 68*a^2*b^3 - 225*a*b^4 + 63*b^5 - 4*(5
12*a^4*b - 672*a^3*b^2 + 1228*a^2*b^3 + 21*a*b^4 - 147*b^5)*cos(4*d*x + 4*c
) - 16*(64*a^3*b^2 - 84*a^2*b^3 - 43*a*b^4 + 21*b^5)*cos(2*d*x + 2*c))*sin(
6*d*x + 6*c) + (24*a^2*b^3 - 99*a*b^4 + 21*b^5 - 4*(32*a^3*b^2 - 84*a^2*b^3
 - 83*a*b^4 + 21*b^5)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (4*a^2*b^3 - 13*
a*b^4 + 3*b^5)*sin(2*d*x + 2*c))/((a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(16*d*
x + 16*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c)^2 + 16*
(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(12*
d*x + 12*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
 49*a*b^7)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*
a^5*b^3 - 67648*a^4*b^4 + 32841*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*cos(
8*d*x + 8*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6
+ 49*a*b^7)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b
^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6
 + a*b^7)*d*cos(2*d*x + 2*c)^2 + (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(16*d*x
 + 16*c)^2 + 64*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(14*d*x + 14*c)^2 + 16*(
```

64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(12*d
*x + 12*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
49*a*b^7)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7*b - 57344*a^6*b^2 + 83712*a
^5*b^3 - 67648*a^4*b^4 + 32841*a^3*b^5 - 9170*a^2*b^6 + 1225*a*b^7)*d*sin(8
*d*x + 8*c)^2 + 64*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 +
49*a*b^7)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^3 - 240*a^4*b^4 + 337*a^3*b^
5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^3*b^
5 - 2*a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c)^2 - 16*(a^3*b^5 - 2*a^2*b^6 + a*b
^7)*d*cos(2*d*x + 2*c) + (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d - 2*(8*(a^3*b^5 -
2*a^2*b^6 + a*b^7)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^
2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2
*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a
^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^
3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^
5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b
^7)*d*cos(2*d*x + 2*c) - (a^3*b^5 - 2*a^2*b^6 + a*b^7)*d)*cos(16*d*x + 16*c
) + 16*(4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(12*d*x + 12
c) - 8(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(10*d*x + 10*
c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d
*cos(8*d*x + 8*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*co
s(6*d*x + 6*c) + 4*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(4*
d*x + 4*c) + 8*(a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*cos(2*d*x + 2*c) - (a^3*b^5
- 2*a^2*b^6 + a*b^7)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^5*b^3 - 424*a^4*b^
4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(10*d*x + 10*c) + 2*(1024*a^
6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b
^7)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a
^2*b^6 + 49*a*b^7)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^3 - 240*a^4*b^4 + 337*a
^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^4 - 23*a^3
*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) + (8*a^4*b^4 - 23*a^3*b^5 +
22*a^2*b^6 - 7*a*b^7)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^6*b^2 - 6528*a
^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(8*d*
x + 8*c) + 8*(256*a^5*b^3 - 736*a^4*b^4 + 753*a^3*b^5 - 322*a^2*b^6 + 49*a*
b^7)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*
a^2*b^6 + 49*a*b^7)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^
2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) + (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6
- 7*a*b^7)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b^2 - 6528*a^5*b^3 + 814
4*a^4*b^4 - 5141*a^3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*cos(6*d*x + 6*c) - 4
*(1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6
- 245*a*b^7)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^
5 - 166*a^2*b^6 + 35*a*b^7)*d*cos(2*d*x + 2*c) + (128*a^5*b^3 - 352*a^4*b^4
+ 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a
^5*b^3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*cos(4*d*x +
4*c) + 8*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c
) - (16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d)*cos(6*d*x + 6*c) +
8*(8*(8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*cos(2*d*x + 2*c) - (
8*a^4*b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d)*cos(4*d*x + 4*c) - 4*(4*(
a^3*b^5 - 2*a^2*b^6 + a*b^7)*d*sin(14*d*x + 14*c) + 2*(8*a^4*b^4 - 23*a^3*b
^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a^3*b^
5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^4*b^4
+ 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^4
- 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^4 - 2
3*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*a^2*b
^6 + a*b^7)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^4 - 23*
a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^4 - 39*a
^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(10*d*x + 10*c) - (128*a^5*b^3 - 352*a^
4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*a*b^7)*d*sin(8*d*x + 8*c) - 4*(16*a^
4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^
4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(4*d*x + 4*c) + 4*(a^3*b^5 - 2*

```
a^2*b^6 + a*b^7)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^
3 - 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(10*d*x + 10*c
) + (1024*a^6*b^2 - 3712*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b
^6 - 245*a*b^7)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^3 - 424*a^4*b^4 + 513*a^3
*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c) - 2*(64*a^5*b^3 - 240*a^4
*b^4 + 337*a^3*b^5 - 210*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4*(8*a^4*
b^4 - 23*a^3*b^5 + 22*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(12*d*x + 1
2*c) + 32*((2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^3*b^5 + 172
2*a^2*b^6 - 245*a*b^7)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^3 - 736*a^4*b^4 +
753*a^3*b^5 - 322*a^2*b^6 + 49*a*b^7)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^3 -
 424*a^4*b^4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) - 4
*(16*a^4*b^4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(1
0*d*x + 10*c) + 16*(2*(2048*a^6*b^2 - 6528*a^5*b^3 + 8144*a^4*b^4 - 5141*a^
3*b^5 + 1722*a^2*b^6 - 245*a*b^7)*d*sin(6*d*x + 6*c) - (1024*a^6*b^2 - 3712
*a^5*b^3 + 5304*a^4*b^4 - 3813*a^3*b^5 + 1442*a^2*b^6 - 245*a*b^7)*d*sin(4*
d*x + 4*c) - 2*(128*a^5*b^3 - 352*a^4*b^4 + 355*a^3*b^5 - 166*a^2*b^6 + 35*
a*b^7)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^3 - 424*a^4*b^
4 + 513*a^3*b^5 - 266*a^2*b^6 + 49*a*b^7)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^
4 - 39*a^3*b^5 + 30*a^2*b^6 - 7*a*b^7)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)
)
```

mupad [B] time = 20.34, size = 6391, normalized size = 18.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^6/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)^3*(19*a*b + 6*a^2 - b^2))/(32*(a^2*b - 2*a*b^2 + b^3)) + (
a*tan(c + d*x)*(a + 2*b))/(16*(a^2*b - 2*a*b^2 + b^3)) + (tan(c + d*x)^7*(1
5*a*b + 2*a^2 + 3*b^2))/(32*a*(a*b - b^2)) + (tan(c + d*x)^5*(14*a*b + 3*a^
2 - 5*b^2))/(16*(a - b)*(a*b - b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2
) + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) +
 4*a^2*tan(c + d*x)^2)) - (atan(((((65536*a^3*b^7 - 163840*a^4*b^6 + 98304*
a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3*a^3*b^5 + 3*a^
4*b^4 - a^5*b^3)) - (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9
)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3
- 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 -
5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*(16384
*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 + 81920*a^7*b^4
- 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*
(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^
5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^
5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a
^9*b^7 - a^10*b^6)))^(1/2) + (tan(c + d*x)*(16*a^5 - 116*a^4*b - 101*a*b^4
+ 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 -
 a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 +
 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(
1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^
9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i - (((65536*a^3*b^7 - 1638
40*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6
 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2
) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^
6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/
(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*
b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b
^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 -
a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 +
30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1
/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9
```

```
 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (tan(c + d*x)*(16*a^5 - 116
*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^
2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(
1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 8
6*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a
^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i)/((((6
5536*a^3*b^7 - 163840*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b
^3)/(32768*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) - (tan(c + d*x)*((9
*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 2
29*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*
b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8
+ 5*a^9*b^7 - a^10*b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5
*b^6 - 163840*a^6*b^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2
*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1
/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86
*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^
6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) + (tan(c +
 d*x)*(16*a^5 - 116*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))
/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) -
 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b
^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16
384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6
)))^(1/2) - (32*a^4 - 424*a^3*b - 381*a*b^3 + 27*b^4 + 1358*a^2*b^2)/(16384
*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (((65536*a^3*b^7 - 163840*a
^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3
*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((9*b^3*(a^5*b^9)^(1/2) -
80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^
4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(163
84*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)
))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 +
 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*
b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a
^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2)
+ 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 1
0*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (tan(c + d*x)*(16*a^5 - 116*a^4
*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^2*b^
4 + 3*a^3*b^3 - a^4*b^2)))*((9*b^3*(a^5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2)
 - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*
b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b
^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)))*((9*b^3*(a^
5*b^9)^(1/2) - 80*a^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b
^5 - 116*a^6*b^4 + 16*a^7*b^3 - 86*a*b^2*(a^5*b^9)^(1/2) + 301*a^2*b*(a^5*b
^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*
b^7 - a^10*b^6)))^(1/2)*2i)/d - (atan(((((65536*a^3*b^7 - 163840*a^4*b^6 +
98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b^6 - 3*a^3*b^5
+ 3*a^4*b^4 - a^5*b^3)) - (tan(c + d*x)*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a
^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^
7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b
^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*
(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6*b^5 + 81920*a^
7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((
80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 +
229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2
*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8
 + 5*a^9*b^7 - a^10*b^6)))^(1/2) + (tan(c + d*x)*(16*a^5 - 116*a^4*b - 101*
a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3
*b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3
*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*
b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*
```

```
a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i - (((65536*a^3*b^7
- 163840*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a
^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((80*a^3*(a^5*b^
9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 -
116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(
1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 -
 a^10*b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840
*a^6*b^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*
b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*
b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b
^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a
^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (tan(c + d*x)*(16*a^5
 - 116*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5
- 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*
b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b
^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11
 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*1i)
/((((65536*a^3*b^7 - 163840*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768
*a^7*b^3)/(32768*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) - (tan(c + d*
x)*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b
^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 30
1*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^
8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 1638
40*a^5*b^6 - 163840*a^6*b^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 -
 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b
^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^
3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11
- 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) + (t
an(c + d*x)*(16*a^5 - 116*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3
*b^2))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^
(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116
*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2
))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^
10*b^6)))^(1/2) - (32*a^4 - 424*a^3*b - 381*a*b^3 + 27*b^4 + 1358*a^2*b^2)/
(16384*(a^2*b^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (((65536*a^3*b^7 - 16
3840*a^4*b^6 + 98304*a^5*b^5 + 32768*a^6*b^4 - 32768*a^7*b^3)/(32768*(a^2*b
^6 - 3*a^3*b^5 + 3*a^4*b^4 - a^5*b^3)) + (tan(c + d*x)*((80*a^3*(a^5*b^9)^(
1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*
a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2)
)/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^1
0*b^6)))^(1/2)*(16384*a^3*b^8 - 81920*a^4*b^7 + 163840*a^5*b^6 - 163840*a^6
*b^5 + 81920*a^7*b^4 - 16384*a^8*b^3))/(256*(a*b^5 - 3*a^2*b^4 + 3*a^3*b^3
- a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7
+ 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^
(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b
^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2) - (tan(c + d*x)*(16*a^5 - 1
16*a^4*b - 101*a*b^4 + 9*b^5 + 331*a^2*b^3 + 149*a^3*b^2))/(256*(a*b^5 - 3*
a^2*b^4 + 3*a^3*b^3 - a^4*b^2)))*((80*a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)
^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 +
 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5
*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 + 5*a^9*b^7 - a^10*b^6)))^(1/2)))*((80*
a^3*(a^5*b^9)^(1/2) - 9*b^3*(a^5*b^9)^(1/2) - 15*a^3*b^7 + 30*a^4*b^6 + 229
*a^5*b^5 - 116*a^6*b^4 + 16*a^7*b^3 + 86*a*b^2*(a^5*b^9)^(1/2) - 301*a^2*b*
(a^5*b^9)^(1/2))/(16384*(a^5*b^11 - 5*a^6*b^10 + 10*a^7*b^9 - 10*a^8*b^8 +
5*a^9*b^7 - a^10*b^6)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**6/(a-b*sin(d*x+c)**4)**3,x)
[Out] Timed out
```

3.232 $\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=313

$$\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\right.}{32ad\left((a-b)\tan^4(c+dx)+\right.}$$

```
[Out] 3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)-b^(1/2))
/a^(7/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-3/64*arctan((a^(1/2)+b^(1/2))^(1
/2)*tan(d*x+c)/a^(1/4))*(2*a^(1/2)+b^(1/2))/a^(7/4)/d/b^(1/2)/(a^(1/2)+b^(1
/2))^(5/2)-1/8*b*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/(a-b)^3/d/(a+2*a*t
an(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*tan(d*x+c)*((9*a^2-24*a*b-b^2)/(a-b)
^3+(17*a+3*b)*tan(d*x+c)^2/(a-b)^2)/a/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c
)^4)
```

Rubi [A] time = 0.70, antiderivative size = 313, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\tan(c+dx)\left(\frac{9a^2-24ab-b^2}{(a-b)^3}+\frac{(17a+3b)\tan^2(c+dx)}{(a-b)^2}\right)}{32ad\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan}{64a^{7/4}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*(2*Sqrt[a] - Sqrt[b])*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1
/4)])/(64*a^(7/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - (3*(2*Sqrt[a] + Sq
rt[b])*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(64*a^(7/4)*
(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*(3*a + b + 4*(a + b)
*Tan[c + d*x]^2))/(8*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c +
d*x]^4)^2) - (Tan[c + d*x]*((9*a^2 - 24*a*b - b^2)/(a - b)^3 + ((17*a + 3*b
)*Tan[c + d*x]^2)/(a - b)^2))/(32*a*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan
[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^4(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^4(1+x^2)^3}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(3a+b)}{(a-b)^3}\cdots}{\cdots}\right)}{\cdots}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{9a^2-\cdots}{(a\cdots}\right.}{32ad\left(a+2a\tan^2(c\cdots\right.}$$

$$= -\frac{b\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{9a^2-\cdots}{(a\cdots}\right.}{32ad\left(a+2a\tan^2(c\cdots\right.}$$

$$= \frac{3\left(2\sqrt{a}-\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{3\left(2\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+\cdots}{\sqrt[4]{a}}\right)}{64a^{7/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\sqrt{b}\,d}$$

Mathematica [A] time = 4.84, size = 316, normalized size = 1.01

$$\frac{-\frac{3\left(2a^{3/2}-3a\sqrt{b}+b^{3/2}\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{3\left(2a^{3/2}+3a\sqrt{b}-b^{3/2}\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{a^{3/2}\sqrt{b}\sqrt{\sqrt{a}\sqrt{b}-a}} + \frac{8\sin(2(c+dx))((2a+b)\cos(2(c+dx))-7a\cdots}{a(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))\cdots}}{64d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^4/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-3*(2*a^(3/2) - 3*a*Sqrt[b] + b^(3/2))*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c
+ d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*Sqrt
[b]) - (3*(2*a^(3/2) + 3*a*Sqrt[b] - b^(3/2))*ArcTanh[((Sqrt[a] - Sqrt[b])*
Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(a^(3/2)*Sqrt[-a + Sqrt[a]*Sqrt[
b]]*Sqrt[b]) + (8*(-7*a - 2*b + (2*a + b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)
])/(a*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (64*(a - b
)*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*b - 4*b*Cos[2*(c + d*
x)] + b*Cos[4*(c + d*x)])^2)/(64*(a - b)^2*d)
```

fricas [B] time = 1.82, size = 5510, normalized size = 17.60

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a
^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b
^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x
+ c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(-((a^8*b -
 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a
^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((
a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12
*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d
^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3
- 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b - 783/4*a^
2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*b^2 + 10*a
*b^3 - b^4)*cos(d*x + c)^2 + 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a^10*b^3 -
60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6 + 160*a^5
*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*
a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a
^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))*cos(d*x
 + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3 + 9*a^3*b
^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*b^2 + 10*a
^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b -
167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*
b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b
^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) + 4*a^3 + 21
*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*
a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7
*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*a^10 - 21*
a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2)*sq
rt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5
+ b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 -
 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^
7*b^11)*d^4))) - 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3
*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b
^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*
d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)*sqrt(
-((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*s
qrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5
 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5
- 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a
^7*b^11)*d^4)) + 4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 1
0*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(432*a^4 + 27*a^3*b
- 783/4*a^2*b^2 + 135/2*a*b^3 - 27/4*b^4 - 27/4*(64*a^4 + 4*a^3*b - 29*a^2*
```

b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 - 27/2*((5*a^12*b - 26*a^11*b^2 + 55*a
^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqrt((256*a^6
 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^
17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b
^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4
))*cos(d*x + c)*sin(d*x + c) - (32*a^7 + 58*a^6*b - 13*a^5*b^2 - 21*a^4*b^3
 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^8*b - 5*a^7*
b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256*a^6 + 16
0*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b
- 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 +
210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4)) +
4*a^3 + 21*a^2*b - 10*a*b^2 + b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^
5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b + 45*a^8*b^
2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c)^2 - (4*
a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^
6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 -
 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*
a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8
*b^10 + a^7*b^11)*d^4))) + 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^
8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b - 5*a^3*b^2
 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 + 3*a^2*b^3
 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b
^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 -
 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*
a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8
*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^
7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-432*a^4 -
 27*a^3*b + 783/4*a^2*b^2 - 135/2*a*b^3 + 27/4*b^4 + 27/4*(64*a^4 + 4*a^3*b
 - 29*a^2*b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 + 27/2*((5*a^12*b - 26*a^11*
b^2 + 55*a^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7)*d^3*sqr
t((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 +
 b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 -
252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7
*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (32*a^7 + 58*a^6*b - 13*a^5*b^2 -
21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^8*b
 - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*sqrt((256
*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/
((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5 - 252*a^
12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)
*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 + 10*a^6*b^
3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21*a^9*b +
45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*cos(d*x + c
)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5
 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*
a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b
^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9
 - 10*a^8*b^10 + a^7*b^11)*d^4))) - 3*((a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(
d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x + c)^6 - 2*(a^4*b -
5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(a^4*b - 3*a^3*b^2 +
3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^
3 + a*b^4)*d)*sqrt(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^
5 - a^3*b^6)*d^2*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*
a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b
^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9
 - 10*a^8*b^10 + a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8
*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2))*log(-
432*a^4 - 27*a^3*b + 783/4*a^2*b^2 - 135/2*a*b^3 + 27/4*b^4 + 27/4*(64*a^4
+ 4*a^3*b - 29*a^2*b^2 + 10*a*b^3 - b^4)*cos(d*x + c)^2 - 27/2*((5*a^12*b -

```
 26*a^11*b^2 + 55*a^10*b^3 - 60*a^9*b^4 + 35*a^8*b^5 - 10*a^7*b^6 + a^6*b^7
)*d^3*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 1
2*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^
13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b
^10 + a^7*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (32*a^7 + 58*a^6*b - 13*a
^5*b^2 - 21*a^4*b^3 + 9*a^3*b^4 - a^2*b^5)*d*cos(d*x + c)*sin(d*x + c))*sqr
t(((a^8*b - 5*a^7*b^2 + 10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2*
sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*b^3 + 46*a^2*b^4 - 12*a*b^
5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 120*a^14*b^4 + 210*a^13*b^5
 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 45*a^9*b^9 - 10*a^8*b^10 +
a^7*b^11)*d^4)) - 4*a^3 - 21*a^2*b + 10*a*b^2 - b^3)/((a^8*b - 5*a^7*b^2 +
10*a^6*b^3 - 10*a^5*b^4 + 5*a^4*b^5 - a^3*b^6)*d^2)) + 27/4*(2*(4*a^10 - 21
*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 - 9*a^5*b^5 + a^4*b^6)*d^2*co
s(d*x + c)^2 - (4*a^10 - 21*a^9*b + 45*a^8*b^2 - 50*a^7*b^3 + 30*a^6*b^4 -
9*a^5*b^5 + a^4*b^6)*d^2)*sqrt((256*a^6 + 160*a^5*b - 167*a^4*b^2 - 28*a^3*
b^3 + 46*a^2*b^4 - 12*a*b^5 + b^6)/((a^17*b - 10*a^16*b^2 + 45*a^15*b^3 - 1
20*a^14*b^4 + 210*a^13*b^5 - 252*a^12*b^6 + 210*a^11*b^7 - 120*a^10*b^8 + 4
5*a^9*b^9 - 10*a^8*b^10 + a^7*b^11)*d^4))) - 8*(2*(2*a*b + b^2)*cos(d*x + c
)^7 - (17*a*b + 7*b^2)*cos(d*x + c)^5 - 8*(a^2 - 3*a*b - b^2)*cos(d*x + c)^
3 + (17*a^2 - 14*a*b - 3*b^2)*cos(d*x + c))*sin(d*x + c))/((a^3*b^2 - 2*a^2
*b^3 + a*b^4)*d*cos(d*x + c)^8 - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*d*cos(d*x
+ c)^6 - 2*(a^4*b - 5*a^3*b^2 + 7*a^2*b^3 - 3*a*b^4)*d*cos(d*x + c)^4 + 4*(
a^4*b - 3*a^3*b^2 + 3*a^2*b^3 - a*b^4)*d*cos(d*x + c)^2 + (a^5 - 4*a^4*b +
6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d)
```

giac [B] time = 2.23, size = 1986, normalized size = 6.35

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 33*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b
)*b^4)*(a^3 - 2*a^2*b + a*b^2)^2*abs(-a + b) - (9*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*a^7*b - 48*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^2 + 93*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 80*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*a^4*b^4 + 27*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^5 - sqrt(a^2 -
 a*b + sqrt(a*b)*(a - b))*a*b^7)*abs(a^3 - 2*a^2*b + a*b^2)*abs(-a + b) - (
6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 27*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 25*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))
*sqrt(a*b)*a^8*b^2 + 53*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b
^3 - 131*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 103*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 29*sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^3*b^7 + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^8)*abs(-
a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - 2*a
^3*b + a^2*b^2 + sqrt((a^4 - 2*a^3*b + a^2*b^2)^2 - (a^4 - 2*a^3*b + a^2*b^
2)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)))/(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3
))))/((3*a^12*b - 27*a^11*b^2 + 104*a^10*b^3 - 224*a^9*b^4 + 294*a^8*b^5 -
238*a^7*b^6 + 112*a^6*b^7 - 24*a^5*b^8 - a^4*b^9 + a^3*b^10)*abs(a^3 - 2*a^
2*b + a*b^2)) - 3*((15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b
- 33*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*b^4)*(a^3 - 2*a^2*b + a*b^2)^2*abs(-a + b) + (9*sqrt(a^2 - a*b -
 sqrt(a*b)*(a - b))*a^7*b - 48*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^2
+ 93*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^3 - 80*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^4*b^4 + 27*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^5 - s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^7)*abs(a^3 - 2*a^2*b + a*b^2)*abs(-a
```

```
 + b) - (6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10 - 27*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b + 25*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^8*b^2 + 53*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^7*b^3 - 131*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 1
03*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 29*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^3*b^7 + sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b
^8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((
a^4 - 2*a^3*b + a^2*b^2 - sqrt((a^4 - 2*a^3*b + a^2*b^2)^2 - (a^4 - 2*a^3*b
 + a^2*b^2)*(a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)))/(a^4 - 3*a^3*b + 3*a^2*b^
2 - a*b^3))))/((3*a^12*b - 27*a^11*b^2 + 104*a^10*b^3 - 224*a^9*b^4 + 294*a
^8*b^5 - 238*a^7*b^6 + 112*a^6*b^7 - 24*a^5*b^8 - a^4*b^9 + a^3*b^10)*abs(a
^3 - 2*a^2*b + a*b^2)) + 2*(17*a^2*tan(d*x + c)^7 - 14*a*b*tan(d*x + c)^7 -
 3*b^2*tan(d*x + c)^7 + 43*a^2*tan(d*x + c)^5 - 18*a*b*tan(d*x + c)^5 - b^2
*tan(d*x + c)^5 + 35*a^2*tan(d*x + c)^3 - 11*a*b*tan(d*x + c)^3 + 9*a^2*tan
(d*x + c) - 3*a*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a
*tan(d*x + c)^2 + a)^2*(a^3 - 2*a^2*b + a*b^2)))/d
```

maple [B] time = 0.33, size = 1624, normalized size = 5.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -17/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x
+c)^7-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b
*tan(d*x+c)^7-43/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/
(a^2-2*a*b+b^2)*tan(d*x+c)^5*a+9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*ta
n(d*x+c)^2+a)^2*b/(a^2-2*a*b+b^2)*tan(d*x+c)^5+1/32/d/(tan(d*x+c)^4*a-tan(d
*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2-35/32/d/
(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(
d*x+c)^3+11/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^
2-2*a*b+b^2)*tan(d*x+c)^3-9/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x
+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)+3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b+15/64/d/(a^2-2*a*b+b^
2)*a/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))-9/32/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-3/32/d/(a
^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-3/64/d*b/(a^2-2*a*b+b^2)*a/
(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))+3/16/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/
(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a
-b))^(1/2))+15/64/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-9/32/d*b/(a^2-2*a*b+b
^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/
2)+a)*(a-b))^(1/2))+3/32/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+
3/64/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/16/d*b^2/(a^2-2*a*
b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x
+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*
b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^
(1/2))-3/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(
1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3+3/64/d/a*
b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*
x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a
-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*
(a-b))^(1/2))*b^3
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^4/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] -1/8*(3*a*b^3*sin(2*d*x + 2*c) - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*cos(4*d*
x + 4*c)*sin(2*d*x + 2*c) - (3*a*b^3*sin(14*d*x + 14*c) - 3*(10*a*b^3 - b^4
)*sin(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4)*sin(10*d*x + 10*c)
 + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*sin(8*d*x + 8*c) + (336*a^2
*b^2 - 95*a*b^3 - 40*b^4)*sin(6*d*x + 6*c) - (64*a^2*b^2 - 54*a*b^3 - 25*b^
4)*sin(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(16*d*x + 16*
c) - 2*(6*(8*a^2*b^2 + 13*a*b^3 - 2*b^4)*sin(12*d*x + 12*c) + 8*(16*a^2*b^2
 - 45*a*b^3 + 8*b^4)*sin(10*d*x + 10*c) - (1408*a^3*b - 544*a^2*b^2 + a*b^3
 + 140*b^4)*sin(8*d*x + 8*c) - 16*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*sin(6*d*
x + 6*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50*b^4)*sin(4*d*x + 4*c) + 8*(11*a*
b^3 + 4*b^4)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) - 2*(2*(640*a^3*b - 488*a
^2*b^2 + 389*a*b^3 - 70*b^4)*sin(10*d*x + 10*c) - (4096*a^4 - 8448*a^3*b +
3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(8*d*x + 8*c) - 2*(2688*a^3*b - 4072
*a^2*b^2 + 861*a*b^3 + 238*b^4)*sin(6*d*x + 6*c) + 4*(256*a^3*b - 560*a^2*b
^2 + 206*a*b^3 + 77*b^4)*sin(4*d*x + 4*c) + 2*(152*a^2*b^2 - 129*a*b^3 - 50
*b^4)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) - 2*((26624*a^4 - 33152*a^3*b +
15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(8*d*x + 8*c) + 8*(3328*a^3*b - 31
04*a^2*b^2 + 529*a*b^3 + 84*b^4)*sin(6*d*x + 6*c) - 2*(2688*a^3*b - 4072*a^
2*b^2 + 861*a*b^3 + 238*b^4)*sin(4*d*x + 4*c) - 16*(96*a^2*b^2 - 29*a*b^3 -
 10*b^4)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) - 2*((26624*a^4 - 33152*a^3*b
 + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*sin(6*d*x + 6*c) - (4096*a^4 - 844
8*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*sin(4*d*x + 4*c) - (1408*a^3*
b - 544*a^2*b^2 + a*b^3 + 140*b^4)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) - 4*(
(640*a^3*b - 488*a^2*b^2 + 389*a*b^3 - 70*b^4)*sin(4*d*x + 4*c) + 4*(16*a^2
*b^2 - 45*a*b^3 + 8*b^4)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 8*((a^3*b^4 -
 2*a^2*b^5 + a*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^
6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 21
0*a^2*b^5 + 49*a*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^
3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384
*a^7 - 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a
^2*b^5 + 1225*a*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 +
 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^
2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^
2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 - 2*a^
2*b^5 + a*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*
sin(14*d*x + 14*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2
*b^5 + 49*a*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 7
53*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7
 - 57344*a^6*b + 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^
5 + 1225*a*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*
a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 2
40*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c)^2 + 6
4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) + (a^3*b^4 - 2*a^2*b^5 + a*b^
6)*d - 2*(8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b
^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^
3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^
2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c)
- 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4
*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^
```

6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a
*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*
b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 16
6*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*
a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*
b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2
*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(1
28*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(10*d
*x + 10*c) + 2*(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1
442*a^2*b^5 - 245*a*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^2 - 424*a^4*b^3
+ 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^2
- 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) -
8*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (8*a
^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(
2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 24
5*a*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 -
322*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^2 - 424*a^4*b^3 +
 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^3 -
 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (16*a^4*b^3 - 39*a
^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b - 6
528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos
(6*d*x + 6*c) - 4*(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4
+ 1442*a^2*b^5 - 245*a*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^2 - 352*a^4*b
^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(2*d*x + 2*c) + (128*a^5*b^
2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d)*cos(8*d*x + 8*c)
 - 16*(4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)
*d*cos(4*d*x + 4*c) + 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*
cos(2*d*x + 2*c) - (16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(
6*d*x + 6*c) + 8*(8*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2
*d*x + 2*c) - (8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(4*d*x
+ 4*c) - 4*(4*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^4
*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*
b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^
2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c)
- 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2
*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(
8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16
*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a
^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x +
8*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c
) + 2*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) +
4*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16
(4(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*si
n(10*d*x + 10*c) + (1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4
 + 1442*a^2*b^5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^2 - 424*a^4*
b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^5*
b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c
) - 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*s
in(12*d*x + 12*c) + 32*((2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^
3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^2 - 736
*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(12
8*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x
 + 4*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x +
2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3
- 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*sin(6*d*x + 6*c) - (1024*a^6*b
 - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d
*sin(4*d*x + 4*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^
5 + 35*a*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^2 - 424

```
*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) + 2*(16
*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x
 + 6*c))*integrate(-3/4*(4*a*b*cos(6*d*x + 6*c)^2 + 4*a*b*cos(2*d*x + 2*c)^
2 + 4*a*b*sin(6*d*x + 6*c)^2 + 4*a*b*sin(2*d*x + 2*c)^2 - 4*(32*a^2 - 20*a*
b + 3*b^2)*cos(4*d*x + 4*c)^2 - a*b*cos(2*d*x + 2*c) - 4*(32*a^2 - 20*a*b +
 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a^2 - 19*a*b + 4*b^2)*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) - (a*b*cos(6*d*x + 6*c) + a*b*cos(2*d*x + 2*c) - 2*(4*a*b -
b^2)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*a*b*cos(2*d*x + 2*c) - a*b + 2
*(8*a^2 - 19*a*b + 4*b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(4*a*b - b
^2 + (8*a^2 - 19*a*b + 4*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b*sin
(6*d*x + 6*c) + a*b*sin(2*d*x + 2*c) - 2*(4*a*b - b^2)*sin(4*d*x + 4*c))*si
n(8*d*x + 8*c) + 2*(4*a*b*sin(2*d*x + 2*c) + (8*a^2 - 19*a*b + 4*b^2)*sin(4
*d*x + 4*c))*sin(6*d*x + 6*c))/(a^3*b^2 - 2*a^2*b^3 + a*b^4 + (a^3*b^2 - 2*
a^2*b^3 + a*b^4)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(
6*d*x + 6*c)^2 + 4*(64*a^5 - 176*a^4*b + 169*a^3*b^2 - 66*a^2*b^3 + 9*a*b^4
)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c)^2
+ (a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 2*a^2*b^
3 + a*b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^5 - 176*a^4*b + 169*a^3*b^2 - 66*a^
2*b^3 + 9*a*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3
 - 3*a*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - 2*a^2*b^3 + a
*b^4)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4 - 4*(a^3*b^2 - 2*
a^2*b^3 + a*b^4)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3 -
3*a*b^4)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c
))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - 2*a^2*b^3 + a*b^4 - 2*(8*a^4*b - 19*a^3*
b^2 + 14*a^2*b^3 - 3*a*b^4)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 2*a^2*b^3 + a*b
^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b
^3 - 3*a*b^4 - 4*(8*a^4*b - 19*a^3*b^2 + 14*a^2*b^3 - 3*a*b^4)*cos(2*d*x +
2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*cos(2*d*x + 2*c) -
 4*(2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(6*d*x + 6*c) + (8*a^4*b - 19*a^3*b^
2 + 14*a^2*b^3 - 3*a*b^4)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4
)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^4*b - 19*a^3*b^2 + 14*a^2*b
^3 - 3*a*b^4)*sin(4*d*x + 4*c) + 2*(a^3*b^2 - 2*a^2*b^3 + a*b^4)*sin(2*d*x
+ 2*c))*sin(6*d*x + 6*c)), x) + (3*a*b^3*cos(14*d*x + 14*c) + 2*a*b^3 + b^4
 - 3*(10*a*b^3 - b^4)*cos(12*d*x + 12*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4
)*cos(10*d*x + 10*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4)*cos(8*d
*x + 8*c) + (336*a^2*b^2 - 95*a*b^3 - 40*b^4)*cos(6*d*x + 6*c) - (64*a^2*b^
2 - 54*a*b^3 - 25*b^4)*cos(4*d*x + 4*c) - (19*a*b^3 + 8*b^4)*cos(2*d*x + 2*
c))*sin(16*d*x + 16*c) - (19*a*b^3 + 8*b^4 - 12*(8*a^2*b^2 + 13*a*b^3 - 2*b
^4)*cos(12*d*x + 12*c) - 16*(16*a^2*b^2 - 45*a*b^3 + 8*b^4)*cos(10*d*x + 10
*c) + 2*(1408*a^3*b - 544*a^2*b^2 + a*b^3 + 140*b^4)*cos(8*d*x + 8*c) + 32*
(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*cos(6*d*x + 6*c) - 4*(152*a^2*b^2 - 129*a*
b^3 - 50*b^4)*cos(4*d*x + 4*c) - 16*(11*a*b^3 + 4*b^4)*cos(2*d*x + 2*c))*si
n(14*d*x + 14*c) - (64*a^2*b^2 - 54*a*b^3 - 25*b^4 - 4*(640*a^3*b - 488*a^2
*b^2 + 389*a*b^3 - 70*b^4)*cos(10*d*x + 10*c) + 2*(4096*a^4 - 8448*a^3*b +
3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*cos(8*d*x + 8*c) + 4*(2688*a^3*b - 4072
*a^2*b^2 + 861*a*b^3 + 238*b^4)*cos(6*d*x + 6*c) - 8*(256*a^3*b - 560*a^2*b
^2 + 206*a*b^3 + 77*b^4)*cos(4*d*x + 4*c) - 4*(152*a^2*b^2 - 129*a*b^3 - 50
*b^4)*cos(2*d*x + 2*c))*sin(12*d*x + 12*c) + (336*a^2*b^2 - 95*a*b^3 - 40*b
^4 + 2*(26624*a^4 - 33152*a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*cos
(8*d*x + 8*c) + 16*(3328*a^3*b - 3104*a^2*b^2 + 529*a*b^3 + 84*b^4)*cos(6*d
*x + 6*c) - 4*(2688*a^3*b - 4072*a^2*b^2 + 861*a*b^3 + 238*b^4)*cos(4*d*x +
 4*c) - 32*(96*a^2*b^2 - 29*a*b^3 - 10*b^4)*cos(2*d*x + 2*c))*sin(10*d*x +
10*c) + (256*a^3*b - 64*a^2*b^2 - 26*a*b^3 + 35*b^4 + 2*(26624*a^4 - 33152*
a^3*b + 15632*a^2*b^2 - 2453*a*b^3 - 420*b^4)*cos(6*d*x + 6*c) - 2*(4096*a^
4 - 8448*a^3*b + 3744*a^2*b^2 - 414*a*b^3 - 385*b^4)*cos(4*d*x + 4*c) - 2*(
1408*a^3*b - 544*a^2*b^2 + a*b^3 + 140*b^4)*cos(2*d*x + 2*c))*sin(8*d*x + 8
*c) - (80*a^2*b^2 - 111*a*b^3 + 16*b^4 - 4*(640*a^3*b - 488*a^2*b^2 + 389*a
*b^3 - 70*b^4)*cos(4*d*x + 4*c) - 16*(16*a^2*b^2 - 45*a*b^3 + 8*b^4)*cos(2*
d*x + 2*c))*sin(6*d*x + 6*c) - 3*(10*a*b^3 - b^4 - 4*(8*a^2*b^2 + 13*a*b^3
```

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- 2*b^4)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c))/((a^3*b^4 - 2*a^2*b^5 + a*b^6)
*d*cos(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 1
4*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^
6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 3
22*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b
+ 83712*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^5 + 1225*a*b^6
)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*
a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 3
37*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^3*b^4 - 2
*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c)^2 + (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*si
n(16*d*x + 16*c)^2 + 64*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c)^
2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*
sin(12*d*x + 12*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^
2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^7 - 57344*a^6*b + 837
12*a^5*b^2 - 67648*a^4*b^3 + 32841*a^3*b^4 - 9170*a^2*b^5 + 1225*a*b^6)*d*s
in(8*d*x + 8*c)^2 + 64*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b
^5 + 49*a*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^5*b^2 - 240*a^4*b^3 + 337*a^
3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^4*b^3 - 23*a
^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^
3*b^4 - 2*a^2*b^5 + a*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*cos(2*d*x + 2*c) + (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d - 2*(8*(a^3*b^
4 - 2*a^2*b^5 + a*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 2
2*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30
*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 3
9*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^
3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*cos(2*d*x + 2*c) - (a^3*b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(16*d*x +
16*c) + 16*(4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(12*d*x
+ 12*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(10*d*x +
 10*c) - 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^
6)*d*cos(8*d*x + 8*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*
d*cos(6*d*x + 6*c) + 4*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*co
s(4*d*x + 4*c) + 8*(a^3*b^4 - 2*a^2*b^5 + a*b^6)*d*cos(2*d*x + 2*c) - (a^3*
b^4 - 2*a^2*b^5 + a*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^5*b^2 - 424*a^
4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(10*d*x + 10*c) + 2*(102
4*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a
*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266
*a^2*b^5 + 49*a*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^5*b^2 - 240*a^4*b^3 + 337
*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^4*b^3 - 23*a
^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (8*a^4*b^3 - 23*a^3*b^4
 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^6*b - 6528*a
^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(8*d*
x + 8*c) + 8*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*b^4 - 322*a^2*b^5 + 49*a*
b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*
a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^
2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) + (16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5
 - 7*a*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^6*b - 6528*a^5*b^2 + 8144*
a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^5 - 245*a*b^6)*d*cos(6*d*x + 6*c) - 4*(
1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 24
5*a*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 -
166*a^2*b^5 + 35*a*b^6)*d*cos(2*d*x + 2*c) + (128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^5*b
^2 - 424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*cos(4*d*x + 4*c)
 + 8*(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) -
(16*a^4*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d)*cos(6*d*x + 6*c) + 8*(8
*(8*a^4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*cos(2*d*x + 2*c) - (8*a^
4*b^3 - 23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^3*
b^4 - 2*a^2*b^5 + a*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^4*b^3 - 23*a^3*b^4 +
```

```
22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b^4 +
30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^3 + 3
55*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^3 - 3
9*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 - 23*a^
3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*b^5 +
 a*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^4*b^3 - 23*a^3*
b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^4*b^3 - 39*a^3*b
^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(10*d*x + 10*c) - (128*a^5*b^2 - 352*a^4*b^
3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^4*b^
3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^4*b^3 -
23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(4*d*x + 4*c) + 4*(a^3*b^4 - 2*a^2*
b^5 + a*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^5*b^2 -
424*a^4*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(10*d*x + 10*c) +
(1024*a^6*b - 3712*a^5*b^2 + 5304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 2
45*a*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*b^4 -
 266*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^5*b^2 - 240*a^4*b^3 +
 337*a^3*b^4 - 210*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^4*b^3 -
23*a^3*b^4 + 22*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) +
 32*((2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 1722*a^2*b^
5 - 245*a*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^5*b^2 - 736*a^4*b^3 + 753*a^3*
b^4 - 322*a^2*b^5 + 49*a*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^5*b^2 - 424*a^4
*b^3 + 513*a^3*b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) - 4*(16*a^4
*b^3 - 39*a^3*b^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(10*d*x +
10*c) + 16*(2*(2048*a^6*b - 6528*a^5*b^2 + 8144*a^4*b^3 - 5141*a^3*b^4 + 17
22*a^2*b^5 - 245*a*b^6)*d*sin(6*d*x + 6*c) - (1024*a^6*b - 3712*a^5*b^2 + 5
304*a^4*b^3 - 3813*a^3*b^4 + 1442*a^2*b^5 - 245*a*b^6)*d*sin(4*d*x + 4*c) -
 2*(128*a^5*b^2 - 352*a^4*b^3 + 355*a^3*b^4 - 166*a^2*b^5 + 35*a*b^6)*d*sin
(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^5*b^2 - 424*a^4*b^3 + 513*a^3*
b^4 - 266*a^2*b^5 + 49*a*b^6)*d*sin(4*d*x + 4*c) + 2*(16*a^4*b^3 - 39*a^3*b
^4 + 30*a^2*b^5 - 7*a*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 19.50, size = 5892, normalized size = 18.82

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^4/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - (atan(((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3
 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(c
+ d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^
4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(
1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3
- a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a
^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3
*a^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a
^4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b
^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11
*b^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9
*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^
3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 +
21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^
7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/
2)*1i - (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3
- 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (tan(c +
 d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1
/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
 a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^
6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*
```

a^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^
4*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^
3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*
b^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*
b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3
*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 2
1*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7
*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2
)*1i)/((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3 -
 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) - (tan(c +
d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
- 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/
2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^6
*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a
^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4
*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3
)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b
^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*b
^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3*
(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 21
*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*
b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)
 - (3*(180*a^2 - 81*a*b + 9*b^2))/(16384*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)) + (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^4*b^4 + 196608*a^5*b^3 -
 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (tan(c +
d*x)*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4
- 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/
2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 -
a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 81920*a^5*b^5 - 163840*a^6
*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a
^3*b^2)))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4
*b^4 - 10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3
)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b
^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(333*a^3*b - 45*a*b^3 + 36*a^4 + 9*b
^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*((9*(16*a^3*
(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 - 10*a^5*b^3 + 21
*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*
b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)
))*((9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) + 4*a^7*b + a^4*b^4 -
10*a^5*b^3 + 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)
))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^
12*b^2)))^(1/2)*2i)/d - (atan(((((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*a^
4*b^4 + 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3 -
 3*a^4*b^2)) - (tan(c + d*x)*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(
1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2
) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10
*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 + 8
1920*a^5*b^5 - 163840*a^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*a^
4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*
b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3
)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^
5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a^3*b
 - 45*a*b^3 + 36*a^4 + 9*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3 -
 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) - 4*a^7*b
- a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(a^
7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5*a
^11*b^3 - a^12*b^2)))^(1/2)*1i - (((3*(49152*a^7*b + 16384*a^3*b^5 - 98304*
a^4*b^4 + 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b^3

```
 - 3*a^4*b^2)) + (tan(c + d*x)*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)
^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1
/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 -
10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6 +
 81920*a^5*b^5 - 163840*a^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(3*
a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^
7*b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b
^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*
b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(333*a^3
*b - 45*a*b^3 + 36*a^4 + 9*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b^3
 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) - 4*a^7*
b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b*(
a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 + 5
*a^11*b^3 - a^12*b^2)))^(1/2)*1i)/((((3*(49152*a^7*b + 16384*a^3*b^5 - 9830
4*a^4*b^4 + 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^3*b
^3 - 3*a^4*b^2)) - (tan(c + d*x)*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^
3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^
(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5
- 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*b^6
 + 81920*a^5*b^5 - 163840*a^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(256*(
3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(
a^7*b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7
*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^
9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2) - (tan(c + d*x)*(333*a
^3*b - 45*a*b^3 + 36*a^4 + 9*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^2*b
^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) - 4*a^
7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^2*b
*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^4 +
 5*a^11*b^3 - a^12*b^2)))^(1/2) - (3*(180*a^2 - 81*a*b + 9*b^2))/(16384*(3*
a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)) + (((3*(49152*a^7*b + 16384*a^3*b^5 - 9
8304*a^4*b^4 + 196608*a^5*b^3 - 163840*a^6*b^2))/(32768*(3*a^5*b - a^6 + a^
3*b^3 - 3*a^4*b^2)) + (tan(c + d*x)*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7
*b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^
3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b
^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)*(16384*a^9*b - 16384*a^4*
b^6 + 81920*a^5*b^5 - 163840*a^6*b^4 + 163840*a^7*b^3 - 81920*a^8*b^2))/(25
6*(3*a^4*b - a^5 + a^2*b^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^
3*(a^7*b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(
a^7*b^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10
*a^9*b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2) + (tan(c + d*x)*(33
3*a^3*b - 45*a*b^3 + 36*a^4 + 9*b^4 - 45*a^2*b^2))/(256*(3*a^4*b - a^5 + a^
2*b^3 - 3*a^3*b^2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^7*b^3)^(1/2) - 4
*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b^3)^(1/2) + 5*a^
2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*b^5 - 10*a^10*b^
4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)))*(-(9*(16*a^3*(a^7*b^3)^(1/2) + b^3*(a^
7*b^3)^(1/2) - 4*a^7*b - a^4*b^4 + 10*a^5*b^3 - 21*a^6*b^2 - 6*a*b^2*(a^7*b
^3)^(1/2) + 5*a^2*b*(a^7*b^3)^(1/2)))/(16384*(a^7*b^7 - 5*a^8*b^6 + 10*a^9*
b^5 - 10*a^10*b^4 + 5*a^11*b^3 - a^12*b^2)))^(1/2)*2i)/d - ((3*tan(c + d*x)
*(3*a - b))/(32*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(35*a - 11*b))/(32*(
a^2 - 2*a*b + b^2)) - (tan(c + d*x)^5*(18*a*b - 43*a^2 + b^2))/(32*a*(a - b
)^2) + (tan(c + d*x)^7*(17*a + 3*b))/(32*a*(a - b)))/(d*(tan(c + d*x)^8*(a^
2 - 2*a*b + b^2) + a^2 - tan(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4
*a*b - 4*a^2) + 4*a^2*tan(c + d*x)^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**4/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.233 $\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=347

$$\frac{\left(-14\sqrt{a}\sqrt{b}+12a+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{\left(14\sqrt{a}\sqrt{b}+12a+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\sqrt{b}\,d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{\tan(c+dx}{32a^2d\left((a\right.}$$

```
[Out] 1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b-14*a^(1/2
)*b^(1/2))/a^(9/4)/d/(a^(1/2)-b^(1/2))^(5/2)/b^(1/2)-1/64*arctan((a^(1/2)+b
^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(12*a+5*b+14*a^(1/2)*b^(1/2))/a^(9/4)/d/b
^(1/2)/(a^(1/2)+b^(1/2))^(5/2)-1/8*b*tan(d*x+c)*(a*(a+3*b)+(a^2+6*a*b+b^2)*
tan(d*x+c)^2)/a/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*ta
n(d*x+c)*(2*a*(5*a^2-9*a*b-4*b^2)/(a-b)^3+5*(2*a^2+3*a*b-b^2)*tan(d*x+c)^2/
(a-b)^2)/a^2/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.72, antiderivative size = 347, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3217, 1333, 1678, 1166, 205}

$$-\frac{\tan(c+dx)\left(\frac{5\left(2a^2+3ab-b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a\left(5a^2-9ab-4b^2\right)}{(a-b)^3}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}-\frac{b\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8ad(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}+$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((12*a - 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
d*x])/a^(1/4)])/(64*a^(9/4)*(Sqrt[a] - Sqrt[b])^(5/2)*Sqrt[b]*d) - ((12*a +
 14*Sqrt[a]*Sqrt[b] + 5*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^
(1/4)])/(64*a^(9/4)*(Sqrt[a] + Sqrt[b])^(5/2)*Sqrt[b]*d) - (b*Tan[c + d*x]*
(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2
*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4)^2) - (Tan[c + d*x]*((2*a*(5*a^2
 - 9*a*b - 4*b^2))/(a - b)^3 + (5*(2*a^2 + 3*a*b - b^2)*Tan[c + d*x]^2)/(a
- b)^2))/(32*a^2*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1333

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
```

```
x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*S
imp[ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d + e*x^2
)^q, a + b*x^2 + c*x^4, x] + b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g +
c*(4*p + 7)*(b*f - 2*a*g)*x^2, x], x], x], x]] /; FreeQ[{a, b, c, d, e}, x]
 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] && IGtQ[m/2, 0]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{x^2\left(1+x^2\right)^4}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-\frac{2a^2b^2(a+3b}{(a-b)^3}\ldots}{\ldots}\right)}{\ldots}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a(5a^2\ldots}{(a\ldots}\right.}{32a^2d\left(a+2a\tan\ldots\right.}$$

$$= -\frac{b\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\tan(c+dx)\left(\frac{2a(5a^2\ldots}{(a\ldots}\right.}{32a^2d\left(a+2a\tan\ldots\right.}$$

$$= \frac{\left(12a-14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{9/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\sqrt{b}\,d} - \frac{\left(12a+14\sqrt{a}\sqrt{b}+5b\right)\tan^{-1}\ldots}{64a^{9/4}\left(\sqrt{a}+\sqrt{b}\ldots\right.}$$

Mathematica [A] time = 6.43, size = 457, normalized size = 1.32

$$\frac{24a^2\sin(2(c+dx))+22ab\sin(2(c+dx))-11ab\sin(4(c+dx))-10b^2\sin(2(c+dx))+5b^2\sin(4(c+dx))}{32a^2d(a-b)^2(-8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b)}+\frac{(11a^3}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] ((-12*a^(5/2)*Sqrt[b] + 10*a^2*b + 11*a^(3/2)*b^(3/2) - 4*a*b^2 - 5*Sqrt[a]
*b^(5/2))*ArcTan[((Sqrt[a]*Sqrt[b] + b)*Tan[c + d*x])/(Sqrt[a + Sqrt[a]*Sqr
t[b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d) - ((1
2*a^(5/2)*Sqrt[b] + 10*a^2*b - 11*a^(3/2)*b^(3/2) - 4*a*b^2 + 5*Sqrt[a]*b^(
5/2))*ArcTanh[((Sqrt[a]*Sqrt[b] - b)*Tan[c + d*x])/(Sqrt[-a + Sqrt[a]*Sqrt[
b]]*Sqrt[b])])/(64*a^(5/2)*Sqrt[-a + Sqrt[a]*Sqrt[b]]*(a - b)^2*b*d) + (-4*
a*Sin[2*(c + d*x)] - 2*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(a*(a - b)*
d*(-8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2) + (24*a^2*Sin
[2*(c + d*x)] + 22*a*b*Sin[2*(c + d*x)] - 10*b^2*Sin[2*(c + d*x)] - 11*a*b*
Sin[4*(c + d*x)] + 5*b^2*Sin[4*(c + d*x)])/(32*a^2*(a - b)^2*d*(-8*a + 3*b
- 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

fricas [B] time = 3.28, size = 6215, normalized size = 17.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] -1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*
a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*
a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*c
os(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-
(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2
+ 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368
640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b
^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17
*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12
*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 +
10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2))*log(13824*a^6 - 24576*
a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 22509/4*a^2*b^4 - 2625/2*a*b^5 + 62
5/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b + 96336*a^4*b^2 - 57820*a^3*b^3 + 22
509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x + c)^2 + 1/2*((22*a^14*b - 125*
a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a^10*b^5 - 147*a^9*b^6 + 40*a^
8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 4
37952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^
7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15
*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b
^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) + (4608*a^9 - 6144*a^8*b +
5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos
(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3
- 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b
^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 +
 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((
a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14
*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)
*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6
)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 17
30*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2*cos(d*x + c)^2 - (
```

144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a
^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((147456*a^8 - 368640*a^7*b + 4
98432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^
2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^
16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^1
1*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*
d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(
a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*
a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^
2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*
b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a
^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b
^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8
)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*
a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b
^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4
*b^6)*d^2))*log(13824*a^6 - 24576*a^5*b + 24084*a^4*b^2 - 14455*a^3*b^3 + 2
2509/4*a^2*b^4 - 2625/2*a*b^5 + 625/4*b^6 - 1/4*(55296*a^6 - 98304*a^5*b +
96336*a^4*b^2 - 57820*a^3*b^3 + 22509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d
x + c)^2 - 1/2((22*a^14*b - 125*a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 +
310*a^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 -
 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a
^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*
a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*
a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*
x + c) + (4608*a^9 - 6144*a^8*b + 5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4
 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 +
76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 - (a^9*b - 5*a^8*b^2 + 10*a^7*b^
3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b +
 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*
a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*
a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a
^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3
- 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b +
 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 -
25*a^5*b^7)*d^2*cos(d*x + c)^2 - (144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2
325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*s
qrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a
^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b -
10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 21
0*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) +
 ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3
 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4
)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x
+ c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^
4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^
7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7
*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35
406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 -
120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 +
45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*
b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2))*log(-13824*a^6 + 24576*a^5*b
- 24084*a^4*b^2 + 14455*a^3*b^3 - 22509/4*a^2*b^4 + 2625/2*a*b^5 - 625/4*b^
6 + 1/4*(55296*a^6 - 98304*a^5*b + 96336*a^4*b^2 - 57820*a^3*b^3 + 22509*a^
2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x + c)^2 + 1/2*((22*a^14*b - 125*a^13*b
^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7
- 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*
a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 62
5*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 -

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252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 +
a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c) - (4608*a^9 - 6144*a^8*b + 5052*a
^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 159*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x +
 c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b
^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^
2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 26964
1*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b
 - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 +
 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))
)/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)
) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8
*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2*cos(d*x + c)^2 - (144*a^
12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5
 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((147456*a^8 - 368640*a^7*b + 498432*
a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6
- 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4
 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9
- 10*a^10*b^10 + a^9*b^11)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(
d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b
- 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^
2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*
a^3*b^3 + a^2*b^4)*d)*sqrt(-(144*a^4 + 76*a^3*b - 155*a^2*b^2 + 94*a*b^3 -
15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6
)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 2
69641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^
19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b
^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d
^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*
d^2))*log(-13824*a^6 + 24576*a^5*b - 24084*a^4*b^2 + 14455*a^3*b^3 - 22509/
4*a^2*b^4 + 2625/2*a*b^5 - 625/4*b^6 + 1/4*(55296*a^6 - 98304*a^5*b + 96336
*a^4*b^2 - 57820*a^3*b^3 + 22509*a^2*b^4 - 5250*a*b^5 + 625*b^6)*cos(d*x +
c)^2 - 1/2*((22*a^14*b - 125*a^13*b^2 + 300*a^12*b^3 - 395*a^11*b^4 + 310*a
^10*b^5 - 147*a^9*b^6 + 40*a^8*b^7 - 5*a^7*b^8)*d^3*sqrt((147456*a^8 - 3686
40*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^
5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*
b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*
b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))*cos(d*x + c)*sin(d*x + c
) - (4608*a^9 - 6144*a^8*b + 5052*a^7*b^2 - 2437*a^6*b^3 + 783*a^5*b^4 - 15
9*a^4*b^5 + 25*a^3*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(144*a^4 + 76*a^
3*b - 155*a^2*b^2 + 94*a*b^3 - 15*b^4 + (a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 1
0*a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2*sqrt((147456*a^8 - 368640*a^7*b + 4984
32*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^4 - 117532*a^3*b^5 + 35406*a^2*b
^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^18*b^2 + 45*a^17*b^3 - 120*a^16*
b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^13*b^7 - 120*a^12*b^8 + 45*a^11*b
^9 - 10*a^10*b^10 + a^9*b^11)*d^4)))/((a^9*b - 5*a^8*b^2 + 10*a^7*b^3 - 10*
a^6*b^4 + 5*a^5*b^5 - a^4*b^6)*d^2)) - 1/4*(2*(144*a^12 - 796*a^11*b + 1845
*a^10*b^2 - 2325*a^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^
5*b^7)*d^2*cos(d*x + c)^2 - (144*a^12 - 796*a^11*b + 1845*a^10*b^2 - 2325*a
^9*b^3 + 1730*a^8*b^4 - 774*a^7*b^5 + 201*a^6*b^6 - 25*a^5*b^7)*d^2)*sqrt((
147456*a^8 - 368640*a^7*b + 498432*a^6*b^2 - 437952*a^5*b^3 + 269641*a^4*b^
4 - 117532*a^3*b^5 + 35406*a^2*b^6 - 6700*a*b^7 + 625*b^8)/((a^19*b - 10*a^
18*b^2 + 45*a^17*b^3 - 120*a^16*b^4 + 210*a^15*b^5 - 252*a^14*b^6 + 210*a^1
3*b^7 - 120*a^12*b^8 + 45*a^11*b^9 - 10*a^10*b^10 + a^9*b^11)*d^4))) + 8*((
11*a*b^2 - 5*b^3)*cos(d*x + c)^7 - 3*(2*a^2*b + 11*a*b^2 - 5*b^3)*cos(d*x +
 c)^5 - 3*(a^2*b - 14*a*b^2 + 5*b^3)*cos(d*x + c)^3 + 5*(2*a^3 + a^2*b - 4*
a*b^2 + b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d
*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a
^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a
^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2
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- 4*a^3*b^3 + a^2*b^4)*d)
```

giac [B] time = 2.29, size = 1184, normalized size = 3.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((30*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^3
+ 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 + 36*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^4 - 105*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b + 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 19*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*
x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (
a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a
^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9*b - 18*a^8*b^2 + 41*a^7*b
^3 - 44*a^6*b^4 + 21*a^5*b^5 - 2*a^4*b^6 - a^3*b^7) + (30*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a^4*b - 72*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^2 +
 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 + 4*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a*b^4 - 36*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4 +
 105*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 69*sqrt(a^2 - a*
b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 19*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^
4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^5 - 2*a^4*b
+ a^3*b^2 - sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a
^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)
)))*abs(-a + b)/(3*a^9*b - 18*a^8*b^2 + 41*a^7*b^3 - 44*a^6*b^4 + 21*a^5*b^
5 - 2*a^4*b^6 - a^3*b^7) - 2*(10*a^3*tan(d*x + c)^7 + 5*a^2*b*tan(d*x + c)^
7 - 20*a*b^2*tan(d*x + c)^7 + 5*b^3*tan(d*x + c)^7 + 30*a^3*tan(d*x + c)^5
+ 12*a^2*b*tan(d*x + c)^5 - 18*a*b^2*tan(d*x + c)^5 + 30*a^3*tan(d*x + c)^3
 + 3*a^2*b*tan(d*x + c)^3 - 9*a*b^2*tan(d*x + c)^3 + 10*a^3*tan(d*x + c) -
4*a^2*b*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x +
 c)^2 + a)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

maple [B] time = 0.47, size = 1906, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] 1/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*ar
ctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3-13/64/d*b/(a^2-2
*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+13/64/d*b/(a^2-2*a*b+b^2)*a/(a*b)^(
1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))-1/64/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
*b^3-5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(
d*x+c)^7+3/16/d/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2-1/32/d*b^2/
(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a
+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-15/16/d/(tan(d*x+c)^4*a-tan(d
*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^5*a-3/16/d/(a^2-
2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*t
an(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/8/d/(tan(d*x+c)^4*a-tan(d*x+
```

```
c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b-15/32/d/(tan(d*x+
c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+9/32/d/(
tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d
*x+c)^3*b^2-3/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(
a^2-2*a*b+b^2)*tan(d*x+c)^3+5/16/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2
)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+
5/16/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-5/64/d/a^2/(a^2-2*a*b+b^2)/(a-
b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)
*(a-b))^(1/2))*b^3-5/64/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3+1/32/d*b^
2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a
-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+9/16/d/(tan(d*x+c)^4*a-tan(d*
x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2+5/32/d/(t
an(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a^2/(a-b)*tan(d*x+c)^7*b
^2-3/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-2*a*b+
b^2)*tan(d*x+c)^5-15/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a
)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)^3-5/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*
a*tan(d*x+c)^2+a)^2*a/(a^2-2*a*b+b^2)*tan(d*x+c)+11/32/d/(a^2-2*a*b+b^2)*a/
(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))+11/32/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(
1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-37/64/d*b/(a^2-
2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))-37/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/16*(4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(4*d*x + 4*c)*sin(
2*d*x + 2*c) + ((12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(14*d*x + 14*c) - (104*a
^2*b^3 - 85*a*b^4 + 35*b^5)*sin(12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*b^3
 + 407*a*b^4 - 105*b^5)*sin(10*d*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b^3 +
 865*a*b^4 - 175*b^5)*sin(8*d*x + 8*c) + (320*a^3*b^2 + 756*a^2*b^3 - 849*a
*b^4 + 175*b^5)*sin(6*d*x + 6*c) - (248*a^2*b^3 - 383*a*b^4 + 105*b^5)*sin(
4*d*x + 4*c) - (12*a^2*b^3 + 77*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(16*d*
x + 16*c) + 2*(2*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*sin(12*d*x +
 12*c) + 8*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(10*d*x + 10*
c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5)*sin(
8*d*x + 8*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^5)*sin(6*d*
x + 6*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*sin(4*d*x + 4
*c) + 24*(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c
) + 2*(2*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*
sin(10*d*x + 10*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b
^4 + 1225*b^5)*sin(8*d*x + 8*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^
3 + 5033*a*b^4 - 735*b^5)*sin(6*d*x + 6*c) + 4*(576*a^3*b^2 - 1696*a^2*b^3
+ 1323*a*b^4 - 245*b^5)*sin(4*d*x + 4*c) + 2*(96*a^3*b^2 + 324*a^2*b^3 - 64
9*a*b^4 + 175*b^5)*sin(2*d*x + 2*c))*cos(12*d*x + 12*c) + 2*((40960*a^5 - 2
4064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(8*
d*x + 8*c) + 8*(5120*a^4*b - 1408*a^3*b^2 - 3900*a^2*b^3 + 2107*a*b^4 - 245
*b^5)*sin(6*d*x + 6*c) - 2*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2*b^3 + 5033*
a*b^4 - 735*b^5)*sin(4*d*x + 4*c) - 16*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b
^4 + 35*b^5)*sin(2*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((40960*a^5 - 24064*a
^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*sin(6*d*x +
6*c) - (9216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)
*sin(4*d*x + 4*c) - 3*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4
```

- 175*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*c) + 4*((2560*a^4*b - 4128*a^3*b
^2 + 3644*a^2*b^3 - 1379*a*b^4 + 245*b^5)*sin(4*d*x + 4*c) + 4*(64*a^3*b^2
- 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 16
*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*
a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^
6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x
+ 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32
841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a
^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x
+ 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^
2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*
x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a
^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240
*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 +
64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*
sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a
^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d
*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*
b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^
3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*
a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^
3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a
^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 3
55*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 -
39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 2
3*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3
*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos
(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d
*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*
d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3
*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b
^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*co
s(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*
(8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*c
os(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*
b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424
*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(
64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*
d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*
x + 2*c) + (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x
+ 12*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 +
1722*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b
^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^
6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x +
4*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x +
2*c) + (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 1
0*c) + 4*(8*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722
*a^3*b^5 - 245*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 +
5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*
c) - 8*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)
*d*cos(2*d*x + 2*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^
5 + 35*a^2*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 51
3*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 -
39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*

```
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 2
3*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3
*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b
^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*
b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^
4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a
^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4
*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a
^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352
*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(
16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8
*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4
*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*
(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(
10*d*x + 10*c) + (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 +
 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*
b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^
6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x +
 4*c) - 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2
*c))*sin(12*d*x + 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5
141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b
^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*
c) - 2*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)
*d*sin(4*d*x + 4*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*
d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 +
 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*
c) - (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^
5 - 245*a^2*b^6)*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^
4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64
*((128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*si
n(4*d*x + 4*c) + 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(-1/8*(4*(12*a^2*b - 11*a*b^2 + 5
*b^3)*cos(6*d*x + 6*c)^2 - 4*(256*a^3 - 248*a^2*b + 97*a*b^2 - 15*b^3)*cos(
4*d*x + 4*c)^2 + 4*(12*a^2*b - 11*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(12
*a^2*b - 11*a*b^2 + 5*b^3)*sin(6*d*x + 6*c)^2 - 4*(256*a^3 - 248*a^2*b + 97
*a*b^2 - 15*b^3)*sin(4*d*x + 4*c)^2 + 2*(96*a^3 - 252*a^2*b + 149*a*b^2 - 3
5*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(12*a^2*b - 11*a*b^2 + 5*b^3)*
sin(2*d*x + 2*c)^2 - ((12*a^2*b - 11*a*b^2 + 5*b^3)*cos(6*d*x + 6*c) - 2*(3
2*a^2*b - 19*a*b^2 + 5*b^3)*cos(4*d*x + 4*c) + (12*a^2*b - 11*a*b^2 + 5*b^3
)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (12*a^2*b - 11*a*b^2 + 5*b^3 - 2*(96
*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*cos(4*d*x + 4*c) - 8*(12*a^2*b - 11*
a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(32*a^2*b - 19*a*b^2
 + 5*b^3 + (96*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*cos(2*d*x + 2*c))*cos(4
*d*x + 4*c) - (12*a^2*b - 11*a*b^2 + 5*b^3)*cos(2*d*x + 2*c) - ((12*a^2*b -
 11*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) - 2*(32*a^2*b - 19*a*b^2 + 5*b^3)*sin(4
*d*x + 4*c) + (12*a^2*b - 11*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + 2*((96*a^3 - 252*a^2*b + 149*a*b^2 - 35*b^3)*sin(4*d*x + 4*c) + 4*(12
*a^2*b - 11*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^4*b^2 - 2
*a^3*b^3 + a^2*b^4 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(8*d*x + 8*c)^2 + 1
6*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*
b + 169*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*cos(4*d*x + 4*c)^2 + 16*(a^4*b^2
 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c)^2 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)
*sin(8*d*x + 8*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c)^2
 + 4*(64*a^6 - 176*a^5*b + 169*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*sin(4*d*x
+ 4*c)^2 + 16*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c)^
```

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2 + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*co
s(6*d*x + 6*c) - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*
x + 4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(8*d*x +
8*c) - 8*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*
b^3 - 3*a^2*b^4)*cos(4*d*x + 4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2
*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^
2*b^4 - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(2*d*x + 2*c))
*cos(4*d*x + 4*c) - 8*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c) - 4*
(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c) + (8*a^5*b - 19*a^4*b^2
 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*
b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^5*b - 19*a^4*b^2 + 14*a^
3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (11*a*b^4 - 5*b^5 + (12*a^2*b^3 - 11
*a*b^4 + 5*b^5)*cos(14*d*x + 14*c) - (104*a^2*b^3 - 85*a*b^4 + 35*b^5)*cos(
12*d*x + 12*c) - (320*a^3*b^2 - 652*a^2*b^3 + 407*a*b^4 - 105*b^5)*cos(10*d
*x + 10*c) + (1408*a^3*b^2 - 1696*a^2*b^3 + 865*a*b^4 - 175*b^5)*cos(8*d*x
+ 8*c) + (320*a^3*b^2 + 756*a^2*b^3 - 849*a*b^4 + 175*b^5)*cos(6*d*x + 6*c)
 - (248*a^2*b^3 - 383*a*b^4 + 105*b^5)*cos(4*d*x + 4*c) - (12*a^2*b^3 + 77*
a*b^4 - 35*b^5)*cos(2*d*x + 2*c))*sin(16*d*x + 16*c) + (12*a^2*b^3 + 77*a*b
^4 - 35*b^5 - 4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(12*d*x +
12*c) - 16*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^5)*cos(10*d*x + 10*
c) + 6*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5)*cos(
8*d*x + 8*c) + 32*(128*a^3*b^2 + 124*a^2*b^3 - 173*a*b^4 + 35*b^5)*cos(6*d*
x + 6*c) - 4*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*cos(4*d*x + 4
*c) - 48*(4*a^2*b^3 + 11*a*b^4 - 5*b^5)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c
) + (248*a^2*b^3 - 383*a*b^4 + 105*b^5 - 4*(2560*a^4*b - 4128*a^3*b^2 + 364
4*a^2*b^3 - 1379*a*b^4 + 245*b^5)*cos(10*d*x + 10*c) + 2*(9216*a^4*b - 2598
4*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)*cos(8*d*x + 8*c) + 4*(25
60*a^4*b + 480*a^3*b^2 - 7908*a^2*b^3 + 5033*a*b^4 - 735*b^5)*cos(6*d*x + 6
*c) - 8*(576*a^3*b^2 - 1696*a^2*b^3 + 1323*a*b^4 - 245*b^5)*cos(4*d*x + 4*c
) - 4*(96*a^3*b^2 + 324*a^2*b^3 - 649*a*b^4 + 175*b^5)*cos(2*d*x + 2*c))*si
n(12*d*x + 12*c) - (320*a^3*b^2 + 756*a^2*b^3 - 849*a*b^4 + 175*b^5 + 2*(40
960*a^5 - 24064*a^4*b - 22080*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*
b^5)*cos(8*d*x + 8*c) + 16*(5120*a^4*b - 1408*a^3*b^2 - 3900*a^2*b^3 + 2107
*a*b^4 - 245*b^5)*cos(6*d*x + 6*c) - 4*(2560*a^4*b + 480*a^3*b^2 - 7908*a^2
*b^3 + 5033*a*b^4 - 735*b^5)*cos(4*d*x + 4*c) - 32*(128*a^3*b^2 + 124*a^2*b
^3 - 173*a*b^4 + 35*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - (1408*a^3*b
^2 - 1696*a^2*b^3 + 865*a*b^4 - 175*b^5 + 2*(40960*a^5 - 24064*a^4*b - 2208
0*a^3*b^2 + 27516*a^2*b^3 - 11095*a*b^4 + 1225*b^5)*cos(6*d*x + 6*c) - 2*(9
216*a^4*b - 25984*a^3*b^2 + 21304*a^2*b^3 - 8575*a*b^4 + 1225*b^5)*cos(4*d*
x + 4*c) - 6*(512*a^4*b + 1024*a^3*b^2 - 1556*a^2*b^3 + 865*a*b^4 - 175*b^5
)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (320*a^3*b^2 - 652*a^2*b^3 + 407*a*b
^4 - 105*b^5 - 4*(2560*a^4*b - 4128*a^3*b^2 + 3644*a^2*b^3 - 1379*a*b^4 + 2
45*b^5)*cos(4*d*x + 4*c) - 16*(64*a^3*b^2 - 196*a^2*b^3 + 125*a*b^4 - 35*b^
5)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (104*a^2*b^3 - 85*a*b^4 + 35*b^5 -
4*(96*a^3*b^2 + 36*a^2*b^3 - 53*a*b^4 + 35*b^5)*cos(2*d*x + 2*c))*sin(4*d*x
 + 4*c) - (12*a^2*b^3 - 11*a*b^4 + 5*b^5)*sin(2*d*x + 2*c))/((a^4*b^4 - 2*a
^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^
6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 21
0*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*
b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(1
6384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 91
70*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5
*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(6
4*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d
*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c)^2 + (a^
4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b
^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a
^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^6*b^2
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 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*
c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a
^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^6*b^
2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c
)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6
)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^
6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)
*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*
c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^
6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^
6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b
^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166
*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30
*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(16*d*x + 16*c)
 + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12
*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 1
0*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^
6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6
)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) -
 (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^6*b^2
 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c
) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b
^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a
^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^6*b^2 - 240
*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(
8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (8*a^
5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*
(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 2
45*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4
 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^6*b^2 - 424*a^5*
b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^
5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (16*a^5*b
^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(204
8*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a
^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 -
3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^6*
b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(2*d*x + 2
*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*
d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*
a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*
a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b
^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*
b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*
b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^
5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin
(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
```

```
 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^6*b^2 - 4
24*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c) +
 (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 -
245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^
4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^6*b^2 - 240*a^5*
b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^5
*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x
+ 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 17
22*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b^2 - 736*a^5*b^3
 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^6*
b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4
*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*
c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 -
5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*c) - (1024*a^7*b
 - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)
*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*
b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^6*b^2 -
 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) +
 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*s
in(6*d*x + 6*c))
```

mupad [B] time = 19.90, size = 6646, normalized size = 19.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)*(5*a - 2*b))/(16*(a^2 - 2*a*b + b^2)) + (3*tan(c + d*x)^3*
(a*b + 10*a^2 - 3*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (5*tan(c + d*x)^7*(3*a
*b + 2*a^2 - b^2))/(32*a^2*(a - b)) + (3*tan(c + d*x)^5*(2*a*b + 5*a^2 - 3*
b^2))/(16*a*(a - b)^2))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 - tan(
c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*tan(c +
 d*x)^2)) - (atan(((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 68812
8*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) -
 (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^
9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9
*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384
*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)
))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 1
63840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2))
)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*
b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) -
 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5
*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (t
an(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a
^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^
(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*
a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1
/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5
 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*1i - (((163840*a^9*b + 6553
6*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7
*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2)
 + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b
^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) -
 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10
*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 +
81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a
^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^
9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b
```

^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) + (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*1i)/(((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (3168*a^4 - 3832*a^3*b - 755*a*b^3 + 125*b^4 + 2410*a^2*b^2)/(16384*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (tan(c + d*x)*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) + (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)))*(-(384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) - 144*a^9*b + 15*a^5*b^5 - 94*a^6*b^4 + 155*a^7*b^3 - 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*2i)/d - (atan(((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) - (tan(c + d*x)*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2)

)/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a
^14*b^2)))^(1/2) - (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5
+ 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*
((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5
 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 13
4*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^
10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*1i - ((
(163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a^7*b^3 - 557056*a^
8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (tan(c + d*x)*((384*
a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*
a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^
3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6
 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*(16384*a^10*b
 - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 163840*a^8*b^3 - 81920*
a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*((384*a^4*(a^9*b^3)^
(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*
a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1
/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5
 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) + (tan(c + d*x)*(460*a^4*b
 - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^2))/(256*(3*a^5*b
 - a^6 + a^3*b^3 - 3*a^4*b^2)))*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)
^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 3
49*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3
)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*
b^3 - a^14*b^2)))^(1/2)*1i)/((((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b
^4 + 688128*a^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a
^6*b^2)) - (tan(c + d*x)*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2)
 + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2
b^2(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2
))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 -
a^14*b^2)))^(1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^
7*b^4 + 163840*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*
a^4*b^2)))*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b -
 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)
^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9
*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1
/2) - (tan(c + d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3
 - 635*a^3*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*((384*a^4*(a^
9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4
 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*
b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a
^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) - (3168*a^4 - 3832*a
^3*b - 755*a*b^3 + 125*b^4 + 2410*a^2*b^2)/(16384*(3*a^7*b - a^8 + a^5*b^3
 - 3*a^6*b^2)) + (((163840*a^9*b + 65536*a^5*b^5 - 360448*a^6*b^4 + 688128*a
^7*b^3 - 557056*a^8*b^2)/(32768*(3*a^7*b - a^8 + a^5*b^3 - 3*a^6*b^2)) + (t
an(c + d*x)*((384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b
 - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3
)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^
9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(
1/2)*(16384*a^10*b - 16384*a^5*b^6 + 81920*a^6*b^5 - 163840*a^7*b^4 + 16384
0*a^8*b^3 - 81920*a^9*b^2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*((
384*a^4*(a^9*b^3)^(1/2) + 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 +
 94*a^6*b^4 - 155*a^7*b^3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*
a*b^3*(a^9*b^3)^(1/2) - 480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10
*b^6 + 10*a^11*b^5 - 10*a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2) + (tan(c
+ d*x)*(460*a^4*b - 149*a*b^4 + 144*a^5 + 25*b^5 + 443*a^2*b^3 - 635*a^3*b^
2))/(256*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))*((384*a^4*(a^9*b^3)^(1/2)
+ 25*b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^
3 + 76*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) -

```
480*a^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*
a^12*b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)))*((384*a^4*(a^9*b^3)^(1/2) + 25*
b^4*(a^9*b^3)^(1/2) + 144*a^9*b - 15*a^5*b^5 + 94*a^6*b^4 - 155*a^7*b^3 + 7
6*a^8*b^2 + 349*a^2*b^2*(a^9*b^3)^(1/2) - 134*a*b^3*(a^9*b^3)^(1/2) - 480*a
^3*b*(a^9*b^3)^(1/2))/(16384*(a^9*b^7 - 5*a^10*b^6 + 10*a^11*b^5 - 10*a^12*
b^4 + 5*a^13*b^3 - a^14*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.234 $\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=319

$$\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}-\frac{b\,\mathrm{ta}}{32a^2}$$

```
[Out] 1/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b-50*a^(1/
2)*b^(1/2))/a^(11/4)/d/(a^(1/2)-b^(1/2))^(5/2)+1/64*arctan((a^(1/2)+b^(1/2)
)^(1/2)*tan(d*x+c)/a^(1/4))*(32*a+21*b+50*a^(1/2)*b^(1/2))/a^(11/4)/d/(a^(1
/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)*(3*a+b+4*(a+b)*tan(d*x+c)^2)/a/(a-b)^
3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*((17*a^2-40
*a*b+7*b^2)/(a-b)^3+(33*a-13*b)*tan(d*x+c)^2/(a-b)^2)/a^2/d/(a+2*a*tan(d*x+
c)^2+(a-b)*tan(d*x+c)^4)
```

Rubi [A] time = 0.65, antiderivative size = 319, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3209, 1205, 1678, 1166, 205}

$$-\frac{b\tan(c+dx)\left(\frac{17a^2-40ab+7b^2}{(a-b)^3}+\frac{(33a-13b)\tan^2(c+dx)}{(a-b)^2}\right)}{32a^2d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}+\frac{\left(-50\sqrt{a}\sqrt{b}+32a+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{(5\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[c + d*x]^4)^(-3),x]
```

```
[Out] ((32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(64*a^(11/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) + ((32*a + 50*Sq
rt[a]*Sqrt[b] + 21*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)
])/(64*a^(11/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - (b^2*Tan[c + d*x]*(3*a + b +
 4*(a + b)*Tan[c + d*x]^2))/(8*a*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a -
 b)*Tan[c + d*x]^4)^2) - (b*Tan[c + d*x]*((17*a^2 - 40*a*b + 7*b^2)/(a - b)
^3 + ((33*a - 13*b)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^2*d*(a + 2*a*Tan[c +
d*x]^2 + (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1205

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> With[{f = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[(d + e*x^2)^q, a + b*x^2 +
c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*g - f*(b^2 - 2*
a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*
```

```
(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[2*a*(p
+ 1)*(b^2 - 4*a*c)*PolynomialQuotient[(d + e*x^2)^q, a + b*x^2 + c*x^4, x]
+ b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5) - a*b*g + c*(4*p + 7)*(b*f - 2*a*g)*x
^2, x], x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[
c*d^2 - b*d*e + a*e^2, 0] && IGtQ[q, 1] && LtQ[p, -1]
```

Rule 1678

```
Int[(Pq_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> With[{d =
 Coeff[PolynomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 0], e = Coeff[Poly
nomialRemainder[Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x
^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^2))/(2*a*(p + 1)*(
b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[(a + b*x^2 + c*
x^4)^(p + 1)*ExpandToSum[2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[Pq, a
 + b*x^2 + c*x^4, x] + b^2*d*(2*p + 3) - 2*a*c*d*(4*p + 5) - a*b*e + c*(4*p
 + 7)*(b*d - 2*a*e)*x^2, x], x], x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^
2] && Expon[Pq, x^2] > 1 && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^5}{\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int -\frac{2ab\left(8a^3-24a^2b+\cdots\right.}{(a-b)\cdots}\right.}{\cdots}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{17a^2-4\cdots}{(a-\cdots}\right.}{32a^2d\left(a+2a\tan^2(c\cdots\right.}$$

$$= -\frac{b^2\tan(c+dx)\left(3a+b+4(a+b)\tan^2(c+dx)\right)}{8a(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{17a^2-4\cdots}{(a-\cdots}\right.}{32a^2d\left(a+2a\tan^2(c\cdots\right.}$$

$$= \frac{\left(32a-50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{11/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} + \frac{\left(32a+50\sqrt{a}\sqrt{b}+21b\right)\tan^{-1}\cdots}{64a^{11/4}\left(\sqrt{a}+\sqrt{\cdots}\right.}$$

Mathematica [A] time = 3.00, size = 333, normalized size = 1.04

$$\frac{\frac{64a^{3/2}b(a-b)(\sin(4(c+dx))-6\sin(2(c+dx)))}{(-8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b)^2} + \frac{\left(50\sqrt{a}\sqrt{b}+32a+21b\right)\left(\sqrt{a}-\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{8\sqrt{a}\,b\sin(2(c+dx))((6a-3b)\cos(2(c+\cdots}{8a+4b\cos(2(c+dx))-b\cos(4(c+\cdots}}{64a^{5/2}d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[c + d*x]^4)^(-3),x]
```

```
[Out] (((Sqrt[a] - Sqrt[b])^2*(32*a + 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTan[((Sqrt[a]
 + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt
[b]] - ((Sqrt[a] + Sqrt[b])^2*(32*a - 50*Sqrt[a]*Sqrt[b] + 21*b)*ArcTanh[((
Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a + Sqr
t[a]*Sqrt[b]] + (8*Sqrt[a]*b*(-19*a + 10*b + (6*a - 3*b)*Cos[2*(c + d*x)])*
Sin[2*(c + d*x)])/(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)]) +
 (64*a^(3/2)*(a - b)*b*(-6*Sin[2*(c + d*x)] + Sin[4*(c + d*x)]))/(-8*a + 3*
b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2)/(64*a^(5/2)*(a - b)^2*d)
```

fricas [B] time = 3.39, size = 6152, normalized size = 19.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")
```

```
[Out] 1/256*(((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a
^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a
^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*co
s(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(
1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*
b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b
- 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5
 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21
- 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 21
0*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/(
(a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(
491520*a^6*b - 1742720*a^5*b^2 + 2747904*a^4*b^3 - 2435877*a^3*b^4 + 510698
9/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4*b^7 - 1/4*(1966080*a^6*b - 6970880*
a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*
b^6 + 194481*b^7)*cos(d*x + c)^2 + 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^
2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*
d^3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^
5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b
^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17
*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b
^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (88320*a^9*b - 319040*a^8
*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4 + 241865*a^5*b^5 - 71421*a^4*b^6 + 9
261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(1024*a^4 - 1916*a^3*b + 15
01*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^
3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458
560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162
574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2
- 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7
+ 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b
^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2)) - 1/4*(2*(1024*a^13 - 6276*a^1
2*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 336
1*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c)^2 - (1024*a^13 - 6276*a^12*b + 16
461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^
6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6
*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2
*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a
^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^
13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)
*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*
(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3
```

```
*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b
^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 -
570*a*b^3 + 105*b^4 - (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4
 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 -
 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 -
 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^
3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8
 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b
^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(491520*a^6*b - 1742720*a^5*b^2 + 274790
4*a^4*b^3 - 2435877*a^3*b^4 + 5106989/4*a^2*b^5 - 750141/2*a*b^6 + 194481/4
*b^7 - 1/4*(1966080*a^6*b - 6970880*a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^
3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 - 1/2*
((32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a
^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*
b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3
*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 4
5*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 12
0*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(
d*x + c) + (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4
+ 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c
))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 - (a^1
0 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686
400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 443358
81*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^
9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^1
6*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10
)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*
d^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 +
 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x +
c)^2 - (1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a
^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8
*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*
b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^
21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 +
 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))
) + ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*
b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*
b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d
*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(102
4*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b +
 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 1
7817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 -
25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 1
0*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a
^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^
10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-49
1520*a^6*b + 1742720*a^5*b^2 - 2747904*a^4*b^3 + 2435877*a^3*b^4 - 5106989/
4*a^2*b^5 + 750141/2*a*b^6 - 194481/4*b^7 + 1/4*(1966080*a^6*b - 6970880*a^
5*b^2 + 10991616*a^4*b^3 - 9743508*a^3*b^4 + 5106989*a^2*b^5 - 1500282*a*b^
6 + 194481*b^7)*cos(d*x + c)^2 + 1/2*((32*a^16 - 193*a^15*b + 498*a^14*b^2
 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^
3*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*
b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8
 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b
^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9
 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (88320*a^9*b - 319040*a^8*b
^2 + 510294*a^7*b^3 - 457551*a^6*b^4 + 241865*a^5*b^5 - 71421*a^4*b^6 + 926
1*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(1024*a^4 - 1916*a^3*b + 1501
*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
```

```
+ 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 3945856
0*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 916257
4*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 -
120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 +
45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2
 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*
b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*
a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c)^2 - (1024*a^13 - 6276*a^12*b + 1646
1*a^11*b^2 - 24005*a^10*b^3 + 21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6
- 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b
^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b
^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^1
8*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13
*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))) - ((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d
*cos(d*x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a
^5*b - 5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a
^4*b^2 + 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2
 - 4*a^3*b^3 + a^2*b^4)*d)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 57
0*a*b^3 + 105*b^4 + (a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 -
 a^5*b^5)*d^2*sqrt((3686400*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 5
1952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1
980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3
+ 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 -
 10*a^12*b^9 + a^11*b^10)*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3
 + 5*a^6*b^4 - a^5*b^5)*d^2))*log(-491520*a^6*b + 1742720*a^5*b^2 - 2747904
*a^4*b^3 + 2435877*a^3*b^4 - 5106989/4*a^2*b^5 + 750141/2*a*b^6 - 194481/4*
b^7 + 1/4*(1966080*a^6*b - 6970880*a^5*b^2 + 10991616*a^4*b^3 - 9743508*a^3
*b^4 + 5106989*a^2*b^5 - 1500282*a*b^6 + 194481*b^7)*cos(d*x + c)^2 - 1/2*(
(32*a^16 - 193*a^15*b + 498*a^14*b^2 - 715*a^13*b^3 + 620*a^12*b^4 - 327*a^
11*b^5 + 98*a^10*b^6 - 13*a^9*b^7)*d^3*sqrt((3686400*a^8*b - 17817600*a^7*b
^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b^5 - 25065628*a^3*
b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^21 - 10*a^20*b + 45
*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 + 210*a^15*b^6 - 120
*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4))*cos(d*x + c)*sin(d
*x + c) - (88320*a^9*b - 319040*a^8*b^2 + 510294*a^7*b^3 - 457551*a^6*b^4 +
 241865*a^5*b^5 - 71421*a^4*b^6 + 9261*a^3*b^7)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(-(1024*a^4 - 1916*a^3*b + 1501*a^2*b^2 - 570*a*b^3 + 105*b^4 + (a^10
 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d^2*sqrt((36864
00*a^8*b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 4433588
1*a^4*b^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9
)/((a^21 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16
*b^5 + 210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)
*d^4)))/((a^10 - 5*a^9*b + 10*a^8*b^2 - 10*a^7*b^3 + 5*a^6*b^4 - a^5*b^5)*d
^2)) - 1/4*(2*(1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 +
21090*a^9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2*cos(d*x + c
)^2 - (1024*a^13 - 6276*a^12*b + 16461*a^11*b^2 - 24005*a^10*b^3 + 21090*a^
9*b^4 - 11214*a^8*b^5 + 3361*a^7*b^6 - 441*a^6*b^7)*d^2)*sqrt((3686400*a^8*
b - 17817600*a^7*b^2 + 39458560*a^6*b^3 - 51952960*a^5*b^4 + 44335881*a^4*b
^5 - 25065628*a^3*b^6 + 9162574*a^2*b^7 - 1980972*a*b^8 + 194481*b^9)/((a^2
1 - 10*a^20*b + 45*a^19*b^2 - 120*a^18*b^3 + 210*a^17*b^4 - 252*a^16*b^5 +
210*a^15*b^6 - 120*a^14*b^7 + 45*a^13*b^8 - 10*a^12*b^9 + a^11*b^10)*d^4)))
 - 8*(6*(2*a*b^2 - b^3)*cos(d*x + c)^7 - (49*a*b^2 - 25*b^3)*cos(d*x + c)^5
 - 8*(2*a^2*b - 9*a*b^2 + 4*b^3)*cos(d*x + c)^3 + (33*a^2*b - 46*a*b^2 + 13
*b^3)*cos(d*x + c))*sin(d*x + c))/((a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*
x + c)^8 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d*cos(d*x + c)^6 - 2*(a^5*b -
5*a^4*b^2 + 7*a^3*b^3 - 3*a^2*b^4)*d*cos(d*x + c)^4 + 4*(a^5*b - 3*a^4*b^2
+ 3*a^3*b^3 - a^2*b^4)*d*cos(d*x + c)^2 + (a^6 - 4*a^5*b + 6*a^4*b^2 - 4*a^
3*b^3 + a^2*b^4)*d)
```

giac [B] time = 0.68, size = 1131, normalized size = 3.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] 1/64*((96*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 - 333*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*a^3*b + 313*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 -
79*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - 21*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*b^4 + 42*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 108
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 34*sqrt(a^2 - a*b +
sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 + 8*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^
5 - 2*a^4*b + a^3*b^2 + sqrt((a^5 - 2*a^4*b + a^3*b^2)^2 - (a^5 - 2*a^4*b +
 a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)))/(a^5 - 3*a^4*b + 3*a^3*b^
2 - a^2*b^3))))*abs(-a + b)/(3*a^9 - 18*a^8*b + 41*a^7*b^2 - 44*a^6*b^3 + 2
1*a^5*b^4 - 2*a^4*b^5 - a^3*b^6) + (96*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
a^4 - 333*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b + 313*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*a^2*b^2 - 79*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^3 -
 21*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 - 42*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^3 + 108*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)
*a^2*b - 34*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2 - 8*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) +
 arctan(tan(d*x + c)/sqrt((a^5 - 2*a^4*b + a^3*b^2 - sqrt((a^5 - 2*a^4*b +
a^3*b^2)^2 - (a^5 - 2*a^4*b + a^3*b^2)*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3
)))/(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3))))*abs(-a + b)/(3*a^9 - 18*a^8*b
+ 41*a^7*b^2 - 44*a^6*b^3 + 21*a^5*b^4 - 2*a^4*b^5 - a^3*b^6) - 2*(33*a^2*b
*tan(d*x + c)^7 - 46*a*b^2*tan(d*x + c)^7 + 13*b^3*tan(d*x + c)^7 + 83*a^2*
b*tan(d*x + c)^5 - 66*a*b^2*tan(d*x + c)^5 + 7*b^3*tan(d*x + c)^5 + 67*a^2*
b*tan(d*x + c)^3 - 43*a*b^2*tan(d*x + c)^3 + 17*a^2*b*tan(d*x + c) - 11*a*b
^2*tan(d*x + c))/((a*tan(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2
 + a)^2*(a^4 - 2*a^3*b + a^2*b^2)))/d
```

maple [B] time = 0.42, size = 1803, normalized size = 5.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -19/16/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3+23/32/d*b/(a^2
-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-23/32/d*b/(a^2-2*a*b+b^2)*a/(a*b)
^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b
)^(1/2)-a)*(a-b))^(1/2))+19/16/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b
)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/
2))*b^3+101/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+21/64/d/
a^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh
((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^4-21/64/d/a^2/(a^2-2*a*
b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x
+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^4-17/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4
*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b-33/32/d/(tan(d*x+c)^4
*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+43/32/d/(tan
(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+
c)^3*b^2-67/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^
2-2*a*b+b^2)*tan(d*x+c)^3+23/32/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)
```

```
-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+2
3/32/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-13/64/d/a^2/(a^2-2*a*b+b^2)/(a
-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a
)*(a-b))^(1/2))*b^3-13/64/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3-101/64/
d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+33/16/d/(tan(d*x+c)^4*a-t
an(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2+13/3
2/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a^2/(a-b)*tan(d*x+
c)^7*b^2-83/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^
2-2*a*b+b^2)*tan(d*x+c)^5-7/32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x
+c)^2+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tan(d*x+c)^5+1/2/d/(a^2-2*a*b+b^2)*a/(a-
b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)
*(a-b))^(1/2))+1/2/d/(a^2-2*a*b+b^2)*a/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-65/64/d*b/(a^2-2*a*b
+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)
^(1/2)-a)*(a-b))^(1/2))-65/64/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a
-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+11/32/d/(
tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b^2/a/(a^2-2*a*b+b^2)*t
an(d*x+c)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] 1/8*(4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ ((7*a*b^4 - 4*b^5)*sin(14*d*x + 14*c) - (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*si
n(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5)*sin(10*d*x + 10*c) + 3*(
256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*sin(8*d*x + 8*c) + (784*a^2
*b^3 - 723*a*b^4 + 140*b^5)*sin(6*d*x + 6*c) - (160*a^2*b^3 - 266*a*b^4 + 9
1*b^5)*sin(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*sin(2*d*x + 2*c))*cos(16*d*x
+ 16*c) + 2*(2*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*sin(12*d*x + 12*c) - 8*(48
*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(10*d*x + 10*c) - (3968*a^3*b^2 - 5024*a^2
*b^3 + 2621*a*b^4 - 560*b^5)*sin(8*d*x + 8*c) - 16*(224*a^2*b^3 - 209*a*b^4
 + 42*b^5)*sin(6*d*x + 6*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(4*d
*x + 4*c) + 8*(31*a*b^4 - 16*b^5)*sin(2*d*x + 2*c))*cos(14*d*x + 14*c) + 2*
(2*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(10*d*x + 10*c) -
(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(8*
d*x + 8*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*sin(6
*d*x + 6*c) + 4*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 343*b^5)*sin(4*d
*x + 4*c) + 2*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*sin(2*d*x + 2*c))*cos(12*
d*x + 12*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4
 + 1960*b^5)*sin(8*d*x + 8*c) + 8*(6400*a^3*b^2 - 8608*a^2*b^3 + 3437*a*b^4
 - 392*b^5)*sin(6*d*x + 6*c) - 2*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4
 - 1078*b^5)*sin(4*d*x + 4*c) - 16*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*sin(2
*d*x + 2*c))*cos(10*d*x + 10*c) + 2*((51200*a^4*b - 84864*a^3*b^2 + 56016*a
^2*b^3 - 18081*a*b^4 + 1960*b^5)*sin(6*d*x + 6*c) - (8192*a^4*b - 23296*a^3
*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*sin(4*d*x + 4*c) - (3968*a^3*
b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*sin(2*d*x + 2*c))*cos(8*d*x + 8*
c) + 4*((1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*sin(4*d*x + 4*c)
 - 4*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) -
8*((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 +
 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a
^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x
 + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 3
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2841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*
a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x
 + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a
^2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d
*x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(
a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 24
0*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2
+ 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d
*sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*
a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2
 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*
d*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3
*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a
^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)
*d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2
*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*
a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 +
355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3
 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*co
s(16*d*x + 16*c) + 16*(4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*
d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^
3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^
3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*c
os(2*d*x + 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8
(8(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*
cos(10*d*x + 10*c) + 2*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4
*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 42
4*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*
(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4
*d*x + 4*c) - 8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d
*x + 2*c) + (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x
 + 12*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 +
 1722*a^3*b^5 - 245*a^2*b^6)*d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*
b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a
^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x
+ 4*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x +
 2*c) + (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x +
10*c) + 4*(8*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 172
2*a^3*b^5 - 245*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2
+ 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4
c) - 8(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6
)*d*cos(2*d*x + 2*c) + (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b
^5 + 35*a^2*b^6)*d)*cos(8*d*x + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 5
13*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 -
 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39
*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a
^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^
3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*
b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3
*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*
a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^

```
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5
 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*
a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39
*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 35
2*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*
(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(
8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^
4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4
*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin
(10*d*x + 10*c) + (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4
+ 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5
*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a
^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x
+ 4*c) - 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x +
2*c))*sin(12*d*x + 12*c) + 32*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 -
5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*
b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6
*c) - 2*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6
)*d*sin(4*d*x + 4*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)
*d*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2
+ 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6
*c) - (1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b
^5 - 245*a^2*b^6)*d*sin(4*d*x + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a
^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 6
4*((128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*s
in(4*d*x + 4*c) + 2*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(1/4*(4*(7*a*b^2 - 4*b^3)*cos(6*
d*x + 6*c)^2 - 4*(256*a^3 - 416*a^2*b + 256*a*b^2 - 51*b^3)*cos(4*d*x + 4*c
)^2 + 4*(7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c)^2 + 4*(7*a*b^2 - 4*b^3)*sin(6*d*
x + 6*c)^2 - 4*(256*a^3 - 416*a^2*b + 256*a*b^2 - 51*b^3)*sin(4*d*x + 4*c)^
2 - 2*(72*a^2*b - 107*a*b^2 + 56*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(7*a*b^2 - 4*b^3)*sin(2*d*x + 2*c)^2 - ((7*a*b^2 - 4*b^3)*cos(6*d*x + 6*c)
 - 2*(32*a^2*b - 40*a*b^2 + 17*b^3)*cos(4*d*x + 4*c) + (7*a*b^2 - 4*b^3)*co
s(2*d*x + 2*c))*cos(8*d*x + 8*c) - (7*a*b^2 - 4*b^3 + 2*(72*a^2*b - 107*a*b
^2 + 56*b^3)*cos(4*d*x + 4*c) - 8*(7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c))*cos(6
*d*x + 6*c) + 2*(32*a^2*b - 40*a*b^2 + 17*b^3 - (72*a^2*b - 107*a*b^2 + 56*
b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (7*a*b^2 - 4*b^3)*cos(2*d*x + 2*c
) - ((7*a*b^2 - 4*b^3)*sin(6*d*x + 6*c) - 2*(32*a^2*b - 40*a*b^2 + 17*b^3)*
sin(4*d*x + 4*c) + (7*a*b^2 - 4*b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 2
*((72*a^2*b - 107*a*b^2 + 56*b^3)*sin(4*d*x + 4*c) - 4*(7*a*b^2 - 4*b^3)*si
n(2*d*x + 2*c))*sin(6*d*x + 6*c))/(a^4*b^2 - 2*a^3*b^3 + a^2*b^4 + (a^4*b^2
 - 2*a^3*b^3 + a^2*b^4)*cos(8*d*x + 8*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*
b^4)*cos(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*b + 169*a^4*b^2 - 66*a^3*b^3
+ 9*a^2*b^4)*cos(4*d*x + 4*c)^2 + 16*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*
d*x + 2*c)^2 + (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(8*d*x + 8*c)^2 + 16*(a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(6*d*x + 6*c)^2 + 4*(64*a^6 - 176*a^5*b + 16
9*a^4*b^2 - 66*a^3*b^3 + 9*a^2*b^4)*sin(4*d*x + 4*c)^2 + 16*(8*a^5*b - 19*a
^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^
4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c)^2 + 2*(a^4*b^2 - 2*a^3*b^3 +
a^2*b^4 - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(6*d*x + 6*c) - 2*(8*a^5*b -
 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*x + 4*c) - 4*(a^4*b^2 - 2*a^3
*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^4*b^2 - 2*a^3*b^3
 + a^2*b^4 - 2*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(4*d*x +
4*c) - 4*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c)
 - 4*(8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4 - 4*(8*a^5*b - 19*a^4*b
^2 + 14*a^3*b^3 - 3*a^2*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 8*(a^4*b^
2 - 2*a^3*b^3 + a^2*b^4)*cos(2*d*x + 2*c) - 4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2
*b^4)*sin(6*d*x + 6*c) + (8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*si
n(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c))*sin(8*
```

d*x + 8*c) + 16*((8*a^5*b - 19*a^4*b^2 + 14*a^3*b^3 - 3*a^2*b^4)*sin(4*d*x + 4*c) + 2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x) - (6*a*b^4 - 3*b^5 + (7*a*b^4 - 4*b^5)*cos(14*d*x + 14*c) - (32*a^2*b^3 + 2*a*b^4 - 7*b^5)*cos(12*d*x + 12*c) - (16*a^2*b^3 - 3*a*b^4 - 28*b^5)*cos(10*d*x + 10*c) + 3*(256*a^3*b^2 - 320*a^2*b^3 + 166*a*b^4 - 35*b^5)*cos(8*d*x + 8*c) + (784*a^2*b^3 - 723*a*b^4 + 140*b^5)*cos(6*d*x + 6*c) - (160*a^2*b^3 - 266*a*b^4 + 91*b^5)*cos(4*d*x + 4*c) - (55*a*b^4 - 28*b^5)*cos(2*d*x + 2*c))*sin(16*d*x + 16*c) + (55*a*b^4 - 28*b^5 - 4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(12*d*x + 12*c) + 16*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*cos(10*d*x + 10*c) + 2*(3968*a^3*b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*cos(8*d*x + 8*c) + 32*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*cos(6*d*x + 6*c) - 4*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*cos(4*d*x + 4*c) - 16*(31*a*b^4 - 16*b^5)*cos(2*d*x + 2*c))*sin(14*d*x + 14*c) + (160*a^2*b^3 - 266*a*b^4 + 91*b^5 - 4*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*cos(10*d*x + 10*c) + 2*(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*cos(8*d*x + 8*c) + 4*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*cos(6*d*x + 6*c) - 8*(512*a^3*b^2 - 1520*a^2*b^3 + 1330*a*b^4 - 343*b^5)*cos(4*d*x + 4*c) - 4*(376*a^2*b^3 - 613*a*b^4 + 210*b^5)*cos(2*d*x + 2*c))*sin(12*d*x + 12*c) - (784*a^2*b^3 - 723*a*b^4 + 140*b^5 + 2*(51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4 + 1960*b^5)*cos(8*d*x + 8*c) + 16*(6400*a^3*b^2 - 8608*a^2*b^3 + 3437*a*b^4 - 392*b^5)*cos(6*d*x + 6*c) - 4*(5248*a^3*b^2 - 10888*a^2*b^3 + 6433*a*b^4 - 1078*b^5)*cos(4*d*x + 4*c) - 32*(224*a^2*b^3 - 209*a*b^4 + 42*b^5)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) - (768*a^3*b^2 - 960*a^2*b^3 + 498*a*b^4 - 105*b^5 + 2*(51200*a^4*b - 84864*a^3*b^2 + 56016*a^2*b^3 - 18081*a*b^4 + 1960*b^5)*cos(6*d*x + 6*c) - 2*(8192*a^4*b - 23296*a^3*b^2 + 21376*a^2*b^3 - 9394*a*b^4 + 1715*b^5)*cos(4*d*x + 4*c) - 2*(3968*a^3*b^2 - 5024*a^2*b^3 + 2621*a*b^4 - 560*b^5)*cos(2*d*x + 2*c))*sin(8*d*x + 8*c) + (16*a^2*b^3 - 3*a*b^4 - 28*b^5 - 4*(1152*a^3*b^2 - 520*a^2*b^3 - 455*a*b^4 + 294*b^5)*cos(4*d*x + 4*c) + 16*(48*a^2*b^3 - 55*a*b^4 + 28*b^5)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c) + (32*a^2*b^3 + 2*a*b^4 - 7*b^5 - 4*(120*a^2*b^3 - 77*a*b^4 + 14*b^5)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) - (7*a*b^4 - 4*b^5)*sin(2*d*x + 2*c))/((a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*cos(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c)^2 + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(16*d*x + 16*c)^2 + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x + 14*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(12*d*x + 12*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c)^2 + 4*(16384*a^8 - 57344*a^7*b + 83712*a^6*b^2 - 67648*a^5*b^3 + 32841*a^4*b^4 - 9170*a^3*b^5 + 1225*a^2*b^6)*d*sin(8*d*x + 8*c)^2 + 64*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c)^2 + 16*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c)^2 + 64*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 64*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c)^2 - 16*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) + (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d - 2*(8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(14*d*x + 14*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x

+ 2*c) - (a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d)*cos(16*d*x + 16*c) + 16*(4*(8*a
^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(12*d*x + 12*c) - 8*(16*
a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(10*d*x + 10*c) - 2*(12
8*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(8*d
*x + 8*c) - 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(6*d*
x + 6*c) + 4*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(4*d*x
+ 4*c) + 8*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*cos(2*d*x + 2*c) - (a^4*b^4 -
2*a^3*b^5 + a^2*b^6)*d)*cos(14*d*x + 14*c) - 8*(8*(128*a^6*b^2 - 424*a^5*b^
3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(10*d*x + 10*c) + 2*(1024*
a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2
*b^6)*d*cos(8*d*x + 8*c) + 8*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266
*a^3*b^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(64*a^6*b^2 - 240*a^5*b^3 + 3
37*a^4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(8*a^5*b^3 -
23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (8*a^5*b^3 - 23*a
^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d)*cos(12*d*x + 12*c) + 16*(2*(2048*a^7*b
- 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*
d*cos(8*d*x + 8*c) + 8*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b^4 - 322*a^3*b
^5 + 49*a^2*b^6)*d*cos(6*d*x + 6*c) - 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^
4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(16*a^5*b^3 - 39*a
^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*cos(2*d*x + 2*c) + (16*a^5*b^3 - 39*a^4*
b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d)*cos(10*d*x + 10*c) + 4*(8*(2048*a^7*b - 65
28*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 - 245*a^2*b^6)*d*co
s(6*d*x + 6*c) - 4*(1024*a^7*b - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4
 + 1442*a^3*b^5 - 245*a^2*b^6)*d*cos(4*d*x + 4*c) - 8*(128*a^6*b^2 - 352*a^
5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d*cos(2*d*x + 2*c) + (128*a
^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2*b^6)*d)*cos(8*d*x
 + 8*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49
*a^2*b^6)*d*cos(4*d*x + 4*c) + 8*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*
a^2*b^6)*d*cos(2*d*x + 2*c) - (16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2
*b^6)*d)*cos(6*d*x + 6*c) + 8*(8*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a
^2*b^6)*d*cos(2*d*x + 2*c) - (8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b
^6)*d)*cos(4*d*x + 4*c) - 4*(4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(14*d*x
 + 14*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(12*d*x
 + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(10*d*
x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2
*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*
b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*b^
6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*d*sin(2*d*x + 2*c
))*sin(16*d*x + 16*c) + 32*(2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a^3*b^5 - 7*a^2*
b^6)*d*sin(12*d*x + 12*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2
*b^6)*d*sin(10*d*x + 10*c) - (128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166
*a^3*b^5 + 35*a^2*b^6)*d*sin(8*d*x + 8*c) - 4*(16*a^5*b^3 - 39*a^4*b^4 + 30
*a^3*b^5 - 7*a^2*b^6)*d*sin(6*d*x + 6*c) + 2*(8*a^5*b^3 - 23*a^4*b^4 + 22*a
^3*b^5 - 7*a^2*b^6)*d*sin(4*d*x + 4*c) + 4*(a^4*b^4 - 2*a^3*b^5 + a^2*b^6)*
d*sin(2*d*x + 2*c))*sin(14*d*x + 14*c) - 16*(4*(128*a^6*b^2 - 424*a^5*b^3 +
 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(10*d*x + 10*c) + (1024*a^7*b
 - 3712*a^6*b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)
*d*sin(8*d*x + 8*c) + 4*(128*a^6*b^2 - 424*a^5*b^3 + 513*a^4*b^4 - 266*a^3*
b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(64*a^6*b^2 - 240*a^5*b^3 + 337*a^
4*b^4 - 210*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(8*a^5*b^3 - 23*a^
4*b^4 + 22*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + 32
*((2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4 + 1722*a^3*b^5 -
 245*a^2*b^6)*d*sin(8*d*x + 8*c) + 4*(256*a^6*b^2 - 736*a^5*b^3 + 753*a^4*b
^4 - 322*a^3*b^5 + 49*a^2*b^6)*d*sin(6*d*x + 6*c) - 2*(128*a^6*b^2 - 424*a^
5*b^3 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) - 4*(16*
a^5*b^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(10*d
*x + 10*c) + 16*(2*(2048*a^7*b - 6528*a^6*b^2 + 8144*a^5*b^3 - 5141*a^4*b^4
 + 1722*a^3*b^5 - 245*a^2*b^6)*d*sin(6*d*x + 6*c) - (1024*a^7*b - 3712*a^6*
b^2 + 5304*a^5*b^3 - 3813*a^4*b^4 + 1442*a^3*b^5 - 245*a^2*b^6)*d*sin(4*d*x

```
 + 4*c) - 2*(128*a^6*b^2 - 352*a^5*b^3 + 355*a^4*b^4 - 166*a^3*b^5 + 35*a^2
*b^6)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) - 64*((128*a^6*b^2 - 424*a^5*b^3
 + 513*a^4*b^4 - 266*a^3*b^5 + 49*a^2*b^6)*d*sin(4*d*x + 4*c) + 2*(16*a^5*b
^3 - 39*a^4*b^4 + 30*a^3*b^5 - 7*a^2*b^6)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6
*c))
```

mupad [B] time = 19.67, size = 6267, normalized size = 19.65

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(c + d*x)^4)^3,x)
```

```
[Out] - ((tan(c + d*x)^5*(83*a^2*b - 66*a*b^2 + 7*b^3))/(32*a^2*(a - b)^2) + (tan
(c + d*x)^7*(33*a*b - 13*b^2))/(32*a^2*(a - b)) + (tan(c + d*x)*(17*a*b - 1
1*b^2))/(32*a*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^3*(67*a*b - 43*b^2))/(32
*a*(a^2 - 2*a*b + b^2)))/(d*(tan(c + d*x)^8*(a^2 - 2*a*b + b^2) + a^2 - tan
(c + d*x)^4*(2*a*b - 6*a^2) - tan(c + d*x)^6*(4*a*b - 4*a^2) + 4*a^2*tan(c
+ d*x)^2)) - (atan(((((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3
866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 +
 a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*
(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*
a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*
b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^
13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5
 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 +
 a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2)
- 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*
a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1
/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^1
4*b^2)))^(1/2) - (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^
2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^
2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024
*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2
) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15
*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*1i -
 (((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 42
27072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^
2)) + (tan(c + d*x)*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 19
16*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^
3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))
/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^
2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 +
 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^
2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024
*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2
) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15
*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (t
an(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^
3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^
11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4
 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^1
1*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^
5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*1i)/((((524288*a^10*b -
 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 234
2912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)
*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^1
0 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) -
4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b -
 a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a
```

^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 8
1920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^
11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4
 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^1
1*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^
5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) - (tan(c + d*x)*(1024*a
^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(25
6*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b
^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 15
01*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a
^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10
*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (((524288*a^10*b - 344064*a^5*b^6 + 1802
240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(
3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) + (tan(c + d*x)*((1920*a^4*(a^11*b)^(
1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*
a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1
/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a
^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6
 + 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(
3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*
(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*
a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*
b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^
13*b^3 - 10*a^14*b^2)))^(1/2) + (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 44
1*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4
*b^3 - 3*a^5*b^2)))*((1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 19
16*a^9*b + 1024*a^10 + 105*a^6*b^4 - 570*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^
3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))
/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^
2)))^(1/2) + (32768*a^4*b - 29581*a*b^4 + 5733*b^5 + 65572*a^2*b^3 - 70784*
a^3*b^2)/(16384*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2))))*((1920*a^4*(a^11*b
)^(1/2) + 441*b^4*(a^11*b)^(1/2) - 1916*a^9*b + 1024*a^10 + 105*a^6*b^4 - 5
70*a^7*b^3 + 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)
^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 -
5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*2i)/d - (atan(((((524288*a^
10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3
 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) - (tan(c
+ d*x)*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1
024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(
1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a
^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(
16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*
b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(-(1920
a^4(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105
*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^
3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 +
 a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) - (tan(c + d*x)
*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b
^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1/2
) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7
*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2)
 + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12
*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*1i - (((524288*a^10*b - 344064*a^
5*b^6 + 1802240*a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b
^2)/(32768*(3*a^8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) + (tan(c + d*x)*(-(1920*a
^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a
^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*
b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a
^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 1

```
6384*a^6*b^6 + 81920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10
*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1
/2) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a
^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/
2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^
12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (tan(c + d*x)*(1024*a^5*b - 2
141*a*b^5 + 441*b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6
*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^1
1*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*
b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*
(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b
^3 - 10*a^14*b^2)))^(1/2)*1i)/((((524288*a^10*b - 344064*a^5*b^6 + 1802240*
a^6*b^5 - 3866624*a^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^
8*b - a^9 + a^6*b^3 - 3*a^7*b^2)) - (tan(c + d*x)*(-(1920*a^4*(a^11*b)^(1/2
) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7
*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2)
 + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12
*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 8
1920*a^7*b^5 - 163840*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a
^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a
^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^
8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^
2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13
*b^3 - 10*a^14*b^2)))^(1/2) - (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*
b^6 + 4099*a^2*b^4 - 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b
^3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 191
6*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3
*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/
(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2
)))^(1/2) + (((524288*a^10*b - 344064*a^5*b^6 + 1802240*a^6*b^5 - 3866624*a
^7*b^4 + 4227072*a^8*b^3 - 2342912*a^9*b^2)/(32768*(3*a^8*b - a^9 + a^6*b^3
 - 3*a^7*b^2)) + (tan(c + d*x)*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b
)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2
 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^
11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3
- 10*a^14*b^2)))^(1/2)*(16384*a^11*b - 16384*a^6*b^6 + 81920*a^7*b^5 - 1638
40*a^8*b^4 + 163840*a^9*b^3 - 81920*a^10*b^2))/(256*(3*a^6*b - a^7 + a^4*b^
3 - 3*a^5*b^2)))*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 1916
*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*
(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(
16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)
))^(1/2) + (tan(c + d*x)*(1024*a^5*b - 2141*a*b^5 + 441*b^6 + 4099*a^2*b^4
- 3139*a^3*b^3 + 4*a^4*b^2))/(256*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)))*(
-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10
 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4
640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b^2*(a^11*b)^(1/2))/(16384*(5*a^15*b -
a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^13*b^3 - 10*a^14*b^2)))^(1/2) + (32768*
a^4*b - 29581*a*b^4 + 5733*b^5 + 65572*a^2*b^3 - 70784*a^3*b^2)/(16384*(3*a
^8*b - a^9 + a^6*b^3 - 3*a^7*b^2))))*(-(1920*a^4*(a^11*b)^(1/2) + 441*b^4*(
a^11*b)^(1/2) + 1916*a^9*b - 1024*a^10 - 105*a^6*b^4 + 570*a^7*b^3 - 1501*a
^8*b^2 - 2246*a*b^3*(a^11*b)^(1/2) - 4640*a^3*b*(a^11*b)^(1/2) + 4669*a^2*b
^2*(a^11*b)^(1/2))/(16384*(5*a^15*b - a^16 + a^11*b^5 - 5*a^12*b^4 + 10*a^1
3*b^3 - 10*a^14*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.235 $\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx$

Optimal. Leaf size=357

$$\frac{3\sqrt{b}\left(-34\sqrt{a}\sqrt{b}+20a+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}-\frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}$$

```
[Out] -cot(d*x+c)/a^3/d+3/64*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b
^(1/2)*(20*a+15*b-34*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)-b^(1/2))^(5/2)-3/
64*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)*(20*a+15*b+34
*a^(1/2)*b^(1/2))/a^(13/4)/d/(a^(1/2)+b^(1/2))^(5/2)-1/8*b^2*tan(d*x+c)*(a*
(a+3*b)+(a^2+6*a*b+b^2)*tan(d*x+c)^2)/a^2/(a-b)^3/d/(a+2*a*tan(d*x+c)^2+(a-
b)*tan(d*x+c)^4)^2-1/32*b*tan(d*x+c)*(2*a^2*(9*a-17*b)/(a-b)^3+(18*a^2+15*a
*b-13*b^2)*tan(d*x+c)^2/(a-b)^2)/a^3/d/(a+2*a*tan(d*x+c)^2+(a-b)*tan(d*x+c)
^4)
```

Rubi [A] time = 1.29, antiderivative size = 357, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3217, 1334, 1669, 1664, 1166, 205}

$$-\frac{b^2\tan(c+dx)\left(\left(a^2+6ab+b^2\right)\tan^2(c+dx)+a(a+3b)\right)}{8a^2d(a-b)^3\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)^2}-\frac{b\tan(c+dx)\left(\frac{\left(18a^2+15ab-13b^2\right)\tan^2(c+dx)}{(a-b)^2}+\frac{2a^2(9a-17b)}{(a-b)^3}\right)}{32a^3d\left((a-b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b
]]*Tan[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] - Sqrt[b])^(5/2)*d) - (3*S
qrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(64*a^(13/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) - Cot[c +
d*x]/(a^3*d) - (b^2*Tan[c + d*x]*(a*(a + 3*b) + (a^2 + 6*a*b + b^2)*Tan[c +
 d*x]^2))/(8*a^2*(a - b)^3*d*(a + 2*a*Tan[c + d*x]^2 + (a - b)*Tan[c + d*x]
^4)^2) - (b*Tan[c + d*x]*((2*a^2*(9*a - 17*b))/(a - b)^3 + ((18*a^2 + 15*a*
b - 13*b^2)*Tan[c + d*x]^2)/(a - b)^2))/(32*a^3*d*(a + 2*a*Tan[c + d*x]^2 +
 (a - b)*Tan[c + d*x]^4))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1334

```
Int[(x_)^(m_)*((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^
4)^(p_), x_Symbol] :> With[{f = Coeff[PolynomialRemainder[x^m*(d + e*x^2)^q
, a + b*x^2 + c*x^4, x], x, 0], g = Coeff[PolynomialRemainder[x^m*(d + e*x^
```

```
2)^q, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(x*(a + b*x^2 + c*x^4)^(p + 1)*(a
*b*g - f*(b^2 - 2*a*c) - c*(b*f - 2*a*g)*x^2))/(2*a*(p + 1)*(b^2 - 4*a*c)),
 x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), Int[x^m*(a + b*x^2 + c*x^4)^(p +
1)*Simp[ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*PolynomialQuotient[x^m*(d +
e*x^2)^q, a + b*x^2 + c*x^4, x])/x^m + (b^2*f*(2*p + 3) - 2*a*c*f*(4*p + 5)
 - a*b*g)/x^m + c*(4*p + 7)*(b*f - 2*a*g)*x^(2 - m), x], x], x], x]] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && IGtQ[q, 1] &
& ILtQ[m/2, 0]
```

Rule 1664

```
Int[(Pq_)*((d_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_.), x_
Symbol] :> Int[ExpandIntegrand[(d*x)^m*Pq*(a + b*x^2 + c*x^4)^p, x], x] /;
FreeQ[{a, b, c, d, m}, x] && PolyQ[Pq, x^2] && IGtQ[p, -2]
```

Rule 1669

```
Int[(Pq_)*(x_)^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :>
 With[{d = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 0],
e = Coeff[PolynomialRemainder[x^m*Pq, a + b*x^2 + c*x^4, x], x, 2]}, Simp[(
x*(a + b*x^2 + c*x^4)^(p + 1)*(a*b*e - d*(b^2 - 2*a*c) - c*(b*d - 2*a*e)*x^
2))/(2*a*(p + 1)*(b^2 - 4*a*c)), x] + Dist[1/(2*a*(p + 1)*(b^2 - 4*a*c)), I
nt[x^m*(a + b*x^2 + c*x^4)^(p + 1)*ExpandToSum[(2*a*(p + 1)*(b^2 - 4*a*c)*P
olynomialQuotient[x^m*Pq, a + b*x^2 + c*x^4, x])/x^m + (b^2*d*(2*p + 3) - 2
*a*c*d*(4*p + 5) - a*b*e)/x^m + c*(4*p + 7)*(b*d - 2*a*e)*x^(2 - m), x], x]
, x]] /; FreeQ[{a, b, c}, x] && PolyQ[Pq, x^2] && GtQ[Expon[Pq, x^2], 1] &&
 NeQ[b^2 - 4*a*c, 0] && LtQ[p, -1] && ILtQ[m/2, 0]
```

Rule 3217

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff^(m + 1
)/f, Subst[Int[(x^m*(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p)/(1 + ff^2*x^2)
^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] &&
 IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\left(a-b\sin^4(c+dx)\right)^3}\,dx = \frac{\text{Subst}\left(\int \frac{(1+x^2)^6}{x^2\left(a+2ax^2+(a-b)x^4\right)^3}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{\text{Subst}\left(\int \frac{-16ab-\frac{2ab(32a}{}}{}\right.}{}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{2a^2(9a-}{(a-b}\right.}{32a^3d\left(a+2a\tan^2(\right.}$$

$$= -\frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(c+dx)\left(\frac{2a^2(9a-}{(a-b}\right.}{32a^3d\left(a+2a\tan^2(\right.}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(}{32a^3}$$

$$= -\frac{\cot(c+dx)}{a^3d} - \frac{b^2\tan(c+dx)\left(a(a+3b)+\left(a^2+6ab+b^2\right)\tan^2(c+dx)\right)}{8a^2(a-b)^3d\left(a+2a\tan^2(c+dx)+(a-b)\tan^4(c+dx)\right)^2} - \frac{b\tan(}{32a^3}$$

$$= \frac{3\sqrt{b}\left(20a-34\sqrt{a}\sqrt{b}+15b\right)\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{64a^{13/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d} - \frac{3\sqrt{b}\left(20a+34\sqrt{a}\sqrt{b}+\right.}{64a^{13/4}\left(\right.}$$

Mathematica [A] time = 5.03, size = 357, normalized size = 1.00

$$-\frac{\frac{4b\sin(2(c+dx))\left(28a^2+b(13b-19a)\cos(2(c+dx))+3ab-13b^2\right)}{(a-b)^2(8a+4b\cos(2(c+dx))-b\cos(4(c+dx))-3b)} + \frac{3\sqrt{b}\left(34\sqrt{a}\sqrt{b}+20a+15b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{128ab\sin(2(c+dx))(2a-}{(a-b)(-8a-4b\cos(2(c+dx)}}{64a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/(a - b*Sin[c + d*x]^4)^3,x]
```

```
[Out] -1/64*((3*Sqrt[b]*(20*a + 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcTan[((Sqrt[a] + Sqr
t[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/((Sqrt[a] + Sqrt[b])^2*Sqrt
[a + Sqrt[a]*Sqrt[b]]) + (3*Sqrt[b]*(20*a - 34*Sqrt[a]*Sqrt[b] + 15*b)*ArcT
anh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/((Sqrt[
a] - Sqrt[b])^2*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 64*Cot[c + d*x] + (4*b*(28*a^
2 + 3*a*b - 13*b^2 + b*(-19*a + 13*b)*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/(
(a - b)^2*(8*a - 3*b + 4*b*Cos[2*(c + d*x)] - b*Cos[4*(c + d*x)])) + (128*a
*b*(2*a + b - b*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])/((a - b)*(-8*a + 3*b -
4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^2))/(a^3*d)
```

fricas [B] time = 3.64, size = 6323, normalized size = 17.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="fricas")

[Out] -1/256*(8*(32*a^2*b^2 - 83*a*b^3 + 45*b^4)*cos(d*x + c)^9 - 48*(19*a^2*b^2 - 54*a*b^3 + 30*b^4)*cos(d*x + c)^7 - 8*(64*a^3*b - 301*a^2*b^2 + 555*a*b^3 - 270*b^4)*cos(d*x + c)^5 + 16*(55*a^3*b - 188*a^2*b^2 + 235*a*b^3 - 90*b^4)*cos(d*x + c)^3 + 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(1728000*a^6*b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 13573305*a^3*b^5 + 31519503/4*a^2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 + 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12800*a^10*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*(400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin(d*x + c) - 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(1728000*a^6*b^2 - 7369920*a^5*b^3 + 13507020*a^4*b^4 - 13573305*a^3*b^5 + 31519503/4*a^2*b^6 - 5011875/2*a*b^7 + 1366875/4*b^8 - 27/4*(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 - 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^

```
6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 5
0625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 -
 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a
^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (12800*a^10*b - 54080*a^9*b^2 +
 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b
^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2
*b^3 - 530*a*b^4 + 105*b^5 - (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5
*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^
6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b
^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^2
0*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15
*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*
a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*(400*a^14 - 2556*a^13*b + 70
05*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6
- 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 - 2556*a^13*b + 7005*a^12*b^2
 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b
^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120
*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b
^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^1
9*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*
b^9 + a^13*b^10)*d^4)))*sin(d*x + c) + 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d
*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a
^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a
^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2
 - 4*a^4*b^3 + a^3*b^4)*d)*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 -
 530*a*b^4 + 105*b^5 + (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b
^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5
- 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 4
92300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3
+ 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 -
 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^
3 + 5*a^7*b^4 - a^6*b^5)*d^2))*log(-1728000*a^6*b^2 + 7369920*a^5*b^3 - 135
07020*a^4*b^4 + 13573305*a^3*b^5 - 31519503/4*a^2*b^6 + 5011875/2*a*b^7 - 1
366875/4*b^8 + 27/4*(256000*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2
010860*a^3*b^5 + 1167389*a^2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2
 + 27/2*((26*a^17 - 167*a^16*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4
 - 361*a^12*b^5 + 112*a^11*b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 23
55200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 53899
80*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22
*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^
6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c
)*sin(d*x + c) - (12800*a^10*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*
b^4 + 56973*a^6*b^5 - 18109*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5
+ (a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqr
t((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8
661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*
b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*
a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b
^10)*d^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b
^5)*d^2)) - 27/4*(2*(400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^
3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x
+ c)^2 - (400*a^14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^
10*b^4 - 5430*a^9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b
^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7
- 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 -
10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*
a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin
(d*x + c) - 3*((a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^
```

```
2 - 2*a^4*b^3 + a^3*b^4)*d*cos(d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^
3 - 3*a^3*b^4)*d*cos(d*x + c)^4 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^
4)*d*cos(d*x + c)^2 + (a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*
sqrt(-(400*a^4*b - 1044*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 + (a^1
1 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409
600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145
*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/
((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b
^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d
^4)))/((a^11 - 5*a^10*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^
2))*log(-1728000*a^6*b^2 + 7369920*a^5*b^3 - 13507020*a^4*b^4 + 13573305*a^
3*b^5 - 31519503/4*a^2*b^6 + 5011875/2*a*b^7 - 1366875/4*b^8 + 27/4*(256000
*a^6*b^2 - 1091840*a^5*b^3 + 2001040*a^4*b^4 - 2010860*a^3*b^5 + 1167389*a^
2*b^6 - 371250*a*b^7 + 50625*b^8)*cos(d*x + c)^2 - 27/2*((26*a^17 - 167*a^1
6*b + 460*a^15*b^2 - 705*a^14*b^3 + 650*a^13*b^4 - 361*a^12*b^5 + 112*a^11*
b^6 - 15*a^10*b^7)*d^3*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054400*a^6
*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086*a^2*b^
9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 120*a^20
*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 45*a^15*
b^8 - 10*a^14*b^9 + a^13*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (12800*a^1
0*b - 54080*a^9*b^2 + 98420*a^8*b^3 - 98415*a^7*b^4 + 56973*a^6*b^5 - 18109
*a^5*b^6 + 2475*a^4*b^7)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(400*a^4*b - 10
44*a^3*b^2 + 1085*a^2*b^3 - 530*a*b^4 + 105*b^5 + (a^11 - 5*a^10*b + 10*a^9
*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2*sqrt((409600*a^8*b^3 - 2355200
*a^7*b^4 + 6054400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^
3*b^8 + 2135086*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b +
45*a^21*b^2 - 120*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 1
20*a^16*b^7 + 45*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))/((a^11 - 5*a^10
*b + 10*a^9*b^2 - 10*a^8*b^3 + 5*a^7*b^4 - a^6*b^5)*d^2)) - 27/4*(2*(400*a^
14 - 2556*a^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^
9*b^5 + 1681*a^8*b^6 - 225*a^7*b^7)*d^2*cos(d*x + c)^2 - (400*a^14 - 2556*a
^13*b + 7005*a^12*b^2 - 10685*a^11*b^3 + 9810*a^10*b^4 - 5430*a^9*b^5 + 168
1*a^8*b^6 - 225*a^7*b^7)*d^2)*sqrt((409600*a^8*b^3 - 2355200*a^7*b^4 + 6054
400*a^6*b^5 - 9073120*a^5*b^6 + 8661145*a^4*b^7 - 5389980*a^3*b^8 + 2135086
*a^2*b^9 - 492300*a*b^10 + 50625*b^11)/((a^23 - 10*a^22*b + 45*a^21*b^2 - 1
20*a^20*b^3 + 210*a^19*b^4 - 252*a^18*b^5 + 210*a^17*b^6 - 120*a^16*b^7 + 4
5*a^15*b^8 - 10*a^14*b^9 + a^13*b^10)*d^4)))*sin(d*x + c) + 8*(32*a^4 - 110
*a^3*b + 189*a^2*b^2 - 156*a*b^3 + 45*b^4)*cos(d*x + c))/(((a^5*b^2 - 2*a^4
*b^3 + a^3*b^4)*d*cos(d*x + c)^8 - 4*(a^5*b^2 - 2*a^4*b^3 + a^3*b^4)*d*cos(
d*x + c)^6 - 2*(a^6*b - 5*a^5*b^2 + 7*a^4*b^3 - 3*a^3*b^4)*d*cos(d*x + c)^4
 + 4*(a^6*b - 3*a^5*b^2 + 3*a^4*b^3 - a^3*b^4)*d*cos(d*x + c)^2 + (a^7 - 4*
a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d)*sin(d*x + c))
```

giac [B] time = 2.27, size = 2203, normalized size = 6.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="giac")
```

```
[Out] -1/64*(3*((78*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b - 267*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 241*sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 53*sqrt(a^2 - a*b + sqrt(a*b)*(a - b)
)*sqrt(a*b)*a*b^4 - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5)*(
a^5 - 2*a^4*b + a^3*b^2)^2*abs(-a + b) - 2*(9*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^10*b - 51*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 + 108*sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*a^8*b^3 - 106*sqrt(a^2 - a*b + sqrt(a*b)*(a -
 b))*a^7*b^4 + 45*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 - 3*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*a^5*b^6 - 2*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*
a^4*b^7)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(-a + b) - (60*sqrt(a^2 - a*b + sq
```

```
rt(a*b)*(a - b))*sqrt(a*b)*a^15 - 441*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*s
qrt(a*b)*a^14*b + 1339*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^13*b
^2 - 2185*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3 + 2059*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 1091*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 265*sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*sqrt(a*b)*a^9*b^6 + 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a
^8*b^7 - 11*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^8)*abs(-a +
 b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^6 - 2*a^5*
b + a^4*b^2 + sqrt((a^6 - 2*a^5*b + a^4*b^2)^2 - (a^6 - 2*a^5*b + a^4*b^2)*
(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)))/(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^
3))))/((3*a^16 - 27*a^15*b + 104*a^14*b^2 - 224*a^13*b^3 + 294*a^12*b^4 - 2
38*a^11*b^5 + 112*a^10*b^6 - 24*a^9*b^7 - a^8*b^8 + a^7*b^9)*abs(a^5 - 2*a^
4*b + a^3*b^2)) - 3*((78*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*
b - 267*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 241*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 53*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^4 - 15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*b^5)*(a^5 - 2*a^4*b + a^3*b^2)^2*abs(-a + b) + 2*(9*sqrt(a^2 - a*b - s
qrt(a*b)*(a - b))*a^10*b - 51*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^9*b^2 +
 108*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^8*b^3 - 106*sqrt(a^2 - a*b - sqr
t(a*b)*(a - b))*a^7*b^4 + 45*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^5 -
3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^6 - 2*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*a^4*b^7)*abs(a^5 - 2*a^4*b + a^3*b^2)*abs(-a + b) - (60*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^15 - 441*sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*a^14*b + 1339*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(
a*b)*a^13*b^2 - 2185*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3
 + 2059*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 1091*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 265*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*sqrt(a*b)*a^9*b^6 + 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
sqrt(a*b)*a^8*b^7 - 11*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^
8)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a
^6 - 2*a^5*b + a^4*b^2 - sqrt((a^6 - 2*a^5*b + a^4*b^2)^2 - (a^6 - 2*a^5*b
+ a^4*b^2)*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)))/(a^6 - 3*a^5*b + 3*a^4*b
^2 - a^3*b^3))))/((3*a^16 - 27*a^15*b + 104*a^14*b^2 - 224*a^13*b^3 + 294*a
^12*b^4 - 238*a^11*b^5 + 112*a^10*b^6 - 24*a^9*b^7 - a^8*b^8 + a^7*b^9)*abs
(a^5 - 2*a^4*b + a^3*b^2)) + 2*(18*a^3*b*tan(d*x + c)^7 - 3*a^2*b^2*tan(d*x
 + c)^7 - 28*a*b^3*tan(d*x + c)^7 + 13*b^4*tan(d*x + c)^7 + 54*a^3*b*tan(d*
x + c)^5 - 4*a^2*b^2*tan(d*x + c)^5 - 26*a*b^3*tan(d*x + c)^5 + 54*a^3*b*ta
n(d*x + c)^3 - 13*a^2*b^2*tan(d*x + c)^3 - 17*a*b^3*tan(d*x + c)^3 + 18*a^3
*b*tan(d*x + c) - 12*a^2*b^2*tan(d*x + c))/((a^5 - 2*a^4*b + a^3*b^2)*(a*ta
n(d*x + c)^4 - b*tan(d*x + c)^4 + 2*a*tan(d*x + c)^2 + a)^2) + 64/(a^3*tan(
d*x + c)))/d
```

maple [B] time = 0.61, size = 1959, normalized size = 5.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x)
```

```
[Out] -57/32/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^3+15/16/d*b/(a^2
-2*a*b+b^2)*a/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*
tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-15/16/d*b/(a^2-2*a*b+b^2)*a/(a*b)
^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b
)^(1/2)-a)*(a-b))^(1/2))+57/32/d/a/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b
)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/
2))*b^3+141/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-
b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+33/64/d/
a^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh
((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*b^4-33/64/d/a^2/(a^2-2*a*
```

```
b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x
+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^4-9/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*
b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)*b-9/16/d/(tan(d*x+c)^4*a
-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)/a*b*tan(d*x+c)^7+13/32/d/(tan(d
*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a/(a^2-2*a*b+b^2)*tan(d*x+c)
^3*b^2-27/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a^2-
2*a*b+b^2)*tan(d*x+c)^3-189/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-
a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-18
9/64/d/a*b^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+39/16/d/a^2/(a^2-2*a*b+b^2)/(a
-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a
)*(a-b))^(1/2))*b^3+39/16/d/a^2/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^3-1/d/a^3
/tan(d*x+c)-141/64/d*b^2/(a^2-2*a*b+b^2)/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)
*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-45/64/
d*b^4/a^3/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b
)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-45/64/d*b^4/a^3/(a^2-2*a*b+b^2)
/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+
a)*(a-b))^(1/2))+1/8/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2
/a/(a^2-2*a*b+b^2)*tan(d*x+c)^5*b^2+17/32/d*b^3/a^2/(tan(d*x+c)^4*a-tan(d*x
+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a^2-2*a*b+b^2)*tan(d*x+c)^3+13/32/d*b^3/a^3/
(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/(a-b)*tan(d*x+c)^7-15/
32/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2/a^2/(a-b)*tan(d*x
+c)^7*b^2-27/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b/(a
^2-2*a*b+b^2)*tan(d*x+c)^5+13/16/d/(tan(d*x+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d
*x+c)^2+a)^2/a^2*b^3/(a^2-2*a*b+b^2)*tan(d*x+c)^5+39/32/d*b/(a^2-2*a*b+b^2)
/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))+39/32/d*b/(a^2-2*a*b+b^2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^
(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+3/8/d/(tan(d*x
+c)^4*a-tan(d*x+c)^4*b+2*a*tan(d*x+c)^2+a)^2*b^2/a/(a^2-2*a*b+b^2)*tan(d*x+
c)
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a-b*sin(d*x+c)^4)^3,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 20.80, size = 7364, normalized size = 20.63

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a - b*sin(c + d*x)^4)^3),x)
```

```
[Out] (atan((((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^1
1*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b
^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(
1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a
^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^14*b^13 - 12079595520*
a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b^9 + 114152177664*a^1
9*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 - 45298483200*a^22*b^5
 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989888*a^25*b^2 + tan(c
+ d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11
*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^
2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1
/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^
```

16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^1
8*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^
21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^
24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27
*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420
800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848
*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^
17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 -
3779592192*a^21*b^3 + 471859200*a^22*b^2))*(-(9*(640*a^4*(a^13*b^3)^(1/2)
+ 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*
a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*
b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b
^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*1i + ((-(9*(640*a^4*(a
^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*
a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 10
94*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b
- a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(201326
592*a^14*b^13 - 2315255808*a^15*b^12 + 12079595520*a^16*b^11 - 37748736000*
a^17*b^10 + 78517370880*a^18*b^9 - 114152177664*a^19*b^8 + 118380036096*a^2
0*b^7 - 87577067520*a^21*b^6 + 45298483200*a^22*b^5 - 15602810880*a^23*b^4
+ 3221225472*a^24*b^3 - 301989888*a^25*b^2 + tan(c + d*x)*(-(9*(640*a^4*(a^
13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a
^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 109
4*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b -
a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483
648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^
19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a
^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a
^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*
b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b^14 - 15574892
544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*a^14*b^10 + 12550511001
6*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^17*b^7 + 484835328*a^18*
b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 - 3779592192*a^21*b^3 + 4
71859200*a^22*b^2))*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/
2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2
+ 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*
(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^1
5*b^3 - 10*a^16*b^2)))^(1/2)*1i)/(((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4
*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 +
1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2
) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^
14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 2013265
92*a^14*b^13 - 12079595520*a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*
a^18*b^9 + 114152177664*a^19*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21
*b^6 - 45298483200*a^22*b^5 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 +
301989888*a^25*b^2 + tan(c + d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*
(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 +
1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2)
- 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^1
4*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*
a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a
^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*
a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a
^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(30
24617472*a^11*b^13 - 265420800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940
032*a^13*b^11 - 94402510848*a^14*b^10 + 125505110016*a^15*b^9 - 10842144768
0*a^16*b^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b
^5 + 12354453504*a^20*b^4 - 3779592192*a^21*b^3 + 471859200*a^22*b^2))*(-(9
*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^

7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3
)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(1638
4*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(
1/2) - ((-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^1
1*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b
^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(
1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a
^16*b^2)))^(1/2)*(201326592*a^14*b^13 - 2315255808*a^15*b^12 + 12079595520*
a^16*b^11 - 37748736000*a^17*b^10 + 78517370880*a^18*b^9 - 114152177664*a^1
9*b^8 + 118380036096*a^20*b^7 - 87577067520*a^21*b^6 + 45298483200*a^22*b^5
 - 15602810880*a^23*b^4 + 3221225472*a^24*b^3 - 301989888*a^25*b^2 + tan(c
+ d*x)*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) - 400*a^11
*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^
2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1
/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^
16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^1
8*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^
21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^
24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27
*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420
800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848
*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^
17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 -
 3779592192*a^21*b^3 + 471859200*a^22*b^2))*(-(9*(640*a^4*(a^13*b^3)^(1/2)
+ 225*b^4*(a^13*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*
a^9*b^3 + 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*
b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b
^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2) + 186624000*a^7*b^14 -
 2227875840*a^8*b^13 + 12162465792*a^9*b^12 - 40050892800*a^10*b^11 + 88332
816384*a^11*b^10 - 136918204416*a^12*b^9 + 152103813120*a^13*b^8 - 12103476
0192*a^14*b^7 + 67571435520*a^15*b^6 - 25193631744*a^16*b^5 + 5643288576*a^
17*b^4 - 575078400*a^18*b^3))*(-(9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^1
3*b^3)^(1/2) - 400*a^11*b - 105*a^7*b^5 + 530*a^8*b^4 - 1085*a^9*b^3 + 1044
*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1
840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^
4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*2i)/d - (1/a + (tan(c + d*x)^2*(64*a
^2 - 119*a*b + 58*b^2))/(16*a*(a^2 - 2*a*b + b^2)) + (tan(c + d*x)^8*(111*a
*b^2 - 78*a^2*b + 32*a^3 - 45*b^3))/(32*a^3*(a - b)) + (tan(c + d*x)^6*(190
*a*b^2 - 165*a^2*b + 64*a^3 - 77*b^3))/(16*a^2*(a - b)^2) + (tan(c + d*x)^4
*(307*a*b^2 - 394*a^2*b + 192*a^3 - 81*b^3))/(32*a^2*(a^2 - 2*a*b + b^2)))/
(d*(tan(c + d*x)^9*(a^2 - 2*a*b + b^2) + a^2*tan(c + d*x) - tan(c + d*x)^5*
(2*a*b - 6*a^2) - tan(c + d*x)^7*(4*a*b - 4*a^2) + 4*a^2*tan(c + d*x)^3)) +
 (atan(((((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^1
1*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b
^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(
1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a
^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^14*b^13 - 12079595520*
a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b^9 + 114152177664*a^1
9*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 - 45298483200*a^22*b^5
 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989888*a^25*b^2 + tan(c
+ d*x)*((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*
b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2
*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/
2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^1
6*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18
*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^2
1*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^2
4*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*
b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 2654208

00*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*
a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^1
7*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 -
3779592192*a^21*b^3 + 471859200*a^22*b^2))*((9*(640*a^4*(a^13*b^3)^(1/2) +
225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^
9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^
3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5
 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*1i + (((9*(640*a^4*(a^13
*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8
*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*
a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a
^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(201326592
*a^14*b^13 - 2315255808*a^15*b^12 + 12079595520*a^16*b^11 - 37748736000*a^1
7*b^10 + 78517370880*a^18*b^9 - 114152177664*a^19*b^8 + 118380036096*a^20*b
^7 - 87577067520*a^21*b^6 + 45298483200*a^22*b^5 - 15602810880*a^23*b^4 + 3
221225472*a^24*b^3 - 301989888*a^25*b^2 + tan(c + d*x)*((9*(640*a^4*(a^13*b
^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b
^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*
b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^1
8 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*
a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 141733920768*a^19*b
^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1700807049216*a^22*
b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*
b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)
) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b^14 - 15574892544*
a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*a^14*b^10 + 125505110016*a^
15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6
 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 - 3779592192*a^21*b^3 + 47185
9200*a^22*b^2))*((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) +
400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 208
5*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13
b^3)^(1/2)))/(16384(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3
 - 10*a^16*b^2)))^(1/2)*1i)/((((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13
*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*
a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 18
40*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4
 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2315255808*a^15*b^12 - 201326592*a^1
4*b^13 - 12079595520*a^16*b^11 + 37748736000*a^17*b^10 - 78517370880*a^18*b
^9 + 114152177664*a^19*b^8 - 118380036096*a^20*b^7 + 87577067520*a^21*b^6 -
 45298483200*a^22*b^5 + 15602810880*a^23*b^4 - 3221225472*a^24*b^3 + 301989
888*a^25*b^2 + tan(c + d*x)*((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b
^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^
10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840
*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 +
 10*a^15*b^3 - 10*a^16*b^2)))^(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^
13 - 25769803776*a^18*b^12 + 141733920768*a^19*b^11 - 472446402560*a^20*b^1
0 + 1063004405760*a^21*b^9 - 1700807049216*a^22*b^8 + 1984274890752*a^23*b^
7 - 1700807049216*a^24*b^6 + 1063004405760*a^25*b^5 - 472446402560*a^26*b^4
 + 141733920768*a^27*b^3 - 25769803776*a^28*b^2)) + tan(c + d*x)*(302461747
2*a^11*b^13 - 265420800*a^10*b^14 - 15574892544*a^12*b^12 + 47520940032*a^1
3*b^11 - 94402510848*a^14*b^10 + 125505110016*a^15*b^9 - 108421447680*a^16*
b^8 + 51536461824*a^17*b^7 + 484835328*a^18*b^6 - 18454413312*a^19*b^5 + 12
354453504*a^20*b^4 - 3779592192*a^21*b^3 + 471859200*a^22*b^2))*((9*(640*a^
4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 -
530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2)
- 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^1
7*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2) - (
((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105
*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*

```
b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(1
6384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2))
)^(1/2)*(201326592*a^14*b^13 - 2315255808*a^15*b^12 + 12079595520*a^16*b^11
 - 37748736000*a^17*b^10 + 78517370880*a^18*b^9 - 114152177664*a^19*b^8 + 1
18380036096*a^20*b^7 - 87577067520*a^21*b^6 + 45298483200*a^22*b^5 - 156028
10880*a^23*b^4 + 3221225472*a^24*b^3 - 301989888*a^25*b^2 + tan(c + d*x)*((
9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2) + 400*a^11*b + 105*a
^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^
3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a^13*b^3)^(1/2)))/(163
84*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^
(1/2)*(2147483648*a^29*b + 2147483648*a^17*b^13 - 25769803776*a^18*b^12 + 1
41733920768*a^19*b^11 - 472446402560*a^20*b^10 + 1063004405760*a^21*b^9 - 1
700807049216*a^22*b^8 + 1984274890752*a^23*b^7 - 1700807049216*a^24*b^6 + 1
063004405760*a^25*b^5 - 472446402560*a^26*b^4 + 141733920768*a^27*b^3 - 257
69803776*a^28*b^2)) + tan(c + d*x)*(3024617472*a^11*b^13 - 265420800*a^10*b
^14 - 15574892544*a^12*b^12 + 47520940032*a^13*b^11 - 94402510848*a^14*b^10
 + 125505110016*a^15*b^9 - 108421447680*a^16*b^8 + 51536461824*a^17*b^7 + 4
84835328*a^18*b^6 - 18454413312*a^19*b^5 + 12354453504*a^20*b^4 - 377959219
2*a^21*b^3 + 471859200*a^22*b^2))*((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(
a^13*b^3)^(1/2) + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1
044*a^10*b^2 + 2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2)
- 1840*a^3*b*(a^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14
*b^4 + 10*a^15*b^3 - 10*a^16*b^2)))^(1/2) + 186624000*a^7*b^14 - 2227875840
*a^8*b^13 + 12162465792*a^9*b^12 - 40050892800*a^10*b^11 + 88332816384*a^11
*b^10 - 136918204416*a^12*b^9 + 152103813120*a^13*b^8 - 121034760192*a^14*b
^7 + 67571435520*a^15*b^6 - 25193631744*a^16*b^5 + 5643288576*a^17*b^4 - 57
5078400*a^18*b^3))*((9*(640*a^4*(a^13*b^3)^(1/2) + 225*b^4*(a^13*b^3)^(1/2)
 + 400*a^11*b + 105*a^7*b^5 - 530*a^8*b^4 + 1085*a^9*b^3 - 1044*a^10*b^2 +
2085*a^2*b^2*(a^13*b^3)^(1/2) - 1094*a*b^3*(a^13*b^3)^(1/2) - 1840*a^3*b*(a
^13*b^3)^(1/2)))/(16384*(5*a^17*b - a^18 + a^13*b^5 - 5*a^14*b^4 + 10*a^15*
b^3 - 10*a^16*b^2)))^(1/2)*2i)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a-b*sin(d*x+c)**4)**3,x)
```

```
[Out] Timed out
```

3.236 $\int \frac{1}{1-\sin^4(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\sqrt{2}\tan(x)\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}$$

```
[Out] 1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

Rubi [A] time = 0.02, antiderivative size = 45, normalized size of antiderivative = 1.80, number of steps used = 3, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3209, 388, 203}

$$\frac{x}{2\sqrt{2}}+\frac{\tan(x)}{2}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{2\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(2*Sqrt[2]
) + Tan[x]/2
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1-\sin^4(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{2}+\frac{1}{2}\text{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)\\ &= \frac{x}{2\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{2\sqrt{2}}+\frac{\tan(x)}{2}\end{aligned}$$

Mathematica [A] time = 0.05, size = 24, normalized size = 0.96

$$\frac{1}{4}\left(\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)+2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^4)^(-1),x]
```

```
[Out] (Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/4
```

fricas [B] time = 0.45, size = 43, normalized size = 1.72

$$-\frac{\sqrt{2}\arctan\left(\frac{3\sqrt{2}\cos(x)^2-2\sqrt{2}}{4\cos(x)\sin(x)}\right)\cos(x)-4\sin(x)}{8\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*(sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*sin(x)))*
cos(x) - 4*sin(x))/cos(x)
```

giac [B] time = 0.12, size = 51, normalized size = 2.04

$$\frac{1}{4}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)+\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - 2*cos(2*x) + 2))) + 1/2*tan(x)
```

maple [A] time = 0.17, size = 18, normalized size = 0.72

$$\frac{\arctan\left(\sqrt{2}\,\tan(x)\right)\sqrt{2}}{4}+\frac{\tan(x)}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^4),x)
```

```
[Out] 1/4*arctan(2^(1/2)*tan(x))*2^(1/2)+1/2*tan(x)
```

maxima [A] time = 0.44, size = 17, normalized size = 0.68

$$\frac{1}{4}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+\frac{1}{2}\tan(x)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^4),x, algorithm="maxima")
```

```
[Out] 1/4*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/2*tan(x)
```

mupad [B] time = 14.34, size = 17, normalized size = 0.68

$$\frac{\tan(x)}{2}+\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^4 - 1),x)
```

```
[Out] tan(x)/2 + (2^(1/2)*atan(2^(1/2)*tan(x)))/4
```

sympy [B] time = 84.42, size = 724, normalized size = 28.96

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**4),x)
```

```
[Out] 54608393*sqrt(2)*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) +
pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 9047
8148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 77227930*sqrt(3 - 2*sqrt(
2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))*tan(x/
2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 639
77712*sqrt(2)) - 77227930*sqrt(3 - 2*sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqr
t(2))) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 9047814
8*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 54608393*sqrt(2)*sqrt(3 - 2*
sqrt(2))*(atan(tan(x/2)/sqrt(3 - 2*sqrt(2))) + pi*floor((x/2 - pi/2)/pi))/(
63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*s
qrt(2)) + 9369319*sqrt(2)*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2)
 + 3)) + pi*floor((x/2 - pi/2)/pi))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)*
*2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) + 13250218*sqrt(2*
sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi
))*tan(x/2)**2/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478
148 - 63977712*sqrt(2)) - 13250218*sqrt(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(
2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2)/pi))/(63977712*sqrt(2)*tan(x/2)**2
+ 90478148*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 9369319*sqrt(2)*sqr
t(2*sqrt(2) + 3)*(atan(tan(x/2)/sqrt(2*sqrt(2) + 3)) + pi*floor((x/2 - pi/2
)/pi))/(63977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63
977712*sqrt(2)) - 90478148*tan(x/2)/(63977712*sqrt(2)*tan(x/2)**2 + 9047814
8*tan(x/2)**2 - 90478148 - 63977712*sqrt(2)) - 63977712*sqrt(2)*tan(x/2)/(6
3977712*sqrt(2)*tan(x/2)**2 + 90478148*tan(x/2)**2 - 90478148 - 63977712*sq
rt(2))
```

3.237 $\int \frac{1}{a+b\sin^4(x)}\,dx$

Optimal. Leaf size=487

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}(a+b)^{3/4}\tan(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{\cdots}}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

```
[Out] 1/8*ln((a+b)^(1/4)*a^(1/2)-a^(1/4)*2^(1/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)*
tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/2)-(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1
/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-1/8*ln((a+b)^(1/4)*a^(1/2)+a^(1/4)*2^(1
/2)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)*tan(x)+(a+b)^(3/4)*tan(x)^2)*(a^(1/2)-(
a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-1/4
*arctan((a^(1/4)*(a+b-a^(1/2)*(a+b)^(1/2))^(1/2)-(a+b)^(3/4)*2^(1/2)*tan(x)
)/a^(1/4)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a
+b)^(1/4)*2^(1/2)/(a+b+a^(1/2)*(a+b)^(1/2))^(1/2)+1/4*arctan((a^(1/4)*(a+b-
a^(1/2)*(a+b)^(1/2))^(1/2)+(a+b)^(3/4)*2^(1/2)*tan(x))/a^(1/4)/(a+b+a^(1/2)
*(a+b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))/a^(3/4)/(a+b)^(1/4)*2^(1/2)/(a+b
+a^(1/2)*(a+b)^(1/2))^(1/2)
```

Rubi [A] time = 1.14, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3209, 1169, 634, 618, 204, 628}

$$-\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{a+b}+a+b}-\sqrt{2}(a+b)^{3/4}\tan(x)}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right)}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}+\frac{\left(\sqrt{a+b}+\sqrt{a}\right)\tan^{-1}\left(\frac{\sqrt{2}(a+b)^{3/4}\tan(x)+\sqrt[4]{a}\sqrt{-\sqrt{a}\sqrt{\cdots}}}{\sqrt[4]{a}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}\right.}{2\sqrt{2}a^{3/4}\sqrt[4]{a+b}\sqrt{\sqrt{a}\sqrt{a+b}+a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^4)^(-1),x]
```

```
[Out] -((Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]
 - Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]
)])/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + (
(Sqrt[a] + Sqrt[a + b])*ArcTan[(a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]] +
 Sqrt[2]*(a + b)^(3/4)*Tan[x])/(a^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]])]
)/(2*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*Sqrt[a + b + Sqrt[a]*Sqrt[a + b]]) + ((S
qrt[a] - Sqrt[a + b])*Log[Sqrt[a]*(a + b)^(1/4) - Sqrt[2]*a^(1/4)*Sqrt[a +
b - Sqrt[a]*Sqrt[a + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/
4)*(a + b)^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a + b]]) - ((Sqrt[a] - Sqrt[a +
b])*Log[Sqrt[a]*(a + b)^(1/4) + Sqrt[2]*a^(1/4)*Sqrt[a + b - Sqrt[a]*Sqrt[a
 + b]]*Tan[x] + (a + b)^(3/4)*Tan[x]^2])/(4*Sqrt[2]*a^(3/4)*(a + b)^(1/4)*S
qrt[a + b - Sqrt[a]*Sqrt[a + b]])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\int \frac{1}{a+b\sin^4(x)}\,dx = \text{Subst}\left(\int \frac{1+x^2}{a+2ax^2+(a+b)x^4}\,dx, x, \tan(x)\right)$$

$$= \frac{\sqrt[4]{a+b}\;\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)x}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}} + \frac{\sqrt[4]{a+b}\;\text{Subst}\left(\int \frac{\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{\ }}}{(a+b)^{3/4}}\cdots}{\frac{\sqrt{a}}{\sqrt{a+b}}+\frac{\sqrt{2}\sqrt[4]{a}\cdots}{}}\cdots\right)}{2\sqrt{2}\,a^{3/4}\sqrt{a+\cdots}}$$

$$= \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\frac{\sqrt{a}}{\sqrt{a+b}}-\frac{\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,x}{(a+b)^{3/4}}+x^2}\,dx, x, \tan(x)\right)}{4(a+b)} + \frac{\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{\cdots}{\frac{\sqrt{a}}{\sqrt{a+b}}\cdots}\cdots\right)}{\cdots}$$

$$= -\frac{\sqrt[4]{a+b}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\log\left(\sqrt{a}\sqrt[4]{a+b}-\sqrt{2}\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}\,\tan(x)+(a+b)^{3/4}\tan\cdots\right)}{4\sqrt{2}\,a^{3/4}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}\cdots$$

$$= -\frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\frac{\sqrt[4]{a}\sqrt{a+b-\sqrt{a}\sqrt{a+b}}}{(a+b)^{3/4}}-\sqrt{2}\tan(x)\right)}{\sqrt[4]{a}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+b+\sqrt{a}\sqrt{a+b}}} + \frac{\left(\sqrt{a}+\sqrt{a+b}\right)\tan^{-1}\left(\frac{(a+b)^{3/4}\left(\cdots\right)}{\cdots}\right)}{2\sqrt{2}\,a^{3/4}\sqrt[4]{a+b}\sqrt{a+\cdots}}$$

Mathematica [C] time = 0.31, size = 148, normalized size = 0.30

$$\frac{\left(\sqrt{a}-i\sqrt{b}\right)\sqrt{a+i\sqrt{a}\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt{a+i\sqrt{a}\sqrt{b}}\,\tan(x)}{\sqrt{a}}\right)-\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{-a+i\sqrt{a}\sqrt{b}}\,\tanh^{-1}\left(\frac{\sqrt{-a+i\sqrt{a}\sqrt{b}}\,\tan(x)}{\sqrt{a}}\right)}{2a(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^4)^(-1),x]
```

```
[Out] ((Sqrt[a] - I*Sqrt[b])*Sqrt[a + I*Sqrt[a]*Sqrt[b]]*ArcTan[(Sqrt[a + I*Sqrt[
a]*Sqrt[b]]*Tan[x])/Sqrt[a]] - (Sqrt[a] + I*Sqrt[b])*Sqrt[-a + I*Sqrt[a]*Sq
rt[b]]*ArcTanh[(Sqrt[-a + I*Sqrt[a]*Sqrt[b]]*Tan[x])/Sqrt[a]])/(2*a*(a + b)
)
```

fricas [B] time = 0.57, size = 823, normalized size = 1.69

$$-\frac{1}{8}\sqrt{-\frac{\left(a^2+ab\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}+1}{a^2+ab}}\,\log\left(\frac{1}{4}\,b\cos(x)^2+\frac{1}{2}\left(ab\cos(x)\sin(x)+\left(a^4+a^3b\right)\sqrt{-\frac{b}{a^5+2\,a^4b+a^3b^2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b)
)*log(1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5
+ 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^
4*b + a^3*b^2)) + 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(
x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(-((a^2 + a*b)*
sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^2 + a*b))*log(1/4*b*cos(x)^2 - 1
/2*(a*b*cos(x)*sin(x) + (a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*co
s(x)*sin(x))*sqrt(-((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1)/(a^
2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1/4*b) + 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b +
a^3*b^2)) - 1)/(a^2 + a*b))*log(-1/4*b*cos(x)^2 + 1/2*(a*b*cos(x)*sin(x) -
(a^4 + a^3*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2
+ a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^
2*b - 2*(a^3 + a^2*b)*cos(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
 - 1/8*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) - 1)/(a^2 + a*b
))*log(-1/4*b*cos(x)^2 - 1/2*(a*b*cos(x)*sin(x) - (a^4 + a^3*b)*sqrt(-b/(a^
5 + 2*a^4*b + a^3*b^2))*cos(x)*sin(x))*sqrt(((a^2 + a*b)*sqrt(-b/(a^5 + 2*a
^4*b + a^3*b^2)) - 1)/(a^2 + a*b)) - 1/4*(a^3 + a^2*b - 2*(a^3 + a^2*b)*cos
(x)^2)*sqrt(-b/(a^5 + 2*a^4*b + a^3*b^2)) + 1/4*b)
```

giac [A] time = 0.39, size = 318, normalized size = 0.65

$$\frac{\left(3\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,a^2+6\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,ab-\sqrt{a^2+ab+\sqrt{-ab}\,(a+b)}\,b^2\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\text{a}\right.}{2\left(3\,a^5+12\,a^4b+14\,a^3b^2+4\,a^2b^3-ab^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b + sqrt(-
a*b)*(a + b))*a*b - sqrt(a^2 + a*b + sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/p
i + 1/2) + arctan(2*tan(x)/sqrt((4*a + sqrt(-16*(a + b)*a + 16*a^2))/(a + b
```

```
))))*abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4) + 1/2*(
3*sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*a^2 + 6*sqrt(a^2 + a*b - sqrt(-a*b)*
(a + b))*a*b - sqrt(a^2 + a*b - sqrt(-a*b)*(a + b))*b^2)*(pi*floor(x/pi + 1
/2) + arctan(2*tan(x)/sqrt((4*a - sqrt(-16*(a + b)*a + 16*a^2))/(a + b))))*
abs(a + b)/(3*a^5 + 12*a^4*b + 14*a^3*b^2 + 4*a^2*b^3 - a*b^4)
```

maple [B] time = 0.41, size = 1677, normalized size = 3.44

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^4),x)

[Out] 1/8/b/(a+b)^(1/2)*ln(-(a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(
1/2)-a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/8/a/b/(a+b)^(1/2)*ln(-(a+b)^(
1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))*(2*(a^2+a*b)^(1
/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/8/a^(3/2)/b*ln(-(a+b)^(1/2)*tan(x)^2+tan(x
)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)-a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2
+a*b)^(1/2)-1/8/a^(1/2)/b*ln(-(a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2
)-2*a)^(1/2)-a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/a^(1/2)/(4*a^(1/2)*(a
+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*
(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2
))-1/4/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+
b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*
(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)/(a+b)^(1/2)*(2*(a^2+
a*b)^(1/2)-2*a)^(1/2)-1/4/a/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)
^(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1
/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)
/(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/4/a^(3/2)/b/(4
*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((-2*(a+b)^(1/2)*ta
n(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2
)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(
a^2+a*b)^(1/2)+1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^
(1/2)*arctan((-2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/
2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*
(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/8/b/(a+b)^(1/2)*ln((a+b)^(1/2)*tan(x)^2+tan
(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)-1/
8/a/b/(a+b)^(1/2)*ln((a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1
/2)+a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/a^(3/2)/b*ln
((a+b)^(1/2)*tan(x)^2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2))*(2*(a^2
+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)+1/8/a^(1/2)/b*ln((a+b)^(1/2)*tan(x)^
2+tan(x)*(2*(a*(a+b))^(1/2)-2*a)^(1/2)+a^(1/2))*(2*(a^2+a*b)^(1/2)-2*a)^(1/
2)+1/a^(1/2)/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*
(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*
(a*(a+b))^(1/2)+2*a)^(1/2))+1/4/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+
2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a
^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1
/2)/(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)+1/4/a/b/(4*a^(1/2)*(a+b)^(1/2
)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1
/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*
(a+b))^(1/2)-2*a)^(1/2)/(a+b)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)
^(1/2)-1/4/a^(3/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2)*ar
ctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)-2*a)^(1/2))/(4*a^(1/2)*(a+b)^
(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+b))^(1/2)-2*a)^(1/2)*(2*(a^2+a
*b)^(1/2)-2*a)^(1/2)*(a^2+a*b)^(1/2)-1/4/a^(1/2)/b/(4*a^(1/2)*(a+b)^(1/2)-2
*(a*(a+b))^(1/2)+2*a)^(1/2)*arctan((2*(a+b)^(1/2)*tan(x)+(2*(a*(a+b))^(1/2)
-2*a)^(1/2))/(4*a^(1/2)*(a+b)^(1/2)-2*(a*(a+b))^(1/2)+2*a)^(1/2))*(2*(a*(a+
b))^(1/2)-2*a)^(1/2)*(2*(a^2+a*b)^(1/2)-2*a)^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^4 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^4 + a), x)
```

mupad [B] time = 15.18, size = 407, normalized size = 0.84

$$\mathrm{atan}\left(\frac{a^3\tan(x)\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^5\tan(x)\left(-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}\right)^{3/2}64\mathrm{i}-a^2\,b\tan(x)\sqrt{-\frac{a^2-\sqrt{-a^3\,b}}{16\,a^4+16\,b\,a^3}}\,4\mathrm{i}+a^4\,b\tan(x)}{\sqrt{-a^3\,b}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^4),x)
```

```
[Out] atan((a^3*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a
^5*tan(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*t
an(x)*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)
*(-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(
-(a^2 - (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*2i - atan((a^3*tan(x)*(-
(a^2 + (-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^5*tan(x)*(-(a^2 +
(-a^3*b)^(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i - a^2*b*tan(x)*(-(a^2 + (-a^
3*b)^(1/2))/(16*a^3*b + 16*a^4))^(1/2)*4i + a^4*b*tan(x)*(-(a^2 + (-a^3*b)^
(1/2))/(16*a^3*b + 16*a^4))^(3/2)*64i)/(-a^3*b)^(1/2))*(-(a^2 + (-a^3*b)^(1
/2))/(16*a^3*b + 16*a^4))^(1/2)*2i
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**4),x)
```

```
[Out] Timed out
```

3.238 $\int \frac{1}{1+\sin^4(x)}\,dx$

Optimal. Leaf size=309

$$\frac{x}{2\sqrt{\sqrt{2}-1}}-\frac{1}{8}\sqrt{\sqrt{2}-1}\,\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\,\tan(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\,\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right.}\right.$$

```
[Out] 1/2*x/(2^(1/2)-1)^(1/2)+1/4*arctan((-cos(x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1)
^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/(2+cos(x)^2*(-2+2^(1/2))-2*cos(x)*sin(
x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/(2^(1/2)-1)^(1/2)-1/4*arctan((cos(
x)*sin(x)*(-2+2^(1/2))+(2^(1/2)-1)^(1/2)-2*cos(x)^2*(2^(1/2)-1)^(1/2))/(2+c
os(x)^2*(-2+2^(1/2))+2*cos(x)*sin(x)*(2^(1/2)-1)^(1/2)+(1+2^(1/2))^(1/2)))/
(2^(1/2)-1)^(1/2)-1/8*ln(2^(1/2)-2*(2^(1/2)-1)^(1/2)*tan(x)+2*tan(x)^2)*(2^
(1/2)-1)^(1/2)+1/8*ln(1+(-2+2*2^(1/2))^(1/2)*tan(x)+2^(1/2)*tan(x)^2)*(2^(1
/2)-1)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.750, Rules used = {3209, 1169, 634, 618, 204, 628}

$$\frac{x}{2\sqrt{\sqrt{2}-1}}-\frac{1}{8}\sqrt{\sqrt{2}-1}\,\log\left(2\tan^2(x)-2\sqrt{\sqrt{2}-1}\,\tan(x)+\sqrt{2}\right)+\frac{1}{8}\sqrt{\sqrt{2}-1}\,\log\left(\sqrt{2}\tan^2(x)+\sqrt{2\left(\sqrt{2}-1\right.}\right.$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] x/(2*Sqrt[-1 + Sqrt[2]]) + ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]
]*Cos[x]^2 - (-2 + Sqrt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + S
qrt[2])*Cos[x]^2 - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2
]]) - ArcTan[(Sqrt[-1 + Sqrt[2]] - 2*Sqrt[-1 + Sqrt[2]]*Cos[x]^2 + (-2 + Sq
rt[2])*Cos[x]*Sin[x])/(2 + Sqrt[1 + Sqrt[2]] + (-2 + Sqrt[2])*Cos[x]^2 + 2*
Sqrt[-1 + Sqrt[2]]*Cos[x]*Sin[x])]/(4*Sqrt[-1 + Sqrt[2]]) - (Sqrt[-1 + Sqrt
[2]]*Log[Sqrt[2] - 2*Sqrt[-1 + Sqrt[2]]*Tan[x] + 2*Tan[x]^2])/8 + (Sqrt[-1
+ Sqrt[2]]*Log[1 + Sqrt[2*(-1 + Sqrt[2])]*Tan[x] + Sqrt[2]*Tan[x]^2])/8
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1169

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[a/c, 2]}, With[{r = Rt[2*q - b/c, 2]}, Dist[1/(2*c*q*r), Int
[(d*r - (d - e*q)*x)/(q - r*x + x^2), x], x] + Dist[1/(2*c*q*r), Int[(d*r +
 (d - e*q)*x)/(q + r*x + x^2), x], x]]] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NegQ[b^2 - 4*a*c]
```

Rule 3209

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a
 + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*x]/ff], x]] /;
 FreeQ[{a, b, e, f}, x] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{1}{1+\sin^4(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{1+2x^2+2x^4}\,dx, x, \tan(x)\right)\\
&= \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}-\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}} + \frac{\text{Subst}\left(\int \frac{\sqrt{-1+\sqrt{2}}+\left(1-\frac{1}{\sqrt{2}}\right)x}{\frac{1}{\sqrt{2}}+\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \tan(x)\right)}{2\sqrt{2\left(-1+\sqrt{2}\right)}}\\
&= -\left(\frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\left(\int \frac{-\sqrt{-1+\sqrt{2}}+2x}{\frac{1}{\sqrt{2}}-\sqrt{-1+\sqrt{2}}\,x+x^2}\,dx, x, \tan(x)\right)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\text{Subst}\Big(\\
&= -\frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log\left(\sqrt{2}-2\sqrt{-1+\sqrt{2}}\,\tan(x)+2\tan^2(x)\right) + \frac{1}{8}\sqrt{-1+\sqrt{2}}\,\log\Big(1+\sqrt{2\Big(}\\
&= \frac{1}{2}\sqrt{1+\sqrt{2}}\,x + \frac{1}{4}\sqrt{1+\sqrt{2}}\,\tan^{-1}\left(\frac{\sqrt{-1+\sqrt{2}}-2\sqrt{-1+\sqrt{2}}\cos^2(x)+\left(2-\sqrt{2}\right)\cos(x}{2+\sqrt{1+\sqrt{2}}-\left(2-\sqrt{2}\right)\cos^2(x)-2\sqrt{-1+\sqrt{2}}\cos(}\right.
\end{aligned}$$

Mathematica [C] time = 0.07, size = 45, normalized size = 0.15

$$\frac{\tan^{-1}\left(\sqrt{1-i}\tan(x)\right)}{2\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\tan(x)\right)}{2\sqrt{1+i}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^4)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - I]*Tan[x]]/(2*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]/(2*
Sqrt[1 + I])
```

fricas [B] time = 32.59, size = 3830, normalized size = 12.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="fricas")
```

```
[Out] -1/32*2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)
^4 + 2*(2*sqrt(2) - 3)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(
1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) + 1/32*2^(1/4)*s
qrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*log(-(4*sqrt(2) - 5)*cos(x)^4 + 2*(2*sqrt(
2) - 3)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) -
 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) - 1/16*2^(1/4)*sqrt(2*sqrt(2) +
 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16
*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt(2)*(51*
sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2) - 36) -
 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqrt(2) + 1
32)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(x)^6 - 4
*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2) - 15)*co
s(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^
15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(x)^13 +
4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^11 - 8*(
2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2*(2^(3/4
)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/4)*(9*sq
rt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt(2) - 2)
 + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt(2) + 4)
*sin(x) + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqr
t(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) -
62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2
) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6
 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqr
t(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt
(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) -
 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94)
)*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(
x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 -
 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(
sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5
)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)
^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8) + 4)/(
112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^
8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) + 1/16*2^(1/4)*sqrt(2*sq
rt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)
^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt
(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2)
 - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqr
t(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(
x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2)
 - 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))
*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(
x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^
11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2
*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/
4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt
(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt
(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 -
 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sq
rt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 1
0*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*
cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt
(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)
*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*s
qrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(
2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 3
4))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*co
```

```
s(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2
^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqr
t(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4
)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8
) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152
*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) - 1/16*2^(1/4)*s
qrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6
)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 +
16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41
*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90)
- 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 2
16)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*
sqrt(2) - 15)*cos(x)^2 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(
2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) +
4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))
*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(
x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 -
2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/
4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqr
t(2*sqrt(2) + 4)*sin(x) + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos
(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2
)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) -
 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2
) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 +
 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sqrt(2) - 11) - 2
*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1
/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(
73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqr
t(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2)
- 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(
x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-
4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 - 4*(2^(1/4)*(3*sqr
t(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*si
n(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^
10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1)) + 1/16*2
^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt
(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x
)^14 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt
(2)*(41*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2)
 - 90) - 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt
(2) + 216)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4
+ 4*(2*sqrt(2) - 15)*cos(x)^2 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(
3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqr
t(2) + 4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2)
 - 14))*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31
))*cos(x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(
x)^7 - 2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (
2*2^(3/4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(
x))*sqrt(2*sqrt(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) +
 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*
(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sq
rt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32
*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos
(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sqrt(2) -
11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) -
 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^
(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*
(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*s
qrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2
```

```
))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)
*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 - 4*(2^(1/4)
*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2)
+ 4)*sin(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*
cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))
```

giac [A] time = 0.36, size = 170, normalized size = 0.55

$$\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+\frac{1}{4}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(-\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\right.}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="giac")
```

```
[Out] 1/4*(pi*floor(x/pi + 1/2) + arctan(2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2)
 + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/4*(pi*floor(x/p
i + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) - 2*tan(x)
)/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1/8*sqrt(sqrt(2) - 1)*log(tan(x)^
2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2)) - 1/8*sqrt(sqrt(2) -
 1)*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x) + sqrt(1/2))
```

maple [A] time = 0.45, size = 239, normalized size = 0.77

$$-\frac{\sqrt{2}\,\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\left(\tan^{2}(x)\right)+\sqrt{2}\right)}{16}+\frac{\arctan\left(\frac{-\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)\sqrt{2}}{4\sqrt{1+\sqrt{2}}}+\frac{\text{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^4),x)
```

```
[Out] -1/16*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(-(-2+2*2^(1/2))^(1/2)*2^(1/2)*tan(x)+
2*tan(x)^2+2^(1/2))+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2
))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))*2^(1/2)+1/4/(1+2^(1/2))^(1/2)*arctan(
1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))+1/16*2^(1/2
)*(-2+2*2^(1/2))^(1/2)*ln(2^(1/2)+2*tan(x)^2+(-2+2*2^(1/2))^(1/2)*2^(1/2)*t
an(x))+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(4*tan(x)+2^(1/2)*(-2+2*2^(1/2))^(1
/2))/(1+2^(1/2))^(1/2))*2^(1/2)+1/4/(1+2^(1/2))^(1/2)*arctan(1/2*(4*tan(x)+
2^(1/2)*(-2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x)^4+1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^4),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^4 + 1), x)
```

mupad [B] time = 14.35, size = 236, normalized size = 0.76

$$\operatorname{atanh}\left(\frac{\tan(x)}{8\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}-\frac{\tan(x)}{8\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\tan(x)}{16\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}}+\frac{\sqrt{2}\tan(x)}{16\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}}\right)\left(2\sqrt{-\frac{\sqrt{2}}{64}-\frac{1}{64}}-2\sqrt{\frac{\sqrt{2}}{64}-\frac{1}{64}}\right)+\text{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^4 + 1),x)
```

```
[Out] atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2)) - tan(x)/(8*(2^(1/2)/64 - 1/64
)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*tan
(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^
(1/2)/64 - 1/64)^(1/2)) + atanh(tan(x)/(8*(- 2^(1/2)/64 - 1/64)^(1/2)) + ta
n(x)/(8*(2^(1/2)/64 - 1/64)^(1/2)) + (2^(1/2)*tan(x))/(16*(- 2^(1/2)/64 - 1
/64)^(1/2)) - (2^(1/2)*tan(x))/(16*(2^(1/2)/64 - 1/64)^(1/2)))*(2*(- 2^(1/2
)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2)) - ((x - atan(tan(x)))*(pi
*(2*(- 2^(1/2)/64 - 1/64)^(1/2) - 2*(2^(1/2)/64 - 1/64)^(1/2))*1i + pi*(2*(
- 2^(1/2)/64 - 1/64)^(1/2) + 2*(2^(1/2)/64 - 1/64)^(1/2))*1i))/pi
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**4),x)
```

```
[Out] Timed out
```

3.239 $\int \sin(c + dx)\sqrt{a + b\sin^4(c + dx)}\, dx$

Optimal. Leaf size=477

$$-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d}+\frac{2\sqrt{b}\,\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)}}{3d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

```
[Out] -1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d+2/3*cos(d*x+c
)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/(1+cos(d*x+c)^2*b^(
1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*b^(1/4)*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)
*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1
/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1
/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(
d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2
)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/3*(a+b)^(3/4)*(cos(2*arct
an(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c
)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2
*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(
1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c
)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x
+c)^4)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3215, 1091, 1197, 1103, 1195}

$$-\frac{\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{3d}+\frac{2\sqrt{b}\,\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)}}{3d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*d) +
 (2*Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4
])/(3*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*b^(1/4
)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4]) + ((a + b)^(3/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(3*b^
(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1091

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol] :> Simp[(x*(a + b*
x^2 + c*x^4)^p)/(4*p + 1), x] + Dist[(2*p)/(4*p + 1), Int[(2*a + b*x^2)*(a
+ b*x^2 + c*x^4)^(p - 1), x], x] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
 0] && GtQ[p, 0] && IntegerQ[2*p]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \sin(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \sqrt{a+b-2bx^2+bx^4}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\text{Subst}\left(\int -\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} - \frac{\left(2\sqrt{b}\sqrt{a+\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3d} + \frac{2\sqrt{b}\cos(c}{}$$

Mathematica [C] time = 31.59, size = 47242, normalized size = 99.04

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}\,\sin\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sin(d*x + c),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(dx+c\right)^{4}+a}\,\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

maple [C] time = 3.67, size = 439, normalized size = 0.92

$$-\frac{\frac{4\cos(dx+c)\sqrt{a+b-2b\left(\cos^{2}(dx+c)\right)+b\left(\cos^{4}(dx+c)\right)}}{3}+\frac{4\left(\frac{2a}{3}+\frac{2b}{3}\right)\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^{2}(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^{2}(dx+c)\right)}{a+b}}\,\text{EllipticF}\left(\cos(dx+c)\sqrt{\frac{i\sqrt{}}{}}\right.}{\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^{2}(dx+c)\right)+b\left(\cos^{4}(dx+c)\right)}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/4/d*(4/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+4*(2/3*a
+2/3*b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*
cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b
-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^
(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))+16/3*b*(a+b
)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*
x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*c
os(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos
(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+
b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(dx+c\right)^{4}+a}\,\sin\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*sin(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\sin\left(c+d\,x\right)\,\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Timed out
```

3.240 $\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=521

$$\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+\frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d}-\frac{\sqrt[4]{b}(a+b}{}$$

```
[Out] 1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2
))*(-a)^(1/2)/d+cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1
/2)/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-b^(1/4)*(a+b)^(3/4)*
(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^
(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^
(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)
/(a+b)^(1/2))^2)^(1/2)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/4*(a
+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*cos(d*
x+c)/(a+b)^(1/4))),1/4*((a+b)^(1/2)+b^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2
*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-
(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2
*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)
^4)^(1/2)
```

Rubi [A] time = 0.68, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3215, 1208, 1197, 1103, 1195, 1216, 1706}

$$\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{d\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}+\frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d}-\frac{\sqrt[4]{b}(a+b}{}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[-a]*ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 +
b*Cos[c + d*x]^4]])/(2*d) + (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c +
d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sq
rt[a + b])) - (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a +
 b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c +
d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(d*Sqrt[a + b - 2*b*Cos
[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(
b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*b^(1
/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1208

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_)/((d_) + (e_.)*(x_)^2), x_Symb
ol] :> -Dist[(e^2)^(-1), Int[(c*d - b*e - c*e*x^2)*(a + b*x^2 + c*x^4)^(p -
 1), x], x] + Dist[(c*d^2 - b*d*e + a*e^2)/e^2, Int[(a + b*x^2 + c*x^4)^(p
- 1)/(d + e*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0
] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IGtQ[p + 1/2, 0]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \csc(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = -\frac{\text{Subst}\left(\int \frac{\sqrt{a+b-2bx^2+bx^4}}{1-x^2}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{-b+bx^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+b-2bx^2+}}\right.}{d}$$

$$= -\frac{\left(\sqrt{b}\sqrt{a+b}\right)\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d} + \frac{\left((a+b)\left(-1\right.\right.}{}$$

$$= \frac{\sqrt{-a}\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2d} + \frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b}}{\sqrt{a+b}\,d\left(1\right.}$$

Mathematica [C] time = 32.00, size = 118912, normalized size = 228.24

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}\,\csc\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*csc(d*x + c),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(dx+c\right)^4+a}\,\csc\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \csc\left(dx+c\right)\sqrt{a+b\left(\sin^4\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin(dx+c)^4 + a} \csc(dx+c)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(d*x + c)^4 + a)*csc(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{b \sin(c+d\,x)^4 + a}}{\sin(c+d\,x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x),x)
[Out] int((a + b*sin(c + d*x)^4)^(1/2)/sin(c + d*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin^4(c+dx)} \csc(c+dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*csc(c + d*x), x)
```

3.241 $\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=484

$$\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}\right.\right.}{6b^{5/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

```
[Out] -1/3*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/b/d+2/3*cos(d*x
+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*x+c)^2*b
^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-2/3*(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*cos(d
*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*
EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a
+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)
^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(3
/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)+1/6*(a+b)^(1/4)*(cos(2*ar
ctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x
+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1
/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(a
-2*b+2*b^(1/2)*(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1
+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(5/4)/d/(a+b-2*b*cos(d*x+c)^2
+b*cos(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.43, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3215, 1206, 1197, 1103, 1195}

$$\frac{\sqrt[4]{a+b}\left(2\sqrt{b}\sqrt{a+b}+a-2b\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}\right.\right.}{6b^{5/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*b*d)
 + (2*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(3*
Sqrt[b]*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - (2*(a +
 b)^(3/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[
c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[
a + b])^2)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 +
Sqrt[b]/Sqrt[a + b])/2])/(3*b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*C
os[c + d*x]^4]) + ((a + b)^(1/4)*(a - 2*b + 2*Sqrt[b]*Sqrt[a + b])*(1 + (Sq
rt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos
[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*Ellipt
icF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b
])/2])/(6*b^(5/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1206

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x
_Symbol] :> Simp[(e^q*x^(2*q - 3)*(a + b*x^2 + c*x^4)^(p + 1))/(c*(4*p + 2*
q + 1)), x] + Dist[1/(c*(4*p + 2*q + 1)), Int[(a + b*x^2 + c*x^4)^p*ExpandT
oSum[c*(4*p + 2*q + 1)*(d + e*x^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - b*(2*p
 + 2*q - 1)*e^q*x^(2*q - 2) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x], x] /;
FreeQ[{a, b, c, d, e, p}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e +
a*e^2, 0] && IGtQ[q, 1]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\text{Subst}\left(\int \frac{-a+2b-2bx^2}{\sqrt{a+b-2bx^2+bx}}\right.}{3b}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} - \frac{\left(2\sqrt{a+b}\right)\text{Subst}\left(\int\right.}{}$$

$$= -\frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{3bd} + \frac{2\cos(c+dx)\sqrt{a+b-}}{3\sqrt{b}\sqrt{a+}}$$

Mathematica [C] time = 31.72, size = 47246, normalized size = 97.62

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] Result too large to show
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\left(\cos\left(dx+c\right)^{4}-2\,\cos\left(dx+c\right)^{2}+1\right)\sin\left(dx+c\right)}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral((cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)*sin(d*x + c)/sqrt(b*cos(d*
x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{5}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [C] time = 1.40, size = 837, normalized size = 1.73

$$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^{2}\left(dx+c\right)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^{2}\left(dx+c\right)\right)}{a+b}}\,\operatorname{EllipticF}\left(\cos\left(dx+c\right)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{a}\,\sqrt{b}-b\right)}{a+b}}\right)}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^{2}\left(dx+c\right)\right)+b\left(\cos^{4}\left(dx+c\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-4/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))-4/d*(1/12/b*cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+
b*cos(d*x+c)^4)^(1/2)-1/12*(a+b)/b/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(
I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+
b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*Elliptic
F(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b
)/(a+b))^(1/2))-2/3*(a+b)/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)
*b^(1/2)+b)/(a+b)*cos(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*
x+c)^2)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)
*b^(1/2))*(EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(
```

```
I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)
+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^5}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^5}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(sin(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Timed out
```

3.242 $\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=431

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

```
[Out] cos(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)/d/b^(1/2)/(1+cos(d*x
+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)-(a+b)^(3/4)*(cos(2*arctan(b^(1/4)*co
s(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)
))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)
/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b
^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/2*(a+b)^(1/4)*(cos(2
*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(
d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))
),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))
*(b^(1/2)-(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(
d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(3/4)/d/(a+b-2*b*cos(d*x+c)^2+b*co
s(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.30, antiderivative size = 431, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3215, 1197, 1103, 1195}

$$-\frac{\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2b^{3/4}d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])/(Sqrt[b]
*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])) - ((a + b)^(3/4)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^
2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2
)]*EllipticE[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/S
qrt[a + b])/2])/(b^(3/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]
^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)
*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)
*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(3/4)*d*S
qrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b}\,\text{Subst}\left(\int \frac{1-\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{\sqrt{b}\,d} - \frac{\left(1-\frac{\sqrt{a+b}}{\sqrt{b}}\right)\text{Subst}\left(\int \frac{}{\sqrt{a+b}\cdots}\right.}{}$$

$$= \frac{\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{\sqrt{b}\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{(a+b)^{3/4}\left(1+\frac{\sqrt{b}\cos^2(c+\cdots}{\sqrt{a+b}}\right.}{}$$

Mathematica [C] time = 31.83, size = 89374, normalized size = 207.36

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(\cos(dx+c)^2-1\right)\sin(dx+c)}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)*sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos
(d*x + c)^2 + a + b), x)
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [C] time = 1.33, size = 398, normalized size = 0.92

$$-\frac{\sqrt{1-\frac{\left(i\sqrt{a}\,\sqrt{b}+b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\sqrt{1+\frac{\left(i\sqrt{a}\,\sqrt{b}-b\right)\left(\cos^2(dx+c)\right)}{a+b}}\,\mathrm{EllipticF}\left(\cos\left(dx+c\right)\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}},\sqrt{-1-\frac{2\left(i\sqrt{a}\,\sqrt{b}-b\right)}{a+b}}\right)}{d\sqrt{\frac{i\sqrt{a}\,\sqrt{b}+b}{a+b}}\,\sqrt{a+b-2b\left(\cos^2\left(dx+c\right)\right)+b\left(\cos^4\left(dx+c\right)\right)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))-2/d*(a+b)/((I*
a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos(d*x+c)^2
)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*b*cos(d*x
+c)^2+b*cos(d*x+c)^4)^(1/2)/(-2*b+2*I*a^(1/2)*b^(1/2))*(EllipticF(cos(d*x+c
)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1
/2))-EllipticE(cos(d*x+c)*((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1
/2)*b^(1/2)-b)/(a+b))^(1/2)))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin\left(c+d\,x\right)^{3}}{\sqrt{b\sin\left(c+d\,x\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.243 $\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=171

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

```
[Out] -1/2*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/co
s(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(sin(2*arctan(b^(1/4)*
cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos
(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*c
os(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3215, 1103}

$$-\frac{\sqrt[4]{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)|\frac{1}{2}\left(\frac{\sqrt{b}}{\sqrt{a+b}}+1\right)\right)}{2\sqrt[4]{b}\,d\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((a + b)^(1/4)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*
b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)],
 (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2
 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\sin(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt[4]{a+b}\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)\sqrt{\frac{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}{(a+b)\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{b}\cos(c+dx)}{\sqrt[4]{a+b}}\right)\right.}{2\sqrt[4]{b}\,d\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}$$

Mathematica [C] time = 25.36, size = 13300, normalized size = 77.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sin[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sin(dx+c)}{\sqrt{b\cos(dx+c)^4 - 2b\cos(dx+c)^2 + a + b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(sin(d*x + c)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b),
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [C] time = 0.86, size = 163, normalized size = 0.95

$$-\frac{\sqrt{1-\frac{(i\sqrt{a}\sqrt{b}+b)(\cos^2(dx+c))}{a+b}}\sqrt{1+\frac{(i\sqrt{a}\sqrt{b}-b)(\cos^2(dx+c))}{a+b}}\,\text{EllipticF}\left(\cos(dx+c)\sqrt{\frac{i\sqrt{a}\sqrt{b}+b}{a+b}}, \sqrt{-1-\frac{2(i\sqrt{a}\sqrt{b}-b)}{a+b}}\right.}{d\sqrt{\frac{i\sqrt{a}\sqrt{b}+b}{a+b}}\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] -1/d/((I*a^(1/2)*b^(1/2)+b)/(a+b))^(1/2)*(1-(I*a^(1/2)*b^(1/2)+b)/(a+b)*cos
(d*x+c)^2)^(1/2)*(1+(I*a^(1/2)*b^(1/2)-b)/(a+b)*cos(d*x+c)^2)^(1/2)/(a+b-2*
b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)*EllipticF(cos(d*x+c)*((I*a^(1/2)*b^(1/
2)+b)/(a+b))^(1/2),(-1-2*(I*a^(1/2)*b^(1/2)-b)/(a+b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(sin(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sin(c+d\,x)}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(sin(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Timed out
```

3.244 $\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=469

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2\sqrt{-a}\,d}+\frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\,\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+d\ldots}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}}{2ad\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c\ldots}}$$

```
[Out] -1/2*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/
2))/d/(-a)^(1/2)+1/2*b^(1/4)*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(
a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*Ellipti
cF(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/
2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2))*((a+b
-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2)
)^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)-1/4*(a+b)^(1/4)*
(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4
)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)
^(1/4))),1/4*((a+b)^(1/2)+b^(1/2))^2/b^(1/2)/(a+b)^(1/2),1/2*(2+2*b^(1/2)/(
a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(b^(1/2)-(a+b)^(1/2
))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(
a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2
)
```

Rubi [A] time = 0.43, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3215, 1216, 1103, 1706}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2\sqrt{-a}\,d}+\frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\,\left(\sqrt{b}-\sqrt{a+b}\right)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)\sqrt{\frac{a+b\cos^4(c+dx)-2b\cos^2(c+d\ldots}{(a+b)\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)^2}}}{2ad\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(2*Sqrt[-a]*d) + (b^(1/4)*(a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])*(
1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2
+ b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]
*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqr
t[a + b])/2])/(2*a*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) -
 ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqr
t[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1
+ (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticPi[(Sqrt[b] + Sqrt[a +
b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4
)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(4*a*b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1216

```
Int[1/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4]), x_S
ymbol] :> With[{q = Rt[c/a, 2]}, Dist[(c*d + a*e*q)/(c*d^2 - a*e^2), Int[1/
Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[(a*e*(e + d*q))/(c*d^2 - a*e^2), Int
[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
 c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[c/a]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= \frac{\left((a+b)\left(-1+\frac{\sqrt{b}}{\sqrt{a+b}}\right)\right)\text{Subst}\left(\int \frac{1+\frac{\sqrt{b}x^2}{\sqrt{a+b}}}{\left(1-x^2\right)\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{ad} + \frac{\left(\sqrt{b}\left(\sqrt{b}\right.\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{2\sqrt{-a}\,d} + \frac{\sqrt[4]{b}\,\sqrt[4]{a+b}\left(\sqrt{b}-\sqrt{a+b}\right)\left(1+\frac{\sqrt{b}\cos^2(c+d}{\sqrt{a+b}}\right.}{2ad\sqrt{a+}}$$

Mathematica [C] time = 31.47, size = 63281, normalized size = 134.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(csc(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(c+d\,x)\,\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)),x)
[Out] int(1/(sin(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(csc(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

3.245 $\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=776

$$-\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2ad\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{4\sqrt{-a}\,d}-\cot(c+dx)\,\mathrm{cs}$$

```
[Out] -1/4*arctan(cos(d*x+c)*(-a)^(1/2)/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/
2))/d/(-a)^(1/2)-1/2*cot(d*x+c)*csc(d*x+c)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+
c)^4)^(1/2)/a/d-1/2*cos(d*x+c)*b^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4
)^(1/2)/a/d/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))/(a+b)^(1/2)+1/2*b^(1/4)*(a
+b)^(3/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/2)/cos(2*arc
tan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticE(sin(2*arctan(b^(1/4)*cos(d*x
+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1
/2)/(a+b)^(1/2))*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)
^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^
(1/2)-1/8*(a+b)^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))^2)^(1/
2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticPi(sin(2*arctan(b^
(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/4*((a+b)^(1/2)+b^(1/2))^2/b^(1/2)/(a+b)^(1
/2),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2
))*(b^(1/2)-(a+b)^(1/2))^2*((a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+
cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))^2)^(1/2)/a/b^(1/4)/d/(a+b-2*b*cos(d*x+c)^
2+b*cos(d*x+c)^4)^(1/2)-1/2*b^(1/4)*(cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^
(1/4)))^2)^(1/2)/cos(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4)))*EllipticF(si
n(2*arctan(b^(1/4)*cos(d*x+c)/(a+b)^(1/4))),1/2*(2+2*b^(1/2)/(a+b)^(1/2))^(
1/2))*(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))*(a+b-b^(1/2)*(a+b)^(1/2))*((a+b-
2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)/(a+b)/(1+cos(d*x+c)^2*b^(1/2)/(a+b)^(1/2))
^2)^(1/2)/a/(a+b)^(1/4)/d/(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2)
```

Rubi [A] time = 1.04, antiderivative size = 776, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3215, 1223, 1714, 1195, 1708, 1103, 1706}

$$-\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}{2ad\sqrt{a+b}\left(\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}+1\right)}-\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{4\sqrt{-a}\,d}-\cot(c+dx)\,\mathrm{cs}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -ArcTan[(Sqrt[-a]*Cos[c + d*x])/Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c +
 d*x]^4]]/(4*Sqrt[-a]*d) - (Sqrt[b]*Cos[c + d*x]*Sqrt[a + b - 2*b*Cos[c + d
*x]^2 + b*Cos[c + d*x]^4])/(2*a*Sqrt[a + b]*d*(1 + (Sqrt[b]*Cos[c + d*x]^2)
/Sqrt[a + b])) - (Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]*Cot[c
 + d*x]*Csc[c + d*x])/(2*a*d) + (b^(1/4)*(a + b)^(3/4)*(1 + (Sqrt[b]*Cos[c
+ d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)
/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])^2)]*EllipticE[2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*
d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4]) - (b^(1/4)*(a + b -
Sqrt[b]*Sqrt[a + b])*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b
 - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*
x]^2)/Sqrt[a + b])^2)]*EllipticF[2*ArcTan[(b^(1/4)*Cos[c + d*x])/(a + b)^(1
```

```
/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(2*a*(a + b)^(1/4)*d*Sqrt[a + b - 2*b*C
os[c + d*x]^2 + b*Cos[c + d*x]^4]) - ((a + b)^(1/4)*(Sqrt[b] - Sqrt[a + b])
^2*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])*Sqrt[(a + b - 2*b*Cos[c + d*x
]^2 + b*Cos[c + d*x]^4)/((a + b)*(1 + (Sqrt[b]*Cos[c + d*x]^2)/Sqrt[a + b])
^2)]*EllipticPi[(Sqrt[b] + Sqrt[a + b])^2/(4*Sqrt[b]*Sqrt[a + b]), 2*ArcTan
[(b^(1/4)*Cos[c + d*x])/(a + b)^(1/4)], (1 + Sqrt[b]/Sqrt[a + b])/2])/(8*a*
b^(1/4)*d*Sqrt[a + b - 2*b*Cos[c + d*x]^2 + b*Cos[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1223

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_
Symbol] :> -Simp[(e^2*x*(d + e*x^2)^(q + 1)*Sqrt[a + b*x^2 + c*x^4])/(2*d*(
q + 1)*(c*d^2 - b*d*e + a*e^2)), x] + Dist[1/(2*d*(q + 1)*(c*d^2 - b*d*e +
a*e^2)), Int[((d + e*x^2)^(q + 1)*Simp[a*e^2*(2*q + 3) + 2*d*(c*d - b*e)*(q
 + 1) - 2*e*(c*d*(q + 1) - b*e*(q + 2))*x^2 + c*e^2*(2*q + 5)*x^4, x])/Sqrt
[a + b*x^2 + c*x^4], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c
, 0] && ILtQ[q, -1]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 1708

```
Int[((A_.) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2
+ (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[c/a, 2]}, Dist[(A*(c*d + a*e*q)
 - a*B*(e + d*q))/(c*d^2 - a*e^2), Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] +
Dist[(a*(B*d - A*e)*(e + d*q))/(c*d^2 - a*e^2), Int[(1 + q*x^2)/((d + e*x^2
)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && N
eQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0
] && PosQ[c/a] && NeQ[c*A^2 - a*B^2, 0]
```

Rule 1714

```
Int[(P4x_)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2], A = Coeff[P4x, x, 0], B = Coeff[P4x, x
```

```
, 2], C = Coeff[P4x, x, 4]}, -Dist[C/(e*q), Int[(1 - q*x^2)/Sqrt[a + b*x^2
+ c*x^4], x], x] + Dist[1/(c*e), Int[(A*c*e + a*C*d*q + (B*c*e - C*(c*d - a
*e*q))*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[{a, b,
c, d, e}, x] && PolyQ[P4x, x^2, 2] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*
d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] &&  !GtQ[b^2 - 4*a*c,
 0]
```

Rule 3215

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Cos[e + f*x], x]}, -Dist[ff/f, S
ubst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b - 2*b*ff^2*x^2 + b*ff^4*x^4)^p,
x], x, Cos[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m -
1)/2]
```

Rubi steps

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = -\frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+b-2bx^2+bx^4}}\,dx, x, \cos(c+dx)\right)}{d}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\,\cot(c+dx)\csc(c+dx)}{2ad} + \frac{\text{Subst}\left(\int \frac{}{(1-}\right.}{}$$

$$= -\frac{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}\,\cot(c+dx)\csc(c+dx)}{2ad} - \frac{\text{Subst}\left(\int \frac{-b(-}{}\right.}{}$$

$$= -\frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}{2a\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right)} - \frac{\sqrt{a+b-2b\cos^2(c+}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{-a}\cos(c+dx)}{\sqrt{a+b-2b\cos^2(c+dx)+b\cos^4(c+dx)}}\right)}{4\sqrt{-a}\,d} - \frac{\sqrt{b}\cos(c+dx)\sqrt{a+b-2b\cos^2(c+dx)+}}{2a\sqrt{a+b}\,d\left(1+\frac{\sqrt{b}\cos^2(c+dx)}{\sqrt{a+b}}\right.}$$

Mathematica [C] time = 32.63, size = 119171, normalized size = 153.57

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Csc[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
[Out] Result too large to show
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc(dx+c)^3}{\sqrt{b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + a + b}}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^3/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 2.48Not invertible Error: B
ad Argument Value
```

maple [F] time = 1.62, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^3}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(c+d\,x)^3\,\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^3*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.246 $\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=499

$$-\frac{\cos^2(c+dx)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\right.}{}$$

```
[Out] -1/2*arctan(b^(1/2)*tan(d*x+c)/(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/2
))*cos(d*x+c)^2*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)^(1/2)/d/b^(1/2)/(a+
b*sin(d*x+c)^4)^(1/2)-1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*ta
n(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*E
llipticF(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+
b)^(1/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c
)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d
*x+c)^2)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)+1/4*cos(d*x+c)^2*(cos(2*a
rctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*ta
n(d*x+c)/a^(1/4)))*EllipticPi(sin(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))
,-1/4*(a^(1/2)-(a+b)^(1/2))^2/a^(1/2)/(a+b)^(1/2),1/2*(2-2*a^(1/2)/(a+b)^(1
/2))^(1/2))*(a^(1/2)+(a+b)^(1/2))^2*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4
)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+
c)^2)/a^(1/4)/b/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
```

Rubi [A] time = 0.66, antiderivative size = 499, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3219, 1319, 1103, 1706}

$$-\frac{\cos^2(c+dx)\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}\right)\sqrt{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}-\frac{\sqrt[4]{a}\left(\sqrt{a+b}+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(ArcTan[(Sqrt[b]*Tan[c + d*x])/Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c
 + d*x]^4]]*Cos[c + d*x]^2*Sqrt[a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4])/(2*Sqrt[b]*d*Sqrt[a + b*Sin[c + d*x]^4]) - (a^(1/4)*(Sqrt[a] + Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((
Sqrt[a] + Sqrt[a + b])^2*Cos[c + d*x]^2*EllipticPi[-(Sqrt[a] - Sqrt[a + b])
^2/(4*Sqrt[a]*Sqrt[a + b]), 2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)],
 (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(
a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan
[c + d*x]^2)^2])/(4*a^(1/4)*b*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1319

```
Int[(x_)^2/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4])
, x_Symbol] :> With[{q = Rt[c/a, 2]}, -Dist[(a*(e + d*q))/(c*d^2 - a*e^2),
Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] + Dist[(a*d*(e + d*q))/(c*d^2 - a*e^2
), Int[(1 + q*x^2)/((d + e*x^2)*Sqrt[a + b*x^2 + c*x^4]), x], x]] /; FreeQ[
{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0]
 && PosQ[c/a] && NeQ[c*d^2 - a*e^2, 0]
```

Rule 1706

```
Int[((A_) + (B_.)*(x_)^2)/(((d_) + (e_.)*(x_)^2)*Sqrt[(a_) + (b_.)*(x_)^2 +
 (c_.)*(x_)^4]), x_Symbol] :> With[{q = Rt[B/A, 2]}, -Simp[((B*d - A*e)*Arc
Tan[(Rt[-b + (c*d)/e + (a*e)/d, 2]*x)/Sqrt[a + b*x^2 + c*x^4]])/(2*d*e*Rt[-
b + (c*d)/e + (a*e)/d, 2]), x] + Simp[((B*d + A*e)*(A + B*x^2)*Sqrt[(A^2*(a
 + b*x^2 + c*x^4))/(a*(A + B*x^2)^2)]*EllipticPi[Cancel[-((B*d - A*e)^2/(4*
d*e*A*B))], 2*ArcTan[q*x], 1/2 - (b*A)/(4*a*B)])/(4*d*e*A*q*Sqrt[a + b*x^2
+ c*x^4]), x]] /; FreeQ[{a, b, c, d, e, A, B}, x] && NeQ[b^2 - 4*a*c, 0] &&
 NeQ[c*d^2 - b*d*e + a*e^2, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[c/a] && EqQ
[c*A^2 - a*B^2, 0]
```

Rule 3219

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff^(m + 1
)*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[(x^m*ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
2]
```

Rubi steps

$$\int \frac{\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{\left(1+x^2\right)\sqrt{a+2a\cdots}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\left(a\left(1+\frac{\sqrt{a+b}}{\sqrt{a}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\cdots}{bd\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{b}\tan(c+dx)}{\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}}\right)\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+\cdots}}{2\sqrt{b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] time = 2.88, size = 287, normalized size = 0.58

$$-\frac{2i\cos^2(c+dx)\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{2+\left(2-\frac{2i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\left(F\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+d\cdots\right.\right.\right.}{d\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sin[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Cos[c + d*x]^2*(EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*T
an[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])] - EllipticPi[Sqr
```

```
t[a]/(Sqrt[a] - I*Sqrt[b]), I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c +
 d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 + (I*Sqrt
[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[2 + (2 - ((2*I)*Sqrt[b])/Sqrt[a])*Tan[c
+ d*x]^2])/(Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c +
 d*x)] + b*Cos[4*(c + d*x)]])
```

fricas [F] time = 0.63, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\cos\left(dx+c\right)^{2}-1}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-(cos(d*x + c)^2 - 1)/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin\left(dx+c\right)^{2}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [A] time = 7.36, size = 881, normalized size = 1.77

$$-\frac{\sqrt{\left(4a+\left(\cos^{2}\left(2dx+2c\right)\right)b+b-2b\cos\left(2dx+2c\right)\right)\left(\sin^{2}\left(2dx+2c\right)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos(2dx+2c)\right)}{\sqrt{-ab}\,\left(\cos(2dx+2c)+1\right)}}\,\left(\cos(2\right.}{2\left(-b+\sqrt{-ab}\right)\sqrt{\frac{(-1+\cos(2dx+2c))(\cos(2dx+2c)+}{}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)
*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*
x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b
)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(EllipticF(((-b+(-a*b)^(1/2))*(-1+
cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-
b+(-a*b)^(1/2)))^(1/2))-2*EllipticPi(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))
/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),(-a*b)^(1/2)/(-b+(-a*b)^(1/2)),((b+
(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2)))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d
*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d
*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2
*b*cos(2*d*x+2*c))^(1/2)/d-1/2*((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c
))*sin(2*d*x+2*c)^2)^(1/2)*(-a*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*
c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*
d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2
*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*Elliptic
F(((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(
1/2),((b+(-a*b)^(1/2))/(-b+(-a*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1
+cos(2*d*x+2*c))*(cos(2*d*x+2*c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b
```

```
*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)
^2*b+b-2*b*cos(2*d*x+2*c))^(1/2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sin(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sin(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sin(c+d\,x)^2}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(sin(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sin(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Timed out
```

3.247 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/co
s(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(1
/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*tan(d
*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.125$, Rules used = {3210, 1103}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3210

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f
*x]^2)^(2*p))/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p), Subs
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx &= \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a+b)x^4}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}\\ &= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)}{2\sqrt[4]{a}\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}\end{aligned}$$

Mathematica [C] time = 8.70, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(\sqrt{b}+i\sqrt{a}\right)\sin^2(c+dx)\tan(c+dx)\left(2\sqrt{a}+i\sqrt{b}\cos(2(c+dx))-i\sqrt{b}\right)\left(2i\sqrt{a}+\sqrt{b}\cos(2(c+dx))-\sqrt{\ }\right.}{\sqrt{a}\,d(8a-4b\cos(2(c+}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [B] time = 2.94, size = 396, normalized size = 2.44

$$-\frac{\sqrt{\left(4a+\left(\cos^2\left(2dx+2c\right)\right)b+b-2b\cos\left(2dx+2c\right)\right)\left(\sin^2\left(2dx+2c\right)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos(2dx+2c)\right)}{\sqrt{-ab}\,\left(\cos(2dx+2c)+1\right)}}\,\left(\cos\right.}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{\left(-1+\cos(2dx+2c)\right)\left(\cos(2dx+2c)+1\right)\left(-b\cos(2dx+2c)+2\sqrt{-ab}+b\right)\left(b\cos(2dx\right.}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
```

```
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b \sin(dx+c)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b \sin(c+d\,x)^4 + a}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b \sin^4(c+dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

3.248 $\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=493

$$\frac{\left(\sqrt{a}\sqrt{a+b}+a+b\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}}{\cdots}\right.\right.}{2a^{3/4}d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/c
os(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(
1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*(a+b+a^(1/2)
*(a+b)^(1/2))*((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)
*tan(d*x+c)^2)^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/(a+b)^(3
/4)/d/(a+b*sin(d*x+c)^4)^(1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*tan(d*x+c)^2+
(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+b
)^(1/2)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2
)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
```

Rubi [A] time = 0.42, antiderivative size = 493, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3219, 1281, 1197, 1103, 1195}

$$\frac{\left(\sqrt{a}\sqrt{a+b}+a+b\right)\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}}{\cdots}\right.\right.}{2a^{3/4}d(a+b)^{3/4}\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b + Sqrt[a]*Sqrt[
a + b])*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1
/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*S
qrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b
]*Tan[c + d*x]^2)^2])/(2*a^(3/4)*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 1197

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 2]}, Dist[(e + d*q)/q, Int[1/Sqrt[a + b*x^2 + c*x^4
], x], x] - Dist[e/q, Int[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x] /; Ne
Q[e + d*q, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[
c/a]
```

Rule 1281

```
Int[((f_.)*(x_))^(m_.)*((d_) + (e_.)*(x_)^2)*((a_) + (b_.)*(x_)^2 + (c_.)*(
x_)^4)^(p_), x_Symbol] :> Simp[(d*(f*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1)
)/(a*f*(m + 1)), x] + Dist[1/(a*f^2*(m + 1)), Int[(f*x)^(m + 2)*(a + b*x^2
+ c*x^4)^p*Simp[a*e*(m + 1) - b*d*(m + 2*p + 3) - c*d*(m + 4*p + 5)*x^2, x]
, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && NeQ[b^2 - 4*a*c, 0] && LtQ[m
, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 3219

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(ff^(m + 1
)*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[(x^m*ExpandToSum[a*(1 + ff^2*x^
2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x
]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[m/2] && IntegerQ[p - 1/
2]
```

Rubi steps

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1+x^2}{x^2\sqrt{a+2ax^2+(a+b)}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - \frac{\left(\cos^2(c+\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} - \frac{\left(\sqrt{a+b}\ \mathrm{c}\right.}{}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\sqrt{a+b}\ \mathrm{co}}{}$$

Mathematica [C] time = 16.20, size = 498, normalized size = 1.01

$$-\frac{\cot(c+dx)\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b}}{2\sqrt{2}ad}-\frac{\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c+dx)\left(a\left(\tan^2(c+dx)+1\right)^2+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Csc[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c + d
*x])/(Sqrt[2]*a*d) - (Sqrt[a]*(I*Sqrt[a] + Sqrt[b])*EllipticE[I*ArcSinh[Sqr
t[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] -
I*Sqrt[b])]*(1 + Tan[c + d*x]^2)*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Tan[c +
 d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2] - Sqrt[a]*Sqrt[
b]*EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a
] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]*(1 + Tan[c + d*x]^2)*Sqrt[1 + (1 - (I
*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c
 + d*x]^2] + Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]*(b*Tan[c + d*x]^4 +
 a*(1 + Tan[c + d*x]^2)^2))/(a*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*(1 + Tan[c +
 d*x]^2)^2*Sqrt[(b*Tan[c + d*x]^4 + a*(1 + Tan[c + d*x]^2)^2)/(1 + Tan[c +
d*x]^2)^2])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\csc\left(dx+c\right)^2}{\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(csc(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\csc\left(dx+c\right)^2}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.50, size = 0, normalized size = 0.00

$$\int\frac{\csc^2\left(dx+c\right)}{\sqrt{a+b\left(\sin^4\left(dx+c\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(csc(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\sin(c+d\,x)^2\,\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)),x)
```

```
[Out] int(1/(sin(c + d*x)^2*(a + b*sin(c + d*x)^4)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\csc^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(csc(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(csc(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.249 $\int \frac{1}{a+b\sin^5(x)}\,dx$

Optimal. Leaf size=384

$$\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\,\mathrm{ta}\ldots\right.}{\sqrt{a^{2/5}+\sqrt[5]{-1}\ldots}}\right.}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}\ldots}}$$

```
[Out] 2/5*arctan((b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(a
^(2/5)-b^(2/5))^(1/2)-2/5*arctan((-1)^(3/5)*(b^(1/5)+(-1)^(2/5)*a^(1/5)*tan
(1/2*x))/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^
(2/5))^(1/2)-2/5*arctan((-1)^(1/5)*(b^(1/5)+(-1)^(4/5)*a^(1/5)*tan(1/2*x))/
(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1
/2)+2/5*arctan(((-1)^(4/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*
b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)+2/5*arctan(((-1)
^(2/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4
/5)/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2)
```

Rubi [A] time = 0.71, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{b}}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{2/5}\sqrt[5]{b}}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+(-1)^{4/5}\sqrt[5]{b}}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\sqrt[5]{a}\,\mathrm{ta}\ldots\right.}{\sqrt{a^{2/5}+\sqrt[5]{-1}\ldots}}\right.}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}\ldots}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5)
*Sqrt[a^(2/5) - b^(2/5)]) + (2*ArcTan[((-1)^(2/5)*b^(1/5) + a^(1/5)*Tan[x/2
])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/5
)*b^(2/5)]) + (2*ArcTan[((-1)^(4/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5
) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) -
(2*ArcTan[((-1)^(3/5)*(b^(1/5) + (-1)^(2/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
 + (-1)^(1/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]) - (
2*ArcTan[((-1)^(1/5)*(b^(1/5) + (-1)^(4/5)*a^(1/5)*Tan[x/2]))/Sqrt[a^(2/5)
- (-1)^(2/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sin^5(x)}\,dx = \int \left(-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}-\frac{1}{5a^{4/5}\left(-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\right.}\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[5]{a}-\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}-\frac{\int \frac{1}{-\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}}}{5a^{4/5}}$$

$$= -\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}-2\sqrt[5]{b}x-\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{2\,\text{Subst}\left(\int \frac{1}{-\sqrt[5]{a}+2\sqrt[5]{-1}\sqrt[5]{b}x-\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right.\right.}{}$$

$$= \frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}\,dx,x,-2\sqrt[5]{b}-2\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{4\,\text{Subst}\left(\int \frac{1}{-4\left(a^{2/5}+\sqrt[5]{-1}b^{2/5}\right)-x^2}\,dx,\right.}{5a^{4/5}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+\sqrt[5]{-1}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}+\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}+\frac{2\,\text{t}}{5a^{4/5}}$$

Mathematica [C] time = 0.21, size = 149, normalized size = 0.39

$$\frac{8}{5}i\text{RootSum}\left[-i\#1^{10}b+5i\#1^8b-10i\#1^6b+32\#1^5a+10i\#1^4b-5i\#1^2b+ib\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\right.}{\#1^8b-4\#1^6b+6\#1^4b+16i\#}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^5)^(-1),x]
```

```
[Out] ((8*I)/5)*RootSum[I*b - (5*I)*b*#1^2 + (10*I)*b*#1^4 + 32*a*#1^5 - (10*I)*b
*#1^6 + (5*I)*b*#1^8 - I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 -
 I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 + (16*I)*a*#1^3 + 6*b*#1
^4 - 4*b*#1^6 + b*#1^8) & ]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^5+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

maple [C] time = 0.26, size = 109, normalized size = 0.28

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}+32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a+16\_R^{4}b+4\_R^{3}a+\_Ra}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a+16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a+32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^5 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^5 + a), x)
```

mupad [B] time = 19.75, size = 1515, normalized size = 3.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^5),x)
```

```
[Out] symsum(log(-10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 1
25*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^1
0 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k
)^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 3171
875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 15625
0*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*root(976
5625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 -
6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^8*b^2*d^1
0 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 1
25*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^2*d^10 - 97
65625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2
*d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d^10 - 9765625
*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2
- 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)^8*a^8*tan(x/2) + 14000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^4*a^3*b + 175000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a
^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b + 5
46875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156
250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b + 128*root(9765
```

```
625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6
250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)*b*tan(x/2) + 1000000*root(9765625*a^8*
b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*
d^4 - 125*a^2*d^2 - 1, d, k)^7*a^5*b^2 - 18750000*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^9*a^7*b^2 + 320*root(9765625*a^8*b^2*d^10 - 9765625*a^
10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1
, d, k)^2*a*b + 6400*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 195312
5*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b*
tan(x/2) + 100000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a
^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b*tan
(x/2) + 500000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*
d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x/
2) + 390625*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^8*b*tan(x/2)
+ 400000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*tan(x/2) -
 5000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 -
156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*tan(x/2)))
*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a
^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^5(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**5),x)
```

```
[Out] Integral(1/(a + b*sin(x)**5), x)
```

3.250 $\int \frac{1}{a+b\sin^6(x)}\,dx$

Optimal. Leaf size=171

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctan((a^(1/3)+b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+b^(1/3)
)^(1/2)+1/3*arctan((a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/
6)/(a^(1/3)-(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)+(-1)^(2/3)*b^(1/3
))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)+(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.300, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\,\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) + b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) +
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) - (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) + (-1)^(2/3)*b^(1/3)
])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sin^6(x)}\,dx = \frac{\int \frac{1}{1+\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int -\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.22, size = 148, normalized size = 0.87

$$-\frac{8}{3}\text{RootSum}\left[\#1^6b-6\#1^5b+15\#1^4b-64\#1^3a-20\#1^3b+15\#1^2b-6\#1b+b\&, \frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^2\,\text{lo}}{\#1^5b-5\#1^4b+10\#1^3b-32\#}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[x]^6)^(-1),x]
[Out] (-8*RootSum[b - 6*b*#1 + 15*b*#1^2 - 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*
b*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*
Cos[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 - 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3
- 5*b*#1^4 + b*#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^6+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="giac")
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

maple [C] time = 1.54, size = 68, normalized size = 0.40

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^5a+\_R^5b+2\_R^3a+\_Ra}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^6),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a+_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a+b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^6 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^6 + a), x)
```

mupad [B] time = 15.71, size = 513, normalized size = 3.00

$$\sum_{k=1}^{6} \ln\left(-\frac{b^3\,(a+b)\,\left(-\cot(x) + \mathrm{root}\left(46656\,a^5\,b\,d^6 + 46656\,a^6\,d^6 + 3888\,a^4\,d^4 + 108\,a^2\,d^2 + 1, d, k\right)\,a\,8 + \mathrm{root}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^6),x)
```

```
[Out] symsum(log(-(3*b^3*(a + b)*(8*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a
^4*d^4 + 108*a^2*d^2 + 1, d, k)*a - cot(x) + 2*root(46656*a^5*b*d^6 + 46656
*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)*b + 504*root(46656*a^5*b*d
^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^3*a^3 + 7776*roo
t(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^5
*a^5 - 144*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^
2 + 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4
*d^4 + 108*a^2*d^2 + 1, d, k)^5*a^4*b - 60*root(46656*a^5*b*d^6 + 46656*a^6
*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a^2*cot(x) - 864*root(46656*
a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^4*a^4*cot
(x) - 864*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2
 + 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 + 46656*a^6*d^6 + 3888
*a^4*d^4 + 108*a^2*d^2 + 1, d, k)^2*a*b*cot(x)))/cot(x))*root(46656*a^5*b*d
^6 + 46656*a^6*d^6 + 3888*a^4*d^4 + 108*a^2*d^2 + 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**6),x)
```

```
[Out] Integral(1/(a + b*sin(x)**6), x)
```

3.251 $\int \frac{1}{a+b\sin^8(x)}\,dx$

Optimal. Leaf size=245

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}$$

```
[Out] -1/4*arctan(((-a)^(1/4)-b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^
(1/4)-b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)-I*b^(1/4))^(1/2)*tan(x)/(-a)^(1
/8))/(-a)^(7/8)/((-a)^(1/4)-I*b^(1/4))^(1/2)-1/4*arctan(((-a)^(1/4)+I*b^(1/
4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+I*b^(1/4))^(1/2)-1/4*ar
ctan(((-a)^(1/4)+b^(1/4))^(1/2)*tan(x)/(-a)^(1/8))/(-a)^(7/8)/((-a)^(1/4)+b
^(1/4))^(1/2)
```

Rubi [A] time = 0.53, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.300$, Rules used = {3211, 3181, 203}

$$-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}}-\frac{\tan^{-1}\left(\frac{\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}\,\tan(x)}{(-a)^{5/8}}\right)}{4(-a)^{3/8}\sqrt{a\sqrt[4]{b}+(-a)^{5/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] -ArcTan[(Sqrt[(-a)^(1/4) - I*b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqr
t[(-a)^(1/4) - I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^(1/4) + I*b^(1/4)]*Tan[x])/(
-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + I*b^(1/4)]) - ArcTan[(Sqrt[(-a)^
(1/4) + b^(1/4)]*Tan[x])/(-a)^(1/8)]/(4*(-a)^(7/8)*Sqrt[(-a)^(1/4) + b^(1/4
)]) - ArcTan[(Sqrt[(-a)^(5/4) + a*b^(1/4)]*Tan[x])/(-a)^(5/8)]/(4*(-a)^(3/8
)*Sqrt[(-a)^(5/4) + a*b^(1/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a+b\sin^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{-a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{-a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= -\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}-i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+i\sqrt[4]{b}}} - \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{-a}}\right)}{4(-a)^{7/8}\sqrt{\sqrt[4]{-a}+\sqrt[4]{b}}} - \frac{\tan^{-1}}{4(-a}$$

Mathematica [C] time = 0.26, size = 174, normalized size = 0.71

$$8\text{RootSum}\left[\#1^8b - 8\#1^7b + 28\#1^6b - 56\#1^5b + 256\#1^4a + 70\#1^4b - 56\#1^3b + 28\#1^2b - 8\#1b + b\&, \frac{}{\#1^7b - 7}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[x]^8)^(-1),x]
```

```
[Out] 8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 + 256*a*#1^4 + 70*b*#1^4 - 56*
b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #
1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2 +
 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^8 + a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="giac")
```

```
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

maple [C] time = 0.26, size = 85, normalized size = 0.35

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a+b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7a+\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a+_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a+b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(x)^8 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(x)^8 + a), x)
```

mupad [B] time = 16.95, size = 816, normalized size = 3.33

$$\sum_{k=1}^{8} \ln\left(-b^5\ (a+b)\ \left(\mathrm{root}\left(16777216\, a^7\, b\, d^8 + 16777216\, a^8\, d^8 + 1048576\, a^6\, d^6 + 24576\, a^4\, d^4 + 256\, a^2\, d^2 + 1, d, k\right)\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(x)^8),x)
```

```
[Out] symsum(log(-2*b^5*(a + b)*(800*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^2*a^2 + 43008*roo
t(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 +
 256*a^2*d^2 + 1, d, k)^4*a^4 + 786432*root(16777216*a^7*b*d^8 + 16777216*a
^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^6 + 4
*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d
^4 + 256*a^2*d^2 + 1, d, k)*b*tan(x) - 6144*root(16777216*a^7*b*d^8 + 16777
216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^4*a^
3*b + 786432*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 +
 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^6*a^5*b - 9984*root(16777216*a^7*b*
d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1,
 d, k)^3*a^3*tan(x) - 557056*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1
048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^5*tan(x) - 1048
5760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a
^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^7*tan(x) + 32*root(16777216*a^7*b*d^8 +
 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k
)^2*a*b - 60*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 +
 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)*a*tan(x) - 768*root(16777216*a^7*b*
d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1,
 d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 +
1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^5*a^4*b*tan(x) - 1
0485760*root(16777216*a^7*b*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 2457
6*a^4*d^4 + 256*a^2*d^2 + 1, d, k)^7*a^6*b*tan(x) + 5))*root(16777216*a^7*b
*d^8 + 16777216*a^8*d^8 + 1048576*a^6*d^6 + 24576*a^4*d^4 + 256*a^2*d^2 + 1
, d, k), k, 1, 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a + b\sin^{8}(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(x)**8),x)
```

```
[Out] Integral(1/(a + b*sin(x)**8), x)
```

3.252 $\int \frac{1}{a-b\sin^5(x)}\,dx$

Optimal. Leaf size=379

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\ \ldots}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

```
[Out] -2/5*arctan((b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)-b^(2/5))^(1/2))/a^(4/5)/(
a^(2/5)-b^(2/5))^(1/2)+2/5*arctan(((-1)^(3/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(
a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)+(-1)^(1/5)*b^(2/5))^(1/
2)+2/5*arctan(((-1)^(1/5)*b^(1/5)+a^(1/5)*tan(1/2*x))/(a^(2/5)-(-1)^(2/5)*b
^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(2/5)*b^(2/5))^(1/2)-2/5*arctan(((-1)^
(4/5)*b^(1/5)-a^(1/5)*tan(1/2*x))/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2))/a^(4/
5)/(a^(2/5)+(-1)^(3/5)*b^(2/5))^(1/2)-2/5*arctan(((-1)^(2/5)*b^(1/5)-a^(1/5
)*tan(1/2*x))/(a^(2/5)-(-1)^(4/5)*b^(2/5))^(1/2))/a^(4/5)/(a^(2/5)-(-1)^(4/
5)*b^(2/5))^(1/2)
```

Rubi [A] time = 0.48, antiderivative size = 379, normalized size of antiderivative = 1.00, number of steps used = 17, number of rules used = 4, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.364, Rules used = {3213, 2660, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[5]{a}\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}\ \ldots}{\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{2/5}b^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[(b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - b^(2/5)]])/(5*a^(4/5
)*Sqrt[a^(2/5) - b^(2/5)]) - (2*ArcTan[((-1)^(2/5)*b^(1/5) - a^(1/5)*Tan[x/
2])/Sqrt[a^(2/5) - (-1)^(4/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(4/
5)*b^(2/5)]) - (2*ArcTan[((-1)^(4/5)*b^(1/5) - a^(1/5)*Tan[x/2])/Sqrt[a^(2/
5) + (-1)^(3/5)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(3/5)*b^(2/5)]) +
 (2*ArcTan[((-1)^(1/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) - (-1)^(2/5
)*b^(2/5)]])/(5*a^(4/5)*Sqrt[a^(2/5) - (-1)^(2/5)*b^(2/5)]) + (2*ArcTan[((-
1)^(3/5)*b^(1/5) + a^(1/5)*Tan[x/2])/Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)]])/(
5*a^(4/5)*Sqrt[a^(2/5) + (-1)^(1/5)*b^(2/5)])
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
```

```
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a-b\sin^5(x)}\,dx = \int \left(\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)\right)}+\frac{1}{5a^{4/5}\left(\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x\right.}\right.$$

$$=\frac{\int\frac{1}{\sqrt[5]{a}-\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}+\frac{\int\frac{1}{\sqrt[5]{a}+\sqrt[5]{-1}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}+\frac{\int\frac{1}{\sqrt[5]{a}-(-1)^{2/5}\sqrt[5]{b}\sin(x)}\,dx}{5a^{4/5}}+\frac{\int\frac{1}{\sqrt[5]{a}+(-1)^{3/5}\sqrt[5]{b}\sin(x)}\,d}{5a^{4/5}}$$

$$=\frac{2\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[5]{a}-2\sqrt[5]{b}x+\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+\frac{2\,\mathrm{Subst}\left(\int\frac{1}{\sqrt[5]{a}+2\sqrt[5]{-1}\sqrt[5]{b}x+\sqrt[5]{a}x^2}\,dx,x,\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}+$$

$$=-\frac{4\,\mathrm{Subst}\left(\int\frac{1}{-4\left(a^{2/5}-b^{2/5}\right)-x^2}\,dx,x,-2\sqrt[5]{b}+2\sqrt[5]{a}\tan\left(\frac{x}{2}\right)\right)}{5a^{4/5}}-\frac{4\,\mathrm{Subst}\left(\int\frac{1}{-4\left(a^{2/5}+\sqrt[5]{-1}b^{2/5}\right)-x^2}\,dx\right.}{5a^{4}}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}-(-1)^{4/5}b^{2/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}\sqrt[5]{b}-\sqrt[5]{a}\tan\left(\frac{x}{2}\right)}{\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}\right)}{5a^{4/5}\sqrt{a^{2/5}+(-1)^{3/5}b^{2/5}}}+-2$$

Mathematica [C] time = 0.19, size = 149, normalized size = 0.39

$$-\frac{8}{5}i\mathrm{RootSum}\left[i\#1^{10}b-5i\#1^8b+10i\#1^6b+32\#1^5a-10i\#1^4b+5i\#1^2b-ib\&,\frac{2\#1^3\tan^{-1}\left(\frac{\sin(x)}{\cos(x)-\#1}\right)-i\#1^3\log\left(\right.}{\#1^8b-4\#1^6b+6\#1^4b-16i\#}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^5)^(-1),x]
```

```
[Out] ((-8*I)/5)*RootSum[(-I)*b + (5*I)*b*#1^2 - (10*I)*b*#1^4 + 32*a*#1^5 + (10*
I)*b*#1^6 - (5*I)*b*#1^8 + I*b*#1^10 & , (2*ArcTan[Sin[x]/(Cos[x] - #1)]*#1
^3 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3)/(b - 4*b*#1^2 - (16*I)*a*#1^3 + 6*
b*#1^4 - 4*b*#1^6 + b*#1^8) & ]
```

fricas [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="fricas")
```

```
[Out] Exception raised: RuntimeError >> no explicit roots found
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\sin(x)^5-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^5 - a), x)
```

maple [C] time = 0.25, size = 109, normalized size = 0.29

$$\frac{\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^{10}+5a\_Z^{8}+10a\_Z^{6}-32b\_Z^{5}+10a\_Z^{4}+5a\_Z^{2}+a\right)}\frac{\left(\_R^{8}+4\_R^{6}+6\_R^{4}+4\_R^{2}+1\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{\_R^{9}a+4\_R^{7}a+6\_R^{5}a-16\_R^{4}b+4\_R^{3}a+\_Ra}\right)}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^5),x)
```

```
[Out] 1/5*sum((_R^8+4*_R^6+6*_R^4+4*_R^2+1)/(_R^9*a+4*_R^7*a+6*_R^5*a-16*_R^4*b+4
*_R^3*a+_R*a)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^10*a+5*_Z^8*a+10*_Z^6*a-32*_Z^
5*b+10*_Z^4*a+5*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{1}{b\sin(x)^{5}-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^5),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^5 - a), x)
```

mupad [B] time = 20.14, size = 1515, normalized size = 4.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^5),x)
```

```
[Out] symsum(log(10995116277760*a*b^7*(16*tan(x/2) + 56*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)*a + 5425*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)
^3*a^3 + 196875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8
*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^5 + 31718
75*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250
*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^7 + 19140625*root(9765
625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6
250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^9 + 1560*root(9765625*a^8*b^2*d^10
 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 12
5*a^2*d^2 - 1, d, k)^2*a^2*tan(x/2) + 57000*root(9765625*a^8*b^2*d^10 - 976
5625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*
d^2 - 1, d, k)^4*a^4*tan(x/2) + 925000*root(9765625*a^8*b^2*d^10 - 9765625*
a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 -
 1, d, k)^6*a^6*tan(x/2) + 5625000*root(9765625*a^8*b^2*d^10 - 9765625*a^10
*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
d, k)^8*a^8*tan(x/2) - 14000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10
- 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^
4*a^3*b - 175000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^5*b - 54
6875*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1562
50*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^7*b - 128*root(97656
```

```
25*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 62
50*a^4*d^4 - 125*a^2*d^2 - 1, d, k)*b*tan(x/2) + 1000000*root(9765625*a^8*b
^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d
^4 - 125*a^2*d^2 - 1, d, k)^7*a^5*b^2 - 18750000*root(9765625*a^8*b^2*d^10
- 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125
*a^2*d^2 - 1, d, k)^9*a^7*b^2 - 320*root(9765625*a^8*b^2*d^10 - 9765625*a^1
0*d^10 - 1953125*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1,
 d, k)^2*a*b - 6400*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125
*a^8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^3*a^2*b*t
an(x/2) - 100000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^
8*d^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^5*a^4*b*tan(
x/2) - 500000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d
^8 - 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x/2
) - 390625*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8
- 156250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^9*a^8*b*tan(x/2) +
 400000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^6*a^4*b^2*tan(x/2) -
5000000*root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 1
56250*a^6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k)^8*a^6*b^2*tan(x/2)))*
root(9765625*a^8*b^2*d^10 - 9765625*a^10*d^10 - 1953125*a^8*d^8 - 156250*a^
6*d^6 - 6250*a^4*d^4 - 125*a^2*d^2 - 1, d, k), k, 1, 10)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\sin^5(x)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**5),x)
```

```
[Out] Integral(1/(a - b*sin(x)**5), x)
```

3.253 $\int \frac{1}{a-b\sin^6(x)}\,dx$

Optimal. Leaf size=175

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

```
[Out] 1/3*arctan((a^(1/3)-b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-b^(1/3)
)^(1/2)+1/3*arctan((a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)*tan(x)/a^(1/6))/a^(5/
6)/(a^(1/3)+(-1)^(1/3)*b^(1/3))^(1/2)+1/3*arctan((a^(1/3)-(-1)^(2/3)*b^(1/3
))^(1/2)*tan(x)/a^(1/6))/a^(5/6)/(a^(1/3)-(-1)^(2/3)*b^(1/3))^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/3) - b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) -
b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*
a^(5/6)*Sqrt[a^(1/3) + (-1)^(1/3)*b^(1/3)]) + ArcTan[(Sqrt[a^(1/3) - (-1)^(
2/3)*b^(1/3)]*Tan[x])/a^(1/6)]/(3*a^(5/6)*Sqrt[a^(1/3) - (-1)^(2/3)*b^(1/3)
])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\sin^6(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1+\frac{\sqrt[3]{-1}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a} + \frac{\int \frac{1}{1-\frac{(-1)^{2/3}\sqrt[3]{b}\sin^2(x)}{\sqrt[3]{a}}}\,dx}{3a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1+\frac{\sqrt[3]{-1}\sqrt[3]{b}}{\sqrt[3]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{3a} + \frac{\text{Subst}\left(\int \frac{}{1+}\right.}{}$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}\,\tan(x)}{\sqrt[6]{a}}\right)}{3a^{5/6}\sqrt{\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}}}$$

Mathematica [C] time = 0.18, size = 148, normalized size = 0.85

$$\frac{8}{3}\text{RootSum}\left[\#1^6 b - 6\#1^5 b + 15\#1^4 b + 64\#1^3 a - 20\#1^3 b + 15\#1^2 b - 6\#1 b + b\&, \frac{2\#1^2\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right) - i\#1^2 \log}{\#1^5 b - 5\#1^4 b + 10\#1^3 b + 32\#1}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(a - b*Sin[x]^6)^(-1),x]
```

```
[Out] (8*RootSum[b - 6*b*#1 + 15*b*#1^2 + 64*a*#1^3 - 20*b*#1^3 + 15*b*#1^4 - 6*b
*#1^5 + b*#1^6 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^2 - I*Log[1 - 2*C
os[2*x]*#1 + #1^2]*#1^2)/(-b + 5*b*#1 + 32*a*#1^2 - 10*b*#1^2 + 10*b*#1^3 -
 5*b*#1^4 + b*#1^5) & ])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\sin(x)^6 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="giac")
```

```
[Out] integrate(-1/(b*sin(x)^6 - a), x)
```

maple [C] time = 1.42, size = 71, normalized size = 0.41

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a-b)\_Z^6+3a\_Z^4+3a\_Z^2+a\right)} \frac{\left(\_R^4+2\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^5 a-\_R^5 b+2\_R^3 a+\_R a}\right)}{6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^6),x)
```

```
[Out] 1/6*sum((_R^4+2*_R^2+1)/(_R^5*a-_R^5*b+2*_R^3*a+_R*a)*ln(tan(x)-_R),_R=Root
Of((a-b)*_Z^6+3*a*_Z^4+3*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\sin(x)^6 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^6),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^6 - a), x)
```

mupad [B] time = 16.07, size = 513, normalized size = 2.93

$$\sum_{k=1}^{6}\ln\left(-\frac{b^3\,(a-b)\,\left(\cot(x)-\mathrm{root}\left(46656\,a^5\,b\,d^6-46656\,a^6\,d^6-3888\,a^4\,d^4-108\,a^2\,d^2-1,d,k\right)\,a\,8+\mathrm{root}\left(4\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^6),x)
```

```
[Out] symsum(log(-(3*b^3*(a - b)*(cot(x) - 8*root(46656*a^5*b*d^6 - 46656*a^6*d^6
 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*a + 2*root(46656*a^5*b*d^6 - 46656
*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)*b - 504*root(46656*a^5*b*d
^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^3*a^3 - 7776*roo
t(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^5
*a^5 - 144*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^
2 - 1, d, k)^3*a^2*b + 7776*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4
*d^4 - 108*a^2*d^2 - 1, d, k)^5*a^4*b + 60*root(46656*a^5*b*d^6 - 46656*a^6
*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a^2*cot(x) + 864*root(46656*
a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^4*a^4*cot
(x) - 864*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2
 - 1, d, k)^4*a^3*b*cot(x) + 12*root(46656*a^5*b*d^6 - 46656*a^6*d^6 - 3888
*a^4*d^4 - 108*a^2*d^2 - 1, d, k)^2*a*b*cot(x)))/cot(x))*root(46656*a^5*b*d
^6 - 46656*a^6*d^6 - 3888*a^4*d^4 - 108*a^2*d^2 - 1, d, k), k, 1, 6)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\sin^6(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**6),x)
```

```
[Out] Integral(1/(a - b*sin(x)**6), x)
```

3.254 $\int \frac{1}{a-b\sin^8(x)}\,dx$

Optimal. Leaf size=213

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

```
[Out] 1/4*arctan((a^(1/4)-b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(1/4)-b^(1/4)
)^(1/2)+1/4*arctan((a^(1/4)-I*b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^(7/8)/(a^(1/
4)-I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+I*b^(1/4))^(1/2)*tan(x)/a^(1/8))/a^
(7/8)/(a^(1/4)+I*b^(1/4))^(1/2)+1/4*arctan((a^(1/4)+b^(1/4))^(1/2)*tan(x)/a
^(1/8))/a^(7/8)/(a^(1/4)+b^(1/4))^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 3, integrand size = 11, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}}+\frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+\sqrt[4]{b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a - b*Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[(Sqrt[a^(1/4) - b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) -
b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) - I*b^(1/4)]*Tan[x])/a^(1/8)]/(4*a^(7/8)*S
qrt[a^(1/4) - I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + I*b^(1/4)]*Tan[x])/a^(1/
8)]/(4*a^(7/8)*Sqrt[a^(1/4) + I*b^(1/4)]) + ArcTan[(Sqrt[a^(1/4) + b^(1/4)]
*Tan[x])/a^(1/8)]/(4*a^(7/8)*Sqrt[a^(1/4) + b^(1/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{a-b\sin^8(x)}\,dx = \frac{\int \frac{1}{1-\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1-\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{i\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a} + \frac{\int \frac{1}{1+\frac{\sqrt[4]{b}\sin^2(x)}{\sqrt[4]{a}}}\,dx}{4a}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \frac{\text{Subst}\left(\int \frac{1}{1+\left(1-\frac{i\sqrt[4]{b}}{\sqrt[4]{a}}\right)x^2}\,dx, x, \tan(x)\right)}{4a} + \text{Subst}\left(\int \frac{\quad}{1+}\right.$$

$$= \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}-i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}\,\tan(x)}{\sqrt[8]{a}}\right)}{4a^{7/8}\sqrt{\sqrt[4]{a}+i\sqrt[4]{b}}} + \frac{\tan^{-1}\left(\frac{\sqrt{}}{}\right.}{4a^{7/8}\sqrt{}}$$

Mathematica [C] time = 0.21, size = 174, normalized size = 0.82

$$-8\text{RootSum}\left[\#1^8b-8\#1^7b+28\#1^6b-56\#1^5b-256\#1^4a+70\#1^4b-56\#1^3b+28\#1^2b-8\#1b+b\&, \frac{}{\#1^7b-}\right.$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a - b*Sin[x]^8)^(-1),x]
[Out] -8*RootSum[b - 8*b*#1 + 28*b*#1^2 - 56*b*#1^3 - 256*a*#1^4 + 70*b*#1^4 - 56
*b*#1^5 + 28*b*#1^6 - 8*b*#1^7 + b*#1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] -
#1)]*#1^3 - I*Log[1 - 2*Cos[2*x]*#1 + #1^2]*#1^3)/(-b + 7*b*#1 - 21*b*#1^2
- 128*a*#1^3 + 35*b*#1^3 - 35*b*#1^4 + 21*b*#1^5 - 7*b*#1^6 + b*#1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int -\frac{1}{b\sin(x)^8-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="giac")
[Out] integrate(-1/(b*sin(x)^8 - a), x)
```

maple [C] time = 0.26, size = 88, normalized size = 0.41

$$\frac{\left(\sum_{\_R=\text{RootOf}\left((a-b)\_Z^8+4a\_Z^6+6a\_Z^4+4a\_Z^2+a\right)} \frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{\_R^7a-\_R^7b+3\_R^5a+3\_R^3a+\_Ra}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a-b*sin(x)^8),x)
```

```
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(_R^7*a-_R^7*b+3*_R^5*a+3*_R^3*a+_R*a)*ln(ta
n(x)-_R),_R=RootOf((a-b)*_Z^8+4*a*_Z^6+6*a*_Z^4+4*a*_Z^2+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{b\sin(x)^8 - a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)^8),x, algorithm="maxima")
```

```
[Out] -integrate(1/(b*sin(x)^8 - a), x)
```

mupad [B] time = 16.54, size = 818, normalized size = 3.84

$$\sum_{k=1}^{8}\ln\Big(-b^5\ (a-b)\ \Big(-\mathrm{root}\Big(16777216\,a^7\,b\,d^8 - 16777216\,a^8\,d^8 - 1048576\,a^6\,d^6 - 24576\,a^4\,d^4 - 256\,a^2\,d^2 - 1, d, k\Big)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a - b*sin(x)^8),x)
```

```
[Out] symsum(log(-2*b^5*(a - b)*(4*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1
048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*b*tan(x) - 43008*ro
ot(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4
- 256*a^2*d^2 - 1, d, k)^4*a^4 - 786432*root(16777216*a^7*b*d^8 - 16777216*
a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^6 -
800*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^
4*d^4 - 256*a^2*d^2 - 1, d, k)^2*a^2 - 6144*root(16777216*a^7*b*d^8 - 16777
216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^4*a^
3*b + 786432*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 -
 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^6*a^5*b + 9984*root(16777216*a^7*b*
d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1,
 d, k)^3*a^3*tan(x) + 557056*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1
048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^5*tan(x) + 1048
5760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a
^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^7*tan(x) + 32*root(16777216*a^7*b*d^8 -
 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k
)^2*a*b + 60*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 -
 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)*a*tan(x) - 768*root(16777216*a^7*b*
d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1,
 d, k)^3*a^2*b*tan(x) + 98304*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 -
1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^5*a^4*b*tan(x) - 1
0485760*root(16777216*a^7*b*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 2457
6*a^4*d^4 - 256*a^2*d^2 - 1, d, k)^7*a^6*b*tan(x) - 5))*root(16777216*a^7*b
*d^8 - 16777216*a^8*d^8 - 1048576*a^6*d^6 - 24576*a^4*d^4 - 256*a^2*d^2 - 1
, d, k), k, 1, 8)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a - b\sin^8(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a-b*sin(x)**8),x)
```

```
[Out] Integral(1/(a - b*sin(x)**8), x)
```

3.255 $\int \frac{1}{1+\sin^5(x)}\,dx$

Optimal. Leaf size=195

$$\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}$$

```
[Out] -1/5*cos(x)/(1+sin(x))-2/5*arctan((-1)^(3/5)*(1+(-1)^(2/5)*tan(1/2*x))/(1+(
-1)^(1/5))^(1/2))/(1+(-1)^(1/5))^(1/2)-2/5*arctan((-1)^(1/5)*(1+(-1)^(4/5)*
tan(1/2*x))/(1-(-1)^(2/5))^(1/2))/(1-(-1)^(2/5))^(1/2)+2/5*arctan(((-1)^(4/
5)+tan(1/2*x))/(1+(-1)^(3/5))^(1/2))/(1+(-1)^(3/5))^(1/2)+2/5*arctan(((-1)^
(2/5)+tan(1/2*x))/(1-(-1)^(4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

Rubi [A] time = 0.38, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}} = 0.625$, Rules used = {3213, 2648, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{2/5}}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{4/5}}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{3/5}\left((-1)^{2/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}\left((-1)^{4/5}\tan\left(\frac{x}{2}\right)+1\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (2*ArcTan[((-1)^(2/5) + Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^(
4/5)]) + (2*ArcTan[((-1)^(4/5) + Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[1
 + (-1)^(3/5)]) - (2*ArcTan[((-1)^(3/5)*(1 + (-1)^(2/5)*Tan[x/2]))/Sqrt[1 +
 (-1)^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) - (2*ArcTan[((-1)^(1/5)*(1 + (-1)^(
4/5)*Tan[x/2]))/Sqrt[1 - (-1)^(2/5)]])/(5*Sqrt[1 - (-1)^(2/5)]) - Cos[x]/(5
*(1 + Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1+\sin^5(x)}\,dx = \int \left(-\frac{1}{5(-1-\sin(x))} - \frac{1}{5\left(-1+\sqrt[5]{-1}\sin(x)\right)} - \frac{1}{5\left(-1-(-1)^{2/5}\sin(x)\right)} - \frac{1}{5\left(-1+(-1)^{3/5}\sin(x\right.}\right.$$

$$= -\left(\frac{1}{5}\int \frac{1}{-1-\sin(x)}\,dx\right) - \frac{1}{5}\int \frac{1}{-1+\sqrt[5]{-1}\sin(x)}\,dx - \frac{1}{5}\int \frac{1}{-1-(-1)^{2/5}\sin(x)}\,dx - \frac{1}{5}\int \frac{}{-1}$$

$$= -\frac{\cos(x)}{5(1+\sin(x))} - \frac{2}{5}\text{Subst}\left(\int \frac{1}{-1+2\sqrt[5]{-1}\,x-x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) - \frac{2}{5}\text{Subst}\left(\int \frac{1}{-1-2(-1)^{2/5}}\right.$$

$$= -\frac{\cos(x)}{5(1+\sin(x))} + \frac{4}{5}\text{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\right)-x^2}\,dx, x, 2(-1)^{3/5}-2\tan\left(\frac{x}{2}\right)\right) + \frac{4}{5}\text{Subst}\left(\int\right.$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}} - \frac{2\tan^{-1}\left(\frac{(-1)^{3/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{2/5}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{4/5}+t}{\sqrt{1+(-1}}\right)}{5\sqrt{1+(-1)^{3}}}$$

Mathematica [C] time = 0.15, size = 411, normalized size = 2.11

$$\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)} - \frac{1}{10}i\text{RootSum}\left[\#1^8-2i\#1^7-8\#1^6+14i\#1^5+30\#1^4-14i\#1^3-8\#1^2+2i\#1+1\&, \frac{2\#1^6\,\text{ta}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^5)^(-1),x]
```

```
[Out] (-1/10*I)*RootSum[1 + (2*I)*#1 - 8*#1^2 - (14*I)*#1^3 + 30*#1^4 + (14*I)*#1
^5 - 8*#1^6 - (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Lo
g[1 - 2*Cos[x]*#1 + #1^2] - (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 - 4*Log[1
 - 2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*L
og[1 - 2*Cos[x]*#1 + #1^2]*#1^2 + (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3
+ 40*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^
4 + (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 - (8*I)*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^5 - 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] -
 #1)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(I - 8*#1 - (21*I)*#1^2 +
60*#1^3 + (35*I)*#1^4 - 24*#1^5 - (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(
5*(Cos[x/2] + Sin[x/2]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0 x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.22, size = 133, normalized size = 0.68

$$\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^8-2\_Z^7+8\_Z^6-14\_Z^5+30\_Z^4-14\_Z^3+8\_Z^2-2\_Z+1\right)}\frac{\left(2\_R^6-3\_R^5+10\_R^4-10\_R^3+10\_R^2-3\_R+2\right)\ln\left(\tan\left(\frac{x}{2}\right)-\_R\right)}{4\_R^7-7\_R^6+24\_R^5-35\_R^4+60\_R^3-21\_R^2+8\_R-1}\right)}{5}-\frac{\cdots}{5\left(\tan\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^5),x)
```

```
[Out] 2/5*sum((2*_R^6-3*_R^5+10*_R^4-10*_R^3+10*_R^2-3*_R+2)/(4*_R^7-7*_R^6+24*_R
^5-35*_R^4+60*_R^3-21*_R^2+8*_R-1)*ln(tan(1/2*x)-_R),_R=RootOf(_Z^8-2*_Z^7+
8*_Z^6-14*_Z^5+30*_Z^4-14*_Z^3+8*_Z^2-2*_Z+1))-2/5/(tan(1/2*x)+1)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^5),x, algorithm="maxima")
```

```
[Out] -1/5*(5*(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)*integrate(-2/5*((4*cos(6*x) -
40*cos(4*x) + 4*cos(2*x) - sin(7*x) + 15*sin(5*x) - 15*sin(3*x) + sin(x))*c
os(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 8*sin(6*x) + 55*sin(4*x
) - 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) - 4
4*sin(5*x) + 44*sin(3*x) - 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*
cos(3*x) - 22*cos(x) - 505*sin(4*x) + 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^
2 + 10*(44*cos(2*x) - 101*sin(3*x) + 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x
)^2 + 44*(cos(x) - 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 - 4*(4*sin(x) - 1)
*cos(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 + (cos(7*x) - 15*cos(5*x) + 15*cos(3
*x) - cos(x) + 4*sin(6*x) - 40*sin(4*x) + 4*sin(2*x))*sin(8*x) + (16*cos(6*
x) - 110*cos(4*x) + 16*cos(2*x) + 44*sin(5*x) - 44*sin(3*x) + 4*sin(x) - 1)
*sin(7*x) - 2*sin(7*x)^2 + 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 55*sin
(4*x) - 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 + (1010*cos(4*x) - 176*cos(2*x
) + 420*sin(3*x) - 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 + 10*(101*cos(
3*x) - 11*cos(x) + 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 + (176*cos(2*x)
+ 44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 + 16*cos(x)*sin(2*x) - 32*sin(2
*x)^2 - 2*sin(x)^2 + sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 2*
sin(7*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2
+ 8*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 4*sin(6*x) + 15*sin(4*x) - 4*sin(2*
x))*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) - 14*sin(5*x) +
14*sin(3*x) - 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos
(x) - 15*sin(4*x) + 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x)
- 14*sin(3*x) + 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) - 4*si
n(2*x))*cos(3*x) - 196*cos(3*x)^2 - 16*(2*sin(x) - 1)*cos(2*x) - 64*cos(2*x
)^2 - 4*cos(x)^2 + 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) + 4*sin(6
*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 4*(8*cos(6*x) - 30*
cos(4*x) + 8*cos(2*x) + 14*sin(5*x) - 14*sin(3*x) + 2*sin(x) - 1)*sin(7*x)
- 4*sin(7*x)^2 + 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 15*sin(4*x) - 4*sin
(2*x))*sin(6*x) - 64*sin(6*x)^2 + 28*(30*cos(4*x) - 8*cos(2*x) + 14*sin(3*x
) - 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 + 120*(7*cos(3*x) - cos(x) + 4*
sin(2*x))*sin(4*x) - 900*sin(4*x)^2 + 28*(8*cos(2*x) + 2*sin(x) - 1)*sin(3*
x) - 196*sin(3*x)^2 + 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 + 4*s
in(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 + 2*sin(x) + 1)
```

mupad [B] time = 15.25, size = 3513, normalized size = 18.02

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^5 + 1),x)
```

```
[Out] 2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019
89888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2
)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) +
 (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25
)) - (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5
*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^
(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 18454
9376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1
/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 -
(452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 -
 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2
))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019898
88*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(
- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(
1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (4
36207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25))
- (5083496448*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (13086
22848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/5
 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 +
 184549376/25)) - (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^
(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364
672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2
))/25 - (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1
/2))/5 - 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*5^(1/2)*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1
/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((
2030043136*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3
01989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5
- 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2
) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376
/25)) - (989855744*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((30
1989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x
/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 -
 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2)
```

```
 - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/
25)) - (1627389952*5^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))
/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622
848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(
1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 -
 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 1
84549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5
- 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/
2)*tan(x/2))/125 - (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (
452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 +
16777216*(- (2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^
(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989
888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*
(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^
(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) - (
436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25))
 + (5083496448*5^(1/2)*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(
1/2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 - (13
08622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (
2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2)
)/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/2
5 + 184549376/25)) - (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (-
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (23
82364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)
^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216
*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan
(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*5^(1/2)
*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1
/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125
- (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^
(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/
2))/25 + 184549376/25)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) -
2/(5*(tan(x/2) + 1)) + 2*atanh((989855744*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 -
 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/12
5 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)
*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2
))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
 + 184549376/25)) - (2030043136*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 -
1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125
 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*
((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2)
)/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)) - (1627389952*5^(1/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/
50)^(1/2))/(25*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125
- (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*(
(2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))
/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 +
 184549376/25)) - (553648128*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5^(1/2))/5 -
1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)
*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (4529
84832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 167772
16*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 -
 1)^(1/2))/25 + 184549376/25)) + (184549376*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/
2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/
5 - (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5
 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777
216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*ta
```

n(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (5083496448*5^(1/2)
tan(x/2)(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(25*((301989888*ta
n(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^
(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
- (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^
(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*
tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^
(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 - (13
08622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^
(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 -
1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 1845
49376/25)) + (553648128*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (238236
4672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)
)/25 - (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2)
)/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*
5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)))*(- ((2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2) - 2*atanh((2030043136*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/
125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/
2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(1
/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/
25 + 184549376/25)) - (989855744*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (130862
2848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^(1/2
))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 - 1)^(
1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 18454937
6/25)) + (1627389952*5^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(
25*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (130862284
8*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^(1/2))/
5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2
) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/2
5)) - (553648128*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 -
1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125
 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*
((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2)
)/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
+ 184549376/25)) + (184549376*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2
))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*
5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
+ (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 -
 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/
2))/5 - 1)^(1/2))/25 + 184549376/25)) - (5083496448*5^(1/2)*tan(x/2)*(((2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(25*((301989888*tan(x/2))/5 - (23823
64672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2
))/25 + (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2
))/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2
*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*tan(x/2)*((2*5^(1/
2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((30198988
8*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((
2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))
/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (43620761
6*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648
128*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/5
0 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))
/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1
/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 - 16777216*((2*5^(
1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))
/25 + 184549376/25)))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{(\sin(x)+1)\left(\sin^4(x)-\sin^3(x)+\sin^2(x)-\sin(x)+1\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**5),x)
```

```
[Out] Integral(1/((sin(x) + 1)*(sin(x)**4 - sin(x)**3 + sin(x)**2 - sin(x) + 1)),
 x)
```

3.256 $\int \frac{1}{1+\sin^6(x)}\,dx$

Optimal. Leaf size=103

$$\frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\ \tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\ \tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

```
[Out] 1/6*x*2^(1/2)+1/6*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(1/2)+1/3*ar
ctan((1-(-1)^(1/3))^(1/2)*tan(x))/(1-(-1)^(1/3))^(1/2)+1/3*arctan((1+(-1)^(
2/3))^(1/2)*tan(x))/(1+(-1)^(2/3))^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$\frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\ \tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\ \tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{3\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] x/(3*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(3*Sqrt[2]
) + ArcTan[Sqrt[1 - (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(1/3)]) + ArcTan[S
qrt[1 + (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(2/3)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{1+\sin^6(x)}\,dx &= \frac{1}{3}\int \frac{1}{1+\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1-\sqrt[3]{-1}\sin^2(x)}\,dx+\frac{1}{3}\int \frac{1}{1+(-1)^{2/3}\sin^2(x)}\,dx\\ &= \frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+2x^2}\,dx,x,\tan(x)\right)+\frac{1}{3}\operatorname{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[3]{-1}\right)x^2}\,dx,x,\tan(x)\right)+\frac{1}{3}\operatorname{Subst}\\ &= \frac{x}{3\sqrt{2}}+\frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{3\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-\sqrt[3]{-1}}\ \tan(x)\right)}{3\sqrt{1-\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{2/3}}\ \tan(x)\right)}{3\sqrt{1+(-1)^{2/3}}}\end{aligned}$$

Mathematica [A] time = 0.16, size = 79, normalized size = 0.77

$$\frac{1}{12}\left(-2\sqrt{3}\tan^{-1}\left(\frac{1-2\tan(x)}{\sqrt{3}}\right)+2\sqrt{2}\tan^{-1}\left(\sqrt{2}\tan(x)\right)+2\sqrt{3}\tan^{-1}\left(\frac{2\tan(x)+1}{\sqrt{3}}\right)-\log(2-\sin(2x))+\mathrm{l}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^6)^(-1),x]
```

```
[Out] (-2*Sqrt[3]*ArcTan[(1 - 2*Tan[x])/Sqrt[3]] + 2*Sqrt[2]*ArcTan[Sqrt[2]*Tan[x
]] + 2*Sqrt[3]*ArcTan[(1 + 2*Tan[x])/Sqrt[3]] - Log[2 - Sin[2*x]] + Log[2 +
 Sin[2*x]])/12
```

fricas [A] time = 0.51, size = 138, normalized size = 1.34

$$\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)+\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)+\frac{1}{12}\sqrt{3}\arctan\left(\frac{4\sqrt{3}\cos(x)\sin(x)-\sqrt{3}}{3\left(2\cos(x)^2-1\right)}\right)-\frac{1}{12}\sqrt{2}\arctan\left(\frac{3\sqrt{2}\,\mathrm{c}}{4\,\mathrm{co}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="fricas")
```

```
[Out] 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) + sqrt(3))/(2*cos(x)^2 - 1
)) + 1/12*sqrt(3)*arctan(1/3*(4*sqrt(3)*cos(x)*sin(x) - sqrt(3))/(2*cos(x)^
2 - 1)) - 1/12*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqrt(2))/(cos(x)*
sin(x))) + 1/24*log(-cos(x)^4 + cos(x)^2 + 2*cos(x)*sin(x) + 1) - 1/24*log(
-cos(x)^4 + cos(x)^2 - 2*cos(x)*sin(x) + 1)
```

giac [B] time = 0.16, size = 185, normalized size = 1.80

$$\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)+\cos(2x)-2\sin(2x)+1}{\sqrt{3}\cos(2x)+\sqrt{3}-2\cos(2x)-\sin(2x)+2}\right)\right)+\frac{1}{6}\sqrt{3}\left(x+\arctan\left(-\frac{\sqrt{3}\sin(2x)-}{\sqrt{3}\cos(2x)+\sqrt{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="giac")
```

```
[Out] 1/6*sqrt(3)*(x + arctan(-(sqrt(3)*sin(2*x) + cos(2*x) - 2*sin(2*x) + 1)/(sq
rt(3)*cos(2*x) + sqrt(3) - 2*cos(2*x) - sin(2*x) + 2))) + 1/6*sqrt(3)*(x +
arctan(-(sqrt(3)*sin(2*x) - cos(2*x) - 2*sin(2*x) - 1)/(sqrt(3)*cos(2*x) +
sqrt(3) - 2*cos(2*x) + sin(2*x) + 2))) + 1/6*sqrt(2)*(x + arctan(-(sqrt(2)*
sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x) + sqrt(2) - 2*cos(2*x) + 2))) + 1/
12*log(tan(x)^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

maple [A] time = 0.18, size = 72, normalized size = 0.70

$$\frac{\arctan\left(\sqrt{2}\tan(x)\right)\sqrt{2}}{6}+\frac{\ln\left(\tan^2(x)+\tan(x)+1\right)}{12}+\frac{\sqrt{3}\arctan\left(\frac{(1+2\tan(x))\sqrt{3}}{3}\right)}{6}-\frac{\ln\left(\tan^2(x)-\tan(x)+1\right)}{12}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^6),x)
```

```
[Out] 1/6*arctan(2^(1/2)*tan(x))*2^(1/2)+1/12*ln(tan(x)^2+tan(x)+1)+1/6*3^(1/2)*a
rctan(1/3*(1+2*tan(x))*3^(1/2))-1/12*ln(tan(x)^2-tan(x)+1)+1/6*3^(1/2)*arct
an(1/3*(2*tan(x)-1)*3^(1/2))
```

maxima [A] time = 0.44, size = 71, normalized size = 0.69

$$\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)+1)\right)+\frac{1}{6}\sqrt{3}\arctan\left(\frac{1}{3}\sqrt{3}(2\tan(x)-1)\right)+\frac{1}{6}\sqrt{2}\arctan\left(\sqrt{2}\tan(x)\right)+\frac{1}{12}\log\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^6),x, algorithm="maxima")
```

```
[Out] 1/6*sqrt(3)*arctan(1/3*sqrt(3)*(2*tan(x) + 1)) + 1/6*sqrt(3)*arctan(1/3*sqr
t(3)*(2*tan(x) - 1)) + 1/6*sqrt(2)*arctan(sqrt(2)*tan(x)) + 1/12*log(tan(x)
^2 + tan(x) + 1) - 1/12*log(tan(x)^2 - tan(x) + 1)
```

mupad [B] time = 14.23, size = 98, normalized size = 0.95

$$\frac{\sqrt{2}\,\mathrm{atan}\left(\sqrt{2}\,\tan(x)\right)}{6}+\mathrm{atan}\left(\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}-\frac{1}{6}\mathrm{i}\right)-\mathrm{atan}\left(-\frac{\sqrt{3}\,\tan(x)}{2}+\frac{\tan(x)\,1\mathrm{i}}{2}\right)\left(\frac{\sqrt{3}}{6}+\frac{1}{6}\mathrm{i}\right)+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^6 + 1),x)
```

```
[Out] atan((tan(x)*1i)/2 + (3^(1/2)*tan(x))/2)*(3^(1/2)/6 - 1i/6) - atan((tan(x)*
1i)/2 - (3^(1/2)*tan(x))/2)*(3^(1/2)/6 + 1i/6) + (2^(1/2)*atan(2^(1/2)*tan(
x)))/6 + ((x - atan(tan(x)))*((2^(1/2)*pi)/6 + pi*(3^(1/2)/6 - 1i/6) + pi*(
3^(1/2)/6 + 1i/6)))/pi
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**6),x)
```

```
[Out] Timed out
```

3.257 $\int \frac{1}{1+\sin^8(x)}\,dx$

Optimal. Leaf size=218

$$\frac{1}{8}\left(\sqrt{1+\sqrt{4-2\sqrt{2}}}+\sqrt{2+2\sqrt[4]{2}+2\sqrt{1+\sqrt{2}}+2\sqrt{2+\sqrt{2}}}+\sqrt{1+\sqrt{4+2\sqrt{2}}}\right)\left(x-\tan^{-1}(\tan(x))\right)+\frac{\text{ta}}{}$$

```
[Out] 1/4*arctan((1-(-1)^(1/4))^(1/2)*tan(x))/(1-(-1)^(1/4))^(1/2)+1/4*arctan((1+
(-1)^(1/4))^(1/2)*tan(x))/(1+(-1)^(1/4))^(1/2)+1/4*arctan((1-(-1)^(3/4))^(1
/2)*tan(x))/(1-(-1)^(3/4))^(1/2)+1/4*arctan((1+(-1)^(3/4))^(1/2)*tan(x))/(1
+(-1)^(3/4))^(1/2)+1/8*(x-arctan(tan(x)))*((1+(4-2*2^(1/2))^(1/2))^(1/2)+(2
+2*2^(1/4)+2*(1+2^(1/2))^(1/2)+2*(2+2^(1/2))^(1/2))^(1/2)+(1+(4+2*2^(1/2))^
(1/2))^(1/2))
```

Rubi [A] time = 0.20, antiderivative size = 129, normalized size of antiderivative = 0.59, number of steps used = 9, number of rules used = 3, integrand size = 8, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3211, 3181, 203}

$$\frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}}+\frac{\tan^{-1}\left(\sqrt{1+(-1)^{3/4}}\,\text{ta}\right.}{4\sqrt{1+(-1)^{3/4}}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 + Sin[x]^8)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 - (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(1/4)]) + ArcTan[Sqrt[
1 + (-1)^(1/4)]*Tan[x]]/(4*Sqrt[1 + (-1)^(1/4)]) + ArcTan[Sqrt[1 - (-1)^(3/
4)]*Tan[x]]/(4*Sqrt[1 - (-1)^(3/4)]) + ArcTan[Sqrt[1 + (-1)^(3/4)]*Tan[x]]/
(4*Sqrt[1 + (-1)^(3/4)])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{1}{1+\sin^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sqrt[4]{-1}\,\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sqrt[4]{-1}\,\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-(-1)^{3/4}\sin^2(x)}\,dx + \frac{1}{4}\int \frac{}{1+}$$
$$= \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\left(1-\sqrt[4]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{4}\,\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[4]{-1}\right)x^2}\,dx, x, \tan(x)\right) +$$
$$= \frac{\tan^{-1}\left(\sqrt{1-\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1-\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1+\sqrt[4]{-1}}\,\tan(x)\right)}{4\sqrt{1+\sqrt[4]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1-(-1)^{3/4}}\,\tan(x)\right)}{4\sqrt{1-(-1)^{3/4}}} + \frac{\text{ta}}{}$$

Mathematica [C] time = 0.15, size = 141, normalized size = 0.65

$$8\text{RootSum}\left[\#1^8-8\#1^7+28\#1^6-56\#1^5+326\#1^4-56\#1^3+28\#1^2-8\#1+1\&, \frac{2\#1^3\tan^{-1}\left(\frac{\sin(2x)}{\cos(2x)-\#1}\right)-i\#1^3\,\text{lo}}{\#1^7-7\#1^6+21\#1^5-35\#1^4+}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 + Sin[x]^8)^(-1),x]
[Out] 8*RootSum[1 - 8*#1 + 28*#1^2 - 56*#1^3 + 326*#1^4 - 56*#1^5 + 28*#1^6 - 8*#
1^7 + #1^8 & , (2*ArcTan[Sin[2*x]/(Cos[2*x] - #1)]*#1^3 - I*Log[1 - 2*Cos[2
*x]*#1 + #1^2]*#1^3)/(-1 + 7*#1 - 21*#1^2 + 163*#1^3 - 35*#1^4 + 21*#1^5 -
7*#1^6 + #1^7) & ]
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$sage_0x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="giac")
[Out] sage0*x
```

maple [C] time = 0.20, size = 71, normalized size = 0.33

$$\frac{\left(\sum_{\_R=\text{RootOf}\left(2\_Z^8+4\_Z^6+6\_Z^4+4\_Z^2+1\right)}\frac{\left(\_R^6+3\_R^4+3\_R^2+1\right)\ln(\tan(x)-\_R)}{2\_R^7+3\_R^5+3\_R^3+\_R}\right)}{8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1+sin(x)^8),x)
[Out] 1/8*sum((_R^6+3*_R^4+3*_R^2+1)/(2*_R^7+3*_R^5+3*_R^3+_R)*ln(tan(x)-_R),_R=R
ootOf(2*_Z^8+4*_Z^6+6*_Z^4+4*_Z^2+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin(x)^8 + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)^8),x, algorithm="maxima")
```

```
[Out] integrate(1/(sin(x)^8 + 1), x)
```

mupad [B] time = 14.92, size = 945, normalized size = 4.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(sin(x)^8 + 1),x)
```

```
[Out] atan((tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/
2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/2)
*tan(x)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(-
 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (tan(x)*(- 2*
2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3
*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) + (2
^(1/2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) -
3)^(1/2)/512)))*((- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(
x)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(- 2*
2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/2)*tan(x)*
(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(- 2*2^(
1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) + (tan(x)*(- 2*2^(1/2
) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3*2^(
1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)^(1/2)/512)) - (2^(1/
2)*tan(x)*(- 2*2^(1/2) - 3)^(1/2)*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(
1/2)*5i)/(256*((3*2^(1/2)*(- 2*2^(1/2) - 3)^(1/2))/2048 - (- 2*2^(1/2) - 3)
^(1/2)/512)))*(- (- 2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i + atan((tan(
x)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1
/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*(- (2*
2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^
(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (tan(x)*(2*2^(1/2) - 3)^(1/2)*(
- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*7i)/(256*((3*2^(1/2)*(2*2^(1/2)
- 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*(2*2^(1/2)
 - 3)^(1/2)*(- (2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*5i)/(256*((3*2^(1/2
)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2)/512)))*(- (2*2^(1/2)
- 3)^(1/2)/128 - 1/128)^(1/2)*2i - atan((tan(x)*((2*2^(1/2) - 3)^(1/2)/128
- 1/128)^(1/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2
) - 3)^(1/2)/512)) + (2^(1/2)*tan(x)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1
/2)*1i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(1/2
)/512)) - (tan(x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)
^(1/2)*7i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(1/2) - 3)^(
1/2)/512)) - (2^(1/2)*tan(x)*(2*2^(1/2) - 3)^(1/2)*((2*2^(1/2) - 3)^(1/2)/1
28 - 1/128)^(1/2)*5i)/(256*((3*2^(1/2)*(2*2^(1/2) - 3)^(1/2))/2048 + (2*2^(
1/2) - 3)^(1/2)/512)))*((2*2^(1/2) - 3)^(1/2)/128 - 1/128)^(1/2)*2i
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sin^{8}(x) + 1}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1+sin(x)**8),x)
[Out] Integral(1/(sin(x)**8 + 1), x)
```

3.258 $\int \frac{1}{1-\sin^5(x)} \, dx$

Optimal. Leaf size=187

$$-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

```
[Out] 1/5*cos(x)/(1-sin(x))+2/5*arctan(((-1)^(3/5)+tan(1/2*x))/(1+(-1)^(1/5))^(1/
2))/(1+(-1)^(1/5))^(1/2)+2/5*arctan(((-1)^(1/5)+tan(1/2*x))/(1-(-1)^(2/5))^
(1/2))/(1-(-1)^(2/5))^(1/2)-2/5*arctan(((-1)^(4/5)-tan(1/2*x))/(1+(-1)^(3/5
))^(1/2))/(1+(-1)^(3/5))^(1/2)-2/5*arctan(((-1)^(2/5)-tan(1/2*x))/(1-(-1)^(
4/5))^(1/2))/(1-(-1)^(4/5))^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 5, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}} = 0.500$, Rules used = {3213, 2648, 2660, 618, 204}

$$-\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}}-\frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+\sqrt[5]{-1}}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}}+\frac{2\tan^{-1}\left(\frac{\tan\left(\frac{x}{2}\right)+(-1)^{3/5}}{\sqrt{1+\sqrt[5]{-1}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}+\frac{\cos(x)}{5(1-\sin(x))}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (-2*ArcTan[((-1)^(2/5) - Tan[x/2])/Sqrt[1 - (-1)^(4/5)]])/(5*Sqrt[1 - (-1)^
(4/5)]) - (2*ArcTan[((-1)^(4/5) - Tan[x/2])/Sqrt[1 + (-1)^(3/5)]])/(5*Sqrt[
1 + (-1)^(3/5)]) + (2*ArcTan[((-1)^(1/5) + Tan[x/2])/Sqrt[1 - (-1)^(2/5)]])
/(5*Sqrt[1 - (-1)^(2/5)]) + (2*ArcTan[((-1)^(3/5) + Tan[x/2])/Sqrt[1 + (-1)
^(1/5)]])/(5*Sqrt[1 + (-1)^(1/5)]) + Cos[x]/(5*(1 - Sin[x]))
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2648

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> -Simp[Cos[c +
 d*x]/(d*(b + a*Sin[c + d*x])), x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b
^2, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{1-\sin^5(x)}\,dx = \int \left(\frac{1}{5(1-\sin(x))} + \frac{1}{5\left(1+\sqrt[5]{-1}\,\sin(x)\right)} + \frac{1}{5\left(1-(-1)^{2/5}\sin(x)\right)} + \frac{1}{5\left(1+(-1)^{3/5}\sin(x)\right)} + \frac{}{5(}\right.$$
$$= \frac{1}{5}\int \frac{1}{1-\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1+\sqrt[5]{-1}\,\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1-(-1)^{2/5}\sin(x)}\,dx + \frac{1}{5}\int \frac{1}{1+(-1)^{3/5}}$$
$$= \frac{\cos(x)}{5(1-\sin(x))} + \frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{1+2\sqrt[5]{-1}\,x+x^2}\,dx, x, \tan\left(\frac{x}{2}\right)\right) + \frac{2}{5}\mathrm{Subst}\left(\int \frac{1}{1-2(-1)^{2/5}x+}\right.$$
$$= \frac{\cos(x)}{5(1-\sin(x))} - \frac{4}{5}\mathrm{Subst}\left(\int \frac{1}{-4\left(1+\sqrt[5]{-1}\right)-x^2}\,dx, x, 2(-1)^{3/5}+2\tan\left(\frac{x}{2}\right)\right) - \frac{4}{5}\mathrm{Subst}\left(\int \frac{}{-}\right.$$
$$= -\frac{2\tan^{-1}\left(\frac{(-1)^{2/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{4/5}}}\right)}{5\sqrt{1-(-1)^{4/5}}} - \frac{2\tan^{-1}\left(\frac{(-1)^{4/5}-\tan\left(\frac{x}{2}\right)}{\sqrt{1+(-1)^{3/5}}}\right)}{5\sqrt{1+(-1)^{3/5}}} + \frac{2\tan^{-1}\left(\frac{\sqrt[5]{-1}+\tan\left(\frac{x}{2}\right)}{\sqrt{1-(-1)^{2/5}}}\right)}{5\sqrt{1-(-1)^{2/5}}} + \frac{2\tan^{-1}\left(\frac{(-1)^{3/5}+\mathrm{t}}{\sqrt{1+\sqrt[5]{}}}\right)}{5\sqrt{1+\sqrt[5]{-1}}}$$

Mathematica [C] time = 0.14, size = 413, normalized size = 2.21

$$\frac{2\sin\left(\frac{x}{2}\right)}{5\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)} + \frac{1}{10}i\mathrm{RootSum}\left[\#1^8+2i\#1^7-8\#1^6-14i\#1^5+30\#1^4+14i\#1^3-8\#1^2-2i\#1+1\&, \frac{2\#1^6\,\mathrm{ta}}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^5)^(-1),x]
```

```
[Out] (I/10)*RootSum[1 - (2*I)*#1 - 8*#1^2 + (14*I)*#1^3 + 30*#1^4 - (14*I)*#1^5
- 8*#1^6 + (2*I)*#1^7 + #1^8 & , (-2*ArcTan[Sin[x]/(Cos[x] - #1)] + I*Log[1
 - 2*Cos[x]*#1 + #1^2] + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1 + 4*Log[1 -
2*Cos[x]*#1 + #1^2]*#1 + 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^2 - (15*I)*Log[
1 - 2*Cos[x]*#1 + #1^2]*#1^2 - (80*I)*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^3 - 4
0*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^3 - 30*ArcTan[Sin[x]/(Cos[x] - #1)]*#1^4 +
 (15*I)*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^4 + (8*I)*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^5 + 4*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^5 + 2*ArcTan[Sin[x]/(Cos[x] - #1
)]*#1^6 - I*Log[1 - 2*Cos[x]*#1 + #1^2]*#1^6)/(-I - 8*#1 + (21*I)*#1^2 + 60
*#1^3 - (35*I)*#1^4 - 24*#1^5 + (7*I)*#1^6 + 4*#1^7) & ] + (2*Sin[x/2])/(5*
(Cos[x/2] - Sin[x/2]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$sage_0x$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="giac")
```

```
[Out] sage0*x
```

maple [C] time = 0.21, size = 133, normalized size = 0.71

$$-\frac{2}{5\left(\tan\left(\frac{x}{2}\right)-1\right)}+\frac{2\left(\sum_{\_R=\mathrm{RootOf}\left(\_Z^{8}+2\_Z^{7}+8\_Z^{6}+14\_Z^{5}+30\_Z^{4}+14\_Z^{3}+8\_Z^{2}+2\_Z+1\right)}\frac{\left(2\_R^{6}+3\_R^{5}+10\_R^{4}+10\_R^{3}+10\_R^{2}+3\_R+2\right)}{4\_R^{7}+7\_R^{6}+24\_R^{5}+35\_R^{4}+60\_R^{3}+21\_}\right.}{5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^5),x)
```

```
[Out] -2/5/(tan(1/2*x)-1)+2/5*sum((2*_R^6+3*_R^5+10*_R^4+10*_R^3+10*_R^2+3*_R+2)/
(4*_R^7+7*_R^6+24*_R^5+35*_R^4+60*_R^3+21*_R^2+8*_R+1)*ln(tan(1/2*x)-_R),_R
=RootOf(_Z^8+2*_Z^7+8*_Z^6+14*_Z^5+30*_Z^4+14*_Z^3+8*_Z^2+2*_Z+1))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^5),x, algorithm="maxima")
```

```
[Out] 1/5*(5*(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)*integrate(2/5*((4*cos(6*x) - 40
*cos(4*x) + 4*cos(2*x) + sin(7*x) - 15*sin(5*x) + 15*sin(3*x) - sin(x))*cos
(8*x) + 2*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) + 8*sin(6*x) - 55*sin(4*x)
+ 8*sin(2*x))*cos(7*x) - 2*cos(7*x)^2 + 4*(110*cos(4*x) - 16*cos(2*x) + 44*
sin(5*x) - 44*sin(3*x) + 4*sin(x) + 1)*cos(6*x) - 32*cos(6*x)^2 + 2*(210*co
s(3*x) - 22*cos(x) + 505*sin(4*x) - 88*sin(2*x))*cos(5*x) - 210*cos(5*x)^2
+ 10*(44*cos(2*x) + 101*sin(3*x) - 11*sin(x) - 4)*cos(4*x) - 1200*cos(4*x)^
2 + 44*(cos(x) + 4*sin(2*x))*cos(3*x) - 210*cos(3*x)^2 + 4*(4*sin(x) + 1)*c
os(2*x) - 32*cos(2*x)^2 - 2*cos(x)^2 - (cos(7*x) - 15*cos(5*x) + 15*cos(3*x
) - cos(x) - 4*sin(6*x) + 40*sin(4*x) - 4*sin(2*x))*sin(8*x) - (16*cos(6*x)
 - 110*cos(4*x) + 16*cos(2*x) - 44*sin(5*x) + 44*sin(3*x) - 4*sin(x) - 1)*s
in(7*x) - 2*sin(7*x)^2 - 8*(22*cos(5*x) - 22*cos(3*x) + 2*cos(x) - 55*sin(4
*x) + 8*sin(2*x))*sin(6*x) - 32*sin(6*x)^2 - (1010*cos(4*x) - 176*cos(2*x)
- 420*sin(3*x) + 44*sin(x) + 15)*sin(5*x) - 210*sin(5*x)^2 - 10*(101*cos(3*
x) - 11*cos(x) - 44*sin(2*x))*sin(4*x) - 1200*sin(4*x)^2 - (176*cos(2*x) -
44*sin(x) - 15)*sin(3*x) - 210*sin(3*x)^2 - 16*cos(x)*sin(2*x) - 32*sin(2*x
)^2 - 2*sin(x)^2 - sin(x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) + 2*si
n(7*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*cos(8*x) - cos(8*x)^2 +
8*(7*cos(5*x) - 7*cos(3*x) + cos(x) + 4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x)
)*cos(7*x) - 4*cos(7*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 14*sin(5*x) - 14
*sin(3*x) + 2*sin(x) + 1)*cos(6*x) - 64*cos(6*x)^2 + 56*(7*cos(3*x) - cos(x
) + 15*sin(4*x) - 4*sin(2*x))*cos(5*x) - 196*cos(5*x)^2 + 60*(8*cos(2*x) +
14*sin(3*x) - 2*sin(x) - 1)*cos(4*x) - 900*cos(4*x)^2 + 56*(cos(x) + 4*sin(
2*x))*cos(3*x) - 196*cos(3*x)^2 + 16*(2*sin(x) + 1)*cos(2*x) - 64*cos(2*x)^
2 - 4*cos(x)^2 - 4*(cos(7*x) - 7*cos(5*x) + 7*cos(3*x) - cos(x) - 4*sin(6*x
) + 15*sin(4*x) - 4*sin(2*x))*sin(8*x) - sin(8*x)^2 - 4*(8*cos(6*x) - 30*co
s(4*x) + 8*cos(2*x) - 14*sin(5*x) + 14*sin(3*x) - 2*sin(x) - 1)*sin(7*x) -
4*sin(7*x)^2 - 32*(7*cos(5*x) - 7*cos(3*x) + cos(x) - 15*sin(4*x) + 4*sin(2
*x))*sin(6*x) - 64*sin(6*x)^2 - 28*(30*cos(4*x) - 8*cos(2*x) - 14*sin(3*x)
+ 2*sin(x) + 1)*sin(5*x) - 196*sin(5*x)^2 - 120*(7*cos(3*x) - cos(x) - 4*si
n(2*x))*sin(4*x) - 900*sin(4*x)^2 - 28*(8*cos(2*x) - 2*sin(x) - 1)*sin(3*x)
 - 196*sin(3*x)^2 - 32*cos(x)*sin(2*x) - 64*sin(2*x)^2 - 4*sin(x)^2 - 4*sin
(x) - 1), x) + 2*cos(x))/(cos(x)^2 + sin(x)^2 - 2*sin(x) + 1)
```

mupad [B] time = 14.44, size = 3513, normalized size = 18.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^5 - 1),x)
```

```
[Out] 2*atanh((989855744*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019
89888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2
)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1
)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) +
 (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25
)) + (2030043136*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5
*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^
(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 18454
9376/25)) + (1627389952*5^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/
2))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
- 184549376/25)) + (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5
 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1
/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 +
(452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 +
 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (184549376*5^(1/2)*(- (2*5^(1/2
))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((3019898
88*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(
- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(
1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (4
36207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25))
+ (5083496448*5^(1/2)*tan(x/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(25*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (13086
22848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5
 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 -
 184549376/25)) + (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^
(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 + (2382364
672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2
))/25 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1
/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) - (553648128*5^(1/2)*tan(
x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1
/2))/(5*((301989888*tan(x/2))/5 + (2382364672*5^(1/2)*tan(x/2))/125 + (1308
622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(- (2*
5^(1/2))/5 - 1)^(1/2))/25 - (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/
5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25
- 184549376/25)))*((- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) - 2*atanh((
989855744*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((1308622848*
tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2))/12
5 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2)
)/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (43620
7616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (2
030043136*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((13
08622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan
(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 -
 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2)
```

```
 + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/
25)) + (1627389952*5^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))
/(25*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^
(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(
1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 -
 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 1
84549376/25)) - (553648128*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5
- 1)^(1/2)/50 - 1/50)^(1/2))/(5*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)
^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (
452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 +
16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^
(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) - (184549376*5^(1/2)*(- (2*5^(1/2)
)/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((130862
2848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2
))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^
(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (
436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25))
 + (5083496448*5^(1/2)*tan(x/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(
1/2))/(25*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (23823646
72*5^(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)*(- (
2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^(1/2)
)/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/2
5 + 184549376/25)) - (553648128*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (-
(2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((1308622848*tan(x/2)*(- (2*5
^(1/2))/5 - 1)^(1/2))/25 - (2382364672*5^(1/2)*tan(x/2))/125 - (301989888*t
an(x/2))/5 + (452984832*5^(1/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216
*5^(1/2))/5 + 16777216*(- (2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan
(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + 184549376/25)) + (553648128*5^(1/2)
*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2)*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1
/50)^(1/2))/(5*((1308622848*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/2))/25 - (238
2364672*5^(1/2)*tan(x/2))/125 - (301989888*tan(x/2))/5 + (452984832*5^(1/2)
*(- (2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*(- (2*5^
(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*(- (2*5^(1/2))/5 - 1)^(1/
2))/25 + 184549376/25)))*(- (- (2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2) -
2/(5*(tan(x/2) - 1)) - 2*atanh((1627389952*5^(1/2)*(- ((2*5^(1/2))/5 - 1)^(
1/2)/50 - 1/50)^(1/2))/(25*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*ta
n(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (4529848
32*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*
((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)
^(1/2))/25 - 184549376/25)) - (2030043136*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(
1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan
(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (45298483
2*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*(
(2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^
(1/2))/25 - 184549376/25)) - (989855744*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1
/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125
- (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*(
(2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))
/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 -
 184549376/25)) + (553648128*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5^(1/2))/5 -
1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)
*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (4529
84832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 167772
16*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 -
 1)^(1/2))/25 - 184549376/25)) - (184549376*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/
2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/
5 - (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5
 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777
216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*ta
```

```
n(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (5083496448*5^(1/2)
*tan(x/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(25*((301989888*ta
n(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^
(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
+ (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^
(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (553648128*
tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^
(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 - (13
08622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (452984832*5^(1/2)*((2*5^
(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 - 16777216*((2*5^(1/2))/5 -
1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 1845
49376/25)) + (553648128*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(- ((2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (238236
4672*5^(1/2)*tan(x/2))/125 - (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)
)/25 + (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2)
)/5 - 16777216*((2*5^(1/2))/5 - 1)^(1/2) + (436207616*5^(1/2)*tan(x/2)*((2*
5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)))*(- ((2*5^(1/2))/5 - 1)^(1/2)/50
 - 1/50)^(1/2) + 2*atanh((989855744*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(
1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (130
8622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(
1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1
)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 18454
9376/25)) + (2030043136*tan(x/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2
))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (130862
2848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(1/2
))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1)^(
1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 18454937
6/25)) - (1627389952*5^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(
25*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (130862284
8*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(1/2))/
5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2
) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/2
5)) + (553648128*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 -
1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125
 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*
((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2)
)/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
- 184549376/25)) - (184549376*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2
))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*
5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25
- (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 +
 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/
2))/5 - 1)^(1/2))/25 - 184549376/25)) - (5083496448*5^(1/2)*tan(x/2)*(((2*5
^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(25*((301989888*tan(x/2))/5 - (23823
64672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2
))/25 - (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2
))/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2
*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (553648128*tan(x/2)*((2*5^(1/
2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2))/(5*((30198988
8*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))/125 + (1308622848*tan(x/2)*((
2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1/2)*((2*5^(1/2))/5 - 1)^(1/2))
/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(1/2))/5 - 1)^(1/2) - (43620761
6*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - 184549376/25)) + (553648
128*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2)*(((2*5^(1/2))/5 - 1)^(1/2)/5
0 - 1/50)^(1/2))/(5*((301989888*tan(x/2))/5 - (2382364672*5^(1/2)*tan(x/2))
/125 + (1308622848*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 - (452984832*5^(1
/2)*((2*5^(1/2))/5 - 1)^(1/2))/25 + (16777216*5^(1/2))/5 + 16777216*((2*5^(
1/2))/5 - 1)^(1/2) - (436207616*5^(1/2)*tan(x/2)*((2*5^(1/2))/5 - 1)^(1/2))
/25 - 184549376/25)))*(((2*5^(1/2))/5 - 1)^(1/2)/50 - 1/50)^(1/2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{1}{\sin^{5}(x) - 1}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**5),x)
[Out] -Integral(1/(sin(x)**5 - 1), x)
```

3.259 $\int \frac{1}{1-\sin^6(x)}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

```
[Out] 1/3*arctan((1+(-1)^(1/3))^(1/2)*tan(x))/(1+(-1)^(1/3))^(1/2)+1/3*arctan((1-
(-1)^(2/3))^(1/2)*tan(x))/(1-(-1)^(2/3))^(1/2)+1/3*tan(x)
```

Rubi [A] time = 0.14, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$\frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}}+\frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}}+\frac{\tan(x)}{3}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^6)^(-1),x]
```

```
[Out] ArcTan[Sqrt[1 + (-1)^(1/3)]*Tan[x]]/(3*Sqrt[1 + (-1)^(1/3)]) + ArcTan[Sqrt[
1 - (-1)^(2/3)]*Tan[x]]/(3*Sqrt[1 - (-1)^(2/3)]) + Tan[x]/3
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin^6(x)}\,dx = \frac{1}{3}\int \frac{1}{1-\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1+\sqrt[3]{-1}\sin^2(x)}\,dx + \frac{1}{3}\int \frac{1}{1-(-1)^{2/3}\sin^2(x)}\,dx$$
$$= \frac{1}{3}\int \sec^2(x)\,dx + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1+\sqrt[3]{-1}\right)x^2}\,dx, x, \tan(x)\right) + \frac{1}{3}\text{Subst}\left(\int \frac{1}{1+\left(1-(-1\right.}\right.$$
$$= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}} - \frac{1}{3}\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)$$
$$= \frac{\tan^{-1}\left(\sqrt{1+\sqrt[3]{-1}}\,\tan(x)\right)}{3\sqrt{1+\sqrt[3]{-1}}} + \frac{\tan^{-1}\left(\sqrt{1-(-1)^{2/3}}\,\tan(x)\right)}{3\sqrt{1-(-1)^{2/3}}} + \frac{\tan(x)}{3}$$

Mathematica [C] time = 0.28, size = 117, normalized size = 1.65

$$\frac{\cos(x)(-8\cos(2x)+\cos(4x)+15)\left(-6\sin(x)+i\sqrt[4]{-3}\left(\sqrt{3}+3i\right)\cos(x)\tan^{-1}\left(\frac{1}{2}\sqrt[4]{-\frac{1}{3}}\left(\sqrt{3}-3i\right)\tan(x)\right)+\sqrt[4]{\cdot}\right.}{144\left(\sin^6(x)-1\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^6)^(-1),x]
[Out] (Cos[x]*(15 - 8*Cos[2*x] + Cos[4*x])*(I*(-3)^(1/4)*(3*I + Sqrt[3])*ArcTan[(
(-1/3)^(1/4)*(-3*I + Sqrt[3])*Tan[x])/2]*Cos[x] + (-3)^(1/4)*(-3*I + Sqrt[3
])*ArcTan[((-1)^(3/4)*(3*I + Sqrt[3])*Tan[x])/(2*3^(1/4))]*Cos[x] - 6*Sin[x
]))/(144*(-1 + Sin[x]^6))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="fricas")
[Out] Timed out
```

giac [B] time = 0.15, size = 197, normalized size = 2.77

$$\frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor - \arctan\left(-\frac{3\left(\frac{1}{3}\right)^{\frac{3}{4}}\left(\left(\frac{1}{3}\right)^{\frac{1}{4}}\left(\sqrt{6}-\sqrt{2}\right)+4\tan(x)\right)}{\sqrt{6}+\sqrt{2}}\right)\right)\sqrt{6\sqrt{3}+9} + \frac{1}{18}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor + \arctan\left(-\cdots\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="giac")
[Out] 1/18*(pi*floor(x/pi + 1/2) - arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) -
sqrt(2)) + 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/18*(pi*f
```

```
loor(x/pi + 1/2) + arctan(-3*(1/3)^(3/4)*((1/3)^(1/4)*(sqrt(6) - sqrt(2)) -
 4*tan(x))/(sqrt(6) + sqrt(2))))*sqrt(6*sqrt(3) + 9) + 1/36*sqrt(6*sqrt(3)
- 9)*log(1/2*(sqrt(6)*(1/3)^(1/4) - sqrt(2)*(1/3)^(1/4))*tan(x) + tan(x)^2
+ sqrt(1/3)) - 1/36*sqrt(6*sqrt(3) - 9)*log(-1/2*(sqrt(6)*(1/3)^(1/4) - sqr
t(2)*(1/3)^(1/4))*tan(x) + tan(x)^2 + sqrt(1/3)) + 1/3*tan(x)
```

maple [B] time = 0.45, size = 255, normalized size = 3.59

$$\frac{\tan(x)}{3}+\frac{\sqrt{3}\,\sqrt{2\sqrt{3}-3}\,\ln\left(\sqrt{3}+3\left(\tan^2(x)\right)+\sqrt{2\sqrt{3}-3}\,\sqrt{3}\,\tan(x)\right)}{36}+\frac{\arctan\left(\frac{6\tan(x)+\sqrt{2\sqrt{3}-3}\,\sqrt{3}}{\sqrt{6\sqrt{3}+9}}\right)\sqrt{3}}{3\sqrt{6\sqrt{3}+9}}+\frac{\mathrm{arct}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^6),x)
```

```
[Out] 1/3*tan(x)+1/36*3^(1/2)*(2*3^(1/2)-3)^(1/2)*ln(3^(1/2)+3*tan(x)^2+(2*3^(1/2
)-3)^(1/2)*3^(1/2)*tan(x))+1/3/(6*3^(1/2)+9)^(1/2)*arctan((6*tan(x)+(2*3^(1
/2)-3)^(1/2)*3^(1/2))/(6*3^(1/2)+9)^(1/2))*3^(1/2)+1/2/(6*3^(1/2)+9)^(1/2)*
arctan((6*tan(x)+(2*3^(1/2)-3)^(1/2)*3^(1/2))/(6*3^(1/2)+9)^(1/2))-1/36*3^(
1/2)*(2*3^(1/2)-3)^(1/2)*ln(-(2*3^(1/2)-3)^(1/2)*3^(1/2)*tan(x)+3*tan(x)^2+
3^(1/2))+1/3/(6*3^(1/2)+9)^(1/2)*arctan((-(2*3^(1/2)-3)^(1/2)*3^(1/2)+6*tan
(x))/(6*3^(1/2)+9)^(1/2))*3^(1/2)+1/2/(6*3^(1/2)+9)^(1/2)*arctan((-(2*3^(1/
2)-3)^(1/2)*3^(1/2)+6*tan(x))/(6*3^(1/2)+9)^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{-4\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\int\frac{(\cos(6x)-10\cos(4x)+\cos(2x))\cos(8x)+(110\cos(4x)-16\cos}{2(8\cos(6x)-30\cos(4x)+8\cos(2x)-1)\cos(8x)-\cos(8x)^2+16(30\cos(4x)-8\cos(2x}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^6),x, algorithm="maxima")
```

```
[Out] -1/3*(3*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(-4/3*((cos(6*x
) - 10*cos(4*x) + cos(2*x))*cos(8*x) + (110*cos(4*x) - 16*cos(2*x) + 1)*cos
(6*x) - 8*cos(6*x)^2 + 10*(11*cos(2*x) - 1)*cos(4*x) - 300*cos(4*x)^2 - 8*c
os(2*x)^2 + (sin(6*x) - 10*sin(4*x) + sin(2*x))*sin(8*x) + 2*(55*sin(4*x) -
 8*sin(2*x))*sin(6*x) - 8*sin(6*x)^2 - 300*sin(4*x)^2 + 110*sin(4*x)*sin(2*
x) - 8*sin(2*x)^2 + cos(2*x))/(2*(8*cos(6*x) - 30*cos(4*x) + 8*cos(2*x) - 1
)*cos(8*x) - cos(8*x)^2 + 16*(30*cos(4*x) - 8*cos(2*x) + 1)*cos(6*x) - 64*c
os(6*x)^2 + 60*(8*cos(2*x) - 1)*cos(4*x) - 900*cos(4*x)^2 - 64*cos(2*x)^2 +
 4*(4*sin(6*x) - 15*sin(4*x) + 4*sin(2*x))*sin(8*x) - sin(8*x)^2 + 32*(15*s
in(4*x) - 4*sin(2*x))*sin(6*x) - 64*sin(6*x)^2 - 900*sin(4*x)^2 + 480*sin(4
*x)*sin(2*x) - 64*sin(2*x)^2 + 16*cos(2*x) - 1), x) - 2*sin(2*x))/(cos(2*x)
^2 + sin(2*x)^2 + 2*cos(2*x) + 1)
```

mupad [B] time = 14.21, size = 99, normalized size = 1.39

$$\frac{\tan(x)}{3}-\frac{\sqrt{6}\,\mathrm{atan}\left(3^{1/4}\sqrt{6}\,\tan(x)\left(\frac{1}{4}-\frac{1}{4}\mathrm{i}\right)+3^{3/4}\sqrt{6}\,\tan(x)\left(\frac{1}{12}+\frac{1}{12}\mathrm{i}\right)\right)\left(3^{1/4}\,(1+1\mathrm{i})+3^{3/4}\,(-1+1\mathrm{i})\right)1\mathrm{i}}{36}+\frac{\sqrt{6}\,\mathrm{a}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^6 - 1),x)
```

```
[Out] tan(x)/3 - (6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 - 1i/4) + 3^(3/4)*6^(1
/2)*tan(x)*(1/12 + 1i/12))*(3^(1/4)*(1 + 1i) - 3^(3/4)*(1 - 1i))*1i)/36 + (
```

```
6^(1/2)*atan(3^(1/4)*6^(1/2)*tan(x)*(1/4 + 1i/4) + 3^(3/4)*6^(1/2)*tan(x)*(
1/12 - 1i/12))*(3^(1/4)*(1 - 1i) - 3^(3/4)*(1 + 1i))*1i)/36
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**6),x)
```

```
[Out] Timed out
```

3.260 $\int \frac{1}{1-\sin^8(x)}\,dx$

Optimal. Leaf size=89

$$\frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

```
[Out] 1/4*arctan((1-I)^(1/2)*tan(x))/(1-I)^(1/2)+1/4*arctan((1+I)^(1/2)*tan(x))/(
1+I)^(1/2)+1/8*x*2^(1/2)+1/8*arctan(cos(x)*sin(x)/(1+sin(x)^2+2^(1/2)))*2^(
1/2)+1/4*tan(x)
```

Rubi [A] time = 0.08, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 10, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.600, Rules used = {3211, 3181, 203, 3175, 3767, 8}

$$\frac{x}{4\sqrt{2}}+\frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}}+\frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}}+\frac{\tan(x)}{4}+\frac{\tan^{-1}\left(\frac{\sin(x)\cos(x)}{\sin^2(x)+\sqrt{2}+1}\right)}{4\sqrt{2}}$$

Antiderivative was successfully verified.

```
[In] Int[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] x/(4*Sqrt[2]) + ArcTan[(Cos[x]*Sin[x])/(1 + Sqrt[2] + Sin[x]^2)]/(4*Sqrt[2]
) + ArcTan[Sqrt[1 - I]*Tan[x]]/(4*Sqrt[1 - I]) + ArcTan[Sqrt[1 + I]*Tan[x]]
/(4*Sqrt[1 + I]) + Tan[x]/4
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3181

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(-1), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[1/(a + (a + b)*ff^2*x^2
), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x]
```

Rule 3211

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(-1), x_Symbol] :> Module[{
k}, Dist[2/(a*n), Sum[Int[1/(1 - Sin[e + f*x]^2/((-1)^((4*k)/n)*Rt[-(a/b),
n/2])), x], {k, 1, n/2}], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[n/2]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
```

```
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\int \frac{1}{1-\sin^8(x)}\,dx = \frac{1}{4}\int \frac{1}{1-\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1-i\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+i\sin^2(x)}\,dx + \frac{1}{4}\int \frac{1}{1+\sin^2(x)}\,dx$$

$$= \frac{1}{4}\int \sec^2(x)\,dx + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1-i)x^2}\,dx, x, \tan(x)\right) + \frac{1}{4}\text{Subst}\left(\int \frac{1}{1+(1+i)x^2}\,dx, \right.$$

$$= \frac{x}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}} + \frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}} - \frac{1}{4}\text{Subst}$$

$$= \frac{x}{4\sqrt{2}} + \frac{\tan^{-1}\left(\frac{\cos(x)\sin(x)}{1+\sqrt{2}+\sin^2(x)}\right)}{4\sqrt{2}} + \frac{\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{4\sqrt{1-i}} + \frac{\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{4\sqrt{1+i}} + \frac{\tan(x)}{4}$$

Mathematica [A] time = 0.17, size = 64, normalized size = 0.72

$$\frac{1}{8}\left(\frac{2\tan^{-1}\left(\sqrt{1-i}\,\tan(x)\right)}{\sqrt{1-i}} + \frac{2\tan^{-1}\left(\sqrt{1+i}\,\tan(x)\right)}{\sqrt{1+i}} + \sqrt{2}\,\tan^{-1}\left(\sqrt{2}\,\tan(x)\right) + 2\tan(x)\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[(1 - Sin[x]^8)^(-1),x]
```

```
[Out] ((2*ArcTan[Sqrt[1 - I]*Tan[x]])/Sqrt[1 - I] + (2*ArcTan[Sqrt[1 + I]*Tan[x]]
)/Sqrt[1 + I] + Sqrt[2]*ArcTan[Sqrt[2]*Tan[x]] + 2*Tan[x])/8
```

fricas [B] time = 33.49, size = 3884, normalized size = 43.64

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="fricas")
```

```
[Out] -1/64*(2^(1/4)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*cos(x)*log(-(4*sqrt(2) - 5
)*cos(x)^4 + 2*(2*sqrt(2) - 3)*cos(x)^2 + (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3
 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) - 2^(1/4
)*sqrt(2*sqrt(2) + 4)*(sqrt(2) - 1)*cos(x)*log(-(4*sqrt(2) - 5)*cos(x)^4 +
2*(2*sqrt(2) - 3)*cos(x)^2 - (2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*
(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 2) + 2*2^(1/4)*sqrt(2*sq
rt(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^
16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14 + 16*(sqrt(
2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*(41*sqrt(2)
- 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 90) - 58*sqrt
(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2) + 216)*cos(x
)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*(2*sqrt(2) -
 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*
cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2) + 4))*cos(x
)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 14))*cos(x)^1
1 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*cos(x)^9 + 2*
(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/4
)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^(3/4)*(sqrt(
2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt(
2) + 4)*sin(x) + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^14 -
56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqr
```

t(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2) - 10) - 10
*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqrt(2) + 92)*c
os(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(
2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*
(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sq
rt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2
) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*sqrt(2) - 34
))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos
(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^
(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt
(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*sqrt(2) - 4)
*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8)
 + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(x)^10 - 152*
cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))*cos(x) - 2*2^(1/4
)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2) - 2*sqrt(2)
 - 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 44)*cos(x)^14
 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 - 16*(sqrt(2)*
(41*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29*sqrt(2) - 9
0) - 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) - 32*sqrt(2)
 + 216)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*cos(x)^4 + 4*
(2*sqrt(2) - 15)*cos(x)^2 + 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2^(1/4)*(3*sq
rt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)*(13*sqrt(2)
 + 4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79*sqrt(2) - 1
4))*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt(2) - 31))*c
os(x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) - 4))*cos(x)^7
 - 2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(x)^5 + (2*2^
(3/4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1/4)*cos(x))*
sqrt(2*sqrt(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*
cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)^12 + 8*(sqr
t(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(2)*(7*sqrt(2
) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) - 46) - 32*sqr
t(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) + 72)*cos(x)^
4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 + (8*(2^(3/4)*(8*sqrt(2) - 11)
 - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2) - 11) - 2*2
^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) - 39) - 2^(1/4
)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) - 2^(1/4)*(23*
sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1/4)*(4*sqrt(
2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqrt(2) - 2))*c
os(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - 2)*sqr
t(-4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 + 4*(2^(1/4)*(3*
sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*sqrt(2) + 4)
*sin(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^12 - 256*cos(
x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2 - 1))*cos(x
) + 2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(-1/4*(32*(sqrt(2)*(3*sqrt(2) + 2)
 - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*sqrt(2) - 4
4)*cos(x)^14 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*cos(x)^12 -
16*(sqrt(2)*(41*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(sqrt(2)*(29
*sqrt(2) - 90) - 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt(2) - 98) -
 32*sqrt(2) + 216)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt(2) - 82)*c
os(x)^4 + 4*(2*sqrt(2) - 15)*cos(x)^2 - 2*(8*(2^(3/4)*(2*sqrt(2) - 1) - 2*2
^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9) - 2*2^(1/4)
*(13*sqrt(2) + 4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) - 2^(1/4)*(79
*sqrt(2) - 14))*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/4)*(27*sqrt
(2) - 31))*cos(x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*(sqrt(2) -
4))*cos(x)^7 - 2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2) - 30))*cos(
x)^5 + (2*2^(3/4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)^3 - 2*2^(1
/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) + (16*(sqrt(2)*(5*sqrt(2) - 6) - 8*s
qrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2) + 4)*cos(x)
^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10 - 40*(sqrt(

```
2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27*sqrt(2) -
46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) - 8*sqrt(2) +
 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^(3/4)*(8*sq
rt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)*(8*sqrt(2)
 - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28*sqrt(2) -
39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(2) - 23) -
2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) - 3) - 8*2^(1
/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2^(1/4)*(sqr
t(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(2) + 4)*sin
(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*cos(x)^2 - 4*
(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(x))*sqrt(2*
sqrt(2) + 4)*sin(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 608*cos(x)^1
2 - 256*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 + 16*cos(x)^2
 - 1))*cos(x) - 2*2^(1/4)*sqrt(2*sqrt(2) + 4)*arctan(1/4*(32*(sqrt(2)*(3*sq
rt(2) + 2) - 2*sqrt(2) - 6)*cos(x)^16 - 16*(sqrt(2)*(29*sqrt(2) + 10) - 24*
sqrt(2) - 44)*cos(x)^14 + 16*(sqrt(2)*(51*sqrt(2) - 4) - 52*sqrt(2) - 46)*c
os(x)^12 - 16*(sqrt(2)*(41*sqrt(2) - 36) - 54*sqrt(2) + 15)*cos(x)^10 + 8*(
sqrt(2)*(29*sqrt(2) - 90) - 58*sqrt(2) + 132)*cos(x)^8 - 4*(sqrt(2)*(5*sqrt
(2) - 98) - 32*sqrt(2) + 216)*cos(x)^6 - 4*(sqrt(2)*(sqrt(2) + 24) + 4*sqrt
(2) - 82)*cos(x)^4 + 4*(2*sqrt(2) - 15)*cos(x)^2 - 2*(8*(2^(3/4)*(2*sqrt(2)
 - 1) - 2*2^(1/4)*(3*sqrt(2) + 2))*cos(x)^15 - 8*(2^(3/4)*(11*sqrt(2) - 9)
- 2*2^(1/4)*(13*sqrt(2) + 4))*cos(x)^13 + 4*(2*2^(3/4)*(21*sqrt(2) - 23) -
2^(1/4)*(79*sqrt(2) - 14))*cos(x)^11 - 8*(2^(3/4)*(19*sqrt(2) - 27) - 2^(1/
4)*(27*sqrt(2) - 31))*cos(x)^9 + 2*(2^(3/4)*(36*sqrt(2) - 65) - 32*2^(1/4)*
(sqrt(2) - 4))*cos(x)^7 - 2*(2^(3/4)*(9*sqrt(2) - 19) - 2*2^(1/4)*(sqrt(2)
- 30))*cos(x)^5 + (2*2^(3/4)*(sqrt(2) - 2) + 2^(1/4)*(sqrt(2) + 26))*cos(x)
^3 - 2*2^(1/4)*cos(x))*sqrt(2*sqrt(2) + 4)*sin(x) - (16*(sqrt(2)*(5*sqrt(2)
 - 6) - 8*sqrt(2) + 4)*cos(x)^14 - 56*(sqrt(2)*(5*sqrt(2) - 6) - 8*sqrt(2)
+ 4)*cos(x)^12 + 8*(sqrt(2)*(49*sqrt(2) - 62) - 76*sqrt(2) + 54)*cos(x)^10
- 40*(sqrt(2)*(7*sqrt(2) - 10) - 10*sqrt(2) + 13)*cos(x)^8 + 4*(sqrt(2)*(27
*sqrt(2) - 46) - 32*sqrt(2) + 92)*cos(x)^6 - 2*(11*sqrt(2)*(sqrt(2) - 2) -
8*sqrt(2) + 72)*cos(x)^4 + 2*(sqrt(2)*(sqrt(2) - 2) + 14)*cos(x)^2 - (8*(2^
(3/4)*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^13 - 24*(2^(3/4)
*(8*sqrt(2) - 11) - 2*2^(1/4)*(5*sqrt(2) - 6))*cos(x)^11 + 4*(2*2^(3/4)*(28
*sqrt(2) - 39) - 2^(1/4)*(73*sqrt(2) - 94))*cos(x)^9 - 8*(2^(3/4)*(16*sqrt(
2) - 23) - 2^(1/4)*(23*sqrt(2) - 34))*cos(x)^7 + 2*(9*2^(3/4)*(2*sqrt(2) -
3) - 8*2^(1/4)*(4*sqrt(2) - 7))*cos(x)^5 - 2*(2^(3/4)*(2*sqrt(2) - 3) - 6*2
^(1/4)*(sqrt(2) - 2))*cos(x)^3 - 2^(1/4)*(sqrt(2) - 2)*cos(x))*sqrt(2*sqrt(
2) + 4)*sin(x) - 2)*sqrt(-4*(4*sqrt(2) - 5)*cos(x)^4 + 8*(2*sqrt(2) - 3)*co
s(x)^2 - 4*(2^(1/4)*(3*sqrt(2) - 4)*cos(x)^3 - 2*2^(1/4)*(sqrt(2) - 1)*cos(
x))*sqrt(2*sqrt(2) + 4)*sin(x) + 8) + 4)/(112*cos(x)^16 - 448*cos(x)^14 + 6
08*cos(x)^12 - 256*cos(x)^10 - 152*cos(x)^8 + 208*cos(x)^6 - 88*cos(x)^4 +
16*cos(x)^2 - 1))*cos(x) + 4*sqrt(2)*arctan(1/4*(3*sqrt(2)*cos(x)^2 - 2*sqr
t(2))/(cos(x)*sin(x)))*cos(x) - 16*sin(x))/cos(x)
```

giac [B] time = 0.35, size = 220, normalized size = 2.47

$$\frac{1}{8}\sqrt{2}\left(x+\arctan\left(-\frac{\sqrt{2}\sin(2x)-2\sin(2x)}{\sqrt{2}\cos(2x)+\sqrt{2}-2\cos(2x)+2}\right)\right)+\frac{1}{8}\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+}\right.}{\sqrt{\sqrt{2}+2}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="giac")
```

```
[Out] 1/8*sqrt(2)*(x + arctan(-(sqrt(2)*sin(2*x) - 2*sin(2*x))/(sqrt(2)*cos(2*x)
+ sqrt(2) - 2*cos(2*x) + 2))) + 1/8*(pi*floor(x/pi + 1/2) + arctan(2*(1/2)^
(3/4)*((1/2)^(1/4)*sqrt(-sqrt(2) + 2) + 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(
```

```
sqrt(2) + 1) + 1/8*(pi*floor(x/pi + 1/2) + arctan(-2*(1/2)^(3/4)*((1/2)^(1/
4)*sqrt(-sqrt(2) + 2) - 2*tan(x))/sqrt(sqrt(2) + 2)))*sqrt(sqrt(2) + 1) + 1
/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 + (1/2)^(1/4)*sqrt(-sqrt(2) + 2)*tan(x)
+ sqrt(1/2)) - 1/16*sqrt(sqrt(2) - 1)*log(tan(x)^2 - (1/2)^(1/4)*sqrt(-sqrt
(2) + 2)*tan(x) + sqrt(1/2)) + 1/4*tan(x)
```

maple [B] time = 0.22, size = 255, normalized size = 2.87

$$\frac{\tan(x)}{4}-\frac{\sqrt{2}\,\sqrt{-2+2\sqrt{2}}\,\ln\left(-\sqrt{-2+2\sqrt{2}}\,\sqrt{2}\,\tan(x)+2\left(\tan^2(x)\right)+\sqrt{2}\right)}{32}+\frac{\arctan\left(\frac{-\sqrt{2}\,\sqrt{-2+2\sqrt{2}}+4\tan(x)}{2\sqrt{1+\sqrt{2}}}\right)\sqrt{2}}{8\sqrt{1+\sqrt{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(1-sin(x)^8),x)
```

```
[Out] 1/4*tan(x)-1/32*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(-(-2+2*2^(1/2))^(1/2)*2^(1/
2)*tan(x)+2*tan(x)^2+2^(1/2))+1/8/(1+2^(1/2))^(1/2)*arctan(1/2*(-2^(1/2)*(-
2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))*2^(1/2)+1/8/(1+2^(1/2))^(1/
2)*arctan(1/2*(-2^(1/2)*(-2+2*2^(1/2))^(1/2)+4*tan(x))/(1+2^(1/2))^(1/2))+1
/32*2^(1/2)*(-2+2*2^(1/2))^(1/2)*ln(2^(1/2)+2*tan(x)^2+(-2+2*2^(1/2))^(1/2)
*2^(1/2)*tan(x))+1/8/(1+2^(1/2))^(1/2)*arctan(1/2*(4*tan(x)+2^(1/2)*(-2+2*2
^(1/2))^(1/2))/(1+2^(1/2))^(1/2))*2^(1/2)+1/8/(1+2^(1/2))^(1/2)*arctan(1/2*
(4*tan(x)+2^(1/2)*(-2+2*2^(1/2))^(1/2))/(1+2^(1/2))^(1/2))+1/8*arctan(2^(1/
2)*tan(x))*2^(1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\left(\cos(2x)^2+\sin(2x)^2+2\cos(2x)+1\right)\left(2\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{2\left(\frac{1}{2}\right)^{\frac{3}{4}}\left(\left(\frac{1}{2}\right)^{\frac{1}{4}}\sqrt{-\sqrt{2}+2}+2\tan(x)\right)}{\sqrt{\sqrt{2}+2}}\right)\right)\sqrt{\sqrt{2}+1}+2\left(\pi\left\lfloor\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)^8),x, algorithm="maxima")
```

```
[Out] 1/16*((sqrt(2)*cos(2*x)^2 + sqrt(2)*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(
2))*arctan2(2*sqrt(2)*sin(x)/(2*(sqrt(2) + 1)*cos(x) + cos(x)^2 + sin(x)^2
+ 2*sqrt(2) + 3), (cos(x)^2 + sin(x)^2 + 2*cos(x) - 1)/(2*(sqrt(2) + 1)*cos
(x) + cos(x)^2 + sin(x)^2 + 2*sqrt(2) + 3)) - (sqrt(2)*cos(2*x)^2 + sqrt(2)
*sin(2*x)^2 + 2*sqrt(2)*cos(2*x) + sqrt(2))*arctan2(2*sqrt(2)*sin(x)/(2*(sq
rt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(2) + 3), (cos(x)^2 + sin(x
)^2 - 2*cos(x) - 1)/(2*(sqrt(2) - 1)*cos(x) + cos(x)^2 + sin(x)^2 - 2*sqrt(
2) + 3)) + 128*(cos(2*x)^2 + sin(2*x)^2 + 2*cos(2*x) + 1)*integrate(((4*cos
(2*x) - 1)*cos(4*x) - cos(8*x)*cos(4*x) + 4*cos(6*x)*cos(4*x) - 22*cos(4*x)
^2 - sin(8*x)*sin(4*x) + 4*sin(6*x)*sin(4*x) - 22*sin(4*x)^2 + 4*sin(4*x)*s
in(2*x))/(2*(4*cos(6*x) - 22*cos(4*x) + 4*cos(2*x) - 1)*cos(8*x) - cos(8*x)
^2 + 8*(22*cos(4*x) - 4*cos(2*x) + 1)*cos(6*x) - 16*cos(6*x)^2 + 44*(4*cos(
2*x) - 1)*cos(4*x) - 484*cos(4*x)^2 - 16*cos(2*x)^2 + 4*(2*sin(6*x) - 11*si
n(4*x) + 2*sin(2*x))*sin(8*x) - sin(8*x)^2 + 16*(11*sin(4*x) - 2*sin(2*x))*
sin(6*x) - 16*sin(6*x)^2 - 484*sin(4*x)^2 + 176*sin(4*x)*sin(2*x) - 16*sin(
2*x)^2 + 8*cos(2*x) - 1), x) + 8*sin(2*x))/(cos(2*x)^2 + sin(2*x)^2 + 2*cos
(2*x) + 1)
```

mupad [B] time = 14.04, size = 141, normalized size = 1.58

$$\frac{\tan(x)}{4}+\mathrm{atan}\left(\sqrt{2}\,\tan(x)\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}-\sqrt{2}\,\tan(x)\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,8\mathrm{i}\right)\left(\sqrt{-\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}+\sqrt{\frac{\sqrt{2}}{256}-\frac{1}{256}}\,2\mathrm{i}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-1/(sin(x)^8 - 1),x)
```

```
[Out] tan(x)/4 + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256)^(1/2)*8i - 2^(1/2)*t
an(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256 - 1/256)^(1/2)*2i + (
2^(1/2)/256 - 1/256)^(1/2)*2i) + atan(2^(1/2)*tan(x)*(- 2^(1/2)/256 - 1/256
)^(1/2)*8i + 2^(1/2)*tan(x)*(2^(1/2)/256 - 1/256)^(1/2)*8i)*((- 2^(1/2)/256
 - 1/256)^(1/2)*2i - (2^(1/2)/256 - 1/256)^(1/2)*2i) + (2^(1/2)*atan(2^(1/2
)*tan(x)))/8
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(1-sin(x)**8),x)
```

```
[Out] Timed out
```

3.261 $\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=38

$$-\frac{\sin^7(x)}{7a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^3(x)}{a}+\frac{\sin(x)}{a}$$

```
[Out] sin(x)/a-sin(x)^3/a+3/5*sin(x)^5/a-1/7*sin(x)^7/a
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$-\frac{\sin^7(x)}{7a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^3(x)}{a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^9/(a - a*Sin[x]^2),x]
```

```
[Out] Sin[x]/a - Sin[x]^3/a + (3*Sin[x]^5)/(5*a) - Sin[x]^7/(7*a)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^9(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^7(x)\,dx}{a}\\
&= -\frac{\text{Subst}\left(\int \left(1-3x^2+3x^4-x^6\right)dx,x,-\sin(x)\right)}{a}\\
&= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{a}+\frac{3\sin^5(x)}{5a}-\frac{\sin^7(x)}{7a}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 35, normalized size = 0.92

$$\frac{\frac{35\sin(x)}{64}+\frac{7}{64}\sin(3x)+\frac{7}{320}\sin(5x)+\frac{1}{448}\sin(7x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2),x]
```

```
[Out] ((35*Sin[x])/64 + (7*Sin[3*x])/64 + (7*Sin[5*x])/320 + Sin[7*x]/448)/a
```

fricas [A] time = 0.45, size = 27, normalized size = 0.71

$$\frac{\left(5\cos(x)^6+6\cos(x)^4+8\cos(x)^2+16\right)\sin(x)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/35*(5*cos(x)^6 + 6*cos(x)^4 + 8*cos(x)^2 + 16)*sin(x)/a
```

giac [A] time = 0.14, size = 28, normalized size = 0.74

$$-\frac{5\,\sin(x)^7 - 21\,\sin(x)^5 + 35\,\sin(x)^3 - 35\,\sin(x)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

maple [A] time = 0.17, size = 26, normalized size = 0.68

$$\frac{-\frac{\left(\sin^7(x)\right)}{7} + \frac{3\left(\sin^5(x)\right)}{5} - \left(\sin^3(x)\right) + \sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(-1/7*sin(x)^7+3/5*sin(x)^5-sin(x)^3+sin(x))
```

maxima [A] time = 0.36, size = 28, normalized size = 0.74

$$-\frac{5\,\sin(x)^7 - 21\,\sin(x)^5 + 35\,\sin(x)^3 - 35\,\sin(x)}{35\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/35*(5*sin(x)^7 - 21*sin(x)^5 + 35*sin(x)^3 - 35*sin(x))/a
```

mupad [B] time = 0.10, size = 34, normalized size = 0.89

$$\frac{\sin(x)}{a} - \frac{\sin(x)^3}{a} + \frac{3\sin(x)^5}{5\,a} - \frac{\sin(x)^7}{7\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a - a*sin(x)^2),x)
```

```
[Out] sin(x)/a - sin(x)^3/a + (3*sin(x)^5)/(5*a) - sin(x)^7/(7*a)
```

sympy [B] time = 41.26, size = 580, normalized size = 15.26

$$\frac{70\tan^{13}\left(\frac{x}{2}\right)}{35a\tan^{14}\left(\frac{x}{2}\right) + 245a\tan^{12}\left(\frac{x}{2}\right) + 735a\tan^{10}\left(\frac{x}{2}\right) + 1225a\tan^{8}\left(\frac{x}{2}\right) + 1225a\tan^{6}\left(\frac{x}{2}\right) + 735a\tan^{4}\left(\frac{x}{2}\right) + 245a\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2),x)
```

```
[Out] 70*tan(x/2)**13/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**1
0 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan
(x/2)**2 + 35*a) + 140*tan(x/2)**11/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12
 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan
```

```
(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 602*tan(x/2)**9/(35*a*tan(x/2)**14 +
 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(
x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 424*tan(x/2)**7/(
35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/
2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
+ 602*tan(x/2)**5/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**12 + 735*a*tan(x/2)*
*10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*tan(x/2)**4 + 245*a*t
an(x/2)**2 + 35*a) + 140*tan(x/2)**3/(35*a*tan(x/2)**14 + 245*a*tan(x/2)**1
2 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2)**6 + 735*a*ta
n(x/2)**4 + 245*a*tan(x/2)**2 + 35*a) + 70*tan(x/2)/(35*a*tan(x/2)**14 + 24
5*a*tan(x/2)**12 + 735*a*tan(x/2)**10 + 1225*a*tan(x/2)**8 + 1225*a*tan(x/2
)**6 + 735*a*tan(x/2)**4 + 245*a*tan(x/2)**2 + 35*a)
```

3.262 $\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5a}-\frac{2\sin^3(x)}{3a}+\frac{\sin(x)}{a}$$

[Out] sin(x)/a-2/3*sin(x)^3/a+1/5*sin(x)^5/a

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin^5(x)}{5a}-\frac{2\sin^3(x)}{3a}+\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^7/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a - (2*Sin[x]^3)/(3*a) + Sin[x]^5/(5*a)

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^5(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right)dx, x, -\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{2\sin^3(x)}{3a}+\frac{\sin^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8}+\frac{5}{48}\sin(3x)+\frac{1}{80}\sin(5x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2),x]

[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a

fricas [A] time = 0.41, size = 21, normalized size = 0.72

$$\frac{\left(3\cos(x)^4+4\cos(x)^2+8\right)\sin(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a
```

giac [A] time = 0.12, size = 22, normalized size = 0.76

$$\frac{3\sin(x)^5 - 10\sin(x)^3 + 15\sin(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

maple [A] time = 0.15, size = 20, normalized size = 0.69

$$\frac{\frac{(\sin^5(x))}{5} - \frac{2(\sin^3(x))}{3} + \sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

maxima [A] time = 0.35, size = 22, normalized size = 0.76

$$\frac{3\sin(x)^5 - 10\sin(x)^3 + 15\sin(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a
```

mupad [B] time = 0.08, size = 19, normalized size = 0.66

$$\frac{\frac{\sin(x)^5}{5} - \frac{2\sin(x)^3}{3} + \sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a - a*sin(x)^2),x)
```

```
[Out] (sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a
```

sympy [B] time = 18.90, size = 311, normalized size = 10.72

$$\frac{30\tan^9\left(\frac{x}{2}\right)}{15a\tan^{10}\left(\frac{x}{2}\right) + 75a\tan^8\left(\frac{x}{2}\right) + 150a\tan^6\left(\frac{x}{2}\right) + 150a\tan^4\left(\frac{x}{2}\right) + 75a\tan^2\left(\frac{x}{2}\right) + 15a} + \frac{}{15a\tan^{10}\left(\frac{x}{2}\right) + 75a\tan^8\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2),x)
```

```
[Out] 30*tan(x/2)**9/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 +
150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x/2)**7/(15*a*tan(x/2
)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan
```

```
(x/2)**2 + 15*a) + 116*tan(x/2)**5/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 +
150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 40*tan(x
/2)**3/(15*a*tan(x/2)**10 + 75*a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*ta
n(x/2)**4 + 75*a*tan(x/2)**2 + 15*a) + 30*tan(x/2)/(15*a*tan(x/2)**10 + 75*
a*tan(x/2)**8 + 150*a*tan(x/2)**6 + 150*a*tan(x/2)**4 + 75*a*tan(x/2)**2 +
15*a)
```

3.263 $\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

```
[Out] sin(x)/a-1/3*sin(x)^3/a
```

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a - a*Sin[x]^2),x]
[Out] Sin[x]/a - Sin[x]^3/(3*a)
```

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^3(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(x)\right)}{a}\\ &= \frac{\sin(x)}{a}-\frac{\sin^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2),x]
[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a
```

fricas [A] time = 0.44, size = 13, normalized size = 0.72

$$\frac{\left(\cos(x)^2+2\right)\sin(x)}{3\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a
```

giac [A] time = 0.14, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a
```

maple [A] time = 0.15, size = 14, normalized size = 0.78

$$\frac{-\frac{\left(\sin^3(x)\right)}{3} + \sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(-1/3*sin(x)^3+sin(x))
```

maxima [A] time = 0.32, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a
```

mupad [B] time = 0.06, size = 16, normalized size = 0.89

$$\frac{3\sin(x) - \sin(x)^3}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a - a*sin(x)^2),x)
```

```
[Out] (3*sin(x) - sin(x)^3)/(3*a)
```

sympy [B] time = 7.95, size = 124, normalized size = 6.89

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} + \frac{4\tan^3\left(\frac{x}{2}\right)}{3a\tan^6\left(\frac{x}{2}\right) + 9a\tan^4\left(\frac{x}{2}\right) + 9a\tan^2\left(\frac{x}{2}\right) + 3a} + \frac{}{3a\tan^6\left(\frac{x}{2}\right) + 9a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a-a*sin(x)**2),x)
```

```
[Out] 6*tan(x/2)**5/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a) +
 4*tan(x/2)**3/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
+ 6*tan(x/2)/(3*a*tan(x/2)**6 + 9*a*tan(x/2)**4 + 9*a*tan(x/2)**2 + 3*a)
```

3.264 $\int \frac{\cos^3(x)}{a-a\sin^2(x)} dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a}$$

[Out] sin(x)/a

Rubi [A] time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2637}

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos(x)\,dx}{a}\\ &= \frac{\sin(x)}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2),x]

[Out] Sin[x]/a

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")

[Out] sin(x)/a

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] sin(x)/a
```

maple [A] time = 0.13, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2),x)
```

```
[Out] sin(x)/a
```

maxima [A] time = 0.33, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] sin(x)/a
```

mupad [B] time = 13.76, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a - a*sin(x)^2),x)
```

```
[Out] sin(x)/a
```

sympy [B] time = 2.86, size = 15, normalized size = 2.50

$$\frac{2\tan\left(\frac{x}{2}\right)}{a\tan^2\left(\frac{x}{2}\right)+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2),x)
```

```
[Out] 2*tan(x/2)/(a*tan(x/2)**2 + a)
```

3.265 $\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

```
[Out] arctanh(sin(x))/a
```

Rubi [A] time = 0.03, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3175, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/a
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec(x)\,dx}{a}\\ &= \frac{\tanh^{-1}(\sin(x))}{a}\end{aligned}$$

Mathematica [B] time = 0.00, size = 37, normalized size = 5.29

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a
```

fricas [B] time = 0.42, size = 20, normalized size = 2.86

$$\frac{\log\left(\sin(x)+1\right)-\log\left(-\sin(x)+1\right)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a
```

giac [B] time = 0.13, size = 23, normalized size = 3.29

$$\frac{\log(\sin(x)+1)}{2\,a}-\frac{\log(-\sin(x)+1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="giac")
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(-sin(x) + 1)/a
```

maple [A] time = 0.10, size = 8, normalized size = 1.14

$$\frac{\operatorname{arctanh}(\sin(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2),x)
[Out] arctanh(sin(x))/a
```

maxima [B] time = 0.35, size = 21, normalized size = 3.00

$$\frac{\log(\sin(x)+1)}{2\,a}-\frac{\log(\sin(x)-1)}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/2*log(sin(x) + 1)/a - 1/2*log(sin(x) - 1)/a
```

mupad [B] time = 13.60, size = 7, normalized size = 1.00

$$\frac{\operatorname{atanh}(\sin(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a - a*sin(x)^2),x)
[Out] atanh(sin(x))/a
```

sympy [B] time = 0.29, size = 19, normalized size = 2.71

$$-\frac{\log(\sin(x)-1)}{2a}+\frac{\log(\sin(x)+1)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2),x)
[Out] -log(sin(x) - 1)/(2*a) + log(sin(x) + 1)/(2*a)
```

3.266 $\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{\tan(x)\sec^3(x)}{4a}+\frac{3\tan(x)\sec(x)}{8a}$$

```
[Out] 3/8*arctanh(sin(x))/a+3/8*sec(x)*tan(x)/a+1/4*sec(x)^3*tan(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {3175, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{\tan(x)\sec^3(x)}{4a}+\frac{3\tan(x)\sec(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a) + (3*Sec[x]*Tan[x])/(8*a) + (Sec[x]^3*Tan[x])/(4*
a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^5(x)\,dx}{a}\\
&= \frac{\sec^3(x)\tan(x)}{4a}+\frac{3\int \sec^3(x)\,dx}{4a}\\
&= \frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}+\frac{3\int \sec(x)\,dx}{8a}\\
&= \frac{3\tanh^{-1}(\sin(x))}{8a}+\frac{3\sec(x)\tan(x)}{8a}+\frac{\sec^3(x)\tan(x)}{4a}
\end{aligned}$$

Mathematica [A] time = 0.13, size = 61, normalized size = 1.74

$$\frac{\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{16a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a - a*Sin[x]^2),x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a)
```

fricas [A] time = 0.43, size = 46, normalized size = 1.31

$$\frac{3\,\cos(x)^4\log\left(\sin(x)+1\right)-3\,\cos(x)^4\log\left(-\sin(x)+1\right)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,a\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a*cos(x)^4)
```

giac [A] time = 0.12, size = 47, normalized size = 1.34

$$\frac{3\,\log\left(\sin(x)+1\right)}{16\,a}-\frac{3\,\log\left(-\sin(x)+1\right)}{16\,a}-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^2-1\right)^2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a - 3/16*log(-sin(x) + 1)/a - 1/8*(3*sin(x)^3 - 5*sin(
x))/((sin(x)^2 - 1)^2*a)
```

maple [B] time = 0.22, size = 66, normalized size = 1.89

$$\frac{1}{16a\left(-1+\sin(x)\right)^2}-\frac{3}{16a\left(-1+\sin(x)\right)}-\frac{3\ln\left(-1+\sin(x)\right)}{16a}-\frac{1}{16a\left(1+\sin(x)\right)^2}-\frac{3}{16a\left(1+\sin(x)\right)}+\frac{3\ln\left(1+\sin(x\right.}{16a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a-a*sin(x)^2),x)
```

```
[Out] 1/16/a/(-1+sin(x))^2-3/16/a/(-1+sin(x))-3/16/a*ln(-1+sin(x))-1/16/a/(1+sin(
x))^2-3/16/a/(1+sin(x))+3/16/a*ln(1+sin(x))
```

maxima [A] time = 0.34, size = 51, normalized size = 1.46

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(a\sin(x)^4-2\,a\sin(x)^2+a\right)}+\frac{3\,\log\left(\sin(x)+1\right)}{16\,a}-\frac{3\,\log\left(\sin(x)-1\right)}{16\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a*sin(x)^4 - 2*a*sin(x)^2 + a) + 3/16*log(sin
(x) + 1)/a - 3/16*log(sin(x) - 1)/a
```

mupad [B] time = 13.88, size = 31, normalized size = 0.89

$$\frac{3\,\mathrm{atanh}\left(\sin(x)\right)}{8\,a}+\frac{3\,\sin(x)}{8\,a\cos(x)^2}+\frac{\sin(x)}{4\,a\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a - a*sin(x)^2)),x)
```

```
[Out] (3*atanh(sin(x)))/(8*a) + (3*sin(x))/(8*a*cos(x)^2) + sin(x)/(4*a*cos(x)^4)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^3(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a-a*sin(x)**2),x)
[Out] -Integral(sec(x)**3/(sin(x)**2 - 1), x)/a
```

3.267 $\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a}+\frac{\sin(x)\cos^3(x)}{4a}+\frac{3\sin(x)\cos(x)}{8a}$$

```
[Out] 3/8*x/a+3/8*cos(x)*sin(x)/a+1/4*cos(x)^3*sin(x)/a
```

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{3x}{8a}+\frac{\sin(x)\cos^3(x)}{4a}+\frac{3\sin(x)\cos(x)}{8a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2),x]
```

```
[Out] (3*x)/(8*a) + (3*Cos[x]*Sin[x])/(8*a) + (Cos[x]^3*Sin[x])/(4*a)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^4(x)\,dx}{a}\\ &= \frac{\cos^3(x)\sin(x)}{4a}+\frac{3\int \cos^2(x)\,dx}{4a}\\ &= \frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}+\frac{3\int 1\,dx}{8a}\\ &= \frac{3x}{8a}+\frac{3\cos(x)\sin(x)}{8a}+\frac{\cos^3(x)\sin(x)}{4a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2),x]
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a
```

fricas [A] time = 0.42, size = 23, normalized size = 0.70

$$\frac{\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="fricas")
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a
```

giac [A] time = 0.12, size = 36, normalized size = 1.09

$$\frac{3\arctan(\tan(x))}{8a}+\frac{\frac{3\tan(x)^3}{a}+\frac{5\tan(x)}{a}}{8\left(\tan(x)^2+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="giac")
[Out] 3/8*arctan(tan(x))/a + 1/8*(3*tan(x)^3/a + 5*tan(x)/a)/(tan(x)^2 + 1)^2
```

maple [A] time = 0.15, size = 40, normalized size = 1.21

$$\frac{\tan(x)}{4a\left(\tan^2(x)+1\right)^2}+\frac{3\tan(x)}{8a\left(\tan^2(x)+1\right)}+\frac{3\arctan(\tan(x))}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2),x)
[Out] 1/4/a*tan(x)/(tan(x)^2+1)^2+3/8/a*tan(x)/(tan(x)^2+1)+3/8/a*arctan(tan(x))
```

maxima [A] time = 0.45, size = 37, normalized size = 1.12

$$\frac{3\tan(x)^3+5\tan(x)}{8\left(a\tan(x)^4+2a\tan(x)^2+a\right)}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a*tan(x)^4 + 2*a*tan(x)^2 + a) + 3/8*x/a
```

mupad [B] time = 13.61, size = 25, normalized size = 0.76

$$\frac{\sin(2x)}{4a}+\frac{\sin(4x)}{32a}+\frac{3x}{8a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a - a*sin(x)^2),x)
[Out] sin(2*x)/(4*a) + sin(4*x)/(32*a) + (3*x)/(8*a)
```

sympy [B] time = 12.93, size = 473, normalized size = 14.33

$$\frac{3x\tan^8\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+32a\tan^2\left(\frac{x}{2}\right)+8a}+\frac{12x\tan^6\left(\frac{x}{2}\right)}{8a\tan^8\left(\frac{x}{2}\right)+32a\tan^6\left(\frac{x}{2}\right)+48a\tan^4\left(\frac{x}{2}\right)+32a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2),x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 12*x*tan(x/2)**6/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**
6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 18*x*tan(x/2)**4/(8*a*tan(
x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 1
2*x*tan(x/2)**2/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32
*a*tan(x/2)**2 + 8*a) + 3*x/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(
x/2)**4 + 32*a*tan(x/2)**2 + 8*a) - 10*tan(x/2)**7/(8*a*tan(x/2)**8 + 32*a*
tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a) + 6*tan(x/2)**5/(8
*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8
*a) - 6*tan(x/2)**3/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6 + 48*a*tan(x/2)**4
+ 32*a*tan(x/2)**2 + 8*a) + 10*tan(x/2)/(8*a*tan(x/2)**8 + 32*a*tan(x/2)**6
 + 48*a*tan(x/2)**4 + 32*a*tan(x/2)**2 + 8*a)
```

3.268 $\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a}+\frac{\sin(x)\cos(x)}{2a}$$

[Out] 1/2*x/a+1/2*cos(x)*sin(x)/a

Rubi [A] time = 0.05, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a}+\frac{\sin(x)\cos(x)}{2a}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^4/(a - a*Sin[x]^2),x]

[Out] x/(2*a) + (Cos[x]*Sin[x])/(2*a)

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \cos^2(x)\,dx}{a}\\ &= \frac{\cos(x)\sin(x)}{2a}+\frac{\int 1\,dx}{2a}\\ &= \frac{x}{2a}+\frac{\cos(x)\sin(x)}{2a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2),x]

[Out] (x/2 + Sin[2*x]/4)/a

fricas [A] time = 0.42, size = 12, normalized size = 0.60

$$\frac{\cos(x)\sin(x) + x}{2\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(cos(x)*sin(x) + x)/a
```

giac [A] time = 0.12, size = 24, normalized size = 1.20

$$\frac{\arctan\left(\tan(x)\right)}{2\,a} + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(tan(x))/a + 1/2*tan(x)/((tan(x)^2 + 1)*a)
```

maple [A] time = 0.16, size = 25, normalized size = 1.25

$$\frac{\tan(x)}{2a\left(\tan^2(x) + 1\right)} + \frac{\arctan\left(\tan(x)\right)}{2a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2),x)
```

```
[Out] 1/2/a*tan(x)/(tan(x)^2+1)+1/2/a*arctan(tan(x))
```

maxima [A] time = 1.40, size = 21, normalized size = 1.05

$$\frac{x}{2\,a} + \frac{\tan(x)}{2\left(a\tan(x)^2 + a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/2*x/a + 1/2*tan(x)/(a*tan(x)^2 + a)
```

mupad [B] time = 13.81, size = 13, normalized size = 0.65

$$\frac{2\,x + \sin\left(2\,x\right)}{4\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a - a*sin(x)^2),x)
```

```
[Out] (2*x + sin(2*x))/(4*a)
```

sympy [B] time = 4.92, size = 153, normalized size = 7.65

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} + \frac{x}{2a\tan^4\left(\frac{x}{2}\right) + 4a\tan^2\left(\frac{x}{2}\right) + 2a} - \frac{2}{2a\tan^4\left(\frac{x}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a-a*sin(x)**2),x)
```

```
[Out] x*tan(x/2)**4/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*x*tan(x/2)**2/(
2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + x/(2*a*tan(x/2)**4 + 4*a*tan(x/2
)**2 + 2*a) - 2*tan(x/2)**3/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a) + 2*t
an(x/2)/(2*a*tan(x/2)**4 + 4*a*tan(x/2)**2 + 2*a)
```

3.269 $\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a}$$

[Out] x/a

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int 1\,dx}{a}\\ &= \frac{x}{a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2),x]
```

```
[Out] x/a
```

fricas [A] time = 0.41, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] x/a
```

giac [B] time = 0.14, size = 14, normalized size = 2.80

$$\frac{\arctan\left(\frac{|a|\tan(x)}{a}\right)}{|a|}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
[Out] arctan(abs(a)*tan(x)/a)/abs(a)
```

maple [C] time = 0.19, size = 8, normalized size = 1.60

$$\frac{\arctan(\tan(x))}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2),x)
[Out] 1/a*arctan(tan(x))
```

maxima [A] time = 0.48, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] x/a
```

mupad [B] time = 13.81, size = 5, normalized size = 1.00

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a - a*sin(x)^2),x)
[Out] x/a
```

sympy [A] time = 1.60, size = 2, normalized size = 0.40

$$\frac{x}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2),x)
[Out] x/a
```

3.270 $\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\tan(x)\sec(x)}{2a}$$

```
[Out] 1/2*arctanh(sin(x))/a+1/2*sec(x)*tan(x)/a
```

Rubi [A] time = 0.04, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\tan(x)\sec(x)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a - a*Sin[x]^2),x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a) + (Sec[x]*Tan[x])/(2*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^3(x)\,dx}{a}\\
&= \frac{\sec(x)\tan(x)}{2a}+\frac{\int \sec(x)\,dx}{2a}\\
&= \frac{\tanh^{-1}(\sin(x))}{2a}+\frac{\sec(x)\tan(x)}{2a}
\end{aligned}$$

Mathematica [B] time = 0.04, size = 45, normalized size = 2.05

$$\frac{\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{2a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2),x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
)
```

fricas [B] time = 0.45, size = 37, normalized size = 1.68

$$\frac{\cos(x)^2\log(\sin(x)+1)-\cos(x)^2\log(-\sin(x)+1)+2\sin(x)}{4a\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a*co
s(x)^2)
```

giac [B] time = 0.13, size = 38, normalized size = 1.73

$$\frac{\log(\sin(x)+1)}{4a}-\frac{\log(-\sin(x)+1)}{4a}-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(-sin(x) + 1)/a - 1/2*sin(x)/((sin(x)^2 - 1)
*a)
```

maple [B] time = 0.21, size = 44, normalized size = 2.00

$$-\frac{1}{4a(-1+\sin(x))}-\frac{\ln(-1+\sin(x))}{4a}-\frac{1}{4a(1+\sin(x))}+\frac{\ln(1+\sin(x))}{4a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2),x)
```

```
[Out] -1/4/a/(-1+sin(x))-1/4/a*ln(-1+sin(x))-1/4/a/(1+sin(x))+1/4/a*ln(1+sin(x))
```

maxima [B] time = 0.35, size = 37, normalized size = 1.68

$$\frac{\log(\sin(x)+1)}{4a}-\frac{\log(\sin(x)-1)}{4a}-\frac{\sin(x)}{2\left(a\sin(x)^2-a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/4*log(sin(x) + 1)/a - 1/4*log(sin(x) - 1)/a - 1/2*sin(x)/(a*sin(x)^2 - a)
```

mupad [B] time = 13.87, size = 25, normalized size = 1.14

$$\frac{\operatorname{atanh}(\sin(x))}{2a}+\frac{\sin(x)}{2\left(a-a\sin(x)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a - a*sin(x)^2)),x)
```

```
[Out] atanh(sin(x))/(2*a) + sin(x)/(2*(a - a*sin(x)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int\frac{\sec(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)/(sin(x)**2 - 1), x)/a
```

3.271 $\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=18

$$\frac{\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

[Out] tan(x)/a+1/3*tan(x)^3/a

Rubi [A] time = 0.05, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

[In] Int[Sec[x]^2/(a - a*Sin[x]^2),x]

[Out] Tan[x]/a + Tan[x]^3/(3*a)

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^4(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+x^2\right)\,dx,x,-\tan(x)\right)}{a}\\ &= \frac{\tan(x)}{a}+\frac{\tan^3(x)}{3a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 21, normalized size = 1.17

$$\frac{\frac{2\tan(x)}{3}+\frac{1}{3}\tan(x)\sec^2(x)}{a}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2),x]

[Out] ((2*Tan[x])/3 + (Sec[x]^2*Tan[x])/3)/a

fricas [A] time = 0.43, size = 19, normalized size = 1.06

$$\frac{\left(2\,\cos(x)^2+1\right)\sin(x)}{3\,a\cos(x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="fricas")
[Out] 1/3*(2*cos(x)^2 + 1)*sin(x)/(a*cos(x)^3)
```

giac [A] time = 0.14, size = 14, normalized size = 0.78

$$\frac{\tan(x)^3 + 3\tan(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="giac")
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

maple [A] time = 0.21, size = 14, normalized size = 0.78

$$\frac{\frac{(\tan^3(x))}{3} + \tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2),x)
[Out] 1/a*(1/3*tan(x)^3+tan(x))
```

maxima [A] time = 0.38, size = 14, normalized size = 0.78

$$\frac{\tan(x)^3 + 3\tan(x)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2),x, algorithm="maxima")
[Out] 1/3*(tan(x)^3 + 3*tan(x))/a
```

mupad [B] time = 13.87, size = 13, normalized size = 0.72

$$\frac{\tan(x)\left(\tan(x)^2 + 3\right)}{3a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a - a*sin(x)^2)),x)
[Out] (tan(x)*(tan(x)^2 + 3))/(3*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^2(x)}{\sin^2(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2),x)
[Out] -Integral(sec(x)**2/(sin(x)**2 - 1), x)/a
```

3.272 $\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a}+\frac{2\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

```
[Out] tan(x)/a+2/3*tan(x)^3/a+1/5*tan(x)^5/a
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a}+\frac{2\tan^3(x)}{3a}+\frac{\tan(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] Tan[x]/a + (2*Tan[x]^3)/(3*a) + Tan[x]^5/(5*a)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{a-a\sin^2(x)}\,dx &= \frac{\int \sec^6(x)\,dx}{a}\\ &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(x)\right)}{a}\\ &= \frac{\tan(x)}{a}+\frac{2\tan^3(x)}{3a}+\frac{\tan^5(x)}{5a}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2),x]
```

```
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a
```

fricas [A] time = 0.43, size = 25, normalized size = 0.86

$$\frac{\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15\,a\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a*cos(x)^5)
```

giac [A] time = 0.19, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^{5}+10\,\tan(x)^{3}+15\,\tan(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

maple [A] time = 0.21, size = 20, normalized size = 0.69

$$\frac{\frac{\left(\tan^{5}(x)\right)}{5}+\frac{2\left(\tan^{3}(x)\right)}{3}+\tan(x)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2),x)
```

```
[Out] 1/a*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

maxima [A] time = 0.39, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^{5}+10\,\tan(x)^{3}+15\,\tan(x)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a
```

mupad [B] time = 13.92, size = 21, normalized size = 0.72

$$\frac{\tan(x)\,\left(3\tan(x)^{4}+10\tan(x)^{2}+15\right)}{15\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a - a*sin(x)^2)),x)
```

```
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$-\frac{\int \frac{\sec^{4}(x)}{\sin^{2}(x)-1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2),x)
```

```
[Out] -Integral(sec(x)**4/(sin(x)**2 - 1), x)/a
```

3.273 $\int \frac{\cos^9(x)}{\left(a-a\sin^2(x)\right)^2} dx$

Optimal. Leaf size=29

$$\frac{\sin^5(x)}{5a^2} - \frac{2\sin^3(x)}{3a^2} + \frac{\sin(x)}{a^2}$$

[Out] sin(x)/a^2-2/3*sin(x)^3/a^2+1/5*sin(x)^5/a^2

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin^5(x)}{5a^2} - \frac{2\sin^3(x)}{3a^2} + \frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^9/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2 - (2*Sin[x]^3)/(3*a^2) + Sin[x]^5/(5*a^2)

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^9(x)}{\left(a-a\sin^2(x)\right)^2} dx &= \frac{\int \cos^5(x)\,dx}{a^2}\\
&= -\frac{\text{Subst}\left(\int \left(1-2x^2+x^4\right) dx, x, -\sin(x)\right)}{a^2}\\
&= \frac{\sin(x)}{a^2} - \frac{2\sin^3(x)}{3a^2} + \frac{\sin^5(x)}{5a^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 27, normalized size = 0.93

$$\frac{\frac{5\sin(x)}{8} + \frac{5}{48}\sin(3x) + \frac{1}{80}\sin(5x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^9/(a - a*Sin[x]^2)^2,x]

[Out] ((5*Sin[x])/8 + (5*Sin[3*x])/48 + Sin[5*x]/80)/a^2

fricas [A] time = 0.42, size = 21, normalized size = 0.72

$$\frac{\left(3\cos(x)^4 + 4\cos(x)^2 + 8\right)\sin(x)}{15a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*(3*cos(x)^4 + 4*cos(x)^2 + 8)*sin(x)/a^2
```

giac [A] time = 0.15, size = 22, normalized size = 0.76

$$\frac{3\,\sin(x)^5 - 10\,\sin(x)^3 + 15\,\sin(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

maple [A] time = 0.16, size = 20, normalized size = 0.69

$$\frac{\frac{\left(\sin^5(x)\right)}{5} - \frac{2\left(\sin^3(x)\right)}{3} + \sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(1/5*sin(x)^5-2/3*sin(x)^3+sin(x))
```

maxima [A] time = 0.35, size = 22, normalized size = 0.76

$$\frac{3\,\sin(x)^5 - 10\,\sin(x)^3 + 15\,\sin(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^9/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*sin(x)^5 - 10*sin(x)^3 + 15*sin(x))/a^2
```

mupad [B] time = 14.00, size = 19, normalized size = 0.66

$$\frac{\frac{\sin(x)^5}{5} - \frac{2\sin(x)^3}{3} + \sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^9/(a - a*sin(x)^2)^2,x)
```

```
[Out] (sin(x) - (2*sin(x)^3)/3 + sin(x)^5/5)/a^2
```

sympy [B] time = 83.59, size = 362, normalized size = 12.48

$$\frac{30\tan^9\left(\frac{x}{2}\right)}{15a^2\tan^{10}\left(\frac{x}{2}\right) + 75a^2\tan^8\left(\frac{x}{2}\right) + 150a^2\tan^6\left(\frac{x}{2}\right) + 150a^2\tan^4\left(\frac{x}{2}\right) + 75a^2\tan^2\left(\frac{x}{2}\right) + 15a^2} + \frac{}{15a^2\tan^{10}\left(\frac{x}{2}\right) + 75a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**9/(a-a*sin(x)**2)**2,x)
```

```
[Out] 30*tan(x/2)**9/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x
/2)**6 + 150*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x/2
)**7/(15*a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 1
```

```
50*a**2*tan(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 116*tan(x/2)**5/(15*
a**2*tan(x/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*t
an(x/2)**4 + 75*a**2*tan(x/2)**2 + 15*a**2) + 40*tan(x/2)**3/(15*a**2*tan(x
/2)**10 + 75*a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(x/2)**4
 + 75*a**2*tan(x/2)**2 + 15*a**2) + 30*tan(x/2)/(15*a**2*tan(x/2)**10 + 75*
a**2*tan(x/2)**8 + 150*a**2*tan(x/2)**6 + 150*a**2*tan(x/2)**4 + 75*a**2*ta
n(x/2)**2 + 15*a**2)
```

3.274 $\int \frac{\cos^7(x)}{(a-a\sin^2(x))^2}\,dx$

Optimal. Leaf size=18

$$\frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}$$

[Out] sin(x)/a^2-1/3*sin(x)^3/a^2

Rubi [A] time = 0.04, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2633}

$$\frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^7/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2 - Sin[x]^3/(3*a^2)

Rule 2633

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^7(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^3(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1-x^2\right)\,dx,x,-\sin(x)\right)}{a^2}\\ &= \frac{\sin(x)}{a^2}-\frac{\sin^3(x)}{3a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 19, normalized size = 1.06

$$\frac{\frac{3\sin(x)}{4}+\frac{1}{12}\sin(3x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^7/(a - a*Sin[x]^2)^2,x]

[Out] ((3*Sin[x])/4 + Sin[3*x]/12)/a^2

fricas [A] time = 0.41, size = 13, normalized size = 0.72

$$\frac{\left(\cos(x)^2+2\right)\sin(x)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(cos(x)^2 + 2)*sin(x)/a^2
```

giac [A] time = 0.14, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

maple [A] time = 0.14, size = 14, normalized size = 0.78

$$\frac{-\frac{(\sin^3(x))}{3} + \sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/a^2*(-1/3*sin(x)^3+sin(x))
```

maxima [A] time = 0.35, size = 14, normalized size = 0.78

$$-\frac{\sin(x)^3 - 3\sin(x)}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/3*(sin(x)^3 - 3*sin(x))/a^2
```

mupad [B] time = 0.04, size = 16, normalized size = 0.89

$$\frac{3\sin(x) - \sin(x)^3}{3a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a - a*sin(x)^2)^2,x)
```

```
[Out] (3*sin(x) - sin(x)^3)/(3*a^2)
```

sympy [B] time = 40.56, size = 144, normalized size = 8.00

$$\frac{6\tan^5\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right) + 9a^2\tan^4\left(\frac{x}{2}\right) + 9a^2\tan^2\left(\frac{x}{2}\right) + 3a^2} + \frac{4\tan^3\left(\frac{x}{2}\right)}{3a^2\tan^6\left(\frac{x}{2}\right) + 9a^2\tan^4\left(\frac{x}{2}\right) + 9a^2\tan^2\left(\frac{x}{2}\right) + 3a^2} + \frac{}{3a^2\tan^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a-a*sin(x)**2)**2,x)
```

```
[Out] 6*tan(x/2)**5/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**2*tan(x/2)**2
 + 3*a**2) + 4*tan(x/2)**3/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)**4 + 9*a**
2*tan(x/2)**2 + 3*a**2) + 6*tan(x/2)/(3*a**2*tan(x/2)**6 + 9*a**2*tan(x/2)*
*4 + 9*a**2*tan(x/2)**2 + 3*a**2)
```

3.275 $\int \frac{\cos^5(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=6

$$\frac{\sin(x)}{a^2}$$

[Out] sin(x)/a^2

Rubi [A] time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 2637}

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^5/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2

Rule 2637

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^5(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos(x)\,dx}{a^2}\\ &= \frac{\sin(x)}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^5/(a - a*Sin[x]^2)^2,x]

[Out] Sin[x]/a^2

fricas [A] time = 0.41, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="fricas")

```
[Out] sin(x)/a^2
```

giac [A] time = 0.12, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] sin(x)/a^2
```

maple [A] time = 0.14, size = 7, normalized size = 1.17

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a-a*sin(x)^2)^2,x)
[Out] sin(x)/a^2
```

maxima [A] time = 0.33, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] sin(x)/a^2
```

mupad [B] time = 0.02, size = 6, normalized size = 1.00

$$\frac{\sin(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a - a*sin(x)^2)^2,x)
[Out] sin(x)/a^2
```

sympy [B] time = 19.21, size = 19, normalized size = 3.17

$$\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right)+a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a-a*sin(x)**2)**2,x)
[Out] 2*tan(x/2)/(a**2*tan(x/2)**2 + a**2)
```

3.276 $\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=7

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

[Out] arctanh(sin(x))/a^2

Rubi [A] time = 0.04, antiderivative size = 7, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^3/(a - a*Sin[x]^2)^2,x]

[Out] ArcTanh[Sin[x]]/a^2

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec(x)\,dx}{a^2}\\ &= \frac{\tanh^{-1}(\sin(x))}{a^2}\end{aligned}$$

Mathematica [B] time = 0.00, size = 37, normalized size = 5.29

$$\frac{\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^3/(a - a*Sin[x]^2)^2,x]

[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]])/a^2

fricas [B] time = 0.42, size = 20, normalized size = 2.86

$$\frac{\log\left(\sin(x)+1\right)-\log\left(-\sin(x)+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/2*(log(sin(x) + 1) - log(-sin(x) + 1))/a^2
```

giac [B] time = 0.13, size = 23, normalized size = 3.29

$$\frac{\log\left(\sin(x)+1\right)}{2\,a^2}-\frac{\log\left(-\sin(x)+1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(-sin(x) + 1)/a^2
```

maple [A] time = 0.18, size = 8, normalized size = 1.14

$$\frac{\operatorname{arctanh}\left(\sin(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a-a*sin(x)^2)^2,x)
```

```
[Out] arctanh(sin(x))/a^2
```

maxima [B] time = 0.33, size = 21, normalized size = 3.00

$$\frac{\log\left(\sin(x)+1\right)}{2\,a^2}-\frac{\log\left(\sin(x)-1\right)}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*log(sin(x) + 1)/a^2 - 1/2*log(sin(x) - 1)/a^2
```

mupad [B] time = 0.06, size = 7, normalized size = 1.00

$$\frac{\operatorname{atanh}\left(\sin(x)\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a - a*sin(x)^2)^2,x)
```

```
[Out] atanh(sin(x))/a^2
```

sympy [B] time = 7.46, size = 22, normalized size = 3.14

$$-\frac{\log\left(\tan\left(\frac{x}{2}\right)-1\right)}{a^2}+\frac{\log\left(\tan\left(\frac{x}{2}\right)+1\right)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(tan(x/2) - 1)/a**2 + log(tan(x/2) + 1)/a**2
```

3.277 $\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=22

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a^2}$$

```
[Out] 1/2*arctanh(sin(x))/a^2+1/2*sec(x)*tan(x)/a^2
```

Rubi [A] time = 0.03, antiderivative size = 22, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\tan(x)\sec(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ArcTanh[Sin[x]]/(2*a^2) + (Sec[x]*Tan[x])/(2*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^3(x)\,dx}{a^2}\\ &= \frac{\sec(x)\tan(x)}{2a^2}+\frac{\int \sec(x)\,dx}{2a^2}\\ &= \frac{\tanh^{-1}(\sin(x))}{2a^2}+\frac{\sec(x)\tan(x)}{2a^2}\end{aligned}$$

Mathematica [B] time = 0.01, size = 45, normalized size = 2.05

$$\frac{\tan(x)\sec(x)-\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/2]] + Sec[x]*Tan[x])/(2*a
^2)
```

fricas [B] time = 0.42, size = 37, normalized size = 1.68

$$\frac{\cos(x)^2 \log(\sin(x)+1) - \cos(x)^2 \log(-\sin(x)+1) + 2\sin(x)}{4a^2\cos(x)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/4*(cos(x)^2*log(sin(x) + 1) - cos(x)^2*log(-sin(x) + 1) + 2*sin(x))/(a^2*
cos(x)^2)
```

giac [B] time = 0.15, size = 38, normalized size = 1.73

$$\frac{\log(\sin(x)+1)}{4a^2} - \frac{\log(-\sin(x)+1)}{4a^2} - \frac{\sin(x)}{2\left(\sin(x)^2-1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*log(sin(x) + 1)/a^2 - 1/4*log(-sin(x) + 1)/a^2 - 1/2*sin(x)/((sin(x)^2
 - 1)*a^2)
```

maple [B] time = 0.12, size = 44, normalized size = 2.00

$$-\frac{1}{4a^2(-1+\sin(x))} - \frac{\ln(-1+\sin(x))}{4a^2} - \frac{1}{4a^2(1+\sin(x))} + \frac{\ln(1+\sin(x))}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a-a*sin(x)^2)^2,x)
```

```
[Out] -1/4/a^2/(-1+sin(x))-1/4/a^2*ln(-1+sin(x))-1/4/a^2/(1+sin(x))+1/4/a^2*ln(1+
sin(x))
```

maxima [B] time = 0.37, size = 41, normalized size = 1.86

$$-\frac{\sin(x)}{2\left(a^2\sin(x)^2-a^2\right)} + \frac{\log(\sin(x)+1)}{4a^2} - \frac{\log(\sin(x)-1)}{4a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*sin(x)/(a^2*sin(x)^2 - a^2) + 1/4*log(sin(x) + 1)/a^2 - 1/4*log(sin(x)
 - 1)/a^2
```

mupad [B] time = 0.08, size = 30, normalized size = 1.36

$$\frac{\mathrm{atanh}(\sin(x))}{2a^2} - \frac{\sin(x)}{2\left(a^2\sin(x)^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a - a*sin(x)^2)^2,x)
```

```
[Out] atanh(sin(x))/(2*a^2) - sin(x)/(2*(a^2*sin(x)^2 - a^2))
```

sympy [B] time = 1.06, size = 117, normalized size = 5.32

$$-\frac{\log\left(\sin\left(x\right)-1\right)\sin^{2}\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}+\frac{\log\left(\sin\left(x\right)-1\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}+\frac{\log\left(\sin\left(x\right)+1\right)\sin^{2}\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}-\frac{\log\left(\sin\left(x\right)+1\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}-\frac{2\sin\left(x\right)}{4a^{2}\sin^{2}\left(x\right)-4a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] -log(sin(x) - 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4*a**2) + log(sin(x) - 1)/(4
*a**2*sin(x)**2 - 4*a**2) + log(sin(x) + 1)*sin(x)**2/(4*a**2*sin(x)**2 - 4
*a**2) - log(sin(x) + 1)/(4*a**2*sin(x)**2 - 4*a**2) - 2*sin(x)/(4*a**2*sin
(x)**2 - 4*a**2)
```

3.278 $\int \frac{\sec(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=35

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a^2}+\frac{3\tan(x)\sec(x)}{8a^2}$$

```
[Out] 3/8*arctanh(sin(x))/a^2+3/8*sec(x)*tan(x)/a^2+1/4*sec(x)^3*tan(x)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3175, 3768, 3770}

$$\frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{\tan(x)\sec^3(x)}{4a^2}+\frac{3\tan(x)\sec(x)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*ArcTanh[Sin[x]])/(8*a^2) + (3*Sec[x]*Tan[x])/(8*a^2) + (Sec[x]^3*Tan[x])
/(4*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^5(x)\,dx}{a^2}\\ &= \frac{\sec^3(x)\tan(x)}{4a^2}+\frac{3\int \sec^3(x)\,dx}{4a^2}\\ &= \frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}+\frac{3\int \sec(x)\,dx}{8a^2}\\ &= \frac{3\tanh^{-1}(\sin(x))}{8a^2}+\frac{3\sec(x)\tan(x)}{8a^2}+\frac{\sec^3(x)\tan(x)}{4a^2}\end{aligned}$$

Mathematica [A] time = 0.01, size = 61, normalized size = 1.74

$$\frac{\frac{1}{2}(11\sin(x)+3\sin(3x))\sec^4(x)-6\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+6\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)}{16a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (-6*Log[Cos[x/2] - Sin[x/2]] + 6*Log[Cos[x/2] + Sin[x/2]] + (Sec[x]^4*(11*S
in[x] + 3*Sin[3*x]))/2)/(16*a^2)
```

fricas [A] time = 0.44, size = 46, normalized size = 1.31

$$\frac{3\,\cos(x)^4\log\left(\sin(x)+1\right)-3\,\cos(x)^4\log\left(-\sin(x)+1\right)+2\left(3\,\cos(x)^2+2\right)\sin(x)}{16\,a^2\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/16*(3*cos(x)^4*log(sin(x) + 1) - 3*cos(x)^4*log(-sin(x) + 1) + 2*(3*cos(x
)^2 + 2)*sin(x))/(a^2*cos(x)^4)
```

giac [A] time = 0.14, size = 47, normalized size = 1.34

$$\frac{3\,\log\left(\sin(x)+1\right)}{16\,a^2}-\frac{3\,\log\left(-\sin(x)+1\right)}{16\,a^2}-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(\sin(x)^2-1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 3/16*log(sin(x) + 1)/a^2 - 3/16*log(-sin(x) + 1)/a^2 - 1/8*(3*sin(x)^3 - 5*
sin(x))/((sin(x)^2 - 1)^2*a^2)
```

maple [B] time = 0.20, size = 66, normalized size = 1.89

$$\frac{1}{16a^2\left(-1+\sin(x)\right)^2}-\frac{3}{16a^2\left(-1+\sin(x)\right)}-\frac{3\ln\left(-1+\sin(x)\right)}{16a^2}-\frac{1}{16a^2\left(1+\sin(x)\right)^2}-\frac{3}{16a^2\left(1+\sin(x)\right)}+\frac{3\ln\left(1+\sin(\right.}{16a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a-a*sin(x)^2)^2,x)
```

```
[Out] 1/16/a^2/(-1+sin(x))^2-3/16/a^2/(-1+sin(x))-3/16/a^2*ln(-1+sin(x))-1/16/a^2
/(1+sin(x))^2-3/16/a^2/(1+sin(x))+3/16/a^2*ln(1+sin(x))
```

maxima [A] time = 0.37, size = 57, normalized size = 1.63

$$-\frac{3\,\sin(x)^3-5\,\sin(x)}{8\left(a^2\sin(x)^4-2\,a^2\sin(x)^2+a^2\right)}+\frac{3\,\log\left(\sin(x)+1\right)}{16\,a^2}-\frac{3\,\log\left(\sin(x)-1\right)}{16\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/8*(3*sin(x)^3 - 5*sin(x))/(a^2*sin(x)^4 - 2*a^2*sin(x)^2 + a^2) + 3/16*l
og(sin(x) + 1)/a^2 - 3/16*log(sin(x) - 1)/a^2
```

mupad [B] time = 13.98, size = 31, normalized size = 0.89

$$\frac{3\,\mathrm{atanh}\left(\sin(x)\right)}{8\,a^2}+\frac{3\,\sin(x)}{8\,a^2\cos(x)^2}+\frac{\sin(x)}{4\,a^2\cos(x)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a - a*sin(x)^2)^2),x)
```

```
[Out] (3*atanh(sin(x)))/(8*a^2) + (3*sin(x))/(8*a^2*cos(x)^2) + sin(x)/(4*a^2*cos
(x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a-a*sin(x)**2)**2,x)
```

```
[Out] Integral(sec(x)/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.279 $\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=33

$$\frac{3x}{8a^2}+\frac{\sin(x)\cos^3(x)}{4a^2}+\frac{3\sin(x)\cos(x)}{8a^2}$$

```
[Out] 3/8*x/a^2+3/8*cos(x)*sin(x)/a^2+1/4*cos(x)^3*sin(x)/a^2
```

Rubi [A] time = 0.05, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{3x}{8a^2}+\frac{\sin(x)\cos^3(x)}{4a^2}+\frac{3\sin(x)\cos(x)}{8a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (3*x)/(8*a^2) + (3*Cos[x]*Sin[x])/(8*a^2) + (Cos[x]^3*Sin[x])/(4*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^8(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^4(x)\,dx}{a^2}\\
&= \frac{\cos^3(x)\sin(x)}{4a^2}+\frac{3\int \cos^2(x)\,dx}{4a^2}\\
&= \frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}+\frac{3\int 1\,dx}{8a^2}\\
&= \frac{3x}{8a^2}+\frac{3\cos(x)\sin(x)}{8a^2}+\frac{\cos^3(x)\sin(x)}{4a^2}
\end{aligned}$$

Mathematica [A] time = 0.00, size = 26, normalized size = 0.79

$$\frac{\frac{3x}{8}+\frac{1}{4}\sin(2x)+\frac{1}{32}\sin(4x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^8/(a - a*Sin[x]^2)^2,x]
[Out] ((3*x)/8 + Sin[2*x]/4 + Sin[4*x]/32)/a^2
```

fricas [A] time = 0.42, size = 23, normalized size = 0.70

$$\frac{\left(2\cos(x)^3+3\cos(x)\right)\sin(x)+3x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] 1/8*((2*cos(x)^3 + 3*cos(x))*sin(x) + 3*x)/a^2
```

giac [A] time = 0.12, size = 31, normalized size = 0.94

$$\frac{3x}{8a^2}+\frac{3\tan(x)^3+5\tan(x)}{8\left(\tan(x)^2+1\right)^2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 3/8*x/a^2 + 1/8*(3*tan(x)^3 + 5*tan(x))/((tan(x)^2 + 1)^2*a^2)
```

maple [A] time = 0.16, size = 40, normalized size = 1.21

$$\frac{\tan(x)}{4a^2\left(\tan^2(x)+1\right)^2}+\frac{3\tan(x)}{8a^2\left(\tan^2(x)+1\right)}+\frac{3\arctan\left(\tan(x)\right)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8/(a-a*sin(x)^2)^2,x)
[Out] 1/4/a^2*tan(x)/(tan(x)^2+1)^2+3/8/a^2*tan(x)/(tan(x)^2+1)+3/8/a^2*arctan(ta
n(x))
```

maxima [A] time = 0.51, size = 43, normalized size = 1.30

$$\frac{3\tan(x)^3+5\tan(x)}{8\left(a^2\tan(x)^4+2a^2\tan(x)^2+a^2\right)}+\frac{3x}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^8/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/8*(3*tan(x)^3 + 5*tan(x))/(a^2*tan(x)^4 + 2*a^2*tan(x)^2 + a^2) + 3/8*x/a
^2
```

mupad [B] time = 13.82, size = 29, normalized size = 0.88

$$\frac{3x}{8a^2}+\frac{3\cos(x)\sin(x)^3}{8a^2}+\frac{5\cos(x)^3\sin(x)}{8a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^8/(a - a*sin(x)^2)^2,x)
[Out] (3*x)/(8*a^2) + (3*cos(x)*sin(x)^3)/(8*a^2) + (5*cos(x)^3*sin(x))/(8*a^2)
```

sympy [B] time = 69.90, size = 549, normalized size = 16.64

$$\frac{3x\tan^8\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right)+32a^2\tan^2\left(\frac{x}{2}\right)+8a^2}+\frac{12x\tan^6\left(\frac{x}{2}\right)}{8a^2\tan^8\left(\frac{x}{2}\right)+32a^2\tan^6\left(\frac{x}{2}\right)+48a^2\tan^4\left(\frac{x}{2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**8/(a-a*sin(x)**2)**2,x)
```

```
[Out] 3*x*tan(x/2)**8/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2
)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**6/(8*a**2*tan(x/2)**8
 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2
) + 18*x*tan(x/2)**4/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*ta
n(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 12*x*tan(x/2)**2/(8*a**2*tan(x/
2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8
*a**2) + 3*x/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 10*tan(x/2)**7/(8*a**2*tan(x/2)**8 + 32
*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 6
*tan(x/2)**5/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**
4 + 32*a**2*tan(x/2)**2 + 8*a**2) - 6*tan(x/2)**3/(8*a**2*tan(x/2)**8 + 32*
a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 + 32*a**2*tan(x/2)**2 + 8*a**2) + 10
*tan(x/2)/(8*a**2*tan(x/2)**8 + 32*a**2*tan(x/2)**6 + 48*a**2*tan(x/2)**4 +
 32*a**2*tan(x/2)**2 + 8*a**2)
```

3.280 $\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=20

$$\frac{x}{2a^2}+\frac{\sin(x)\cos(x)}{2a^2}$$

```
[Out] 1/2*x/a^2+1/2*cos(x)*sin(x)/a^2
```

Rubi [A] time = 0.04, antiderivative size = 20, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 2635, 8}

$$\frac{x}{2a^2}+\frac{\sin(x)\cos(x)}{2a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/(2*a^2) + (Cos[x]*Sin[x])/(2*a^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^6(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \cos^2(x)\,dx}{a^2}\\ &= \frac{\cos(x)\sin(x)}{2a^2}+\frac{\int 1\,dx}{2a^2}\\ &= \frac{x}{2a^2}+\frac{\cos(x)\sin(x)}{2a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 18, normalized size = 0.90

$$\frac{\frac{x}{2}+\frac{1}{4}\sin(2x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a - a*Sin[x]^2)^2,x]
```

```
[Out] (x/2 + Sin[2*x]/4)/a^2
```

fricas [A] time = 0.41, size = 12, normalized size = 0.60

$$\frac{\cos(x)\sin(x) + x}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] 1/2*(cos(x)*sin(x) + x)/a^2
```

giac [A] time = 0.12, size = 22, normalized size = 1.10

$$\frac{x}{2\,a^2} + \frac{\tan(x)}{2\left(\tan(x)^2 + 1\right)a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 1/2*x/a^2 + 1/2*tan(x)/((tan(x)^2 + 1)*a^2)
```

maple [A] time = 0.16, size = 25, normalized size = 1.25

$$\frac{\tan(x)}{2a^2\left(\tan^2(x) + 1\right)} + \frac{\arctan\left(\tan(x)\right)}{2a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a-a*sin(x)^2)^2,x)
[Out] 1/2/a^2*tan(x)/(tan(x)^2+1)+1/2/a^2*arctan(tan(x))
```

maxima [A] time = 0.45, size = 25, normalized size = 1.25

$$\frac{\tan(x)}{2\left(a^2\tan(x)^2 + a^2\right)} + \frac{x}{2\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/2*tan(x)/(a^2*tan(x)^2 + a^2) + 1/2*x/a^2
```

mupad [B] time = 13.93, size = 13, normalized size = 0.65

$$\frac{2\,x + \sin\left(2\,x\right)}{4\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a - a*sin(x)^2)^2,x)
[Out] (2*x + sin(2*x))/(4*a^2)
```

sympy [B] time = 31.08, size = 178, normalized size = 8.90

$$\frac{x\tan^4\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} + \frac{2x\tan^2\left(\frac{x}{2}\right)}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} + \frac{x}{2a^2\tan^4\left(\frac{x}{2}\right) + 4a^2\tan^2\left(\frac{x}{2}\right) + 2a^2} - \frac{}{2a^2\,\mathrm{ta}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a-a*sin(x)**2)**2,x)
```

```
[Out] x*tan(x/2)**4/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + 2*x*tan(
x/2)**2/(2*a**2*tan(x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) + x/(2*a**2*tan(
x/2)**4 + 4*a**2*tan(x/2)**2 + 2*a**2) - 2*tan(x/2)**3/(2*a**2*tan(x/2)**4
+ 4*a**2*tan(x/2)**2 + 2*a**2) + 2*tan(x/2)/(2*a**2*tan(x/2)**4 + 4*a**2*ta
n(x/2)**2 + 2*a**2)
```

3.281 $\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=5

$$\frac{x}{a^2}$$

```
[Out] x/a^2
```

Rubi [A] time = 0.04, antiderivative size = 5, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 8}

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int 1\,dx}{a^2}\\ &= \frac{x}{a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] x/a^2
```

fricas [A] time = 0.39, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] x/a^2
```

giac [A] time = 0.13, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] x/a^2
```

maple [C] time = 0.19, size = 8, normalized size = 1.60

$$\frac{\arctan(\tan(x))}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a-a*sin(x)^2)^2,x)
[Out] 1/a^2*arctan(tan(x))
```

maxima [A] time = 0.44, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] x/a^2
```

mupad [B] time = 13.94, size = 5, normalized size = 1.00

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a - a*sin(x)^2)^2,x)
[Out] x/a^2
```

sympy [A] time = 13.00, size = 3, normalized size = 0.60

$$\frac{x}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a-a*sin(x)**2)**2,x)
[Out] x/a**2
```

3.282 $\int \frac{\cos^2(x)}{(a-a\sin^2(x))^2} dx$

Optimal. Leaf size=6

$$\frac{\tan(x)}{a^2}$$

[Out] tan(x)/a^2

Rubi [A] time = 0.04, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.188, Rules used = {3175, 3767, 8}

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Int[Cos[x]^2/(a - a*Sin[x]^2)^2,x]

[Out] Tan[x]/a^2

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned} \int \frac{\cos^2(x)}{\left(a-a\sin^2(x)\right)^2} dx &= \frac{\int \sec^2(x)\,dx}{a^2} \\ &= -\frac{\text{Subst}\left(\int 1\,dx, x, -\tan(x)\right)}{a^2} \\ &= \frac{\tan(x)}{a^2} \end{aligned}$$

Mathematica [A] time = 0.00, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[x]^2/(a - a*Sin[x]^2)^2,x]

[Out] Tan[x]/a^2

fricas [A] time = 0.39, size = 10, normalized size = 1.67

$$\frac{\sin(x)}{a^2\cos(x)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] sin(x)/(a^2*cos(x))
```

giac [A] time = 0.13, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] tan(x)/a^2
```

maple [A] time = 0.20, size = 7, normalized size = 1.17

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a-a*sin(x)^2)^2,x)
[Out] tan(x)/a^2
```

maxima [A] time = 0.37, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] tan(x)/a^2
```

mupad [B] time = 13.78, size = 6, normalized size = 1.00

$$\frac{\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a - a*sin(x)^2)^2,x)
[Out] tan(x)/a^2
```

sympy [B] time = 5.64, size = 20, normalized size = 3.33

$$-\frac{2\tan\left(\frac{x}{2}\right)}{a^2\tan^2\left(\frac{x}{2}\right)-a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a-a*sin(x)**2)**2,x)
[Out] -2*tan(x/2)/(a**2*tan(x/2)**2 - a**2)
```

3.283 $\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=29

$$\frac{\tan^5(x)}{5a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan(x)}{a^2}$$

```
[Out] tan(x)/a^2+2/3*tan(x)^3/a^2+1/5*tan(x)^5/a^2
```

Rubi [A] time = 0.05, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^5(x)}{5a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
[Out] Tan[x]/a^2 + (2*Tan[x]^3)/(3*a^2) + Tan[x]^5/(5*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^6(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+2x^2+x^4\right)dx, x, -\tan(x)\right)}{a^2}\\ &= \frac{\tan(x)}{a^2}+\frac{2\tan^3(x)}{3a^2}+\frac{\tan^5(x)}{5a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 31, normalized size = 1.07

$$\frac{\frac{8\tan(x)}{15}+\frac{1}{5}\tan(x)\sec^4(x)+\frac{4}{15}\tan(x)\sec^2(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a - a*Sin[x]^2)^2,x]
[Out] ((8*Tan[x])/15 + (4*Sec[x]^2*Tan[x])/15 + (Sec[x]^4*Tan[x])/5)/a^2
```

fricas [A] time = 0.40, size = 25, normalized size = 0.86

$$\frac{\left(8\cos(x)^4+4\cos(x)^2+3\right)\sin(x)}{15\,a^2\cos(x)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] 1/15*(8*cos(x)^4 + 4*cos(x)^2 + 3)*sin(x)/(a^2*cos(x)^5)
```

giac [A] time = 0.15, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

maple [A] time = 0.24, size = 20, normalized size = 0.69

$$\frac{\frac{\left(\tan^5(x)\right)}{5}+\frac{2\left(\tan^3(x)\right)}{3}+\tan(x)}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a-a*sin(x)^2)^2,x)
[Out] 1/a^2*(1/5*tan(x)^5+2/3*tan(x)^3+tan(x))
```

maxima [A] time = 0.35, size = 22, normalized size = 0.76

$$\frac{3\,\tan(x)^5+10\,\tan(x)^3+15\,\tan(x)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/15*(3*tan(x)^5 + 10*tan(x)^3 + 15*tan(x))/a^2
```

mupad [B] time = 13.80, size = 21, normalized size = 0.72

$$\frac{\tan(x)\,\left(3\tan(x)^4+10\tan(x)^2+15\right)}{15\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a - a*sin(x)^2)^2),x)
[Out] (tan(x)*(10*tan(x)^2 + 3*tan(x)^4 + 15))/(15*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^2(x)}{\sin^4(x)-2\sin^2(x)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a-a*sin(x)**2)**2,x)
[Out] Integral(sec(x)**2/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.284 $\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=37

$$\frac{\tan^7(x)}{7a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^3(x)}{a^2}+\frac{\tan(x)}{a^2}$$

```
[Out] tan(x)/a^2+tan(x)^3/a^2+3/5*tan(x)^5/a^2+1/7*tan(x)^7/a^2
```

Rubi [A] time = 0.05, antiderivative size = 37, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3175, 3767}

$$\frac{\tan^7(x)}{7a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^3(x)}{a^2}+\frac{\tan(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] Tan[x]/a^2 + Tan[x]^3/a^2 + (3*Tan[x]^5)/(5*a^2) + Tan[x]^7/(7*a^2)
```

Rule 3175

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Dist[
a^p, Int[ActivateTrig[u*cos[e + f*x]^(2*p)], x], x] /; FreeQ[{a, b, e, f, p
}, x] && EqQ[a + b, 0] && IntegerQ[p]
```

Rule 3767

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> -Dist[d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^4(x)}{\left(a-a\sin^2(x)\right)^2}\,dx &= \frac{\int \sec^8(x)\,dx}{a^2}\\ &= -\frac{\text{Subst}\left(\int \left(1+3x^2+3x^4+x^6\right)\,dx, x, -\tan(x)\right)}{a^2}\\ &= \frac{\tan(x)}{a^2}+\frac{\tan^3(x)}{a^2}+\frac{3\tan^5(x)}{5a^2}+\frac{\tan^7(x)}{7a^2}\end{aligned}$$

Mathematica [A] time = 0.00, size = 41, normalized size = 1.11

$$\frac{\frac{16\tan(x)}{35}+\frac{1}{7}\tan(x)\sec^6(x)+\frac{6}{35}\tan(x)\sec^4(x)+\frac{8}{35}\tan(x)\sec^2(x)}{a^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a - a*Sin[x]^2)^2,x]
```

```
[Out] ((16*Tan[x])/35 + (8*Sec[x]^2*Tan[x])/35 + (6*Sec[x]^4*Tan[x])/35 + (Sec[x]
^6*Tan[x])/7)/a^2
```

fricas [A] time = 0.41, size = 31, normalized size = 0.84

$$\frac{\left(16\,\cos\left(x\right)^{6}+8\,\cos\left(x\right)^{4}+6\,\cos\left(x\right)^{2}+5\right)\sin\left(x\right)}{35\,a^{2}\cos\left(x\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="fricas")
[Out] 1/35*(16*cos(x)^6 + 8*cos(x)^4 + 6*cos(x)^2 + 5)*sin(x)/(a^2*cos(x)^7)
```

giac [A] time = 0.13, size = 28, normalized size = 0.76

$$\frac{5\,\tan\left(x\right)^{7}+21\,\tan\left(x\right)^{5}+35\,\tan\left(x\right)^{3}+35\,\tan\left(x\right)}{35\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="giac")
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

maple [A] time = 0.24, size = 24, normalized size = 0.65

$$\frac{\frac{\left(\tan^{7}\left(x\right)\right)}{7}+\frac{3\left(\tan^{5}\left(x\right)\right)}{5}+\tan^{3}\left(x\right)+\tan\left(x\right)}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a-a*sin(x)^2)^2,x)
[Out] 1/a^2*(1/7*tan(x)^7+3/5*tan(x)^5+tan(x)^3+tan(x))
```

maxima [A] time = 0.36, size = 28, normalized size = 0.76

$$\frac{5\,\tan\left(x\right)^{7}+21\,\tan\left(x\right)^{5}+35\,\tan\left(x\right)^{3}+35\,\tan\left(x\right)}{35\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a-a*sin(x)^2)^2,x, algorithm="maxima")
[Out] 1/35*(5*tan(x)^7 + 21*tan(x)^5 + 35*tan(x)^3 + 35*tan(x))/a^2
```

mupad [B] time = 13.84, size = 33, normalized size = 0.89

$$\frac{\tan\left(x\right)}{a^{2}}+\frac{{\tan\left(x\right)}^{3}}{a^{2}}+\frac{3\,{\tan\left(x\right)}^{5}}{5\,a^{2}}+\frac{{\tan\left(x\right)}^{7}}{7\,a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a - a*sin(x)^2)^2),x)
[Out] tan(x)/a^2 + tan(x)^3/a^2 + (3*tan(x)^5)/(5*a^2) + tan(x)^7/(7*a^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{\sec^{4}\left(x\right)}{\sin^{4}\left(x\right)-2\sin^{2}\left(x\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a-a*sin(x)**2)**2,x)
[Out] Integral(sec(x)**4/(sin(x)**4 - 2*sin(x)**2 + 1), x)/a**2
```

3.285 $\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=109

$$\frac{(8a+b)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5(8a+b)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5(8a+b)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}x(8$$

```
[Out] 5/128*(8*a+b)*x+5/128*(8*a+b)*cos(f*x+e)*sin(f*x+e)/f+5/192*(8*a+b)*cos(f*x
+e)^3*sin(f*x+e)/f+1/48*(8*a+b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e)^
7*sin(f*x+e)/f
```

Rubi [A] time = 0.06, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3191, 385, 199, 203}

$$\frac{(8a+b)\sin(e+fx)\cos^5(e+fx)}{48f}+\frac{5(8a+b)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{5(8a+b)\sin(e+fx)\cos(e+fx)}{128f}+\frac{5}{128}x(8$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (5*(8*a + b)*x)/128 + (5*(8*a + b)*Cos[e + f*x]*Sin[e + f*x])/(128*f) + (5*
(8*a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(192*f) + ((8*a + b)*Cos[e + f*x]^5*
Sin[e + f*x])/(48*f) - (b*Cos[e + f*x]^7*Sin[e + f*x])/(8*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^6(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^5}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^7(e+fx)\sin(e+fx)}{8f} + \frac{(8a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^4}dx, x, \tan(e$$

$$= \frac{(8a+b)\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{b\cos^7(e+fx)\sin(e+fx)}{8f} + \frac{(5}{}$$

$$= \frac{5(8a+b)\cos^3(e+fx)\sin(e+fx)}{192f} + \frac{(8a+b)\cos^5(e+fx)\sin(e+}{48f}$$

$$= \frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f} + \frac{5(8a+b)\cos^3(e+fx)\sin(e+}{192f}$$

$$= \frac{5}{128}(8a+b)x + \frac{5(8a+b)\cos(e+fx)\sin(e+fx)}{128f} + \frac{5(8a+b)\cos^3(}{}$$

Mathematica [A] time = 0.27, size = 87, normalized size = 0.80

$$\frac{48(15a+b)\sin(2(e+fx)) + 24(6a-b)\sin(4(e+fx)) + 16a\sin(6(e+fx)) + 960ae + 960afx - 16b\sin(6(e+}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (960*a*e + 960*a*f*x + 120*b*f*x + 48*(15*a + b)*Sin[2*(e + f*x)] + 24*(6*a
 - b)*Sin[4*(e + f*x)] + 16*a*Sin[6*(e + f*x)] - 16*b*Sin[6*(e + f*x)] - 3*
b*Sin[8*(e + f*x)])/(3072*f)
```

fricas [A] time = 0.43, size = 78, normalized size = 0.72

$$\frac{15\,(8\,a+b)fx - \left(48\,b\cos\left(fx+e\right)^7 - 8\,(8\,a+b)\cos\left(fx+e\right)^5 - 10\,(8\,a+b)\cos\left(fx+e\right)^3 - 15\,(8\,a+b)\cos\left(f\right.\right.}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/384*(15*(8*a + b)*f*x - (48*b*cos(f*x + e)^7 - 8*(8*a + b)*cos(f*x + e)^5
 - 10*(8*a + b)*cos(f*x + e)^3 - 15*(8*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.23, size = 87, normalized size = 0.80

$$\frac{5}{128}(8\,a+b)x - \frac{b\sin\left(8\,fx+8\,e\right)}{1024\,f} + \frac{(a-b)\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{(6\,a-b)\sin\left(4\,fx+4\,e\right)}{128\,f} + \frac{(15\,a+b)\sin\left(2\,fx+2\,e\right.}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 5/128*(8*a + b)*x - 1/1024*b*sin(8*f*x + 8*e)/f + 1/192*(a - b)*sin(6*f*x +
 6*e)/f + 1/128*(6*a - b)*sin(4*f*x + 4*e)/f + 1/64*(15*a + b)*sin(2*f*x +
2*e)/f
```

maple [A] time = 0.54, size = 112, normalized size = 1.03

$$\frac{b\left(-\frac{\sin(fx+e)(\cos^7(fx+e))}{8}+\frac{\left(\cos^5(fx+e)+\frac{5(\cos^3(fx+e))}{4}+\frac{15\cos(fx+e)}{8}\right)\sin(fx+e)}{48}+\frac{5fx}{128}+\frac{5e}{128}\right)+a\left(\frac{\left(\cos^5(fx+e)+\frac{5(\cos^3(fx+e))}{4}+\frac{15\cos(fx+e)}{8}\right.}{6}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/8*sin(f*x+e)*cos(f*x+e)^7+1/48*(cos(f*x+e)^5+5/4*cos(f*x+e)^3+15
/8*cos(f*x+e))*sin(f*x+e)+5/128*f*x+5/128*e)+a*(1/6*(cos(f*x+e)^5+5/4*cos(f
*x+e)^3+15/8*cos(f*x+e))*sin(f*x+e)+5/16*f*x+5/16*e))
```

maxima [A] time = 0.47, size = 122, normalized size = 1.12

$$\frac{15\left(fx+e\right)(8a+b)+\frac{15(8a+b)\tan(fx+e)^7+55(8a+b)\tan(fx+e)^5+73(8a+b)\tan(fx+e)^3+3(88a-5b)\tan(fx+e)}{\tan(fx+e)^8+4\tan(fx+e)^6+6\tan(fx+e)^4+4\tan(fx+e)^2+1}}{384f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/384*(15*(f*x + e)*(8*a + b) + (15*(8*a + b)*tan(f*x + e)^7 + 55*(8*a + b)
*tan(f*x + e)^5 + 73*(8*a + b)*tan(f*x + e)^3 + 3*(88*a - 5*b)*tan(f*x + e)
)/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2
+ 1))/f
```

mupad [B] time = 15.22, size = 119, normalized size = 1.09

$$x\left(\frac{5a}{16}+\frac{5b}{128}\right)+\frac{\left(\frac{5a}{16}+\frac{5b}{128}\right)\tan\left(e+fx\right)^7+\left(\frac{55a}{48}+\frac{55b}{384}\right)\tan\left(e+fx\right)^5+\left(\frac{73a}{48}+\frac{73b}{384}\right)\tan\left(e+fx\right)^3+\left(\frac{11a}{16}-\frac{5b}{128}\right.}{f\left(\tan\left(e+fx\right)^8+4\tan\left(e+fx\right)^6+6\tan\left(e+fx\right)^4+4\tan\left(e+fx\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*((5*a)/16 + (5*b)/128) + (tan(e + f*x)^7*((5*a)/16 + (5*b)/128) + tan(e +
 f*x)^5*((55*a)/48 + (55*b)/384) + tan(e + f*x)^3*((73*a)/48 + (73*b)/384)
+ tan(e + f*x)*((11*a)/16 - (5*b)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e + f*
x)^4 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

sympy [A] time = 14.47, size = 354, normalized size = 3.25

$$\begin{cases}\frac{5ax\sin^6\left(e+fx\right)}{16}+\frac{15ax\sin^4\left(e+fx\right)\cos^2\left(e+fx\right)}{16}+\frac{15ax\sin^2\left(e+fx\right)\cos^4\left(e+fx\right)}{16}+\frac{5ax\cos^6\left(e+fx\right)}{16}+\frac{5a\sin^5\left(e+fx\right)\cos\left(e+fx\right)}{16f}+\frac{5a\sin^3\left(e\right.}{} \\ x\left(a+b\sin^2\left(e\right)\right)\cos^6\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((5*a*x*sin(e + f*x)**6/16 + 15*a*x*sin(e + f*x)**4*cos(e + f*x)**
2/16 + 15*a*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + 5*a*x*cos(e + f*x)**6/16
 + 5*a*sin(e + f*x)**5*cos(e + f*x)/(16*f) + 5*a*sin(e + f*x)**3*cos(e + f*
x)**3/(6*f) + 11*a*sin(e + f*x)*cos(e + f*x)**5/(16*f) + 5*b*x*sin(e + f*x)
```

```
**8/128 + 5*b*x*sin(e + f*x)**6*cos(e + f*x)**2/32 + 15*b*x*sin(e + f*x)**4
*cos(e + f*x)**4/64 + 5*b*x*sin(e + f*x)**2*cos(e + f*x)**6/32 + 5*b*x*cos(
e + f*x)**8/128 + 5*b*sin(e + f*x)**7*cos(e + f*x)/(128*f) + 55*b*sin(e + f
*x)**5*cos(e + f*x)**3/(384*f) + 73*b*sin(e + f*x)**3*cos(e + f*x)**5/(384*
f) - 5*b*sin(e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)*
*2)*cos(e)**6, True))
```

3.286 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=83

$$\frac{(6a+b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(6a+b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(6a+b)-\frac{b\sin(e+fx)\cos^5(e+fx)}{6f}$$

```
[Out] 1/16*(6*a+b)*x+1/16*(6*a+b)*cos(f*x+e)*sin(f*x+e)/f+1/24*(6*a+b)*cos(f*x+e)
^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)/f
```

Rubi [A] time = 0.05, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3191, 385, 199, 203}

$$\frac{(6a+b)\sin(e+fx)\cos^3(e+fx)}{24f}+\frac{(6a+b)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x(6a+b)-\frac{b\sin(e+fx)\cos^5(e+fx)}{6f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((6*a + b)*x)/16 + ((6*a + b)*Cos[e + f*x]*Sin[e + f*x])/(16*f) + ((6*a + b
)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[e + f*x]^5*Sin[e + f*x])/(6*
f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^4}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{(6a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^3}dx, x, \tan(e\right.}{6f}$$

$$= \frac{(6a+b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+fx)}{6f} + \frac{(6}{}$$

$$= \frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(6a+b)\cos^3(e+fx)\sin(e+fx}{24f}$$

$$= \frac{1}{16}(6a+b)x + \frac{(6a+b)\cos(e+fx)\sin(e+fx)}{16f} + \frac{(6a+b)\cos^3(e+}{24}$$

Mathematica [A] time = 0.15, size = 64, normalized size = 0.77

$$\frac{3(16a+b)\sin(2(e+fx)) + (6a-3b)\sin(4(e+fx)) + 72ae + 72afx - b\sin(6(e+fx)) + 12bfx}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (72*a*e + 72*a*f*x + 12*b*f*x + 3*(16*a + b)*Sin[2*(e + f*x)] + (6*a - 3*b)
*Sin[4*(e + f*x)] - b*Sin[6*(e + f*x)])/(192*f)
```

fricas [A] time = 0.41, size = 63, normalized size = 0.76

$$\frac{3\,(6\,a+b)fx - \left(8\,b\cos\left(fx+e\right)^5 - 2\,(6\,a+b)\cos\left(fx+e\right)^3 - 3\,(6\,a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/48*(3*(6*a + b)*f*x - (8*b*cos(f*x + e)^5 - 2*(6*a + b)*cos(f*x + e)^3 -
3*(6*a + b)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 67, normalized size = 0.81

$$\frac{1}{16}\,(6\,a+b)x - \frac{b\sin\left(6\,fx+6\,e\right)}{192\,f} + \frac{(2\,a-b)\sin\left(4\,fx+4\,e\right)}{64\,f} + \frac{(16\,a+b)\sin\left(2\,fx+2\,e\right)}{64\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/16*(6*a + b)*x - 1/192*b*sin(6*f*x + 6*e)/f + 1/64*(2*a - b)*sin(4*f*x +
4*e)/f + 1/64*(16*a + b)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.57, size = 92, normalized size = 1.11

$$\frac{b\left(-\frac{\sin\left(fx+e\right)\left(\cos^5\left(fx+e\right)\right)}{6} + \frac{\left(\cos^3\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{24} + \frac{fx}{16} + \frac{e}{16}\right) + a\left(\frac{\left(\cos^3\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/6*sin(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin
(f*x+e)+1/16*f*x+1/16*e)+a*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/
8*f*x+3/8*e))
```

maxima [A] time = 0.46, size = 97, normalized size = 1.17

$$\frac{3\left(fx+e\right)(6\,a+b)+\frac{3\,(6\,a+b)\tan\left(fx+e\right)^{5}+8\,(6\,a+b)\tan\left(fx+e\right)^{3}+3\,(10\,a-b)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{6}+3\,\tan\left(fx+e\right)^{4}+3\,\tan\left(fx+e\right)^{2}+1}}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*(f*x + e)*(6*a + b) + (3*(6*a + b)*tan(f*x + e)^5 + 8*(6*a + b)*tan
(f*x + e)^3 + 3*(10*a - b)*tan(f*x + e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4
 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 14.20, size = 91, normalized size = 1.10

$$x\left(\frac{3\,a}{8}+\frac{b}{16}\right)+\frac{\left(\frac{3\,a}{8}+\frac{b}{16}\right)\,\tan\left(e+f\,x\right)^5+\left(a+\frac{b}{6}\right)\,\tan\left(e+f\,x\right)^3+\left(\frac{5\,a}{8}-\frac{b}{16}\right)\,\tan\left(e+f\,x\right)}{f\,\left(\tan\left(e+f\,x\right)^6+3\,\tan\left(e+f\,x\right)^4+3\,\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*((3*a)/8 + b/16) + (tan(e + f*x)^5*((3*a)/8 + b/16) + tan(e + f*x)*((5*a)
/8 - b/16) + tan(e + f*x)^3*(a + b/6))/(f*(3*tan(e + f*x)^2 + 3*tan(e + f*x
)^4 + tan(e + f*x)^6 + 1))
```

sympy [A] time = 5.28, size = 250, normalized size = 3.01

$$\begin{cases} \frac{3ax\sin^{4}\left(e+fx\right)}{8}+\frac{3ax\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+\frac{3ax\cos^{4}\left(e+fx\right)}{8}+\frac{3a\sin^{3}\left(e+fx\right)\cos\left(e+fx\right)}{8f}+\frac{5a\sin\left(e+fx\right)\cos^{3}\left(e+fx\right)}{8f}+\frac{bx\sin^{6}\left(e+fx\right)}{16} \\ x\left(a+b\sin^{2}\left(e\right)\right)\cos^{4}\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((3*a*x*sin(e + f*x)**4/8 + 3*a*x*sin(e + f*x)**2*cos(e + f*x)**2/
4 + 3*a*x*cos(e + f*x)**4/8 + 3*a*sin(e + f*x)**3*cos(e + f*x)/(8*f) + 5*a*
sin(e + f*x)*cos(e + f*x)**3/(8*f) + b*x*sin(e + f*x)**6/16 + 3*b*x*sin(e +
 f*x)**4*cos(e + f*x)**2/16 + 3*b*x*sin(e + f*x)**2*cos(e + f*x)**4/16 + b*
x*cos(e + f*x)**6/16 + b*sin(e + f*x)**5*cos(e + f*x)/(16*f) + b*sin(e + f*
x)**3*cos(e + f*x)**3/(6*f) - b*sin(e + f*x)*cos(e + f*x)**5/(16*f), Ne(f,
0)), (x*(a + b*sin(e)**2)*cos(e)**4, True))
```

3.287 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=57

$$\frac{(4a+b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(4a+b)-\frac{b\sin(e+fx)\cos^3(e+fx)}{4f}$$

```
[Out] 1/8*(4*a+b)*x+1/8*(4*a+b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b*cos(f*x+e)^3*sin(f*
x+e)/f
```

Rubi [A] time = 0.04, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3191, 385, 199, 203}

$$\frac{(4a+b)\sin(e+fx)\cos(e+fx)}{8f}+\frac{1}{8}x(4a+b)-\frac{b\sin(e+fx)\cos^3(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] ((4*a + b)*x)/8 + ((4*a + b)*Cos[e + f*x]*Sin[e + f*x])/(8*f) - (b*Cos[e +
f*x]^3*Sin[e + f*x])/(4*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx = \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{(1+x^2)^3}dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{(4a+b)\,\text{Subst}\left(\int \frac{1}{(1+x^2)^2}dx, x, \tan(e+\right.}{4f}$$

$$= \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)\sin(e+fx)}{4f} + \frac{(4a+}{}$$

$$= \frac{1}{8}(4a+b)x + \frac{(4a+b)\cos(e+fx)\sin(e+fx)}{8f} - \frac{b\cos^3(e+fx)\sin(e+}{4f}$$

Mathematica [A] time = 0.08, size = 46, normalized size = 0.81

$$\frac{4(4ae+4afx+bfx)+8a\sin(2(e+fx))-b\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (4*(4*a*e + 4*a*f*x + b*f*x) + 8*a*Sin[2*(e + f*x)] - b*Sin[4*(e + f*x)])/(
32*f)
```

fricas [A] time = 0.42, size = 47, normalized size = 0.82

$$\frac{(4a+b)fx - \left(2b\cos\left(fx+e\right)^3 - (4a+b)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/8*((4*a + b)*f*x - (2*b*cos(f*x + e)^3 - (4*a + b)*cos(f*x + e))*sin(f*x
+ e))/f
```

giac [A] time = 0.16, size = 41, normalized size = 0.72

$$\frac{1}{8}(4a+b)x - \frac{b\sin\left(4fx+4e\right)}{32f} + \frac{a\sin\left(2fx+2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/8*(4*a + b)*x - 1/32*b*sin(4*f*x + 4*e)/f + 1/4*a*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.35, size = 70, normalized size = 1.23

$$\frac{b\left(-\frac{\sin\left(fx+e\right)\left(\cos^3\left(fx+e\right)\right)}{4}+\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{8}+\frac{fx}{8}+\frac{e}{8}\right)+a\left(\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(b*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*
e)+a*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e))
```

maxima [A] time = 0.46, size = 69, normalized size = 1.21

$$\frac{\left(fx+e\right)\left(4\,a+b\right)+\frac{\left(4\,a+b\right)\tan\left(fx+e\right)^{3}+\left(4\,a-b\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^{4}+2\,\tan\left(fx+e\right)^{2}+1}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/8*((f*x + e)*(4*a + b) + ((4*a + b)*tan(f*x + e)^3 + (4*a - b)*tan(f*x +
e))/(tan(f*x + e)^4 + 2*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 13.66, size = 67, normalized size = 1.18

$$x\,\left(\frac{a}{2}+\frac{b}{8}\right)+\frac{\left(\frac{a}{2}+\frac{b}{8}\right)\,\tan\left(e+f\,x\right)^{3}+\left(\frac{a}{2}-\frac{b}{8}\right)\,\tan\left(e+f\,x\right)}{f\,\left(\tan\left(e+f\,x\right)^{4}+2\,\tan\left(e+f\,x\right)^{2}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2),x)
```

```
[Out] x*(a/2 + b/8) + (tan(e + f*x)^3*(a/2 + b/8) + tan(e + f*x)*(a/2 - b/8))/(f*
(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

sympy [A] time = 1.42, size = 150, normalized size = 2.63

$$\begin{cases} \frac{ax\sin^{2}\left(e+fx\right)}{2}+\frac{ax\cos^{2}\left(e+fx\right)}{2}+\frac{a\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{bx\sin^{4}\left(e+fx\right)}{8}+\frac{bx\sin^{2}\left(e+fx\right)\cos^{2}\left(e+fx\right)}{4}+\frac{bx\cos^{4}\left(e+fx\right)}{8}+\frac{b\sin^{3}\left(e- \right.}{} \\ x\left(a+b\sin^{2}\left(e\right)\right)\cos^{2}\left(e\right) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Piecewise((a*x*sin(e + f*x)**2/2 + a*x*cos(e + f*x)**2/2 + a*sin(e + f*x)*c
os(e + f*x)/(2*f) + b*x*sin(e + f*x)**4/8 + b*x*sin(e + f*x)**2*cos(e + f*x
)**2/4 + b*x*cos(e + f*x)**4/8 + b*sin(e + f*x)**3*cos(e + f*x)/(8*f) - b*s
in(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)), (x*(a + b*sin(e)**2)*cos(e)**
2, True))
```

3.288 $\int \left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$ax-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

```
[Out] a*x+1/2*b*x-1/2*b*cos(f*x+e)*sin(f*x+e)/f
```

Rubi [A] time = 0.02, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 12, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {2635, 8}

$$ax-\frac{b\sin(e+fx)\cos(e+fx)}{2f}+\frac{bx}{2}$$

Antiderivative was successfully verified.

```
[In] Int[a + b*Sin[e + f*x]^2,x]
```

```
[Out] a*x + (b*x)/2 - (b*Cos[e + f*x]*Sin[e + f*x])/(2*f)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2635

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Sin[c + d*x])^(n - 1))/(d*n), x] + Dist[(b^2*(n - 1))/n, Int[(b*Sin[c
+ d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2*n
]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(e+fx)\right)\,dx &= ax+b\int \sin^2(e+fx)\,dx\\ &= ax-\frac{b\cos(e+fx)\sin(e+fx)}{2f}+\frac{1}{2}b\int 1\,dx\\ &= ax+\frac{bx}{2}-\frac{b\cos(e+fx)\sin(e+fx)}{2f}\end{aligned}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.10

$$ax+\frac{b(e+fx)}{2f}-\frac{b\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[a + b*Sin[e + f*x]^2,x]
```

```
[Out] a*x + (b*(e + f*x))/(2*f) - (b*Sin[2*(e + f*x)])/(4*f)
```

fricas [A] time = 0.43, size = 29, normalized size = 0.97

$$\frac{(2\,a+b)fx-b\cos\left(fx+e\right)\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] 1/2*((2*a + b)*f*x - b*cos(f*x + e)*sin(f*x + e))/f
```

giac [A] time = 0.13, size = 26, normalized size = 0.87

$$\frac{1}{4}\,b\left(2\,x-\frac{\sin\left(2\,fx+2\,e\right)}{f}\right)+ax$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="giac")
```

```
[Out] 1/4*b*(2*x - sin(2*f*x + 2*e)/f) + a*x
```

maple [A] time = 0.07, size = 32, normalized size = 1.07

$$ax+\frac{b\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2}+\frac{fx}{2}+\frac{e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a+b*sin(f*x+e)^2,x)
```

```
[Out] a*x+b/f*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)
```

maxima [A] time = 0.36, size = 29, normalized size = 0.97

$$ax+\frac{\left(2\,fx+2\,e-\sin\left(2\,fx+2\,e\right)\right)b}{4\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] a*x + 1/4*(2*f*x + 2*e - sin(2*f*x + 2*e))*b/f
```

mupad [B] time = 13.65, size = 27, normalized size = 0.90

$$-\frac{\frac{b\,\sin\left(2\,e+2\,f\,x\right)}{4}-f\,x\,\left(a+\frac{b}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(a + b*sin(e + f*x)^2,x)
```

```
[Out] -((b*sin(2*e + 2*f*x))/4 - f*x*(a + b/2))/f
```

sympy [A] time = 0.25, size = 51, normalized size = 1.70

$$ax+b\left(\begin{cases}\frac{x\sin^{2}\left(e+fx\right)}{2}+\frac{x\cos^{2}\left(e+fx\right)}{2}-\frac{\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f} & \text{for}\: f\neq 0\\ x\sin^{2}\left(e\right) & \text{otherwise}\end{cases}\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(a+b*sin(f*x+e)**2,x)
```

```
[Out] a*x + b*Piecewise((x*sin(e + f*x)**2/2 + x*cos(e + f*x)**2/2 - sin(e + f*x)
*cos(e + f*x)/(2*f), Ne(f, 0)), (x*sin(e)**2, True))
```

3.289 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=18

$$\frac{(a+b)\tan(e+fx)}{f}-bx$$

```
[Out] -b*x+(a+b)*tan(f*x+e)/f
```

Rubi [A] time = 0.03, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3191, 388, 203}

$$\frac{(a+b)\tan(e+fx)}{f}-bx$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] -(b*x) + ((a + b)*Tan[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)dx &= \frac{\text{Subst}\left(\int \frac{a+(a+b)x^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{(a+b)\tan(e+fx)}{f}-\frac{b\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= -bx+\frac{(a+b)\tan(e+fx)}{f}
\end{aligned}$$

Mathematica [A] time = 0.01, size = 36, normalized size = 2.00

$$\frac{a\tan(e+fx)}{f}-\frac{b\tan^{-1}(\tan(e+fx))}{f}+\frac{b\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2),x]
[Out] -((b*ArcTan[Tan[e + f*x]])/f) + (a*Tan[e + f*x])/f + (b*Tan[e + f*x])/f
```

fricas [A] time = 0.43, size = 35, normalized size = 1.94

$$-\frac{bfx\cos\left(fx+e\right)-(a+b)\sin\left(fx+e\right)}{f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
[Out] -(b*f*x*cos(f*x + e) - (a + b)*sin(f*x + e))/(f*cos(f*x + e))
```

giac [B] time = 0.18, size = 49, normalized size = 2.72

$$-\frac{\left(fx-\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+e-\tan\left(fx+e\right)\right)b-a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="giac")
[Out] -((f*x - pi*floor((f*x + e)/pi + 1/2) + e - tan(f*x + e))*b - a*tan(f*x + e
))/f
```

maple [A] time = 0.48, size = 30, normalized size = 1.67

$$\frac{\tan\left(fx+e\right)a+b\left(\tan\left(fx+e\right)-fx-e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x)
[Out] 1/f*(tan(f*x+e)*a+b*(tan(f*x+e)-f*x-e))
```

maxima [A] time = 0.46, size = 30, normalized size = 1.67

$$-\frac{\left(fx+e-\tan\left(fx+e\right)\right)b-a\tan\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
[Out] -((f*x + e - tan(f*x + e))*b - a*tan(f*x + e))/f
```

mupad [B] time = 13.64, size = 26, normalized size = 1.44

$$\frac{a\tan\left(e+f\,x\right)+b\tan\left(e+f\,x\right)-b\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^2,x)
[Out] (a*tan(e + f*x) + b*tan(e + f*x) - b*f*x)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^2\left(e+fx\right)\right)\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.290 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=30

$$\frac{(a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(a+b)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.03, antiderivative size = 30, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 1, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.048, Rules used = {3191}

$$\frac{(a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
[Out] (a*Tan[e + f*x])/f + ((a + b)*Tan[e + f*x]^3)/(3*f)
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx &= \frac{\operatorname{Subst}\left(\int \left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(a+b)\tan^3(e+fx)}{3f}\end{aligned}$$

Mathematica [A] time = 0.07, size = 41, normalized size = 1.37

$$\frac{a\left(\frac{1}{3}\tan^3(e+fx)+\tan(e+fx)\right)}{f}+\frac{b\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2),x]
[Out] (b*Tan[e + f*x]^3)/(3*f) + (a*(Tan[e + f*x] + Tan[e + f*x]^3/3))/f
```

fricas [A] time = 0.41, size = 38, normalized size = 1.27

$$\frac{\left((2\,a-b)\cos\left(fx+e\right)^2+a+b\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/3*((2*a - b)*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(f*cos(f*x + e)^3)
```

giac [A] time = 0.19, size = 38, normalized size = 1.27

$$\frac{a\tan\left(fx+e\right)^{3}+b\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="giac")
[Out] 1/3*(a*tan(f*x + e)^3 + b*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

maple [A] time = 0.46, size = 46, normalized size = 1.53

$$\frac{-a\left(-\frac{2}{3}-\frac{\left(\sec^{2}\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)+\frac{b\left(\sin^{3}\left(fx+e\right)\right)}{3\cos\left(fx+e\right)^{3}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x)
[Out] 1/f*(-a*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+1/3*b*sin(f*x+e)^3/cos(f*x+e)^3)
```

maxima [A] time = 0.36, size = 27, normalized size = 0.90

$$\frac{(a+b)\tan\left(fx+e\right)^{3}+3\,a\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
[Out] 1/3*((a + b)*tan(f*x + e)^3 + 3*a*tan(f*x + e))/f
```

mupad [B] time = 14.16, size = 31, normalized size = 1.03

$$\frac{\tan\left(e+f\,x\right)^{3}\,\left(\frac{a}{3}+\frac{b}{3}\right)}{f}+\frac{a\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^4,x)
[Out] (tan(e + f*x)^3*(a/3 + b/3))/f + (a*tan(e + f*x))/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{2}\left(e+fx\right)\right)\sec^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2),x)
[Out] Integral((a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.291 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=50

$$\frac{(a+b)\tan^5(e+fx)}{5f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(2*a+b)*tan(f*x+e)^3/f+1/5*(a+b)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.04, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a+b)\tan^5(e+fx)}{5f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((2*a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*Tan[e + f*
x]^5)/(5*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx&=\frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{\text{Subst}\left(\int\left(a+(2a+b)x^2+(a+b)x^4\right)\,dx,x,\tan(e+fx)\right)}{f}\\&=\frac{a\tan(e+fx)}{f}+\frac{(2a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)\tan^5(e+fx)}{5f}\end{aligned}$$

Mathematica [A] time = 0.18, size = 64, normalized size = 1.28

$$\frac{\tan(e+fx)\left(3a\tan^4(e+fx)+10a\tan^2(e+fx)+15a+3b\sec^4(e+fx)-b\sec^2(e+fx)-2b\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (Tan[e + f*x]*(15*a - 2*b - b*Sec[e + f*x]^2 + 3*b*Sec[e + f*x]^4 + 10*a*Ta
n[e + f*x]^2 + 3*a*Tan[e + f*x]^4))/(15*f)
```

fricas [A] time = 0.42, size = 59, normalized size = 1.18

$$\frac{\left(2\,(4\,a-b)\cos\left(fx+e\right)^{4}+(4\,a-b)\cos\left(fx+e\right)^{2}+3\,a+3\,b\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/15*(2*(4*a - b)*cos(f*x + e)^4 + (4*a - b)*cos(f*x + e)^2 + 3*a + 3*b)*si
n(f*x + e)/(f*cos(f*x + e)^5)
```

giac [A] time = 0.20, size = 64, normalized size = 1.28

$$\frac{3\,a\tan\left(fx+e\right)^{5}+3\,b\tan\left(fx+e\right)^{5}+10\,a\tan\left(fx+e\right)^{3}+5\,b\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/15*(3*a*tan(f*x + e)^5 + 3*b*tan(f*x + e)^5 + 10*a*tan(f*x + e)^3 + 5*b*t
an(f*x + e)^3 + 15*a*tan(f*x + e))/f
```

maple [A] time = 0.54, size = 76, normalized size = 1.52

$$\frac{-a\left(-\frac{8}{15}-\frac{\left(\sec^{4}\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^{2}\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)+b\left(\frac{\sin^{3}\left(fx+e\right)}{5\cos\left(fx+e\right)^{5}}+\frac{2\left(\sin^{3}\left(fx+e\right)\right)}{15\cos\left(fx+e\right)^{3}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+b*(1/5*sin(f*
x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3))
```

maxima [A] time = 0.35, size = 43, normalized size = 0.86

$$\frac{3\,(a+b)\tan\left(fx+e\right)^{5}+5\,(2\,a+b)\tan\left(fx+e\right)^{3}+15\,a\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a + b)*tan(f*x + e)^5 + 5*(2*a + b)*tan(f*x + e)^3 + 15*a*tan(f*x
+ e))/f
```

mupad [B] time = 13.90, size = 45, normalized size = 0.90

$$\frac{\left(\frac{a}{5}+\frac{b}{5}\right)\,\tan\left(e+f\,x\right)^{5}+\left(\frac{2\,a}{3}+\frac{b}{3}\right)\,\tan\left(e+f\,x\right)^{3}+a\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^6,x)
```

```
[Out] (tan(e + f*x)^3*((2*a)/3 + b/3) + tan(e + f*x)^5*(a/5 + b/5) + a*tan(e + f*
x))/f
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.292 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx$

Optimal. Leaf size=72

$$\frac{(a+b)\tan^7(e+fx)}{7f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

```
[Out] a*tan(f*x+e)/f+1/3*(3*a+b)*tan(f*x+e)^3/f+1/5*(3*a+2*b)*tan(f*x+e)^5/f+1/7*
(a+b)*tan(f*x+e)^7/f
```

Rubi [A] time = 0.06, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3191, 373}

$$\frac{(a+b)\tan^7(e+fx)}{7f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{a\tan(e+fx)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (a*Tan[e + f*x])/f + ((3*a + b)*Tan[e + f*x]^3)/(3*f) + ((3*a + 2*b)*Tan[e
+ f*x]^5)/(5*f) + ((a + b)*Tan[e + f*x]^7)/(7*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+(a+b)x^2\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a+(3a+b)x^2+(3a+2b)x^4+(a+b)x^6\right)\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{a\tan(e+fx)}{f}+\frac{(3a+b)\tan^3(e+fx)}{3f}+\frac{(3a+2b)\tan^5(e+fx)}{5f}+\frac{(a+\end{aligned}$$

Mathematica [A] time = 0.31, size = 86, normalized size = 1.19

$$\frac{\tan(e+fx)\left(15a\tan^6(e+fx)+63a\tan^4(e+fx)+105a\tan^2(e+fx)+105a+15b\sec^6(e+fx)-3b\sec^4(e+f\right.}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2),x]
```

```
[Out] (Tan[e + f*x]*(105*a - 8*b - 4*b*Sec[e + f*x]^2 - 3*b*Sec[e + f*x]^4 + 15*b
*Sec[e + f*x]^6 + 105*a*Tan[e + f*x]^2 + 63*a*Tan[e + f*x]^4 + 15*a*Tan[e +
 f*x]^6))/(105*f)
```

fricas [A] time = 0.40, size = 77, normalized size = 1.07

$$\frac{\left(8\,(6\,a-b)\cos\left(fx+e\right)^{6}+4\,(6\,a-b)\cos\left(fx+e\right)^{4}+3\,(6\,a-b)\cos\left(fx+e\right)^{2}+15\,a+15\,b\right)\sin\left(fx+e\right)}{105\,f\cos\left(fx+e\right)^{7}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="fricas")
```

```
[Out] 1/105*(8*(6*a - b)*cos(f*x + e)^6 + 4*(6*a - b)*cos(f*x + e)^4 + 3*(6*a - b
)*cos(f*x + e)^2 + 15*a + 15*b)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

giac [A] time = 0.22, size = 88, normalized size = 1.22

$$\frac{15\,a\tan\left(fx+e\right)^{7}+15\,b\tan\left(fx+e\right)^{7}+63\,a\tan\left(fx+e\right)^{5}+42\,b\tan\left(fx+e\right)^{5}+105\,a\tan\left(fx+e\right)^{3}+35\,b\,\mathrm{t}}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="giac")
```

```
[Out] 1/105*(15*a*tan(f*x + e)^7 + 15*b*tan(f*x + e)^7 + 63*a*tan(f*x + e)^5 + 42
*b*tan(f*x + e)^5 + 105*a*tan(f*x + e)^3 + 35*b*tan(f*x + e)^3 + 105*a*tan(
f*x + e))/f
```

maple [A] time = 0.50, size = 104, normalized size = 1.44

$$\frac{-a\left(-\frac{16}{35}-\frac{\left(\sec^{6}\left(fx+e\right)\right)}{7}-\frac{6\left(\sec^{4}\left(fx+e\right)\right)}{35}-\frac{8\left(\sec^{2}\left(fx+e\right)\right)}{35}\right)\tan\left(fx+e\right)+b\left(\frac{\sin^{3}\left(fx+e\right)}{7\cos\left(fx+e\right)^{7}}+\frac{4\left(\sin^{3}\left(fx+e\right)\right)}{35\cos\left(fx+e\right)^{5}}+\frac{8\left(\sin^{3}\left(fx+e\right)\right)}{105\cos\left(fx+e\right)^{3}}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x)
```

```
[Out] 1/f*(-a*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan(f
*x+e)+b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5+8/105
*sin(f*x+e)^3/cos(f*x+e)^3))
```

maxima [A] time = 0.38, size = 60, normalized size = 0.83

$$\frac{15\,(a+b)\tan\left(fx+e\right)^{7}+21\,(3\,a+2\,b)\tan\left(fx+e\right)^{5}+35\,(3\,a+b)\tan\left(fx+e\right)^{3}+105\,a\tan\left(fx+e\right)}{105\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(15*(a + b)*tan(f*x + e)^7 + 21*(3*a + 2*b)*tan(f*x + e)^5 + 35*(3*a
+ b)*tan(f*x + e)^3 + 105*a*tan(f*x + e))/f
```

mupad [B] time = 13.84, size = 59, normalized size = 0.82

$$\frac{\left(\frac{a}{7}+\frac{b}{7}\right)\tan\left(e+f\,x\right)^{7}+\left(\frac{3\,a}{5}+\frac{2\,b}{5}\right)\tan\left(e+f\,x\right)^{5}+\left(a+\frac{b}{3}\right)\tan\left(e+f\,x\right)^{3}+a\tan\left(e+f\,x\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/cos(e + f*x)^8,x)
```

```
[Out] (tan(e + f*x)^5*((3*a)/5 + (2*b)/5) + tan(e + f*x)^7*(a/7 + b/7) + a*tan(e
+ f*x) + tan(e + f*x)^3*(a + b/3))/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2),x)
```

```
[Out] Timed out
```

3.293 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2 dx$

Optimal. Leaf size=156

$$\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos(e+fx)}{128f}+\frac{1}{128}x\left(48a^2+16\right.$$

```
[Out] 1/128*(48*a^2+16*a*b+3*b^2)*x+1/128*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)*sin(f*
x+e)/f+1/192*(48*a^2+16*a*b+3*b^2)*cos(f*x+e)^3*sin(f*x+e)/f-1/48*b*(10*a+3
*b)*cos(f*x+e)^5*sin(f*x+e)/f-1/8*b*cos(f*x+e)^7*sin(f*x+e)*(a+(a+b)*tan(f*
x+e)^2)/f
```

Rubi [A] time = 0.15, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 203}

$$\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos^3(e+fx)}{192f}+\frac{\left(48a^2+16ab+3b^2\right)\sin(e+fx)\cos(e+fx)}{128f}+\frac{1}{128}x\left(48a^2+16\right.$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 + 16*a*b + 3*b^2)*x)/128 + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]
*Sin[e + f*x])/(128*f) + ((48*a^2 + 16*a*b + 3*b^2)*Cos[e + f*x]^3*Sin[e +
f*x])/(192*f) - (b*(10*a + 3*b)*Cos[e + f*x]^5*Sin[e + f*x])/(48*f) - (b*Co
s[e + f*x]^7*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(8*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^5}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^7(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{8f} + \frac{\text{Subst}\left(\int \frac{a(8}{}\right.}{}$$

$$= -\frac{b(10a+3b)\cos^5(e+fx)\sin(e+fx)}{48f} - \frac{b\cos^7(e+fx)\sin(e+fx)\left(a\right.}{8f}$$

$$= \frac{\left(48a^2+16ab+3b^2\right)\cos^3(e+fx)\sin(e+fx)}{192f} - \frac{b(10a+3b)\cos^5(e+}{48f}$$

$$= \frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+fx)}{128f} + \frac{\left(48a^2+16ab+3b^2\right)\text{c}}{19}$$

$$= \frac{1}{128}\left(48a^2+16ab+3b^2\right)x + \frac{\left(48a^2+16ab+3b^2\right)\cos(e+fx)\sin(e+}{128f}$$

Mathematica [A] time = 0.32, size = 96, normalized size = 0.62

$$\frac{24\left(48a^2+16ab+3b^2\right)(e+fx)+24\left(4a^2-4ab-b^2\right)\sin(4(e+fx))-32ab\sin(6(e+fx))+96a(8a+b)\sin(2(e}{3072f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (24*(48*a^2 + 16*a*b + 3*b^2)*(e + f*x) + 96*a*(8*a + b)*Sin[2*(e + f*x)] +
 24*(4*a^2 - 4*a*b - b^2)*Sin[4*(e + f*x)] - 32*a*b*Sin[6*(e + f*x)] + 3*b^
2*Sin[8*(e + f*x)])/(3072*f)
```

fricas [A] time = 0.42, size = 114, normalized size = 0.73

$$\frac{3\left(48a^2+16ab+3b^2\right)fx+\left(48b^2\cos\left(fx+e\right)^7-8\left(16ab+9b^2\right)\cos\left(fx+e\right)^5+2\left(48a^2+16ab+3b^2\right)\cos\left(f\right.\right.}{384f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*f*x + (48*b^2*cos(f*x + e)^7 - 8*(16*a*b
 + 9*b^2)*cos(f*x + e)^5 + 2*(48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e)^3 + 3*(
48*a^2 + 16*a*b + 3*b^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.24, size = 108, normalized size = 0.69

$$\frac{1}{128}\left(48\,a^2+16\,ab+3\,b^2\right)x+\frac{b^2\sin\left(8\,fx+8\,e\right)}{1024\,f}-\frac{ab\sin\left(6\,fx+6\,e\right)}{96\,f}+\frac{\left(4\,a^2-4\,ab-b^2\right)\sin\left(4\,fx+4\,e\right)}{128\,f}+\frac{\left(8\,a^2\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/128*(48*a^2 + 16*a*b + 3*b^2)*x + 1/1024*b^2*sin(8*f*x + 8*e)/f - 1/96*a*
b*sin(6*f*x + 6*e)/f + 1/128*(4*a^2 - 4*a*b - b^2)*sin(4*f*x + 4*e)/f + 1/3
2*(8*a^2 + a*b)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.51, size = 167, normalized size = 1.07

$$\frac{b^2\left(-\frac{\left(\sin^3\left(fx+e\right)\right)\left(\cos^5\left(fx+e\right)\right)}{8}-\frac{\sin\left(fx+e\right)\left(\cos^5\left(fx+e\right)\right)}{16}+\frac{\left(\cos^3\left(fx+e\right)+\frac{3\cos\left(fx+e\right)}{2}\right)\sin\left(fx+e\right)}{64}+\frac{3fx}{128}+\frac{3e}{128}\right)+2ab\left(-\frac{\sin\left(fx+e\right)\left(c\right.}{6}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/8*sin(f*x+e)^3*cos(f*x+e)^5-1/16*sin(f*x+e)*cos(f*x+e)^5+1/64*
(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/128*f*x+3/128*e)+2*a*b*(-1/6*sin
(f*x+e)*cos(f*x+e)^5+1/24*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+1/16*f*x
+1/16*e)+a^2*(1/4*(cos(f*x+e)^3+3/2*cos(f*x+e))*sin(f*x+e)+3/8*f*x+3/8*e))
```

maxima [A] time = 0.50, size = 169, normalized size = 1.08

$$\frac{3\left(48\,a^2+16\,ab+3\,b^2\right)\left(fx+e\right)+\frac{3\left(48\,a^2+16\,ab+3\,b^2\right)\tan\left(fx+e\right)^7+11\left(48\,a^2+16\,ab+3\,b^2\right)\tan\left(fx+e\right)^5+\left(624\,a^2+80\,ab-33\,b^2\right)\tan\left(fx\right.}{\tan\left(fx+e\right)^8+4\tan\left(fx+e\right)^6+6\tan\left(fx+e\right)^4+4\tan\left(fx+e\right)^2+1}}{384\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/384*(3*(48*a^2 + 16*a*b + 3*b^2)*(f*x + e) + (3*(48*a^2 + 16*a*b + 3*b^2)
*tan(f*x + e)^7 + 11*(48*a^2 + 16*a*b + 3*b^2)*tan(f*x + e)^5 + (624*a^2 +
80*a*b - 33*b^2)*tan(f*x + e)^3 + 3*(80*a^2 - 16*a*b - 3*b^2)*tan(f*x + e))
/(tan(f*x + e)^8 + 4*tan(f*x + e)^6 + 6*tan(f*x + e)^4 + 4*tan(f*x + e)^2 +
 1))/f
```

mupad [B] time = 15.49, size = 160, normalized size = 1.03

$$x\left(\frac{3\,a^2}{8}+\frac{a\,b}{8}+\frac{3\,b^2}{128}\right)+\frac{\left(\frac{3\,a^2}{8}+\frac{a\,b}{8}+\frac{3\,b^2}{128}\right)\tan\left(e+f\,x\right)^7+\left(\frac{11\,a^2}{8}+\frac{11\,a\,b}{24}+\frac{11\,b^2}{128}\right)\tan\left(e+f\,x\right)^5+\left(\frac{13\,a^2}{8}+\frac{5\,a\,b}{24}-\right.}{f\left(\tan\left(e+f\,x\right)^8+4\tan\left(e+f\,x\right)^6+6\tan\left(e+f\,x\right)^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^2,x)
```

```
[Out] x*((a*b)/8 + (3*a^2)/8 + (3*b^2)/128) + (tan(e + f*x)^7*((a*b)/8 + (3*a^2)/
8 + (3*b^2)/128) - tan(e + f*x)*((a*b)/8 - (5*a^2)/8 + (3*b^2)/128) + tan(e
 + f*x)^3*((5*a*b)/24 + (13*a^2)/8 - (11*b^2)/128) + tan(e + f*x)^5*((11*a*
b)/24 + (11*a^2)/8 + (11*b^2)/128))/(f*(4*tan(e + f*x)^2 + 6*tan(e + f*x)^4
 + 4*tan(e + f*x)^6 + tan(e + f*x)^8 + 1))
```

sympy [A] time = 14.62, size = 481, normalized size = 3.08

$$\begin{cases} \frac{3a^2x\sin^4(e+fx)}{8} + \frac{3a^2x\sin^2(e+fx)\cos^2(e+fx)}{4} + \frac{3a^2x\cos^4(e+fx)}{8} + \frac{3a^2\sin^3(e+fx)\cos(e+fx)}{8f} + \frac{5a^2\sin(e+fx)\cos^3(e+fx)}{8f} + \frac{abx\sin^6(}{8} \\ x\left(a+b\sin^2(e)\right)^2\cos^4(e) \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((3*a**2*x*sin(e + f*x)**4/8 + 3*a**2*x*sin(e + f*x)**2*cos(e + f*
x)**2/4 + 3*a**2*x*cos(e + f*x)**4/8 + 3*a**2*sin(e + f*x)**3*cos(e + f*x)/
(8*f) + 5*a**2*sin(e + f*x)*cos(e + f*x)**3/(8*f) + a*b*x*sin(e + f*x)**6/8
 + 3*a*b*x*sin(e + f*x)**4*cos(e + f*x)**2/8 + 3*a*b*x*sin(e + f*x)**2*cos(
e + f*x)**4/8 + a*b*x*cos(e + f*x)**6/8 + a*b*sin(e + f*x)**5*cos(e + f*x)/
(8*f) + a*b*sin(e + f*x)**3*cos(e + f*x)**3/(3*f) - a*b*sin(e + f*x)*cos(e
+ f*x)**5/(8*f) + 3*b**2*x*sin(e + f*x)**8/128 + 3*b**2*x*sin(e + f*x)**6*c
os(e + f*x)**2/32 + 9*b**2*x*sin(e + f*x)**4*cos(e + f*x)**4/64 + 3*b**2*x*
sin(e + f*x)**2*cos(e + f*x)**6/32 + 3*b**2*x*cos(e + f*x)**8/128 + 3*b**2*
sin(e + f*x)**7*cos(e + f*x)/(128*f) + 11*b**2*sin(e + f*x)**5*cos(e + f*x)
**3/(128*f) - 11*b**2*sin(e + f*x)**3*cos(e + f*x)**5/(128*f) - 3*b**2*sin(
e + f*x)*cos(e + f*x)**7/(128*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)
**4, True))
```

3.294 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=116

$$\frac{\left(8a^2+4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(8a^2+4ab+b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{b\sin(e}{}$$

```
[Out] 1/16*(8*a^2+4*a*b+b^2)*x+1/16*(8*a^2+4*a*b+b^2)*cos(f*x+e)*sin(f*x+e)/f-1/2
4*b*(8*a+3*b)*cos(f*x+e)^3*sin(f*x+e)/f-1/6*b*cos(f*x+e)^5*sin(f*x+e)*(a+(a
+b)*tan(f*x+e)^2)/f
```

Rubi [A] time = 0.15, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3191, 413, 385, 199, 203}

$$\frac{\left(8a^2+4ab+b^2\right)\sin(e+fx)\cos(e+fx)}{16f}+\frac{1}{16}x\left(8a^2+4ab+b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos^3(e+fx)}{24f}-\frac{b\sin(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 4*a*b + b^2)*x)/16 + ((8*a^2 + 4*a*b + b^2)*Cos[e + f*x]*Sin[e +
f*x])/(16*f) - (b*(8*a + 3*b)*Cos[e + f*x]^3*Sin[e + f*x])/(24*f) - (b*Cos[
e + f*x]^5*Sin[e + f*x]*(a + (a + b)*Tan[e + f*x]^2))/(6*f)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^4}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\left(a+(a+b)\tan^2(e+fx)\right)}{6f} + \frac{\text{Subst}\left(\int \frac{a(6a}{}\right.}{}$$

$$= -\frac{b(8a+3b)\cos^3(e+fx)\sin(e+fx)}{24f} - \frac{b\cos^5(e+fx)\sin(e+fx)\left(a\right.}{6f}$$

$$= \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b(8a+3b)\cos^3(e+fx)\sin}{24f}$$

$$= \frac{1}{16}\left(8a^2+4ab+b^2\right)x + \frac{\left(8a^2+4ab+b^2\right)\cos(e+fx)\sin(e+fx)}{16f} - \frac{b}{}$$

Mathematica [C] time = 0.27, size = 79, normalized size = 0.68

$$\frac{12(b+(2-2i)a)(b+(2+2i)a)(e+fx) - 3b(4a+b)\sin(4(e+fx)) + 3(4a-b)(4a+b)\sin(2(e+fx)) + b^2\sin(6(}{192f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (12*((2 - 2*I)*a + b)*((2 + 2*I)*a + b)*(e + f*x) + 3*(4*a - b)*(4*a + b)*S
in[2*(e + f*x)] - 3*b*(4*a + b)*Sin[4*(e + f*x)] + b^2*Sin[6*(e + f*x)])/(1
92*f)
```

fricas [A] time = 0.42, size = 85, normalized size = 0.73

$$\frac{3\left(8a^2+4ab+b^2\right)fx + \left(8b^2\cos\left(fx+e\right)^5 - 2\left(12ab+7b^2\right)\cos\left(fx+e\right)^3 + 3\left(8a^2+4ab+b^2\right)\cos\left(fx+e\right)\right)s}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*f*x + (8*b^2*cos(f*x + e)^5 - 2*(12*a*b + 7*b
^2)*cos(f*x + e)^3 + 3*(8*a^2 + 4*a*b + b^2)*cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.18, size = 84, normalized size = 0.72

$$\frac{1}{16}\left(8a^2+4ab+b^2\right)x + \frac{b^2\sin\left(6fx+6e\right)}{192f} - \frac{\left(4ab+b^2\right)\sin\left(4fx+4e\right)}{64f} + \frac{\left(16a^2-b^2\right)\sin\left(2fx+2e\right)}{64f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/16*(8*a^2 + 4*a*b + b^2)*x + 1/192*b^2*sin(6*f*x + 6*e)/f - 1/64*(4*a*b +
 b^2)*sin(4*f*x + 4*e)/f + 1/64*(16*a^2 - b^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.36, size = 134, normalized size = 1.16

$$\frac{b^2\left(-\frac{(\sin^3(fx+e))(\cos^3(fx+e))}{6}-\frac{\sin(fx+e)(\cos^3(fx+e))}{8}+\frac{\sin(fx+e)\cos(fx+e)}{16}+\frac{fx}{16}+\frac{e}{16}\right)+2ab\left(-\frac{\sin(fx+e)(\cos^3(fx+e))}{4}+\frac{\text{si}}{}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/6*sin(f*x+e)^3*cos(f*x+e)^3-1/8*sin(f*x+e)*cos(f*x+e)^3+1/16*s
in(f*x+e)*cos(f*x+e)+1/16*f*x+1/16*e)+2*a*b*(-1/4*sin(f*x+e)*cos(f*x+e)^3+1
/8*sin(f*x+e)*cos(f*x+e)+1/8*f*x+1/8*e)+a^2*(1/2*sin(f*x+e)*cos(f*x+e)+1/2*
f*x+1/2*e))
```

maxima [A] time = 0.49, size = 127, normalized size = 1.09

$$\frac{3\left(8a^2+4ab+b^2\right)\left(fx+e\right)+\frac{3\left(8a^2+4ab+b^2\right)\tan\left(fx+e\right)^5+8\left(6a^2-b^2\right)\tan\left(fx+e\right)^3+3\left(8a^2-4ab-b^2\right)\tan\left(fx+e\right)}{\tan\left(fx+e\right)^6+3\tan\left(fx+e\right)^4+3\tan\left(fx+e\right)^2+1}}{48f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/48*(3*(8*a^2 + 4*a*b + b^2)*(f*x + e) + (3*(8*a^2 + 4*a*b + b^2)*tan(f*x
+ e)^5 + 8*(6*a^2 - b^2)*tan(f*x + e)^3 + 3*(8*a^2 - 4*a*b - b^2)*tan(f*x +
 e))/(tan(f*x + e)^6 + 3*tan(f*x + e)^4 + 3*tan(f*x + e)^2 + 1))/f
```

mupad [B] time = 14.73, size = 120, normalized size = 1.03

$$x\left(\frac{a^2}{2}+\frac{a\,b}{4}+\frac{b^2}{16}\right)+\frac{\left(\frac{a^2}{2}+\frac{a\,b}{4}+\frac{b^2}{16}\right)\,\tan\left(e+f\,x\right)^5+\left(a^2-\frac{b^2}{6}\right)\,\tan\left(e+f\,x\right)^3+\left(\frac{a^2}{2}-\frac{a\,b}{4}-\frac{b^2}{16}\right)\,\tan\left(e+f\,x\right)}{f\,\left(\tan\left(e+f\,x\right)^6+3\,\tan\left(e+f\,x\right)^4+3\,\tan\left(e+f\,x\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^2,x)
```

```
[Out] x*((a*b)/4 + a^2/2 + b^2/16) + (tan(e + f*x)^3*(a^2 - b^2/6) - tan(e + f*x)
*((a*b)/4 - a^2/2 + b^2/16) + tan(e + f*x)^5*((a*b)/4 + a^2/2 + b^2/16))/(f
*(3*tan(e + f*x)^2 + 3*tan(e + f*x)^4 + tan(e + f*x)^6 + 1))
```

sympy [A] time = 4.84, size = 314, normalized size = 2.71

$$\begin{cases}\frac{a^2x\sin^2\left(e+fx\right)}{2}+\frac{a^2x\cos^2\left(e+fx\right)}{2}+\frac{a^2\sin\left(e+fx\right)\cos\left(e+fx\right)}{2f}+\frac{abx\sin^4\left(e+fx\right)}{4}+\frac{abx\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{2}+\frac{abx\cos^4\left(e+fx\right)}{4}+\frac{ab}{} \\ x\left(a+b\sin^2\left(e\right)\right)^2\cos^2\left(e\right)\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x*sin(e + f*x)**2/2 + a**2*x*cos(e + f*x)**2/2 + a**2*sin(e
 + f*x)*cos(e + f*x)/(2*f) + a*b*x*sin(e + f*x)**4/4 + a*b*x*sin(e + f*x)**
```

```
2*cos(e + f*x)**2/2 + a*b*x*cos(e + f*x)**4/4 + a*b*sin(e + f*x)**3*cos(e +
 f*x)/(4*f) - a*b*sin(e + f*x)*cos(e + f*x)**3/(4*f) + b**2*x*sin(e + f*x)*
*6/16 + 3*b**2*x*sin(e + f*x)**4*cos(e + f*x)**2/16 + 3*b**2*x*sin(e + f*x)
**2*cos(e + f*x)**4/16 + b**2*x*cos(e + f*x)**6/16 + b**2*sin(e + f*x)**5*c
os(e + f*x)/(16*f) - b**2*sin(e + f*x)**3*cos(e + f*x)**3/(6*f) - b**2*sin(
e + f*x)*cos(e + f*x)**5/(16*f), Ne(f, 0)), (x*(a + b*sin(e)**2)**2*cos(e)*
*2, True))
```

3.295 $\int \left(a+b\sin^2(e+fx)\right)^2 \, dx$

Optimal. Leaf size=72

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*x-1/8*b*(8*a+3*b)*cos(f*x+e)*sin(f*x+e)/f-1/4*b^2*c
os(f*x+e)*sin(f*x+e)^3/f
```

Rubi [A] time = 0.02, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.071, Rules used = {3179}

$$\frac{1}{8}x\left(8a^2+8ab+3b^2\right)-\frac{b(8a+3b)\sin(e+fx)\cos(e+fx)}{8f}-\frac{b^2\sin^3(e+fx)\cos(e+fx)}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*x)/8 - (b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(
8*f) - (b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f)
```

Rule 3179

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^2, x_Symbol] :> Simp[((8*a^2 +
 8*a*b + 3*b^2)*x)/8, x] + (-Simp[(b^2*Cos[e + f*x]*Sin[e + f*x]^3)/(4*f),
x] - Simp[(b*(8*a + 3*b)*Cos[e + f*x]*Sin[e + f*x])/(8*f), x]) /; FreeQ[{a,
 b, e, f}, x]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^2 \, dx = \frac{1}{8}\left(8a^2+8ab+3b^2\right)x-\frac{b(8a+3b)\cos(e+fx)\sin(e+fx)}{8f}-\frac{b^2\cos(e+fx)\sin}{4f}$$

Mathematica [A] time = 0.12, size = 58, normalized size = 0.81

$$\frac{4\left(8a^2+8ab+3b^2\right)(e+fx)-8b(2a+b)\sin(2(e+fx))+b^2\sin(4(e+fx))}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (4*(8*a^2 + 8*a*b + 3*b^2)*(e + f*x) - 8*b*(2*a + b)*Sin[2*(e + f*x)] + b^2
*Sin[4*(e + f*x)])/(32*f)
```

fricas [A] time = 0.41, size = 63, normalized size = 0.88

$$\frac{\left(8\,a^2+8\,ab+3\,b^2\right)fx+\left(2\,b^2\cos\left(fx+e\right)^3-\left(8\,ab+5\,b^2\right)\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/8*((8*a^2 + 8*a*b + 3*b^2)*f*x + (2*b^2*cos(f*x + e)^3 - (8*a*b + 5*b^2)*
cos(f*x + e))*sin(f*x + e))/f
```

giac [A] time = 0.13, size = 60, normalized size = 0.83

$$\frac{1}{8}\left(8a^2 + 8ab + 3b^2\right)x + \frac{b^2\sin\left(4fx + 4e\right)}{32f} - \frac{\left(2ab + b^2\right)\sin\left(2fx + 2e\right)}{4f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/8*(8*a^2 + 8*a*b + 3*b^2)*x + 1/32*b^2*sin(4*f*x + 4*e)/f - 1/4*(2*a*b +
b^2)*sin(2*f*x + 2*e)/f
```

maple [A] time = 0.35, size = 78, normalized size = 1.08

$$\frac{b^2\left(-\frac{\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)}{4} + \frac{3fx}{8} + \frac{3e}{8}\right) + 2ab\left(-\frac{\sin\left(fx+e\right)\cos\left(fx+e\right)}{2} + \frac{fx}{2} + \frac{e}{2}\right) + a^2\left(fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(b^2*(-1/4*(sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)+3/8*f*x+3/8*e)+2*a*
b*(-1/2*sin(f*x+e)*cos(f*x+e)+1/2*f*x+1/2*e)+a^2*(f*x+e))
```

maxima [A] time = 0.32, size = 68, normalized size = 0.94

$$a^2x + \frac{\left(2fx + 2e - \sin\left(2fx + 2e\right)\right)ab}{2f} + \frac{\left(12fx + 12e + \sin\left(4fx + 4e\right) - 8\sin\left(2fx + 2e\right)\right)b^2}{32f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] a^2*x + 1/2*(2*f*x + 2*e - sin(2*f*x + 2*e))*a*b/f + 1/32*(12*f*x + 12*e +
sin(4*f*x + 4*e) - 8*sin(2*f*x + 2*e))*b^2/f
```

mupad [B] time = 14.49, size = 77, normalized size = 1.07

$$x\left(a^2 + ab + \frac{3b^2}{8}\right) - \frac{\left(\frac{5b^2}{8} + ab\right)\tan\left(e + fx\right)^3 + \left(\frac{3b^2}{8} + ab\right)\tan\left(e + fx\right)}{f\left(\tan\left(e + fx\right)^4 + 2\tan\left(e + fx\right)^2 + 1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2,x)
```

```
[Out] x*(a*b + a^2 + (3*b^2)/8) - (tan(e + f*x)*(a*b + (3*b^2)/8) + tan(e + f*x)^
3*(a*b + (5*b^2)/8))/(f*(2*tan(e + f*x)^2 + tan(e + f*x)^4 + 1))
```

sympy [A] time = 1.34, size = 168, normalized size = 2.33

$$\begin{cases} a^2x + abx\sin^2\left(e + fx\right) + abx\cos^2\left(e + fx\right) - \frac{ab\sin\left(e+fx\right)\cos\left(e+fx\right)}{f} + \frac{3b^2x\sin^4\left(e+fx\right)}{8} + \frac{3b^2x\sin^2\left(e+fx\right)\cos^2\left(e+fx\right)}{4} + 3b^2 \\ x\left(a + b\sin^2\left(e\right)\right)^2 \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Piecewise((a**2*x + a*b*x*sin(e + f*x)**2 + a*b*x*cos(e + f*x)**2 - a*b*sin
(e + f*x)*cos(e + f*x)/f + 3*b**2*x*sin(e + f*x)**4/8 + 3*b**2*x*sin(e + f*
x)**2*cos(e + f*x)**2/4 + 3*b**2*x*cos(e + f*x)**4/8 - 5*b**2*sin(e + f*x)*
*3*cos(e + f*x)/(8*f) - 3*b**2*sin(e + f*x)*cos(e + f*x)**3/(8*f), Ne(f, 0)
), (x*(a + b*sin(e)**2)**2, True))
```

3.296 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=51

$$\frac{(a+b)^2\tan(e+fx)}{f}-\frac{1}{2}bx(4a+3b)+\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

```
[Out] -1/2*b*(4*a+3*b)*x+1/2*b^2*cos(f*x+e)*sin(f*x+e)/f+(a+b)^2*tan(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3191, 390, 385, 203}

$$\frac{(a+b)^2\tan(e+fx)}{f}-\frac{1}{2}bx(4a+3b)+\frac{b^2\sin(e+fx)\cos(e+fx)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] -(b*(4*a + 3*b)*x)/2 + (b^2*Cos[e + f*x]*Sin[e + f*x])/(2*f) + ((a + b)^2*T
an[e + f*x])/f
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2 dx = \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{\left(1+x^2\right)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a+b)^2 - \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{(a+b)^2\tan(e+fx)}{f} - \frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(1+x^2\right)^2}dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{b^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{(a+b)^2\tan(e+fx)}{f} - \frac{(b(4a+3b))\,\text{Su}}{}$$

$$= -\frac{1}{2}b(4a+3b)x + \frac{b^2\cos(e+fx)\sin(e+fx)}{2f} + \frac{(a+b)^2\tan(e+fx)}{f}$$

Mathematica [A] time = 0.31, size = 48, normalized size = 0.94

$$\frac{-2b(4a+3b)(e+fx)+4(a+b)^2\tan(e+fx)+b^2\sin(2(e+fx))}{4f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (-2*b*(4*a + 3*b)*(e + f*x) + b^2*Sin[2*(e + f*x)] + 4*(a + b)^2*Tan[e + f*
x])/(4*f)
```

fricas [A] time = 0.42, size = 68, normalized size = 1.33

$$-\frac{\left(4\,ab+3\,b^2\right)fx\cos\left(fx+e\right)-\left(b^2\cos\left(fx+e\right)^2+2\,a^2+4\,ab+2\,b^2\right)\sin\left(fx+e\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] -1/2*((4*a*b + 3*b^2)*f*x*cos(f*x + e) - (b^2*cos(f*x + e)^2 + 2*a^2 + 4*a*
b + 2*b^2)*sin(f*x + e))/(f*cos(f*x + e))
```

giac [B] time = 0.19, size = 99, normalized size = 1.94

$$\frac{2\,a^2\tan\left(fx+e\right)+4\,ab\tan\left(fx+e\right)+2\,b^2\tan\left(fx+e\right)-\left(4\,ab+3\,b^2\right)\left(fx-\pi\left\lfloor\frac{fx+e}{\pi}+\frac{1}{2}\right\rfloor+e\right)+\frac{b^2\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(2*a^2*tan(f*x + e) + 4*a*b*tan(f*x + e) + 2*b^2*tan(f*x + e) - (4*a*b
+ 3*b^2)*(f*x - pi*floor((f*x + e)/pi + 1/2) + e) + b^2*tan(f*x + e)/(tan(f
*x + e)^2 + 1))/f
```

maple [A] time = 0.62, size = 87, normalized size = 1.71

$$\frac{a^2\tan\left(fx+e\right)+2ab\left(\tan\left(fx+e\right)-fx-e\right)+b^2\left(\frac{\sin^5\left(fx+e\right)}{\cos\left(fx+e\right)}+\left(\sin^3\left(fx+e\right)+\frac{3\sin\left(fx+e\right)}{2}\right)\cos\left(fx+e\right)-\frac{3fx}{2}-\frac{3e}{2}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(a^2*tan(f*x+e)+2*a*b*(tan(f*x+e)-f*x-e)+b^2*(sin(f*x+e)^5/cos(f*x+e)+(
sin(f*x+e)^3+3/2*sin(f*x+e))*cos(f*x+e)-3/2*f*x-3/2*e))
```

maxima [A] time = 0.46, size = 74, normalized size = 1.45

$$-\frac{4\left(fx+e-\tan\left(fx+e\right)\right)ab+\left(3\,fx+3\,e-\frac{\tan\left(fx+e\right)}{\tan\left(fx+e\right)^2+1}-2\,\tan\left(fx+e\right)\right)b^2-2\,a^2\tan\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] -1/2*(4*(f*x + e - tan(f*x + e))*a*b + (3*f*x + 3*e - tan(f*x + e)/(tan(f*x
 + e)^2 + 1) - 2*tan(f*x + e))*b^2 - 2*a^2*tan(f*x + e))/f
```

mupad [B] time = 14.09, size = 74, normalized size = 1.45

$$\frac{\tan\left(e+f\,x\right)\,{\left(a+b\right)}^2}{f}+\frac{b^2\,\sin\left(2\,e+2\,f\,x\right)}{4\,f}-\frac{b\,\mathrm{atan}\left(\frac{b\,\tan\left(e+f\,x\right)\,\left(4\,a+3\,b\right)}{2\,\left(\frac{3\,b^2}{2}+2\,a\,b\right)}\right)\,\left(4\,a+3\,b\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^2,x)
```

```
[Out] (tan(e + f*x)*(a + b)^2)/f + (b^2*sin(2*e + 2*f*x))/(4*f) - (b*atan((b*tan(
e + f*x)*(4*a + 3*b))/(2*(2*a*b + (3*b^2)/2)))*(4*a + 3*b))/(2*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(e+fx\right)\right)^2\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**2*sec(e + f*x)**2, x)
```

3.297 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=45

$$\frac{\left(a^2-b^2\right)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+b^2x$$

[Out] b^2*x+(a^2-b^2)*tan(f*x+e)/f+1/3*(a+b)^2*tan(f*x+e)^3/f

Rubi [A] time = 0.06, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3191, 390, 203}

$$\frac{\left(a^2-b^2\right)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+b^2x$$

Antiderivative was successfully verified.

[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]

[Out] b^2*x + ((a^2 - b^2)*Tan[e + f*x])/f + ((a + b)^2*Tan[e + f*x]^3)/(3*f)

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+(a+b)x^2\right)^2}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2-b^2+(a+b)^2x^2+\frac{b^2}{1+x^2}\right)dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\left(a^2-b^2\right)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{1+x^2}\,d\right.}{f}\\
&= b^2x+\frac{\left(a^2-b^2\right)\tan(e+fx)}{f}+\frac{(a+b)^2\tan^3(e+fx)}{3f}
\end{aligned}$$

Mathematica [A] time = 0.33, size = 57, normalized size = 1.27

$$\frac{(a+b)\tan(e+fx)\sec^2(e+fx)((a-2b)\cos(2(e+fx))+2a-b)+3b^2(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (3*b^2*(e + f*x) + (a + b)*(2*a - b + (a - 2*b)*Cos[2*(e + f*x)])*Sec[e + f
*x]^2*Tan[e + f*x])/(3*f)
```

fricas [A] time = 0.42, size = 70, normalized size = 1.56

$$\frac{3\,b^2fx\cos\left(fx+e\right)^3+\left(2\left(a^2-ab-2\,b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sin\left(fx+e\right)}{3\,f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*b^2*f*x*cos(f*x + e)^3 + (2*(a^2 - a*b - 2*b^2)*cos(f*x + e)^2 + a^2
 + 2*a*b + b^2)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [A] time = 0.22, size = 80, normalized size = 1.78

$$\frac{a^2\tan\left(fx+e\right)^3+2\,ab\tan\left(fx+e\right)^3+b^2\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\,a^2\tan\left(fx+e\right)-3\,b^2\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/3*(a^2*tan(f*x + e)^3 + 2*a*b*tan(f*x + e)^3 + b^2*tan(f*x + e)^3 + 3*(f*
x + e)*b^2 + 3*a^2*tan(f*x + e) - 3*b^2*tan(f*x + e))/f
```

maple [A] time = 0.53, size = 76, normalized size = 1.69

$$\frac{-a^2\left(-\frac{2}{3}-\frac{\left(\sec^2\left(fx+e\right)\right)}{3}\right)\tan\left(fx+e\right)+\frac{2ab\left(\sin^3\left(fx+e\right)\right)}{3\cos\left(fx+e\right)^3}+b^2\left(\frac{\left(\tan^3\left(fx+e\right)\right)}{3}-\tan\left(fx+e\right)+fx+e\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-2/3-1/3*sec(f*x+e)^2)*tan(f*x+e)+2/3*a*b*sin(f*x+e)^3/cos(f*x+e
)^3+b^2*(1/3*tan(f*x+e)^3-tan(f*x+e)+f*x+e))
```

maxima [A] time = 0.47, size = 53, normalized size = 1.18

$$\frac{\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^3+3\left(fx+e\right)b^2+3\left(a^2-b^2\right)\tan\left(fx+e\right)}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/3*((a^2 + 2*a*b + b^2)*tan(f*x + e)^3 + 3*(f*x + e)*b^2 + 3*(a^2 - b^2)*t
an(f*x + e))/f
```

mupad [B] time = 13.76, size = 46, normalized size = 1.02

$$\frac{\frac{\tan\left(e+f\,x\right)^3\,{\left(a+b\right)}^2}{3}-\tan\left(e+f\,x\right)\,\left({\left(a+b\right)}^2-2\,a\,\left(a+b\right)\right)+b^2\,f\,x}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^4,x)
```

```
[Out] ((tan(e + f*x)^3*(a + b)^2)/3 - tan(e + f*x)*((a + b)^2 - 2*a*(a + b)) + b^
2*f*x)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.298 $\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=53

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}$$

```
[Out] a^2*tan(f*x+e)/f+2/3*a*(a+b)*tan(f*x+e)^3/f+1/5*(a+b)^2*tan(f*x+e)^5/f
```

Rubi [A] time = 0.06, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 194}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(a + b)*Tan[e + f*x]^3)/(3*f) + ((a + b)^2*Tan[
e + f*x]^5)/(5*f)
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \sec^6(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \left(a^2+2a(a+b)x^2+(a+b)^2x^4\right)\,dx,x,\tan(e+fx)\right)}{f}\\
&= \frac{a^2\tan(e+fx)}{f}+\frac{2a(a+b)\tan^3(e+fx)}{3f}+\frac{(a+b)^2\tan^5(e+fx)}{5f}
\end{aligned}$$

Mathematica [A] time = 0.36, size = 67, normalized size = 1.26

$$\frac{\tan(e+fx)\left(\left(4a^2-2ab-6b^2\right)\sec^2(e+fx)+8a^2+3(a+b)^2\sec^4(e+fx)-4ab+3b^2\right)}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^6*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((8*a^2 - 4*a*b + 3*b^2 + (4*a^2 - 2*a*b - 6*b^2)*Sec[e + f*x]^2 + 3*(a + b
)^2*Sec[e + f*x]^4)*Tan[e + f*x])/(15*f)
```

fricas [A] time = 0.43, size = 83, normalized size = 1.57

$$\frac{\left(\left(8\,a^2-4\,ab+3\,b^2\right)\cos\left(fx+e\right)^4+2\left(2\,a^2-ab-3\,b^2\right)\cos\left(fx+e\right)^2+3\,a^2+6\,ab+3\,b^2\right)\sin\left(fx+e\right)}{15\,f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/15*((8*a^2 - 4*a*b + 3*b^2)*cos(f*x + e)^4 + 2*(2*a^2 - a*b - 3*b^2)*cos(
f*x + e)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(f*x + e)/(f*cos(f*x + e)^5)
```

giac [A] time = 0.21, size = 86, normalized size = 1.62

$$\frac{3\,a^2\tan\left(fx+e\right)^5+6\,ab\tan\left(fx+e\right)^5+3\,b^2\tan\left(fx+e\right)^5+10\,a^2\tan\left(fx+e\right)^3+10\,ab\tan\left(fx+e\right)^3+15\,a^2}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/15*(3*a^2*tan(f*x + e)^5 + 6*a*b*tan(f*x + e)^5 + 3*b^2*tan(f*x + e)^5 +
10*a^2*tan(f*x + e)^3 + 10*a*b*tan(f*x + e)^3 + 15*a^2*tan(f*x + e))/f
```

maple [B] time = 0.60, size = 101, normalized size = 1.91

$$\frac{-a^2\left(-\frac{8}{15}-\frac{\left(\sec^4\left(fx+e\right)\right)}{5}-\frac{4\left(\sec^2\left(fx+e\right)\right)}{15}\right)\tan\left(fx+e\right)+2ab\left(\frac{\sin^3\left(fx+e\right)}{5\cos\left(fx+e\right)^5}+\frac{2\left(\sin^3\left(fx+e\right)\right)}{15\cos\left(fx+e\right)^3}\right)+\frac{b^2\left(\sin^5\left(fx+e\right)\right)}{5\cos\left(fx+e\right)^5}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-8/15-1/5*sec(f*x+e)^4-4/15*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/5*
sin(f*x+e)^3/cos(f*x+e)^5+2/15*sin(f*x+e)^3/cos(f*x+e)^3)+1/5*b^2*sin(f*x+e
)^5/cos(f*x+e)^5)
```

maxima [A] time = 0.32, size = 55, normalized size = 1.04

$$\frac{3\left(a^2+2\,ab+b^2\right)\tan\left(fx+e\right)^5+10\left(a^2+ab\right)\tan\left(fx+e\right)^3+15\,a^2\tan\left(fx+e\right)}{15\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^6*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/15*(3*(a^2 + 2*a*b + b^2)*tan(f*x + e)^5 + 10*(a^2 + a*b)*tan(f*x + e)^3
+ 15*a^2*tan(f*x + e))/f
```

mupad [B] time = 15.83, size = 44, normalized size = 0.83

$$\frac{a^2\tan\left(e+fx\right)+\frac{\tan\left(e+fx\right)^5\,(a+b)^2}{5}+\frac{2\,a\tan\left(e+fx\right)^3\,(a+b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^6,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^5*(a + b)^2)/5 + (2*a*tan(e + f*x)^3*(a +
 b))/3)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**6*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.299 $\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=80

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}$$

```
[Out] a^2*tan(f*x+e)/f+1/3*a*(3*a+2*b)*tan(f*x+e)^3/f+1/5*(a+b)*(3*a+b)*tan(f*x+e
)^5/f+1/7*(a+b)^2*tan(f*x+e)^7/f
```

Rubi [A] time = 0.08, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 373}

$$\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^7(e+fx)}{7f}+\frac{(a+b)(3a+b)\tan^5(e+fx)}{5f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (a*(3*a + 2*b)*Tan[e + f*x]^3)/(3*f) + ((a + b)*(3*a
 + b)*Tan[e + f*x]^5)/(5*f) + ((a + b)^2*Tan[e + f*x]^7)/(7*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^8(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int\left(1+x^2\right)\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int\left(a^2+a(3a+2b)x^2+(a+b)(3a+b)x^4+(a+b)^2x^6\right)\,dx,\right.}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{a(3a+2b)\tan^3(e+fx)}{3f}+\frac{(a+b)(3a+b)\tan^5(e}{5f}\end{aligned}$$

Mathematica [A] time = 0.48, size = 92, normalized size = 1.15

$$\frac{\tan(e+fx)\left(6\left(3a^2-ab-4b^2\right)\sec^4(e+fx)+\left(24a^2-8ab+3b^2\right)\sec^2(e+fx)+48a^2+15(a+b)^2\sec^6(e+f\right.}{105f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^8*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] ((48*a^2 - 16*a*b + 6*b^2 + (24*a^2 - 8*a*b + 3*b^2)*Sec[e + f*x]^2 + 6*(3*
a^2 - a*b - 4*b^2)*Sec[e + f*x]^4 + 15*(a + b)^2*Sec[e + f*x]^6)*Tan[e + f*
x])/(105*f)
```

fricas [A] time = 0.41, size = 108, normalized size = 1.35

$$\frac{\left(2\left(24a^2 - 8ab + 3b^2\right)\cos\left(fx+e\right)^6 + \left(24a^2 - 8ab + 3b^2\right)\cos\left(fx+e\right)^4 + 6\left(3a^2 - ab - 4b^2\right)\cos\left(fx+e\right)^2 + 1\right.}{105 f\cos\left(fx+e\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/105*(2*(24*a^2 - 8*a*b + 3*b^2)*cos(f*x + e)^6 + (24*a^2 - 8*a*b + 3*b^2)
*cos(f*x + e)^4 + 6*(3*a^2 - a*b - 4*b^2)*cos(f*x + e)^2 + 15*a^2 + 30*a*b
+ 15*b^2)*sin(f*x + e)/(f*cos(f*x + e)^7)
```

giac [A] time = 0.25, size = 127, normalized size = 1.59

$$\frac{15a^2\tan\left(fx+e\right)^7 + 30ab\tan\left(fx+e\right)^7 + 15b^2\tan\left(fx+e\right)^7 + 63a^2\tan\left(fx+e\right)^5 + 84ab\tan\left(fx+e\right)^5 + 21b^2}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/105*(15*a^2*tan(f*x + e)^7 + 30*a*b*tan(f*x + e)^7 + 15*b^2*tan(f*x + e)^
7 + 63*a^2*tan(f*x + e)^5 + 84*a*b*tan(f*x + e)^5 + 21*b^2*tan(f*x + e)^5 +
 105*a^2*tan(f*x + e)^3 + 70*a*b*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

maple [A] time = 0.60, size = 149, normalized size = 1.86

$$\frac{-a^2\left(-\frac{16}{35} - \frac{\left(\sec^6\left(fx+e\right)\right)}{7} - \frac{6\left(\sec^4\left(fx+e\right)\right)}{35} - \frac{8\left(\sec^2\left(fx+e\right)\right)}{35}\right)\tan\left(fx+e\right) + 2ab\left(\frac{\sin^3\left(fx+e\right)}{7\cos\left(fx+e\right)^7} + \frac{4\left(\sin^3\left(fx+e\right)\right)}{35\cos\left(fx+e\right)^5} + \frac{8\left(\sin^3\left(fx+e\right)\right)}{105\cos\left(fx+e\right)^3}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-16/35-1/7*sec(f*x+e)^6-6/35*sec(f*x+e)^4-8/35*sec(f*x+e)^2)*tan
(f*x+e)+2*a*b*(1/7*sin(f*x+e)^3/cos(f*x+e)^7+4/35*sin(f*x+e)^3/cos(f*x+e)^5
+8/105*sin(f*x+e)^3/cos(f*x+e)^3)+b^2*(1/7*sin(f*x+e)^5/cos(f*x+e)^7+2/35*s
in(f*x+e)^5/cos(f*x+e)^5))
```

maxima [A] time = 0.35, size = 81, normalized size = 1.01

$$\frac{15\left(a^2 + 2ab + b^2\right)\tan\left(fx+e\right)^7 + 21\left(3a^2 + 4ab + b^2\right)\tan\left(fx+e\right)^5 + 35\left(3a^2 + 2ab\right)\tan\left(fx+e\right)^3 + 105a^2\tan}{105f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^8*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/105*(15*(a^2 + 2*a*b + b^2)*tan(f*x + e)^7 + 21*(3*a^2 + 4*a*b + b^2)*tan
(f*x + e)^5 + 35*(3*a^2 + 2*a*b)*tan(f*x + e)^3 + 105*a^2*tan(f*x + e))/f
```

mupad [B] time = 15.14, size = 72, normalized size = 0.90

$$\frac{a^2\tan\left(e+fx\right) + \frac{\tan\left(e+fx\right)^7(a+b)^2}{7} + \tan\left(e+fx\right)^5\left(\frac{3a^2}{5} + \frac{4ab}{5} + \frac{b^2}{5}\right) + \frac{a\tan\left(e+fx\right)^3(3a+2b)}{3}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^8,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^7*(a + b)^2)/7 + tan(e + f*x)^5*((4*a*b)/
5 + (3*a^2)/5 + b^2/5) + (a*tan(e + f*x)^3*(3*a + 2*b))/3)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**8*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.300 $\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx$

Optimal. Leaf size=106

$$\frac{\left(6a^2+6ab+b^2\right)\tan^5(e+fx)}{5f}+\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{2a(2a+b)}{}$$

```
[Out] a^2*tan(f*x+e)/f+2/3*a*(2*a+b)*tan(f*x+e)^3/f+1/5*(6*a^2+6*a*b+b^2)*tan(f*x
+e)^5/f+2/7*(a+b)*(2*a+b)*tan(f*x+e)^7/f+1/9*(a+b)^2*tan(f*x+e)^9/f
```

Rubi [A] time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3191, 373}

$$\frac{\left(6a^2+6ab+b^2\right)\tan^5(e+fx)}{5f}+\frac{a^2\tan(e+fx)}{f}+\frac{(a+b)^2\tan^9(e+fx)}{9f}+\frac{2(a+b)(2a+b)\tan^7(e+fx)}{7f}+\frac{2a(2a+b)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (a^2*Tan[e + f*x])/f + (2*a*(2*a + b)*Tan[e + f*x]^3)/(3*f) + ((6*a^2 + 6*a
*b + b^2)*Tan[e + f*x]^5)/(5*f) + (2*(a + b)*(2*a + b)*Tan[e + f*x]^7)/(7*f
) + ((a + b)^2*Tan[e + f*x]^9)/(9*f)
```

Rule 373

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol
] :> Int[ExpandIntegrand[(a + b*x^n)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b
, c, d, n}, x] && NeQ[b*c - a*d, 0] && IGtQ[p, 0] && IGtQ[q, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \sec^{10}(e+fx)\left(a+b\sin^2(e+fx)\right)^2\,dx &= \frac{\text{Subst}\left(\int \left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(a^2+2a(2a+b)x^2+\left(6a^2+6ab+b^2\right)x^4+2(a+b)(2a+b\right.\right.}{f}\\ &= \frac{a^2\tan(e+fx)}{f}+\frac{2a(2a+b)\tan^3(e+fx)}{3f}+\frac{\left(6a^2+6ab+b^2\right)\tan^5(e}{5f}\end{aligned}$$

Mathematica [A] time = 0.47, size = 107, normalized size = 1.01

$$\frac{\sec^9(e+fx)\left(252\left(8a^2+8ab+3b^2\right)\sin(e+fx)+336\left(4a^2-ab-b^2\right)\sin(3(e+fx))+\left(16a^2-4ab+b^2\right)(36\sin\right.}{10080f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^10*(a + b*Sin[e + f*x]^2)^2,x]
```

```
[Out] (Sec[e + f*x]^9*(252*(8*a^2 + 8*a*b + 3*b^2)*Sin[e + f*x] + 336*(4*a^2 - a*
b - b^2)*Sin[3*(e + f*x)] + (16*a^2 - 4*a*b + b^2)*(36*Sin[5*(e + f*x)] + 9
*Sin[7*(e + f*x)] + Sin[9*(e + f*x)])))/(10080*f)
```

fricas [A] time = 0.42, size = 128, normalized size = 1.21

$$\frac{\left(8\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right)^8+4\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right)^6+3\left(16a^2-4ab+b^2\right)\cos\left(fx+e\right)^4 \cdots\right.}{315f\cos\left(fx+e\right)^9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="fricas")
```

```
[Out] 1/315*(8*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^8 + 4*(16*a^2 - 4*a*b + b^2)*c
os(f*x + e)^6 + 3*(16*a^2 - 4*a*b + b^2)*cos(f*x + e)^4 + 10*(4*a^2 - a*b -
 5*b^2)*cos(f*x + e)^2 + 35*a^2 + 70*a*b + 35*b^2)*sin(f*x + e)/(f*cos(f*x
+ e)^9)
```

giac [A] time = 0.29, size = 168, normalized size = 1.58

$$\frac{35a^2\tan\left(fx+e\right)^9+70ab\tan\left(fx+e\right)^9+35b^2\tan\left(fx+e\right)^9+180a^2\tan\left(fx+e\right)^7+270ab\tan\left(fx+e\right)^7 \cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="giac")
```

```
[Out] 1/315*(35*a^2*tan(f*x + e)^9 + 70*a*b*tan(f*x + e)^9 + 35*b^2*tan(f*x + e)^
9 + 180*a^2*tan(f*x + e)^7 + 270*a*b*tan(f*x + e)^7 + 90*b^2*tan(f*x + e)^7
 + 378*a^2*tan(f*x + e)^5 + 378*a*b*tan(f*x + e)^5 + 63*b^2*tan(f*x + e)^5
+ 420*a^2*tan(f*x + e)^3 + 210*a*b*tan(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

maple [A] time = 0.61, size = 195, normalized size = 1.84

$$\frac{-a^2\left(-\frac{128}{315}-\frac{\left(\sec^8\left(fx+e\right)\right)}{9}-\frac{8\left(\sec^6\left(fx+e\right)\right)}{63}-\frac{16\left(\sec^4\left(fx+e\right)\right)}{105}-\frac{64\left(\sec^2\left(fx+e\right)\right)}{315}\right)\tan\left(fx+e\right)+2ab\left(\frac{\sin^3\left(fx+e\right)}{9\cos\left(fx+e\right)^9}+\frac{2\left(\sin^3\left(f\cdots\right.\right.}{21\cos\left(f\cdots\right.}\cdots\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x)
```

```
[Out] 1/f*(-a^2*(-128/315-1/9*sec(f*x+e)^8-8/63*sec(f*x+e)^6-16/105*sec(f*x+e)^4-
64/315*sec(f*x+e)^2)*tan(f*x+e)+2*a*b*(1/9*sin(f*x+e)^3/cos(f*x+e)^9+2/21*s
in(f*x+e)^3/cos(f*x+e)^7+8/105*sin(f*x+e)^3/cos(f*x+e)^5+16/315*sin(f*x+e)^
3/cos(f*x+e)^3)+b^2*(1/9*sin(f*x+e)^5/cos(f*x+e)^9+4/63*sin(f*x+e)^5/cos(f*
x+e)^7+8/315*sin(f*x+e)^5/cos(f*x+e)^5))
```

maxima [A] time = 0.35, size = 103, normalized size = 0.97

$$\frac{35\left(a^2+2ab+b^2\right)\tan\left(fx+e\right)^9+90\left(2a^2+3ab+b^2\right)\tan\left(fx+e\right)^7+63\left(6a^2+6ab+b^2\right)\tan\left(fx+e\right)^5+\cdots}{315f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^10*(a+b*sin(f*x+e)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/315*(35*(a^2 + 2*a*b + b^2)*tan(f*x + e)^9 + 90*(2*a^2 + 3*a*b + b^2)*tan
(f*x + e)^7 + 63*(6*a^2 + 6*a*b + b^2)*tan(f*x + e)^5 + 210*(2*a^2 + a*b)*t
an(f*x + e)^3 + 315*a^2*tan(f*x + e))/f
```

mupad [B] time = 14.22, size = 94, normalized size = 0.89

$$\frac{a^2 \tan\left(e+f\,x\right)+\frac{{\tan\left(e+f\,x\right)}^9\,{\left(a+b\right)}^2}{9}+{\tan\left(e+f\,x\right)}^5\,\left(\frac{6\,a^2}{5}+\frac{6\,a\,b}{5}+\frac{b^2}{5}\right)+{\tan\left(e+f\,x\right)}^7\,\left(\frac{4\,a^2}{7}+\frac{6\,a\,b}{7}+\frac{2\,b^2}{7}\right)+\frac{2\,a\,\tan\left(e-\right.}{}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^2/cos(e + f*x)^10,x)
```

```
[Out] (a^2*tan(e + f*x) + (tan(e + f*x)^9*(a + b)^2)/9 + tan(e + f*x)^5*((6*a*b)/
5 + (6*a^2)/5 + b^2/5) + tan(e + f*x)^7*((6*a*b)/7 + (4*a^2)/7 + (2*b^2)/7)
 + (2*a*tan(e + f*x)^3*(2*a + b))/3)/f
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**10*(a+b*sin(f*x+e)**2)**2,x)
```

```
[Out] Timed out
```

3.301 $\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=78

$$-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}$$

```
[Out] -(a^2+3*a*b+3*b^2)*sin(x)/b^3+1/3*(a+3*b)*sin(x)^3/b^2-1/5*sin(x)^5/b+(a+b)
^3*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(7/2)/a^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}+\frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{\sin^5(x)}{5b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^3*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(7/2)) - ((a^2 + 3*
a*b + 3*b^2)*Sin[x])/b^3 + ((a + 3*b)*Sin[x]^3)/(3*b^2) - Sin[x]^5/(5*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^7(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a+bx^2}\,dx,x,\sin(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{a^2+3ab+3b^2}{b^3}+\frac{(a+3b)x^2}{b^2}-\frac{x^4}{b}+\frac{a^3+3a^2b+3ab^2+b^3}{b^3\left(a+bx^2\right)}\right)dx,x,\sin(x)\right)\\
&= -\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}+\frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\text{s}\right.}{b^3}\\
&= \frac{(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{7/2}}-\frac{\left(a^2+3ab+3b^2\right)\sin(x)}{b^3}+\frac{(a+3b)\sin^3(x)}{3b^2}-\frac{\sin^5(x)}{5b}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 109, normalized size = 1.40

$$\frac{-2\sqrt{a}\sqrt{b}\sin(x)\left(120a^2+4b(5a+12b)\cos(2x)+340ab+3b^2\cos(4x)+309b^2\right)+120(a+b)^3\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-}{240\sqrt{a}b^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^7/(a + b*Sin[x]^2),x]
```

```
[Out] (-120*(a + b)^3*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 120*(a + b)^3*ArcTan[(Sq
rt[b]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(120*a^2 + 340*a*b + 309*b^2 + 4
*b*(5*a + 12*b)*Cos[2*x] + 3*b^2*Cos[4*x])*Sin[x])/(240*Sqrt[a]*b^(7/2))
```

fricas [A] time = 0.48, size = 233, normalized size = 2.99

$$\left[-\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(3ab^3\cos(x)^4+15a^3b+40a^2b^2+33ab\right.}{30ab^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/30*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(3*a*b^3*cos(x)^4 + 15
*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a
*b^4), 1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b)*arctan(sqrt(a*b)*
sin(x)/a) - (3*a*b^3*cos(x)^4 + 15*a^3*b + 40*a^2*b^2 + 33*a*b^3 + (5*a^2*b
^2 + 9*a*b^3)*cos(x)^2)*sin(x))/(a*b^4)]
```

giac [A] time = 0.14, size = 98, normalized size = 1.26

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}-\frac{3b^4\sin(x)^5-5ab^3\sin(x)^3-15b^4\sin(x)^3+15a^2b^2\sin(x)+45ab^3\sin}{15b^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
- 1/15*(3*b^4*sin(x)^5 - 5*a*b^3*sin(x)^3 - 15*b^4*sin(x)^3 + 15*a^2*b^2*si
n(x) + 45*a*b^3*sin(x) + 45*b^4*sin(x))/b^5
```

maple [B] time = 0.20, size = 136, normalized size = 1.74

$$-\frac{\sin^5(x)}{5b}+\frac{a\left(\sin^3(x)\right)}{3b^2}+\frac{\sin^3(x)}{b}-\frac{a^2\sin(x)}{b^3}-\frac{3a\sin(x)}{b^2}-\frac{3\sin(x)}{b}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)a^3}{b^3\sqrt{ab}}+\frac{3\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)a^2}{b^2\sqrt{ab}}+\frac{3\,\mathrm{arcta}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a+b*sin(x)^2),x)
```

```
[Out] -1/5*sin(x)^5/b+1/3/b^2*a*sin(x)^3+sin(x)^3/b-1/b^3*a^2*sin(x)-3/b^2*a*sin(
x)-3*sin(x)/b+1/b^3/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a^3+3/b^2/(a*b
)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a^2+3/b/(a*b)^(1/2)*arctan(sin(x)*b/(a
*b)^(1/2))*a+1/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.45, size = 86, normalized size = 1.10

$$\frac{\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^3}-\frac{3b^2\sin(x)^5-5\left(ab+3b^2\right)\sin(x)^3+15\left(a^2+3ab+3b^2\right)\sin(x)}{15b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^7/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^3)
- 1/15*(3*b^2*sin(x)^5 - 5*(a*b + 3*b^2)*sin(x)^3 + 15*(a^2 + 3*a*b + 3*b^2
)*sin(x))/b^3
```

mupad [B] time = 0.12, size = 99, normalized size = 1.27

$$\sin(x)^3\left(\frac{a}{3\,b^2}+\frac{1}{b}\right)-\sin(x)\left(\frac{3}{b}+\frac{a\left(\frac{a}{b^2}+\frac{3}{b}\right)}{b}\right)-\frac{\sin(x)^5}{5\,b}+\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^3}{\sqrt{a}\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}\right)\,(a+b)^3}{\sqrt{a}\,b^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^7/(a + b*sin(x)^2),x)
```

```
[Out] sin(x)^3*(a/(3*b^2) + 1/b) - sin(x)*(3/b + (a*(a/b^2 + 3/b))/b) - sin(x)^5/
(5*b) + (atan((b^(1/2)*sin(x)*(a + b)^3)/(a^(1/2)*(3*a*b^2 + 3*a^2*b + a^3
+ b^3)))*(a + b)^3)/(a^(1/2)*b^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**7/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.302 $\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{\sin(x)\cos^3(x)}{4b}$$

```
[Out] -1/8*(8*a^2+20*a*b+15*b^2)*x/b^3-1/8*(4*a+7*b)*cos(x)*sin(x)/b^2-1/4*cos(x)
^3*sin(x)/b+(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/b^3/a^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = $\{3191, 414, 527, 522, 203, 205\}$

$$-\frac{x\left(8a^2+20ab+15b^2\right)}{8b^3}+\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3}-\frac{(4a+7b)\sin(x)\cos(x)}{8b^2}-\frac{\sin(x)\cos^3(x)}{4b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] -((8*a^2 + 20*a*b + 15*b^2)*x)/(8*b^3) + ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]
*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - ((4*a + 7*b)*Cos[x]*Sin[x])/(8*b^2) - (C
os[x]^3*Sin[x])/(4*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
```

```
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)$$

$$= -\frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{a+4b-3(a+b)x^2}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx, x, \tan(x)\right)}{4b}$$

$$= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2} - \frac{\cos^3(x)\sin(x)}{4b} + \frac{\text{Subst}\left(\int \frac{4a^2+9ab+8b^2-(a+b)(4a+7b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx, x, \tan\right.}{8b^2}$$

$$= -\frac{(4a+7b)\cos(x)\sin(x)}{8b^2} - \frac{\cos^3(x)\sin(x)}{4b} + \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{b^3} -$$

$$= -\frac{\left(8a^2+20ab+15b^2\right)x}{8b^3} + \frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3} - \frac{(4a+7b)\cos(x)\sin(x)}{8b^2} - \cos$$

Mathematica [A] time = 0.19, size = 79, normalized size = 0.91

$$\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^3} - \frac{4x\left(8a^2+20ab+15b^2\right)+8b(a+2b)\sin(2x)+b^2\sin(4x)}{32b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b^3) - (4*(8*
a^2 + 20*a*b + 15*b^2)*x + 8*b*(a + 2*b)*Sin[2*x] + b^2*Sin[4*x])/(32*b^3)
```

fricas [A] time = 0.46, size = 312, normalized size = 3.59

$$\left[\frac{2\left(a^2+2ab+b^2\right)\sqrt{-\frac{a+b}{a}}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2-4\left(\left(2a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\,\sin(x)+}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right.}{8b^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/8*(2*(a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos
(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2
 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4
```

```
- 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - (8*a^2 + 20*a*b + 15*b^2)*
x - (2*b^2*cos(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3, -1/8*(4*(a^2 + 2
*a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(x)*sin(x))) + (8*a^2 + 20*a*b + 15*b^2)*x + (2*b^2*co
s(x)^3 + (4*a*b + 7*b^2)*cos(x))*sin(x))/b^3]
```

giac [A] time = 0.16, size = 131, normalized size = 1.51

$$-\frac{\left(8\,a^2+20\,ab+15\,b^2\right)x}{8\,b^3}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,b^3}-\frac{4\,a\tan$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*
floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 +
a*b)))/(sqrt(a^2 + a*b)*b^3) - 1/8*(4*a*tan(x)^3 + 7*b*tan(x)^3 + 4*a*tan(x
) + 9*b*tan(x))/((tan(x)^2 + 1)^2*b^2)
```

maple [B] time = 0.24, size = 202, normalized size = 2.32

$$-\frac{\left(\tan^3(x)\right)a}{2b^2\left(\tan^2(x)+1\right)^2}-\frac{7\left(\tan^3(x)\right)}{8b\left(\tan^2(x)+1\right)^2}-\frac{\tan(x)a}{2b^2\left(\tan^2(x)+1\right)^2}-\frac{9\tan(x)}{8b\left(\tan^2(x)+1\right)^2}-\frac{\arctan\left(\tan(x)\right)a^2}{b^3}-\frac{5\arctan\left(\tan\right.}{2b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2),x)
```

```
[Out] -1/2/b^2/(tan(x)^2+1)^2*tan(x)^3*a-7/8/b/(tan(x)^2+1)^2*tan(x)^3-1/2/b^2/(t
an(x)^2+1)^2*tan(x)*a-9/8/b/(tan(x)^2+1)^2*tan(x)-1/b^3*arctan(tan(x))*a^2-
5/2/b^2*arctan(tan(x))*a-15/8/b*arctan(tan(x))+1/b^3/(a*(a+b))^(1/2)*arctan
((a+b)*tan(x)/(a*(a+b))^(1/2))*a^3+3/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x
)/(a*(a+b))^(1/2))*a^2+3/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1
/2))*a+1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.47, size = 114, normalized size = 1.31

$$-\frac{\left(4\,a+7\,b\right)\tan(x)^3+\left(4\,a+9\,b\right)\tan(x)}{8\left(b^2\tan(x)^4+2\,b^2\tan(x)^2+b^2\right)}-\frac{\left(8\,a^2+20\,ab+15\,b^2\right)x}{8\,b^3}+\frac{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/8*((4*a + 7*b)*tan(x)^3 + (4*a + 9*b)*tan(x))/(b^2*tan(x)^4 + 2*b^2*tan(
x)^2 + b^2) - 1/8*(8*a^2 + 20*a*b + 15*b^2)*x/b^3 + (a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^3)
```

mupad [B] time = 15.45, size = 1804, normalized size = 20.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a + b*sin(x)^2),x)
```

```
[Out] (atan(((((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5
 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) - ((((25*a*b^9)/2
```

```
+ 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 - (tan(x)*(a*b*20i
+ a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(5
12*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*8i + b^2*15
i)*1i)/(16*b^3) + (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4
459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))/(32*b^4) + ((((2
5*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/b^6 + (tan(x
)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a
^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3))*(a*b*20i + a^2*
8i + b^2*15i)*1i)/(16*b^3))/(((725*a*b^7)/32 + (37*a^7*b)/4 + a^8 + (105*b^
8)/32 + (1093*a^2*b^6)/16 + (1881*a^3*b^5)/16 + (4045*a^4*b^4)/32 + (2785*a
^5*b^3)/32 + (75*a^6*b^2)/2)/b^6 - (((tan(x)*(1723*a*b^6 + 960*a^6*b + 128*
a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a^5*b^2))
/(32*b^4) - ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*
b^6)/b^6 - (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b^9 + 128
0*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3)
)*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3) + (((tan(x)*(1723*a*b^6 + 960*a^6*
b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3136*a
^5*b^2))/(32*b^4) + ((((25*a*b^9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2
+ 2*a^4*b^6)/b^6 + (tan(x)*(a*b*20i + a^2*8i + b^2*15i)*(1024*a*b^8 + 256*b
^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(512*b^7))*(a*b*20i + a^2*8i + b^2*15i))/
(16*b^3))*(a*b*20i + a^2*8i + b^2*15i))/(16*b^3)))*(a*b*20i + a^2*8i + b^2*
15i)*1i)/(8*b^3) - ((tan(x)^3*(4*a + 7*b))/(8*b^2) + (tan(x)*(4*a + 9*b))/(
8*b^2))/(2*tan(x)^2 + tan(x)^4 + 1) + (atan((((-a*(a + b)^5)^(1/2)*((tan(x)
*(1723*a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4
+ 5784*a^4*b^3 + 3136*a^5*b^2))/(64*b^4) - ((-a*(a + b)^5)^(1/2)*(((25*a*b^
9)/2 + 4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*
(-a*(a + b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(
128*a*b^7)))/(2*a*b^3))*1i)/(a*b^3) + ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*
a*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*
a^4*b^3 + 3136*a^5*b^2))/(64*b^4) + ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 +
4*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(-a*(a
+ b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b
^7)))/(2*a*b^3))*1i)/(a*b^3))/(((725*a*b^7)/32 + (37*a^7*b)/4 + a^8 + (105*
b^8)/32 + (1093*a^2*b^6)/16 + (1881*a^3*b^5)/16 + (4045*a^4*b^4)/32 + (2785
*a^5*b^3)/32 + (75*a^6*b^2)/2)/b^6 - ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a
*b^6 + 960*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a
^4*b^3 + 3136*a^5*b^2))/(64*b^4) - ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4
*b^10 + 15*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) - (tan(x)*(-a*(a +
 b)^5)^(1/2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^
7)))/(2*a*b^3)))/(a*b^3) + ((-a*(a + b)^5)^(1/2)*((tan(x)*(1723*a*b^6 + 960
*a^6*b + 128*a^7 + 289*b^7 + 4459*a^2*b^5 + 6505*a^3*b^4 + 5784*a^4*b^3 + 3
136*a^5*b^2))/(64*b^4) + ((-a*(a + b)^5)^(1/2)*(((25*a*b^9)/2 + 4*b^10 + 15
*a^2*b^8 + (17*a^3*b^7)/2 + 2*a^4*b^6)/(2*b^6) + (tan(x)*(-a*(a + b)^5)^(1/
2)*(1024*a*b^8 + 256*b^9 + 1280*a^2*b^7 + 512*a^3*b^6))/(128*a*b^7)))/(2*a*
b^3)))/(a*b^3)))*(-a*(a + b)^5)^(1/2)*1i)/(a*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.303 $\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=54

$$\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}$$

```
[Out] -(a+2*b)*sin(x)/b^2+1/3*sin(x)^3/b+(a+b)^2*arctan(sin(x)*b^(1/2)/a^(1/2))/b
^(5/2)/a^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 390, 205}

$$-\frac{(a+2b)\sin(x)}{b^2}+\frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}+\frac{\sin^3(x)}{3b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)^2*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(5/2)) - ((a + 2*b)
*Sin[x])/b^2 + Sin[x]^3/(3*b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^5(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a+bx^2}\,dx,x,\sin(x)\right)\\
&= \text{Subst}\left(\int \left(-\frac{a+2b}{b^2}+\frac{x^2}{b}+\frac{a^2+2ab+b^2}{b^2\left(a+bx^2\right)}\right)dx,x,\sin(x)\right)\\
&= -\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{b^2}\\
&= \frac{(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{5/2}}-\frac{(a+2b)\sin(x)}{b^2}+\frac{\sin^3(x)}{3b}
\end{aligned}$$

Mathematica [A] time = 0.17, size = 84, normalized size = 1.56

$$\frac{6(a+b)^2\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)-2\sqrt{a}\sqrt{b}\sin(x)(6a+b\cos(2x)+11b)-6(a+b)^2\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{12\sqrt{a}b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (-6*(a + b)^2*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]] + 6*(a + b)^2*ArcTan[(Sqrt[b
]*Sin[x])/Sqrt[a]] - 2*Sqrt[a]*Sqrt[b]*(6*a + 11*b + b*Cos[2*x])*Sin[x])/(1
2*Sqrt[a]*b^(5/2))
```

fricas [A] time = 0.46, size = 159, normalized size = 2.94

$$\left[-\frac{3\left(a^2+2ab+b^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab^2\cos(x)^2+3a^2b+5ab^2\right)\sin(x)}{6ab^3}, \frac{3\left(a^2+2al\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/6*(3*(a^2 + 2*a*b + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin
(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)
*sin(x))/(a*b^3), 1/3*(3*(a^2 + 2*a*b + b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin
(x)/a) - (a*b^2*cos(x)^2 + 3*a^2*b + 5*a*b^2)*sin(x))/(a*b^3)]
```

giac [A] time = 0.15, size = 58, normalized size = 1.07

$$\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b^2\sin(x)^3-3ab\sin(x)-6b^2\sin(x)}{3b^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b^2*s
in(x)^3 - 3*a*b*sin(x) - 6*b^2*sin(x))/b^3
```

maple [A] time = 0.20, size = 85, normalized size = 1.57

$$\frac{\sin^3(x)}{3b}-\frac{a\sin(x)}{b^2}-\frac{2\sin(x)}{b}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)a^2}{b^2\sqrt{ab}}+\frac{2\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)a}{b\sqrt{ab}}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2),x)
```

```
[Out] 1/3*sin(x)^3/b-1/b^2*a*sin(x)-2*sin(x)/b+1/b^2/(a*b)^(1/2)*arctan(sin(x)*b/
(a*b)^(1/2))*a^2+2/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a+1/(a*b)^(1/
2)*arctan(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.48, size = 52, normalized size = 0.96

$$\frac{\left(a^2+2ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}b^2}+\frac{b\sin(x)^3-3(a+2b)\sin(x)}{3b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a^2 + 2*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b^2) + 1/3*(b*sin
(x)^3 - 3*(a + 2*b)*sin(x))/b^2
```

mupad [B] time = 14.30, size = 65, normalized size = 1.20

$$\frac{\sin(x)^3}{3\,b} - \sin(x)\left(\frac{a}{b^2} + \frac{2}{b}\right) + \frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)^2}{\sqrt{a}\,\left(a^2+2\,a\,b+b^2\right)}\right)\,(a+b)^2}{\sqrt{a}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*sin(x)^2),x)
```

```
[Out] sin(x)^3/(3*b) - sin(x)*(a/b^2 + 2/b) + (atan((b^(1/2)*sin(x)*(a + b)^2)/(a
^(1/2)*(2*a*b + a^2 + b^2)))*(a + b)^2)/(a^(1/2)*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.304 $\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{\sin(x)\cos(x)}{2b}$$

```
[Out] -1/2*(2*a+3*b)*x/b^2-1/2*cos(x)*sin(x)/b+(a+b)^(3/2)*arctan((a+b)^(1/2)*tan
(x)/a^(1/2))/b^2/a^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 203, 205}

$$-\frac{x(2a+3b)}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{\sin(x)\cos(x)}{2b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] -((2*a + 3*b)*x)/(2*b^2) + ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[
a]])/(Sqrt[a]*b^2) - (Cos[x]*Sin[x])/(2*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^4(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\cos(x)\sin(x)}{2b}+\frac{\text{Subst}\left(\int \frac{a+2b+(-a-b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx,x,\tan(x)\right)}{2b}\\
&= -\frac{\cos(x)\sin(x)}{2b}+\frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(x)\right)}{b^2}-\frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\right.}{2b^2}\\
&= -\frac{(2a+3b)x}{2b^2}+\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^2}-\frac{\cos(x)\sin(x)}{2b}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 55, normalized size = 0.93

$$\frac{\frac{4(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}}-2(2ax+3bx+b\sin(x)\cos(x))}{4b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] ((4*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/Sqrt[a] - 2*(2*a*x
+ 3*b*x + b*Cos[x]*Sin[x]))/(4*b^2)
```

fricas [A] time = 0.45, size = 239, normalized size = 4.05

$$\left[-\frac{2\,b\cos(x)\sin(x)-(a+b)\sqrt{-\frac{a+b}{a}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2-4\left(\left(2\,a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2\,ab+b^2}\right.}{4\,b^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(2*b*cos(x)*sin(x) - (a + b)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b
^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3
 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*co
s(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) + 2*(2*a + 3*b)*x)/b^
2, -1/2*(b*cos(x)*sin(x) + (a + b)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*co
s(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + (2*a + 3*b)*x)/b
^2]
```

giac [A] time = 0.15, size = 92, normalized size = 1.56

$$-\frac{(2\,a+3\,b)x}{2\,b^2}+\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)\left(a^2+2\,ab+b^2\right)}{\sqrt{a^2+ab}\,b^2}-\frac{\tan(x)}{2\left(\tan(x)^2+1\right)b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 + (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*t
an(x) + b*tan(x))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*b^
2) - 1/2*tan(x)/((tan(x)^2 + 1)*b)
```

maple [B] time = 0.21, size = 111, normalized size = 1.88

$$-\frac{\tan(x)}{2b\left(\tan^2(x)+1\right)}-\frac{3\arctan\left(\tan(x)\right)}{2b}-\frac{\arctan\left(\tan(x)\right)a}{b^2}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)a^2}{b^2\sqrt{a\,(a+b)}}+\frac{2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)a}{b\sqrt{a\,(a+b)}}+\frac{\text{arc}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2),x)
```

```
[Out] -1/2/b*tan(x)/(tan(x)^2+1)-3/2/b*arctan(tan(x))-1/b^2*arctan(tan(x))*a+1/b^
2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))*a^2+2/b/(a*(a+b))^(1
/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))*a+1/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.46, size = 64, normalized size = 1.08

$$-\frac{(2\,a+3\,b)x}{2\,b^2}-\frac{\tan(x)}{2\left(b\tan(x)^2+b\right)}+\frac{\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*a + 3*b)*x/b^2 - 1/2*tan(x)/(b*tan(x)^2 + b) + (a^2 + 2*a*b + b^2)*
arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b^2)
```

mupad [B] time = 14.67, size = 119, normalized size = 2.02

$$-\frac{3\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{2\,b}-\frac{a\,\mathrm{atan}\left(\frac{\sin(x)}{\cos(x)}\right)}{b^2}-\frac{\cos(x)\,\sin(x)}{2\,b}-\frac{\mathrm{atanh}\left(\frac{\sin(x)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-a\,b^3}}{\cos(x)\,a^2+b\,\cos(x)\,a}\right)\,\sqrt{-a^4-3\,a^3\,b-3\,a^2\,b^2-}}{a\,b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*sin(x)^2),x)
```

```
[Out] - (3*atan(sin(x)/cos(x)))/(2*b) - (a*atan(sin(x)/cos(x)))/b^2 - (cos(x)*sin
(x))/(2*b) - (atanh((sin(x)*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a
^2*cos(x) + a*b*cos(x)))*(- a*b^3 - 3*a^3*b - a^4 - 3*a^2*b^2)^(1/2))/(a*b^
2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.305 $\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=36

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

```
[Out] -sin(x)/b+(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/b^(3/2)/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 388, 205}

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^3(x)}{a+b\sin^2(x)}\,dx&=\text{Subst}\left(\int \frac{1-x^2}{a+bx^2}\,dx,x,\sin(x)\right)\\&=-\frac{\sin(x)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{b}\\&=\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}\end{aligned}$$

Mathematica [A] time = 0.02, size = 36, normalized size = 1.00

$$\frac{(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*b^(3/2)) - Sin[x]/b
```

fricas [A] time = 0.45, size = 101, normalized size = 2.81

$$\left[-\frac{2\,ab\sin(x)+\sqrt{-ab}\,(a+b)\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab^2}, -\frac{ab\sin(x)-\sqrt{ab}\,(a+b)\arctan\left(\frac{\sqrt{ab}\,\sin(x)}{a}\right)}{ab^2}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*(2*a*b*sin(x) + sqrt(-a*b)*(a + b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*si
n(x) + a - b)/(b*cos(x)^2 - a - b)))/(a*b^2), -(a*b*sin(x) - sqrt(a*b)*(a +
 b)*arctan(sqrt(a*b)*sin(x)/a))/(a*b^2)]
```

giac [A] time = 0.12, size = 30, normalized size = 0.83

$$\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

maple [A] time = 0.24, size = 45, normalized size = 1.25

$$-\frac{\sin(x)}{b}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)a}{b\sqrt{ab}}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2),x)
```

```
[Out] -sin(x)/b+1/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))*a+1/(a*b)^(1/2)*arct
an(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.47, size = 30, normalized size = 0.83

$$\frac{(a+b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,b}-\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*b) - sin(x)/b
```

mupad [B] time = 0.09, size = 28, normalized size = 0.78

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)(a+b)}{\sqrt{a}\,b^{3/2}}-\frac{\sin(x)}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*sin(x)^2),x)
```

```
[Out] (atan((b^(1/2)*sin(x))/a^(1/2))*(a + b))/(a^(1/2)*b^(3/2)) - sin(x)/b
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.306 $\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

```
[Out] -x/b+arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/b/a^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 391, 203, 205}

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx,x,\tan(x)\right)\\
&= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)}{b}+\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(x)\right)}{b}\\
&= -\frac{x}{b}+\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}
\end{aligned}$$

Mathematica [A] time = 0.06, size = 39, normalized size = 1.00

$$\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,b}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] -(x/b) + (Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*b)
```

fricas [A] time = 0.46, size = 206, normalized size = 5.28

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2-4\left(\left(2\,a^2+ab\right)\cos(x)^3-\left(a^2+ab\right)\cos(x)\right)\sqrt{-\frac{a+b}{a}}\,\sin(x)+a^2+2\,ab+b^2}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2\,ab+b^2}\right)-4\,x}{4\,b},-\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a
*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 + a*b)*cos(x))*sqrt(-
(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)
^2 + a^2 + 2*a*b + b^2)) - 4*x)/b, -1/2*(sqrt((a + b)/a)*arctan(1/2*((2*a +
 b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) + 2*x)/b]
```

giac [A] time = 0.13, size = 62, normalized size = 1.59

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*b) - x/b
```

maple [A] time = 0.21, size = 58, normalized size = 1.49

$$-\frac{\arctan\left(\tan(x)\right)}{b}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)a}{b\sqrt{a\left(a+b\right)}}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{\sqrt{a\left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2),x)
```

```
[Out] -1/b*arctan(tan(x))+1/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2)
)*a+1/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 1.44, size = 35, normalized size = 0.90

$$\frac{(a+b)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,b}-\frac{x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] (a + b)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*b) - x/b
```

mupad [B] time = 14.71, size = 272, normalized size = 6.97

$$-\frac{\operatorname{atan}\left(\frac{2\,a^2\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{2\,b^2\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}+\frac{4\,a\,b\,\tan(x)}{2\,a^2+4\,a\,b+2\,b^2}\right)}{b}-\frac{\operatorname{atanh}\left(\frac{6\,b^2\,\tan(x)\,\sqrt{-a^2-b\,a}}{2\,a^3+6\,a^2\,b+6\,a\,b^2+2\,b^3}+\frac{2\,a\,\tan(x)\,\sqrt{-a^2-b\,a}}{6\,a\,b+2\,a^2+6\,b^2+\frac{2\,b^3}{a}}+\frac{6\,b\,\tan(x)\,\sqrt{-}}{6\,a\,b+2\,a^2+6}\right.}{a\,b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(x)^2),x)
```

```
[Out] - atan((2*a^2*tan(x))/(4*a*b + 2*a^2 + 2*b^2) + (2*b^2*tan(x))/(4*a*b + 2*a
^2 + 2*b^2) + (4*a*b*tan(x))/(4*a*b + 2*a^2 + 2*b^2))/b - (atanh((6*b^2*tan
(x)*(- a*b - a^2)^(1/2))/(6*a*b^2 + 6*a^2*b + 2*a^3 + 2*b^3) + (2*a*tan(x)*
(- a*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (6*b*tan(x)*(- a
*b - a^2)^(1/2))/(6*a*b + 2*a^2 + 6*b^2 + (2*b^3)/a) + (2*b^3*tan(x)*(- a*b
 - a^2)^(1/2))/(2*a*b^3 + 6*a^3*b + 2*a^4 + 6*a^2*b^2))*(-a*(a + b))^(1/2))
/(a*b)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.307 $\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=25

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

```
[Out] arctan(sin(x)*b^(1/2)/a^(1/2))/a^(1/2)/b^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3190, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2),x]
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}\end{aligned}$$

Mathematica [A] time = 0.01, size = 25, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\sqrt{b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2),x]
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(Sqrt[a]*Sqrt[b])
```

fricas [A] time = 0.45, size = 78, normalized size = 3.12

$$\left[-\frac{\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{2\,ab}, \frac{\sqrt{ab}\arctan\left(\frac{\sqrt{ab}\sin(x)}{a}\right)}{ab}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^
2 - a - b))/(a*b), sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a)/(a*b)]
```

giac [A] time = 0.14, size = 16, normalized size = 0.64

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

maple [A] time = 0.11, size = 17, normalized size = 0.68

$$\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)^2),x)
```

```
[Out] 1/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.46, size = 16, normalized size = 0.64

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] arctan(b*sin(x)/sqrt(a*b))/sqrt(a*b)
```

mupad [B] time = 14.64, size = 17, normalized size = 0.68

$$\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)^2),x)
```

```
[Out] atan((b^(1/2)*sin(x))/a^(1/2))/(a^(1/2)*b^(1/2))
```

sympy [A] time = 1.34, size = 87, normalized size = 3.48

$$\begin{cases} \frac{\tilde{\infty}}{\sin(x)} & \text{for } a = 0 \wedge b = 0 \\ -\frac{1}{b\sin(x)} & \text{for } a = 0 \\ \frac{\sin(x)}{a} & \text{for } b = 0 \\ -\frac{i\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} + \frac{i\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{2\sqrt{a}\,b\sqrt{\frac{1}{b}}} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)**2),x)
```

```
[Out] Piecewise((zoo/sin(x), Eq(a, 0) & Eq(b, 0)), (-1/(b*sin(x)), Eq(a, 0)), (si
n(x)/a, Eq(b, 0)), (-I*log(-I*sqrt(a)*sqrt(1/b) + sin(x))/(2*sqrt(a)*b*sqrt
(1/b)) + I*log(I*sqrt(a)*sqrt(1/b) + sin(x))/(2*sqrt(a)*b*sqrt(1/b)), True)
)
```

3.308 $\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=40

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

```
[Out] arctanh(sin(x))/(a+b)+arctan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/(a+b)/a^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 40, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.308, Rules used = {3190, 391, 206, 205}

$$\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)}+\frac{\tanh^{-1}(\sin(x))}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (Sqrt[b]*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)) + ArcTanh[Sin[
x]]/(a + b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 391

```
Int[1/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))), x_Symbol] :> Dis
t[b/(b*c - a*d), Int[1/(a + b*x^n), x], x] - Dist[d/(b*c - a*d), Int[1/(c +
 d*x^n), x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{(1-x^2)\,(a+bx^2)}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx, x, \sin(x)\right)}{a+b} + \frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{a+b}$$
$$= \frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)} + \frac{\tanh^{-1}(\sin(x))}{a+b}$$

Mathematica [B] time = 0.13, size = 96, normalized size = 2.40

$$\frac{\sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right) - \sqrt{b}\,\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right) + 2\sqrt{a}\left(\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right) - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)\right)}{2\sqrt{a}\,(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2),x]
```

```
[Out] (-(Sqrt[b]*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]]) + Sqrt[b]*ArcTan[(Sqrt[b]*Sin[
x])/Sqrt[a]] + 2*Sqrt[a]*(-Log[Cos[x/2] - Sin[x/2]] + Log[Cos[x/2] + Sin[x/
2]]))/(2*Sqrt[a]*(a + b))
```

fricas [A] time = 0.46, size = 116, normalized size = 2.90

$$\left[\frac{\sqrt{-\frac{b}{a}}\log\left(-\frac{b\cos(x)^2 - 2a\sqrt{-\frac{b}{a}}\sin(x) + a - b}{b\cos(x)^2 - a - b}\right) + \log(\sin(x)+1) - \log(-\sin(x)+1)}{2\,(a+b)}, \frac{2\sqrt{\frac{b}{a}}\arctan\left(\sqrt{\frac{b}{a}}\sin(x)\right) + \log(\text{s}}{2\,(a+}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x
)^2 - a - b)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b), 1/2*(2*sqrt(b/
a)*arctan(sqrt(b/a)*sin(x)) + log(sin(x) + 1) - log(-sin(x) + 1))/(a + b)]
```

giac [A] time = 0.14, size = 49, normalized size = 1.22

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)} + \frac{\log(\sin(x)+1)}{2\,(a+b)} - \frac{\log(-\sin(x)+1)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a +
 b) - 1/2*log(-sin(x) + 1)/(a + b)
```

maple [A] time = 0.21, size = 55, normalized size = 1.38

$$-\frac{\ln(-1+\sin(x))}{2a+2b} + \frac{b\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{(a+b)\sqrt{ab}} + \frac{\ln(1+\sin(x))}{2a+2b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2),x)
```

```
[Out] -1/(2*a+2*b)*ln(-1+sin(x))+b/(a+b)/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
+1/(2*a+2*b)*ln(1+sin(x))
```

maxima [A] time = 0.46, size = 47, normalized size = 1.18

$$\frac{b\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\sqrt{ab}\,(a+b)}+\frac{\log\left(\sin(x)+1\right)}{2\,(a+b)}-\frac{\log\left(\sin(x)-1\right)}{2\,(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*(a + b)) + 1/2*log(sin(x) + 1)/(a +
 b) - 1/2*log(sin(x) - 1)/(a + b)
```

mupad [B] time = 14.69, size = 856, normalized size = 21.40

$$-\frac{\mathrm{atan}\left(\frac{\frac{\left(4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}\right)1\mathrm{i}}{2\,(a+b)}+\frac{\left(4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}\right)1\mathrm{i}}{2\,(a+b)}}{\frac{4\,b^3\,\sin(x)+\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2-\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}-\frac{4\,b^3\,\sin(x)-\frac{8\,a\,b^3+4\,b^4+4\,a^2\,b^2+\frac{\sin(x)\,\left(-8\,a^3\,b^2-8\,a^2\,b^3+8\,a\,b^4+8\,b^5\right)}{2\,(a+b)}}{2\,(a+b)}}{2\,(a+b)}}\right)1\mathrm{i}}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*sin(x)^2)),x)
```

```
[Out] - (atan((((4*b^3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4 +
 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b))
+ ((4*b^3*sin(x) - (8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b^5
- 8*a^2*b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))*1i)/(2*(a + b)))/((4*b^
3*sin(x) + (8*a*b^3 + 4*b^4 + 4*a^2*b^2 - (sin(x)*(8*a*b^4 + 8*b^5 - 8*a^2*
b^3 - 8*a^3*b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b)) - (4*b^3*sin(x) - (
8*a*b^3 + 4*b^4 + 4*a^2*b^2 + (sin(x)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*
b^2))/(2*(a + b)))/(2*(a + b)))/(2*(a + b))))*1i)/(a + b) - (atan((((2*b^3*
sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (sin(x)*(-a*b)^(1/2)*
(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)
))*(-a*b)^(1/2)*1i)/(a*b + a^2) + ((2*b^3*sin(x) - ((-a*b)^(1/2)*(4*a*b^3 +
 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*
a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2)*1i)/(a*b + a^2))/
(((2*b^3*sin(x) + ((-a*b)^(1/2)*(4*a*b^3 + 2*b^4 + 2*a^2*b^2 - (sin(x)*(-a*
b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a
*b + a^2)))*(-a*b)^(1/2))/(a*b + a^2) - ((2*b^3*sin(x) - ((-a*b)^(1/2)*(4*a
*b^3 + 2*b^4 + 2*a^2*b^2 + (sin(x)*(-a*b)^(1/2)*(8*a*b^4 + 8*b^5 - 8*a^2*b^
3 - 8*a^3*b^2))/(4*(a*b + a^2))))/(2*(a*b + a^2)))*(-a*b)^(1/2))/(a*b + a^2
)))*(-a*b)^(1/2)*1i)/(a*(a + b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec(x)}{a+b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)/(a + b*sin(x)**2), x)
```

3.309 $\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=39

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

```
[Out] b*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(3/2)/a^(1/2)+tan(x)/(a+b)
```

Rubi [A] time = 0.06, antiderivative size = 39, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 388, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1+x^2}{a+(a+b)x^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{a+b}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(x)\right)}{a+b}\\ &= \frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}\end{aligned}$$

Mathematica [A] time = 0.08, size = 39, normalized size = 1.00

$$\frac{b\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{3/2}}+\frac{\tan(x)}{a+b}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2),x]
```

```
[Out] (b*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(3/2)) + Tan[x]/(
a + b)
```

fricas [B] time = 0.47, size = 255, normalized size = 6.54

$$\left[-\frac{\sqrt{-a^2-ab}\,b\cos(x)\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2+4\left((2\,a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2+2\,ab+}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2\,ab+b^2}\right.}{4\left(a^3+2\,a^2b+ab^2\right)\cos(x)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/4*(sqrt(-a^2 - a*b)*b*cos(x)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4
*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt
(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(
x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^2 + a*b)*sin(x))/((a^3 + 2*a^2*b + a*b^2)
*cos(x)), -1/2*(sqrt(a^2 + a*b)*b*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/(
sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x) - 2*(a^2 + a*b)*sin(x))/((a^3 + 2*a^
2*b + a*b^2)*cos(x))]
```

giac [A] time = 0.13, size = 45, normalized size = 1.15

$$\frac{b\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{\sqrt{a^2+ab}\,(a+b)}+\frac{\tan(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/(sqrt(a^2 + a*b)*(a + b)) +
 tan(x)/(a + b)
```

maple [A] time = 0.24, size = 38, normalized size = 0.97

$$\frac{\tan(x)}{a+b}+\frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)\sqrt{a\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2),x)
```

```
[Out] tan(x)/(a+b)+b/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.46, size = 37, normalized size = 0.95

$$\frac{b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)}+\frac{\tan(x)}{a+b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b)) + tan(x)
/(a + b)
```

mupad [B] time = 14.66, size = 39, normalized size = 1.00

$$\frac{\tan(x)}{a+b}+\frac{b\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{\sqrt{a}\,(a+b)^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*sin(x)^2)),x)
```

```
[Out] tan(x)/(a + b) + (b*atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2))))/(
a^(1/2)*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sec^2(x)}{a+b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**2/(a + b*sin(x)**2), x)
```

3.310 $\int \frac{\sec^3(x)}{a+b\sin^2(x)} \, dx$

Optimal. Leaf size=61

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\tan(x)\sec(x)}{2(a+b)}$$

```
[Out] 1/2*(a+3*b)*arctanh(sin(x))/(a+b)^2+b^(3/2)*arctan(sin(x)*b^(1/2)/a^(1/2))/
(a+b)^2/a^(1/2)+1/2*sec(x)*tan(x)/(a+b)
```

Rubi [A] time = 0.09, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3190, 414, 522, 206, 205}

$$\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\tan(x)\sec(x)}{2(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(3/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^2) + ((a + 3*b)
*ArcTanh[Sin[x]])/(2*(a + b)^2) + (Sec[x]*Tan[x])/(2*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(x)}{a+b\sin^2(x)}\,dx &= \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx,x,\sin(x)\right)\\ &= \frac{\sec(x)\tan(x)}{2(a+b)}+\frac{\text{Subst}\left(\int \frac{a+2b+bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx,x,\sin(x)\right)}{2(a+b)}\\ &= \frac{\sec(x)\tan(x)}{2(a+b)}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{(a+b)^2}+\frac{(a+3b)\,\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(x)\right)}{2(a+b)^2}\\ &= \frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^2}+\frac{(a+3b)\tanh^{-1}(\sin(x))}{2(a+b)^2}+\frac{\sec(x)\tan(x)}{2(a+b)}\end{aligned}$$

Mathematica [B] time = 0.32, size = 147, normalized size = 2.41

$$\frac{\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{2b^{3/2}\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{\sqrt{a}}+\frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2}-\frac{a+b}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2}-2(a+3b)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)+2(a}{4(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2),x]
```

```
[Out] ((-2*b^(3/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] + (2*b^(3/2)*ArcTan[
(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] - 2*(a + 3*b)*Log[Cos[x/2] - Sin[x/2]] +
 2*(a + 3*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2] - Sin[x/2])^2 - (
a + b)/(Cos[x/2] + Sin[x/2])^2)/(4*(a + b)^2)
```

fricas [A] time = 0.52, size = 203, normalized size = 3.33

$$\left[\frac{2\,b\sqrt{-\frac{b}{a}}\cos(x)^2\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+(a+3\,b)\cos(x)^2\log(\sin(x)+1)-(a+3\,b)\cos(x)^2\log(-\sin}{4\left(a^2+2\,ab+b^2\right)\cos(x)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(2*b*sqrt(-b/a)*cos(x)^2*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a
- b)/(b*cos(x)^2 - a - b)) + (a + 3*b)*cos(x)^2*log(sin(x) + 1) - (a + 3*b)
*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x))/((a^2 + 2*a*b + b^2)*cos(x)^
2), 1/4*(4*b*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))*cos(x)^2 + (a + 3*b)*cos(x)
^2*log(sin(x) + 1) - (a + 3*b)*cos(x)^2*log(-sin(x) + 1) + 2*(a + b)*sin(x)
)/((a^2 + 2*a*b + b^2)*cos(x)^2)]
```

giac [B] time = 0.12, size = 102, normalized size = 1.67

$$\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{ab}}+\frac{(a+3\,b)\log(\sin(x)+1)}{4\left(a^2+2\,ab+b^2\right)}-\frac{(a+3\,b)\log(-\sin(x)+1)}{4\left(a^2+2\,ab+b^2\right)}-\frac{\sin(x)}{2\left(\sin(x)^2-1\right)(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a + 3
*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(-sin(x) + 1)/(a
^2 + 2*a*b + b^2) - 1/2*sin(x)/((sin(x)^2 - 1)*(a + b))
```

maple [B] time = 0.24, size = 112, normalized size = 1.84

$$-\frac{1}{(4a+4b)(-1+\sin(x))}-\frac{\ln(-1+\sin(x))\,a}{4(a+b)^2}-\frac{3\ln(-1+\sin(x))\,b}{4(a+b)^2}+\frac{b^2\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{(a+b)^2\sqrt{ab}}-\frac{1}{(4a+4b)(1+\sin(x))}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2),x)
```

```
[Out] -1/(4*a+4*b)/(-1+sin(x))-1/4/(a+b)^2*ln(-1+sin(x))*a-3/4/(a+b)^2*ln(-1+sin(
x))*b+b^2/(a+b)^2/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))-1/(4*a+4*b)/(1+s
in(x))+1/4/(a+b)^2*ln(1+sin(x))*a+3/4/(a+b)^2*ln(1+sin(x))*b
```

maxima [B] time = 0.59, size = 104, normalized size = 1.70

$$\frac{b^2\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^2+2\,ab+b^2\right)\sqrt{ab}}+\frac{(a+3\,b)\log(\sin(x)+1)}{4\left(a^2+2\,ab+b^2\right)}-\frac{(a+3\,b)\log(\sin(x)-1)}{4\left(a^2+2\,ab+b^2\right)}-\frac{\sin(x)}{2\left((a+b)\sin(x)^2-a-b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan(b*sin(x)/sqrt(a*b))/((a^2 + 2*a*b + b^2)*sqrt(a*b)) + 1/4*(a + 3
*b)*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/4*(a + 3*b)*log(sin(x) - 1)/(a^
2 + 2*a*b + b^2) - 1/2*sin(x)/((a + b)*sin(x)^2 - a - b)
```

mupad [B] time = 15.36, size = 1139, normalized size = 18.67

$$\frac{\sin(x)}{2\cos(x)^2\,(a+b)}-\ln(\sin(x)-1)\left(\frac{b}{2(a+b)^2}+\frac{1}{4\,(a+b)}\right)+\frac{\ln(\sin(x)+1)\,(a+3\,b)}{4(a+b)^2}+\frac{\mathrm{atan}\left(\frac{\sqrt{-a\,b^3}\left(\frac{\sin(x)\,(a^{\cdots}}{4\,(\cdots}\right)\cdots}{\frac{\frac{3\,b^5}{2}+\frac{a\,b^4}{2}}{a^3+3\,a^2\,b+3\,a\,b^2+b^3}-\cdots}\right)\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*sin(x)^2)),x)
```

```
[Out] sin(x)/(2*cos(x)^2*(a + b)) - log(sin(x) - 1)*(b/(2*(a + b)^2) + 1/(4*(a +
b))) + (log(sin(x) + 1)*(a + 3*b))/(4*(a + b)^2) + (atan((((-a*b^3)^(1/2)*(
(sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6
 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 +
3*a^2*b + a^3 + b^3)) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*
b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2
+ 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2
+ 2*a^2*b + a^3) + ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(
4*(2*a*b + a^2 + b^2)) - (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12
*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (sin(x)*(-a*b^3
```

```
)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*
b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(
a*b^2 + 2*a^2*b + a^3)))*1i)/(a*b^2 + 2*a^2*b + a^3))/(((a*b^4)/2 + (3*b^5)
/2)/(3*a*b^2 + 3*a^2*b + a^3 + b^3) - ((-a*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 1
3*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2)) + (((18*a*b^6 + 4*b^7 + 32*a^2*b^
5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)
) - (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 4
8*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 + b^2)*(a*b^2 + 2*a^2*b + a^3)))*(
-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))))/(a*b^2 + 2*a^2*b + a^3) + ((-a
*b^3)^(1/2)*((sin(x)*(6*a*b^4 + 13*b^5 + a^2*b^3))/(4*(2*a*b + a^2 + b^2))
- (((18*a*b^6 + 4*b^7 + 32*a^2*b^5 + 28*a^3*b^4 + 12*a^4*b^3 + 2*a^5*b^2)/(
2*(3*a*b^2 + 3*a^2*b + a^3 + b^3)) + (sin(x)*(-a*b^3)^(1/2)*(48*a*b^6 + 16*
b^7 + 32*a^2*b^5 - 32*a^3*b^4 - 48*a^4*b^3 - 16*a^5*b^2))/(8*(2*a*b + a^2 +
 b^2)*(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2))/(2*(a*b^2 + 2*a^2*b + a^3))
))/(a*b^2 + 2*a^2*b + a^3)))*(-a*b^3)^(1/2)*1i)/(a*b^2 + 2*a^2*b + a^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(x)}{a + b\sin^2(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**3/(a + b*sin(x)**2), x)
```

3.311 $\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=59

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

```
[Out] b^2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(5/2)/a^(1/2)+(a+2*b)*tan(x)/(
a+b)^2+1/3*tan(x)^3/(a+b)
```

Rubi [A] time = 0.08, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 390, 205}

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{\tan^3(x)}{3(a+b)}+\frac{(a+2b)\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((a +
2*b)*Tan[x])/(a + b)^2 + Tan[x]^3/(3*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a+2b}{(a+b)^2}+\frac{x^2}{a+b}+\frac{b^2}{(a+b)^2\left(a+(a+b)x^2\right)}\right)dx, x, \tan(x)\right)$$
$$= \frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}+\frac{b^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{(a+b)^2}$$
$$= \frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{(a+2b)\tan(x)}{(a+b)^2}+\frac{\tan^3(x)}{3(a+b)}$$

Mathematica [A] time = 0.22, size = 59, normalized size = 1.00

$$\frac{b^2\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{5/2}}+\frac{\tan(x)\left((a+b)\sec^2(x)+2a+5b\right)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2),x]
```

```
[Out] (b^2*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(5/2)) + ((2*a
+ 5*b + (a + b)*Sec[x]^2)*Tan[x])/(3*(a + b)^2)
```

fricas [B] time = 0.46, size = 343, normalized size = 5.81

$$\left[-\frac{3\sqrt{-a^2-ab}\,b^2\cos(x)^3\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2+2ab}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right.}{12\left(a^4+3a^3b+3a^2b^2+ab^3\right)\cos(x)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*sqrt(-a^2 - a*b)*b^2*cos(x)^3*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4
 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x
))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^
2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(a^3 + 2*a^2*b + a*b^2 + (2*a^3 + 7*a
^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*cos(
x)^3), -1/6*(3*sqrt(a^2 + a*b)*b^2*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)/
(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^3 - 2*(a^3 + 2*a^2*b + a*b^2 + (2*a
^3 + 7*a^2*b + 5*a*b^2)*cos(x)^2)*sin(x))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b
^3)*cos(x)^3)]
```

giac [B] time = 0.16, size = 134, normalized size = 2.27

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^2}{\left(a^2+2ab+b^2\right)\sqrt{a^2+ab}}+\frac{a^2\tan(x)^3+2ab\tan(x)^3+b^2\tan(x)^3+3a^2\tan(x)+9a}{3\left(a^3+3a^2b+3ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a*b)) + 1/3*(a^2*tan(x)^3 +
```

```
2*a*b*tan(x)^3 + b^2*tan(x)^3 + 3*a^2*tan(x) + 9*a*b*tan(x) + 6*b^2*tan(x))
/(a^3 + 3*a^2*b + 3*a*b^2 + b^3)
```

maple [A] time = 0.26, size = 75, normalized size = 1.27

$$\frac{\left(\tan^{3}(x)\right) a}{3 \left(a+b\right)^{2}}+\frac{\left(\tan^{3}(x)\right) b}{3 \left(a+b\right)^{2}}+\frac{\tan(x) a}{\left(a+b\right)^{2}}+\frac{2 \tan(x) b}{\left(a+b\right)^{2}}+\frac{b^{2} \arctan\left(\frac{(a+b) \tan(x)}{\sqrt{a(a+b)}}\right)}{\left(a+b\right)^{2} \sqrt{a \left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2),x)
```

```
[Out] 1/3/(a+b)^2*tan(x)^3*a+1/3/(a+b)^2*tan(x)^3*b+1/(a+b)^2*tan(x)*a+2/(a+b)^2*
tan(x)*b+b^2/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.47, size = 72, normalized size = 1.22

$$\frac{b^{2} \arctan\left(\frac{(a+b) \tan(x)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a} \left(a^{2}+2 \, a b+b^{2}\right)}+\frac{(a+b) \tan(x)^{3}+3 \, (a+2 \, b) \tan(x)}{3 \, \left(a^{2}+2 \, a b+b^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^2*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a^2 + 2*a*b +
b^2)) + 1/3*((a + b)*tan(x)^3 + 3*(a + 2*b)*tan(x))/(a^2 + 2*a*b + b^2)
```

mupad [B] time = 15.02, size = 77, normalized size = 1.31

$$\frac{\tan(x)^{3}}{3\,\left(a+b\right)}-\tan(x)\,\left(\frac{a}{{\left(a+b\right)}^{2}}-\frac{2}{a+b}\right)+\frac{b^{2}\,\mathrm{atan}\left(\frac{\tan(x)\,\left(2\,a+2\,b\right)\,\left(a^{2}+2\,a\,b+b^{2}\right)}{2\,\sqrt{a}\,{\left(a+b\right)}^{5/2}}\right)}{\sqrt{a}\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*sin(x)^2)),x)
```

```
[Out] tan(x)^3/(3*(a + b)) - tan(x)*(a/(a + b)^2 - 2/(a + b)) + (b^2*atan((tan(x)
*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2))))/(a^(1/2)*(a +
 b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{4}(x)}{a+b\sin^{2}(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2),x)
```

```
[Out] Integral(sec(x)**4/(a + b*sin(x)**2), x)
```

3.312 $\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=93

$$\frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}+\frac{\tan(x)\sec^3(x)}{4(a+b)}+\frac{(3a+7b)\tan(x)\sec(x)}{8(a+b)^2}$$

```
[Out] 1/8*(3*a^2+10*a*b+15*b^2)*arctanh(sin(x))/(a+b)^3+b^(5/2)*arctan(sin(x)*b^(
1/2)/a^(1/2))/(a+b)^3/a^(1/2)+1/8*(3*a+7*b)*sec(x)*tan(x)/(a+b)^2+1/4*sec(x
)^3*tan(x)/(a+b)
```

Rubi [A] time = 0.14, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.400, Rules used = {3190, 414, 527, 522, 206, 205}

$$\frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\sin(x))}{8(a+b)^3}+\frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3}+\frac{\tan(x)\sec^3(x)}{4(a+b)}+\frac{(3a+7b)\tan(x)\sec(x)}{8(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] (b^(5/2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^3) + ((3*a^2 +
10*a*b + 15*b^2)*ArcTanh[Sin[x]])/(8*(a + b)^3) + ((3*a + 7*b)*Sec[x]*Tan[x
])/(8*(a + b)^2) + (Sec[x]^3*Tan[x])/(4*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
```

```
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^5(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a+bx^2\right)}\,dx, x, \sin(x)\right)$$

$$= \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a+4b+3bx^2}{\left(1-x^2\right)^2\left(a+bx^2\right)}\,dx, x, \sin(x)\right)}{4(a+b)}$$

$$= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{\text{Subst}\left(\int \frac{3a^2+7ab+8b^2+b(3a+7b)x^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx, x, \sin(x)\right)}{8(a+b)^2}$$

$$= \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} + \frac{\sec^3(x)\tan(x)}{4(a+b)} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{(a+b)^3} + \frac{\left(3a^2+10a\right.}{}$$

$$= \frac{b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^3} + \frac{\left(3a^2+10ab+15b^2\right)\tanh^{-1}(\sin(x))}{8(a+b)^3} + \frac{(3a+7b)\sec(x)\tan(x)}{8(a+b)^2} +$$

Mathematica [B] time = 1.25, size = 214, normalized size = 2.30

$$-\frac{2\left(3a^2+10ab+15b^2\right)\log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)-2\left(3a^2+10ab+15b^2\right)\log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)-\frac{8b^{5/2}\tan^{-1}\left(\frac{\sqrt{b}}{\cdot}\right.}{\sqrt{a}}}{16(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^5/(a + b*Sin[x]^2),x]
```

```
[Out] -1/16*((8*b^(5/2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/Sqrt[a] - (8*b^(5/2)*Ar
cTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/Sqrt[a] + 2*(3*a^2 + 10*a*b + 15*b^2)*Log[C
os[x/2] - Sin[x/2]] - 2*(3*a^2 + 10*a*b + 15*b^2)*Log[Cos[x/2] + Sin[x/2]]
- (a + b)^2/(Cos[x/2] - Sin[x/2])^4 + (a + b)^2/(Cos[x/2] + Sin[x/2])^4 + (
(a + b)*(3*a + 7*b))/(Cos[x/2] + Sin[x/2])^2 + ((a + b)*(3*a + 7*b))/(-1 +
Sin[x]))/(a + b)^3
```

fricas [A] time = 0.56, size = 327, normalized size = 3.52

$$\left[\frac{8\,b^2\sqrt{-\frac{b}{a}}\cos(x)^4\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left(3\,a^2+10\,ab+15\,b^2\right)\cos(x)^4\log\left(\sin(x)+1\right)-\left(3\,a^2+10\right.}{16\left(a^3+3\,a^2b+3\,ab^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [1/16*(8*b^2*sqrt(-b/a)*cos(x)^4*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) +
 a - b)/(b*cos(x)^2 - a - b)) + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(
x) + 1) - (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 +
 10*a*b + 7*b^2)*cos(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*cos(x)^4), 1/16*(16*b^2*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
*cos(x)^4 + (3*a^2 + 10*a*b + 15*b^2)*cos(x)^4*log(sin(x) + 1) - (3*a^2 + 1
0*a*b + 15*b^2)*cos(x)^4*log(-sin(x) + 1) + 2*((3*a^2 + 10*a*b + 7*b^2)*cos
(x)^2 + 2*a^2 + 4*a*b + 2*b^2)*sin(x))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos
(x)^4)]
```

giac [B] time = 0.14, size = 177, normalized size = 1.90

$$\frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}}+\frac{\left(3a^2+10ab+15b^2\right)\log\left(\sin(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(3a^2+10ab+15b^2\right)\log\left(-\sin(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
+ 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*
a*b^2 + b^3) - 1/8*(3*a*sin(x)^3 + 7*b*sin(x)^3 - 5*a*sin(x) - 9*b*sin(x))/
((a^2 + 2*a*b + b^2)*(sin(x)^2 - 1)^2)
```

maple [B] time = 0.32, size = 204, normalized size = 2.19

$$\frac{1}{2\left(8a+8b\right)\left(-1+\sin(x)\right)^2}-\frac{3a}{16\left(a+b\right)^2\left(-1+\sin(x)\right)}-\frac{7b}{16\left(a+b\right)^2\left(-1+\sin(x)\right)}-\frac{3\ln\left(-1+\sin(x)\right)a^2}{16\left(a+b\right)^3}-\frac{5\ln\left(-1+\right.}{8\left(a+\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^5/(a+b*sin(x)^2),x)
```

```
[Out] 1/2/(8*a+8*b)/(-1+sin(x))^2-3/16/(a+b)^2/(-1+sin(x))*a-7/16/(a+b)^2/(-1+sin
(x))*b-3/16/(a+b)^3*ln(-1+sin(x))*a^2-5/8/(a+b)^3*ln(-1+sin(x))*a*b-15/16/(
a+b)^3*ln(-1+sin(x))*b^2+b^3/(a+b)^3/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2
))-1/2/(8*a+8*b)/(1+sin(x))^2-3/16/(a+b)^2/(1+sin(x))*a-7/16/(a+b)^2/(1+sin
(x))*b+3/16/(a+b)^3*ln(1+sin(x))*a^2+5/8/(a+b)^3*ln(1+sin(x))*a*b+15/16/(a+
b)^3*ln(1+sin(x))*b^2
```

maxima [B] time = 0.48, size = 199, normalized size = 2.14

$$\frac{b^3\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{ab}}+\frac{\left(3a^2+10ab+15b^2\right)\log\left(\sin(x)+1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{\left(3a^2+10ab+15b^2\right)\log\left(\sin(x)-1\right)}{16\left(a^3+3a^2b+3ab^2+b^3\right)}-\frac{}{8(}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^5/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a*b))
+ 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 +
 b^3) - 1/16*(3*a^2 + 10*a*b + 15*b^2)*log(sin(x) - 1)/(a^3 + 3*a^2*b + 3*a
*b^2 + b^3) - 1/8*((3*a + 7*b)*sin(x)^3 - (5*a + 9*b)*sin(x))/((a^2 + 2*a*b
 + b^2)*sin(x)^4 - 2*(a^2 + 2*a*b + b^2)*sin(x)^2 + a^2 + 2*a*b + b^2)
```

mupad [B] time = 17.45, size = 832, normalized size = 8.95

$$\frac{5\,a^3\,\sin(x)-3\,a^3\,\sin(x)^3+3\,a^3\,\mathrm{atanh}\left(\sin(x)\right)+9\,a\,b^2\,\sin(x)+14\,a^2\,b\,\sin(x)-6\,a^3\,\mathrm{atanh}\left(\sin(x)\right)\,\sin(x)^2+}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^5*(a + b*sin(x)^2)),x)
```

```
[Out] (5*a^3*sin(x) - 3*a^3*sin(x)^3 + 3*a^3*atanh(sin(x)) + atan((a*sin(x)*(-a*b
^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)*9
i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*300i
 + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*60i)/
(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*
b^3))*(-a*b^5)^(1/2)*8i + 9*a*b^2*sin(x) + 14*a^2*b*sin(x) - 6*a^3*atanh(si
n(x))*sin(x)^2 + 3*a^3*atanh(sin(x))*sin(x)^4 - 7*a*b^2*sin(x)^3 - 10*a^2*b
*sin(x)^3 + 15*a*b^2*atanh(sin(x)) + 10*a^2*b*atanh(sin(x)) - atan((a*sin(x
)*(-a*b^5)^(3/2)*64i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^
(1/2)*9i + a^2*b^5*sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/
2)*300i + a^4*b^3*sin(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2
)*60i)/(64*a^2*b^8 + 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 +
 9*a^7*b^3))*sin(x)^2*(-a*b^5)^(1/2)*16i + atan((a*sin(x)*(-a*b^5)^(3/2)*64
i - b*sin(x)*(-a*b^5)^(3/2)*64i + a^6*b*sin(x)*(-a*b^5)^(1/2)*9i + a^2*b^5*
sin(x)*(-a*b^5)^(1/2)*289i + a^3*b^4*sin(x)*(-a*b^5)^(1/2)*300i + a^4*b^3*s
in(x)*(-a*b^5)^(1/2)*190i + a^5*b^2*sin(x)*(-a*b^5)^(1/2)*60i)/(64*a^2*b^8
+ 225*a^3*b^7 + 300*a^4*b^6 + 190*a^5*b^5 + 60*a^6*b^4 + 9*a^7*b^3))*sin(x)
^4*(-a*b^5)^(1/2)*8i - 30*a*b^2*atanh(sin(x))*sin(x)^2 - 20*a^2*b*atanh(sin
(x))*sin(x)^2 + 15*a*b^2*atanh(sin(x))*sin(x)^4 + 10*a^2*b*atanh(sin(x))*si
n(x)^4)/(8*a^4*sin(x)^4 - 16*a^4*sin(x)^2 + 8*a*b^3 + 24*a^3*b + 8*a^4 + 24
*a^2*b^2 - 48*a^2*b^2*sin(x)^2 + 24*a^2*b^2*sin(x)^4 - 16*a*b^3*sin(x)^2 -
48*a^3*b*sin(x)^2 + 8*a*b^3*sin(x)^4 + 24*a^3*b*sin(x)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**5/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.313 $\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx$

Optimal. Leaf size=87

$$\frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

```
[Out] b^3*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/(a+b)^(7/2)/a^(1/2)+(a^2+3*a*b+3*b^2
)*tan(x)/(a+b)^3+1/3*(2*a+3*b)*tan(x)^3/(a+b)^2+1/5*tan(x)^5/(a+b)
```

Rubi [A] time = 0.11, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 390, 205}

$$\frac{(a^2+3ab+3b^2)\tan(x)}{(a+b)^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}+\frac{\tan^5(x)}{5(a+b)}+\frac{(2a+3b)\tan^3(x)}{3(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((a^2
+ 3*a*b + 3*b^2)*Tan[x])/(a + b)^3 + ((2*a + 3*b)*Tan[x]^3)/(3*(a + b)^2) +
 Tan[x]^5/(5*(a + b))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(x)}{a+b\sin^2(x)}\,dx = \text{Subst}\left(\int \frac{(1+x^2)^3}{a+(a+b)x^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a^2+3ab+3b^2}{(a+b)^3} + \frac{(2a+3b)x^2}{(a+b)^2} + \frac{x^4}{a+b} + \frac{b^3}{(a+b)^3\left(a+(a+b)x^2\right)}\right) dx, x, \tan$$
$$= \frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3} + \frac{(2a+3b)\tan^3(x)}{3(a+b)^2} + \frac{\tan^5(x)}{5(a+b)} + \frac{b^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \text{ta}\right.}{(a+b)^3}$$
$$= \frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}} + \frac{\left(a^2+3ab+3b^2\right)\tan(x)}{(a+b)^3} + \frac{(2a+3b)\tan^3(x)}{3(a+b)^2} + \frac{\tan^5(x)}{5(a+b)}$$

Mathematica [A] time = 0.38, size = 90, normalized size = 1.03

$$\frac{\tan(x)\left(\left(4a^2+13ab+9b^2\right)\sec^2(x)+8a^2+3(a+b)^2\sec^4(x)+26ab+33b^2\right)}{15(a+b)^3}+\frac{b^3\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{\sqrt{a}\,(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^6/(a + b*Sin[x]^2),x]
```

```
[Out] (b^3*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(Sqrt[a]*(a + b)^(7/2)) + ((8*a^
2 + 26*a*b + 33*b^2 + (4*a^2 + 13*a*b + 9*b^2)*Sec[x]^2 + 3*(a + b)^2*Sec[x
]^4)*Tan[x])/(15*(a + b)^3)
```

fricas [B] time = 0.48, size = 459, normalized size = 5.28

$$\left[-\frac{15\sqrt{-a^2-ab}\,b^3\cos(x)^5\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2+4\left((2a+b)\cos(x)^3-(a+b)\cos(x)\right)\sqrt{-a^2-ab}\,\sin(x)+a^2}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+a^2+2ab+b^2}\right.}{60\left(a^5+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(15*sqrt(-a^2 - a*b)*b^3*cos(x)^5*log(((8*a^2 + 8*a*b + b^2)*cos(x)^
4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(
x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b
^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*((8*a^4 + 34*a^3*b + 59*a^2*b^2 + 33
*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b^2 + 3*a*b^3 + (4*a^4 + 17*a^3*
b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/((a^5 + 4*a^4*b + 6*a^3*b^2 + 4
*a^2*b^3 + a*b^4)*cos(x)^5), -1/30*(15*sqrt(a^2 + a*b)*b^3*arctan(1/2*((2*a
 + b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x)))*cos(x)^5 - 2*((8*a
^4 + 34*a^3*b + 59*a^2*b^2 + 33*a*b^3)*cos(x)^4 + 3*a^4 + 9*a^3*b + 9*a^2*b
^2 + 3*a*b^3 + (4*a^4 + 17*a^3*b + 22*a^2*b^2 + 9*a*b^3)*cos(x)^2)*sin(x))/
((a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^5)]
```

giac [B] time = 0.13, size = 254, normalized size = 2.92

$$\frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)b^3}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{a^2+ab}}+\frac{3a^4\tan(x)^5+12a^3b\tan(x)^5+18a^2b^2\tan(x)^5+12a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="giac")
```

```
[Out] (pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^
2 + a*b)))*b^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(a^2 + a*b)) + 1/15*(3*
a^4*tan(x)^5 + 12*a^3*b*tan(x)^5 + 18*a^2*b^2*tan(x)^5 + 12*a*b^3*tan(x)^5
+ 3*b^4*tan(x)^5 + 10*a^4*tan(x)^3 + 45*a^3*b*tan(x)^3 + 75*a^2*b^2*tan(x)^
3 + 55*a*b^3*tan(x)^3 + 15*b^4*tan(x)^3 + 15*a^4*tan(x) + 75*a^3*b*tan(x) +
 150*a^2*b^2*tan(x) + 135*a*b^3*tan(x) + 45*b^4*tan(x))/(a^5 + 5*a^4*b + 10
*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
```

maple [A] time = 0.22, size = 147, normalized size = 1.69

$$\frac{\left(\tan^5(x)\right)a^2}{5\left(a+b\right)^3}+\frac{2\left(\tan^5(x)\right)ab}{5\left(a+b\right)^3}+\frac{b^2\left(\tan^5(x)\right)}{5\left(a+b\right)^3}+\frac{2\left(\tan^3(x)\right)a^2}{3\left(a+b\right)^3}+\frac{5ab\left(\tan^3(x)\right)}{3\left(a+b\right)^3}+\frac{\left(\tan^3(x)\right)b^2}{\left(a+b\right)^3}+\frac{a^2\tan(x)}{\left(a+b\right)^3}+\frac{3ab\tan(x)}{\left(a+b\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^6/(a+b*sin(x)^2),x)
```

```
[Out] 1/5/(a+b)^3*tan(x)^5*a^2+2/5/(a+b)^3*tan(x)^5*a*b+1/5/(a+b)^3*b^2*tan(x)^5+
2/3/(a+b)^3*tan(x)^3*a^2+5/3/(a+b)^3*a*b*tan(x)^3+1/(a+b)^3*tan(x)^3*b^2+1/
(a+b)^3*a^2*tan(x)+3/(a+b)^3*a*b*tan(x)+3/(a+b)^3*b^2*tan(x)+b^3/(a+b)^3/(a
*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.46, size = 126, normalized size = 1.45

$$\frac{b^3\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{(a+b)a}}+\frac{3\left(a^2+2ab+b^2\right)\tan(x)^5+5\left(2a^2+5ab+3b^2\right)\tan(x)^3+15\left(a^2+3ab+\right.}{15\left(a^3+3a^2b+3ab^2+b^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^6/(a+b*sin(x)^2),x, algorithm="maxima")
```

```
[Out] b^3*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)
*sqrt((a + b)*a)) + 1/15*(3*(a^2 + 2*a*b + b^2)*tan(x)^5 + 5*(2*a^2 + 5*a*b
 + 3*b^2)*tan(x)^3 + 15*(a^2 + 3*a*b + 3*b^2)*tan(x))/(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)
```

mupad [B] time = 13.99, size = 121, normalized size = 1.39

$$\frac{\tan(x)^5}{5\,(a+b)}-\tan(x)^3\left(\frac{a}{3\,(a+b)^2}-\frac{1}{a+b}\right)+\tan(x)\left(\frac{3}{a+b}+\frac{a\left(\frac{a}{(a+b)^2}-\frac{3}{a+b}\right)}{a+b}\right)+\frac{b^3\,\mathrm{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b\right.}{2\,\sqrt{a}\,(a+b)^{7/2}}\right.}{\sqrt{a}\,(a+b)^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^6*(a + b*sin(x)^2)),x)
```

```
[Out] tan(x)^5/(5*(a + b)) - tan(x)^3*(a/(3*(a + b)^2) - 1/(a + b)) + tan(x)*(3/(
a + b) + (a*(a/(a + b)^2 - 3/(a + b)))/(a + b)) + (b^3*atan((tan(x)*(2*a +
2*b)*(3*a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(a^(1/2)*
(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**6/(a+b*sin(x)**2),x)
```

```
[Out] Timed out
```

3.314 $\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=113

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}+\frac{x(4a+5b)}{2b^3}+\frac{(2a+b)(a+b)\tan(x)}{2ab^2\left((a+b)\tan^2(x)+a\right)}-\frac{\sin(x)\cos(x)}{2b\left((a+b)\tan^2(x)+a\right)}$$

```
[Out] 1/2*(4*a+5*b)*x/b^3-1/2*(4*a-b)*(a+b)^(3/2)*arctan((a+b)^(1/2)*tan(x)/a^(1/
2))/a^(3/2)/b^3-1/2*cos(x)*sin(x)/b/(a+(a+b)*tan(x)^2)+1/2*(a+b)*(2*a+b)*ta
n(x)/a/b^2/(a+(a+b)*tan(x)^2)
```

Rubi [A] time = 0.22, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3191, 414, 527, 522, 203, 205}

$$-\frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3}+\frac{x(4a+5b)}{2b^3}+\frac{(2a+b)(a+b)\tan(x)}{2ab^2\left((a+b)\tan^2(x)+a\right)}-\frac{\sin(x)\cos(x)}{2b\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((4*a + 5*b)*x)/(2*b^3) - ((4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[
x])/Sqrt[a]])/(2*a^(3/2)*b^3) - (Cos[x]*Sin[x])/(2*b*(a + (a + b)*Tan[x]^2)
) + ((a + b)*(2*a + b)*Tan[x])/(2*a*b^2*(a + (a + b)*Tan[x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b-3(a+b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{2b}$$
$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{2\left(2a^2+2ab-b^2\right)-2(a+b)(2a-}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\right.}{4ab^2}$$
$$= -\frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b)(2a+b)\tan(x)}{2ab^2\left(a+(a+b)\tan^2(x)\right)} - \frac{\left((4a-b)(a+b)^2\right)\text{Subst}\left(\int \frac{}{a+(}\right.}{2ab^3}$$
$$= \frac{(4a+5b)x}{2b^3} - \frac{(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^3} - \frac{\cos(x)\sin(x)}{2b\left(a+(a+b)\tan^2(x)\right)} + \frac{(a+b}{2ab^2\left(a\right.}$$

Mathematica [A] time = 0.30, size = 90, normalized size = 0.80

$$\frac{-\frac{2(4a-b)(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}} + 2x(4a+5b) + \frac{2b(a+b)^2\sin(2x)}{a(2a-b\cos(2x)+b)} + b\sin(2x)}{4b^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^6/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (2*(4*a + 5*b)*x - (2*(4*a - b)*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[x])/S
qrt[a]])/a^(3/2) + b*Sin[2*x] + (2*b*(a + b)^2*Sin[2*x])/(a*(2*a + b - b*Co
s[2*x])))/(4*b^3)
```

fricas [B] time = 0.49, size = 491, normalized size = 4.35

$$\left[\frac{4\left(4a^2b+5ab^2\right)x\cos(x)^2 + \left(4a^3+7a^2b+2ab^2-b^3-\left(4a^2b+3ab^2-b^3\right)\cos(x)^2\right)\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos}{}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(4*a^2*b + 5*a*b^2)*x*cos(x)^2 + (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 -
 (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b +
 b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)
^3 - (a^2 + a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*
cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 4*(4*a^3 + 9*a^2*
b + 5*a*b^2)*x + 4*(a*b^2*cos(x)^3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(
x))/(a*b^4*cos(x)^2 - a^2*b^3 - a*b^4), 1/4*(2*(4*a^2*b + 5*a*b^2)*x*cos(x)
^2 - (4*a^3 + 7*a^2*b + 2*a*b^2 - b^3 - (4*a^2*b + 3*a*b^2 - b^3)*cos(x)^2)
*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((
a + b)*cos(x)*sin(x))) - 2*(4*a^3 + 9*a^2*b + 5*a*b^2)*x + 2*(a*b^2*cos(x)^
3 - (2*a^2*b + 3*a*b^2 + b^3)*cos(x))*sin(x))/(a*b^4*cos(x)^2 - a^2*b^3 - a
*b^4)]
```

giac [A] time = 0.16, size = 175, normalized size = 1.55

$$\frac{(4\,a+5\,b)x}{2\,b^3}-\frac{\left(4\,a^3+7\,a^2b+2\,ab^2-b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}\left(2\,a+2\,b\right)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)}{2\,\sqrt{a^2+ab}\,ab^3}+\frac{2\,a^2\tan(x)^3+3}{2\left(a\,t\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(4*a + 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*(pi*floor(x/p
i + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/(s
qrt(a^2 + a*b)*a*b^3) + 1/2*(2*a^2*tan(x)^3 + 3*a*b*tan(x)^3 + b^2*tan(x)^3
 + 2*a^2*tan(x) + 2*a*b*tan(x) + b^2*tan(x))/((a*tan(x)^4 + b*tan(x)^4 + 2*
a*tan(x)^2 + b*tan(x)^2 + a)*a*b^2)
```

maple [B] time = 0.23, size = 211, normalized size = 1.87

$$\frac{\tan(x)}{2b^2\left(\tan^2(x)+1\right)}+\frac{5\arctan\left(\tan(x)\right)}{2b^2}+\frac{2\arctan\left(\tan(x)\right)a}{b^3}+\frac{a\tan(x)}{2b^2\left(\left(\tan^2(x)\right)a+\left(\tan^2(x)\right)b+a\right)}+\frac{t}{b\left(\left(\tan^2(x)\right)a\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/b^2*tan(x)/(tan(x)^2+1)+5/2/b^2*arctan(tan(x))+2/b^3*arctan(tan(x))*a+1
/2/b^2*a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/b*tan(x)/(tan(x)^2*a+tan(x)^2*b
+a)+1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)-2/b^3/(a*(a+b))^(1/2)*arctan((a+
b)*tan(x)/(a*(a+b))^(1/2))*a^2-7/2/b^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/
(a*(a+b))^(1/2))*a-1/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.48, size = 150, normalized size = 1.33

$$\frac{\left(2\,a^2+3\,ab+b^2\right)\tan(x)^3+\left(2\,a^2+2\,ab+b^2\right)\tan(x)}{2\left(\left(a^2b^2+ab^3\right)\tan(x)^4+a^2b^2+\left(2\,a^2b^2+ab^3\right)\tan(x)^2\right)}+\frac{(4\,a+5\,b)x}{2\,b^3}-\frac{\left(4\,a^3+7\,a^2b+2\,ab^2-b^3\right)\arctan\left(\frac{(a+}{\sqrt{}}\right.}{2\,\sqrt{(a+b)a}\,ab^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^6/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*((2*a^2 + 3*a*b + b^2)*tan(x)^3 + (2*a^2 + 2*a*b + b^2)*tan(x))/((a^2*b
^2 + a*b^3)*tan(x)^4 + a^2*b^2 + (2*a^2*b^2 + a*b^3)*tan(x)^2) + 1/2*(4*a +
 5*b)*x/b^3 - 1/2*(4*a^3 + 7*a^2*b + 2*a*b^2 - b^3)*arctan((a + b)*tan(x)/s
qrt((a + b)*a))/(sqrt((a + b)*a)*a*b^3)
```

mupad [B] time = 14.70, size = 463, normalized size = 4.10

$$\frac{\frac{\tan(x)\left(2a^2+2ab+b^2\right)}{2ab^2}+\frac{\tan(x)^3(a+b)(2a+b)}{2ab^2}}{(a+b)\tan(x)^4+(2a+b)\tan(x)^2+a}-\frac{\ln\left(a^2b-\tan(x)\sqrt{-a^3(a+b)^3}+a^3\right)\sqrt{-a^3(a+b)^3}\,(4a-b)}{4a^3b^3}+\frac{\ln\left(\tan(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^6/(a + b*sin(x)^2)^2,x)
```

```
[Out] ((tan(x)*(2*a*b + 2*a^2 + b^2))/(2*a*b^2) + (tan(x)^3*(a + b)*(2*a + b))/(2
*a*b^2))/(a + tan(x)^2*(2*a + b) + tan(x)^4*(a + b)) - (atan((41*tan(x))/(2
*((131*a)/(4*b) + (11*b)/(4*a) - (5*b^2)/(4*a^2) + (85*a^2)/(4*b^2) + (5*a^
3)/b^3 + 41/2)) + (11*tan(x))/(4*((41*a)/(2*b) - (5*b)/(4*a) + (131*a^2)/(4
*b^2) + (85*a^3)/(4*b^3) + (5*a^4)/b^4 + 11/4)) + (131*a*tan(x))/(4*((131*a
)/4 + (41*b)/2 + (11*b^2)/(4*a) + (85*a^2)/(4*b) - (5*b^3)/(4*a^2) + (5*a^3
)/b^2)) - (5*b*tan(x))/(4*((11*a)/4 - (5*b)/4 + (41*a^2)/(2*b) + (131*a^3)/
(4*b^2) + (85*a^4)/(4*b^3) + (5*a^5)/b^4)) + (85*a^2*tan(x))/(4*((131*a*b)/
4 + (85*a^2)/4 + (41*b^2)/2 + (11*b^3)/(4*a) + (5*a^3)/b - (5*b^4)/(4*a^2))
) + (5*a^3*tan(x))/((131*a*b^2)/4 + (85*a^2*b)/4 + 5*a^3 + (41*b^3)/2 + (11
*b^4)/(4*a) - (5*b^5)/(4*a^2)))*(a*1i + (b*5i)/4)*2i)/b^3 - (log(a^2*b - ta
n(x)*(-a^3*(a + b)^3)^(1/2) + a^3)*(-a^3*(a + b)^3)^(1/2)*(4*a - b))/(4*a^3
*b^3) + (log(tan(x)*(-a^3*(a + b)^3)^(1/2) + a^2*b + a^3)*(a - b/4)*(-a^3*(
a + b)^3)^(1/2))/(a^3*b^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**6/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.315 $\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=72

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

```
[Out] -1/2*(3*a-b)*(a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(5/2)+sin(x)/b^
2+1/2*(a+b)^2*sin(x)/a/b^2/(a+b*sin(x)^2)
```

Rubi [A] time = 0.12, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3190, 390, 385, 205}

$$-\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}}+\frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}+\frac{\sin(x)}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -((3*a - b)*(a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(5/2)) +
 Sin[x]/b^2 + ((a + b)^2*Sin[x])/(2*a*b^2*(a + b*Sin[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^5(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{b^2} - \frac{a^2-b^2+2b(a+b)x^2}{b^2\left(a+bx^2\right)^2}\right)dx, x, \sin(x)\right)$$
$$= \frac{\sin(x)}{b^2} - \frac{\text{Subst}\left(\int \frac{a^2-b^2+2b(a+b)x^2}{\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{b^2}$$
$$= \frac{\sin(x)}{b^2} + \frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)} - \frac{((3a-b)(a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx, x, \sin(x)\right)}{2ab^2}$$
$$= -\frac{(3a-b)(a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{5/2}} + \frac{\sin(x)}{b^2} + \frac{(a+b)^2\sin(x)}{2ab^2\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.34, size = 118, normalized size = 1.64

$$\frac{\frac{\left(-3a^2-2ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} + \frac{\left(3a^2+2ab-b^2\right)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{4\sqrt{b}(a+b)^2\sin(x)}{a(2a-b\cos(2x)+b)} + 4\sqrt{b}\sin(x)}{4b^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^5/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (((3*a^2 + 2*a*b - b^2)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2) + ((-3*a^
2 - 2*a*b + b^2)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*Sqrt[b]*Sin[
x] + (4*Sqrt[b]*(a + b)^2*Sin[x])/(a*(2*a + b - b*Cos[2*x])))/(4*b^(5/2))
```

fricas [B] time = 0.46, size = 296, normalized size = 4.11

$$\left[-\frac{\left(3a^3+5a^2b+ab^2-b^3-\left(3a^2b+2ab^2-b^3\right)\cos(x)^2\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right) - 2\left(2a^2b^2\cos(x)\right.}{4\left(a^2b^4\cos(x)^2-a^3b^3-a^2b^4\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)
*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a
 - b)) - 2*(2*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*
b^4*cos(x)^2 - a^3*b^3 - a^2*b^4), 1/2*((3*a^3 + 5*a^2*b + a*b^2 - b^3 - (3
*a^2*b + 2*a*b^2 - b^3)*cos(x)^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (2
*a^2*b^2*cos(x)^2 - 3*a^3*b - 4*a^2*b^2 - a*b^3)*sin(x))/(a^2*b^4*cos(x)^2
- a^3*b^3 - a^2*b^4)]
```

giac [A] time = 0.12, size = 82, normalized size = 1.14

$$\frac{\sin(x)}{b^2} - \frac{\left(3a^2+2ab-b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab^2} + \frac{a^2\sin(x)+2ab\sin(x)+b^2\sin(x)}{2\left(b\sin(x)^2+a\right)ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] sin(x)/b^2 - 1/2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b
)*a*b^2) + 1/2*(a^2*sin(x) + 2*a*b*sin(x) + b^2*sin(x))/((b*sin(x)^2 + a)*a
*b^2)
```

maple [A] time = 0.22, size = 120, normalized size = 1.67

$$\frac{\sin(x)}{b^2}+\frac{a\sin(x)}{2b^2\left(a+b\left(\sin^2(x)\right)\right)}+\frac{\sin(x)}{b\left(a+b\left(\sin^2(x)\right)\right)}+\frac{\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}-\frac{3a\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{2b^2\sqrt{ab}}-\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{b\sqrt{ab}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a+b*sin(x)^2)^2,x)
```

```
[Out] sin(x)/b^2+1/2/b^2*a*sin(x)/(a+b*sin(x)^2)+1/b*sin(x)/(a+b*sin(x)^2)+1/2*si
n(x)/a/(a+b*sin(x)^2)-3/2/b^2*a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))-1/
b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2/a/(a*b)^(1/2)*arctan(sin(x)*
b/(a*b)^(1/2))
```

maxima [A] time = 0.51, size = 79, normalized size = 1.10

$$\frac{\left(a^2+2\,ab+b^2\right)\sin(x)}{2\left(ab^3\sin(x)^2+a^2b^2\right)}+\frac{\sin(x)}{b^2}-\frac{\left(3\,a^2+2\,ab-b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^5/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a^2 + 2*a*b + b^2)*sin(x)/(a*b^3*sin(x)^2 + a^2*b^2) + sin(x)/b^2 - 1/
2*(3*a^2 + 2*a*b - b^2)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b^2)
```

mupad [B] time = 14.40, size = 96, normalized size = 1.33

$$\frac{\sin(x)}{b^2}+\frac{\sin(x)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a\,\left(b^3\,{\sin(x)}^2+a\,b^2\right)}-\frac{\mathrm{atan}\left(\frac{\sqrt{b}\,\sin(x)\,(a+b)\,(3\,a-b)}{\sqrt{a}\,\left(3\,a^2+2\,a\,b-b^2\right)}\right)\,(a+b)\,(3\,a-b)}{2\,a^{3/2}\,b^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^5/(a + b*sin(x)^2)^2,x)
```

```
[Out] sin(x)/b^2 + (sin(x)*(2*a*b + a^2 + b^2))/(2*a*(b^3*sin(x)^2 + a*b^2)) - (a
tan((b^(1/2)*sin(x)*(a + b)*(3*a - b))/(a^(1/2)*(2*a*b + 3*a^2 - b^2)))*(a
+ b)*(3*a - b))/(2*a^(3/2)*b^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**5/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.316 $\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=75

$$-\frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}+\frac{x}{b^2}$$

```
[Out] x/b^2-1/2*(2*a-b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/b^
2+1/2*(a+b)*tan(x)/a/b/(a+(a+b)*tan(x)^2)
```

Rubi [A] time = 0.11, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3191, 414, 522, 203, 205}

$$-\frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2}+\frac{(a+b)\tan(x)}{2ab\left((a+b)\tan^2(x)+a\right)}+\frac{x}{b^2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] x/b^2 - ((2*a - b)*Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(
3/2)*b^2) + ((a + b)*Tan[x])/(2*a*b*(a + (a + b)*Tan[x]^2))
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
```

```
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1+x^2\right)\left(a+(a+b)x^2\right)^2}\,dx,x,\tan(x)\right)$$
$$= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{a-b+(-a-b)x^2}{\left(1+x^2\right)\left(a+(a+b)x^2\right)}\,dx,x,\tan(x)\right)}{2ab}$$
$$= \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx,x,\tan(x)\right)}{b^2} - \frac{((2a-b)(a+b))\,\text{Subst}\left(\int\right.}{2ab}$$
$$= \frac{x}{b^2} - \frac{(2a-b)\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}b^2} + \frac{(a+b)\tan(x)}{2ab\left(a+(a+b)\tan^2(x)\right)}$$

Mathematica [A] time = 0.32, size = 79, normalized size = 1.05

$$\frac{\frac{\left(-2a^2-ab+b^2\right)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{a^{3/2}\sqrt{a+b}} + \frac{b(a+b)\sin(2x)}{a(2a-b\cos(2x)+b)} + 2x}{2b^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^4/(a + b*Sin[x]^2)^2,x]
[Out] (2*x + ((-2*a^2 - a*b + b^2)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)
*Sqrt[a + b]) + (b*(a + b)*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])))/(2*b^2)
```

fricas [B] time = 0.46, size = 367, normalized size = 4.89

$$\left[\frac{8\,abx\cos(x)^2 - 4\left(ab+b^2\right)\cos(x)\sin(x) - \left(\left(2\,ab-b^2\right)\cos(x)^2 - 2\,a^2 - ab + b^2\right)\sqrt{-\frac{a+b}{a}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x\right.}{}\right.}{8\left(ab^3\cos(x)^2 - a^2b^2 - a\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
[Out] [1/8*(8*a*b*x*cos(x)^2 - 4*(a*b + b^2)*cos(x)*sin(x) - ((2*a*b - b^2)*cos(x
)^2 - 2*a^2 - a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(x)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 - 4*((2*a^2 + a*b)*cos(x)^3 - (a^2 +
a*b)*cos(x))*sqrt(-(a + b)/a)*sin(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2
*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)) - 8*(a^2 + a*b)*x)/(a*b^3*cos(x
)^2 - a^2*b^2 - a*b^3), 1/4*(4*a*b*x*cos(x)^2 - 2*(a*b + b^2)*cos(x)*sin(x)
 + ((2*a*b - b^2)*cos(x)^2 - 2*a^2 - a*b + b^2)*sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(x)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(x)*sin(x))) - 4*(
a^2 + a*b)*x)/(a*b^3*cos(x)^2 - a^2*b^2 - a*b^3)]
```

giac [A] time = 0.16, size = 109, normalized size = 1.45

$$\frac{x}{b^2} - \frac{\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}\,(2\,a+2\,b) + \arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)\right)\left(2\,a^2+ab-b^2\right)}{2\sqrt{a^2+ab}\,ab^2} + \frac{a\tan(x)+b\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] x/b^2 - 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan
(x))/sqrt(a^2 + a*b)))*(2*a^2 + a*b - b^2)/(sqrt(a^2 + a*b)*a*b^2) + 1/2*(a
*tan(x) + b*tan(x))/((a*tan(x)^2 + b*tan(x)^2 + a)*a*b)
```

maple [B] time = 0.21, size = 132, normalized size = 1.76

$$\frac{\tan(x)}{2b\left(\left(\tan^2(x)\right)a+\left(\tan^2(x)\right)b+a\right)}-\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)a}{b^2\sqrt{a\,(a+b)}}-\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2b\sqrt{a\,(a+b)}}+\frac{\tan(x)}{2a\left(\left(\tan^2(x)\right)a+\left(\tan^2(x)\right)b+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/b*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)-1/b^2/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))*a-1/2/b/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b)
)^(1/2))+1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/2/a/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(x)/(a*(a+b))^(1/2))+x/b^2
```

maxima [A] time = 0.95, size = 80, normalized size = 1.07

$$\frac{(a+b)\tan(x)}{2\left(a^2b+\left(a^2b+ab^2\right)\tan(x)^2\right)}+\frac{x}{b^2}-\frac{\left(2\,a^2+ab-b^2\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\,\sqrt{(a+b)a}\,ab^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a + b)*tan(x)/(a^2*b + (a^2*b + a*b^2)*tan(x)^2) + x/b^2 - 1/2*(2*a^2
+ a*b - b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a*b^2)
```

mupad [B] time = 14.33, size = 533, normalized size = 7.11

$$\frac{\operatorname{atan}\left(\frac{5\tan(x)}{2\left(\frac{3a}{2b}+\frac{b}{2a}-\frac{b^2}{2a^2}+\frac{5}{2}\right)}+\frac{\tan(x)}{2\left(\frac{5a}{2b}-\frac{b}{2a}+\frac{3a^2}{2b^2}+\frac{1}{2}\right)}+\frac{3a\tan(x)}{2\left(\frac{3a}{2}+\frac{5b}{2}+\frac{b^2}{2a}-\frac{b^3}{2a^2}\right)}-\frac{b\tan(x)}{2\left(\frac{a}{2}-\frac{b}{2}+\frac{5a^2}{2b}+\frac{3a^3}{2b^2}\right)}\right)}{b^2}+\frac{\operatorname{atanh}\left(\frac{\tan(x)\sqrt{-a^4-b\,a^3}}{a^2-\frac{3ab}{2}-\frac{b^2}{2}+\frac{b^3}{4a}+\frac{13a^3}{4b}+\frac{3a^4}{2b^2}}+\cdots\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^4/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((5*tan(x))/(2*((3*a)/(2*b) + b/(2*a) - b^2/(2*a^2) + 5/2)) + tan(x)/(2
*((5*a)/(2*b) - b/(2*a) + (3*a^2)/(2*b^2) + 1/2)) + (3*a*tan(x))/(2*((3*a)/
2 + (5*b)/2 + b^2/(2*a) - b^3/(2*a^2))) - (b*tan(x))/(2*(a/2 - b/2 + (5*a^2
)/(2*b) + (3*a^3)/(2*b^2)))))/b^2 + (atanh((tan(x)*(- a^3*b - a^4)^(1/2))/(a
^2 - (3*a*b)/2 - b^2/2 + b^3/(4*a) + (13*a^3)/(4*b) + (3*a^4)/(2*b^2)) + (3
*tan(x)*(- a^3*b - a^4)^(1/2))/(2*((13*a*b)/4 + (3*a^2)/2 + b^2 - (3*b^3)/(
2*a) - b^4/(2*a^2) + b^5/(4*a^3))) + (13*tan(x)*(- a^3*b - a^4)^(1/2))/(4*(
a*b + (13*a^2)/4 - (3*b^2)/2 - b^3/(2*a) + (3*a^3)/(2*b) + b^4/(4*a^2))) -
(3*b*tan(x)*(- a^3*b - a^4)^(1/2))/(2*(a^3 - (3*a^2*b)/2 - (a*b^2)/2 + b^3/
4 + (13*a^4)/(4*b) + (3*a^5)/(2*b^2))) - (b^2*tan(x)*(- a^3*b - a^4)^(1/2))
/(2*((a*b^3)/4 - (3*a^3*b)/2 + a^4 - (a^2*b^2)/2 + (13*a^5)/(4*b) + (3*a^6)
/(2*b^2))) + (b^3*tan(x)*(- a^3*b - a^4)^(1/2))/(4*(a^5 - (3*a^4*b)/2 + (a^
2*b^3)/4 - (a^3*b^2)/2 + (13*a^6)/(4*b) + (3*a^7)/(2*b^2))))*(-a^3*(a + b))
^(1/2)*(2*a - b))/(2*a^3*b^2) + (tan(x)*(a + b))/(2*a*b*(a + tan(x)^2*(a +
b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.317 $\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=59

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

```
[Out] -1/2*(a-b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(3/2)+1/2*(a+b)*sin(x)/
a/b/(a+b*sin(x)^2)
```

Rubi [A] time = 0.06, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 385, 205}

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*b^(3/2)) + ((a + b)*
Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^2}\,dx,x,\sin(x)\right)\\
&= \frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{2ab}\\
&= -\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}+\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}
\end{aligned}$$

Mathematica [A] time = 0.07, size = 59, normalized size = 1.00

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\sin^2(x)\right)}-\frac{(a-b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}b^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] -1/2*((a - b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(a^(3/2)*b^(3/2)) + ((a + b
)*Sin[x])/(2*a*b*(a + b*Sin[x]^2))
```

fricas [A] time = 0.43, size = 206, normalized size = 3.49

$$\left[\frac{\left(\left(ab-b^2\right)\cos(x)^2-a^2+b^2\right)\sqrt{-ab}\log\left(-\frac{b\cos(x)^2+2\sqrt{-ab}\sin(x)+a-b}{b\cos(x)^2-a-b}\right)-2\left(a^2b+ab^2\right)\sin(x)}{4\left(a^2b^3\cos(x)^2-a^3b^2-a^2b^3\right)},-\frac{\left(\left(ab-b^2\right)\cos(x)^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((a*b - b^2)*cos(x)^2 - a^2 + b^2)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*sq
rt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) - 2*(a^2*b + a*b^2)*sin(x))/
(a^2*b^3*cos(x)^2 - a^3*b^2 - a^2*b^3), -1/2*(((a*b - b^2)*cos(x)^2 - a^2 +
 b^2)*sqrt(a*b)*arctan(sqrt(a*b)*sin(x)/a) + (a^2*b + a*b^2)*sin(x))/(a^2*b
^3*cos(x)^2 - a^3*b^2 - a^2*b^3)]
```

giac [A] time = 0.13, size = 56, normalized size = 0.95

$$-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}+\frac{a\sin(x)+b\sin(x)}{2\left(b\sin(x)^2+a\right)ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] -1/2*(a - b)*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a*b) + 1/2*(a*sin(x) + b
*sin(x))/((b*sin(x)^2 + a)*a*b)
```

maple [A] time = 0.23, size = 65, normalized size = 1.10

$$\frac{(a+b)\sin(x)}{2ab\left(a+b\left(\sin^2(x)\right)\right)}-\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{2b\sqrt{ab}}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2*(a+b)*sin(x)/a/b/(a+b*sin(x)^2)-1/2/b/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)
^(1/2))+1/2/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 1.83, size = 53, normalized size = 0.90

$$\frac{(a+b)\sin(x)}{2\left(ab^2\sin(x)^2+a^2b\right)}-\frac{(a-b)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\sqrt{ab}\,ab}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*(a + b)*sin(x)/(a*b^2*sin(x)^2 + a^2*b) - 1/2*(a - b)*arctan(b*sin(x)/s
qrt(a*b))/(sqrt(a*b)*a*b)
```

mupad [B] time = 0.14, size = 47, normalized size = 0.80

$$\frac{\sin(x)\ (a+b)}{2\,a\,b\,\left(b\sin(x)^2+a\right)}-\frac{\operatorname{atan}\left(\frac{\sqrt{b}\ \sin(x)}{\sqrt{a}}\right)\,(a-b)}{2\,a^{3/2}\,b^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^3/(a + b*sin(x)^2)^2,x)
```

```
[Out] (sin(x)*(a + b))/(2*a*b*(a + b*sin(x)^2)) - (atan((b^(1/2)*sin(x))/a^(1/2))
*(a - b))/(2*a^(3/2)*b^(3/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.318 $\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=54

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

```
[Out] 1/2*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(1/2)+1/2*tan(x)/a/(a+
(a+b)*tan(x)^2)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3191, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left((a+b)\tan^2(x)+a\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) + Tan[x]/(2*a*
(a + (a + b)*Tan[x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+(a+b)x^2\right)^2}\,dx,x,\tan(x)\right)\\ &= \frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(x)\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}+\frac{\tan(x)}{2a\left(a+(a+b)\tan^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.13, size = 59, normalized size = 1.09

$$\frac{\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{a+b}}-\frac{\sin(2x)}{2a(-2a+b\cos(2x)-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[a + b]) - Sin[2*x]/(2*
a*(-2*a - b + b*Cos[2*x]))
```

fricas [B] time = 0.47, size = 313, normalized size = 5.80

$$\left[\frac{4\left(a^2+ab\right)\cos(x)\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(x)^4-2\left(4a^2+5ab+b^2\right)\cos(x)^2+4\left((2a+b)\cos(x\right.}{b^2\cos(x)^4-2\left(ab+b^2\right)\cos(x)^2+}\right.}{8\left(a^4+2a^3b+a^2b^2-\left(a^3b+a^2b^2\right)\cos(x)^2\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/8*(4*(a^2 + a*b)*cos(x)*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a^2 - a*b)*l
og(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(x)^2 + 4*(
(2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) + a^2 + 2*a*b
+ b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b^2)))/(a^4 +
 2*a^3*b + a^2*b^2 - (a^3*b + a^2*b^2)*cos(x)^2), 1/4*(2*(a^2 + a*b)*cos(x)
*sin(x) + (b*cos(x)^2 - a - b)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)
^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))))/(a^4 + 2*a^3*b + a^2*b^2 - (a
^3*b + a^2*b^2)*cos(x)^2)]
```

giac [A] time = 0.15, size = 77, normalized size = 1.43

$$\frac{\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\sqrt{a^2+ab}\,a}+\frac{\tan(x)}{2\left(a\tan(x)^2+b\tan(x)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(x) + b*tan(x))/sqr
t(a^2 + a*b)))/(sqrt(a^2 + a*b)*a) + 1/2*tan(x)/((a*tan(x)^2 + b*tan(x)^2 +
 a)*a)
```

maple [A] time = 0.22, size = 51, normalized size = 0.94

$$\frac{\tan(x)}{2a\left(\left(\tan^2(x)\right)a+\left(\tan^2(x)\right)b+a\right)}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2a\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+1/2/a/(a*(a+b))^(1/2)*arctan((a+b)*t
an(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.72, size = 49, normalized size = 0.91

$$\frac{\tan(x)}{2\left(\left(a^2+ab\right)\tan(x)^2+a^2\right)}+\frac{\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\sqrt{(a+b)a}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*tan(x)/((a^2 + a*b)*tan(x)^2 + a^2) + 1/2*arctan((a + b)*tan(x)/sqrt((a
 + b)*a))/(sqrt((a + b)*a)*a)
```

mupad [B] time = 14.31, size = 50, normalized size = 0.93

$$\frac{\operatorname{atan}\left(\frac{\tan(x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{2\,a^{3/2}\,\sqrt{a+b}}+\frac{\tan(x)}{2\,a\,\left((a+b)\,\tan(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)^2/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((tan(x)*(2*a + 2*b))/(2*a^(1/2)*(a + b)^(1/2)))/(2*a^(3/2)*(a + b)^(1/
2)) + tan(x)/(2*a*(a + tan(x)^2*(a + b)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)**2/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.319 $\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=48

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

```
[Out] 1/2*sin(x)/a/(a+b*sin(x)^2)+1/2*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/b^(1
/2)
```

Rubi [A] time = 0.03, antiderivative size = 48, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3190, 199, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(x)}{\left(a+b\sin^2(x)\right)^2}\,dx &= \text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^2}\,dx,x,\sin(x)\right)\\ &= \frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}+\frac{\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,x,\sin(x)\right)}{2a}\\ &= \frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 48, normalized size = 1.00

$$\frac{\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}\sqrt{b}}+\frac{\sin(x)}{2a\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[x]/(a + b*Sin[x]^2)^2,x]
[Out] ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]]/(2*a^(3/2)*Sqrt[b]) + Sin[x]/(2*a*(a + b*S
in[x]^2))
```

fricas [A] time = 0.44, size = 165, normalized size = 3.44

$$\left[-\frac{2\,ab\sin(x)+\left(b\cos(x)^2-a-b\right)\sqrt{-ab}\,\log\left(-\frac{b\cos(x)^2+2\,\sqrt{-ab}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)}{4\left(a^2b^2\cos(x)^2-a^3b-a^2b^2\right)},-\frac{ab\sin(x)-\left(b\cos(x)^2-a-b\right)\sqrt{ab}}{2\left(a^2b^2\cos(x)^2-a^3b-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
[Out] [-1/4*(2*a*b*sin(x) + (b*cos(x)^2 - a - b)*sqrt(-a*b)*log(-(b*cos(x)^2 + 2*
sqrt(-a*b)*sin(x) + a - b)/(b*cos(x)^2 - a - b)))/(a^2*b^2*cos(x)^2 - a^3*b
 - a^2*b^2), -1/2*(a*b*sin(x) - (b*cos(x)^2 - a - b)*sqrt(a*b)*arctan(sqrt(
a*b)*sin(x)/a))/(a^2*b^2*cos(x)^2 - a^3*b - a^2*b^2)]
```

giac [A] time = 0.14, size = 38, normalized size = 0.79

$$\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}+\frac{\sin(x)}{2\left(b\sin(x)^2+a\right)a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
[Out] 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)*a) + 1/2*sin(x)/((b*sin(x)^2 + a)
*a)
```

maple [A] time = 0.11, size = 39, normalized size = 0.81

$$\frac{\sin(x)}{2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{2a\sqrt{ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a+b*sin(x)^2)^2,x)
[Out] 1/2*sin(x)/a/(a+b*sin(x)^2)+1/2/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))
```

maxima [A] time = 0.70, size = 38, normalized size = 0.79

$$\frac{\sin(x)}{2\left(ab\sin(x)^2+a^2\right)}+\frac{\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\,\sqrt{ab}\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sin(x)/(a*b*sin(x)^2 + a^2) + 1/2*arctan(b*sin(x)/sqrt(a*b))/(sqrt(a*b)
*a)
```

mupad [B] time = 15.38, size = 36, normalized size = 0.75

$$\frac{\operatorname{atan}\left(\frac{\sqrt{b}\,\sin(x)}{\sqrt{a}}\right)}{2\,a^{3/2}\,\sqrt{b}}+\frac{\sin(x)}{2\,a\,\left(b\sin(x)^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(x)/(a + b*sin(x)^2)^2,x)
```

```
[Out] atan((b^(1/2)*sin(x))/a^(1/2))/(2*a^(3/2)*b^(1/2)) + sin(x)/(2*a*(a + b*sin
(x)^2))
```

sympy [A] time = 15.38, size = 340, normalized size = 7.08

$$\begin{cases} \frac{\tilde{\infty}}{\sin^3(x)} \\ -\frac{1}{3b^2\sin^3(x)} \\ \frac{\sin(x)}{a^2} \\ \frac{2i\sqrt{a}\,b\sqrt{\frac{1}{b}}\sin(x)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)}+\frac{a\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)}-\frac{a\log\left(i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)}+\frac{b\log\left(-i\sqrt{a}\sqrt{\frac{1}{b}}+\sin(x)\right)\sin^2(x)}{4ia^{\frac{5}{2}}b\sqrt{\frac{1}{b}}+4ia^{\frac{3}{2}}b^2\sqrt{\frac{1}{b}}\sin^2(x)}-\frac{b\log}{4ia^{\frac{5}{2}}} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Piecewise((zoo/sin(x)**3, Eq(a, 0) & Eq(b, 0)), (-1/(3*b**2*sin(x)**3), Eq(
a, 0)), (sin(x)/a**2, Eq(b, 0)), (2*I*sqrt(a)*b*sqrt(1/b)*sin(x)/(4*I*a**(5
/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2) + a*log(-I*sqrt(a)
*sqrt(1/b) + sin(x))/(4*I*a**(5/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b
)*sin(x)**2) - a*log(I*sqrt(a)*sqrt(1/b) + sin(x))/(4*I*a**(5/2)*b*sqrt(1/b
) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2) + b*log(-I*sqrt(a)*sqrt(1/b) + s
in(x))*sin(x)**2/(4*I*a**(5/2)*b*sqrt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*si
n(x)**2) - b*log(I*sqrt(a)*sqrt(1/b) + sin(x))*sin(x)**2/(4*I*a**(5/2)*b*sq
rt(1/b) + 4*I*a**(3/2)*b**2*sqrt(1/b)*sin(x)**2), True))
```

3.320 $\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2} \, dx$

Optimal. Leaf size=73

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}$$

```
[Out] arctanh(sin(x))/(a+b)^2+1/2*b*sin(x)/a/(a+b)/(a+b*sin(x)^2)+1/2*(3*a+b)*arc
tan(sin(x)*b^(1/2)/a^(1/2))*b^(1/2)/a^(3/2)/(a+b)^2
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 13, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.385, Rules used = {3190, 414, 522, 206, 205}

$$\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2}+\frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}+\frac{\tanh^{-1}(\sin(x))}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^2)
+ ArcTanh[Sin[x]]/(a + b)^2 + (b*Sin[x])/(2*a*(a + b)*(a + b*Sin[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
```

```
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx,x,\sin(x)\right)$$
$$= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{b-2(a+b)+bx^2}{\left(1-x^2\right)\left(a+bx^2\right)}\,dx,x,\sin(x)\right)}{2a(a+b)}$$
$$= \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{1}{1-x^2}\,dx,x,\sin(x)\right)}{(a+b)^2} + \frac{(b(3a+b))\,\text{Subst}\left(\int \frac{1}{a+bx^2}\,dx,\right.}{2a(a+b)^2}$$
$$= \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^2} + \frac{\tanh^{-1}(\sin(x))}{(a+b)^2} + \frac{b\sin(x)}{2a(a+b)\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 0.53, size = 130, normalized size = 1.78

$$\frac{\frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\sqrt{b}\,(3a+b)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + 4\left(\frac{b(a+b)\sin(x)}{a(2a-b\cos(2x)+b)} - \log\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right) + \log\left(\sin\left(\frac{x}{2}\right)+\cos\left(\right.\right.\right.}{4(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((Sqrt[b]*(3*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (Sqrt[b]
*(3*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) + 4*(-Log[Cos[x/2] - S
in[x/2]] + Log[Cos[x/2] + Sin[x/2]] + (b*(a + b)*Sin[x])/(a*(2*a + b - b*Co
s[2*x]))))/(4*(a + b)^2)
```

fricas [B] time = 0.52, size = 354, normalized size = 4.85

$$\left[-\frac{\left(\left(3\,ab+b^2\right)\cos(x)^2-3\,a^2-4\,ab-b^2\right)\sqrt{-\frac{b}{a}}\,\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+2\left(ab\cos(x)^2-a^2-ab\right)\log\left(\sin\right.}{4\left(a^4+3\,a^3b+3\,a^2b^2+ab^3-\left(a^3b+2\,a^2b^2+ab\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/4*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a*b - b^2)*sqrt(-b/a)*log(-(b*co
s(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b)) + 2*(a*b*cos(
x)^2 - a^2 - a*b)*log(sin(x) + 1) - 2*(a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x
) + 1) - 2*(a*b + b^2)*sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b
+ 2*a^2*b^2 + a*b^3)*cos(x)^2), -1/2*(((3*a*b + b^2)*cos(x)^2 - 3*a^2 - 4*a
*b - b^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x)) + (a*b*cos(x)^2 - a^2 - a*b)*l
og(sin(x) + 1) - (a*b*cos(x)^2 - a^2 - a*b)*log(-sin(x) + 1) - (a*b + b^2)*
sin(x))/(a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 + a*b^3)*co
s(x)^2)]
```

giac [A] time = 0.13, size = 109, normalized size = 1.49

$$\frac{\left(3\,ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}}+\frac{\log\left(\sin(x)+1\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{\log\left(-\sin(x)+1\right)}{2\left(a^2+2\,ab+b^2\right)}+\frac{b\sin(x)}{2\left(b\sin(x)^2+a\right)\left(a^2+ab\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*(3*a*b + b^2)*arctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(
a*b)) + 1/2*log(sin(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(-sin(x) + 1)/(a^2
 + 2*a*b + b^2) + 1/2*b*sin(x)/((b*sin(x)^2 + a)*(a^2 + a*b))
```

maple [A] time = 0.22, size = 122, normalized size = 1.67

$$-\frac{\ln\left(-1+\sin(x)\right)}{2\left(a+b\right)^2}+\frac{b\sin(x)}{2\left(a+b\right)^2\left(a+b\left(\sin^2(x)\right)\right)}+\frac{b^2\sin(x)}{2\left(a+b\right)^2a\left(a+b\left(\sin^2(x)\right)\right)}+\frac{3b\arctan\left(\frac{\sin(x)b}{\sqrt{ab}}\right)}{2\left(a+b\right)^2\sqrt{ab}}+\frac{b^2\arctan\left(\right.}{2\left(a+b\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)/(a+b*sin(x)^2)^2,x)
```

```
[Out] -1/2/(a+b)^2*ln(-1+sin(x))+1/2*b/(a+b)^2*sin(x)/(a+b*sin(x)^2)+1/2*b^2/(a+b
)^2/a*sin(x)/(a+b*sin(x)^2)+3/2*b/(a+b)^2/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)
^(1/2))+1/2*b^2/(a+b)^2/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2/(a+b
)^2*ln(1+sin(x))
```

maxima [A] time = 0.78, size = 115, normalized size = 1.58

$$\frac{b\sin(x)}{2\left(a^3+a^2b+\left(a^2b+ab^2\right)\sin(x)^2\right)}+\frac{\left(3\,ab+b^2\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{ab}}+\frac{\log\left(\sin(x)+1\right)}{2\left(a^2+2\,ab+b^2\right)}-\frac{\log\left(\sin(x)-1\right)}{2\left(a^2+2\,ab+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b*sin(x)/(a^3 + a^2*b + (a^2*b + a*b^2)*sin(x)^2) + 1/2*(3*a*b + b^2)*a
rctan(b*sin(x)/sqrt(a*b))/((a^3 + 2*a^2*b + a*b^2)*sqrt(a*b)) + 1/2*log(sin
(x) + 1)/(a^2 + 2*a*b + b^2) - 1/2*log(sin(x) - 1)/(a^2 + 2*a*b + b^2)
```

mupad [B] time = 15.84, size = 2213, normalized size = 30.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)*(a + b*sin(x)^2)^2),x)
```

```
[Out] (b*sin(x))/(2*a*(a + b)*(a + b*sin(x)^2)) - (atan((((3*a + b)*(-a^3*b)^(1/2
)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((
3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 +
18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) - (sin(x)*(3*
a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 -
48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3
*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*a^4*b + a^5 + a^3*b^2)) +
 ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3
*b + a^4 + a^2*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 2
8*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 +
 3*a^3*b^2) + (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 3
```

```
2*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*
b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2)))*1i)/(4*(2*
a^4*b + a^5 + a^3*b^2)))/(((3*a*b^3)/2 + b^4/2)/(3*a^4*b + a^5 + a^2*b^3 +
3*a^3*b^2) + ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3
))/(2*(2*a^3*b + a^4 + a^2*b^2)) + ((3*a + b)*(-a^3*b)^(1/2)*((2*a*b^7 + 12
*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(3*a^4*b + a^5
 + a^2*b^3 + 3*a^3*b^2) - (sin(x)*(3*a + b)*(-a^3*b)^(1/2)*(16*a^2*b^7 + 48
*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(2*a^3*b
+ a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))
))/(4*(2*a^4*b + a^5 + a^3*b^2)) - ((3*a + b)*(-a^3*b)^(1/2)*((sin(x)*(6*a*
b^4 + b^5 + 13*a^2*b^3))/(2*(2*a^3*b + a^4 + a^2*b^2)) - ((3*a + b)*(-a^3*b
)^(1/2)*((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a
^6*b^2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + (sin(x)*(3*a + b)*(-a^3*b)^
(1/2)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*
a^7*b^2))/(8*(2*a^3*b + a^4 + a^2*b^2)*(2*a^4*b + a^5 + a^3*b^2))))/(4*(2*a
^4*b + a^5 + a^3*b^2))))/(4*(2*a^4*b + a^5 + a^3*b^2))))*(3*a + b)*(-a^3*b)
^(1/2)*1i)/(2*(2*a^4*b + a^5 + a^3*b^2)) - (atan((((((2*a*b^7 + 12*a^2*b^6
+ 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2
*b^3 + 3*a^3*b^2)) - (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5
*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))*1
i)/(2*(a + b)^2) + (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a
^4 + a^2*b^2)))/(a + b)^2 - ((((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*
b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) + (
sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16
*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))*1i)/(2*(a + b)^2) - (si
n(x)*(6*a*b^4 + b^5 + 13*a^2*b^3)*1i)/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a + b
)^2)/(((3*a*b^3)/2 + b^4/2)/(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2) + (((2*a*
b^7 + 12*a^2*b^6 + 28*a^3*b^5 + 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*
a^4*b + a^5 + a^2*b^3 + 3*a^3*b^2)) - (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32
*a^4*b^5 - 32*a^5*b^4 - 48*a^6*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a
^4 + a^2*b^2)))/(2*(a + b)^2) + (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(4*(2
*a^3*b + a^4 + a^2*b^2)))/(a + b)^2 + (((2*a*b^7 + 12*a^2*b^6 + 28*a^3*b^5
+ 32*a^4*b^4 + 18*a^5*b^3 + 4*a^6*b^2)/(2*(3*a^4*b + a^5 + a^2*b^3 + 3*a^3*
b^2)) + (sin(x)*(16*a^2*b^7 + 48*a^3*b^6 + 32*a^4*b^5 - 32*a^5*b^4 - 48*a^6
*b^3 - 16*a^7*b^2))/(8*(a + b)^2*(2*a^3*b + a^4 + a^2*b^2)))/(2*(a + b)^2)
- (sin(x)*(6*a*b^4 + b^5 + 13*a^2*b^3))/(4*(2*a^3*b + a^4 + a^2*b^2)))/(a +
 b)^2))*1i)/(a + b)^2
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.321 $\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=76

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan(x)}{(a+b)^2}$$

```
[Out] 1/2*b*(4*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(5/2)+tan(x)
/(a+b)^2+1/2*b^2*tan(x)/a/(a+b)^2/(a+(a+b)*tan(x)^2)
```

Rubi [A] time = 0.12, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.267$, Rules used = {3191, 390, 385, 205}

$$\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}}+\frac{b^2\tan(x)}{2a(a+b)^2\left((a+b)\tan^2(x)+a\right)}+\frac{\tan(x)}{(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(5/2)
) + Tan[x]/(a + b)^2 + (b^2*Tan[x])/(2*a*(a + b)^2*(a + (a + b)*Tan[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$

$$= \text{Subst}\left(\int \left(\frac{1}{(a+b)^2} + \frac{b(2a+b)+2b(a+b)x^2}{(a+b)^2\left(a+(a+b)x^2\right)^2}\right)dx, x, \tan(x)\right)$$

$$= \frac{\tan(x)}{(a+b)^2} + \frac{\text{Subst}\left(\int \frac{b(2a+b)+2b(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^2}$$

$$= \frac{\tan(x)}{(a+b)^2} + \frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)} + \frac{(b(4a+b))\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(x)\right)}{2a(a+b)^2}$$

$$= \frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{5/2}} + \frac{\tan(x)}{(a+b)^2} + \frac{b^2\tan(x)}{2a(a+b)^2\left(a+(a+b)\tan^2(x)\right)}$$

Mathematica [**A**] time = 0.51, size = 76, normalized size = 1.00

$$\frac{1}{2}\left(\frac{b(4a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{5/2}} + \frac{\frac{b^2\sin(2x)}{a(2a-b\cos(2x)+b)}+2\tan(x)}{(a+b)^2}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^2/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((b*(4*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(5/2))
 + ((b^2*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 2*Tan[x])/(a + b)^2)/2
```

fricas [**B**] time = 0.48, size = 505, normalized size = 6.64

$$\left[-\frac{\left(\left(4\,ab^2+b^3\right)\cos(x)^3-\left(4\,a^2b+5\,ab^2+b^3\right)\cos(x)\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2+4\left((2\,a\right.}{b^2\cos(x)^4-2\left(ab+b^2\right)}\right.}{8\left(\left(a^5b+3\,a^4b^2+3\,a^3b^3+a^2b^4\right)\cos(x)^3-\left(a\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/8*(((4*a*b^2 + b^3)*cos(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(-
a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b + b^2)*co
s(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*sin(x) +
a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*a*b + b
^2)) + 4*(2*a^4 + 4*a^3*b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^
2)*sin(x))/((a^5*b + 3*a^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a
^5*b + 6*a^4*b^2 + 4*a^3*b^3 + a^2*b^4)*cos(x)), -1/4*(((4*a*b^2 + b^3)*cos
(x)^3 - (4*a^2*b + 5*a*b^2 + b^3)*cos(x))*sqrt(a^2 + a*b)*arctan(1/2*((2*a
+ b)*cos(x)^2 - a - b)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^4 + 4*a^3*
b + 2*a^2*b^2 - (2*a^3*b + a^2*b^2 - a*b^3)*cos(x)^2)*sin(x))/((a^5*b + 3*a
^4*b^2 + 3*a^3*b^3 + a^2*b^4)*cos(x)^3 - (a^6 + 4*a^5*b + 6*a^4*b^2 + 4*a^3
*b^3 + a^2*b^4)*cos(x))]
```

giac [**A**] time = 0.13, size = 113, normalized size = 1.49

$$\frac{b^2\tan(x)}{2\left(a^3+2\,a^2b+ab^2\right)\left(a\tan(x)^2+b\tan(x)^2+a\right)} + \frac{\left(4\,ab+b^2\right)\arctan\left(\frac{a\tan(x)+b\tan(x)}{\sqrt{a^2+ab}}\right)}{2\left(a^3+2\,a^2b+ab^2\right)\sqrt{a^2+ab}} + \frac{\tan(x)}{a^2+2\,ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^2*tan(x)/((a^3 + 2*a^2*b + a*b^2)*(a*tan(x)^2 + b*tan(x)^2 + a)) + 1/
2*(4*a*b + b^2)*arctan((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b))/((a^3 + 2*a^2
*b + a*b^2)*sqrt(a^2 + a*b)) + tan(x)/(a^2 + 2*a*b + b^2)
```

maple [A] time = 0.25, size = 112, normalized size = 1.47

$$\frac{\tan(x)}{a^2+2ab+b^2}+\frac{b^2\tan(x)}{2(a+b)^2 a\left(\left(\tan^2(x)\right)a+\left(\tan^2(x)\right)b+a\right)}+\frac{2b\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{(a+b)^2\sqrt{a(a+b)}}+\frac{b^2\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{a(a+b)}}\right)}{2(a+b)^2 a\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^2/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/(a^2+2*a*b+b^2)*tan(x)+1/2*b^2/(a+b)^2/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)
+2*b/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))+1/2*b^2/(
a+b)^2/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.55, size = 119, normalized size = 1.57

$$\frac{b^2\tan(x)}{2\left(a^4+2a^3b+a^2b^2+\left(a^4+3a^3b+3a^2b^2+ab^3\right)\tan(x)^2\right)}+\frac{\left(4ab+b^2\right)\arctan\left(\frac{(a+b)\tan(x)}{\sqrt{(a+b)a}}\right)}{2\left(a^3+2a^2b+ab^2\right)\sqrt{(a+b)a}}+\frac{\tan(x)}{a^2+2ab+b^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^2/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^2*tan(x)/(a^4 + 2*a^3*b + a^2*b^2 + (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*tan(x)^2) + 1/2*(4*a*b + b^2)*arctan((a + b)*tan(x)/sqrt((a + b)*a))/((a
^3 + 2*a^2*b + a*b^2)*sqrt((a + b)*a)) + tan(x)/(a^2 + 2*a*b + b^2)
```

mupad [B] time = 14.78, size = 123, normalized size = 1.62

$$\frac{\tan(x)}{(a+b)^2}+\frac{b^2\tan(x)}{2a\left(ab^2+2a^2b+\tan(x)^2\left(a^3+3a^2b+3ab^2+b^3\right)+a^3\right)}+\frac{b\,\mathrm{atan}\left(\frac{b\tan(x)(4a+b)(2a+2b)\left(a^2+2ab+b^2\right)}{2\sqrt{a}\,(a+b)^{5/2}\left(b^2+4ab\right)}\right)}{2a^{3/2}(a+b)^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^2*(a + b*sin(x)^2)^2),x)
```

```
[Out] tan(x)/(a + b)^2 + (b^2*tan(x))/(2*a*(a*b^2 + 2*a^2*b + tan(x)^2*(3*a*b^2 +
 3*a^2*b + a^3 + b^3) + a^3)) + (b*atan((b*tan(x)*(4*a + b)*(2*a + 2*b)*(2*
a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2)*(4*a*b + b^2)))*(4*a + b))/(2*a^
(3/2)*(a + b)^(5/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**2/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.322 $\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx$

Optimal. Leaf size=109

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}-\frac{b(a-b)\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}+\frac{\tan(x)\sec(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

```
[Out] 1/2*b^(3/2)*(5*a+b)*arctan(sin(x)*b^(1/2)/a^(1/2))/a^(3/2)/(a+b)^3+1/2*(a+5
*b)*arctanh(sin(x))/(a+b)^3-1/2*(a-b)*b*sin(x)/a/(a+b)^2/(a+b*sin(x)^2)+1/2
*sec(x)*tan(x)/(a+b)/(a+b*sin(x)^2)
```

Rubi [A] time = 0.16, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3190, 414, 527, 522, 206, 205}

$$\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3}-\frac{b(a-b)\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}+\frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3}+\frac{\tan(x)\sec(x)}{2(a+b)\left(a+b\sin^2(x)\right)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^(3/2)*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^3)
+ ((a + 5*b)*ArcTanh[Sin[x]])/(2*(a + b)^3) - ((a - b)*b*Sin[x])/(2*a*(a +
b)^2*(a + b*Sin[x]^2)) + (Sec[x]*Tan[x])/(2*(a + b)*(a + b*Sin[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 522

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)$$

$$= \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\text{Subst}\left(\int \frac{a+2b+3bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^2}\,dx, x, \sin(x)\right)}{2(a+b)}$$

$$= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} - \frac{\text{Subst}\left(\int \frac{-2\left(a^2+4ab+b^2\right)-2(a-b)}{\left(1-x^2\right)\left(a+bx^2\right)}\right.}{4a(a+b)^2}$$

$$= -\frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)} + \frac{\sec(x)\tan(x)}{2(a+b)\left(a+b\sin^2(x)\right)} + \frac{\left(b^2(5a+b)\right)\text{Subst}\left(\int \frac{1}{a+bx^2}\right.}{2a(a+b)^3}$$

$$= \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^3} + \frac{(a+5b)\tanh^{-1}(\sin(x))}{2(a+b)^3} - \frac{(a-b)b\sin(x)}{2a(a+b)^2\left(a+b\sin^2(x)\right)}$$

Mathematica [A] time = 1.04, size = 183, normalized size = 1.68

$$\frac{\frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{b}\sin(x)}{\sqrt{a}}\right)}{a^{3/2}} - \frac{b^{3/2}(5a+b)\tan^{-1}\left(\frac{\sqrt{a}\csc(x)}{\sqrt{b}}\right)}{a^{3/2}} + \frac{4b^2(a+b)\sin(x)}{a(2a-b\cos(2x)+b)} + \frac{a+b}{\left(\cos\left(\frac{x}{2}\right)-\sin\left(\frac{x}{2}\right)\right)^2} - \frac{a+b}{\left(\sin\left(\frac{x}{2}\right)+\cos\left(\frac{x}{2}\right)\right)^2} - 2(a+5b)\,\mathrm{l}}{4(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^3/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (-((b^(3/2)*(5*a + b)*ArcTan[(Sqrt[a]*Csc[x])/Sqrt[b]])/a^(3/2)) + (b^(3/2)
*(5*a + b)*ArcTan[(Sqrt[b]*Sin[x])/Sqrt[a]])/a^(3/2) - 2*(a + 5*b)*Log[Cos[
x/2] - Sin[x/2]] + 2*(a + 5*b)*Log[Cos[x/2] + Sin[x/2]] + (a + b)/(Cos[x/2]
 - Sin[x/2])^2 - (a + b)/(Cos[x/2] + Sin[x/2])^2 + (4*b^2*(a + b)*Sin[x])/(
a*(2*a + b - b*Cos[2*x])))/(4*(a + b)^3)
```

fricas [B] time = 0.57, size = 560, normalized size = 5.14

$$\left[\frac{\left(\left(5\,ab^2+b^3\right)\cos(x)^4-\left(5\,a^2b+6\,ab^2+b^3\right)\cos(x)^2\right)\sqrt{-\frac{b}{a}}\,\log\left(-\frac{b\cos(x)^2-2\,a\sqrt{-\frac{b}{a}}\,\sin(x)+a-b}{b\cos(x)^2-a-b}\right)+\left(\left(a^2b+5\,ab^2\right)\mathrm{co}\right.}{4\left(\left(a^4b+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [1/4*(((5*a*b^2 + b^3)*cos(x)^4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(
-b/a)*log(-(b*cos(x)^2 - 2*a*sqrt(-b/a)*sin(x) + a - b)/(b*cos(x)^2 - a - b
)) + ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(
sin(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x
)^2)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*
sin(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b
+ 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2), 1/4*(2*((5*a*b^2 + b^3)*cos(x)^
4 - (5*a^2*b + 6*a*b^2 + b^3)*cos(x)^2)*sqrt(b/a)*arctan(sqrt(b/a)*sin(x))
+ ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2)*log(sin
(x) + 1) - ((a^2*b + 5*a*b^2)*cos(x)^4 - (a^3 + 6*a^2*b + 5*a*b^2)*cos(x)^2
)*log(-sin(x) + 1) - 2*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - b^3)*cos(x)^2)*sin
(x))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*cos(x)^4 - (a^5 + 4*a^4*b + 6
*a^3*b^2 + 4*a^2*b^3 + a*b^4)*cos(x)^2)]
```

giac [B] time = 0.13, size = 194, normalized size = 1.78

$$\frac{(a+5\,b)\log\left(\sin(x)+1\right)}{4\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}-\frac{(a+5\,b)\log\left(-\sin(x)+1\right)}{4\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}+\frac{\left(5\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)\sqrt{ab}}-\frac{ab\sin(x)^{3}-b^{2}\ldots}{2\left(b\sin(x)^{4}+a\sin(\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5*
b)*log(-sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*a
rctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
 1/2*(a*b*sin(x)^3 - b^2*sin(x)^3 + a^2*sin(x) + b^2*sin(x))/((b*sin(x)^4 +
 a*sin(x)^2 - b*sin(x)^2 - a)*(a^3 + 2*a^2*b + a*b^2))
```

maple [A] time = 0.29, size = 180, normalized size = 1.65

$$-\frac{1}{4\left(a+b\right)^{2}\left(-1+\sin(x)\right)}-\frac{\ln\left(-1+\sin(x)\right)a}{4\left(a+b\right)^{3}}-\frac{5\ln\left(-1+\sin(x)\right)b}{4\left(a+b\right)^{3}}+\frac{b^{2}\sin(x)}{2\left(a+b\right)^{3}\left(a+b\left(\sin^{2}(x)\right)\right)}+\frac{b^{3}\sin(\ldots}{2\left(a+b\right)^{3}a\left(a+\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^3/(a+b*sin(x)^2)^2,x)
```

```
[Out] -1/4/(a+b)^2/(-1+sin(x))-1/4/(a+b)^3*ln(-1+sin(x))*a-5/4/(a+b)^3*ln(-1+sin(
x))*b+1/2/(a+b)^3*b^2*sin(x)/(a+b*sin(x)^2)+1/2/(a+b)^3*b^3/a*sin(x)/(a+b*s
in(x)^2)+5/2/(a+b)^3*b^2/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))+1/2/(a+b)
^3*b^3/a/(a*b)^(1/2)*arctan(sin(x)*b/(a*b)^(1/2))-1/4/(a+b)^2/(1+sin(x))+1/
4/(a+b)^3*ln(1+sin(x))*a+5/4/(a+b)^3*ln(1+sin(x))*b
```

maxima [B] time = 0.65, size = 220, normalized size = 2.02

$$\frac{(a+5\,b)\log\left(\sin(x)+1\right)}{4\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}-\frac{(a+5\,b)\log\left(\sin(x)-1\right)}{4\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)}+\frac{\left(5\,ab^{2}+b^{3}\right)\arctan\left(\frac{b\sin(x)}{\sqrt{ab}}\right)}{2\left(a^{4}+3\,a^{3}b+3\,a^{2}b^{2}+ab^{3}\right)\sqrt{ab}}-\frac{\ldots}{2\left(\left(a^{3}b+2\,a^{2}b^{2}+ab\ldots\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^3/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/4*(a + 5*b)*log(sin(x) + 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 1/4*(a + 5*
b)*log(sin(x) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + 1/2*(5*a*b^2 + b^3)*ar
```

```
ctan(b*sin(x)/sqrt(a*b))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt(a*b)) -
1/2*((a*b - b^2)*sin(x)^3 + (a^2 + b^2)*sin(x))/((a^3*b + 2*a^2*b^2 + a*b^3
)*sin(x)^4 - a^4 - 2*a^3*b - a^2*b^2 + (a^4 + a^3*b - a^2*b^2 - a*b^3)*sin(
x)^2)
```

mupad [B] time = 15.06, size = 2009, normalized size = 18.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^3*(a + b*sin(x)^2)^2),x)
```

```
[Out] (log(sin(x) + 1)*(a + 5*b))/(4*(a + b)^3) - log(sin(x) - 1)*(b/(a + b)^3 +
1/(4*(a + b)^2)) - ((sin(x)*(a^2 + b^2))/(2*a*(2*a*b + a^2 + b^2)) + (b*sin
(x)^3*(a - b))/(2*a*(2*a*b + a^2 + b^2)))/(b*sin(x)^4 - a + sin(x)^2*(a - b
)) - (atan(((((sin(x)*(10*a*b^6 + b^7 + 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))
/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)) + ((5*a + b)*(-a^3*b
^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6
+ 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b
^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) - (sin(x)*(5*a + b)*
(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a
^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^
3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a
^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3*b^3)^(1/2)*1i)/(4*(3*a^
5*b + a^6 + a^3*b^3 + 3*a^4*b^2)) + (((sin(x)*(10*a*b^6 + b^7 + 50*a^2*b^5
+ 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^
2)) - ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80*a^3*b^8 + 17
2*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2
)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b
^2) + (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4
*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8
*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3
 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3
*b^3)^(1/2)*1i)/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))/(((21*a*b^6)/4 +
 (5*b^7)/4 - (21*a^2*b^5)/4 - (5*a^3*b^4)/4)/(6*a^7*b + a^8 + a^2*b^6 + 6*a
^3*b^5 + 15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) + (((sin(x)*(10*a*b^6 + b^7
+ 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^6 + a^2*b^4 + 4*a^3*b
^3 + 6*a^4*b^2)) + ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b^10 + 20*a^2*b^9 + 80
*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 80*a^7*b^4 + 20*a^8*b^
3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 + 15*a^4*b^4 + 20*a^5*b
^3 + 15*a^6*b^2) - (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*(16*a^2*b^9 + 80*a^3*
b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*b^4 - 80*a^8*b^3 - 16
*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4*a^5*b + a^6 + a^2*b^
4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)))*(5
*a + b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)) - (((si
n(x)*(10*a*b^6 + b^7 + 50*a^2*b^5 + 10*a^3*b^4 + a^4*b^3))/(2*(4*a^5*b + a^
6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2)) - ((5*a + b)*(-a^3*b^3)^(1/2)*((2*a*b
^10 + 20*a^2*b^9 + 80*a^3*b^8 + 172*a^4*b^7 + 220*a^5*b^6 + 172*a^6*b^5 + 8
0*a^7*b^4 + 20*a^8*b^3 + 2*a^9*b^2)/(6*a^7*b + a^8 + a^2*b^6 + 6*a^3*b^5 +
15*a^4*b^4 + 20*a^5*b^3 + 15*a^6*b^2) + (sin(x)*(5*a + b)*(-a^3*b^3)^(1/2)*
(16*a^2*b^9 + 80*a^3*b^8 + 144*a^4*b^7 + 80*a^5*b^6 - 80*a^6*b^5 - 144*a^7*
b^4 - 80*a^8*b^3 - 16*a^9*b^2))/(8*(3*a^5*b + a^6 + a^3*b^3 + 3*a^4*b^2)*(4
*a^5*b + a^6 + a^2*b^4 + 4*a^3*b^3 + 6*a^4*b^2))))/(4*(3*a^5*b + a^6 + a^3*
b^3 + 3*a^4*b^2)))*(5*a + b)*(-a^3*b^3)^(1/2))/(4*(3*a^5*b + a^6 + a^3*b^3
+ 3*a^4*b^2))))*(5*a + b)*(-a^3*b^3)^(1/2)*1i)/(2*(3*a^5*b + a^6 + a^3*b^3
+ 3*a^4*b^2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**3/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.323 $\int \frac{\sec^4(x)}{(a+b\sin^2(x))^2}\,dx$

Optimal. Leaf size=96

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

```
[Out] 1/2*b^2*(6*a+b)*arctan((a+b)^(1/2)*tan(x)/a^(1/2))/a^(3/2)/(a+b)^(7/2)+(a+3
*b)*tan(x)/(a+b)^3+1/3*tan(x)^3/(a+b)^2+1/2*b^3*tan(x)/a/(a+b)^3/(a+(a+b)*t
an(x)^2)
```

Rubi [A] time = 0.15, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3191, 390, 385, 205}

$$\frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{b^3\tan(x)}{2a(a+b)^3\left((a+b)\tan^2(x)+a\right)}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{(a+3b)\tan(x)}{(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] (b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(2*a^(3/2)*(a + b)^(7/
2)) + ((a + 3*b)*Tan[x])/(a + b)^3 + Tan[x]^3/(3*(a + b)^2) + (b^3*Tan[x])/
(2*a*(a + b)^3*(a + (a + b)*Tan[x]^2))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 390

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Int[PolynomialDivide[(a + b*x^n)^p, (c + d*x^n)^(-q), x], x] /; FreeQ[{a
, b, c, d}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && IGtQ[p, 0] && ILtQ[q,
0] && GeQ[p, -q]
```

Rule 3191

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + (a + b)*ff^2*x^2)^p/(1 + ff^2*x^2)^(m/2 + p + 1), x], x, Tan[e
+ f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2] && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(x)}{\left(a+b\sin^2(x)\right)^2}\,dx = \text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{a+3b}{(a+b)^3}+\frac{x^2}{(a+b)^2}+\frac{b^2(3a+b)+3b^2(a+b)x^2}{(a+b)^3\left(a+(a+b)x^2\right)^2}\right)dx, x, \tan(x)\right)$$
$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{\text{Subst}\left(\int \frac{b^2(3a+b)+3b^2(a+b)x^2}{\left(a+(a+b)x^2\right)^2}\,dx, x, \tan(x)\right)}{(a+b)^3}$$
$$= \frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\tan^2(x)\right)}+\frac{\left(b^2(6a+b)\right)\text{Subst}\left(\int\right.}{2a(a}$$
$$= \frac{b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{2a^{3/2}(a+b)^{7/2}}+\frac{(a+3b)\tan(x)}{(a+b)^3}+\frac{\tan^3(x)}{3(a+b)^2}+\frac{b^3\tan(x)}{2a(a+b)^3\left(a+(a+b)\,\text{ta}\right.}$$

Mathematica [A] time = 0.96, size = 97, normalized size = 1.01

$$\frac{1}{6}\left(\frac{3b^2(6a+b)\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(x)}{\sqrt{a}}\right)}{a^{3/2}(a+b)^{7/2}}+\frac{\frac{3b^3\sin(2x)}{a(2a-b\cos(2x)+b)}+2(a+b)\tan(x)\sec^2(x)+4a\tan(x)+16b\tan(x)}{(a+b)^3}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[x]^4/(a + b*Sin[x]^2)^2,x]
```

```
[Out] ((3*b^2*(6*a + b)*ArcTan[(Sqrt[a + b]*Tan[x])/Sqrt[a]])/(a^(3/2)*(a + b)^(7
/2)) + ((3*b^3*Sin[2*x])/(a*(2*a + b - b*Cos[2*x])) + 4*a*Tan[x] + 16*b*Tan
[x] + 2*(a + b)*Sec[x]^2*Tan[x])/(a + b)^3)/6
```

fricas [B] time = 0.50, size = 653, normalized size = 6.80

$$\left[-\frac{3\left(\left(6\,ab^3+b^4\right)\cos(x)^5-\left(6\,a^2b^2+7\,ab^3+b^4\right)\cos(x)^3\right)\sqrt{-a^2-ab}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(x)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(x)^2+4}{b^2\cos(x)^4-2\left(ab\right.}\right.}{24\left(\left(a^6b+4\,a^5b^2+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="fricas")
```

```
[Out] [-1/24*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*b^3 + b^4)*cos(x)^3)
*sqrt(-a^2 - a*b)*log(((8*a^2 + 8*a*b + b^2)*cos(x)^4 - 2*(4*a^2 + 5*a*b +
b^2)*cos(x)^2 + 4*((2*a + b)*cos(x)^3 - (a + b)*cos(x))*sqrt(-a^2 - a*b)*si
n(x) + a^2 + 2*a*b + b^2)/(b^2*cos(x)^4 - 2*(a*b + b^2)*cos(x)^2 + a^2 + 2*
a*b + b^2)) + 4*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2*b^3 - (4*a^4*b + 20*a^
3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 + 11*a^4*b + 16*a^3*b^2 +
 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 +
 a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 +
 a^2*b^5)*cos(x)^3), -1/12*(3*((6*a*b^3 + b^4)*cos(x)^5 - (6*a^2*b^2 + 7*a*
b^3 + b^4)*cos(x)^3)*sqrt(a^2 + a*b)*arctan(1/2*((2*a + b)*cos(x)^2 - a - b
)/(sqrt(a^2 + a*b)*cos(x)*sin(x))) + 2*(2*a^5 + 6*a^4*b + 6*a^3*b^2 + 2*a^2
*b^3 - (4*a^4*b + 20*a^3*b^2 + 13*a^2*b^3 - 3*a*b^4)*cos(x)^4 + 2*(2*a^5 +
11*a^4*b + 16*a^3*b^2 + 7*a^2*b^3)*cos(x)^2)*sin(x))/((a^6*b + 4*a^5*b^2 +
```

```
6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*cos(x)^5 - (a^7 + 5*a^6*b + 10*a^5*b^2 + 1
0*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)*cos(x)^3)]
```

giac [B] time = 0.14, size = 270, normalized size = 2.81

$$\frac{b^3\tan(x)}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\left(a\tan(x)^2+b\tan(x)^2+a\right)}+\frac{\left(6ab^2+b^3\right)\left(\pi\left\lfloor\frac{x}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan}{}\right.\right.}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\sqrt{a^2+ab}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="giac")
```

```
[Out] 1/2*b^3*tan(x)/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*(a*tan(x)^2 + b*tan(x)^
2 + a)) + 1/2*(6*a*b^2 + b^3)*(pi*floor(x/pi + 1/2)*sgn(2*a + 2*b) + arctan
((a*tan(x) + b*tan(x))/sqrt(a^2 + a*b)))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^
3)*sqrt(a^2 + a*b)) + 1/3*(a^4*tan(x)^3 + 4*a^3*b*tan(x)^3 + 6*a^2*b^2*tan(
x)^3 + 4*a*b^3*tan(x)^3 + b^4*tan(x)^3 + 3*a^4*tan(x) + 18*a^3*b*tan(x) + 3
6*a^2*b^2*tan(x) + 30*a*b^3*tan(x) + 9*b^4*tan(x))/(a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)
```

maple [B] time = 0.27, size = 193, normalized size = 2.01

$$\frac{\left(\tan^3(x)\right)a}{3\left(a^2+2ab+b^2\right)(a+b)}+\frac{\left(\tan^3(x)\right)b}{3\left(a^2+2ab+b^2\right)(a+b)}+\frac{\tan(x)a}{\left(a^2+2ab+b^2\right)(a+b)}+\frac{3\tan(x)b}{\left(a^2+2ab+b^2\right)(a+b)}+\frac{}{2(a+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(x)^4/(a+b*sin(x)^2)^2,x)
```

```
[Out] 1/3/(a^2+2*a*b+b^2)/(a+b)*tan(x)^3*a+1/3/(a^2+2*a*b+b^2)/(a+b)*tan(x)^3*b+1
/(a^2+2*a*b+b^2)/(a+b)*tan(x)*a+3/(a^2+2*a*b+b^2)/(a+b)*tan(x)*b+1/2*b^3/(a
+b)^3/a*tan(x)/(tan(x)^2*a+tan(x)^2*b+a)+3*b^2/(a+b)^3/(a*(a+b))^(1/2)*arct
an((a+b)*tan(x)/(a*(a+b))^(1/2))+1/2*b^3/(a+b)^3/a/(a*(a+b))^(1/2)*arctan((
a+b)*tan(x)/(a*(a+b))^(1/2))
```

maxima [B] time = 0.70, size = 170, normalized size = 1.77

$$\frac{b^3\tan(x)}{2\left(a^5+3a^4b+3a^3b^2+a^2b^3+\left(a^5+4a^4b+6a^3b^2+4a^2b^3+ab^4\right)\tan(x)^2\right)}+\frac{\left(6ab^2+b^3\right)\arctan\left(\frac{(a+b)\tan}{\sqrt{(a+b)}}\right.}{2\left(a^4+3a^3b+3a^2b^2+ab^3\right)\sqrt{(}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)^4/(a+b*sin(x)^2)^2,x, algorithm="maxima")
```

```
[Out] 1/2*b^3*tan(x)/(a^5 + 3*a^4*b + 3*a^3*b^2 + a^2*b^3 + (a^5 + 4*a^4*b + 6*a^
3*b^2 + 4*a^2*b^3 + a*b^4)*tan(x)^2) + 1/2*(6*a*b^2 + b^3)*arctan((a + b)*t
an(x)/sqrt((a + b)*a))/((a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*sqrt((a + b)*a)
) + 1/3*((a + b)*tan(x)^3 + 3*(a + 3*b)*tan(x))/(a^3 + 3*a^2*b + 3*a*b^2 +
b^3)
```

mupad [B] time = 14.30, size = 176, normalized size = 1.83

$$\frac{\tan(x)^3}{3(a+b)^2}-\tan(x)\left(\frac{2a}{(a+b)^3}-\frac{3}{(a+b)^2}\right)+\frac{b^3\tan(x)}{2a\left(\tan(x)^2\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)+ab^3+3a^3b+a^4\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(x)^4*(a + b*sin(x)^2)^2),x)
```

```
[Out] tan(x)^3/(3*(a + b)^2) - tan(x)*((2*a)/(a + b)^3 - 3/(a + b)^2) + (b^3*tan(
x))/(2*a*(tan(x)^2*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b^2) + a*b^3 + 3*
a^3*b + a^4 + 3*a^2*b^2)) + (b^2*atan((b^2*tan(x)*(6*a + b)*(2*a + 2*b)*(3*
a*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2)*(6*a*b^2 + b^3)))*(6
*a + b))/(2*a^(3/2)*(a + b)^(7/2))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(x)**4/(a+b*sin(x)**2)**2,x)
```

```
[Out] Timed out
```

3.324 $\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=117

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}$$

```
[Out] 1/8*a*(a+4*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/
f-1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+1/8*(a+4*b)*sin(f*x+e)*(a+b*s
in(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3190, 388, 195, 217, 206}

$$\frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf}+\frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(a + 4*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8
*b^(3/2)*f) + ((a + 4*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*b*f) -
 (Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(4*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
```

```
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\sqrt{a+bx^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4bf} + \frac{(a+4b)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,\right.}{4bf}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+\right.}{4bf}$$

$$= \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8bf} - \frac{\sin(e+fx)\left(a+b\sin^2(e+\right.}{4bf}$$

$$= \frac{a(a+4b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8b^{3/2}f} + \frac{(a+4b)\sin(e+fx)\sqrt{a+b\sin^2(e}}{8bf}$$

Mathematica [A] time = 0.44, size = 125, normalized size = 1.07

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(\sqrt{a}\,(a+4b)\sinh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a}}\right)-\sqrt{b}\,\sin(e+fx)\left(a+2b\sin^2(e+fx)-4b\right)\sqrt{\frac{b\sin^2(e+fx)}{a}}\right.}{8b^{3/2}f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(Sqrt[a]*(a + 4*b)*ArcSinh[(Sqrt[b]*Sin[e + f*x
])/Sqrt[a]] - Sqrt[b]*Sin[e + f*x]*(a - 4*b + 2*b*Sin[e + f*x]^2)*Sqrt[1 +
(b*Sin[e + f*x]^2)/a]))/(8*b^(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

fricas [A] time = 0.81, size = 511, normalized size = 4.37

$$\left[\frac{\left(a^2+4\,ab\right)\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/64*((a^2 + 4*a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
```

```
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) + 8*(2*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^
2 + a + b)*sin(f*x + e))/(b^2*f), -1/32*((a^2 + 4*a*b)*sqrt(-b)*arctan(1/4*
(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^
2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b
+ 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*(2
*b^2*cos(f*x + e)^2 - a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(2*(-8*b^2/64/b^2*sin(f*x+exp(1))*sin(f*x+exp(1))-(-16*b^2+4*b*
a)/64/b^2)*sin(f*x+exp(1))*sqrt(a+b*sin(f*x+exp(1))^2)+2*(-a^2-4*a*b)/16/b/
sqrt(b)*ln(abs(sqrt(a+b*sin(f*x+exp(1))^2)-sqrt(b)*sin(f*x+exp(1)))))
```

maple [A] time = 1.54, size = 155, normalized size = 1.32

$$-\frac{\left(\sin^3\left(fx+e\right)\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{4f}-\frac{a\sin\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{8fb}+\frac{a^2\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a}\right.}{8f\,b^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/4/f*sin(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2)-1/8/f*a/b*sin(f*x+e)*(a+b*sin(
f*x+e)^2)^(1/2)+1/8/f/b^(3/2)*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^
(1/2))+1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/2/f*a*ln(sin(f*x+e)*b^(1
/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

maxima [A] time = 0.64, size = 119, normalized size = 1.02

$$\frac{\frac{a^2\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{4\,a\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}}+4\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)-\frac{2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)}{b}+\frac{\sqrt{b\sin}}{}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/8*(a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 4*a*arcsinh(b*sin(f*x
+ e)/sqrt(a*b))/sqrt(b) + 4*sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e) - 2*(b*
sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e)/b + sqrt(b*sin(f*x + e)^2 + a)*a*sin
(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.325 $\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=72

$$\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f}$$

```
[Out] 1/2*a*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+1/2*si
n(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.05, antiderivative size = 72, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3190, 195, 217, 206}

$$\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[b]*f
) + (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \sqrt{a+bx^2}\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{2f}$$
$$= \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx,x,\frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$
$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{b}f} + \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Mathematica [A] time = 0.26, size = 96, normalized size = 1.33

$$\frac{a^{3/2}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{2\sqrt{b}f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[b]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2) + a^(3/2)*ArcSinh[(Sqrt[b]*Sin
[e + f*x])/Sqrt[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(2*Sqrt[b]*f*Sqrt[a + b
*Sin[e + f*x]^2])
```

fricas [B] time = 0.53, size = 453, normalized size = 6.29

$$\left[\frac{a\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(a*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x +
 e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b
+ 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 1
6*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f
*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24
*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))
 + 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f), -1/8*(a*sqrt(-b
)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 +
 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x +
e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x
+ e))) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(1/2*sin(f*x+exp(1))*sqrt(a+b*sin(f*x+exp(1))^2)-2*a/4/sqrt(b)*
ln(abs(sqrt(a+b*sin(f*x+exp(1))^2)-sqrt(b)*sin(f*x+exp(1)))))
```

maple [A] time = 0.17, size = 62, normalized size = 0.86

$$\frac{\sin\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{2f}+\frac{a\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{2f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+1/2/f*a*ln(sin(f*x+e)*b^(1/2)+(a+
b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

maxima [A] time = 0.34, size = 46, normalized size = 0.64

$$\frac{\frac{a\,\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}}+\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + sqrt(b*sin(f*x + e)^2 +
a)*sin(f*x + e))/f
```

mupad [B] time = 14.48, size = 61, normalized size = 0.85

$$\frac{\sin\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}{2\,f}+\frac{a\,\ln\left(\sqrt{b}\,\sin\left(e+f\,x\right)+\sqrt{b\sin\left(e+f\,x\right)^2+a}\right)}{2\,\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2))/(2*f) + (a*log(b^(1/2)*sin(e +
f*x) + (a + b*sin(e + f*x)^2)^(1/2)))/(2*b^(1/2)*f)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a+b\sin^2\left(e+fx\right)}\,\cos\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cos(e + f*x), x)
```

3.326 $\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

```
[Out] -arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f+arctanh(sin
(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3190, 402, 217, 206, 377}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[b]*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f) +
 (Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 402

```
Int[((a_) + (b_.)*(x_)^2)^(p_.)/((c_) + (d_.)*(x_)^2), x_Symbol] :> Dist[b/
d, Int[(a + b*x^2)^(p - 1), x], x] - Dist[(b*c - a*d)/d, Int[(a + b*x^2)^(p
 - 1)/(c + d*x^2), x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0] &&
 GtQ[p, 0] && (EqQ[p, 1/2] || EqQ[Denominator[p], 4])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+bx}}\right.}{f}$$

$$= -\frac{b\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\right.}{f}$$

$$= -\frac{\sqrt{b}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f} + \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

Mathematica [A] time = 0.27, size = 129, normalized size = 1.57

$$\frac{\frac{\sqrt{a}\sqrt{-b}\sin^{-1}\left(\frac{\sqrt{-b}\sin(e+fx)}{\sqrt{a}}\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}}{\sqrt{2a-b\cos(2(e+fx))+b}} + \sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{2a+2b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*
(e + f*x)]]] + (Sqrt[a]*Sqrt[-b]*ArcSin[(Sqrt[-b]*Sin[e + f*x])/Sqrt[a]]*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]])/f
```

fricas [B] time = 0.68, size = 1246, normalized size = 15.20

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)
^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 1
60*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b
^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x
+ e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^
3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) +
2*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b +
2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/co
s(f*x + e)^4))/f, -1/8*(4*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(b)*log(128*b^4*cos(
f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3
+ 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b
^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3
*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a
*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)))/f, 1/4*(sqrt(-b)*arctan(1/4*(8*
```

```
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + sqrt(a
+ b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4))/f, -1/4*(2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e
)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a} \sec\left(fx+e\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e), x)
```

maple [B] time = 4.14, size = 155, normalized size = 1.89

$$-\frac{\sqrt{b}\ \ln\left(\frac{\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\ \sqrt{b}+b\sin\left(fx+e\right)}{\sqrt{b}}\right)}{f}+\frac{\sqrt{a+b}\ \ln\left(\frac{2\sqrt{a+b}\ \sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)}{2f}-\frac{\sqrt{a+b}\ \ln\left(\frac{2\sqrt{a+}\cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*b^(1/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))+
1/2/f*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))-1/2/f*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

maxima [A] time = 0.49, size = 126, normalized size = 1.54

$$-\frac{2\sqrt{b}\ \operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)-\sqrt{a+b}\ \operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)-\sqrt{a+b}\ \operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-\cdots\right.}\right.}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(2*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) - sqrt(a + b)*arcsinh(b*s
in(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1
))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) -
a/(sqrt(a*b)*(sin(f*x + e) - 1))))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b \sin\left(e+f\,x\right)^2 + a}}{\cos\left(e+f\,x\right)} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,\sec\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x), x)
```

3.327 $\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=82

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f\sqrt{a+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

```
[Out] 1/2*a*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/2
)+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.09, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 378, 377, 206}

$$\frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f\sqrt{a+b}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*Sqrt[
a + b]*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+}}\right.}{2}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f} + \frac{a\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\right.}{2}$$

$$= \frac{a\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2\sqrt{a+b}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+}{2f}$$

Mathematica [**A**] time = 2.17, size = 164, normalized size = 2.00

$$\frac{\sin(e+fx)\left(\sqrt{2}a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\tanh^{-1}\left(\frac{\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}}{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}\right)+\sec^2(e+fx)\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}(2a-b\cos(2(e+\right.}{4f\sqrt{\frac{(a+b)\sin^2(e+fx)}{a}}\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sin[e + f*x]*(Sqrt[2]*a*ArcTanh[Sqrt[((a + b)*Sin[e + f*x]^2)/a]/Sqrt[1 +
(b*Sin[e + f*x]^2)/a]]*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a] + (2*a + b -
b*Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[((a + b)*Sin[e + f*x]^2)/a]))/(4*f*
Sqrt[((a + b)*Sin[e + f*x]^2)/a]*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [**B**] time = 0.53, size = 337, normalized size = 4.11

$$\left[\frac{\sqrt{a+b}\,a\cos(fx+e)^2\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2}}{\cos(fx+e)^4}\right.}{8(a+b)f\cos(fx+e)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*(sqrt(a + b)*a*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^
4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 -
2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*(a +
 b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^2), -1/4*(a*sqrt(-a - b)*arctan(1
/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))
)*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/
((a + b)*f*cos(f*x + e)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

maple [B] time = 4.43, size = 291, normalized size = 3.55

$$\frac{2\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\sqrt{a+b}\,b\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+2\left(a+b-b\left(\cos^{2}\left(fx+e\right)\right)\right)^{\frac{3}{2}}\sqrt{a+b}\,\sin\left(\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+2*(
a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*sin(f*x+e)-a*(ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a+ln(2/(1+sin(f*x+e))
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b-ln(2/(sin(f*x+e
)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a-ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b)*cos(f*
x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^{2}+a}}{\cos\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^3, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{2}\left(e+fx\right)}\,\sec^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**3, x)
```

3.328 $\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=143

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f(a+b)}+\frac{(3a+4b)\tan(e+fx)\mathrm{se}}{8}$$

```
[Out] 1/8*a*(3*a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a
+b)^(3/2)/f+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/(a+b)/f+1/
8*(3*a+4*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 382, 378, 377, 206}

$$\frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f(a+b)}+\frac{(3a+4b)\tan(e+fx)\mathrm{se}}{8}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*(3*a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2
]])/(8*(a + b)^(3/2)*f) + ((3*a + 4*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]
^2]*Tan[e + f*x])/(8*(a + b)*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3
/2)*Tan[e + f*x])/(4*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4(a+b)f} + \frac{(3a+4b)\,\text{Subst}\left(\int \frac{}{(}\right.}{4(}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e+fx)\,(}{}$$

$$= \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{8(a+b)f} + \frac{\sec^3(e+fx)\,(}{}$$

$$= \frac{a(3a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8(a+b)^{3/2}f} + \frac{(3a+4b)\sec(e+fx)\sqrt{a+b\sin^2}}{8(a+b)f}$$

Mathematica [C] time = 13.30, size = 669, normalized size = 4.68

$$-\frac{\tan(e+fx)\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)\left(10b\sin^2(e+fx)\sqrt{-\frac{(a+b)\tan^2(e+fx)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)}{a^2}}+15a\sqrt{-\frac{(a+}{}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/40*(Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(-15*a*ArcSin
[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] - 10*b*ArcSin[Sqrt[-(((a + b)*Tan[e +
 f*x]^2)/a)]]*Sin[e + f*x]^2 - 30*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) - 20*b*Sin[e + f*x]^2*Sqrt[(
Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(
3/2) - 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqr
t[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a)
)^(5/2) - 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*
Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*
Tan[e + f*x]^2)/a))^(5/2) + 32*a*Hypergeometric2F1[2, 4, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(7/2) + 32*b*Hypergeometric2F1[2, 4, 7/2, -(((a + b)
*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 15*a*Sqrt[-(((a + b)*Sec[e
 + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + 10*b*Sin[e + f*x]^
2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2
```

)]))/(f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]
^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))

fricas [A] time = 0.86, size = 443, normalized size = 3.10

$$\left[\frac{\left(3\,a^2+4\,ab\right)\sqrt{a+b}\,\cos\left(fx+e\right)^4\log\left(\frac{\left(a^2+8\,ab+8\,b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3\,ab+2\,b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2\,b\right)\cos\left(fx+e\right)^2-2\,a-2\,b\right)\sqrt{\cdots}}{\cos\left(fx+e\right)^4}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")

[Out] [1/32*((3*a^2 + 4*a*b)*sqrt(a + b)*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2
)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*co
s(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f
*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + 5*a*b + 2*b
^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/16*((3*a^2 + 4*a*
b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*
a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^4 - 2*((3*a^2 + 5*a*b + 2*b^2)*cos(f
*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
+ e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^5\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)

maple [B] time = 4.85, size = 570, normalized size = 3.99

$$\frac{2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,\left(a+b\right)^{\frac{3}{2}}b\left(3a+4b\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+2\left(a+b-b\left(\cos^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)

[Out] 1/16*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*b*(3*a+4*b)*sin(f*x+e)*cos(f
x+e)^4+2(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*(3*a+4*b)*cos(f*x+e)^2*sin
(f*x+e)+4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(5/2)*sin(f*x+e)+a*(3*ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+10*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^2*b+11*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin
(f*x+e)+a))*a*b^2+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))*b^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3-10*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b-11*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^2-4*ln(2/(1+sin(f*x+e

```
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)
^4)/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b+b^2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\,\sec\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^{2}+a}}{\cos\left(e+f\,x\right)^{5}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.329 $\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=220

$$-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a+b)(a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\text{si}}{}$$

```
[Out] -1/5*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f+2/15*(a+3*b)*cos(f*
x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-1/15*(2*a^2+7*a*b-3*b^2)*(cos(
f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+
e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/15*a*(a+b)*(a+3*b)*(cos(f*x+
e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2
/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 260, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(2a^2+7ab-3b^2\right)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{15b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{2a(a+b)(a+}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f)
 - (Cos[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(5*b*f) - ((2*a
^2 + 7*a*b - 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + (2*a*(a + b)*(a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSi
n[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b
^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}\,dx,x,\text{si}\right.}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{5bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\ \text{s}\right.}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx}{}$$

$$= \frac{2(a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15bf} - \frac{\cos(e+fx}{}$$

Mathematica [A] time = 1.38, size = 199, normalized size = 0.90

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(8a^2-4b(4a-3b)\cos(2(e+fx))-32ab+3b^2\cos(4(e+fx))-15b^2\right)+32a\left(a^2+4ab+\right.}{240b^2f\sqrt{2a-b\cos(2(e}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-16*a*(2*a^2 + 7*a*b - 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 32*a*(a^2 + 4*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] - Sqrt[2]*b*(8*a^2 - 32*a*b - 15*
b^2 - 4*(4*a - 3*b)*b*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

maple [A] time = 1.64, size = 432, normalized size = 1.96

$$\frac{-3b^3\sin\left(fx+e\right)\left(\cos^6\left(fx+e\right)\right)+4a\,b^2\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(-a^2b+2a\,b^2+3b^3\right)\left(\cos^2\left(fx+e\right)\right)\sin}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)*cos(f*x+e)^6+4*a*b^2*sin(f*x+e)*cos(f*x+e)^4+(-a^2*
b+2*a*b^2+3*b^3)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f
*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*(cos(f*x+
e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)
^(1/2))*a^2*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elli
pticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-7*(cos(f*x+e)
^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b+3*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/
2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\cos\left(e+f\,x\right)^4\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.330 $\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=159

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3bf\sqrt{a+b\sin^2(e+fx)}}-\frac{(a-b)\sqrt{a+b\sin^2(e+}}{3bf\sqrt{\frac{b\sin^2(}{a}}}$$

```
[Out] 1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(a-b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(
1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*
x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b
*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 199, normalized size of antiderivative = 1.25, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {3192, 417, 524, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin\right.}{3bf\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((a - b)*Sqr
t[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt
[a + b*Sin[e + f*x]^2])/(3*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)
*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*
Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 417

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(x*(a + b*x^n)^p*(c + d*x^n)^q)/(n*(p + q) + 1), x] + Dist[n/(n*(p
+ q) + 1), Int[(a + b*x^n)^(p - 1)*(c + d*x^n)^(q - 1)*Simp[a*c*(p + q) + (
q*(b*c - a*d) + a*d*(p + q))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] &
& NeQ[b*c - a*d, 0] && GtQ[q, 0] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, n
, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x^2}\,\sqrt{a+bx^2}\,dx, x, \sin(e+\right.}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2(e+fx)}\,\text{s}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\left((a-b)\sqrt{\cos^2(e+fx)}\,\text{s}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(a-b)\sqrt{\cos^2(e+fx)}\,E}{}$$

Mathematica [A] time = 0.86, size = 158, normalized size = 0.99

$$\frac{b\sin(2(e+fx))(2a-b\cos(2(e+fx))+b)+2\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a(a-b)\sqrt{\frac{2a-b}{}}}{6\sqrt{2}\,bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
f*x, -(b/a)] + 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + b*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x
)])/(6*Sqrt[2]*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

maple [A] time = 1.64, size = 265, normalized size = 1.67

$$-b^{2}\left(\sin^{5}\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\operatorname{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}+a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(-b^2*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-(cos(f*x+e)^2)^(1/2)
*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+(cos
(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)
^(1/2))*a*b-a*b*sin(f*x+e)^3+b^2*sin(f*x+e)^3+a*b*sin(f*x+e))/b/cos(f*x+e)/
(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^{2}+a}\,\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{2}\,\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.331 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 0.70, size = 71, normalized size = 1.39

$$\frac{a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.332 $\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=131

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(cos(f*x+e)^2)^(1/2)/cos(f
*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*
sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.17, antiderivative size = 171, normalized size of antiderivative = 1.31, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 412, 12, 493, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\left(1-x^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a}\right.}{}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.47, size = 134, normalized size = 1.02

$$\frac{\sqrt{2}\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+f\right.}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt
[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

maple [A] time = 2.30, size = 294, normalized size = 2.24

$$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-a*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\sec^2\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**2, x)
```

3.333 $\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=196

$$\frac{(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{2a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+\right.}{3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -1/3*(2*a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+2/3*a*(cos
(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x
+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a+b)*(a+b*sin(f*x+e)^2)^(1
/2)*tan(f*x+e)/(a+b)/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)
/f
```

Rubi [A] time = 0.22, antiderivative size = 236, normalized size of antiderivative = 1.20, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2a\sqrt{\cos^2(e+fx)}\sec(e}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x
]^2)/a]) + (2*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + ((2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f) +
(Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{(1-x^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e-\right.}{}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= \frac{(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e}}{3f}$$

$$= -\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 1.83, size = 187, normalized size = 0.95

$$\frac{\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(8a^2-4b^2\right)\cos(2(e+fx))+(2a+b)(8a-b\cos(4(e+fx))+5b)\right)}{2\sqrt{2}} + 4a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right) - 2a(2}{6f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + 4*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + (((8*a^2 - 4*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(8*a + 5*b - b
*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt[2]))/(6*(a + b)*f*
Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

maple [A] time = 2.58, size = 368, normalized size = 1.88

$$\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b\left(2a+b\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos(f*
x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*si
n(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x
+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^
(1/2)*(cos(f*x+e)^2)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+2*El
lipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*a-EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+b*sin(f*x+e)^
2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)/(sin(f*x+e)-1)/(1+sin(f*x+e))
/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b \sin\left(fx+e\right)^2 + a}\, \sec\left(fx+e\right)^4\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{b\sin\left(e+f\,x\right)^2+a}}{\cos\left(e+f\,x\right)^4}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(1/2)/cos(e + f*x)^4, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\, \sec^4\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*sec(e + f*x)**4, x)
```

3.334 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=157

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{24bf}$$

```
[Out] 1/16*a^2*(a+6*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/
2)/f+1/24*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/b/f-1/6*sin(f*x+e)*(a
+b*sin(f*x+e)^2)^(5/2)/b/f+1/16*a*(a+6*b)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/
2)/b/f
```

Rubi [A] time = 0.14, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3190, 388, 195, 217, 206}

$$\frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f}-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf}+\frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)}{24bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(a + 6*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(16*b^(3/2)*f) + (a*(a + 6*b)*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(16*
b*f) + ((a + 6*b)*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/(24*b*f) - (Si
n[e + f*x]*(a + b*Sin[e + f*x]^2)^(5/2))/(6*b*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)^{3/2}dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6bf} + \frac{(a+6b)\,\text{Subst}\left(\int \left(a+bx^2\right.\right.}{6bf}$$

$$= \frac{(a+6b)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24bf} - \frac{\sin(e+fx)\left(a+b\sin\right.}{6bf}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+fx)}{2}$$

$$= \frac{a(a+6b)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{16bf} + \frac{(a+6b)\sin(e+fx)}{2}$$

$$= \frac{a^2(a+6b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16b^{3/2}f} + \frac{a(a+6b)\sin(e+fx)\sqrt{a+}}{16bf}$$

Mathematica [A] time = 0.81, size = 149, normalized size = 0.95

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(3a^{3/2}(a+6b)\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)+\sqrt{b}\sin(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\left(-2b(7a-6b)\sin^2\right.\right.}{48b^{3/2}f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(3*a^(3/2)*(a + 6*b)*ArcSinh[(Sqrt[b]*Sin[e + f
*x])/Sqrt[a]] + Sqrt[b]*Sin[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a]*(-3*a*(
a - 10*b) - 2*(7*a - 6*b)*b*Sin[e + f*x]^2 - 8*b^2*Sin[e + f*x]^4)))/(48*b^
(3/2)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

fricas [A] time = 2.13, size = 577, normalized size = 3.68

$$\left[\frac{3\left(a^3+6a^2b\right)\sqrt{b}\log\left(128b^4\cos\left(fx+e\right)^8-256\left(ab^3+2b^4\right)\cos\left(fx+e\right)^6+32\left(5a^2b^2+24ab^3+24b^4\right)\cos\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/384*(3*(a^3 + 6*a^2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 +
 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4
+ a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b
^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b
^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*
b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt
(b)*sin(f*x + e)) - 8*(8*b^3*cos(f*x + e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 -
2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x
 + e))/(b^2*f), -1/192*(3*(a^3 + 6*a^2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(8*b^3*cos(f*x
+ e)^4 + 3*a^2*b - 16*a*b^2 - 4*b^3 - 2*(7*a*b^2 + 2*b^3)*cos(f*x + e)^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(2*((-192*b^5/2304/b^4*sin(f*x+exp(1))*sin(f*x+exp(1))+(288*b^5
-336*b^4*a)/2304/b^4)*sin(f*x+exp(1))*sin(f*x+exp(1))+(720*b^4*a-72*b^3*a^2
)/2304/b^4)*sin(f*x+exp(1))*sqrt(a+b*sin(f*x+exp(1))^2)+2*(-a^3-6*a^2*b)/32
/b/sqrt(b)*ln(abs(sqrt(a+b*sin(f*x+exp(1))^2)-sqrt(b)*sin(f*x+exp(1)))))
```

maple [B] time = 1.97, size = 277, normalized size = 1.76

$$-\frac{b\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\ \sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)}{6f}+\frac{7\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\ \left(\cos^2\left(fx+e\right)\right)\sin\left(f\right.}{24f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/6*b/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)*cos(f*x+e)^4+7/24/f*(a+b-b*c
os(f*x+e)^2)^(1/2)*cos(f*x+e)^2*sin(f*x+e)*a+1/12/f*(a+b-b*cos(f*x+e)^2)^(1
/2)*cos(f*x+e)^2*sin(f*x+e)*b-1/16/b/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e
)*a^2+1/3/f*(a+b-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)*a+1/12*b/f*(a+b-b*cos(f*x
+e)^2)^(1/2)*sin(f*x+e)+1/16/b^(3/2)/f*a^3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos
(f*x+e)^2)^(1/2))+3/8/b^(1/2)/f*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)
^2)^(1/2))
```

maxima [A] time = 0.37, size = 174, normalized size = 1.11

$$\frac{\frac{3\,a^3\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{18\,a^2\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}}+12\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)+18\sqrt{b\sin\left(fx+e\right)^2+a}\,a\sin\left(\right.}{48\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/48*(3*a^3*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 18*a^2*arcsinh(b*si
n(f*x + e)/sqrt(a*b))/sqrt(b) + 12*(b*sin(f*x + e)^2 + a)^(3/2)*sin(f*x + e
) + 18*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e) - 8*(b*sin(f*x + e)^2 + a)
```

```
^(5/2)*sin(f*x + e)/b + 2*(b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)/b + 3
*sqrt(b*sin(f*x + e)^2 + a)*a^2*sin(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.335 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=104

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

```
[Out] 1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a^2*arctanh(sin(f*x+e)*b^(1/2
)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(
1/2)/f
```

Rubi [A] time = 0.07, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 195, 217, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f}+\frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}+\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*Sqrt[
b]*f) + (3*a*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(8*f) + (Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(3/2))/(4*f)
```

Rule 195

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^p)/(n*p
 + 1), x] + Dist[(a*n*p)/(n*p + 1), Int[(a + b*x^n)^(p - 1), x], x] /; Free
Q[{a, b}, x] && IGtQ[n, 0] && GtQ[p, 0] && (IntegerQ[2*p] || (EqQ[n, 2] &&
IntegerQ[4*p]) || (EqQ[n, 2] && IntegerQ[3*p]) || LtQ[Denominator[p + 1/n],
 Denominator[p]])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \left(a+bx^2\right)^{3/2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f} + \frac{(3a)\,\text{Subst}\left(\int \sqrt{a+bx^2}\,dx, x\right.}{4f}$$
$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e+f\right.}{4f}$$
$$= \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f} + \frac{\sin(e+fx)\left(a+b\sin^2(e+f\right.}{4f}$$
$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{b}f} + \frac{3a\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8f}$$

Mathematica [A] time = 0.49, size = 93, normalized size = 0.89

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(\frac{3a^{3/2}\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{\sqrt{b}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}} + 5a\sin(e+fx) + 2b\sin^3(e+fx)\right)}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a + b*Sin[e + f*x]^2]*(5*a*Sin[e + f*x] + 2*b*Sin[e + f*x]^3 + (3*a^(
3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a]])/(Sqrt[b]*Sqrt[1 + (b*Sin[e +
f*x]^2)/a])))/(8*f)
```

fricas [B] time = 0.82, size = 503, normalized size = 4.84

$$\left[\frac{3\,a^2\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/64*(3*a^2*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f
*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^
3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3
 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*c
os(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2
+ 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x +
 e)) - 8*(2*b^2*cos(f*x + e)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sin(f*x + e))/(b*f), -1/32*(3*a^2*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x +
 e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^
```

```
3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(2*b^2*cos(f*x + e
)^2 - 5*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(b*f)]
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(2*(4*b^3/32/b^2*sin(f*x+exp(1))*sin(f*x+exp(1))+10*b^2*a/32/b^
2)*sin(f*x+exp(1))*sqrt(a+b*sin(f*x+exp(1))^2)-6*a^2/16/sqrt(b)*ln(abs(sqrt
(a+b*sin(f*x+exp(1))^2)-sqrt(b)*sin(f*x+exp(1)))))
```

maple [**A**] time = 0.22, size = 90, normalized size = 0.87

$$\frac{\sin\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}{4f}+\frac{3a\sin\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{8f}+\frac{3a^2\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{8f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+3/8*a*sin(f*x+e)*(a+b*sin(f*x+e)^
2)^(1/2)/f+3/8/f*a^2*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2
)
```

maxima [**A**] time = 0.36, size = 73, normalized size = 0.70

$$\frac{\frac{3\,a^2\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}}+2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sin\left(fx+e\right)+3\sqrt{b\sin\left(fx+e\right)^2+a}\,a\sin\left(fx+e\right)}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/8*(3*a^2*arcsinh(b*sin(f*x + e)/sqrt(a*b))/sqrt(b) + 2*(b*sin(f*x + e)^2
+ a)^(3/2)*sin(f*x + e) + 3*sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e))/f
```

mupad [**B**] time = 14.54, size = 60, normalized size = 0.58

$$\frac{\sin\left(e+f\,x\right)\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,{}_2\mathrm{F}_1\left(-\frac{3}{2},\frac{1}{2};\ \frac{3}{2};\ -\frac{b\sin\left(e+f\,x\right)^2}{a}\right)}{f\,{\left(\frac{b\sin\left(e+f\,x\right)^2}{a}+1\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)*hypergeom([-3/2, 1/2], 3/2, -(b*
sin(e + f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^(3/2))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.336 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=121

$$-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$

```
[Out] (a+b)^(3/2)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f-1/2*
(3*a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*b^(1/2)/f-1/
2*b*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.14, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3190, 416, 523, 217, 206, 377}

$$-\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}-\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Sqrt[b]*(3*a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x
]^2]])/(2*f) + ((a + b)^(3/2)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b
*Sin[e + f*x]^2]])/f - (b*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{1-x^2}dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} - \frac{\text{Subst}\left(\int \frac{-a(2a+b)-b(3a+2b)x^2}{\left(1-x^2\right)\sqrt{a+bx^2}} d$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)\sqrt{a+b}}$$

$$= -\frac{b\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2} d$$

$$= -\frac{\sqrt{b}\,(3a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f} + \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b}\,s}{\sqrt{a+b\,\mathrm{si}}}\right.}{f}$$

Mathematica [A] time = 0.67, size = 233, normalized size = 1.93

$$\frac{\sqrt{2}\left(4a^2+5ab+2b^2\right)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\,\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right) - 2\sqrt{b}\,\sqrt{a+b}\left(\sqrt{b}\,\sin(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}\right.}{4\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*b*(3*a + 2*b)*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]]] + Sqrt[2]*(4*a^2 + 5*a*b + 2*b^2)*ArcTanh[(Sqrt[
2*a + 2*b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)]]] - 2*Sqrt[b]*Sq
rt[a + b]*(Sqrt[2]*(3*a + 2*b)*Log[Sqrt[2*a + b - b*Cos[2*(e + f*x)]] + Sqr
t[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[b]*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Sin
[e + f*x]))/(4*Sqrt[2]*Sqrt[a + b]*f)
```

fricas [B] time = 0.95, size = 1381, normalized size = 11.41

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*((3*a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)
*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 +
```

```
 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24
*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*
b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*
a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin
(f*x + e)) + 4*(a + b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*
(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a -
2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*
a*b + 8*b^2)/cos(f*x + e)^4) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x
+ e))/f, -1/16*(8*(a + b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(b)*log(
128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2
 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a
*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2
 + 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*
a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) + 8*sqrt(-b*cos(f*x
+ e)^2 + a + b)*b*sin(f*x + e))/f, 1/8*((3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*
b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3
*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 2*(a +
b)^(3/2)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2
)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x
 + e)^4) - 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/f, -1/8*(4*(a
+ b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b
*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 -
2*a*b - b^2)*sin(f*x + e))) - (3*a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*
x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2
*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x
 + e)^2 + a + b)*b*sin(f*x + e))/f]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e), x)
```

maple [B] time = 3.71, size = 451, normalized size = 3.73

$$-\frac{b^{\frac{3}{2}}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\right)}{f}-\frac{b\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\sin\left(fx+e\right)}{2f}-3a\sqrt{b}\,\ln(\mathrm{si}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/f*b^(3/2)*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))-1/2*b/f*(a+b
-b*cos(f*x+e)^2)^(1/2)*sin(f*x+e)-3/2/f*a*b^(1/2)*ln(sin(f*x+e)*b^(1/2)+(a+
b-b*cos(f*x+e)^2)^(1/2))+1/2/(a+b)^(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+1/(a+b)^(1/2)/f*ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+1/2/
(a+b)^(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b
```

```
*sin(f*x+e)+a))*b^2-1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2-1/(a+b)^(1/2)/f*ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b-1/2/(a+b)
^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a))*b^2
```

maxima [A] time = 0.47, size = 168, normalized size = 1.39

$$-\frac{3\,a\sqrt{b}\,\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)+2\,b^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)-(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)-}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(3*a*sqrt(b)*arcsinh(b*sin(f*x + e)/sqrt(a*b)) + 2*b^(3/2)*arcsinh(b*s
in(f*x + e)/sqrt(a*b)) - (a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(s
in(f*x + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - (a + b)^(3/2)*arcsi
nh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x +
 e) - 1))) + sqrt(b*sin(f*x + e)^2 + a)*b*sin(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.337 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=127

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin}}{2f}$$

```
[Out] b^(3/2)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f+1/2*(a-2*b)*
arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))*(a+b)^(1/2)/f+1/2*
(a+b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.15, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3190, 413, 523, 217, 206, 377}

$$\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f}+\frac{(a+b)\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin}}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (b^(3/2)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/f + ((
a - 2*b)*Sqrt[a + b]*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e +
f*x]^2]])/(2*f) + ((a + b)*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 523

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*Sqrt[(c_) + (d_.)*(x
_)^(n_)]), x_Symbol] :> Dist[f/b, Int[1/Sqrt[c + d*x^n], x], x] + Dist[(b*e
```

```
 - a*f)/b, Int[1/((a + b*x^n)*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d
, e, f, n}, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}-\frac{\text{Subst}\left(\int \frac{}{(}\right.}{}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{b^2\,\text{Subst}\left(\right.}{}$$

$$=\frac{(a+b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2f}+\frac{b^2\,\text{Subst}\left(\right.}{}$$

$$=\frac{b^{3/2}\tanh^{-1}\left(\frac{\sqrt{b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}+\frac{(a-2b)\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e}{\sqrt{a+b\sin^2(e}}\right.}{2f}$$

Mathematica [A] time = 0.95, size = 210, normalized size = 1.65

$$\frac{2\left(a^2-ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{2a+2b}\,\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)+\sqrt{a+b}\,\left(4b^{3/2}\log\left(\sqrt{2a-b\cos(2(e+fx))+b}\,+\sqrt{2}\,\sqrt{b}\,\sin(e\right.\right.}{4f\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b^2*ArcTanh[(Sqrt[2]*Sqrt[a + b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(
e + f*x)]]] + 2*(a^2 - a*b - b^2)*ArcTanh[(Sqrt[2*a + 2*b]*Sin[e + f*x])/Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]] + Sqrt[a + b]*(4*b^(3/2)*Log[Sqrt[2*a + b
 - b*Cos[2*(e + f*x)]] + Sqrt[2]*Sqrt[b]*Sin[e + f*x]] + Sqrt[2]*(a + b)*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]]*Sec[e + f*x]*Tan[e + f*x]))/(4*Sqrt[a + b]
*f)
```

fricas [B] time = 1.04, size = 1471, normalized size = 11.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(b^(3/2)*cos(f*x + e)^2*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^
4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4
```

```
 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 2
4*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*s
in(f*x + e)) - sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b
^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*
cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin
(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e
)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*(a - 2*b)*sq
rt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b -
 b^2)*sin(f*x + e)))*cos(f*x + e)^2 - b^(3/2)*cos(f*x + e)^2*log(128*b^4*co
s(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^
3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128
*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b
^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24
*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e)) - 4*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/8*(2*sqrt(-b)*b*arctan(1
/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8
*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2
*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos
(f*x + e)^2 + sqrt(a + b)*(a - 2*b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 + 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2), -1/4*((a - 2*b)*sqrt(
-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x +
 e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^
2)*sin(f*x + e)))*cos(f*x + e)^2 + sqrt(-b)*b*arctan(1/4*(8*b^2*cos(f*x + e
)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3
- (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*(a + b)*sin(f*x + e))/(f*cos(f*x + e)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

maple [B] time = 3.87, size = 402, normalized size = 3.17

$$\frac{2\sin\left(fx+e\right)\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\left(a+b\right)^{\frac{5}{2}}-\left(-4b^{\frac{3}{2}}\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\right)(a\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)-(-4*b^(3/2)*ln(sin
(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*(a+b)^(3/2)-ln(2/(sin(f*x+e)-1)
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3+3*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^2+2*l
```

```
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*b^3+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))*a^3-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*
sin(f*x+e)+a))*a*b^2-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*b^3)*cos(f*x+e)^2)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx + e\right)^2 + a\right)^{\frac{3}{2}} \sec \left(fx + e\right)^3 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sin \left(e + f\,x\right)^2 + a\right)^{3/2}}{\cos \left(e + f\,x\right)^3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.338 $\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=122

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tan(e+fx)\sec(e+fx)\sqrt{a+b}}{8f}$$

```
[Out] 3/8*a^2*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1
/2)+1/4*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f+3/8*a*sec(f*x+e)
*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 378, 377, 206}

$$\frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8f\sqrt{a+b}}+\frac{\tan(e+fx)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4f}+\frac{3a\tan(e+fx)\sec(e+fx)\sqrt{a+b}}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*a^2*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(8*S
qrt[a + b]*f) + (3*a*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/
(8*f) + (Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^{3/2}}{(1-x^2)^3}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{4f} + \frac{(3a)\,\text{Subst}\left(\int \cdots\right.}{\cdots}$$

$$= \frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8f} + \frac{\sec^3(e+fx)\left(a \cdots\right.}{\cdots}$$

$$= \frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{8f} + \frac{\sec^3(e+fx)\left(a \cdots\right.}{\cdots}$$

$$= \frac{3a^2\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{8\sqrt{a+b}f} + \frac{3a\sec(e+fx)\sqrt{a+b\sin^2(e+fx)} \cdots}{8f}$$

Mathematica [C] time = 0.13, size = 63, normalized size = 0.52

$$\frac{a^2\sin(e+fx)\,_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{(a+b)\sin^2(e+fx)}{b\sin^2(e+fx)+a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Hypergeometric2F1[1/2, 3, 3/2, ((a + b)*Sin[e + f*x]^2)/(a + b*Sin[e +
 f*x]^2)]*Sin[e + f*x])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [A] time = 0.98, size = 413, normalized size = 3.39

$$\left[\frac{3\sqrt{a+b}\,a^2\cos\left(fx+e\right)^4\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2b\right)\cos\left(fx+e\right)^2-2a-2b\right)\sqrt{-b\cos\left(fx+\cdots\right.}}{\cos\left(fx+e\right)^4}\right.}{3\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/32*(3*sqrt(a + b)*a^2*cos(f*x + e)^4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x
+ e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)
^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) +
8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((3*a^2 + a*b - 2*b^2)*cos(f*x
+ e)^2 + 2*a^2 + 4*a*b + 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e
))/((a + b)*f*cos(f*x + e)^4), -1/16*(3*a^2*sqrt(-a - b)*arctan(1/2*((a + 2
*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b
)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x
+ e)^4 - 2*((3*a^2 + a*b - 2*b^2)*cos(f*x + e)^2 + 2*a^2 + 4*a*b + 2*b^2)*s
qrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a + b)*f*cos(f*x + e)^4)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sec \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

maple [B] time = 3.19, size = 406, normalized size = 3.33

$$2\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\ (a+b)^{\frac{5}{2}} (3a-2b) \sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+4\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\ (a+b$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/16*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(5/2)*(3*a-2*b)*sin(f*x+e)*cos(f*x
+e)^2+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*sin(f*x+e)-3*a^2*(ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+2*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a))*b^2-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin
(f*x+e)+a))*a^2-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a*b-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)+b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^4)/(a+b)^(5/2)/cos(f*x+e)^4/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \sec \left(fx+e\right)^{5} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sin\left(e+f\,x\right)^{2}+a\right)^{3/2}}{\cos\left(e+f\,x\right)^{5}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^5, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.339 $\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=195

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6f(a+b)}+\frac{(5a+6b)\tan(e+fx)\,s}{}$$

```
[Out] 1/16*a^2*(5*a+6*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))
/(a+b)^(3/2)/f+1/24*(5*a+6*b)*sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x
+e)/(a+b)/f+1/6*sec(f*x+e)^5*(a+b*sin(f*x+e)^2)^(5/2)*tan(f*x+e)/(a+b)/f+1/
16*a*(5*a+6*b)*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.18, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3190, 382, 378, 377, 206}

$$\frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{6f(a+b)}+\frac{(5a+6b)\tan(e+fx)\,s}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*(5*a + 6*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]
^2]])/(16*(a + b)^(3/2)*f) + (a*(5*a + 6*b)*Sec[e + f*x]*Sqrt[a + b*Sin[e +
 f*x]^2]*Tan[e + f*x])/(16*(a + b)*f) + ((5*a + 6*b)*Sec[e + f*x]^3*(a + b*
Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/(24*(a + b)*f) + (Sec[e + f*x]^5*(a + b
*Sin[e + f*x]^2)^(5/2)*Tan[e + f*x])/(6*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
```

```
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^7(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^4}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}\tan(e+fx)}{6(a+b)f} + \frac{(5a+6b)\,\text{Subst}\left(\right.}{}$$

$$= \frac{(5a+6b)\sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{24(a+b)f} + \frac{\sec^5(e+}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{16(a+b)f} + \frac{(5a+6b)}{}$$

$$= \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{16(a+b)f} + \frac{(5a+6b)}{}$$

$$= \frac{a^2(5a+6b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{16(a+b)^{3/2}f} + \frac{a(5a+6b)\sec(e+fx)\sqrt{a+b}}{16(a+}$$

Mathematica [C] time = 15.22, size = 938, normalized size = 4.81

$$\frac{a^2\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^2\tan(e+fx)\left(256b\,{}_2F_1\left(2,5;\frac{7}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)\sin^2(e+fx)\sqrt{\frac{\sec^2(e+fx)\left(b\sin^2(e+fx\right.}{a}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^7*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a^2*Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)^2*Tan[e + f*x]*(45*a*ArcSin[
Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e +
f*x]^2)/a)]]*Sin[e + f*x]^2 + 210*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + 140*b*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(3/2) - 120*a*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Ta
n[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(5/2) - 80*b*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a
+ b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) + 256*b*Hyper
```

```
geometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[
(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^
(5/2) - 512*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*S
qrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/
a))^(7/2) - 512*b*Hypergeometric2F1[2, 5, 7/2, -(((a + b)*Tan[e + f*x]^2)/a
)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a +
b)*Tan[e + f*x]^2)/a))^(7/2) + 256*a*Hypergeometric2F1[2, 5, 7/2, -(((a + b
)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((
a + b)*Tan[e + f*x]^2)/a))^(9/2) + 256*b*Hypergeometric2F1[2, 5, 7/2, -(((a
 + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e
+ f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(9/2) - 45*a*Sqrt[-(((a + b)*
Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - 30*b*Sin[e +
f*x]^2*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2
)/a^2)]))/(240*f*(a + b*Sin[e + f*x]^2)^(3/2)*Sqrt[(Sec[e + f*x]^2*(a + b*S
in[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

fricas [A] time = 3.56, size = 545, normalized size = 2.79

$$\left[\frac{3\left(5a^3+6a^2b\right)\sqrt{a+b}\,\cos\left(fx+e\right)^6\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2-4\left(\left(a+2b\right)\cos\left(fx+e\right)^2-2a-2b\cdots}{\cos\left(fx+e\right)^4}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/192*(3*(5*a^3 + 6*a^2*b)*sqrt(a + b)*cos(f*x + e)^6*log(((a^2 + 8*a*b +
8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*
b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*
sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*((15*a^3 + 23*a^
2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^3
 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)
^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6), -1/96*(3*
(5*a^3 + 6*a^2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a -
 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x +
e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^6 - 2*((15*a^3 + 23*a
^2*b + 4*a*b^2 - 4*b^3)*cos(f*x + e)^4 + 8*a^3 + 24*a^2*b + 24*a*b^2 + 8*b^
3 + 2*(5*a^3 + 8*a^2*b + a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^6)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

maple [B] time = 4.21, size = 693, normalized size = 3.55

$$\frac{2\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,\left(a+b\right)^{\frac{7}{2}}\left(15a^2+8ab-4b^2\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+4\sqrt{a+b-b\left(\cos^2\left(f\cdots\right.\right.}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/96*(2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(15*a^2+8*a*b-4*b^2)*sin(f*x
+e)*cos(f*x+e)^4+4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(5*a^2+3*a*b-2*b^
2)*cos(f*x+e)^2*sin(f*x+e)+16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(7/2)*(a^2+2
*a*b+b^2)*sin(f*x+e)+3*a^2*(5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+21*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b+33*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^2+23*ln(2/(sin(f*
x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+6*ln
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
b^4-5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))*a^4-21*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-
b*sin(f*x+e)+a))*a^3*b-33*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2-23*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3-6*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^6)/(a+b)
^(9/2)/cos(f*x+e)^6/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^7\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^7*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^7, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^7}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^7, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**7*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.340 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=321

$$-\frac{\left(a^2-9ab-2b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\cos^2(e+fx)}\ \sec(}{35b^2f\sqrt{a-}}$$

```
[Out] -1/35*(a^2-9*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f+
2/35*(4*a+b)*cos(f*x+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/7*b*cos(f
*x+e)^5*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2/35*(a-b)*(a^2+6*a*b+b^2)*El
lipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/35*a*(a+b)*(2*a^2+9*a*b-b
^2)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b
*sin(f*x+e)^2/a)^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.39, antiderivative size = 321, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(a^2-9ab-2b^2\right)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf}+\frac{a(a+b)\left(2a^2+9ab-b^2\right)\sqrt{\cos^2(e+fx)}\ \sec(}{35b^2f\sqrt{a-}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^2 - 9*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(35*b*f) + (2*(4*a + b)*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f
*x]^2])/(35*f) - (b*Cos[e + f*x]^5*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])
/(7*f) - (2*(a - b)*(a^2 + 6*a*b + b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcS
in[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(35*b^2*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*(a + b)*(2*a^2 + 9*a*b - b^2)*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(35*b^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}dx\right.}{f}$$

$$= -\frac{b\cos^5(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{7f} - \frac{\left(\sqrt{\cos^2(e+f\right.}}{}$$

$$= \frac{2(4a+b)\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35f} - \frac{b\cos^5(}{}$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} +$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} +$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} +$$

$$= -\frac{\left(a^2-9ab-2b^2\right)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{35bf} +$$

Mathematica [A] time = 2.57, size = 247, normalized size = 0.77

$$\frac{\sqrt{2}\,b\sin(2(e+fx))\left(-32a^3+b\left(144a^2-192ab-37b^2\right)\cos(2(e+fx))+400a^2b+2b^2(b-26a)\cos(4(e+fx)\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-128*a*(a^3 + 5*a^2*b - 5*a*b^2 - b^3)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])
/a]*EllipticE[e + f*x, -(b/a)] + 64*a*(2*a^3 + 11*a^2*b + 8*a*b^2 - b^3)*Sq
rt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b
*(-32*a^3 + 400*a^2*b + 212*a*b^2 + 30*b^3 + b*(144*a^2 - 192*a*b - 37*b^2)
*Cos[2*(e + f*x)] + 2*b^2*(-26*a + b)*Cos[4*(e + f*x)] + 5*b^3*Cos[6*(e + f
*x)])*Sin[2*(e + f*x)])/(2240*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^6-(a+b)\cos\left(fx+e\right)^4\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^6 - (a + b)*cos(f*x + e)^4)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

maple [A] time = 1.69, size = 590, normalized size = 1.84

$$\frac{5b^4\sin\left(fx+e\right)\left(\cos^8\left(fx+e\right)\right)+\left(-13a\,b^3-7b^4\right)\left(\cos^6\left(fx+e\right)\right)\sin\left(fx+e\right)+\left(9a^2b^2+a\,b^3\right)\left(\cos^4\left(fx+e\right)\right)\mathrm{s}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/35*(5*b^4*sin(f*x+e)*cos(f*x+e)^8+(-13*a*b^3-7*b^4)*cos(f*x+e)^6*sin(f*x+
e)+(9*a^2*b^2+a*b^3)*cos(f*x+e)^4*sin(f*x+e)+(-a^3*b+8*a^2*b^2+11*a*b^3+2*b
^4)*cos(f*x+e)^2*sin(f*x+e)+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^4+11*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*
b+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*
x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3-2*(cos(f*x+e)^2)^(1/2)*
(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4-
10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))*a^3*b+10*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/
a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(cos(f*x+e)^2)^(1/2
)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*
b^3)/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.341 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=259

$$-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{b\sin(e+fx)\cos}{}$$

```
[Out] 2/15*(3*a+b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/5*b*cos(f*x
+e)^3*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/15*(3*a^2-7*a*b-2*b^2)*Ellipt
icE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/b/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/15*a*(3*a-b)*(a+b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 259, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 416, 528, 524, 426, 424, 421, 419}

$$-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{15bf\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{b\sin(e+fx)\cos}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*(3*a + b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*f) -
 (b*Cos[e + f*x]^3*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(5*f) - ((3*a^2
 - 7*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(15*b*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) + (a*(3*a - b)*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[
e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(15*b*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \sqrt{1-x^2}\,\left(a+bx^2\right)^{3/2}\,dx,x,\text{si}\right.}{f}$$

$$= -\frac{b\cos^3(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{5f} - \frac{\left(\sqrt{\cos^2(e+fx)}\right.\ \text{s}}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+}{}$$

$$= \frac{2(3a+b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{15f} - \frac{b\cos^3(e+}{}$$

Mathematica [A] time = 1.34, size = 200, normalized size = 0.77

$$\frac{\sqrt{2}\,b\sin(2(e+fx))\left(48a^2-4b(9a+2b)\cos(2(e+fx))+28ab+3b^2\cos(4(e+fx))+5b^2\right)+16a\left(3a^2+2ab-b^2\right.}{240bf\sqrt{2a-b\cos(2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-16*a*(3*a^2 - 7*a*b - 2*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellip
ticE[e + f*x, -(b/a)] + 16*a*(3*a^2 + 2*a*b - b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(48*a^2 + 28*a*b + 5*
b^2 - 4*b*(9*a + 2*b)*Cos[2*(e + f*x)] + 3*b^2*Cos[4*(e + f*x)])*Sin[2*(e +
 f*x)])/(240*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^4-(a+b)\cos\left(fx+e\right)^2\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^4 - (a + b)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^
2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

maple [A] time = 1.80, size = 429, normalized size = 1.66

$$\frac{-3b^3\left(\sin^7\left(fx+e\right)\right)-9a\,b^2\left(\sin^5\left(fx+e\right)\right)+4b^3\left(\sin^5\left(fx+e\right)\right)+3\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\ \mathrm{Ellip}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/15*(-3*b^3*sin(f*x+e)^7-9*a*b^2*sin(f*x+e)^5+4*b^3*sin(f*x+e)^5+3*(cos(f*
x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a^3+2*a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(
sin(f*x+e),(-1/a*b)^(1/2))*b-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*(cos(f*x+e)^2)^(1/2)*((a+b
*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*(cos(f*x
+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/
2))*a^2*b+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*a*b^2-6*a^2*b*sin(f*x+e)^3+10*a*b^2*sin(f*x+e)^3-b^
3*sin(f*x+e)^3+6*a^2*b*sin(f*x+e)-a*b^2*sin(f*x+e))/b/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^2\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.342 $\int \left(a+b\sin^2(e+fx)\right)^{3/2} dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(\ldots}}{3f\sqrt{\frac{b\sin^2\ldots}{}}}$$

```
[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(\ldots}}{3f\sqrt{\frac{b\sin^2\ldots}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)}{\sqrt{a+b\sin^2(e+}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+f}\right.}{3\sqrt{1+\frac{b\,\mathrm{s}}{}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a}}{3f\sqrt{1+\frac{b\sin^2(e}{a}}}$$

Mathematica [A] time = 0.76, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}\,a(2a+b)}{6\sqrt{2}\,f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.50, size = 266, normalized size = 1.73

$$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+\frac{4\sqrt{\mathrm{co}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*a
*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.343 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=182

$$\frac{(a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+\right.}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)
*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*(a+b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)
/f
```

Rubi [A] time = 0.18, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 413, 524, 426, 424, 421, 419}

$$\frac{(a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+\right.}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]))
 + (a*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
 + ((a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+f\right.}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} + \frac{\left(a(a+b)\sqrt{\cos^2(e+fx)}\,\mathrm{s}\right.}{}$$

$$= \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left((a+2b)\sqrt{\cos^2(e+fx)}\,\mathrm{s}\right.}{}$$

$$= -\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.86, size = 144, normalized size = 0.79

$$\frac{(a+b)\left(\sqrt{2}\,\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)\right)-2a(a+2b)\sqrt{\frac{2a-b\cos}{}}}{2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + (a + b)*(2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] + Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x]))/(2*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sec\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*
x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

maple [B] time = 2.40, size = 466, normalized size = 2.56

$$-\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)+(a+b)\left(\cos^{2}\left(fx+e\right)\right)}\,b\,(a+b)\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)+\sqrt{-b\left(\cos^{4}\left(fx+e\right)\right)-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)+(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+a*b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2)*(-b/
a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x
+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+
(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b \sin\left(e + f x\right)^2 + a\right)^{3/2}}{\cos\left(e + f x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.344 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=236

$$\frac{2(a-b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(2a-b)\sqrt{\text{co}}}{}$$

```
[Out] -2/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/
2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*a*(2*a-b)*Elli
pticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x
+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a-b)*(a+b*sin(f*x+e)^2)^(1/2
)*tan(f*x+e)/f+1/3*(a+b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.26, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 413, 527, 524, 426, 424, 421, 419}

$$\frac{2(a-b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(a+b)\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{a(2a-b)\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (a*(2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)
]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]
^2]) + (2*(a - b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f) + ((a + b)
*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*f)
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{5/2}}dx,x,\sin(e+fx)\right.}{f}$$

$$= \frac{(a+b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{}}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{}}{}$$

$$= \frac{2(a-b)\sqrt{a+b\sin^2(e+fx)}\tan(e+fx)}{3f} + \frac{(a+b)\sec^2(e+fx)\sqrt{}}{}$$

$$= -\frac{2(a-b)\sqrt{\cos^2(e+fx)}E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{}}{3f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 2.03, size = 190, normalized size = 0.81

$$\frac{\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(4a^2-6ab-2b^2\right)\cos(2(e+fx))+8a^2+b(b-a)\cos(4(e+fx))+3ab+b^2\right)}{\sqrt{2}} + 2a(2a-b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\right|}{6f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-4*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + 2*a*(2*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] + ((8*a^2 + 3*a*b + b^2 + (4*a^2 - 6*a*b - 2*b^2)*Cos[2*(e + f*
x)] + b*(-a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*
x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

maple [A] time = 2.69, size = 375, normalized size = 1.59

$$\frac{2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b\,(a-b)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*
x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2-a*b-3*b^2)*cos(f*x
+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)
*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2)
)*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+b*sin
(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(1+sin(f*x+e
))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*sec(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^(3/2)/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.345 $\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=79

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

```
[Out] 1/2*(a+2*b)*arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f-
1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f
```

Rubi [A] time = 0.09, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 388, 217, 206}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f}-\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/(2*b
^(3/2)*f) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{2bf}$$
$$= -\frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2bf}$$
$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}f} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2bf}$$

Mathematica [A] time = 0.11, size = 79, normalized size = 1.00

$$\frac{-\frac{(-a-2b)\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2b^{3/2}} - \frac{\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-1/2*((-a - 2*b)*ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]
])/b^(3/2) - (Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(2*b))/f
```

fricas [B] time = 0.56, size = 461, normalized size = 5.84

$$\left[\frac{(a+2\,b)\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((a + 2*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*c
os(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 3
2*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a
*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^
3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*
b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f
*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e))/(b^2*f), -1/8*
((a + 2*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(
f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/
((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*
x + e)^2)*sin(f*x + e))) + 4*sqrt(-b*cos(f*x + e)^2 + a + b)*b*sin(f*x + e)
)/(b^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(-1/2/b*sin(f*x+exp(1))*sqrt(b*sin(f*x+exp(1))^2+a)+2*(-a-2*b)/
4/b/sqrt(b)*ln(abs(sqrt(b*sin(f*x+exp(1))^2+a)-sqrt(b)*sin(f*x+exp(1)))))
```

maple [A] time = 1.49, size = 98, normalized size = 1.24

$$-\frac{\sin\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{2bf}+\frac{a\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{2f\,b^{\frac{3}{2}}}+\frac{\ln\left(\sin\left(fx+e\right)\sqrt{b}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f+1/2/f*a/b^(3/2)*ln(sin(f*x+e)*
b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))+1/f*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)
^2)^(1/2))/b^(1/2)
```

maxima [A] time = 0.34, size = 69, normalized size = 0.87

$$\frac{\frac{a\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}+\frac{2\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}}-\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\,\sin\left(fx+e\right)}{b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(a*arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) + 2*arcsinh(b*sin(f*x + e)
/sqrt(a*b))/sqrt(b) - sqrt(b*sin(f*x + e)^2 + a)*sin(f*x + e)/b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Timed out
```

3.346 $\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=38

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

```
[Out] arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/b^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3190, 217, 206\}$

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}\\
&= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}\,f}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{b}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[b]*f)
```

fricas [B] time = 0.51, size = 394, normalized size = 10.37

$$\left[\frac{\log\left(128\,b^4\cos\left(fx+e\right)^8-256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6+32\left(5\,a^2b^2+24\,ab^3+24\,b^4\right)\cos\left(fx+e\right)^4+a^4+\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(
5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^
2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f
*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 -
a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*
x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*sin(f*x + e))/(sqrt(b)*f)
, -1/4*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x
+ e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*
b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x +
e)^2)*sin(f*x + e)))/(b*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)-1/f/sqrt(b)*ln(abs(sqrt(b*sin(f*x+exp(1))^2+a)-sqrt(b)*sin(f*x+exp
(1))))
```

maple [A] time = 0.13, size = 34, normalized size = 0.89

$$\frac{\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{f\sqrt{b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f*ln(sin(f*x+e)*b^(1/2)+(a+b*sin(f*x+e)^2)^(1/2))/b^(1/2)
```

maxima [**A**] time = 0.35, size = 21, normalized size = 0.55

$$\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{\sqrt{b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] arcsinh(b*sin(f*x + e)/sqrt(a*b))/(sqrt(b)*f)
```

mupad [**B**] time = 14.74, size = 33, normalized size = 0.87

$$\frac{\ln\left(\sqrt{b}\ \sin\left(e+f\,x\right)+\sqrt{b\sin\left(e+f\,x\right)^2+a}\right)}{\sqrt{b}\ f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] log(b^(1/2)*sin(e + f*x) + (a + b*sin(e + f*x)^2)^(1/2))/(b^(1/2)*f)
```

sympy [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cos\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.347 $\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

```
[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3190, 377, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{\sqrt{a+b}\,f}$$

Mathematica [A] time = 0.03, size = 42, normalized size = 1.00

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(Sqrt[a + b]
*f)
```

fricas [B] time = 0.51, size = 240, normalized size = 5.71

$$\left[\frac{\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4-8(a^2+3ab+2b^2)\cos(fx+e)^2-4\left((a+2b)\cos(fx+e)^2-2a-2b\right)\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}\sin(fx+e)+8a^2+16a}{\cos(fx+e)^4}\right.}{4\sqrt{a+b}\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*co
s(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e
)^4)/(sqrt(a + b)*f), -1/2*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^
2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*c
os(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e)))/((a + b)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(fx+e)}{\sqrt{b\sin(fx+e)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [B] time = 3.10, size = 113, normalized size = 2.69

$$\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)}{2\sqrt{a+b}\,f}-\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)}{2\sqrt{a+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))-1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

maxima [B] time = 0.46, size = 105, normalized size = 2.50

$$\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)}{\sqrt{a+b}}+\frac{\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)}{\sqrt{a+b}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*(
sin(f*x + e) + 1)))/sqrt(a + b) + arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(f
*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\cos\left(e+f\,x\right)\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.348 $\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}$$

```
[Out] 1/2*(a+2*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^
(3/2)/f+1/2*sec(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.11, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {3190, 382, 377, 206}

$$\frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{3/2}}+\frac{\tan(e+fx)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((a + 2*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(3/2)*f) + (Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]
)/(2*(a + b)*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] ||  !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx\right.}{2(a+b)f}$$

$$= \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2(a+b)f} + \frac{(a+2b)\,\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x,\right.}{2(a+b)f}$$

$$= \frac{(a+2b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{3/2}f} + \frac{\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{2(a+b)f}$$

Mathematica [C] time = 9.75, size = 436, normalized size = 4.79

$$\frac{\tan(e+fx)\sec^3(e+fx)\left(\frac{b\sin^2(e+fx)}{a}+1\right)\left(-30b\sin^2(e+fx)\sqrt{-\frac{\tan^2(e+fx)\sec^2(e+fx)\left(a^2+ab\left(\sin^2(e+fx)+1\right)+b^2\sin^2(e+fx\right.}{a^2}}\right.}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sec[e + f*x]^3*(1 + (b*Sin[e + f*x]^2)/a)*Tan[e + f*x]*(45*a*ArcSin[Sqrt[-
(((a + b)*Tan[e + f*x]^2)/a)]] + 30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2
)/a)]]*Sin[e + f*x]^2 + 16*a*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan[e
+ f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*T
an[e + f*x]^2)/a))^(5/2) + 16*b*Hypergeometric2F1[2, 3, 7/2, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(5/2) - 45*a*Sqrt[-((Sec[e + f*x]^2*(a
^2 + b^2*Sin[e + f*x]^2 + a*b*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)] -
 30*b*Sin[e + f*x]^2*Sqrt[-((Sec[e + f*x]^2*(a^2 + b^2*Sin[e + f*x]^2 + a*b
*(1 + Sin[e + f*x]^2))*Tan[e + f*x]^2)/a^2)]))/(30*a*f*Sqrt[a + b*Sin[e + f
*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e +
f*x]^2)/a))^(3/2))
```

fricas [B] time = 0.54, size = 361, normalized size = 3.97

$$\left[\frac{(a+2b)\sqrt{a+b}\,\cos\left(fx+e\right)^2\log\left(\frac{\left(a^2+8ab+8b^2\right)\cos\left(fx+e\right)^4-8\left(a^2+3ab+2b^2\right)\cos\left(fx+e\right)^2-4\left((a+2b)\cos\left(fx+e\right)^2-2a-2b\right)\sqrt{-b\cos\right.}{\cos\left(fx+e\right)^4}\right.}{8\left(a^2+2ab+b^2\right)f\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a + 2*b)*sqrt(a + b)*cos(f*x + e)^2*log(((a^2 + 8*a*b + 8*b^2)*cos(f
*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x +
 e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e)
```

```
 + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2), -1/4*((a
+ 2*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2
- 2*a*b - b^2)*sin(f*x + e)))*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
 + b)*(a + b)*sin(f*x + e))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [B] time = 3.29, size = 360, normalized size = 3.96

$$2\sin\left(fx+e\right)\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\,(a+b)^{\frac{3}{2}}-\left(-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)a^2-3\ln\left(\frac{2\sqrt{a\cdots}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)-(-ln(2/(sin(f*x+e)
-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2-3*ln(2/(si
n(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b-2*
ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a)
)*b^2+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b
*sin(f*x+e)+a))*a*b+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2)/(a+b)^(5/2)/cos(f*x+e)^2/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^3}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(sec(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.349 $\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=168

$$\frac{(a+b)(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{\sin(e+fx)\cos(e}{}$$

```
[Out] -1/3*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/b/f-2/3*(a+2*b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(a+b)*(2*a+3*b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.24, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 416, 524, 426, 424, 421, 419}

$$\frac{(a+b)(2a+3b)\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{3b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{\cos^2(e+fx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -(Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b*f) - (2*(a + 2
*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (
(a + b)*(2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*b^2*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{\left(2(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e\right.}{}$$

$$= -\frac{\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3bf} - \frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-}\right.}{3}$$

Mathematica [A] time = 0.85, size = 170, normalized size = 1.01

$$\frac{2\sqrt{2}\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)+b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-4\sqrt{2}a}{6\sqrt{2}b^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*Sqrt[2]*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e
+ f*x, -(b/a)] + 2*Sqrt[2]*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)
])*Sin[2*(e + f*x)])/(6*Sqrt[2]*b^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.54, size = 316, normalized size = 1.88

$$\frac{b^2\left(\sin^5\left(fx+e\right)\right)+2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2+5a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*(b^2*sin(f*x+e)^5+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+3*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2-4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+a*b*sin(f*x+e)^3-b^2*sin(f*x+e)^3-a*b*
sin(f*x+e))/b^2/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^4}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Timed out
```

3.350 $\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=114

$$\frac{(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{a\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -a*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*
sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(a+b)*(cos(f*x+e)^2)^(1/
2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)
/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 153, normalized size of antiderivative = 1.34, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3192, 423, 426, 424, 421, 419}

$$\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\ldots}}{bf\sqrt{\frac{b\ldots}{}}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) + ((a +
 b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{bf} + \frac{\left((a+b)\sqrt{c\right.}}{}$$

$$= -\frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{Subst}\left(\int \frac{\sqrt{1+\frac{bx^2}{a}}}{\sqrt{1-x^2}}\,dx, x, \sin(e\right.}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{bf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + -\ (\ldots$$

Mathematica [A] time = 0.20, size = 83, normalized size = 0.73

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\left((a+b)F\left(e+fx\middle|-\frac{b}{a}\right)-aE\left(e+fx\middle|-\frac{b}{a}\right)\right)}{bf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*(-(a*EllipticE[e + f*x, -(b/a)]) +
(a + b)*EllipticF[e + f*x, -(b/a)]))/(b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.14, size = 111, normalized size = 0.97

$$\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)b-\mathrm{Ellipti}\right.}{b\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*(EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a)/b/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^{2}}{\sqrt{b\sin\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^{2}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cos(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.351 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\ &= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.07, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [C] time = 0.36, size = 60, normalized size = 1.18

$$\frac{\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)-a-b}{a}}\;\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sin\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.352 $\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=140

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}+\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*si
n(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(cos(f*x+e)^2)^(1/2)/c
os(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(
a+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.17, antiderivative size = 180, normalized size of antiderivative = 1.29, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 414, 21, 423, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}+\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]
*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a +
 b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 423

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
b/d, Int[Sqrt[c + d*x^2]/Sqrt[a + b*x^2], x], x] - Dist[(b*c - a*d)/d, Int[
1/(Sqrt[a + b*x^2]*Sqrt[c + d*x^2]), x], x] /; FreeQ[{a, b, c, d}, x] && Po
sQ[d/c] && NegQ[b/a]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{b-bx^2}{\sqrt{1-x^2}\,\sqrt{a}}\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(b\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\sqrt{a+bx^2}}\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{a}}\right.}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx}\right.}{(a+b)f\sqrt{1+\frac{b\,\mathrm{s}}{}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{\mathrm{co}}}{}$$

Mathematica [A] time = 0.62, size = 141, normalized size = 1.01

$$\frac{\sqrt{2}\,\tan(e+fx)(2a-b\cos(2(e+fx))+b)+2(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E}{2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + 2
*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)]
+ Sqrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*
a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 2.82, size = 278, normalized size = 1.99

$$\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,\left(a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\ \mathrm{EllipticF}\left(\sin\left(fx+\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))+b*(cos(f*x+e
)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^
(1/2))-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(s
in(f*x+e),(-1/a*b)^(1/2))-sin(f*x+e)*cos(f*x+e)^2*b+a*sin(f*x+e)+b*sin(f*x+
e))/(a+b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x
+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.353 $\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2(a+2b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{(2a+3b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a+2b)\sqrt{a+b\sin^2(e+}}{3f(a+b)^2\sqrt{\frac{b\sin^2}{}}}$$

```
[Out] -2/3*(a+2*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1
/2))*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(2*a
+3*b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1
+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(a+2*b)*(a+b*
sin(f*x+e)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2
)^(1/2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.25, antiderivative size = 252, normalized size of antiderivative = 1.19, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$\frac{2(a+2b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}+\frac{(2a+3b)\sqrt{\cos^2(e+f}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
 f*x]^2)/a]) + ((2*a + 3*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqr
t[a + b*Sin[e + f*x]^2]) + (2*(a + 2*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e +
f*x])/(3*(a + b)*f)
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 && !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
```

```
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{5/2}\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subs}}{3(a}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(}{3(a+b)f}$$

$$= \frac{2(a+2b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(}{3(a+b)f}$$

$$= -\frac{2(a+2b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+f}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 2.19, size = 205, normalized size = 0.97

$$\frac{2\left(2a^2+5ab+3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(4a^2+6ab-2b^2\right)\cos(2(e+fx))+8a^2-b(a+2b)\cos(4(e+}{\sqrt{2}}}{6f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-4*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] + 2*(2*a^2 + 5*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*E
llipticF[e + f*x, -(b/a)] + ((8*a^2 + 15*a*b + 4*b^2 + (4*a^2 + 6*a*b - 2*b
^2)*Cos[2*(e + f*x)] - b*(a + 2*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e +
 f*x])/Sqrt[2])/(6*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")

[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)

maple [A] time = 3.03, size = 405, normalized size = 1.91

$$\frac{2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\, b\,(a+2b)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)

[Out] 1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+2*b)*sin(f*x+e)*cos(f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*cos(f*x+e)^2*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b+3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-4*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^2)/(1+sin(f*x+e))/(sin(f*x+e)-1)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")

[Out] integrate(sec(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)),x)

[Out] int(1/(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2)), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^4\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)

[Out] Integral(sec(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)

3.354 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=75

$$\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}$$

```
[Out] -arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(3/2)/f+(a+b)*sin(f
*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3190, 385, 217, 206}

$$\frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(3/2)*f)) +
 ((a + b)*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{bf}$$

$$= \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-bx^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{bf}$$

$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{3/2}f} + \frac{(a+b)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.17, size = 88, normalized size = 1.17

$$\frac{\sqrt{b}(a+b)\sin(e+fx) - a^{3/2}\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,\sinh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a}}\right)}{ab^{3/2}f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[b]*(a + b)*Sin[e + f*x] - a^(3/2)*ArcSinh[(Sqrt[b]*Sin[e + f*x])/Sqrt
[a]]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*b^(3/2)*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

fricas [B] time = 0.62, size = 559, normalized size = 7.45

$$\left[\frac{\left(ab\cos\left(fx+e\right)^2 - a^2 - ab\right)\sqrt{b}\,\log\left(128\,b^4\cos\left(fx+e\right)^8 - 256\left(ab^3+2\,b^4\right)\cos\left(fx+e\right)^6 + 32\left(5\,a^2b^2 + 24\,a\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 -
 256*(a*b^3 + 2*b^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*co
s(f*x + e)^4 + a^4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3
*b + 10*a^2*b^2 + 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 + 8*(16*b^3*cos(f*x + e
)^6 - 24*(a*b^2 + 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^
3 + 2*(5*a^2*b + 24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(b)*sin(f*x + e)) - 8*sqrt(-b*cos(f*x + e)^2 + a + b)*(a*b + b
^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f), 1/4*((a*b
*cos(f*x + e)^2 - a^2 - a*b)*sqrt(-b)*arctan(1/4*(8*b^2*cos(f*x + e)^4 - 8*
(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*b^2 + 2*b^3 - (3*a*b
```

```
^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) - 4*sqrt(-b*cos(f*x + e)^2 + a +
 b)*(a*b + b^2)*sin(f*x + e))/(a*b^3*f*cos(f*x + e)^2 - (a^2*b^2 + a*b^3)*f
)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2
*pi/x/2)1/f*(-2*(-2*b-2*a)/4/b/a*sin(f*x+exp(1))*sqrt(b*sin(f*x+exp(1))^2+a
)/(b*sin(f*x+exp(1))^2+a)+1/b/sqrt(b)*ln(abs(sqrt(b*sin(f*x+exp(1))^2+a)-sq
rt(b)*sin(f*x+exp(1)))))
```

maple [A] time = 1.48, size = 90, normalized size = 1.20

$$\frac{\sin\left(fx+e\right)}{fb\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}-\frac{\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\right)}{f\,b^{\frac{3}{2}}}+\frac{\sin\left(fx+e\right)}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/f*sin(f*x+e)/b/(a+b*sin(f*x+e)^2)^(1/2)-1/f/b^(3/2)*ln(sin(f*x+e)*b^(1/2)
+(a+b*sin(f*x+e)^2)^(1/2))+sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.34, size = 74, normalized size = 0.99

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{b^{\frac{3}{2}}}-\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a}-\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,b}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsinh(b*sin(f*x + e)/sqrt(a*b))/b^(3/2) - sin(f*x + e)/(sqrt(b*sin(f*x
+ e)^2 + a)*a) - sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^3}{{\left(b\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.355 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=29

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.04, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3190, 191}

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}\\ &= \frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 1.00

$$\frac{\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
[Out] Sin[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [A] time = 0.44, size = 49, normalized size = 1.69

$$-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{abf\cos\left(fx+e\right)^2-\left(a^2+ab\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a*b*f*cos(f*x + e)^2 - (a^2
+ a*b)*f)
```

giac [A] time = 0.63, size = 29, normalized size = 1.00

$$\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

maple [A] time = 0.16, size = 28, normalized size = 0.97

$$\frac{\sin\left(fx+e\right)}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.33, size = 27, normalized size = 0.93

$$\frac{\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*f)
```

mupad [B] time = 15.61, size = 117, normalized size = 4.03

$$\frac{\sqrt{2}\,\sqrt{2\,a+b-b\,\cos\left(2\,e+2\,f\,x\right)}\,\left(4\,a\,\sin\left(e+f\,x\right)+3\,b\,\sin\left(e+f\,x\right)-b\,\sin\left(3\,e+3\,f\,x\right)\right)}{a\,f\,\left(8\,a\,b+8\,a^2+3\,b^2-4\,b^2\,\cos\left(2\,e+2\,f\,x\right)+b^2\,\cos\left(4\,e+4\,f\,x\right)-8\,a\,b\,\cos\left(2\,e+2\,f\,x\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (2^(1/2)*(2*a + b - b*cos(2*e + 2*f*x))^(1/2)*(4*a*sin(e + f*x) + 3*b*sin(e
 + f*x) - b*sin(3*e + 3*f*x)))/(a*f*(8*a*b + 8*a^2 + 3*b^2 - 4*b^2*cos(2*e
+ 2*f*x) + b^2*cos(4*e + 4*f*x) - 8*a*b*cos(2*e + 2*f*x)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cos(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.356 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=78

$$\frac{b\sin(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

```
[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(3/2)/f+b*si
n(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 382, 377, 206}

$$\frac{b\sin(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(3/
2)*f) + (b*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 382

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[(b*c + n*(p + 1)*(b*c - a*d))/(a*n*(p + 1)*(b*c - a*d)), I
nt[(a + b*x^n)^(p + 1)*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, q}, x
] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 2) + 1, 0] && (LtQ[p, -1] || !LtQ
[q, -1]) && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{(a+b)f}$$

$$= \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-(a+b)x^2}\,dx, x, \frac{\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{3/2}f} + \frac{b\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 7.41, size = 480, normalized size = 6.15

$$\frac{\tan(e+fx)\sec(e+fx)\left(-\frac{30b(a+b)\sin^2(e+fx)\tan^2(e+fx)\sin^{-1}\left(\sqrt{-\frac{(a+b)\tan^2(e+fx)}{a}}\right)}{a^2} + \frac{30b\sin^2(e+fx)\sqrt{-\frac{(a+b)\tan^2(e+fx)\sec^2(e+fx)(a+\cdots}{a^2}}}{a}\cdots\right.}{\cdots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(-45*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]
 - (30*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a - (4
5*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/a - (
30*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan
[e + f*x]^2)/a^2 + 4*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x]^2
)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f
*x]^2)/a))^(5/2) + (4*b*Hypergeometric2F1[2, 2, 7/2, -(((a + b)*Tan[e + f*x
]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(
((a + b)*Tan[e + f*x]^2)/a))^(5/2))/a + 45*Sqrt[-(((a + b)*Sec[e + f*x]^2*(
a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] + (30*b*Sin[e + f*x]^2*Sqrt[-((
(a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a))/(1
5*a*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2
))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))
```

fricas [B] time = 0.56, size = 453, normalized size = 5.81

$$\left[\frac{\left(ab\cos(fx+e)^2 - a^2 - ab\right)\sqrt{a+b}\,\log\left(\frac{(a^2+8ab+8b^2)\cos(fx+e)^4 - 8(a^2+3ab+2b^2)\cos(fx+e)^2 - 4\left((a+2b)\cos(fx+e)^2 - 2a - 2b\right)\cdots}{\cos(fx+e)^4}\right)\cdots}{4\left((a^3b+2a^2b^2+ab^3)f\cos(fx+e)^2 - \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*((a*b*cos(f*x + e)^2 - a^2 - a*b)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^
2)*cos(f*x + e)^4 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*c
os(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(
f*x + e) + 8*a^2 + 16*a*b + 8*b^2)/cos(f*x + e)^4) - 4*sqrt(-b*cos(f*x + e)
^2 + a + b)*(a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*
x + e)^2 - (a^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f), -1/2*((a*b*cos(f*x + e)^
2 - a^2 - a*b)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*
b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*cos(f*x + e)^
2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*(
a*b + b^2)*sin(f*x + e))/((a^3*b + 2*a^2*b^2 + a*b^3)*f*cos(f*x + e)^2 - (a
^4 + 3*a^3*b + 3*a^2*b^2 + a*b^3)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 4.63, size = 397, normalized size = 5.09

$$\frac{2\sqrt{-b\left(\cos^{2}\left(fx+e\right)\right)+\frac{a\,b^{2}+b^{3}}{b^{2}}}\,\sqrt{a+b}\,b\sin\left(fx+e\right)+ab\left(-\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)+\ln\left(\frac{2\sqrt{}}{}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/a/(-a*b*cos(f*x+e)^2-b^2*cos(f*x+e)^2+a^2+2*a*b+b^2)*(2*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)*b*sin(f*x+e)+a*b*(-ln(2/(s
in(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*co
s(f*x+e)^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*a^2-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b)/f
```

maxima [B] time = 0.45, size = 152, normalized size = 1.95

$$\frac{\frac{2\,b\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}+\sqrt{b\sin\left(fx+e\right)^{2}+a}\,ab}+\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)}{(a+b)^{\frac{3}{2}}}+\frac{\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)}{(a+b)^{\frac{3}{2}}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + sqrt(b*sin(f*x + e)
^2 + a)*a*b) + arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(s
qrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(3/2) + arcsinh(-b*sin(f*x + e)/(sqrt
```

```
(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(3/2)
)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)),x)
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.357 $\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=134

$$-\frac{b(a-2b)\sin(e+fx)}{2af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{5/2}}+\frac{\tan(e+fx)\sec(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/2*(a+4*b)*arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^
(5/2)/f-1/2*(a-2*b)*b*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2*s
ec(f*x+e)*tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3190, 414, 527, 12, 377, 206}

$$-\frac{b(a-2b)\sin(e+fx)}{2af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2f(a+b)^{5/2}}+\frac{\tan(e+fx)\sec(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + 4*b)*ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]])/
(2*(a + b)^(5/2)*f) - ((a - 2*b)*b*Sin[e + f*x])/(2*a*(a + b)^2*f*Sqrt[a +
b*Sin[e + f*x]^2]) + (Sec[e + f*x]*Tan[e + f*x])/(2*(a + b)*f*Sqrt[a + b*Si
n[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{a+2b+2bx^2}{\left(1-x^2\right)\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\Big(\ldots}{\ldots}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+4b)\ldots}{\ldots}$$

$$= -\frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(e+fx)\tan(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+4b)\ldots}{\ldots}$$

$$= \frac{(a+4b)\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{2(a+b)^{5/2}f} - \frac{(a-2b)b\sin(e+fx)}{2a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec(\ldots}{2(a+b\ldots}$$

Mathematica [C] time = 5.62, size = 224, normalized size = 1.67

$$-\frac{\tan(e+fx)\sec^5(e+fx)\left(16(a+b)\sin^2(e+fx)\left(a+b\sin^2(e+fx)\right)^2 {}_3F_2\left(2,2,3;1,\frac{9}{2};-\frac{(a+b)\tan^2(e+fx)}{a}\right)+16(\ldots}{\ldots}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/105*(Sec[e + f*x]^5*(16*(a + b)*HypergeometricPFQ[{2, 2, 3}, {1, 9/2}, -
(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*(a + b*Sin[e + f*x]^2)^2 + 16*
```

```
(a + b)*Hypergeometric2F1[2, 3, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e +
 f*x]^2*(4*a^2 + 7*a*b*Sin[e + f*x]^2 + 3*b^2*Sin[e + f*x]^4) - 7*a*Cos[e +
 f*x]^2*Hypergeometric2F1[1, 2, 7/2, -(((a + b)*Tan[e + f*x]^2)/a)]*(15*a^2
 + 20*a*b*Sin[e + f*x]^2 + 8*b^2*Sin[e + f*x]^4))*Tan[e + f*x])/(a^4*f*Sqrt
[a + b*Sin[e + f*x]^2])
```

fricas [B] time = 0.91, size = 625, normalized size = 4.66

$$\left[\frac{\left(\left(a^{2}b+4\,ab^{2}\right)\cos\left(fx+e\right)^{4}-\left(a^{3}+5\,a^{2}b+4\,ab^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a+b}\,\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-8\left(a^{2}+3\,ab+2\,b^{2}\right.}{}\right.}{8\left(\left(a^{4}b+3\,a^{3}b^{2}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/8*(((a^2*b + 4*a*b^2)*cos(f*x + e)^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x
 + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4 - 8*(a^2 + 3
*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2 + 16*a*b + 8*
b^2)/cos(f*x + e)^4) - 4*(a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^
3*b^2 + 3*a^2*b^3 + a*b^4)*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 +
4*a^2*b^3 + a*b^4)*f*cos(f*x + e)^2), -1/4*(((a^2*b + 4*a*b^2)*cos(f*x + e)
^4 - (a^3 + 5*a^2*b + 4*a*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a
+ 2*b)*cos(f*x + e)^2 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a
- b)/(((a*b + b^2)*cos(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) + 2*(
a^3 + 2*a^2*b + a*b^2 - (a^2*b - a*b^2 - 2*b^3)*cos(f*x + e)^2)*sqrt(-b*cos
(f*x + e)^2 + a + b)*sin(f*x + e))/((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)
*f*cos(f*x + e)^4 - (a^5 + 4*a^4*b + 6*a^3*b^2 + 4*a^2*b^3 + a*b^4)*f*cos(f
*x + e)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{3}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 11.57, size = 3217, normalized size = 24.01

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/(a+b)^(1/2)/a/b^5/cos(f*x+e)^2/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+
e)^4+6*a^2*b^4*cos(f*x+e)^4+4*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*c
os(f*x+e)^2-10*a^4*b^2*cos(f*x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(
f*x+e)^2-10*a*b^5*cos(f*x+e)^2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20
*a^3*b^3+15*a^2*b^4+6*a*b^5+b^6)*(-2*a*(8*(a+b)^(1/2)*b^(19/2)*ln(((-b*cos(
f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))-8*(a+b)^(1
```

```
/2)*b^(19/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))+
16*(a+b)^(1/2)*b^(17/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)
+sin(f*x+e)*b^2)/b^(3/2))*a-16*(a+b)^(1/2)*b^(17/2)*ln(((a+b-b*cos(f*x+e)^2
)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*a+8*(a+b)^(1/2)*b^(15/2)*ln(((-b*cos
(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*a^2-8*(a+
b)^(1/2)*b^(15/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1
/2))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^4*b^6+7*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+b*sin(f*x+e)+a))*a^3*b^7+15*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*c
os(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^8+13*ln(2/(sin(f*x+e)-1)*((a+b)^(
1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^9+4*ln(2/(sin(f*x+e)-1
)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^10-ln(2/(1+sin
(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^6-7
*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a
))*a^3*b^7-15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b
*sin(f*x+e)+a))*a^2*b^8-13*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9-4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^10)*cos(f*x+e)^4+a*(8*(a+b)^(1/2)*
b^(19/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2
)/b^(3/2))-8*(a+b)^(1/2)*b^(19/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*
sin(f*x+e))/b^(1/2))+8*(a+b)^(1/2)*b^(17/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*a-8*(a+b)^(1/2)*b^(17/2)*ln((
(a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*a+ln(2/(sin(f*x+e
)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7+6*ln(
2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a
^2*b^8+9*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a*b^9+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+b*sin(f*x+e)+a))*b^10-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^7-6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^8-9*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9-4*ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^10)*cos(f*x
+e)^6+a*(8*(a+b)^(1/2)*b^(19/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)
*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))-8*(a+b)^(1/2)*b^(19/2)*ln(((a+b-b*cos(f*x
+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))+24*(a+b)^(1/2)*b^(17/2)*ln(((-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*a-24*
(a+b)^(1/2)*b^(17/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b
^(1/2))*a+24*(a+b)^(1/2)*b^(15/2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/
2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3/2))*a^2-24*(a+b)^(1/2)*b^(15/2)*ln(((a+b-b*
cos(f*x+e)^2)^(1/2)*b^(1/2)+b*sin(f*x+e))/b^(1/2))*a^2+8*(a+b)^(1/2)*b^(13/
2)*ln(((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(3/2)+sin(f*x+e)*b^2)/b^(3
/2))*a^3-8*(a+b)^(1/2)*b^(13/2)*ln(((a+b-b*cos(f*x+e)^2)^(1/2)*b^(1/2)+b*si
n(f*x+e))/b^(1/2))*a^3+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*a^5*b^5+8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^6+22*ln(2/(sin(f*x+e)-1)*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^7+28*ln(2/(sin(f*x+
e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^8+17*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a*b^9+4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*b^10-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-b*sin(f*x+e)+a))*a^5*b^5-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^6-22*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^7-28*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^8-17*ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^9-
4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+
a))*b^10)*cos(f*x+e)^2+2*sin(f*x+e)*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2)*
a*b^5*(a^3+3*a^2*b+3*a*b^2+b^3)-2*cos(f*x+e)^4*sin(f*x+e)*(a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)*a*b^7*(a*b*cos(f*x+e)^2+b^2*cos(f*x+e)^2+a^2+2*a*b+b^
```

```
2)+2*sin(f*x+e)*cos(f*x+e)^6*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a*b^8-2
*sin(f*x+e)*cos(f*x+e)^2*(a+b)^(1/2)*b^6*(2*(a+b-b*cos(f*x+e)^2)^(3/2)*a^3+
4*(a+b-b*cos(f*x+e)^2)^(3/2)*a^2*b+2*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b^2-2*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2*b-4*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(3/2)*a*b^2-2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b^3-(a+b-b*cos(f*
x+e)^2)^(1/2)*a^4-3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^3*b-3*(a+b-b*cos(f*x+e)^2)
^(1/2)*a^2*b^2-(a+b-b*cos(f*x+e)^2)^(1/2)*a*b^3))/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^{3}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^3/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)^3\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{3}{\left(e+fx\right)}}{\left(a+b\sin^{2}{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.358 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=274

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{(a+b)(8a+9}{}$$

```
[Out] (a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(4*a+3*b)*
cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/b^2/f+1/3*(8*a^2+13*a*b+3*
b^2)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+
b*sin(f*x+e)^2)^(1/2)/a/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*(8*a+9*b
)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*s
in(f*x+e)^2/a)^(1/2)/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 413, 528, 524, 426, 424, 421, 419}

$$\frac{\left(8a^2+13ab+3b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3ab^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(4a+3b)\sin(}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((4*a + 3*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^2
*f) + ((8*a^2 + 13*a*b + 3*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*b^3*f*Sqrt[
1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*(8*a + 9*b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(3*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-}}{}\right.}{abf}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3ab^2f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(4a+3b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin}}{3ab^2f}$$

Mathematica [A] time = 1.12, size = 184, normalized size = 0.67

$$\frac{\sqrt{2}\,b\sin(2(e+fx))\left(8a^2-ab\cos(2(e+fx))+13ab+6b^2\right)-4a\left(8a^2+17ab+9b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+\right.}{12ab^3f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*a*(8*a^2 + 13*a*b + 3*b^2)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ellipt
icE[e + f*x, -(b/a)] - 4*a*(8*a^2 + 17*a*b + 9*b^2)*Sqrt[(2*a + b - b*Cos[2
*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)] + Sqrt[2]*b*(8*a^2 + 13*a*b + 6*
b^2 - a*b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)])/(12*a*b^3*f*Sqrt[2*a + b - b*
Cos[2*(e + f*x)]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^6}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.81, size = 415, normalized size = 1.51

$$-\frac{a\,b^{2}\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(-4a^{2}b-7a\,b^{2}-3b^{3}\right)\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)+8\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(a*b^2*sin(f*x+e)*cos(f*x+e)^4+(-4*a^2*b-7*a*b^2-3*b^3)*cos(f*x+e)^2*s
in(f*x+e)+8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3+17*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+9*(cos(f*x+e)^2)
^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2
))*a*b^2-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a^3-13*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+
(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-3*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a*b^2)/a/b^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^{6}}{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.359 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=202

$$-\frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+$$

```
[Out] (a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+(2*a+b)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^
2)^(1/2)/a/b^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-2*(a+b)*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b^2/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.21, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 413, 524, 426, 424, 421, 419}

$$-\frac{2(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) + ((
2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*b^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]
) - (2*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]
*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(b^2*f*Sqrt[a + b*Sin[e + f*x
]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a+(\ldots}{\sqrt{1-x^2}\ldots}\right.\ldots}{abf}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\ldots\right.}{b^2f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\,\mathrm{s}\ldots}\right.}{ab^2f\sqrt{1\ldots}}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\ldots\right.}{ab^2f\sqrt{1+\frac{b\,\mathrm{si}\ldots}{\ldots}}}$$

Mathematica [A] time = 0.56, size = 139, normalized size = 0.69

$$\frac{2a(2a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}E\left(e+fx\middle|-\frac{b}{a}\right)-(a+b)\left(4a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}b\sin(2(e+fx)\right)}{2ab^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - (a + b)*(4*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)] - Sqrt[2]*b*Sin[2*(e + f*x)]))/(2*a*b^2*f*Sqrt[2*a + b - b*Cos[
2*(e + f*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\cos\left(fx+e\right)^4}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.42, size = 274, normalized size = 1.36

$$-\frac{\left(-ab-b^2\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -((-a*b-b^2)*sin(f*x+e)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(
sin(f*x+e),(-1/a*b)^(1/2))*a^2-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)/a/b^2/cos(f*x+e)/(a+b*
sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.360 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=188

$$\frac{\sin(e+fx)\cos(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}}{}$$

```
[Out] cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)+EllipticE(sin(f*x+e),(-b
/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/b/f/(
1+b*sin(f*x+e)^2/a)^(1/2)-EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(co
s(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/b/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3192, 412, 493, 426, 424, 421, 419}

$$\frac{\sin(e+fx)\cos(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{bf\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[Cos[e
+ f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*S
in[e + f*x]^2])/(a*b*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]
^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a])/(b*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 493

```
Int[(x_)^(n_)/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.)*(x_)^(n_)]),
x_Symbol] :> Dist[1/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n], x], x] - Dist[a
/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x] /; FreeQ[{a, b, c, d},
x] && NeQ[b*c - a*d, 0] && (EqQ[n, 2] || EqQ[n, 4]) &&  !(EqQ[n, 2] && Simp
lerSqrtQ[-(b/a), -(d/c)])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{1-x^2}\,\sqrt{a+bx^2}}\right.}{af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x^2}\,\sqrt{a+bx^2}}\right.}{bf}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\right)\text{S}}{abf\sqrt{1+\frac{b\sin^2(e+f}{a}}}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+f}{abf\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.30, size = 133, normalized size = 0.71

$$\frac{-\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+b\sin(2(e+fx))}{\sqrt{2}\,abf\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)
] - Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b
/a)] + b*Sin[2*(e + f*x)])/(Sqrt[2]*a*b*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]
])
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cos\left(fx+e\right)^{2}}{b^{2}\cos\left(fx+e\right)^{4}-2\left(ab+b^{2}\right)\cos\left(fx+e\right)^{2}+a^{2}+2\,ab+b^{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.28, size = 145, normalized size = 0.77

$$-\frac{a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\;\mathrm{Ellipt}}{ab\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\;f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -(a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),
(-1/a*b)^(1/2))-a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipti
cE(sin(f*x+e),(-1/a*b)^(1/2))+b*sin(f*x+e)^3-b*sin(f*x+e))/a/b/cos(f*x+e)/(
a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.361 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.15, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

maple [A] time = 1.43, size = 103, normalized size = 1.02

$$\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sin\left(fx+e\right)\left(\cos^{2}\left(fx+e\right)\right)b}{a\left(a+b\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] ((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.362 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=240

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{b(a-b)\sin(e+fx)\cos(e+fx)}{af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F}{f(a+b)\sqrt{a+b\sin^2(e+}}$$

```
[Out] -(a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-(a-b)*E
llipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2)/a/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e)
,(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(
a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{b(a-b)\sin(e+fx)\cos(e+fx)}{af(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F}{f(a+b)\sqrt{a+b\sin^2(e+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(((a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])) - ((a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)^2*f*Sqrt[1 + (b*S
in[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]],
-(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{(1-x^2)^{3/2}(a+bx^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{b+}{\sqrt{1-x^2}\,(}\right.}{(a+b)f}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((a-b)\sqrt{\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left((a-b)\sqrt{\right.}{}$$

$$= -\frac{(a-b)b\cos(e+fx)\sin(e+fx)}{a(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{(a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx\right.}{a(a+b)^2f\sqrt{1}}$$

Mathematica [A] time = 1.23, size = 167, normalized size = 0.70

$$\frac{\tan(e+fx)\left(2a^2+b(b-a)\cos(2(e+fx))+ab+b^2\right)+\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\sqrt{2}\,a(}{\sqrt{2}\,af(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(Sqrt[2]*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f
*x, -(b/a)]) + Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*Ell
ipticF[e + f*x, -(b/a)] + (2*a^2 + a*b + b^2 + b*(-a + b)*Cos[2*(e + f*x)])
*Tan[e + f*x])/(Sqrt[2]*a*(a + b)^2*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [B] time = 2.75, size = 468, normalized size = 1.95

$$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b\left(a-b\right)\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a-b)*sin(f*x+e)*cos(f*x+e)^
2+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*sin(f*x+e)+(cos(f*x+e)
^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+a*b*(cos(f*x+e)^2)^(1/
2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*(
-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*
x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4
+(a+b)*cos(f*x+e)^2)^(1/2)*a^2+(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos(f*x+e)^4+(a+b)*cos(
f*x+e)^2)^(1/2)*a*b)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x
+e)))^(1/2)/a/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.363 $\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=130

$$-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\sin(e+fx)\cos^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] arctanh(sin(f*x+e)*b^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/b^(5/2)/f+1/3*(a+b)*co
s(f*x+e)^2*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(3*a-2*b)*(a+b)*si
n(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = $\{3190, 413, 385, 217, 206\}$

$$-\frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f}+\frac{(a+b)\sin(e+fx)\cos^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/(b^(5/2)*f) + ((
a + b)*Cos[e + f*x]^2*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- ((3*a - 2*b)*(a + b)*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b*Sin[e + f*x]^2
])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 217

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Subst[Int[1/(1 - b*x^2), x],
x, x/Sqrt[a + b*x^2]] /; FreeQ[{a, b}, x] &&  !GtQ[a, 0]
```

Rule 385

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> -S
imp[((b*c - a*d)*x*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] - Dist[(a*d - b
*c*(n*(p + 1) + 1))/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && (LtQ[p, -1] || ILtQ[1/n +
 p, 0])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{-a+2b+3ax^2}{\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3abf}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{\sqrt{a}}\right.}{}$$

$$= \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1-b}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{b^{5/2}f} + \frac{(a+b)\cos^2(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(3a-2b)(a+b)}{3a^2b^2f\sqrt{a+b\sin}}$$

Mathematica [A] time = 0.81, size = 128, normalized size = 0.98

$$\frac{\frac{2\sqrt{2}(a+b)\sin(e+fx)\left(-3a^2+b(2a-b)\cos(2(e+fx))+ab+b^2\right)}{a^2(2a-b\cos(2(e+fx))+b)^{3/2}} + \frac{3\tan^{-1}\left(\frac{\sqrt{2}\sqrt{-b}\sin(e+fx)}{\sqrt{2a-b\cos(2(e+fx))+b}}\right)}{\sqrt{-b}}}{3b^2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((3*ArcTan[(Sqrt[2]*Sqrt[-b]*Sin[e + f*x])/Sqrt[2*a + b - b*Cos[2*(e + f*x)
]]])/Sqrt[-b] + (2*Sqrt[2]*(a + b)*(-3*a^2 + a*b + b^2 + (2*a - b)*b*Cos[2*
(e + f*x)])*Sin[e + f*x])/(a^2*(2*a + b - b*Cos[2*(e + f*x)])^(3/2)))/(3*b^
2*f)
```

fricas [B] time = 1.62, size = 799, normalized size = 6.15

$$\left[\frac{3\left(a^2b^2\cos\left(fx+e\right)^4 + a^4 + 2a^3b + a^2b^2 - 2\left(a^3b + a^2b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{b}\log\left(128b^4\cos\left(fx+e\right)^8 - 256\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/24*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2
*b^2)*cos(f*x + e)^2)*sqrt(b)*log(128*b^4*cos(f*x + e)^8 - 256*(a*b^3 + 2*b
^4)*cos(f*x + e)^6 + 32*(5*a^2*b^2 + 24*a*b^3 + 24*b^4)*cos(f*x + e)^4 + a^
4 + 32*a^3*b + 160*a^2*b^2 + 256*a*b^3 + 128*b^4 - 32*(a^3*b + 10*a^2*b^2 +
 24*a*b^3 + 16*b^4)*cos(f*x + e)^2 - 8*(16*b^3*cos(f*x + e)^6 - 24*(a*b^2 +
 2*b^3)*cos(f*x + e)^4 - a^3 - 10*a^2*b - 24*a*b^2 - 16*b^3 + 2*(5*a^2*b +
24*a*b^2 + 24*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(b)*
sin(f*x + e)) - 8*(3*a^3*b + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b
^3 - b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^
2*b^5*f*cos(f*x + e)^4 - 2*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3
+ 2*a^3*b^4 + a^2*b^5)*f), -1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b
 + a^2*b^2 - 2*(a^3*b + a^2*b^2)*cos(f*x + e)^2)*sqrt(-b)*arctan(1/4*(8*b^2
*cos(f*x + e)^4 - 8*(a*b + 2*b^2)*cos(f*x + e)^2 + a^2 + 8*a*b + 8*b^2)*sqr
t(-b*cos(f*x + e)^2 + a + b)*sqrt(-b)/((2*b^3*cos(f*x + e)^4 + a^2*b + 3*a*
b^2 + 2*b^3 - (3*a*b^2 + 4*b^3)*cos(f*x + e)^2)*sin(f*x + e))) + 4*(3*a^3*b
 + 4*a^2*b^2 - a*b^3 - 2*b^4 - 2*(2*a^2*b^2 + a*b^3 - b^4)*cos(f*x + e)^2)*
sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e))/(a^2*b^5*f*cos(f*x + e)^4 - 2
*(a^3*b^4 + a^2*b^5)*f*cos(f*x + e)^2 + (a^4*b^3 + 2*a^3*b^4 + a^2*b^5)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{5}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^5/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 4.00, size = 383, normalized size = 2.95

$$\frac{3\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\right)a^{4}b^{4}+6\ln\left(\sin\left(fx+e\right)\sqrt{b}+\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\right)}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/b^(13/2)/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^
2+2*a*b+b^2)*(3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^4*b^4+6
*ln(sin(f*x+e)*b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^3*b^5+3*ln(sin(f*x+e)*
b^(1/2)+(a+b-b*cos(f*x+e)^2)^(1/2))*a^2*b^6+3*ln(sin(f*x+e)*b^(1/2)+(a+b-b*
cos(f*x+e)^2)^(1/2))*a^2*b^6*cos(f*x+e)^4-6*ln(sin(f*x+e)*b^(1/2)+(a+b-b*co
s(f*x+e)^2)^(1/2))*a^2*b^5*(a+b)*cos(f*x+e)^2+2*(-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(1/2)*b^(11/2)*(2*a^2+a*b-b^2)*sin(f*x+e)*cos(f*x+e)^2-sin(f*x+e)*(-
b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^(9/2)*(3*a^3+4*a^2*b-a*b^2-2*b^3))/
f
```

maxima [A] time = 1.36, size = 207, normalized size = 1.59

$$-\frac{\left(\frac{3\sin\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}b}+\frac{2a}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}b^{2}}\right)\sin\left(fx+e\right)-\frac{3\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}}\right)}{b^{\frac{5}{2}}}-\frac{2\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{\sin\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a}+-}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*((3*sin(f*x + e)^2/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*a/((b*sin(f*x
+ e)^2 + a)^(3/2)*b^2))*sin(f*x + e) - 3*arcsinh(b*sin(f*x + e)/sqrt(a*b))/
b^(5/2) - 2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - sin(f*x + e)/((
b*sin(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*b
^2) - 2*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) + 2*sin(f*x + e)/(sqr
t(b*sin(f*x + e)^2 + a)*a*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(e+f\,x\right)^5}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.364 $\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=73

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos^2(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3*cos(f*x+e)^2*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2
/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3190, 378, 191}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos^2(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]^2*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin
[e + f*x])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 378

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.), x_Symbol]
 :> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] - Dist[(c
*q)/(a*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1), x], x] /; Fre
eQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d, 0] && EqQ[n*(p + q + 1) + 1, 0]
 && GtQ[q, 0] && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cos^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx=\frac{\text{Subst}\left(\int \frac{1-x^2}{\left(a+bx^2\right)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,x,\sin(e+fx)\right)}{3af}$$
$$=\frac{\cos^2(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.70

$$\frac{3a\sin(e+fx)-(a-2b)\sin^3(e+fx)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Sin[e + f*x] - (a - 2*b)*Sin[e + f*x]^3)/(3*a^2*f*(a + b*Sin[e + f*x]^
2)^(3/2))
```

fricas [A] time = 0.72, size = 107, normalized size = 1.47

$$\frac{\left((a-2\,b)\cos\left(fx+e\right)^2+2\,a+2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{3\left(a^2b^2f\cos\left(fx+e\right)^4-2\left(a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^4+2\,a^3b+a^2b^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] 1/3*((a - 2*b)*cos(f*x + e)^2 + 2*a + 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*
sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)
^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)
```

giac [A] time = 0.78, size = 58, normalized size = 0.79

$$-\frac{\left(\frac{\left(ab-2\,b^2\right)\sin\left(fx+e\right)^2}{a^2b}-\frac{3}{a}\right)\sin\left(fx+e\right)}{3\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*((a*b - 2*b^2)*sin(f*x + e)^2/(a^2*b) - 3/a)*sin(f*x + e)/((b*sin(f*x
+ e)^2 + a)^(3/2)*f)
```

maple [A] time = 3.89, size = 120, normalized size = 1.64

$$\frac{\sin\left(fx+e\right)\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{a\,b^2+b^3}{b^2}}\;\left(a\left(\cos^2\left(fx+e\right)\right)-2b\left(\cos^2\left(fx+e\right)\right)+2a+2b\right)}{3a^2\left(b^2\left(\cos^4\left(fx+e\right)\right)-2ab\left(\cos^2\left(fx+e\right)\right)-2b^2\left(\cos^2\left(fx+e\right)\right)+a^2+2ab+b^2\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3/a^2/(b^2*cos(f*x+e)^4-2*a*b*cos(f*x+e)^2-2*b^2*cos(f*x+e)^2+a^2+2*a*b+b
^2)*sin(f*x+e)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a*cos(f*x+e)^2-2*b*
cos(f*x+e)^2+2*a+2*b)/f
```

maxima [A] time = 0.78, size = 107, normalized size = 1.47

$$\frac{\frac{2\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,a^2}+\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}a}+\frac{\sin(fx+e)}{\left(b\sin(fx+e)^2+a\right)^{\frac{3}{2}}b}-\frac{\sin(fx+e)}{\sqrt{b\sin(fx+e)^2+a}\,ab}}{3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin
(f*x + e)^2 + a)^(3/2)*a) + sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*b) -
 sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a*b))/f
```

mupad [B] time = 23.81, size = 183, normalized size = 2.51

$$-\frac{2\,e^{e\,1i+f\,x\,1i}\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\sqrt{a+b\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,\left(a\,1i-b\,2i+a\,e^{e\,2i+f\,x\,2i}\,10i+a\,e^{e\,4i+f\,x\,4i}\,1i+b\,e^{e}\right.}{3\,a^2\,f\,{\left(b-4\,a\,e^{e\,2i+f\,x\,2i}-2\,b\,e^{e\,2i+f\,x\,2i}+b\,e^{e\,4i+f\,x\,4i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] -(2*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*1
i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(a*1i - b*2i + a*exp(e*2i +
f*x*2i)*10i + a*exp(e*4i + f*x*4i)*1i + b*exp(e*2i + f*x*2i)*4i - b*exp(e*4
i + f*x*4i)*2i))/(3*a^2*f*(b - 4*a*exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*
2i) + b*exp(e*4i + f*x*4i))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.365 $\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=65

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*sin(f*x+e)/a^2/f/(a+b*sin(f
*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 192, 191}

$$\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Sin[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*Sin[e + f*x])/(3*a^2
*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 191

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(x*(a + b*x^n)^(p + 1)
)/a, x] /; FreeQ[{a, b, n, p}, x] && EqQ[1/n + p + 1, 0]
```

Rule 192

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b, n, p}, x] && ILtQ[Simplify[1/n + p + 1], 0]
 && NeQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{5/2}}\,dx,x,\sin(e+fx)\right)}{f}\\
&= \frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\,\text{Subst}\left(\int \frac{1}{\left(a+bx^2\right)^{3/2}}\,dx,x,\sin(e+fx)\right)}{3af}\\
&= \frac{\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{2\sin(e+fx)}{3a^2f\sqrt{a+b\sin^2(e+fx)}}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 47, normalized size = 0.72

$$\frac{\sin(e+fx)\left(3a+2b\sin^2(e+fx)\right)}{3a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]

[Out] (Sin[e + f*x]*(3*a + 2*b*Sin[e + f*x]^2))/(3*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))

fricas [A] time = 0.61, size = 104, normalized size = 1.60

$$-\frac{\left(2\,b\cos\left(fx+e\right)^2-3\,a-2\,b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sin\left(fx+e\right)}{3\left(a^2b^2f\cos\left(fx+e\right)^4-2\left(a^3b+a^2b^2\right)f\cos\left(fx+e\right)^2+\left(a^4+2\,a^3b+a^2b^2\right)f\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")

[Out] -1/3*(2*b*cos(f*x + e)^2 - 3*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sin(f*x + e)/(a^2*b^2*f*cos(f*x + e)^4 - 2*(a^3*b + a^2*b^2)*f*cos(f*x + e)^2 + (a^4 + 2*a^3*b + a^2*b^2)*f)

giac [A] time = 0.73, size = 48, normalized size = 0.74

$$\frac{\left(\frac{2\,b\sin\left(fx+e\right)^2}{a^2}+\frac{3}{a}\right)\sin\left(fx+e\right)}{3\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")

[Out] 1/3*(2*b*sin(f*x + e)^2/a^2 + 3/a)*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*f)

maple [A] time = 0.15, size = 56, normalized size = 0.86

$$\frac{\frac{\sin\left(fx+e\right)}{3a\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}+\frac{2\sin\left(fx+e\right)}{3a^2\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)

[Out] 1/f*(1/3*sin(f*x+e)/a/(a+b*sin(f*x+e)^2)^(3/2)+2/3/a^2*sin(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2))

maxima [A] time = 0.34, size = 55, normalized size = 0.85

$$\frac{\frac{2\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2}+\frac{\sin\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/3*(2*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^2) + sin(f*x + e)/((b*sin
(f*x + e)^2 + a)^(3/2)*a))/f
```

mupad [B] time = 21.84, size = 164, normalized size = 2.52

$$\frac{4\,e^{e\,1i+f\,x\,1i}\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\sqrt{a+b\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,\left(b\,1i-a\,e^{e\,2i+f\,x\,2i}\,6i-b\,e^{e\,2i+f\,x\,2i}\,2i+b\,e^{e\,4i+f\,x\,4}\right.}{3\,a^2\,f\,{\left(b-4\,a\,e^{e\,2i+f\,x\,2i}-2\,b\,e^{e\,2i+f\,x\,2i}+b\,e^{e\,4i+f\,x\,4i}\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] (4*exp(e*1i + f*x*1i)*(exp(e*2i + f*x*2i) - 1)*(a + b*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*(b*1i - a*exp(e*2i + f*x*2i)*
6i - b*exp(e*2i + f*x*2i)*2i + b*exp(e*4i + f*x*4i)*1i))/(3*a^2*f*(b - 4*a*
exp(e*2i + f*x*2i) - 2*b*exp(e*2i + f*x*2i) + b*exp(e*4i + f*x*4i))^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.366 $\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{b(5a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b\sin(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

```
[Out] arctanh(sin(f*x+e)*(a+b)^(1/2)/(a+b*sin(f*x+e)^2)^(1/2))/(a+b)^(5/2)/f+1/3*
b*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*b*(5*a+2*b)*sin(f*x+e)/
a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3190, 414, 527, 12, 377, 206}

$$\frac{b(5a+2b)\sin(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b\sin(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[(Sqrt[a + b]*Sin[e + f*x])/Sqrt[a + b*Sin[e + f*x]^2]]/((a + b)^(5/
2)*f) + (b*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (b*
(5*a + 2*b)*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 377

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)/((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Su
bst[Int[1/(c - (b*c - a*d)*x^n), x], x, x/(a + b*x^n)^(1/n)] /; FreeQ[{a, b
, c, d}, x] && NeQ[b*c - a*d, 0] && EqQ[n*p + 1, 0] && IntegerQ[n]
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\sec(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{b-3(a+b)+2bx^2}{(1-x^2)\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{3a(a+b)f}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subs}}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b}\,\sin(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\right)}{(a+b)^{5/2}f} + \frac{b\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{b(5a+2b}{3a^2(a+b)^2f\sqrt{}}$$

Mathematica [C] time = 9.37, size = 1291, normalized size = 10.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Sec[e + f*x]*Tan[e + f*x]*(1575*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]
] + (2100*b*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2)/a +
 (840*b^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^4)/a^2 +
 (3150*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^2)/
a + (4200*b*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x
]^2*Tan[e + f*x]^2)/a^2 + (1680*b^2*(a + b)*ArcSin[Sqrt[-(((a + b)*Tan[e +
```

```
f*x]^2)/a)]]*Sin[e + f*x]^4*Tan[e + f*x]^2)/a^3 + (1575*(a + b)^2*ArcSin[Sq
rt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Tan[e + f*x]^4)/a^2 + (2100*b*(a + b)^2*
ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e + f*x]^2*Tan[e + f*x]^4)/
a^3 + (840*b^2*(a + b)^2*ArcSin[Sqrt[-(((a + b)*Tan[e + f*x]^2)/a)]]*Sin[e
+ f*x]^4*Tan[e + f*x]^4)/a^4 + 2100*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2) + (2800*b*Sin[e + f*x]^2*Sqr
t[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a)
)^(3/2))/a + (1120*b^2*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f
*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(3/2))/a^2 + 96*Hypergeometric2F
1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2*(a + b*Sin
[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + 24*HypergeometricP
FQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)]*Sqrt[(Sec[e + f*x]^2
*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2) + (168*b*
Hypergeometric2F1[2, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^2*
Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)
/a))^(7/2))/a + (48*b*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan
[e + f*x]^2)/a)]*Sin[e + f*x]^2*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)
)/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a + (72*b^2*Hypergeometric2F1[2
, 2, 9/2, -(((a + b)*Tan[e + f*x]^2)/a)]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^
2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b)*Tan[e + f*x]^2)/a))^(7/2))/a^2 + (
24*b^2*HypergeometricPFQ[{2, 2, 2}, {1, 9/2}, -(((a + b)*Tan[e + f*x]^2)/a)
]*Sin[e + f*x]^4*Sqrt[(Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(-(((a + b
)*Tan[e + f*x]^2)/a))^(7/2))/a^2 - 1575*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a +
 b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)] - (2100*b*Sin[e + f*x]^2*Sqrt[-(((
a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*Tan[e + f*x]^2)/a^2)])/a - (84
0*b^2*Sin[e + f*x]^4*Sqrt[-(((a + b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)*
Tan[e + f*x]^2)/a^2)])/a^2))/(315*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]*Sqrt[(Se
c[e + f*x]^2*(a + b*Sin[e + f*x]^2))/a]*(1 + (b*Sin[e + f*x]^2)/a)*(-(((a +
 b)*Tan[e + f*x]^2)/a))^(5/2))
```

fricas [B] time = 0.94, size = 775, normalized size = 6.15

$$\left[\frac{3\left(a^{2}b^{2}\cos\left(fx+e\right)^{4}+a^{4}+2\,a^{3}b+a^{2}b^{2}-2\left(a^{3}b+a^{2}b^{2}\right)\cos\left(fx+e\right)^{2}\right)\sqrt{a+b}\,\log\left(\frac{\left(a^{2}+8\,ab+8\,b^{2}\right)\cos\left(fx+e\right)^{4}-8\left(a^{2}+\cdots\right.}{\cdots}\right.}{12\left(\left(a^{5}b^{2}+3\,a^{4}b^{3}+3\,a^{3}b^{4}+a^{2}b^{5}\right)f\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b + a^2
*b^2)*cos(f*x + e)^2)*sqrt(a + b)*log(((a^2 + 8*a*b + 8*b^2)*cos(f*x + e)^4
 - 8*(a^2 + 3*a*b + 2*b^2)*cos(f*x + e)^2 - 4*((a + 2*b)*cos(f*x + e)^2 - 2
*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)*sin(f*x + e) + 8*a^2
+ 16*a*b + 8*b^2)/cos(f*x + e)^4) + 4*(6*a^3*b + 14*a^2*b^2 + 10*a*b^3 + 2*
b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2
+ a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*f*cos(f
*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3*b^4 + a^2*b^5)
*f), -1/6*(3*(a^2*b^2*cos(f*x + e)^4 + a^4 + 2*a^3*b + a^2*b^2 - 2*(a^3*b +
 a^2*b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(1/2*((a + 2*b)*cos(f*x + e)^2
 - 2*a - 2*b)*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(((a*b + b^2)*co
s(f*x + e)^2 - a^2 - 2*a*b - b^2)*sin(f*x + e))) - 2*(6*a^3*b + 14*a^2*b^2
+ 10*a*b^3 + 2*b^4 - (5*a^2*b^2 + 7*a*b^3 + 2*b^4)*cos(f*x + e)^2)*sqrt(-b*
cos(f*x + e)^2 + a + b)*sin(f*x + e))/((a^5*b^2 + 3*a^4*b^3 + 3*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^4 - 2*(a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a
^2*b^5)*f*cos(f*x + e)^2 + (a^7 + 5*a^6*b + 10*a^5*b^2 + 10*a^4*b^3 + 5*a^3
*b^4 + a^2*b^5)*f)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 5.68, size = 899, normalized size = 7.13

$$-3a^{4}b^{2}\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)+3a^{4}b^{2}\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)-3\ln\left(\frac{2}{-}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/b^2/(a+b)^(1/2)/a^2/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-3*a^4*b^2*ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*a^4*b^
2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+
a))-3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x
+e)+a))*a^2*b^4+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a^2*b^4-6*a^3*b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+6*a^3*b^3*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+3*a^2*b^4*(ln(2/(sin(f*
x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))-ln(2/(1+si
n(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*cos(f*x
+e)^4-2*sin(f*x+e)*cos(f*x+e)^2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^
2)^(1/2)*b^4*(5*a+2*b)+4*sin(f*x+e)*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3
)/b^2)^(1/2)*b^3*(3*a^2+4*a*b+b^2)-6*a^2*b^3*(ln(2/(sin(f*x+e)-1)*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a+ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b-ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a-ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b)*cos(f*x+e)^2
)/f
```

maxima [B] time = 0.58, size = 272, normalized size = 2.16

$$\frac{\frac{2\,b\sin\left(fx+e\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}+\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}ab}+\frac{6\,b\sin\left(fx+e\right)}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{3}+2\,\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}b+\sqrt{b\sin\left(fx+e\right)^{2}+a}\,ab^{2}}+\frac{4\,b\sin}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{3}+}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(2*b*sin(f*x + e)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2 + (b*sin(f*x + e)^2
 + a)^(3/2)*a*b) + 6*b*sin(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + 2*sqr
t(b*sin(f*x + e)^2 + a)*a^2*b + sqrt(b*sin(f*x + e)^2 + a)*a*b^2) + 4*b*sin
(f*x + e)/(sqrt(b*sin(f*x + e)^2 + a)*a^3 + sqrt(b*sin(f*x + e)^2 + a)*a^2*
b) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)
```

```
*(sin(f*x + e) + 1)))/(a + b)^(5/2) + 3*arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*
(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\cos\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.367 $\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=243

$$-\frac{2(2a-b)(a+b)\sin(e+fx)\cos(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2b^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{(8a-b)(a}{3}$$

```
[Out] 1/3*(a+b)*cos(f*x+e)^3*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(2*a-b
)*(a+b)*cos(f*x+e)*sin(f*x+e)/a^2/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(8*a^2
+3*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+1/3*(8*
a-b)*(a+b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2
))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 283, normalized size of antiderivative = 1.16, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 413, 526, 524, 426, 424, 421, 419}

$$-\frac{2(2a-b)(a+b)\sin(e+fx)\cos(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+3ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E}{3a^2b^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]^3*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (2*(2*a - b)*(a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b^2*f*Sqrt[a + b
*Sin[e + f*x]^2]) - ((8*a^2 + 3*a*b - 2*b^2)*Sqrt[Cos[e + f*x]^2]*EllipticE
[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^2*b^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((8*a - b)*(a + b)*Sqrt[Cos[e +
 f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*S
in[e + f*x]^2)/a])/(3*a*b^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 526

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^q)/(a*b*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p
+ 1)*(c + d*x^n)^(q - 1)*Simp[c*(b*e*n*(p + 1) + b*e - a*f) + d*(b*e*n*(p +
 1) + (b*e - a*f)*(n*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, n}
, x] && LtQ[p, -1] && GtQ[q, 0]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{\left(1-x^2\right)^{5/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\operatorname{Subst}\left(\int \frac{\sqrt{1-}}{}\right.}{3abf}$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} +$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} -$$

$$= \frac{(a+b)\cos^3(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)(a+b)\cos(e+fx)\sin(e+fx)}{3a^2b^2f\sqrt{a+b\sin^2(e+fx)}} -$$

<u>**Mathematica**</u> **[A]** time = 2.12, size = 194, normalized size = 0.80

$$\frac{\frac{1}{2}(a+b)\left(4a^2(8a-b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,b\sin(2(e+fx))\left(8a^2+b(2b-5a)\cos(2(e+\right.\right.}{6a^2b^3f(2a-b\cos(2(e+fx))+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^6/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(8*a^2 + 3*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*El
lipticE[e + f*x, -(b/a)] + ((a + b)*(4*a^2*(8*a - b)*((2*a + b - b*Cos[2*(e
 + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] - 2*Sqrt[2]*b*(8*a^2 - a*b -
2*b^2 + b*(-5*a + 2*b)*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/2)/(6*a^2*b^3*f
*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

<u>**fricas**</u> **[F]** time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^6}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^6/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 1.90, size = 712, normalized size = 2.93

$$\frac{\left(5a^{2}b^{2}+3a\,b^{3}-2b^{4}\right)\sin\left(fx+e\right)\left(\cos^{4}\left(fx+e\right)\right)+\left(-4a^{3}b-6a^{2}b^{2}+2b^{4}\right)\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{\frac{\cos(2}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((5*a^2*b^2+3*a*b^3-2*b^4)*sin(f*x+e)*cos(f*x+e)^4+(-4*a^3*b-6*a^2*b^2+
2*b^4)*cos(f*x+e)^2*sin(f*x+e)-(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b
)/a)^(1/2)*a*b*(8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+7*EllipticF(sin(
f*x+e),(-1/a*b)^(1/2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-3*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b+2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+8*(cos(f*x+e)^
2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a^4+15*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+6*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2-(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^3-8*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
E(sin(f*x+e),(-1/a*b)^(1/2))*a^4-11*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2
+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b-(cos(f*x+e)^2)^(
1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))
*a^2*b^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE
(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/a^2/(a+b*sin(f*x+e)^2)^(3/2)/b^3/cos(f*x
+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{6}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^6/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^6/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^6}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^6/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**6/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.368 $\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{2(a-b)\sin(e+fx)\cos(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{2(a-b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,E\left(e+fx\middle|-\right.}{3ab^2f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/3*(a+b)*cos(f*x+e)*sin(f*x+e)/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)-2/3*(a-b)*co
s(f*x+e)*sin(f*x+e)/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(a-b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/a/b^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(2*a-b)*(cos(f*x+e)^2)^(1/2)/co
s(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b^
2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 263, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3192, 413, 527, 524, 426, 424, 421, 419}

$$-\frac{2(a-b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{3a^2b^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}-\frac{2(a-b)\sin(e+fx)\cos(e+}{3a^2bf\sqrt{a+b\sin^2(e+f}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2))
- (2*(a - b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*b*f*Sqrt[a + b*Sin[e + f*x]^
2]) - (2*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b^2*f*Sqrt[1 + (b*Sin[e
+ f*x]^2)/a]) + ((2*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b^2*f*Sqrt[a
 + b*Sin[e + f*x]^2])
```

Rule 413

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[((a*d - c*b)*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(a*b*n*(p +
 1)), x] - Dist[1/(a*b*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q -
 2)*Simp[c*(a*d - c*b*(n*(p + 1) + 1)) + d*(a*d*(n*(q - 1) + 1) - b*c*(n*(p
 + q) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a+2b+(2}{\sqrt{1-x^2}\,(a+}\right.}{3abf}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((2a-b)\sqrt{c\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\left(a^2-b^2\right)\right.}{}$$

$$= \frac{(a+b)\cos(e+fx)\sin(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(a-b)\cos(e+fx)\sin(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{2(a-b)\sqrt{\cos}}{}$$

Mathematica [A] time = 1.41, size = 171, normalized size = 0.77

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(a^2+b(b-a)\cos(2(e+fx))-2ab-b^2\right)+a^2(2a-b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right)-}{3a^2b^2f(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x,
 -(b/a)] + a^2*(2*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF
[e + f*x, -(b/a)] - Sqrt[2]*b*(a^2 - 2*a*b - b^2 + b*(-a + b)*Cos[2*(e + f*
x)])*Sin[2*(e + f*x)])/(3*a^2*b^2*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^4}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.83, size = 485, normalized size = 2.17

$$\frac{\left(2a\,b^2-2b^3\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+\left(-a^2b+a\,b^2+2b^3\right)\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((2*a*b^2-2*b^3)*sin(f*x+e)*cos(f*x+e)^4+(-a^2*b+a*b^2+2*b^3)*cos(f*x+e
)^2*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(
2*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a-EllipticF(sin(f*x+e),(-1/a*b)^(1/2
))*b-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+2*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*b)*cos(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a
)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+(cos(f*x+e)^2)^(1/2)*(-b/a
*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-(co
s(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a*b^2-2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2
)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*(-b/a*cos
(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)/a^2/(a
+b*sin(f*x+e)^2)^(3/2)/b^2/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(e+f\,x\right)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.369 $\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=217

$$\frac{(a+2b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sin(e+fx)\cos(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3abf\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\ldots}{3abf(a+\ldots}$$

```
[Out] 1/3*cos(f*x+e)*sin(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(a+2*b)*cos(f*x+
e)*sin(f*x+e)/a^2/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(a+2*b)*(cos(f*x+e)^
2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)
^(1/2)/a/b/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(cos(f*x+e)^2)^(1/2)/cos(f*
x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/b/f/(a
+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 257, normalized size of antiderivative = 1.18, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 412, 527, 524, 426, 424, 421, 419}

$$\frac{(a+2b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx\ldots\right.}{3a^2bf(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (Cos[e + f*x]*Sin[e + f*x])/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((a + 2*
b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
+ ((a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*(a + b)*f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a]) - (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b
/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*b*f*Sqrt[a + b*Sin[e
 + f*x]^2])
```

Rule 412

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*n*(p + 1)), x] + Dist[1/(
a*n*(p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(n*(p + 1)
 + 1) + d*(n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, n}, x]
&& NeQ[b*c - a*d, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-2+x^2}{\sqrt{1-x^2}\,(a+b}\right.}{3af}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+f\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left((a+2b)\sqrt{\text{co}\right.}{}$$

$$= \frac{\cos(e+fx)\sin(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(a+2b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{(a+2b)\sqrt{\cos}}{}$$

Mathematica [A] time = 1.43, size = 175, normalized size = 0.81

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(-4a^2+b(a+2b)\cos(2(e+fx))-7ab-2b^2\right)-2a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+f\right.}{6a^2bf(a+b)(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*(a + 2*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - 2*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elliptic
F[e + f*x, -(b/a)] - Sqrt[2]*b*(-4*a^2 - 7*a*b - 2*b^2 + b*(a + 2*b)*Cos[2*
(e + f*x)])*Sin[2*(e + f*x)])/(6*a^2*b*(a + b)*f*(2*a + b - b*Cos[2*(e + f*
x)])^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cos\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\left(a^2b+2\,ab^2+b^3\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cos(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 1.85, size = 552, normalized size = 2.54

$$\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^{2}(fx+e))}{a}}\,\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}b\left(\sin^{2}\left(fx+e\right)\right)+2\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e)
,(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)
^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-(cos(f
*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(
1/2))*a^2*b*sin(f*x+e)^2-(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*
EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-a*b^2*sin(f*x+e)^5-
2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^3+2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-(cos(f*x+e)^2)^(1/2)
*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3-a^2*
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/
a*b)^(1/2))*b-2*a^2*b*sin(f*x+e)^3-2*a*b^2*sin(f*x+e)^3+2*b^3*sin(f*x+e)^3+
2*a^2*b*sin(f*x+e)+3*a*b^2*sin(f*x+e))/a^2/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/b
/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(cos(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(e+f\,x\right)}^2}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cos(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Timed out
```

3.370 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\left|-\frac{b}{a}\right.\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx\right)}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\left|-\frac{b}{a}\right.\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+fx)}{3af(a+b)\left(a+b\sin^2(e+fx\right)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
 + (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int -\ldots}{\ldots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \ldots}{\ldots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2\ldots\right.}{\ldots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2\ldots}{\ldots}$$

Mathematica [A] time = 1.23, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\right|\ldots}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

maple [B] time = 1.96, size = 547, normalized size = 2.45

$$-\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}b\left(\sin^{2}\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b}{}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.371 $\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=288

$$-\frac{b\left(3a^2-7ab-2b^2\right)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(}{f(a+b)\left(a+b\right.}$$

```
[Out] -1/3*(3*a-b)*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1
/3*b*(3*a^2-7*a*b-2*b^2)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^3/f/(a+b*sin(f*x+e
)^2)^(1/2)-1/3*(3*a^2-7*a*b-2*b^2)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*Elliptic
E(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^3/f/(a+b*sin(
f*x+e)^2)^(1/2)+1/3*(3*a-b)*(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f
*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/a/(a+b)^2/f/(a+b*sin(f*x+e)^
2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.36, antiderivative size = 328, normalized size of antiderivative = 1.14, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3192, 414, 527, 524, 426, 424, 421, 419}

$$-\frac{b\left(3a^2-7ab-2b^2\right)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(3a^2-7ab-2b^2\right)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E}{3a^2f(a+b)^3\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -((3*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^2*f*(a + b*Sin[e + f*
x]^2)^(3/2)) - (b*(3*a^2 - 7*a*b - 2*b^2)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2
*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((3*a^2 - 7*a*b - 2*b^2)*Sqrt[Co
s[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a +
 b*Sin[e + f*x]^2])/(3*a^2*(a + b)^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((
3*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)^2*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rule 414

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> -Simp[(b*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*n*(p + 1)*(b*c -
a*d)), x] + Dist[1/(a*n*(p + 1)*(b*c - a*d)), Int[(a + b*x^n)^(p + 1)*(c +
d*x^n)^q*Simp[b*c + n*(p + 1)*(b*c - a*d) + d*b*(n*(p + q + 2) + 1)*x^n, x]
, x], x] /; FreeQ[{a, b, c, d, n, q}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1]
 &&  !( !IntegerQ[p] && IntegerQ[q] && LtQ[q, -1]) && IntBinomialQ[a, b, c,
 d, n, p, q, x]
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{b+3b}{\sqrt{1-x^2}\,(a}\right.}{(a+b)f}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(\right.}{}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

$$= -\frac{(3a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{b\left(3a^2-7ab-2b^2\right)\cos(e+fx)\sin(e+fx}{3a^2(a+b)^3f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [A] time = 3.35, size = 245, normalized size = 0.85

$$\frac{2a^2\left(3a^2+2ab-b^2\right)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right) - 2a^2\left(3a^2-7ab-2b^2\right)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}E\left(e+fx\right.}{6a^2f(a+b)^3(2a-b\cos(2(e+f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(3*a^2 - 7*a*b - 2*b^2)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*El
lipticE[e + f*x, -(b/a)] + 2*a^2*(3*a^2 + 2*a*b - b^2)*((2*a + b - b*Cos[2*
(e + f*x)])/a)^(3/2)*EllipticF[e + f*x, -(b/a)] + ((24*a^4 + 24*a^3*b + 41*
a^2*b^2 + 19*a*b^3 + 2*b^4 - 4*a*b*(6*a^2 - 5*a*b - 3*b^2)*Cos[2*(e + f*x)]
 + b^2*(3*a^2 - 7*a*b - 2*b^2)*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2])/(6*
a^2*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sec\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3\,a^2b-3\,ab^2-b^3+3\left(a^2b+2\,ab^2+b^3\right)\cos\left(fx-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*sec(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 3.25, size = 1082, normalized size = 3.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(3*a^2-7*a*b-2*b^2)*sin
(f*x+e)*cos(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a^3-
a^2*b-5*a*b^2-b^3)*cos(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x
+e)^2)^(1/2)*a^2*(a^2+2*a*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b*(3*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-3*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^2+7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b+2*EllipticE(sin
(f*x+e),(-1/a*b)^(1/2))*b^2)*cos(f*x+e)^2+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+
e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*a^4+5*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/
2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^3*b+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f
*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b^2-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2
)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b^3-3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)
*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^4
+4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*b+9*(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e
)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b^2+2*(-b*cos(f
*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(
a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^3)/(a+b*sin(f*x+e)^2
)^(3/2)/a^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)
^3/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate(sec(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\cos\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)),x)
```

```
[Out] int(1/(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^(5/2)), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^{2}{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(sec(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.372 $\int (d\cos(e+fx))^m \left(a+b\sin^2(e+fx)\right)^p \, dx$

Optimal. Leaf size=115

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2\right.}{f}$$

```
[Out] d*AppellF1(1/2,1/2-1/2*m,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(d*cos(f*x+
e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/f/((1
+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.09, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3193, 430, 429}

$$\frac{d\sin(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(d\cos(e+fx))^{m-1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2\right.}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (d*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a
)]*(d*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3193

```
Int[(cos[(e_.) + (f_.)*(x_)]*(d_.))^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x
_)]^2)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*d^(2*IntPart[(m - 1)/2] + 1)*(d*Cos[e + f*x])^(2*FracPart[(m - 1)/2]))/(
f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/
2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f
, m, p}, x] &&  !IntegerQ[m]
```

Rubi steps

$$\int (d\cos(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Subst}\left(\int\left(1-x^2\right)^{\frac{1}{2}(-1+}\right.}{f}$$

$$= \frac{\left(d(d\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+b\sin^2(e+fx)\right)^p\left(1\right.\right.}{}$$

$$= \frac{dF_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)(d\cos(e+fx))^{-1+m}\,\text{c}}{}$$

Mathematica [A] time = 0.90, size = 228, normalized size = 1.98

$$\frac{3a\tan(e+fx)(d\cos(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};\frac{1-m}{2},-p;\frac{3}{2};\sin^2(\right.}{f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};\frac{1-m}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-a(m-1)F_1\left(\frac{3}{2};\frac{3-m}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin}{}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)
/a)]*(d*Cos[e + f*x])^m*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(3*a*Appe
llF1[1/2, (1 - m)/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2
*b*p*AppellF1[3/2, (1 - m)/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]
^2)/a)] - a*(-1 + m)*AppellF1[3/2, (3 - m)/2, -p, 5/2, Sin[e + f*x]^2, -((b
*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\cos\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*cos(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\cos\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

maple [F] time = 2.56, size = 0, normalized size = 0.00

$$\int\left(d\cos\left(fx+e\right)\right)^m\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^2 + a\right)^p \left(d \cos\left(fx+e\right)\right)^m \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))^m*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*cos(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\cos\left(e+f\,x\right)\right)}^m\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
[Out] int((d*cos(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((d*cos(f*x+e))**m*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.373 $\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=214

$$\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin}{}\right.}{b^2f(2p+3)(2p+5)}$$

```
[Out] -(3*a+b*(7+2*p))*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b^2/f/(4*p^2+16*p+15)-
cos(f*x+e)^2*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b/f/(5+2*p)+(3*a^2+2*a*b*(
5+2*p)+b^2*(4*p^2+16*p+15))*hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*si
n(f*x+e)*(a+b*sin(f*x+e)^2)^p/b^2/f/(4*p^2+16*p+15)/((1+b*sin(f*x+e)^2/a)^p
)
```

Rubi [A] time = 0.21, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3190, 416, 388, 246, 245}

$$\frac{\left(3a^2+2ab(2p+5)+b^2\left(4p^2+16p+15\right)\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin}{}\right.}{b^2f(2p+3)(2p+5)}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -(((3*a + b*(7 + 2*p))*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b^2*f*
(3 + 2*p)*(5 + 2*p))) - (Cos[e + f*x]^2*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)
^(1 + p))/(b*f*(5 + 2*p)) + ((3*a^2 + 2*a*b*(5 + 2*p) + b^2*(15 + 16*p + 4*
p^2))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^p)/(b^2*f*(3 + 2*p)*(5 + 2*p)*(1 + (b*Sin[e + f*x]^
2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 416

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[(d*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1))/(b*(n*(p + q) + 1)),
x] + Dist[1/(b*(n*(p + q) + 1)), Int[(a + b*x^n)^p*(c + d*x^n)^(q - 2)*Simp
[c*(b*c*(n*(p + q) + 1) - a*d) + d*(b*c*(n*(p + 2*q - 1) + 1) - a*d*(n*(q -
```

```
 1) + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, n, p}, x] && NeQ[b*c - a*d,
 0] && GtQ[q, 1] && NeQ[n*(p + q) + 1, 0] &&  !IGtQ[p, 1] && IntBinomialQ[a
, b, c, d, n, p, q, x]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+bx^2\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=-\frac{\cos^2(e+fx)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(5+2p)}+\frac{\text{Subst}\left(\int\left(a+\right.\right.}{}$$

$$=-\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}-\frac{\cos^2(e+f}{}$$

$$=-\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}-\frac{\cos^2(e+f}{}$$

$$=-\frac{(3a+b(7+2p))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{b^2f(3+2p)(5+2p)}-\frac{\cos^2(e+f}{}$$

Mathematica [C] time = 0.40, size = 191, normalized size = 0.89

$$\frac{3a\sin(e+fx)\cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};-2,-p;\frac{3}{2};\sin^2(e+\right.}{f\left(2\sin^2(e+fx)\left(bpF_1\left(\frac{3}{2};-2,1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-2aF_1\left(\frac{3}{2};-1,-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+f}{a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Co
s[e + f*x]^4*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2, -
2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + 2*(b*p*AppellF1[3/2,
 -2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 2*a*AppellF1[3/
2, -1, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^5,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{p} \cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

maple [F] time = 2.93, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{2}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{p} \cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(e+f\,x\right)^{5}\,{\left(b\,\sin\left(e+f\,x\right)^{2}+a\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.374 $\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=124

$$\frac{(a+b(2p+3))\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{bf(2p+3)}-\frac{\sin(e+fx)\left(a\right.}{bf}$$

```
[Out] -sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1+p)/b/f/(3+2*p)+(a+b*(3+2*p))*hypergeom([1
/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^p/b/f/(3+2*p
)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 119, normalized size of antiderivative = 0.96, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3190, 388, 246, 245}

$$\frac{\left(\frac{a}{2bp+3b}+1\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}-\frac{\sin(e+fx)\left(a+\right.}{bf(2}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(1 + p))/(b*f*(3 + 2*p))) + ((1 + a/
(3*b + 2*b*p))*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin
[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 388

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)), x_Symbol] :> Si
mp[(d*x*(a + b*x^n)^(p + 1))/(b*(n*(p + 1) + 1)), x] - Dist[(a*d - b*c*(n*(
p + 1) + 1))/(b*(n*(p + 1) + 1)), Int[(a + b*x^n)^p, x], x] /; FreeQ[{a, b,
 c, d, n}, x] && NeQ[b*c - a*d, 0] && NeQ[n*(p + 1) + 1, 0]
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)\left(a+bx^2\right)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right)\text{Subst}\left(\int \left(a+bx^2\right.\right.}{f}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(\left(1+\frac{a}{3b+2bp}\right)\left(a+b\sin^2(e+f\right.\right.}{}$$

$$= -\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^{1+p}}{bf(3+2p)} + \frac{\left(1+\frac{a}{3b+2bp}\right){}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin}{}\right.}{}$$

Mathematica [A] time = 0.19, size = 120, normalized size = 0.97

$$-\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}\left(\left(a+b\sin^2(e+fx)\right)\left(\frac{b\sin^2(e+fx)}{a}+1\right)^p-(a+b(2p+3))\,{}_2F_1\right.}{bf(2p+3)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
[Out] -((Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p*(-((a + b*(3 + 2*p))*Hypergeometri
c2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]) + (a + b*Sin[e + f*x]^2)*(1 +
(b*Sin[e + f*x]^2)/a)^p))/(b*f*(3 + 2*p)*(1 + (b*Sin[e + f*x]^2)/a)^p))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

maple [F] time = 7.10, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{3}\,\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.375 $\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] hypergeom([1/2, -p],[3/2],-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^
p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3190, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(a+bx^2\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^2}{a}\right)^p\,dx,x,\text{si}\right.}{f}$$

$$=\frac{{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(\right.}{a}}{f}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.00

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[e + f*x]^2)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.42, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

maple [F] time = 2.60, size = 0, normalized size = 0.00

$$\int\cos\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e), x)
```

mupad [B] time = 15.22, size = 64, normalized size = 0.96

$$\frac{\sin\left(e+f\,x\right)\left(b\sin\left(e+f\,x\right)^2+a\right)^p{}_2\mathrm{F}_1\left(\frac{1}{2},-p;\ \frac{3}{2};\ -\frac{b\sin\left(e+f\,x\right)^2}{a}\right)}{f\left(\frac{b\sin\left(e+f\,x\right)^2}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^2)^p*hypergeom([1/2, -p], 3/2, -(b*sin(e
+ f*x)^2)/a))/(f*((b*sin(e + f*x)^2)/a + 1)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.376 $\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.07, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3190, 430, 429}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{1-x^2}\,dx, x, \sin(e\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e+fx\right.}{f}$$

Mathematica [F] time = 4.45, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

maple [F] time = 1.94, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x),x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.377 $\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=76

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^2)^p/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3190, 430, 429}

$$\frac{\sin(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3190

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^
(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Su
bst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[(m - 1)/2]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx^2)^p}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^2}{a}\right)^p}{(1-x^2)^2}\,dx, x, s\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};2,-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^2(e\right.}{f}$$

Mathematica [F] time = 6.56, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

maple [F] time = 1.96, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^3, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^3, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.378 $\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,-3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^p\,dx,x,\sin\right.}{f}$$
$$=\frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)}{f}$$
$$=\frac{F_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(\right.}{f}$$

Mathematica [B] time = 0.58, size = 199, normalized size = 2.21

$$\frac{3a\sin(e+fx)\cos^3(e+fx)\left(a+b\sin^2(e+fx)\right)^pF_1\left(\frac{1}{2};-\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx\right.}{f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};-\frac{3}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-3aF_1\left(\frac{3}{2};-\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
Cos[e + f*x]^3*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^p)/(f*(3*a*AppellF1[1/2,
 -3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[
3/2, -3/2, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - 3*a*Appel
lF1[3/2, -1/2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f
*x]^2))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

maple [F] time = 4.61, size = 0, normalized size = 0.00

$$\int\left(\cos^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^{2}+a\right)^{p} \cos \left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos \left(e+f\,x\right)^{4}\,\left(b\,\sin \left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.379 $\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,-1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)
^p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt
[Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int\sqrt{1-x^2}\left(a+bx^2\right)^p\,dx,x,\text{si}\right.}{f}$$
$$= \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-}\right.}{f}$$
$$= \frac{F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\,\text{si}\right.}{f}$$

Mathematica [B] time = 0.47, size = 195, normalized size = 2.17

$$\frac{3a\sin(2(e+fx))\left(a+b\sin^2(e+fx)\right)^p F_1\left(\frac{1}{2};-\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\right.}{2f\left(\sin^2(e+fx)\left(2bpF_1\left(\frac{3}{2};-\frac{1}{2},1-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)-aF_1\left(\frac{3}{2};\frac{1}{2},-p;\frac{5}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right.\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (3*a*AppellF1[1/2, -1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*
(a + b*Sin[e + f*x]^2)^p*Sin[2*(e + f*x)])/(2*f*(3*a*AppellF1[1/2, -1/2, -p
, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] + (2*b*p*AppellF1[3/2, -1/2
, 1 - p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)] - a*AppellF1[3/2, 1/
2, -p, 5/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)])*Sin[e + f*x]^2))
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

maple [F] time = 6.17, size = 0, normalized size = 0.00

$$\int\left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\cos\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*cos(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{2}\,\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
[Out] Timed out
```

3.380 $\int \left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,1/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.214, Rules used = {3185, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 1/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3185

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f
*x]), Subst[Int[(a + b*ff^2*x^2)^p/Sqrt[1 - ff^2*x^2], x], x, Sin[e + f*x]/
ff], x]] /; FreeQ[{a, b, e, f, p}, x] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{\sqrt{1-x^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{(1+}{\sqrt{1}}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{1}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e+fx)\right)^p\left(1+\right.}{f}$$

Mathematica [A] time = 0.50, size = 145, normalized size = 1.61

$$\frac{2^{-p-1}\csc(2(e+fx))\sqrt{-\frac{b\sin^2(e+fx)}{a}}\sqrt{\frac{b\cos^2(e+fx)}{a+b}}\,(2a-b\cos(2(e+fx))+b)^{p+1}F_1\left(p+1;\frac{1}{2},\frac{1}{2};p+2;\frac{2a+b-b\cos(2(e+f\right.}{2(a+b)}}{bf(p+1)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (2^(-1 - p)*AppellF1[1 + p, 1/2, 1/2, 2 + p, (2*a + b - b*Cos[2*(e + f*x)])
/(2*(a + b)), (2*a + b - b*Cos[2*(e + f*x)])/(2*a)]*Sqrt[(b*Cos[e + f*x]^2)
/(a + b)]*(2*a + b - b*Cos[2*(e + f*x)])^(1 + p)*Csc[2*(e + f*x)]*Sqrt[-((b
*Sin[e + f*x]^2)/a)])/(b*f*(1 + p))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

maple [F] time = 1.36, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \left(b\sin\left(e+fx\right)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.381 $\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,3/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\,\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 3/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{(a+bx^2)^p}{(1-x^2)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-}\right.}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{3}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\,\left(a+b\sin\right.}{f}$$

Mathematica [F] time = 4.55, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

maple [F] time = 1.77, size = 0, normalized size = 0.00

$$\int \left(\sec^2\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.382 $\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$

Optimal. Leaf size=90

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

```
[Out] AppellF1(1/2,5/2,-p,3/2,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(a+b*sin(f*x+e)^2)^
p*(cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.08, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3192, 430, 429}

$$\frac{\sqrt{\cos^2(e+fx)}\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[1/2, 5/2, -p, 3/2, Sin[e + f*x]^2, -((b*Sin[e + f*x]^2)/a)]*Sqrt[
Cos[e + f*x]^2]*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + (b*Sin[e + f
*x]^2)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3192

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff*Sqrt[
Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a
+ b*ff^2*x^2)^p, x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x]
&& IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{\left(1-x^2\right)^{5/2}}dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e+fx)}{a}\right)^{-p}\right) S}{f}$$

$$= \frac{F_1\left(\frac{1}{2};\frac{5}{2},-p;\frac{3}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\sqrt{\cos^2(e+fx)}\left(a+b\sin^2(e\right.}{f}$$

Mathematica [F] time = 6.78, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p,x]
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^p, x]
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

maple [F] time = 1.24, size = 0, normalized size = 0.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*sec(f*x + e)^4, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^2+a\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^2)^p/cos(e + f*x)^4, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.383 $\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=219

$$-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}\right.}{}$$

```
[Out] 1/3*(a^(4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(5/3)/d-1/6*
(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^
2)/a^(2/3)/b^(5/3)/d-2/3*ln(a+b*sin(d*x+c)^3)/b/d+1/2*sin(d*x+c)^2/b/d+1/3*
(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.29, antiderivative size = 219, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3223, 1887, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d}+\frac{\left(a^{4/3}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(2/3)*b^(5/3)*d) + ((a^(4/3) + b^(4/3))*Log[a^(1/3) + b^
(1/3)*Sin[c + d*x]])/(3*a^(2/3)*b^(5/3)*d) - ((a^(4/3) + b^(4/3))*Log[a^(2/
3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*b^(
5/3)*d) - (2*Log[a + b*Sin[c + d*x]^3])/(3*b*d) + Sin[c + d*x]^2/(2*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
```

```
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{x}{b}+\frac{b-ax-2bx^2}{b(a+bx^3)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\sin^2(c+dx)}{2bd} + \frac{\text{Subst}\left(\int \frac{b-ax-2bx^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= \frac{\sin^2(c+dx)}{2bd} - \frac{2\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d} + \frac{\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= -\frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\sin^2(c+dx)}{2bd} + \frac{\left(\frac{1}{a^{2/3}}+\frac{a^{2/3}}{b^{4/3}}\right)\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}x}\,dx, x, \sin(\right.}{3d}$$
$$= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{2\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\sin^2(c+dx)}{2bd} -$$
$$= \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)\right.}{6a^{2/3}b^{5/3}d}$$
$$= \frac{\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}b^{5/3}d} + \frac{\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}b^{5/3}d} - \frac{\left(a^{4/3}+\right.}{}$$

Mathematica [C] time = 0.24, size = 203, normalized size = 0.93

$$\frac{-b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right) - 3a^{2/3}\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right) - 4a^{2/3}\log\left(\right.}{6a^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*Sqrt[3]*b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1
/3))] + 2*b^(2/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - b^(2/3)*Log[a^(2/3)
 - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 4*a^(2/3)*Log[a
 + b*Sin[c + d*x]^3] + 3*a^(2/3)*Sin[c + d*x]^2 - 3*a^(2/3)*Hypergeometric2
F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*b*d)
```

fricas [C] time = 1.31, size = 3216, normalized size = 14.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(2*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4
)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3
)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d*log(1/4*((1/2)^(1/3)*(I*sqrt
(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)
```

```
/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(
2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*
d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*(
(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5
*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt
(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a
^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d + (a^4 - b^4)*sin(d*x + c)) + 6*cos(d*x
+ c)^2 - (((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*b*d + 3*sqrt(1/3)*b*d*sqrt(-(((
1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(
3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^
4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b
^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3)
)^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3)
+ (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3))
)/(b*d)) - 12)*log(1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2
*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d)
+ 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 +
 b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 + 2*
a^3*b - 2*a*b^3 - 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(
b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3
))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) +
 (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))
*d - 3/4*sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^
2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2
*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^
3*b^2 + a*b^4)*d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d
) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2
 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^
(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
- (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3)
+ 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
 b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 2*(a^4 - b^4)*sin(d*x + c)) - (((1/2)
^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3)
 - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) +
 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 -
b^4)/(a^2*b^5*d^3))^(1/3)))*b*d - 3*sqrt(1/3)*b*d*sqrt(-(((1/2)^(1/3)*(I*sq
rt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^
4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2
*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^
5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 -
 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) - 32/(b*
d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))/(b*d)) - 12)*l
og(-1/4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)
/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)
*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*
d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*a^3*b^3*d^2 - 2*a^3*b + 2*a*b^3
 + 1/2*(4*a^3*b^2 - a*b^4)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4
 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b
*d) + 2*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b
^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*d - 3/4*sqrt(1
/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a
^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/3)*(-
```

```
I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3
) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))*a^3*b^3*d^2 - 2*(2*a^3*b^2 + a*b^4)*
d)*sqrt(-(((1/2)^(1/3)*(I*sqrt(3) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^
4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3) + 4/(b*d) + 2*(1/2)^(2/
3)*(-I*sqrt(3) + 1)/(b^2*d^2*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^
5*d^3) - (a^4 - b^4)/(a^2*b^5*d^3))^(1/3)))^2*b*d - 8*(1/2)^(1/3)*(I*sqrt(3
) + 1)*(2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(
a^2*b^5*d^3))^(1/3) - 32/(b*d) - 16*(1/2)^(2/3)*(-I*sqrt(3) + 1)/(b^2*d^2*(
2/(b^3*d^3) + (a^4 - 2*a^2*b^2 + b^4)/(a^2*b^5*d^3) - (a^4 - b^4)/(a^2*b^5*
d^3))^(1/3)))/(b*d)) + 2*(a^4 - b^4)*sin(d*x + c)))/(b*d)
```

giac [A] time = 0.19, size = 221, normalized size = 1.01

$$\frac{\frac{3\sin(dx+c)^2}{b}-\frac{4\log\left(\left|b\sin(dx+c)^3+a\right|\right)}{b}+\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2+\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3}+\frac{\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\log\left(\sin(dx+c)\right.}{ab^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/6*(3*sin(d*x + c)^2/b - 4*log(abs(b*sin(d*x + c)^3 + a))/b + 2*sqrt(3)*((
-a*b^2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*
sin(d*x + c))/(-a/b)^(1/3))/(a*b^3) + ((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*
a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b^3) +
 2*(a*b^4*(-a/b)^(1/3) - b^5)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x
+ c)))/(a*b^5))/d
```

maple [A] time = 0.92, size = 278, normalized size = 1.27

$$\frac{\sin^2(dx+c)}{2bd}+\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6db\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\right.}{3}\right.}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/2*sin(d*x+c)^2/b/d+1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b
/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a
/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+1/3/d/b^
2*a/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b^2*a/(a/b)^(1/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/3/d/b^2*a*3^(1/2)/(a/b)^(1/3)
*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-2/3*ln(a+b*sin(d*x+c)^3)/
b/d
```

maxima [A] time = 0.89, size = 210, normalized size = 0.96

$$\frac{\frac{9\sin(dx+c)^2}{b}-\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-4\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{4a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(b\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)+a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\right.\right.}{b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(9*sin(d*x + c)^2/b - 2*sqrt(3)*(a*(3*(a/b)^(2/3) - 4) - b*(3*(a/b)^(1
/3) - 4*a/b))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3
))/(a*b) - 3*(b*(4*(a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log(sin(d*x + c)^2 - (
a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b^2*(a/b)^(2/3)) - 6*(b*(2*(a/b)^(2
/3) - 1) - a*(a/b)^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(b^2*(a/b)^(2/3))
)/d
```

mupad [B] time = 15.02, size = 229, normalized size = 1.05

$$\frac{\left(\sum_{k=1}^{3} \ln\left(3\,a + \mathrm{root}\left(27\,a^2\,b^5\,d^3 + 54\,a^2\,b^4\,d^2 + 27\,a^2\,b^3\,d + 2\,a^2\,b^2 - b^4 - a^4, d, k\right)\,\left(12\,a\,b + 3\,b^2\,\sin\left(c + d\,x\right)\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3),x)
```

```
[Out] (symsum(log(3*a + root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*a
^2*b^2 - b^4 - a^4, d, k)*(12*a*b + 3*b^2*sin(c + d*x) + 9*root(27*a^2*b^5*
d^3 + 54*a^2*b^4*d^2 + 27*a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k)*a*b^2) +
 (sin(c + d*x)*(a^2 + 2*b^2))/b)*root(27*a^2*b^5*d^3 + 54*a^2*b^4*d^2 + 27*
a^2*b^3*d + 2*a^2*b^2 - b^4 - a^4, d, k), k, 1, 3) + sin(c + d*x)^2/(2*b))/
d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.384 $\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=167

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}-\frac{\log}{}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*ln(a+b*sin(
d*x+c)^3)/b/d-1/3*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2)
)/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1871, 200, 31, 634, 617, 204, 628, 260}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}-\frac{\log}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d) - Log[a + b*Sin[c + d*x]^3]/(3*b*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
```

```
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\ldots}{a^{2/3}-\ldots}\right.}{\ldots}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a+b\sin^3(c+dx)\right)}{3bd} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}x+b^{2/3}x\ldots}\right.}{2\sqrt[3]{a}\,\ldots}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d} - \ldots$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+\ldots\right.}{6a^{2/3}\sqrt[3]{\ldots}}$$

Mathematica [A] time = 0.14, size = 139, normalized size = 0.83

$$\frac{\left((-1)^{2/3}b^{2/3}-a^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)+\left(b^{2/3}-a^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\left(a^{2/3}+\sqrt[3]{-1}b\right.}{3a^{2/3}bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-a^(2/3) + (-1)^(2/3)*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[c
+ d*x]] + (-a^(2/3) + b^(2/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - (a^(2/
3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]])/(3
*a^(2/3)*b*d)
```

fricas [C] time = 70.62, size = 2298, normalized size = 13.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(6*sqrt(1/3)*b*d*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(
a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/
2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^
3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt(
((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^
2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*s
in(d*x + c) + 2*(a*b^2*d*sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b
*d)) + 4*a^2 + 4*b^2)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b
*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*a*b*d^2 - 2*a*d)*sqrt((
((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^
2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/
(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d
 + 4)/(b^2*d^2)) + sqrt(1/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1
/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^3 -
8*a*b*d*sin(d*x + c) + 4*a^2*d + 4*(a*b^2*d^2*sin(d*x + c) - a^2*b*d^2)*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b
^3*d^3))^(1/3) + 2/(b*d)))*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3)
+ 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4
*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a
^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)))/b) - 6*sqrt(1/3)*b*d*sqr
t((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/
(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*
(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*
b*d + 4)/(b^2*d^2))*arctan(-1/8*(2*sqrt(1/3)*sqrt(((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*
d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) + 2*(a*b^2*d*
sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^
2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + 4*a^2 + 4*b^2)*(((
1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*
b^3*d^3))^(1/3) + 2/(b*d))*a*b*d^2 - 2*a*d)*sqrt((((1/2)^(1/3)*(I*sqrt(3) +
 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*
d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3)
 - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d + 4)/(b^2*d^2)) - sqrt(1
/3)*(((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2
)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^3 - 8*a*b*d*sin(d*x + c) + 4*
a^2*d + 4*(a*b^2*d^2*sin(d*x + c) - a^2*b*d^2)*((1/2)^(1/3)*(I*sqrt(3) + 1)
*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))
)*sqrt((((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 -
```

```
b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))^2*b^2*d^2 - 4*((1/2)^(1/3)*(I*sqrt(3)
+ 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b
*d))*b*d + 4)/(b^2*d^2)))/b) - ((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) +
1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d*log(1/4*((1
/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b
^3*d^3))^(1/3) + 2/(b*d))^2*a^2*b^2*d^2 - b^2*cos(d*x + c)^2 + 2*a*b*sin(d*
x + c) - (a*b^2*d*sin(d*x + c) + a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(
b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + a^
2 + b^2) + (((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^
2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d))*b*d - 6)*log(((1/2)^(1/3)*(I*sqrt(
3) + 1)*(1/(b^3*d^3) + 1/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2
/(b*d))^2*a^2*b^2*d^2 - 4*b^2*cos(d*x + c)^2 - 4*a*b*sin(d*x + c) + 2*(a*b^
2*d*sin(d*x + c) - 2*a^2*b*d)*((1/2)^(1/3)*(I*sqrt(3) + 1)*(1/(b^3*d^3) + 1
/(a^2*b*d^3) - (a^2 - b^2)/(a^2*b^3*d^3))^(1/3) + 2/(b*d)) + 4*a^2 + 4*b^2)
)/(b*d)
```

giac [A] time = 0.18, size = 156, normalized size = 0.93

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a}+\frac{2\log\left(\left|b\sin(dx+c)^{3}+a\right|\right)}{b}-\frac{2\sqrt{3}\left(-ab^{2}\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^{2}\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^{2}+\right.}{a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a + 2*log(abs(b
*sin(d*x + c)^3 + a))/b - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/
b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x
 + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a*b))/d
```

maple [A] time = 0.84, size = 141, normalized size = 0.84

$$\frac{\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^{2}(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6db\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(a+\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b/(a/b)^(2/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3*ln(a+b*sin(d*x+c)^3)/b/d
```

maxima [A] time = 0.45, size = 159, normalized size = 0.95

$$\frac{\frac{2\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{3\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(dx+c)^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\mathrm{si}\right.}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{18d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b
)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b) - 3*(2*(a/b)^(2/3) + 1)*log(si
n(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(b*(a/b)^(2/3)) - 6*
((a/b)^(2/3) - 1)*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/3)))/d
```

mupad [B] time = 15.02, size = 153, normalized size = 0.92

$$\frac{\sum_{k=1}^{3} \ln\left(\left(\mathrm{root}\left(27\,a^2\,b^3\,d^3+27\,a^2\,b^2\,d^2+9\,a^2\,b\,d-b^2+a^2,d,k\right)\,b\,3+1\right)\,\left(a+b\,\sin\left(c+d\,x\right)+\mathrm{root}\left(27\,a^2\,b^3\,d^3\right.\right.\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log((3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2 + 9*a^2*b*d - b^2 + a^2,
 d, k)*b + 1)*(a + b*sin(c + d*x) + 3*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2
+ 9*a^2*b*d - b^2 + a^2, d, k)*a*b))*root(27*a^2*b^3*d^3 + 27*a^2*b^2*d^2 +
 9*a^2*b*d - b^2 + a^2, d, k), k, 1, 3)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Timed out
```

3.385 $\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=144

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

```
[Out] 1/3*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/b^(1/3)/d-1/6*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/b^(1/3)/d-1/3*arctan(1/3*
(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3223, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d}+\frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d}-\frac{\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -(ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))]/(Sqrt[3]*a^(
2/3)*b^(1/3)*d)) + Log[a^(1/3) + b^(1/3)*Sin[c + d*x]]/(3*a^(2/3)*b^(1/3)*d
) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]/(6
*a^(2/3)*b^(1/3)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d} + \frac{\text{Subst}\left(\int \frac{2\sqrt[3]{a}-\sqrt[3]{b}\,x}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \sin(c+dx)\right)}{3a^{2/3}d}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\,x+b^{2/3}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt[3]{a}\,d} - \frac{\text{Subst}(}{}$$

$$= \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}\sqrt[3]{b}\,d} + \frac{\text{Su}}{}$$

$$= -\frac{\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\sqrt[3]{b}\,d} + \frac{\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3a^{2/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\sin(c+dx)\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

Mathematica [A] time = 0.05, size = 116, normalized size = 0.81

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\,\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)-2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)+2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx}{\sqrt{3}\sqrt[3]{a}}\right.}{6a^{2/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/6*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))
] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*S
in[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(a^(2/3)*b^(1/3)*d)
```

fricas [A] time = 0.51, size = 401, normalized size = 2.78

$$\left[\frac{3\sqrt{\frac{1}{3}}ab\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\left(a^2b\right)^{\frac{1}{3}}a\sin(dx+c)+a^2+3\sqrt{\frac{1}{3}}\left(2ab\cos(dx+c)^2-2ab-\left(a^2b\right)^{\frac{2}{3}}\sin(dx+c)+\left(a^2b\right)^{\frac{1}{3}}a\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}+2\left(ab\cos(dx+c)\right.}{\left(b\cos(dx+c)^2-b\right)\sin(dx+c)-a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] [1/6*(3*sqrt(1/3)*a*b*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*
x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*si
n(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^
2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) - (a^2*b)
^(2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)
^(1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d
), 1/6*(6*sqrt(1/3)*a*b*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) - (a^2*b)^(
2/3)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(
1/3)*a) + 2*(a^2*b)^(2/3)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^2*b*d)]
```

giac [A] time = 0.15, size = 137, normalized size = 0.95

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a - 2*sqrt(3)*(
-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/
3))/(a*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) +
 (-a/b)^(2/3))/(a*b))/d
```

maple [A] time = 0.47, size = 120, normalized size = 0.83

$$\frac{\ln\left(\sin\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2\left(dx+c\right)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6db\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3db\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d/b/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/6/d/b/(a/b)^(2/3)*ln(sin(d
*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/b/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))
```

maxima [A] time = 0.44, size = 121, normalized size = 0.84

$$\frac{\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{b\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/6*(2*sqrt(3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/
3))/(b*(a/b)^(2/3)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)
^(2/3))/(b*(a/b)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(b*(a/b)^(2/3))
)/d
```

mupad [B] time = 0.28, size = 123, normalized size = 0.85

$$\frac{\ln\left(b^{1/3}\,\sin\left(c+d\,x\right)+a^{1/3}\right)}{3\,a^{2/3}\,b^{1/3}\,d}+\frac{\ln\left(3\,b^2\,\sin\left(c+d\,x\right)+\frac{3\,a^{1/3}\,b^{5/3}\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{2}\right)\,\left(-1+\sqrt{3}\,1{}\mathrm{i}\right)}{6\,a^{2/3}\,b^{1/3}\,d}-\frac{\ln\left(3\,b^2\,\sin\left(c+d\,x\right)-\frac{3\,a}{}\right.}{6\,a^{2/3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)^3),x)
```

```
[Out] log(b^(1/3)*sin(c + d*x) + a^(1/3))/(3*a^(2/3)*b^(1/3)*d) + (log(3*b^2*sin(
c + d*x) + (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i - 1))/2)*(3^(1/2)*1i - 1))/(6*a^(
2/3)*b^(1/3)*d) - (log(3*b^2*sin(c + d*x) - (3*a^(1/3)*b^(5/3)*(3^(1/2)*1i
+ 1))/2)*(3^(1/2)*1i + 1))/(6*a^(2/3)*b^(1/3)*d)
```

sympy [A] time = 8.85, size = 250, normalized size = 1.74

$$\begin{cases} \frac{\tilde{\infty}x\cos(c)}{\sin^3(c)} \\ -\frac{1}{2bd\sin^2(c+dx)} \\ \frac{\sin(c+dx)}{ad} \\ \frac{x\cos(c)}{a+b\sin^3(c)} \\ -\frac{\sqrt[3]{-1}\sqrt[3]{\frac{1}{b}}\log\left(-\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}+\sin(c+dx)\right)}{3a^{\frac{2}{3}}d}+\frac{\sqrt[3]{-1}\sqrt[3]{\frac{1}{b}}\log\left(4(-1)^{\frac{2}{3}}a^{\frac{2}{3}}\left(\frac{1}{b}\right)^{\frac{2}{3}}+4\sqrt[3]{-1}\sqrt[3]{a}\sqrt[3]{\frac{1}{b}}\sin(c+dx)+4\sin^2(c+dx)\right)}{6a^{\frac{2}{3}}d}+\frac{\sqrt[3]{-1}\sqrt{3}\sqrt[3]{\frac{1}{b}}\operatorname{atan}\left(\cdots\right)}{3\cdots} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**3, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (-1/(2*
b*d*sin(c + d*x)**2), Eq(a, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (x*cos(c)/
(a + b*sin(c)**3), Eq(d, 0)), (-(-1)**(1/3)*(1/b)**(1/3)*log(-(-1)**(1/3)*a
**(1/3)*(1/b)**(1/3) + sin(c + d*x))/(3*a**(2/3)*d) + (-1)**(1/3)*(1/b)**(1
```

```
/3)*log(4*(-1)**(2/3)*a**(2/3)*(1/b)**(2/3) + 4*(-1)**(1/3)*a**(1/3)*(1/b)*
*(1/3)*sin(c + d*x) + 4*sin(c + d*x)**2)/(6*a**(2/3)*d) + (-1)**(1/3)*sqrt(
3)*(1/b)**(1/3)*atan(sqrt(3)/3 - 2*(-1)**(2/3)*sqrt(3)*sin(c + d*x)/(3*a**(
1/3)*(1/b)**(1/3)))/(3*a**(2/3)*d), True))
```

3.386 $\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=290

$$-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)}+\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)}{3}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)/d+1/2*ln(1+sin(d*x+c))/(a-b)/d-1/3*b^(1/3)*(a^(
4/3)+b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(2/3)/(a^2-b^2)/d+1/6*b^(1/3
)*(a^(4/3)+b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c
)^2)/a^(2/3)/(a^2-b^2)/d-1/3*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)/d-1/3*b^(1/3)
*(a^(4/3)-b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2
))/a^(2/3)/(a^2-b^2)/d*3^(1/2)
```

Rubi [A] time = 0.33, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.476, Rules used = {3223, 2074, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)}-\frac{b\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)}-\frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)}{3a^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((b^(1/3)*(a^(4/3) - b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)*d)) - Log[1 - Sin[c + d*x]]/
(2*(a + b)*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)*d) - (b^(1/3)*(a^(4/3) + b
^(4/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)*d) + (b
^(1/3)*(a^(4/3) + b^(4/3))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(
2/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)*d) - (b*Log[a + b*Sin[c + d*x]
^3])/(3*(a^2 - b^2)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)(-1+x)} + \frac{1}{2(a-b)(1+x)} + \frac{b(b-ax+bx^2)}{(-a^2+b^2)(a+bx^3)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{b-ax+bx^2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{b-ax}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{(a^2-b^2)\,d} -$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{b\log\left(a+b\sin^3(c+dx)\right)}{3(a^2-b^2)\,d} - \frac{b^{2/3}\,\text{Subs}}{}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c\right.}{3a^{2/3}(a^2-b^2)\,d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d} - \frac{\sqrt[3]{b}\left(a^{4/3}+b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c\right.}{3a^{2/3}(a^2-b^2)\,d}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}(a^2-b^2)\,d} - \frac{\log(1-\sin(c+dx))}{2(a+b)d} + \frac{\log(1+\sin(c+dx))}{2(a-b)d}$$

Mathematica [C] time = 0.20, size = 268, normalized size = 0.92

$$\frac{b^{5/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)+3a^{2/3}b\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)-2a^{2/3}b\log}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/
3))] - 3*a^(5/3)*Log[1 - Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 - Sin[c + d*x]]
+ 3*a^(5/3)*Log[1 + Sin[c + d*x]] + 3*a^(2/3)*b*Log[1 + Sin[c + d*x]] - 2*b
^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] + b^(5/3)*Log[a^(2/3) - a^(1/3)*
b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2] - 2*a^(2/3)*b*Log[a + b*Sin[
c + d*x]^3] + 3*a^(2/3)*b*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(6*a^(2/3)*(a - b)*(a + b)*d)
```

fricas [C] time = 1.48, size = 4396, normalized size = 15.16

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] -1/36*(2*(a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sqrt(3)
 + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*
(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d
 - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 +
 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^
2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3
- a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^
2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^
4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^
2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - (a^2*b + b^3)*sin(d*
x + c)) - ((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3)
- 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^
(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2
*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*
d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a
^4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(
1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*
d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d
^3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a
^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
 b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1
/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d
- 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 18*b)*log(1/36*(a^5 - a^3*b^2)*
(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2
*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3)
 + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d
 - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2
*d - b^2*d))^2*d^2 - a*b^2 - 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/5
4*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)
*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2
*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2
+ b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d + 1/12*sq
rt(1/3)*((a^5 - a^3*b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3
) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3)
)^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b
^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^
2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d^2 - 6*(a^3*b - a*b^3)*d)*sqrt(-((a^
4 - 2*a^2*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1
/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d
^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^
3))^(1/3)) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) +
1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^
2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d -
b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/
54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d -
 108*b^2)/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 2*(a^2*b + b^3)*sin(d*x + c)) -
((a^2 - b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3
/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^
2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/
27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3
)) + 6*b/(a^2*d - b^2*d))*d + 3*sqrt(1/3)*(a^2 - b^2)*d*sqrt(-((a^4 - 2*a^2
*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/
```

```
(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2
*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/2
7*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)
) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) + 1)*(-1/54
*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*
b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 +
 b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 108*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*d^2)) - 18*b)*log(-1/36*(a^5 - a^3*b^2)*(9*(I*sqr
t(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1
/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a
^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)
^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*
d))^2*d^2 + a*b^2 + 1/6*(2*a^3*b + a*b^3)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*
d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2
- b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b
/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/
((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d + 1/12*sqrt(1/3)*(
(a^5 - a^3*b^2)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*
b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) +
 b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) -
 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(
1/3)) + 6*b/(a^2*d - b^2*d))*d^2 - 6*(a^3*b - a*b^3)*d)*sqrt(-((a^4 - 2*a^2
*b^2 + b^4)*(9*(I*sqrt(3) + 1)*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/
(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2
*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/2
7*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)
) + 6*b/(a^2*d - b^2*d))^2*d^2 - 12*(a^2*b - b^3)*(9*(I*sqrt(3) + 1)*(-1/54
*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 + b^2)*
b/((a^2 - b^2)^2*a^2*d^3))^(1/3) + b^2*(-I*sqrt(3) + 1)/((a^2*d - b^2*d)^2*
(-1/54*b/(a^4*d^3 - a^2*b^2*d^3) - 1/27*b^3/(a^2*d - b^2*d)^3 + 1/54*(a^2 +
 b^2)*b/((a^2 - b^2)^2*a^2*d^3))^(1/3)) + 6*b/(a^2*d - b^2*d))*d - 108*b^2)
/((a^4 - 2*a^2*b^2 + b^4)*d^2)) + 2*(a^2*b + b^3)*sin(d*x + c)) - 18*(a + b
)*log(sin(d*x + c) + 1) + 18*(a - b)*log(-sin(d*x + c) + 1))/((a^2 - b^2)*d
)
```

giac [A] time = 0.33, size = 309, normalized size = 1.07

$$-\frac{\frac{2\left(a^3b^2\left(-\frac{a}{b}\right)^{\frac{1}{3}}-ab^4\left(-\frac{a}{b}\right)^{\frac{1}{3}}-a^2b^3+b^5\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^5b-2a^3b^3+ab^5}+\frac{2\left(\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}b^2+\sqrt{3}\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^3b-ab^3}+\cdots}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] -1/6*(2*(a^3*b^2*(-a/b)^(1/3) - a*b^4*(-a/b)^(1/3) - a^2*b^3 + b^5)*(-a/b)^
(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^5*b - 2*a^3*b^3 + a*b^5) +
2*(sqrt(3)*(-a*b^2)^(1/3)*b^2 + sqrt(3)*(-a*b^2)^(2/3)*a)*arctan(1/3*sqrt(3
)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^3*b - a*b^3) + ((-a*b^2)
^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^3*b - a*b^3) + 2*b*log(abs(b*sin(d*x + c)^3 + a))/(a^
2 - b^2) - 3*log(abs(sin(d*x + c) + 1))/(a - b) + 3*log(abs(sin(d*x + c) -
1))/(a + b))/d
```

maple [A] time = 0.96, size = 374, normalized size = 1.29

$$-\frac{\ln(\sin(dx+c)-1)}{d(2a+2b)}-\frac{b\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3d(a-b)(a+b)\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{b\ln\left(\sin^{2}(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6d(a-b)(a+b)\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{b\sqrt{3}\ \mathrm{arcta}}{3d(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/d/(2*a+2*b)*ln(sin(d*x+c)-1)-1/3/d*b/(a-b)/(a+b)/(a/b)^(2/3)*ln(sin(d*x+
c)+(a/b)^(1/3))+1/6/d*b/(a-b)/(a+b)/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)
*sin(d*x+c)+(a/b)^(2/3))-1/3/d*b/(a-b)/(a+b)/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3/d/(a-b)/(a+b)*a/(a/b)^(1/3)*ln(s
in(d*x+c)+(a/b)^(1/3))+1/6/d/(a-b)/(a+b)*a/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b
)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/3/d/(a-b)/(a+b)*a*3^(1/2)/(a/b)^(1/3)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-1/3/d*b/(a-b)/(a+b)*ln(a+b*si
n(d*x+c)^3)+1/d/(2*a-2*b)*ln(1+sin(d*x+c))
```

maxima [A] time = 0.51, size = 288, normalized size = 0.99

$$\frac{\frac{2\sqrt{3}\left(a\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}+2\right)-b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{2a}{b}\right)\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{3\left(b\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-a\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log\left(\sin(dx+c)^{2}-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}-b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{6\left(b\right.}{}}{18\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] 1/18*(2*sqrt(3)*(a*(3*(a/b)^(2/3) + 2) - b*(3*(a/b)^(1/3) + 2*a/b))*arctan(
-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/((a^2*(a/b)^(2/3)
- b^2*(a/b)^(2/3))*(a/b)^(1/3)) - 3*(b*(2*(a/b)^(2/3) - 1) - a*(a/b)^(1/3))
*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^2*(a/b)^(2
/3) - b^2*(a/b)^(2/3)) - 6*(b*((a/b)^(2/3) + 1) + a*(a/b)^(1/3))*log((a/b)^
(1/3) + sin(d*x + c))/(a^2*(a/b)^(2/3) - b^2*(a/b)^(2/3)) + 9*log(sin(d*x +
 c) + 1)/(a - b) - 9*log(sin(d*x + c) - 1)/(a + b))/d
```

mupad [B] time = 0.27, size = 600, normalized size = 2.07

$$\left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}-27\,a^{4}\,z^{3}-27\,a^{2}\,b\,z^{2}-b,z,k\right)^{2}\,a\,b^{4}\,13-\mathrm{root}\left(27\,a^{2}\,b^{2}\,z^{3}-27\,a^{4}\,z^{3}-27\,a^{2}\,b\,z^{2}-\right.\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (symsum(log(- 13*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)
^2*a*b^4 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^3*
a*b^5 - 36*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a*b
^6 - 16*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^2*b^5*si
n(c + d*x) - 12*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^
```

```
3*b^6*sin(c + d*x) - 27*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b
, z, k)^3*a^3*b^3 - 180*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b
, z, k)^4*a^3*b^4 - 5*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b,
z, k)*b^4*sin(c + d*x) - 69*root(27*a^2*b^2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2
 - b, z, k)^3*a^2*b^4*sin(c + d*x) - 162*root(27*a^2*b^2*z^3 - 27*a^4*z^3 -
 27*a^2*b*z^2 - b, z, k)^4*a^2*b^5*sin(c + d*x) - 54*root(27*a^2*b^2*z^3 -
27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k)^4*a^4*b^3*sin(c + d*x))*root(27*a^2*b^
2*z^3 - 27*a^4*z^3 - 27*a^2*b*z^2 - b, z, k), k, 1, 3) - log(sin(c + d*x) -
 1)/(2*a + 2*b) + log(sin(c + d*x) + 1)/(2*a - 2*b))/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)/(a + b*sin(c + d*x)**3), x)
```

3.387 $\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=385

$$\frac{b\left(a^2+2b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+\right.}{6a^{2/3}d\left(a^2-b^2\right)^2}$$

```
[Out] -1/4*(a+4*b)*ln(1-sin(d*x+c))/(a+b)^2/d+1/4*(a-4*b)*ln(1+sin(d*x+c))/(a-b)^
2/d+1/3*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c)
)/a^(2/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*ln(a^(2/3
)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(2/3)/(a^2-b^2)^2/d+1/
3*b*(a^2+2*b^2)*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1/4/(a+b)/d/(1-sin(d*x+c
))-1/4/(a-b)/d/(1+sin(d*x+c))-1/3*b^(5/3)*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(2/3)/(a^2-b^2)^2
/d*3^(1/2)
```

Rubi [A] time = 0.50, antiderivative size = 385, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3223, 2074, 1871, 1860, 31, 634, 617, 204, 628, 260}

$$-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6a^{2/3}d\left(a^2-b^2\right)^2}+\frac{b\left(a^2+2b^2\right)\log\left(a+b\sin^3(c\right.}{3d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -((b^(5/3)*(2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Si
n[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(2/3)*(a^2 - b^2)^2*d)) - ((a +
4*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^2*d) + ((a - 4*b)*Log[1 + Sin[c + d*
x]])/(4*(a - b)^2*d) + (b^(5/3)*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(3*a^(2/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(2*a^2 +
 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2
/3)*Sin[c + d*x]^2])/(6*a^(2/3)*(a^2 - b^2)^2*d) + (b*(a^2 + 2*b^2)*Log[a +
 b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + 1/(4*(a + b)*d*(1 - Sin[c + d*x])
) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^3)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)(-1+x)^2} + \frac{-a-4b}{4(a+b)^2(-1+x)} + \frac{1}{4(a-b)(1+x)^2} + \frac{a-4b}{4(a-b)^2(1+x)} + \frac{b^2(2a^2+b^2-3abx+(a^2}{(a^2-b^2)^2(a+bx}\right.\right.}{d}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\sin(}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{1}{4(a+b)d(1-\sin(}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b\left(a^2+2b^2\right)\log\left(\right.}{3\left(a^2-\right.}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}\left(2a^2+3a^{4/3}b\right.}{}$$

$$= -\frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} + \frac{(a-4b)\log(1+\sin(c+dx))}{4(a-b)^2d} + \frac{b^{5/3}\left(2a^2+3a^{4/3}b\right.}{}$$

$$= -\frac{b^{5/3}\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{2/3}\left(a^2-b^2\right)^2d} - \frac{(a+4b)\log(1-\sin(c+dx))}{4(a+b)^2d} +$$

Mathematica [C] time = 2.19, size = 333, normalized size = 0.86

$$-\frac{-\frac{4b\left(a^2+2b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{\left(a^2-b^2\right)^2} + \frac{18b^3\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{\left(a^2-b^2\right)^2} - \frac{4b^{5/3}\left(2a^2+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{2/3}\left(a^2-b^2\right)^2} + \frac{2b^{5/3}\left(2a^2+b^2\right)\left(1\right.}{}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/12*((3*(a + 4*b)*Log[1 - Sin[c + d*x]])/(a + b)^2 - (3*(a - 4*b)*Log[1 +
 Sin[c + d*x]])/(a - b)^2 - (4*b^(5/3)*(2*a^2 + b^2)*Log[a^(1/3) + b^(1/3)*
Sin[c + d*x]])/(a^(2/3)*(a^2 - b^2)^2) + (2*b^(5/3)*(2*a^2 + b^2)*(2*Sqrt[3
]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + Log[a^(2/3
) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^(2/3)*(a^2
- b^2)^2) - (4*b*(a^2 + 2*b^2)*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2 + 3
/((a + b)*(-1 + Sin[c + d*x])) + (18*b^3*Hypergeometric2F1[2/3, 1, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^2 - b^2)^2 + 3/((a - b)*(1 + Sin[c
 + d*x]))))/d
```

fricas [C] time = 2.96, size = 10135, normalized size = 26.32

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] 1/12*(2*(a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^
2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(
3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^
2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(
a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*
b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*
d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2
*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*cos(d*x + c)^2*log(7*a^3*
b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^
2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4
*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a
^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
 b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^
4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
+ b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*
sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1/2
*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d
^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt
(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b
^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*
(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2
 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2
*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4
*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4
*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) +
2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - (8*a^2*b^3 + b^5)*sin(
d*x + c)) + 3*(a^3 - 2*a^2*b - 7*a*b^2 - 4*b^3)*cos(d*x + c)^2*log(sin(d*x
+ c) + 1) - 3*(a^3 + 2*a^2*b - 7*a*b^2 + 4*b^3)*cos(d*x + c)^2*log(-sin(d*x
 + c) + 1) - 6*a^2*b + 6*b^3 - ((a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2
/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2
*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3
) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a
^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 +
(8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^
3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*
b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^
4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(
1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*c
os(d*x + c)^2 - 3*sqrt(1/3)*(a^4 - 2*a^2*b^2 + b^4)*d*sqrt(-(4*a^4*b^2 - 80
*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2
)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 +
 a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*
(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*
d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b +
2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4
*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d +
 b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*
d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^
```

```
2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d
+ b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 +
b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4
*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 +
 b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a
^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*s
qrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4
*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))*cos(d*x + c)^2 - 6*(a^2*b + 2
*b^3)*cos(d*x + c)^2)*log(7*a^3*b^2 + 2*a*b^4 + 3/4*(a^7 - 2*a^5*b^2 + a^3*
b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b
^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a
^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) -
(1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3
)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) +
 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((
a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2
*a^2*b^2*d + b^4*d))^2*d^2 - 1/2*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(
2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d -
2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^
2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^
4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^
4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^
3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2
 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b
^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^
2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^
4*d))*d + 3/4*sqrt(1/3)*(3*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/
(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*
d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3)
 - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^
2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (
8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3
 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b
^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4
*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1
/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2
+ 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d)*sqrt(-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 +
 (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d
^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^
4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(
a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*
d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2
+ b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a
^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2
 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I
*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*
(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 +
b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) +
 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((
a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*
b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^
2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*
d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d -
2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^
2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^
```

4 - 4*a^2*b^6 + b^8)*d^2)) + 2*(8*a^2*b^3 + b^5)*sin(d*x + c)) - ((a^4 - 2*a^2*b^2 + b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d*cos(d*x + c)^2 + 3*sqrt(1/3)*(a^4 - 2*a^2*b^2 + b^4)*d*sqrt(-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2))*cos(d*x + c)^2 - 6*(a^2*b + 2*b^3)*cos(d*x + c)^2)*log(-7*a^3*b^2 - 2*a*b^4 - 3/4*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 1/2*(10*a^5*b + 16*a^3*b^3 + a*b^5)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*d))*d + 3/4*sqrt(1/3)*(3*(a^7 - 2*a^5*b^2 + a^3*b^4)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2

```
*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d +
 b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b
^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/
(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2 + 2*(a^5*b - 2*a^3*b^3 + a*b^5)*d)*sqrt(
-(4*a^4*b^2 - 80*a^2*b^4 - 32*b^6 + (a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^
6 + b^8)*(2*(1/2)^(2/3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b +
 2*b^3)^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 -
 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2
*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d
 - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3
) - (1/2)^(1/3)*(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2
*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d
)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^
5/((a^2 - b^2)^4*a^2*d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d
 - 2*a^2*b^2*d + b^4*d))^2*d^2 - 4*(a^6*b - 3*a^2*b^5 + 2*b^7)*(2*(1/2)^(2/
3)*(b^2/(a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2) - (a^2*b + 2*b^3)^2/(a^4*d - 2*
a^2*b^2*d + b^4*d)^2)*(-I*sqrt(3) + 1)/(b^3/(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*
b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 - 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*
d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3)^3/(a^4*d - 2*a^2*b^2*d + b^4*
d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*d^3))^(1/3) - (1/2)^(1/3)*(b^3/
(a^6*d^3 - 2*a^4*b^2*d^3 + a^2*b^4*d^3) - 3*(a^2*b + 2*b^3)*b^2/((a^4*d^2 -
 2*a^2*b^2*d^2 + b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) + 2*(a^2*b + 2*b^3
)^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + (8*a^2 + b^2)*b^5/((a^2 - b^2)^4*a^2*
d^3))^(1/3)*(I*sqrt(3) + 1) + 2*(a^2*b + 2*b^3)/(a^4*d - 2*a^2*b^2*d + b^4*
d))*d)/((a^8 - 4*a^6*b^2 + 6*a^4*b^4 - 4*a^2*b^6 + b^8)*d^2)) - 2*(8*a^2*b^
3 + b^5)*sin(d*x + c)) + 6*(a^3 - a*b^2)*sin(d*x + c))/((a^4 - 2*a^2*b^2 +
b^4)*d*cos(d*x + c)^2)
```

giac [A] time = 0.23, size = 510, normalized size = 1.32

$$\frac{4\left(3\,a^{5}b^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-6\,a^{3}b^{6}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+3\,ab^{8}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^{6}b^{3}+3\,a^{4}b^{5}-b^{9}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin\left(dx+c\right)\right|\right)}{a^{9}b-4\,a^{7}b^{3}+6\,a^{5}b^{5}-4\,a^{3}b^{7}+ab^{9}}+\frac{4\left(3\,\sqrt{3}\left(-ab^{2}\right)^{\frac{2}{3}}ab+\left(2\,\sqrt{3}\,a^{2}b+\sqrt{3}\,b^{3}\right)\left(-ab^{2}\right.\right.}{a^{5}-2\,a^{3}b^{2}+\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] 1/12*(4*(3*a^5*b^4*(-a/b)^(1/3) - 6*a^3*b^6*(-a/b)^(1/3) + 3*a*b^8*(-a/b)^(
1/3) - 2*a^6*b^3 + 3*a^4*b^5 - b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + si
n(d*x + c)))/(a^9*b - 4*a^7*b^3 + 6*a^5*b^5 - 4*a^3*b^7 + a*b^9) + 4*(3*sqr
t(3)*(-a*b^2)^(2/3)*a*b + (2*sqrt(3)*a^2*b + sqrt(3)*b^3)*(-a*b^2)^(1/3))*a
rctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^5 - 2*a^
3*b^2 + a*b^4) - 2*(3*(-a*b^2)^(2/3)*a*b - (2*a^2*b + b^3)*(-a*b^2)^(1/3))*
log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^5 - 2*a^3
*b^2 + a*b^4) + 4*(a^2*b + 2*b^3)*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*a
^2*b^2 + b^4) + 3*(a - 4*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*a*b + b^2)
- 3*(a + 4*b)*log(abs(sin(d*x + c) - 1))/(a^2 + 2*a*b + b^2) + 6*(a^2*b*sin
(d*x + c)^2 + 2*b^3*sin(d*x + c)^2 - a^3*sin(d*x + c) + a*b^2*sin(d*x + c)
- 3*b^3)/((a^4 - 2*a^2*b^2 + b^4)*(sin(d*x + c)^2 - 1)))/d
```

maple [B] time = 0.97, size = 668, normalized size = 1.74

$$-\frac{1}{d\left(4a+4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a+b\right)^{2}}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)b}{d\left(a+b\right)^{2}}+\frac{2b\ln\left(\sin\left(dx+c\right)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)a^{2}}{3d\left(a-b\right)^{2}\left(a+b\right)^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{b^{3}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -1/d/(4*a+4*b)/(sin(d*x+c)-1)-1/4/d/(a+b)^2*ln(sin(d*x+c)-1)*a-1/d/(a+b)^2*
ln(sin(d*x+c)-1)*b+2/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^
(1/3))*a^2+1/3/d*b^3/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))
-1/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)
+(a/b)^(2/3))*a^2-1/6/d*b^3/(a-b)^2/(a+b)^2/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/
b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/3/d*b/(a-b)^2/(a+b)^2/(a/b)^(2/3)*3^(1/2
)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))*a^2+1/3/d*b^3/(a-b)^2/(a
+b)^2/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+
1/d*b^2/(a-b)^2/(a+b)^2*a/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))-1/2/d*b^2/
(a-b)^2/(a+b)^2*a/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^
(2/3))-1/d*b^2/(a-b)^2/(a+b)^2*a*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/
(a/b)^(1/3)*sin(d*x+c)-1))+1/3/d*b/(a-b)^2/(a+b)^2*ln(a+b*sin(d*x+c)^3)*a^2
+2/3/d*b^3/(a-b)^2/(a+b)^2*ln(a+b*sin(d*x+c)^3)-1/d/(4*a-4*b)/(1+sin(d*x+c)
)+1/4*a*ln(1+sin(d*x+c))/(a-b)^2/d-b*ln(1+sin(d*x+c))/(a-b)^2/d
```

maxima [A] time = 0.44, size = 470, normalized size = 1.22

$$-\frac{\frac{4\sqrt{3}\left(ab^{2}\left(9\left(\frac{a}{b}\right)^{\frac{2}{3}}+4\right)-b^{3}\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{4a}{b}\right)-2a^{2}b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+2a^{3}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin\left(dx+c\right)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^{2}b^{2}\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{6\left(b^{3}\left(4\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)+2a^{2}b\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}-1\right)-3ab^{2}\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\log}{a^{4}\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^{2}b^{2}\left(\frac{a}{b}\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] -1/36*(4*sqrt(3)*(a*b^2*(9*(a/b)^(2/3) + 4) - b^3*(3*(a/b)^(1/3) + 4*a/b) -
 2*a^2*b*(3*(a/b)^(1/3) + a/b) + 2*a^3)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) -
2*sin(d*x + c))/(a/b)^(1/3))/((a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) + b^
4*(a/b)^(2/3))*(a/b)^(1/3)) - 6*(b^3*(4*(a/b)^(2/3) - 1) + 2*a^2*b*((a/b)^(
2/3) - 1) - 3*a*b^2*(a/b)^(1/3))*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x +
 c) + (a/b)^(2/3))/(a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^(2/3) + b^4*(a/b)^(2/
3)) - 12*(b^3*(2*(a/b)^(2/3) + 1) + a^2*b*((a/b)^(2/3) + 2) + 3*a*b^2*(a/b)
^(1/3))*log((a/b)^(1/3) + sin(d*x + c))/(a^4*(a/b)^(2/3) - 2*a^2*b^2*(a/b)^
(2/3) + b^4*(a/b)^(2/3)) - 9*(a - 4*b)*log(sin(d*x + c) + 1)/(a^2 - 2*a*b +
 b^2) + 9*(a + 4*b)*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2) + 18*(a*sin(d
*x + c) - b)/((a^2 - b^2)*sin(d*x + c)^2 - a^2 + b^2))/d
```

mupad [B] time = 15.27, size = 898, normalized size = 2.33

$$\frac{\left(\sum_{k=1}^{3}\ln\left(-\mathrm{root}\left(54\,a^{4}\,b^{2}\,z^{3}-27\,a^{2}\,b^{4}\,z^{3}-27\,a^{6}\,z^{3}+54\,a^{2}\,b^{3}\,z^{2}+27\,a^{4}\,b\,z^{2}-9\,a^{2}\,b^{2}\,z+b^{3},z,k\right)\left(-\frac{28\,a\,b^{7}-6\,a^{3}\,b^{5}}{a^{4}-2\,a^{2}\,b^{2}+b^{4}}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (symsum(log((a*b^6)/(2*(a^4 + b^4 - 2*a^2*b^2)) - root(54*a^4*b^2*z^3 - 27*
a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^
3, z, k)*(root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^
2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*(root(54*a^4*b^2*z^3 - 27*a^2*b
^4*z^3 - 27*a^6*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z,
 k)*((48*a*b^9 + (51*a^3*b^7)/2 - 87*a^5*b^5 + (27*a^7*b^3)/2)/(a^4 + b^4 -
 2*a^2*b^2) + root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6*z^3 + 54*a^2*b^
3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k)*((36*a*b^10 + 108*a^3*b^8 -
 324*a^5*b^6 + 180*a^7*b^4)/(a^4 + b^4 - 2*a^2*b^2) + (sin(c + d*x)*(648*a^
2*b^9 - 1080*a^4*b^7 + 216*a^6*b^5 + 216*a^8*b^3))/(4*(a^4 + b^4 - 2*a^2*b^
2))) + (sin(c + d*x)*(48*b^10 + 552*a^2*b^8 - 600*a^4*b^6))/(4*(a^4 + b^4 -
 2*a^2*b^2))) - (12*a*b^8 - (219*a^3*b^6)/4 + 18*a^5*b^4)/(a^4 + b^4 - 2*a^
2*b^2) + (sin(c + d*x)*(96*b^9 + 120*a^2*b^7 - 171*a^4*b^5))/(4*(a^4 + b^4
- 2*a^2*b^2))) - (28*a*b^7 - 6*a^3*b^5)/(a^4 + b^4 - 2*a^2*b^2) + (sin(c +
d*x)*(40*b^8 + 61*a^2*b^6))/(4*(a^4 + b^4 - 2*a^2*b^2))) + (2*b^7*sin(c + d
*x))/(a^4 + b^4 - 2*a^2*b^2))*root(54*a^4*b^2*z^3 - 27*a^2*b^4*z^3 - 27*a^6
*z^3 + 54*a^2*b^3*z^2 + 27*a^4*b*z^2 - 9*a^2*b^2*z + b^3, z, k), k, 1, 3) +
 (b/(2*(a^2 - b^2)) - (a*sin(c + d*x))/(2*(a^2 - b^2)))/(sin(c + d*x)^2 - 1
) - (log(sin(c + d*x) - 1)*(a + 4*b))/(8*a*b + 4*a^2 + 4*b^2) + (log(sin(c
+ d*x) + 1)*(a - 4*b))/(4*a^2 - 8*a*b + 4*b^2))/d
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^3(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)**3/(a + b*sin(c + d*x)**3), x)
```

3.388 $\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=764

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\tan\ldots}{3\ldots}$$

```
[Out] -cos(d*x+c)/b/d+2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^
(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*a^(2/3)*arctan((b^(1/3)
+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3)-b^
(2/3))^(1/2)-4/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/
3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)+(-1)^(2/3
)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d
/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+4/3*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)
*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2))/b^(2/3)/d/(-(-1)^
(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)^(1
/3)*b^(2/3))^(1/2)-2/3*(-1)^(1/3)*a^(2/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3
)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/b^(4/3)/d/(a^(2/3
)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*(-1)^(2/3)*a^(2/3)*arctan(((-1)^(1/3)*b^(1/
3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/b^(4/3)/
d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2
/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)
/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 1.53, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 38, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.348, Rules used = {3226, 3213, 2660, 618, 204, 3220, 206, 2638}

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3b^{4/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{4\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2a^{2/3}\tan\ldots}{3\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-2*(-1)^(2/3)*a^(2/3)*ArcTan[((-1)^(1/3)*b^(1/3) - a^(1/3)*Tan[(c + d*x)/2
])/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*b^(4/3)*d) + (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3)
 - b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*a^(2/3)*ArcTan[(b^
(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3)
 - b^(2/3)]*b^(4/3)*d) - (4*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqr
t[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) + (2*ArcTan[((
-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(
2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*(-1)^(1/3)*a^
(2/3)*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) +
 (-1)^(1/3)*b^(2/3)]])/(3*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*b^(4/3)*d) - (
2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[
a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3
)]*d) + (4*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((
-1)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^
(2/3)*d) + (4*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[
(-1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/
3)*d) - Cos[c + d*x]/(b*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2638

```
Int[sin[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[Cos[c + d*x]/d, x] /; FreeQ
[{c, d}, x]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3226

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx = \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{2\sin^2(c+dx)}{a+b\sin^3(c+dx)} + \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\right)dx$$

$$= -\left(2\int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx\right) + \int \frac{1}{a+b\sin^3(c+dx)}\,dx + \int \frac{\sin^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

$$= -\left(2\int\left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}\left(\right.}\right.\right.$$

$$= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} + \int$$

$$= -\frac{\cos(c+dx)}{bd} - \frac{a\int\left(-\frac{1}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} - \frac{(-1)^{2/3}}{3\sqrt[3]{a}\sqrt[3]{b}\left(\sqrt[3]{a}-\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3\sqrt[3]{a}\sqrt[3]{b}\left(\right.}\right.}{b}$$

$$= -\frac{\cos(c+dx)}{bd} + \frac{a^{2/3}\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} - \frac{\left(\sqrt[3]{-1}\,a^{2/3}\right)\int \frac{1}{\sqrt[3]{a}+(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3b^{4/3}} + \frac{((-}{}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{4\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan}{\sqrt{a^{2/3}}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{4\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan}{\sqrt{a^{2/3}}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}}$$

$$= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} - \frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,b^{4/3}d} + \frac{2\tan}{}$$

Mathematica [C] time = 0.15, size = 300, normalized size = 0.39

$$-\frac{3\cos(c+dx) + i\text{RootSum}\left[i\#1^6b - 3i\#1^4b + 8\#1^3a + 3i\#1^2b - ib\&, \frac{2\#1^4b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right) + 2i\#1^3a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right) - }{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] -1/3*(3*Cos[c + d*x] + I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b
*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 - a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + (2*I)*a*ArcTan[Sin[c
+ d*x]/(Cos[c + d*x] - #1)]*#1^3 + a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3
 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(b*d
)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^4}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(cos(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 1.01, size = 123, normalized size = 0.16

$$-\frac{2}{db\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}+\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4b-2\_R^3a-6\_R^2b-2\_Ra+b\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3db}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
[Out] -2/d/b/(1+tan(1/2*d*x+1/2*c)^2)+1/3/d/b*sum((_R^4*b-2*_R^3*a-6*_R^2*b-2*_R*
a+b)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z
^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 18.04, size = 2338, normalized size = 3.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3),x)
[Out] symsum(log(172032*a^4*b^9 - 81920*a^2*b^11 - 98304*a^6*b^7 + 8192*a^8*b^5 -
 319488*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b
^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^2*b^12 - 688128*root(72
9*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*
b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a^4*b^10 + 344064*root(729*a^4*b^8*d^6 -
 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4
 + a^6 - b^6, d, k)*a^6*b^8 - 98304*a*b^12*tan(c/2 + (d*x)/2) - 294912*root
(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a
^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^2*a^2*b^13 - 1400832*root(729*a^4*b^8
*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a
^2*b^4 + a^6 - b^6, d, k)^2*a^4*b^11 + 1695744*root(729*a^4*b^8*d^6 - 729*a
^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6
 - b^6, d, k)^2*a^6*b^9 + 5750784*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 +
162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)
```

```
^3*a^4*b^12 - 3760128*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*
d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^6*b^10
+ 3317760*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2
*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^4*b^13 - 3317760*ro
ot(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3
*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^6*b^11 - 7962624*root(729*a^4*b
^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3
*a^2*b^4 + a^6 - b^6, d, k)^5*a^4*b^14 + 5971968*root(729*a^4*b^8*d^6 - 729
*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a
^6 - b^6, d, k)^5*a^6*b^12 + 196608*a^3*b^10*tan(c/2 + (d*x)/2) - 98304*a^5
*b^8*tan(c/2 + (d*x)/2) - 1277952*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 +
162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)
*a^3*b^11*tan(c/2 + (d*x)/2) + 712704*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^
4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d
, k)*a^5*b^9*tan(c/2 + (d*x)/2) + 98304*root(729*a^4*b^8*d^6 - 729*a^4*b^6*
d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6,
 d, k)*a^7*b^7*tan(c/2 + (d*x)/2) + 589824*root(729*a^4*b^8*d^6 - 729*a^4*b
^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b
^6, d, k)^2*a^3*b^12*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^8*d^6 - 729
*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a
^6 - b^6, d, k)^2*a^5*b^10*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^8*d^6
 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b
^4 + a^6 - b^6, d, k)^2*a^7*b^8*tan(c/2 + (d*x)/2) + 5308416*root(729*a^4*b
^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3
*a^2*b^4 + a^6 - b^6, d, k)^3*a^3*b^13*tan(c/2 + (d*x)/2) - 3538944*root(72
9*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*
b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^5*b^11*tan(c/2 + (d*x)/2) + 221184*r
oot(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*d^2 -
3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^3*a^7*b^9*tan(c/2 + (d*x)/2) - 530
8416*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a^2*b^6*
d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^3*b^14*tan(c/2 + (d*x)/2
) + 5308416*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2 + 81*a
^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^4*a^5*b^12*tan(c/2 +
(d*x)/2) - 1990656*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2
 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)^5*a^5*b^13*tan
(c/2 + (d*x)/2) - 196608*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b
^4*d^2 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k)*a*b^13*t
an(c/2 + (d*x)/2))*root(729*a^4*b^8*d^6 - 729*a^4*b^6*d^4 + 162*a^4*b^4*d^2
 + 81*a^2*b^6*d^2 - 3*a^4*b^2 + 3*a^2*b^4 + a^6 - b^6, d, k), k, 1, 6)/d -
2/(d*(b + b*tan(c/2 + (d*x)/2)^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**4/(a + b*sin(c + d*x)**3), x)
```

3.389 $\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=484

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\,(\cdots}{\cdots}\right)}{3a^{2/3}d\sqrt{\cdots}}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2
*c))/(a^(2/3)-b^(2/3))^(1/2))/b^(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctanh
((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(2/3
))^(1/2))/b^(2/3)/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctanh((b^(1/3)
-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)
)/b^(2/3)/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1
/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)
/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(
2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3
)/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.63, antiderivative size = 484, normalized size of antiderivative = 1.00, number of steps used = 24, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3226, 3213, 2660, 618, 204, 3220, 206}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3b^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\,(\cdots}{\cdots}\right)}{3a^{2/3}d\sqrt{\cdots}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) - (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c +
 d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3*Sqrt[a^(2/3) - b^(2/3)]*b^(2/3)*d) +
 (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (
-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*
ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^
(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]
*d) + (2*ArcTanh[(b^(1/3) - (-1)^(1/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[-((-1
)^(2/3)*a^(2/3)) + b^(2/3)]])/(3*Sqrt[-((-1)^(2/3)*a^(2/3)) + b^(2/3)]*b^(2
/3)*d) + (2*ArcTanh[(b^(1/3) + (-1)^(2/3)*a^(1/3)*Tan[(c + d*x)/2])/Sqrt[(-
1)^(1/3)*a^(2/3) + b^(2/3)]])/(3*Sqrt[(-1)^(1/3)*a^(2/3) + b^(2/3)]*b^(2/3)
*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3220

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_
))^(p_.), x_Symbol] :> Int[ExpandTrig[sin[e + f*x]^m*(a + b*sin[e + f*x]^n)
^p, x], x] /; FreeQ[{a, b, e, f}, x] && IntegersQ[m, p] && (EqQ[n, 4] || Gt
Q[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rule 3226

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)/((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_)
), x_Symbol] :> Int[Expand[(1 - Sin[e + f*x]^2)^(m/2)/(a + b*Sin[e + f*x]^n
), x], x] /; FreeQ[{a, b, e, f}, x] && IGtQ[m/2, 0] && IntegerQ[(n - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos^2(c+dx)}{a+b\sin^3(c+dx)}\,dx &= \int \left(\frac{1}{a+b\sin^3(c+dx)} - \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\right) dx \\
&= \int \frac{1}{a+b\sin^3(c+dx)}\,dx - \int \frac{\sin^2(c+dx)}{a+b\sin^3(c+dx)}\,dx \\
&= -\int \left(\frac{1}{3b^{2/3}\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{1}{3b^{2/3}\left(-\sqrt[3]{-1}\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)} + \frac{}{3b^{2/3}\left((-1\right.}\right. \\
&= -\frac{\int \frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int \frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}} - \frac{\int}{} \\
&= -\frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx, x, \tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} - \frac{2\,\mathrm{Subst}\left(\int \frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\right.}{3a^{2/3}d} \\
&= \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx, x, -2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d} + \frac{4\,\mathrm{Subst}\left(\int \frac{1}{-4\left(a^{2/3}+\sqrt[3]{}\right.}\right.}{} \\
&= -\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d} + \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d} - \frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan}{\sqrt{a^{2/3}}}\right.}{3\sqrt{a^{2/3}-b^{2/3}}}
\end{aligned}$$

Mathematica [C] time = 0.10, size = 231, normalized size = 0.48

$$-\frac{i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-2i\#1^2\log\left(\#1^2-2\#1\cos(c+dx)+1\right)-i\log\left(\#1^2-2\#\right.}{}\right.}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((-1/6*I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^
6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + 4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (2*I)*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] -
#1)]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2
- 2*b*#1^3 + b*#1^5) & ])/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.89, size = 83, normalized size = 0.17

$$\frac{\sum_{\_R=\text{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4-2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4-2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos(dx+c)^2}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

mupad [B] time = 17.40, size = 951, normalized size = 1.96

$$\sum_{k=1}^{6} \ln\Big(24576\,a^4 - 24576\,a^2\,b^2 - \mathrm{root}\Big(729\,a^4\,b^4\,d^6 + 27\,a^2\,b^2\,d^2 + a^2 - b^2, d, k\Big)\,a^2\,b^3\,122880 - \mathrm{root}\Big(729\,a^4\,b^4\,d$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(24576*a^4 - 24576*a^2*b^2 - 122880*root(729*a^4*b^4*d^6 + 27*a^2
*b^2*d^2 + a^2 - b^2, d, k)*a^2*b^3 - 24576*root(729*a^4*b^4*d^6 + 27*a^2*b
^2*d^2 + a^2 - b^2, d, k)*a^5*tan(c/2 + (d*x)/2) - 32768*a*b^3*tan(c/2 + (d
*x)/2) + 32768*a^3*b*tan(c/2 + (d*x)/2) - 294912*root(729*a^4*b^4*d^6 + 27*
a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^2*b^4 + 294912*root(729*a^4*b^4*d^6 + 27
*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^4*b^2 + 663552*root(729*a^4*b^4*d^6 + 2
7*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^4*b^4 - 663552*root(729*a^4*b^4*d^6 +
27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^6*b^2 - 7962624*root(729*a^4*b^4*d^6
+ 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^4*b^5 + 5971968*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^5*a^6*b^3 + 49152*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^4*b + 147456*root(729*a^4*b^4*d^6 +
 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a^3*b^2*tan(c/2 + (d*x)/2) + 294912*root
(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^5*b*tan(c/2 + (d*x
)/2) - 294912*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^2*a^
3*b^3*tan(c/2 + (d*x)/2) + 1769472*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 +
a^2 - b^2, d, k)^3*a^3*b^4*tan(c/2 + (d*x)/2) - 1769472*root(729*a^4*b^4*d^
6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^3*a^5*b^2*tan(c/2 + (d*x)/2) - 530841
6*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)^4*a^3*b^5*tan(c/
2 + (d*x)/2) + 5308416*root(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d
, k)^4*a^5*b^3*tan(c/2 + (d*x)/2) - 1990656*root(729*a^4*b^4*d^6 + 27*a^2*b
^2*d^2 + a^2 - b^2, d, k)^5*a^5*b^4*tan(c/2 + (d*x)/2) - 196608*root(729*a^
4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k)*a*b^4*tan(c/2 + (d*x)/2))*roo
t(729*a^4*b^4*d^6 + 27*a^2*b^2*d^2 + a^2 - b^2, d, k), k, 1, 6)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos^2(c + dx)}{a + b\sin^3(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(cos(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.390 $\int \frac{1}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

```
[Out] 2/3*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/a^
(2/3)/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(
1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)+(-1)
^(1/3)*b^(2/3))^(1/2)-2/3*arctan((-1)^(1/3)*(b^(1/3)+(-1)^(2/3)*a^(1/3)*tan
(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/d/(a^(2/3)-(-1
)^(2/3)*b^(2/3))^(1/2)
```

Rubi [A] time = 0.25, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 4, integrand size = 14, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3213, 2660, 618, 204}

$$\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-b^{2/3}}}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1)^{2/3}\sqrt[3]{b}}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\left((-1)^{2/3}\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-1),x]
```

```
[Out] (2*ArcTan[(b^(1/3) + a^(1/3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) - b^(2/3)]])/(3
*a^(2/3)*Sqrt[a^(2/3) - b^(2/3)]*d) + (2*ArcTan[((-1)^(2/3)*b^(1/3) + a^(1/
3)*Tan[(c + d*x)/2])/Sqrt[a^(2/3) + (-1)^(1/3)*b^(2/3)]])/(3*a^(2/3)*Sqrt[a
^(2/3) + (-1)^(1/3)*b^(2/3)]*d) - (2*ArcTan[((-1)^(1/3)*(b^(1/3) + (-1)^(2/
3)*a^(1/3)*Tan[(c + d*x)/2]))/Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]])/(3*a^(2/
3)*Sqrt[a^(2/3) - (-1)^(2/3)*b^(2/3)]*d)
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 618

```
Int[((a_.) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> Dist[-2, Subst[I
nt[1/Simp[b^2 - 4*a*c - x^2, x], x], x, b + 2*c*x], x] /; FreeQ[{a, b, c},
x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 2660

```
Int[((a_) + (b_.)*sin[(c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = Fre
eFactors[Tan[(c + d*x)/2], x]}, Dist[(2*e)/d, Subst[Int[1/(a + 2*b*e*x + a*
e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[
a^2 - b^2, 0]
```

Rule 3213

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_), x_Symbol] :>
Int[ExpandTrig[(a + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, e, f
```

```
, n}, x] && (IGtQ[p, 0] || (EqQ[p, -1] && IntegerQ[n]))
```

Rubi steps

$$\int \frac{1}{a+b\sin^3(c+dx)}\,dx = \int \left(-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}-\frac{1}{3a^{2/3}\left(-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)\right)}-\frac{}{3a^{2/3}\left(-\right.}$$

$$=-\frac{\int\frac{1}{-\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int\frac{1}{-\sqrt[3]{a}+\sqrt[3]{-1}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}-\frac{\int\frac{1}{-\sqrt[3]{a}-(-1)^{2/3}\sqrt[3]{b}\sin(c+dx)}\,dx}{3a^{2/3}}$$

$$=-\frac{2\,\text{Subst}\left(\int\frac{1}{-\sqrt[3]{a}-2\sqrt[3]{b}x-\sqrt[3]{a}x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}-\frac{2\,\text{Subst}\left(\int\frac{1}{-\sqrt[3]{a}+2\sqrt[3]{-1}\sqrt[3]{b}x-\sqrt[3]{a}x^2}\right.}{3a^{2/3}d}$$

$$=\frac{4\,\text{Subst}\left(\int\frac{1}{-4\left(a^{2/3}-b^{2/3}\right)-x^2}\,dx,x,-2\sqrt[3]{b}-2\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a^{2/3}d}+\frac{4\,\text{Subst}\left(\int\frac{1}{-4\left(a^{2/3}+\sqrt[3]{}\right.}\right.}{}$$

$$=-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{\sqrt[3]{b}+\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}\sqrt{a^{2/3}-b^{2/3}}\,d}+\frac{2\tan^{-1}\left(\frac{(-1)^{2/3}\sqrt[3]{b}+}{\sqrt{a^{2/3}}}\right.}{3a^{2/3}\sqrt{a^{2/3}+}}$$

Mathematica [C] time = 0.16, size = 126, normalized size = 0.51

$$-\frac{2i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-i\#1\log\left(\#1^2-2\#1\cos(c+dx)+1\right)}{\#1^4b-2\#1^2b-4i\#1a+b}\&\right]}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-1),x]
[Out] (((-2*I)/3)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#
1^6 & , (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 - I*Log[1 - 2*Cos[c
+ d*x]*#1 + #1^2]*#1)/(b - (4*I)*a*#1 - 2*b*#1^2 + b*#1^4) & ])/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin(dx+c)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.57, size = 83, normalized size = 0.34

$$\frac{\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{\left(\_R^4+2\_R^2+1\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\_R\right)}{\_R^5a+2\_R^3a+4\_R^2b+\_Ra}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3),x)
```

```
[Out] 1/3/d*sum((_R^4+2*_R^2+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/
2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{b\sin\left(dx+c\right)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] integrate(1/(b*sin(d*x + c)^3 + a), x)
```

mupad [B] time = 16.71, size = 609, normalized size = 2.49

$$\sum_{k=1}^{6}\ln\left(-\frac{8192\,a\,b^3\left(-729\,a^5+243\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,b-324\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)a^4\,\mathrm{root}\left(d^6+27\,a^2\,d^4+243\,a^4\,d^2+729\,a^4\left(a^2-b^2\right),d,k\right)+972\,a^3\,b^2+a^3\,b\,\mathrm{root}(d\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3),x)
```

```
[Out] symsum(log(-(8192*a*b^3*(972*a^3*b^2 - 729*a^5 - 9*a*root(d^6 + 27*a^2*d^4
+ 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 162*a^3*root(d^6 + 27*a^2*d^
4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2 - 4*tan(c/2 + (d*x)/2)*root(
d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5 + 243*a^4*b*t
an(c/2 + (d*x)/2) - 324*tan(c/2 + (d*x)/2)*a^4*root(d^6 + 27*a^2*d^4 + 243*
a^4*d^2 + 729*a^4*(a^2 - b^2), d, k) + 24*b*tan(c/2 + (d*x)/2)*root(d^6 + 2
7*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^4 - 72*a^2*tan(c/2 + (
d*x)/2)*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
+ 36*a*b*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^3
 + 243*b*a^3*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d,
k) + 648*tan(c/2 + (d*x)/2)*a^2*b^2*root(d^6 + 27*a^2*d^4 + 243*a^4*d^2 + 7
29*a^4*(a^2 - b^2), d, k) + 216*a^2*b*tan(c/2 + (d*x)/2)*root(d^6 + 27*a^2*
d^4 + 243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^2))/root(d^6 + 27*a^2*d^4 +
243*a^4*d^2 + 729*a^4*(a^2 - b^2), d, k)^5)*root(729*a^4*b^2*d^6 - 729*a^6*
d^6 - 243*a^4*d^4 - 27*a^2*d^2 - 1, d, k), k, 1, 6)/d
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{a+b\sin^3\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(1/(a + b*sin(c + d*x)**3), x)
```

3.391 $\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=299

$$\frac{2(-1)^{2/3}b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)^{3/2}}-\frac{2b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}-b^{2/3}\right)^{3/2}}+\frac{2\sqrt[3]{-1}\,b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+(-1\ldots}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}\right)}{3a^{2/3}d\left(a^{2/3}+\sqrt[3]{-1}\,b^{2/3}\right)^{3/2}}$$

```
[Out] 2/3*(-1)^(2/3)*b^(2/3)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c
))/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-(-1)^(2/3)*b^(2/3))
^(3/2)/d-2/3*b^(2/3)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b
^(2/3))^(1/2))/a^(2/3)/(a^(2/3)-b^(2/3))^(3/2)/d+2/3*(-1)^(1/3)*b^(2/3)*arc
tan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(
2/3))^(1/2))/a^(2/3)/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(3/2)/d+sec(d*x+c)*(b-a*s
in(d*x+c))/(-a^2+b^2)/d
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] time = 0.26, size = 432, normalized size = 1.44

$$-ib\cos(c+dx)\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-4i\#1^3a\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)+2\#1\ldots}{\ldots}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] (-6*b + 6*b*Cos[c + d*x] - I*b*Cos[c + d*x]*RootSum[(-I)*b + (3*I)*b*#1^2 +
 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & , (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d
*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c +
 d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 -
12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (6*I)*b*Log[1 - 2*Cos[
c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x
]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(
b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + 6*a*Sin[c + d*x])/(6*(a - b)
*(a + b)*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2] + Sin[(c
 + d*x)/2]))
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^2}{b\sin(dx+c)^3 + a}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="giac")
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 0.96, size = 164, normalized size = 0.55

$$-\frac{2}{d(2a+2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{2}{d(2a-2b)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}-\frac{b\left(\sum_{\_R=\mathrm{RootOf}\left(a\_Z^6+3a\_Z^4+8b\_Z^3+3a\_Z^2+a\right)}\frac{(\_R^4 b-}{}\right.}{3d(a-b)(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x)
[Out] -2/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)-1)-2/d/(2*a-2*b)/(tan(1/2*d*x+1/2*c)+1)-
1/3/d*b/(a-b)/(a+b)*sum((_R^4*b-2*_R^3*a+6*_R^2*b-2*_R*a+b)/(_R^5*a+2*_R^3*
a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3
*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
[Out] Timed out
```

mupad [B] time = 18.52, size = 19737, normalized size = 66.01

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)),x)
[Out] (a^2*symsum(log((8192*(a^2*b^20*cos(c/2 + (d*x)/2) - 12*a*b^21*sin(c/2 + (d
*x)/2) - 7*a^4*b^18*cos(c/2 + (d*x)/2) + 21*a^6*b^16*cos(c/2 + (d*x)/2) - 3
5*a^8*b^14*cos(c/2 + (d*x)/2) + 35*a^10*b^12*cos(c/2 + (d*x)/2) - 21*a^12*b
^10*cos(c/2 + (d*x)/2) + 7*a^14*b^8*cos(c/2 + (d*x)/2) - a^16*b^6*cos(c/2 +
 (d*x)/2) + 84*a^3*b^19*sin(c/2 + (d*x)/2) - 252*a^5*b^17*sin(c/2 + (d*x)/2
) + 420*a^7*b^15*sin(c/2 + (d*x)/2) - 420*a^9*b^13*sin(c/2 + (d*x)/2) + 252
*a^11*b^11*sin(c/2 + (d*x)/2) - 84*a^13*b^9*sin(c/2 + (d*x)/2) + 12*a^15*b^
```

7*sin(c/2 + (d*x)/2) - 198*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^3*b^20*sin(c/2 + (d*x)/2) + 714*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^5*b^18*sin(c/2 + (d*x)/2) - 1470*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^7*b^16*sin(c/2 + (d*x)/2) + 1890*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^9*b^14*sin(c/2 + (d*x)/2) - 1554*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^11*b^12*sin(c/2 + (d*x)/2) + 798*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^13*b^10*sin(c/2 + (d*x)/2) - 234*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^15*b^8*sin(c/2 + (d*x)/2) + 30*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^17*b^6*sin(c/2 + (d*x)/2) + 36*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^2*b^22*cos(c/2 + (d*x)/2) - 369*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^4*b^20*cos(c/2 + (d*x)/2) + 1575*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^6*b^18*cos(c/2 + (d*x)/2) - 3717*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^8*b^16*cos(c/2 + (d*x)/2) + 5355*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^10*b^14*cos(c/2 + (d*x)/2) - 4851*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^12*b^12*cos(c/2 + (d*x)/2) + 2709*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^14*b^10*cos(c/2 + (d*x)/2) - 855*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^16*b^8*cos(c/2 + (d*x)/2) + 117*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^18*b^6*cos(c/2 + (d*x)/2) - 1188*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^4*b^21*cos(c/2 + (d*x)/2) + 7803*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^6*b^19*cos(c/2 + (d*x)/2) - 21357*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^8*b^17*cos(c/2 + (d*x)/2) + 30807*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^10*b^15*cos(c/2 + (d*x)/2) - 23625*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^12*b^13*cos(c/2 + (d*x)/2) + 6993*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^14*b^11*cos(c/2 + (d*x)/2) + 2457*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^16*b^9*cos(c/2 + (d*x)/2) - 2403*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^

3*a^18*b^7*cos(c/2 + (d*x)/2) + 513*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^20*b^5*cos(c/2 + (d*x)/2) + 567*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^4*b^22*cos(c/2 + (d*x)/2) - 4374*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^6*b^20*cos(c/2 + (d*x)/2) + 14580*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^8*b^18*cos(c/2 + (d*x)/2) - 27216*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^10*b^16*cos(c/2 + (d*x)/2) + 30618*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^12*b^14*cos(c/2 + (d*x)/2) - 20412*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^14*b^12*cos(c/2 + (d*x)/2) + 6804*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^16*b^10*cos(c/2 + (d*x)/2) - 729*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^20*b^6*cos(c/2 + (d*x)/2) + 162*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^22*b^4*cos(c/2 + (d*x)/2) + 972*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^4*b^23*cos(c/2 + (d*x)/2) - 9477*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^6*b^21*cos(c/2 + (d*x)/2) + 41553*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^8*b^19*cos(c/2 + (d*x)/2) - 107892*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^10*b^17*cos(c/2 + (d*x)/2) + 183708*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^12*b^15*cos(c/2 + (d*x)/2) - 214326*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^14*b^13*cos(c/2 + (d*x)/2) + 173502*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^16*b^11*cos(c/2 + (d*x)/2) - 96228*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^18*b^9*cos(c/2 + (d*x)/2) + 34992*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^20*b^7*cos(c/2 + (d*x)/2) - 7533*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^22*b^5*cos(c/2 + (d*x)/2) + 729*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^24*b^3*cos(c/2 + (d*x)/2) - 396*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^3*b^21*sin(c/2 + (d*x)/2) + 2736*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^5*b^19*sin(c/2 + (d*x)/2) - 8064*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^7*b^17*sin(c/2 + (d*x)/2) + 13104*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*

a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^9*b^15*sin(c/2 + (d*x)/2) - 1
2600*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*
d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^
11*b^13*sin(c/2 + (d*x)/2) + 7056*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6
+ 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*
a^2*b^4*d^2 - b^4, d, k)^2*a^13*b^11*sin(c/2 + (d*x)/2) - 2016*root(2187*a^
8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^
4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^15*b^9*sin(c/2 +
(d*x)/2) + 144*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 -
729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4,
d, k)^2*a^17*b^7*sin(c/2 + (d*x)/2) + 36*root(2187*a^8*b^2*d^6 - 2187*a^6*b
^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^
4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^19*b^5*sin(c/2 + (d*x)/2) + 648*root(21
87*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a
^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^3*b^22*sin(c
/2 + (d*x)/2) - 3456*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6
*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 -
b^4, d, k)^3*a^5*b^20*sin(c/2 + (d*x)/2) + 6021*root(2187*a^8*b^2*d^6 - 21
87*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^
6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^7*b^18*sin(c/2 + (d*x)/2) + 189
*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6
- 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^9*b^
16*sin(c/2 + (d*x)/2) - 15687*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 72
9*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*
b^4*d^2 - b^4, d, k)^3*a^11*b^14*sin(c/2 + (d*x)/2) + 25137*root(2187*a^8*b
^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d
^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^13*b^12*sin(c/2 + (d
*x)/2) - 19089*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 -
729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4,
d, k)^3*a^15*b^10*sin(c/2 + (d*x)/2) + 7479*root(2187*a^8*b^2*d^6 - 2187*a^
6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2
*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^17*b^8*sin(c/2 + (d*x)/2) - 1269*roo
t(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 14
58*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^19*b^6*s
in(c/2 + (d*x)/2) + 27*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b
^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2
- b^4, d, k)^3*a^21*b^4*sin(c/2 + (d*x)/2) + 648*root(2187*a^8*b^2*d^6 - 2
187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a
^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^3*b^23*sin(c/2 + (d*x)/2) - 48
60*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^
6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^5*
b^21*sin(c/2 + (d*x)/2) + 15552*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 +
729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^
2*b^4*d^2 - b^4, d, k)^4*a^7*b^19*sin(c/2 + (d*x)/2) - 27216*root(2187*a^8*
b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*
d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^9*b^17*sin(c/2 + (d
*x)/2) + 27216*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 -
729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4,
d, k)^4*a^11*b^15*sin(c/2 + (d*x)/2) - 13608*root(2187*a^8*b^2*d^6 - 2187*a
^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^
2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^13*b^13*sin(c/2 + (d*x)/2) + 3888*r
oot(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 -
1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^17*b^9
*sin(c/2 + (d*x)/2) - 1944*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a
^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4
*d^2 - b^4, d, k)^4*a^19*b^7*sin(c/2 + (d*x)/2) + 324*root(2187*a^8*b^2*d^6
- 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 7
29*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^21*b^5*sin(c/2 + (d*x)/2)
+ 243*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10

*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^5*b^22*sin(c/2 + (d*x)/2) - 2187*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^7*b^20*sin(c/2 + (d*x)/2) + 8748*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^9*b^18*sin(c/2 + (d*x)/2) - 20412*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^11*b^16*sin(c/2 + (d*x)/2) + 30618*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^13*b^14*sin(c/2 + (d*x)/2) - 30618*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^15*b^12*sin(c/2 + (d*x)/2) + 20412*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^17*b^10*sin(c/2 + (d*x)/2) - 8748*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^19*b^8*sin(c/2 + (d*x)/2) + 2187*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^21*b^6*sin(c/2 + (d*x)/2) - 243*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^23*b^4*sin(c/2 + (d*x)/2) + 24*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a*b^22*sin(c/2 + (d*x)/2) - 33*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^2*b^21*cos(c/2 + (d*x)/2) + 231*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^4*b^19*cos(c/2 + (d*x)/2) - 693*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^6*b^17*cos(c/2 + (d*x)/2) + 1155*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^8*b^15*cos(c/2 + (d*x)/2) - 1155*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^10*b^13*cos(c/2 + (d*x)/2) + 693*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^12*b^11*cos(c/2 + (d*x)/2) - 231*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^14*b^9*cos(c/2 + (d*x)/2) + 33*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^16*b^7*cos(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k), k, 1, 6))/(d*(a^2 - b^2)) - (b^2*symsum(log((8192*(a^2*b^20*cos(c/2 + (d*x)/2) - 12*a*b^21*sin(c/2 + (d*x)/2) - 7*a^4*b^18*cos(c/2 + (d*x)/2) + 21*a^6*b^16*cos(c/2 + (d*x)/2) - 35*a^8*b^14*cos(c/2 + (d*x)/2) + 35*a^10*b^12*cos(c/2 + (d*x)/2) - 21*a^12*b^10*cos(c/2 + (d*x)/2) + 7*a^14*b^8*cos(c/2 + (d*x)/2) - a^16*b^6*cos(c/2 + (d*x)/2) + 84*a^3*b^19*sin(c/2 + (d*x)/2) - 252*a^5*b^17*sin(c/2 + (d*x)/2) + 420*a^7*b^15*sin(c/2 + (d*x)/2) - 420*a^9*b^13*sin(c/2 + (d*x)/2) + 252*a^11*b^11*sin(c/2 + (d*x)/2) - 84*a^13*b^9*sin(c/2 + (d*x)/2) + 12*a^15*b^7*sin(c/2 + (d*x)/2) - 198*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^3*b^20*sin(c/2 + (d*x)/2) + 714*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^5*b^18*sin(c/2 + (d*x)/2) - 1470*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 72

9*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^7*b^16*sin(c/2 + (d*x)/2) + 1890*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^9*b^14*sin(c/2 + (d*x)/2) - 1554*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^11*b^12*sin(c/2 + (d*x)/2) + 798*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^13*b^10*sin(c/2 + (d*x)/2) - 234*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^15*b^8*sin(c/2 + (d*x)/2) + 30*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^17*b^6*sin(c/2 + (d*x)/2) + 36*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^2*b^22*cos(c/2 + (d*x)/2) - 369*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^4*b^20*cos(c/2 + (d*x)/2) + 1575*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^6*b^18*cos(c/2 + (d*x)/2) - 3717*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^8*b^16*cos(c/2 + (d*x)/2) + 5355*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^10*b^14*cos(c/2 + (d*x)/2) - 4851*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^12*b^12*cos(c/2 + (d*x)/2) + 2709*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^14*b^10*cos(c/2 + (d*x)/2) - 855*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^16*b^8*cos(c/2 + (d*x)/2) + 117*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^18*b^6*cos(c/2 + (d*x)/2) - 1188*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^4*b^21*cos(c/2 + (d*x)/2) + 7803*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^6*b^19*cos(c/2 + (d*x)/2) - 21357*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^8*b^17*cos(c/2 + (d*x)/2) + 30807*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^10*b^15*cos(c/2 + (d*x)/2) - 23625*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^12*b^13*cos(c/2 + (d*x)/2) + 6993*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^14*b^11*cos(c/2 + (d*x)/2) + 2457*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^16*b^9*cos(c/2 + (d*x)/2) - 2403*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^18*b^7*cos(c/2 + (d*x)/2) + 513*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^20*b^5*cos(c/2 + (d*x)/2) + 567*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^4*b^22*cos(c/2 + (d*x)/2) - 4374*root(2187*a^8*b^2*d^6 - 2187*a^6 *b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^6*b^20*cos(c/2 + (d*x)/2) + 14580*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^8*b^18*cos(c/2 + (d*x)/2) - 27216*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^10*b^16*cos(c/2 + (d*x)/2) + 30618*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^12*b^14*cos(c/2 + (d*x)/2) - 20412*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^14*b^12*cos(c/2 + (d*x)/2) + 6804*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^16*b^10*cos(c/2 + (d*x)/2) - 729*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^20*b^6*cos(c/2 + (d*x)/2) + 162*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^22*b^4*cos(c/2 + (d*x)/2) + 972*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^4*b^23*cos(c/2 + (d*x)/2) - 9477*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^6*b^21*cos(c/2 + (d*x)/2) + 41553*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^8*b^19*cos(c/2 + (d*x)/2) - 107892*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^10*b^17*cos(c/2 + (d*x)/2) + 183708*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^12*b^15*cos(c/2 + (d*x)/2) - 214326*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^14*b^13*cos(c/2 + (d*x)/2) + 173502*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^16*b^11*cos(c/2 + (d*x)/2) - 96228*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^18*b^9*cos(c/2 + (d*x)/2) + 34992*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^20*b^7*cos(c/2 + (d*x)/2) - 7533*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^22*b^5*cos(c/2 + (d*x)/2) + 729*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^24*b^3*cos(c/2 + (d*x)/2) - 396*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^3*b^21*sin(c/2 + (d*x)/2) + 2736*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^5*b^19*sin(c/2 + (d*x)/2) - 8064*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^7*b^17*sin(c/2 + (d*x)/2) + 13104*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^9*b^15*sin(c/2 + (d*x)/2) - 12600*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^11*b^13*sin(c/2 + (d*x)/2) + 7056*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^13*b^11*sin(c/2 + (

d*x)/2) - 2016*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^15*b^9*sin(c/2 + (d*x)/2) + 144*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^17*b^7*sin(c/2 + (d*x)/2) + 36*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^2*a^19*b^5*sin(c/2 + (d*x)/2) + 648*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^3*b^22*sin(c/2 + (d*x)/2) - 3456*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^5*b^20*sin(c/2 + (d*x)/2) + 6021*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^7*b^18*sin(c/2 + (d*x)/2) + 189*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^9*b^16*sin(c/2 + (d*x)/2) - 15687*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^11*b^14*sin(c/2 + (d*x)/2) + 25137*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^13*b^12*sin(c/2 + (d*x)/2) - 19089*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^15*b^10*sin(c/2 + (d*x)/2) + 7479*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^17*b^8*sin(c/2 + (d*x)/2) - 1269*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^19*b^6*sin(c/2 + (d*x)/2) + 27*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^3*a^21*b^4*sin(c/2 + (d*x)/2) + 648*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^3*b^23*sin(c/2 + (d*x)/2) - 4860*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^5*b^21*sin(c/2 + (d*x)/2) + 15552*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^7*b^19*sin(c/2 + (d*x)/2) - 27216*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^9*b^17*sin(c/2 + (d*x)/2) + 27216*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^11*b^15*sin(c/2 + (d*x)/2) - 13608*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^13*b^13*sin(c/2 + (d*x)/2) + 3888*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^17*b^9*sin(c/2 + (d*x)/2) - 1944*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^19*b^7*sin(c/2 + (d*x)/2) + 324*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^4*a^21*b^5*sin(c/2 + (d*x)/2) + 243*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^5*b^22*sin(c/2 + (d*x)/2) - 2187*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^7*b^20*sin(c/2 + (d*x)/2) + 8748*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d

```
, k)^5*a^9*b^18*sin(c/2 + (d*x)/2) - 20412*root(2187*a^8*b^2*d^6 - 2187*a^6
*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*
d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^11*b^16*sin(c/2 + (d*x)/2) + 30618*ro
ot(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1
458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^13*b^14
*sin(c/2 + (d*x)/2) - 30618*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*
a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^
4*d^2 - b^4, d, k)^5*a^15*b^12*sin(c/2 + (d*x)/2) + 20412*root(2187*a^8*b^2
*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4
 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^17*b^10*sin(c/2 + (d*x
)/2) - 8748*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 72
9*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d,
k)^5*a^19*b^8*sin(c/2 + (d*x)/2) + 2187*root(2187*a^8*b^2*d^6 - 2187*a^6*b^
4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4
 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^21*b^6*sin(c/2 + (d*x)/2) - 243*root(218
7*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^
4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)^5*a^23*b^4*sin(c/
2 + (d*x)/2) + 24*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^
6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^
4, d, k)*a*b^22*sin(c/2 + (d*x)/2) - 33*root(2187*a^8*b^2*d^6 - 2187*a^6*b^
4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4
 - 81*a^2*b^4*d^2 - b^4, d, k)*a^2*b^21*cos(c/2 + (d*x)/2) + 231*root(2187*
a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*
b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^4*b^19*cos(c/2 +
(d*x)/2) - 693*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 -
 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4,
d, k)*a^6*b^17*cos(c/2 + (d*x)/2) + 1155*root(2187*a^8*b^2*d^6 - 2187*a^6*b
^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^
4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^8*b^15*cos(c/2 + (d*x)/2) - 1155*root(218
7*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^
4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^10*b^13*cos(c/2
 + (d*x)/2) + 693*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^
6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^
4, d, k)*a^12*b^11*cos(c/2 + (d*x)/2) - 231*root(2187*a^8*b^2*d^6 - 2187*a^
6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2
*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^14*b^9*cos(c/2 + (d*x)/2) + 33*root(21
87*a^8*b^2*d^6 - 2187*a^6*b^4*d^6 + 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a
^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*a^2*b^4*d^2 - b^4, d, k)*a^16*b^7*cos(c/2
 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(2187*a^8*b^2*d^6 - 2187*a^6*b^4*d^6
+ 729*a^4*b^6*d^6 - 729*a^10*d^6 - 1458*a^4*b^4*d^4 - 729*a^6*b^2*d^4 - 81*
a^2*b^4*d^2 - b^4, d, k), k, 1, 6))/(d*(a^2 - b^2)) - b/(d*cos(c + d*x)*(a^
2 - b^2)) + (a*sin(c + d*x))/(d*cos(c + d*x)*(a^2 - b^2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec^2(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3),x)
```

```
[Out] Integral(sec(c + d*x)**2/(a + b*sin(c + d*x)**3), x)
```

3.392 $\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$

Optimal. Leaf size=1093

$$-\frac{2(-1)^{2/3}a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{-1}\sqrt[3]{b}-\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}}\right)b^{8/3}}{\sqrt{a^{2/3}-(-1)^{2/3}b^{2/3}}\left(a^2-b^2\right)^2 d}+\frac{2a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}(c+dx)\right)+\sqrt[3]{b}}{\sqrt{a^{2/3}-b^{2/3}}}\right)b^{8/3}}{\sqrt{a^{2/3}-b^{2/3}}\left(a^2-b^2\right)^2 d}-\frac{2\sqrt[3]{-1}\,a^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}\tan\left(\frac{1}{2}\right.}{\sqrt{a^2}}\right.}{\sqrt{a^{2/3}+\sqrt[3]{-1}\,b^{2/3}}}$$

```
[Out] 1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*x+c))^2+1/12*cos(d*x+c)/(a+b)/d/(1-sin(d*x
+c))+1/4*(a+4*b)*cos(d*x+c)/(a+b)^2/d/(1-sin(d*x+c))-1/12*cos(d*x+c)/(a-b)/
d/(1+sin(d*x+c))^2-1/4*(a-4*b)*cos(d*x+c)/(a-b)^2/d/(1+sin(d*x+c))-1/12*cos
(d*x+c)/(a-b)/d/(1+sin(d*x+c))+2*a^(2/3)*b^(8/3)*arctan((b^(1/3)+a^(1/3)*ta
n(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^
(1/2)+2/3*b^2*(2*a^2+b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2
/3)-b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)+2/3*b^(4/
3)*(a^2+2*b^2)*arctan((b^(1/3)+a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-b^(2/3)
)^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-b^(2/3))^(1/2)-2/3*b^(4/3)*(a^2+2*b^2)*arct
anh((b^(1/3)+(-1)^(2/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/((-1)^(1/3)*a^(2/3)+b^(
2/3))^(1/2))/(a^2-b^2)^2/d/((-1)^(1/3)*a^(2/3)+b^(2/3))^(1/2)-2/3*b^(4/3)*(
a^2+2*b^2)*arctanh((b^(1/3)-(-1)^(1/3)*a^(1/3)*tan(1/2*d*x+1/2*c))/(-(-1)^(
2/3)*a^(2/3)+b^(2/3))^(1/2))/(a^2-b^2)^2/d/(-(-1)^(2/3)*a^(2/3)+b^(2/3))^(1
/2)-2*(-1)^(1/3)*a^(2/3)*b^(8/3)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/3)*tan(1/2
*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)+(-1
)^(1/3)*b^(2/3))^(1/2)+2/3*b^2*(2*a^2+b^2)*arctan(((-1)^(2/3)*b^(1/3)+a^(1/
3)*tan(1/2*d*x+1/2*c))/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2
)^2/d/(a^(2/3)+(-1)^(1/3)*b^(2/3))^(1/2)-2*(-1)^(2/3)*a^(2/3)*b^(8/3)*arcta
n(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(2/3)*b^(2/
3))^(1/2))/(a^2-b^2)^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/2)-2/3*b^2*(2*a^2+
b^2)*arctan(((-1)^(1/3)*b^(1/3)-a^(1/3)*tan(1/2*d*x+1/2*c))/(a^(2/3)-(-1)^(
2/3)*b^(2/3))^(1/2))/a^(2/3)/(a^2-b^2)^2/d/(a^(2/3)-(-1)^(2/3)*b^(2/3))^(1/
2)
```

Rubi [F] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3), x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx=\int \frac{\sec^4(c+dx)}{a+b\sin^3(c+dx)}\,dx$$

Mathematica [C] time = 1.70, size = 679, normalized size = 0.62

$$\frac{\sec^3(c+dx)\left(12a^3\sin(c+dx)+4a^3\sin(3(c+dx))-3b\left(5a^2+13b^2\right)\cos(c+dx)+12b\left(a^2+2b^2\right)\cos(2(c+dx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3),x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 4*b^2*ArcTan[Sin[c
 + d*x]/(Cos[c + d*x] - #1)] - I*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (2
*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (12*I)*a*b*ArcTan[Sin[c + d*x]/
(Cos[c + d*x] - #1)]*#1 + 6*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 20*a
^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 16*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + (10*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]
*#1^2 + (8*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (12*I)*a*b*ArcTa
n[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 6*a*b*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*a^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 4*b^2*
ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*a^2*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2]*#1^4 - (2*I)*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1
 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ] + Sec[c + d*x]^3*(4*a^2*b + 32*b^3
 - 3*b*(5*a^2 + 13*b^2)*Cos[c + d*x] + 12*b*(a^2 + 2*b^2)*Cos[2*(c + d*x)]
- 5*a^2*b*Cos[3*(c + d*x)] - 13*b^3*Cos[3*(c + d*x)] + 12*a^3*Sin[c + d*x]
- 30*a*b^2*Sin[c + d*x] + 4*a^3*Sin[3*(c + d*x)] - 22*a*b^2*Sin[3*(c + d*x)
]))/(24*(a - b)^2*(a + b)^2*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^{4}}{b\sin\left(dx+c\right)^{3}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a), x)
```

maple [C] time = 1.13, size = 346, normalized size = 0.32

$$-\frac{2}{3d\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}(2a+2b)}-\frac{1}{d\left(2a+2b\right)\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{a}{d\left(a+b\right)^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}-\frac{}{2d\left(a+b\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)-1)^3/(2*a+2*b)-1/d/(2*a+2*b)/(tan(1/2*d*x+1/2*c)
-1)^2-1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)*a-5/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c
)-1)*b-2/3/d/(tan(1/2*d*x+1/2*c)+1)^3/(2*a-2*b)+1/d/(2*a-2*b)/(tan(1/2*d*x+
1/2*c)+1)^2-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)*a+5/2/d/(a-b)^2/(tan(1/2*d*x
+1/2*c)+1)*b+1/3/d*b^2/(a-b)^2/(a+b)^2*sum(((2*a^2+b^2)*_R^4-6*_R^3*a*b+2*(
4*a^2+5*b^2)*_R^2-6*a*_R*b+2*a^2+b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(ta
n(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [B] time = 25.85, size = 323390, normalized size = 295.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)),x)
```

```
[Out] (14*b^3*cos(c/2 + (d*x)/2)^6)/(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/
2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 -
2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4
*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4
*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 -
 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^2)) - (4*a^3*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)/(3*(a^4*
d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)
/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a
^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x
)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/
2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2
+ (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^
2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) + (6*b^3*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 +
(d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^
2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b
^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d
*x)/2)^2) - (8*b^3*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/(a^4*d*cos(c/
2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 -
b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d
*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 -
 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x
)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/
2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (4*a^2*b*cos(c/2 + (d*x)/2)^6)
/(3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/
2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2
)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c
/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4
*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4
*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)
^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) + (2*a^3*cos(c
/2 + (d*x)/2)*sin(c/2 + (d*x)/2)^5)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos
(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6
 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*
a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2
)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^
4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2
```

+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c
/2 + (d*x)/2)^2) + (2*a^3*cos(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/(a^4*d*c
os(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)
^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*
b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 +
(d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b
^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (a^4*cos(c/2 + (d*x)/2)^6
symsum(log(-(8192(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*
x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) -
2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 1201
2*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592
*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*
a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^2
4*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12
*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2
+ (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d
*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x
)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)
/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/
2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2)
- 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 2
0*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*
sin(c/2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2
+ (d*x)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*
x)/2) - 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2)
- 815100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 26
6994*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a

^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)*a^23*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*
b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c
/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2)
- 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 6608
7*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2
*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a
^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*
a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*
a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8

*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28
*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^1
0*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/
2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 +
(d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (
d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*
d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^1
0*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*
d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 +
(d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2
 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c
/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^

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12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c
/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/
2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d
*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2)
+ 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 8
40780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 9
72972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4
934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) -
9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290
```

*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root

(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*
b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^
26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 2186125
2*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 30238
92*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^
3*cos(c/2 + (d*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c
/2 + (d*x)/2) - 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 +
(d*x)/2) - 907452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 +

(d*x)/2) - 2630628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^

30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^

8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^

6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 773323
2*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 1181
4660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7
733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) -
2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) -
198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 437
4*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36

45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(
c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 -
 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2)
 - 5859*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)*a^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10
*b^31*cos(c/2 + (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27
*cos(c/2 + (d*x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(
c/2 + (d*x)/2) + 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 +
 (d*x)/2) + 68796*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/
2) + 5166*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
 d, k)*a^28*b^13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*
b^9*cos(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a

```
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)
/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin
(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2
 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*
cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*si
n(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*
a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 +
 (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (b^4*cos(c/2 + (d*x)/2)^6*symsum(log(-(
8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4
*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*
cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*co
s(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos
(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(
c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2
 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*
x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) -
1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020
*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a
^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^
17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^2
1*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b
^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*si
n(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x
)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 3
01392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^
15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^
22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
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5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*si
n(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 +
(d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2)
+ 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38
*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(
c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c
/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(
c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(
c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*

b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364

5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1

2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^

10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 +
 (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/
2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos
(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*co
s(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c
/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d
*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2)
- 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 90
7452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 26
30628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*

a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2 *d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d

^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^
15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 2390731
2*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4
*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 2863
3176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 1
2737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) -
 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) -
 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^
39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*s
in(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^1

0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a ^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^ 4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^ 12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6 *b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2* b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^1 0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a ^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^ 4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a ^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^ 6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2 *b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^ 10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729* a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a ^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin (c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645 *a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393* a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a ^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290 *a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7 29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364 5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22* sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3 645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123 93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13 5*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(72 90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3 645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^1 8*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12 393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1 35*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7 290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^ 14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12 393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1 35*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(72 90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3 645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^1 0*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36 45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239 3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135 *a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^ 10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729* a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a ^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12* b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^ 6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8 *d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d ^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2)

- 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*
cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2
+ (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*
x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2)
+ 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68
796*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^
13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 +
(d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*

cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2
)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 +
 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x
)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/
2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(
c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*si
n(c/2 + (d*x)/2)^2) - (a^4*sin(c/2 + (d*x)/2)^6*symsum(log(-(8192*(6*a^2*b^
38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2
+ (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x
)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/
2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2
) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2)
 + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) +
546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30
*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*
sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(
c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c
/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/
2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2
 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 +
(d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/
2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*
b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^30*s
in(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/
2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (
d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/

2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) -

133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 2
4597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37
*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*c
os(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*c
os(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25
*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b
^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*

b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b ^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1 2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(72 90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3 645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9 *cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645 *a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393* a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a ^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10* b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4 *b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4* b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b ^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6 *d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8* d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d ^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10 *d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d ^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d *x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d ^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d ^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d* x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d ^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d ^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d* x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d ^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^ 6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10* d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^ 4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d *x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2* d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^ 4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d ^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10 *d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d ^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2 *d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d ^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^

2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4

*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77
976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) +
 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
 d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2
) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/
2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2)
+ 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^3
5*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*
sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36

45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27
*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^2
3*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^1
9*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*
sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c
/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)
/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 2
3625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 17575
11*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830
457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*

b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12
675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) -
1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) +
1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) +
318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4
347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^4
1*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(
c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2
+ (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b

^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +

729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -

729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*co
s(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 + (d*x
)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2) + 1
62162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 339768
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^
16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^20*b^
21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17*cos
(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2 + (
d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))/cos
(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x
)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^

2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) - (b^4*sin(c/2 + (d*x)/2)^6*symsum(log(-(8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d

^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6

- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^

2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8 *b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(72

90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(c/2 + (d*

x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d
*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d
*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x
)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2)
- 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*
b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*s
in(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*si
n(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*s

in(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d

, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2

+ 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*
b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17
*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^
21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 1063319
4*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 45110
52*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 7435
80*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4
*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6
 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4

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 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/
2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
 d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 + (d*x)/2) + 25116*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*
b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2) + 162162*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*c
os(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 339768*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/
2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^20*b^21*cos(c/2 + (
d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(7290*a^10*b^4*d^6 -
 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2
) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
 d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2 + (d*x)/2) - 729*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3
0*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2
))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k),
 k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d
*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 +
(d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^
```

2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (3*a^4*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4*symsum(log(-(8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^

8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*
b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 +
(d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2)
+ 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 23
5053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 101
8017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 11
93049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 20
7207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 1334
97*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b

^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a
^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^
8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(
c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/
2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(c/
2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(
c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*c
os(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*
cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393

*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6

+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)
/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)
/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2)
+ 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4
*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^3
7*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^3
3*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*
b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^
20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 7797675
6*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*

b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 4124
3904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9
220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) +
743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 140
94*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^3*b^3
9*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^35*sin
(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*sin(c
/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(
c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a

^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6

*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 1267566
3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3
*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 19420
83*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796
067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 3187
89*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^
35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin
(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 +
(d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d
*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d

^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (
d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2
+ (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c
/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*si
n(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*s
in(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(
c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d
*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) +
37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 74

3580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 45
11052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 1
0633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2)
+ 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2
) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/
2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2
) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 24
3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c/2

+ (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d

*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(
c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*si
n(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)
 - (3*a^4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2*symsum(log(-(8192*(6*a^
2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(
c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 +
(d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d
*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*
x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x
)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2
) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*
a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b
^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*
sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*s
in(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*si
n(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin
(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/
2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d
*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 317
10*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*
a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^
30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*si
n(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2
 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*
d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^1
0*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*
d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d
*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +

b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^

8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2)
 - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 3
69*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*
b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^
33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^
29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*
b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^
22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a
^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d

^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^
30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34
*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos
(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2
+ (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 +
 (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 +
 (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2
+ (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2
 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*

b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/
2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/
2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 +
 (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2)
 + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 1618
38*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 356918
4*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 2429
0280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 6
5732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2)
+ 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36

45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/
2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*
x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d
*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)
/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 9
36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7
*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b
^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*
b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +

12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7

29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2)
- 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2
) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/
2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/
2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2)
 + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3
*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*
sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin
(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729

```
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*si
n(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25
*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b
^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^2
7*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^
31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35
*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin
(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*
d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^1
0*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*
d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (
d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x
)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
```

+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645

*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 +
(d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2)
 + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
 d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 33
9768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2
0*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17
*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2
 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))
/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 +
(d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^
2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*c
os(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b

^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (3*b^4*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4*symsum(log(-(8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^

4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4 *d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) +

109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) +
26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4
*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36
*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*
cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*
cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24
*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^2
0*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^

16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*

d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2
 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(
c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos
(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/
2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*
x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) -
 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907
452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 263
0628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^2*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4

*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 24
55596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 6
84684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22
932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*si
n(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 +
(d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*
x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d
*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*
d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^1
0*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*

d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^

4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^1 5*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6* d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^ 6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d ^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312 *root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b ^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4 *d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4* a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b ^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14 *d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^ 6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633 176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^ 6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8* b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k) ^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^ 8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a ^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4 *b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12 737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645 *a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a ^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290* a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729 *a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a ^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364 5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645* a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d , k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290* a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729 *a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a ^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645* a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^ 8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b ^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14 *d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^ 6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*ro ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8* d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^ 4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3 9*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12 393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1 35*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290* a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72 9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645 *a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*si n(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a ^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^ 6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2 *b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10 *b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^ 4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4 *b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c /2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1 2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*

b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6

+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*c
os(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 +
(d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x
)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2)
+ 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 687
96*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*
a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^1
3*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (
d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*c
os(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)
^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 +
3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c

/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2) - (3*b^4*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2*symsu
m(log(-(8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2)
- 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a
^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12
*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*
b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b
^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16
*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c
/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*
x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2)
- 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) -
60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) -
60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 2
0020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 182
0*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29
*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/
2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x
)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2)
- 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815
100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23
*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -

729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7

29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*
cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(
c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d
*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/
2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2
) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/
2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*
x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (
d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (
d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*

d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*

b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 8116
20*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*
a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*
cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2
+ (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 +
(d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2
+ (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(
c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*c
os(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(72

90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x

)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*
x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x
)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 72
90*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 13
5*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2)
+ 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*
sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(
c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin
(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*s

in(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +

135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b
^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*
b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^
23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 208202
4*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*
a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*
b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 +
(d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a

^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 585
9*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a
^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*
cos(c/2 + (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c
/2 + (d*x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 +
(d*x)/2) + 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6
 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)
/2) + 68796*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5
166*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
*a^28*b^13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*co
s(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6
+ b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 +
 (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x
)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/
2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2
+ (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^
2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)

/2)^4*sin(c/2 + (d*x)/2)^2) - (8*a*b^2*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2
)^5)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c
/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/
2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(
c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^
4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^
4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) - (8*a*b^2*cos
(c/2 + (d*x)/2)^5*sin(c/2 + (d*x)/2))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*c
os(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)
^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 +
3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c
/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2) + (40*a*b^2*cos(c/2 + (d*x)/2)^3*sin(c/2 + (d*x)/2)^3)/(
3*(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2
 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^
6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2
 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d
*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*s
in(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4
 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)) - (4*a^2*b*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2)/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*c
os(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)
^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 +
3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)
/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2
)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c
/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin
(c/2 + (d*x)/2)^2) - (2*a^2*b^2*cos(c/2 + (d*x)/2)^6*symsum(log(-(8192*(6*a
^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos
(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 +
 (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (
d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d
*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*
x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/
2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6
*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*
b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31
*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*
sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*s
in(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*si
n(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c
/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (
d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31
710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*roo

t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b
^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*s
in(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/
2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (
d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2)
 + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8
*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2
+ (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*
x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
 b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6

- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x
)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*
x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d
^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*
x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)
/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2
) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3
645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 364
5*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) -
369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6
*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b
^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(

7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2

+ (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)

^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*

a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) -
936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*r
oot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8
*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d
^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^
7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*
b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15
*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^1
9*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2
3*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^
6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*
b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*
sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8

*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645 *a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^

6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4

*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/
2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2
 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*
x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3
645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^
8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^
39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*
b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4
*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*
b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 +
 (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 -
 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2
) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 3
39768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^
20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6

+ 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^1
7*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/
2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2))
)/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 +
 (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a
^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*
cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*si
n(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*
b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*
x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (
d*x)/2)^2) + (2*a^2*b^2*sin(c/2 + (d*x)/2)^6*symsum(log(-(8192*(6*a^2*b^38*
cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2 + (
d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x)/2
) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/2)
+ 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2) +
 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2) +
6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) + 546
*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30*b^
10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*sin
(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(c/2
 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c/2
+ (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/2 +
 (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2 +
(d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 + (d*
x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/2)
- 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*b^3
4*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729

*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^11*b^30*sin(
c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/2 +
(d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (d*x
)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/2)
+ 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2
9*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2
+ (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
+ b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/
2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) +
235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*

a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a

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^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^14*b^29*cos(
c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a
^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^
6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2
*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*co
s(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21
*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^1
7*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13
*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*co
s(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8
*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 +
(d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6
- 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^
6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)
/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/
2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
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+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -

3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root

(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^3*b
^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12
393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 1
35*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7290
*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 7
29*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 364
5*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^35*s
in(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*sin
(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*si
n(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*s
in(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364
5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393
*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*
a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*s
in(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin
(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^1
2*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*
b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b
^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^
8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2
+ (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7
290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 +
 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 1
35*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2)

 + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 2362
5*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3
*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a
^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*
a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675
663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 194
2083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 17
96067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 31
8789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*

a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135 *a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4

- 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x) /2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k) ^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d ^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^ 2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a ^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729 *a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645* a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c /2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b ^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6 *d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8* d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^ 6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 - 7290*a^ 8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a ^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4 *b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2) + 1621 62*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6 *b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b ^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)* a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6 *d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d ^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6* d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 339768*ro ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8* d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^ 4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^16* b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*root(72 90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^20*b^21* cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 364 5*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393 *a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*

a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17*cos(c/
2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2 + (d*x
)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))/cos(c/
2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2
)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d
*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2
+ (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 +
(d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*co
s(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*
sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^
2) - (6*a^2*b^2*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4*symsum(log(-(8192
*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^3
6*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(
c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/
2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2
 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2
+ (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (
d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2
) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820
*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9
*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*
b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b
^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^
19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*
sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/
2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 -
 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2)
 - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36
45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 30139
2*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^

4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a
^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b
^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(729
0*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 36
45*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*s
in(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645
*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*
a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a
^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/
2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*
b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^
6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8
*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*
x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*
root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^
8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*
d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3
3*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos
(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2
+ (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b
^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6
*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*
d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4
*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^
10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8

*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +

d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a
^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 1319303
7*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3
*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 1388
8017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 33
39063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*
a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^
8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^
8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a
^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4
*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 10
9431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 266
49*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^
40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*
a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 72
9*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645
*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*co
s(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^
10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*
a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a
^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos
(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a

^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 4934358*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 9660222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 30861*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729 *a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 36

45*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645
*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8,
d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a
^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*
a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^
4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84
132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*
a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 263062
8*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2
*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 24555
96*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 6846
84*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*
a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root
(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^
6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
+ 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*
b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c
/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*

b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290

*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2 + (d*x)/2) +
648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/2) + 35964*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^7
*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) - 311040*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^11*b^
33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 -
3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1
2393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 +
135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) - 6722352*root(
7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^15*b
^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6
- 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +
12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2
+ 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2) - 23907312*ro
ot(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*
d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^
4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^1
9*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/2) - 28633176
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*
a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*x)/2) - 12737
088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (d*x)/2) - 196
3440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (d*x)/2) - 797

04*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*s in(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 + (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 5859*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31*cos(c/2 + (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(c/2 + (d*x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 + (d*x)/2) + 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x)/2) + 68796* root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) + 5166*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^28*b^13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*cos(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2 + (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2) + (6*a^2*b^2*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2*symsum(log(-(8192*(6*a^2*b^38*cos(c/2 + (d*x)/2) - 20*a*b^39*sin(c/2 + (d*x)/2) - 84*a^4*b^36*cos(c/2 + (d*x)/2) + 546*a^6*b^34*cos(c/2 + (d*x)/2) - 2184*a^8*b^32*cos(c/2 + (d*x)/2) + 6006*a^10*b^30*cos(c/2 + (d*x)/2) - 12012*a^12*b^28*cos(c/2 + (d*x)/2) + 18018*a^14*b^26*cos(c/2 + (d*x)/2) - 20592*a^16*b^24*cos(c/2 + (d*x)/2) + 18018*a^18*b^22*cos(c/2 + (d*x)/2) - 12012*a^20*b^20*cos(c/2 + (d*x)/2) + 6006*a^22*b^18*cos(c/2 + (d*x)/2) - 2184*a^24*b^16*cos(c/2 + (d*x)/2) + 546*a^26*b^14*cos(c/2 + (d*x)/2) - 84*a^28*b^12*cos(c/2 + (d*x)/2) + 6*a^30*b^10*cos(c/2 + (d*x)/2) + 280*a^3*b^37*sin(c/2 + (d*x)/2) - 1820*a^5*b^35*sin(c/2 + (d*x)/2) + 7280*a^7*b^33*sin(c/2 + (d*x)/2) - 20020*a^9*b^31*sin(c/2 + (d*x)/2) + 40040*a^11*b^29*sin(c/2 + (d*x)/2) - 60060*a^13*b^27*sin(c/2 + (d*x)/2) + 68640*a^15*b^25*sin(c/2 + (d*x)/2) - 60060*a^17*b^23*sin(c/2 + (d*x)/2) + 40040*a^19*b^21*sin(c/2 + (d*x)/2) - 20020*a^21*b^19*sin(c/2 + (d*x)/2) + 7280*a^23*b^17*sin(c/2 + (d*x)/2) - 1820*a^25*b^15*sin(c/2 + (d*x)/2) + 280*a^27*b^13*sin(c/2 + (d*x)/2) - 20*a^29*b^11*sin(c/2 + (d*x)/2) - 588*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^3*b^38*sin(c/2 + (d*x)/2) + 5715*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^5*b^36*sin(c/2 + (d*x)/2) - 31710*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^7*b^34*sin(c/2 + (d*x)/2) + 116025*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^9*b^32*sin(c/2 + (d*x)/2) - 301392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +

b^8, d, k)*a^11*b^30*sin(c/2 + (d*x)/2) + 579579*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^13*b^28*sin(c/2 + (d*x)/2) - 845130*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^15*b^26*sin(c/2 + (d*x)/2) + 945945*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^17*b^24*sin(c/2 + (d*x)/2) - 815100*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^19*b^22*sin(c/2 + (d*x)/2) + 537537*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^21*b^20*sin(c/2 + (d*x)/2) - 266994*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^23*b^18*sin(c/2 + (d*x)/2) + 96915*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^25*b^16*sin(c/2 + (d*x)/2) - 24360*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^27*b^14*sin(c/2 + (d*x)/2) + 3825*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^29*b^12*sin(c/2 + (d*x)/2) - 294*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^31*b^10*sin(c/2 + (d*x)/2) + 3*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^33*b^8*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^2*b^40*cos(c/2 + (d*x)/2) - 1143*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^4*b^38*cos(c/2 + (d*x)/2) + 11853*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^6*b^36*cos(c/2 + (d*x)/2) - 66087*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^8*b^34*cos(c/2 + (d*x)/2) + 235053*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^10*b^32*cos(c/2 + (d*x)/2) - 577395*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^12*b

^30*cos(c/2 + (d*x)/2) + 1018017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^14*b^28*cos(c/2 + (d*x)/2) - 1303731*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^16*b^26*cos(c/2 + (d*x)/2) + 1193049*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^18*b^24*cos(c/2 + (d*x)/2) - 724581*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^20*b^22*cos(c/2 + (d*x)/2) + 207207*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^22*b^20*cos(c/2 + (d*x)/2) + 85995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^24*b^18*cos(c/2 + (d*x)/2) - 133497*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^26*b^16*cos(c/2 + (d*x)/2) + 75663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^28*b^14*cos(c/2 + (d*x)/2) - 24597*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^30*b^12*cos(c/2 + (d*x)/2) + 4527*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^32*b^10*cos(c/2 + (d*x)/2) - 369*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^34*b^8*cos(c/2 + (d*x)/2) - 3078*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^4*b^39*cos(c/2 + (d*x)/2) + 33453*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^6*b^37*cos(c/2 + (d*x)/2) - 147744*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^8*b^35*cos(c/2 + (d*x)/2) + 279531*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^10*b^33*cos(c/2 + (d*x)/2) + 191646*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^12*b^31*cos(c/2 + (d*x)/2) - 2542995*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b

^8, d, k)^3*a^14*b^29*cos(c/2 + (d*x)/2) + 7459452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^16*b^27*cos(c/2 + (d*x)/2) - 13193037*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^18*b^25*cos(c/2 + (d*x)/2) + 16054038*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^20*b^23*cos(c/2 + (d*x)/2) - 13888017*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^22*b^21*cos(c/2 + (d*x)/2) + 8432424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^24*b^19*cos(c/2 + (d*x)/2) - 3339063*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^26*b^17*cos(c/2 + (d*x)/2) + 633906*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^28*b^15*cos(c/2 + (d*x)/2) + 109431*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^30*b^13*cos(c/2 + (d*x)/2) - 104004*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^32*b^11*cos(c/2 + (d*x)/2) + 26649*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^34*b^9*cos(c/2 + (d*x)/2) - 2592*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^36*b^7*cos(c/2 + (d*x)/2) + 891*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^4*b^40*cos(c/2 + (d*x)/2) - 12879*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^6*b^38*cos(c/2 + (d*x)/2) + 84807*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^8*b^36*cos(c/2 + (d*x)/2) - 332424*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^10*b^34*cos(c/2 + (d*x)/2) + 840780*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^12*b^32*cos(c/2 + (d*x)/2) - 1340388*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6

*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^14*b^30*cos(c/2 + (d*x)/2) + 972972
*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b
^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4
*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*
a^16*b^28*cos(c/2 + (d*x)/2) + 1187784*root(7290*a^10*b^4*d^6 - 7290*a^8*b^
6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*
d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6
*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^18*b^26*cos(c/2 + (d*x)/2) - 493435
8*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4
*a^20*b^24*cos(c/2 + (d*x)/2) + 8455590*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^22*b^22*cos(c/2 + (d*x)/2) - 96602
22*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^
4*a^24*b^20*cos(c/2 + (d*x)/2) + 8061768*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^26*b^18*cos(c/2 + (d*x)/2) - 5041
764*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^4*a^28*b^16*cos(c/2 + (d*x)/2) + 2360988*root(7290*a^10*b^4*d^6 - 7290*a^8
*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^
14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*
b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^30*b^14*cos(c/2 + (d*x)/2) - 811
620*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^
6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*
b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)
^4*a^32*b^12*cos(c/2 + (d*x)/2) + 196344*root(7290*a^10*b^4*d^6 - 7290*a^8*
b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^1
4*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b
^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^34*b^10*cos(c/2 + (d*x)/2) - 3086
1*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4
*a^36*b^8*cos(c/2 + (d*x)/2) + 2673*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^38*b^6*cos(c/2 + (d*x)/2) - 81*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^40*b^4
*cos(c/2 + (d*x)/2) + 972*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*
a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a
^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^
2*b^8*d^2 + b^8, d, k)^5*a^4*b^41*cos(c/2 + (d*x)/2) - 18225*root(7290*a^10
*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^
4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4
*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^6*b^39*cos(c/
2 + (d*x)/2) + 161838*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12
*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b
^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^
8*d^2 + b^8, d, k)^5*a^8*b^37*cos(c/2 + (d*x)/2) - 904689*root(7290*a^10*b^
4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b
^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^

8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^10*b^35*cos(c/2 + (d*x)/2) + 3569184*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^12*b^33*cos(c/2 + (d*x)/2) - 10558836*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^14*b^31*cos(c/2 + (d*x)/2) + 24290280*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^16*b^29*cos(c/2 + (d*x)/2) - 44466084*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^18*b^27*cos(c/2 + (d*x)/2) + 65732472*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^20*b^25*cos(c/2 + (d*x)/2) - 79158222*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^22*b^23*cos(c/2 + (d*x)/2) + 77976756*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^24*b^21*cos(c/2 + (d*x)/2) - 62832510*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^26*b^19*cos(c/2 + (d*x)/2) + 41243904*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^28*b^17*cos(c/2 + (d*x)/2) - 21861252*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^30*b^15*cos(c/2 + (d*x)/2) + 9220392*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^32*b^13*cos(c/2 + (d*x)/2) - 3023892*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^34*b^11*cos(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^36*b^9*cos(c/2 + (d*x)/2) - 129033*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^38*b^7*cos(c/2 + (d*x)/2) + 14094*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^40*b^5*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^42*b^3*cos(c/2 + (d*x)/2) - 936*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b

^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^3*b^39*sin(c/2 + (d*x)/2) + 13032*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^5*b^37*sin(c/2 + (d*x)/2) - 84132*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^7*b^35*sin(c/2 + (d*x)/2) + 333648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^9*b^33*sin(c/2 + (d*x)/2) - 907452*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^11*b^31*sin(c/2 + (d*x)/2) + 1788696*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^13*b^29*sin(c/2 + (d*x)/2) - 2630628*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^15*b^27*sin(c/2 + (d*x)/2) + 2924064*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^17*b^25*sin(c/2 + (d*x)/2) - 2455596*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^19*b^23*sin(c/2 + (d*x)/2) + 1534104*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^21*b^21*sin(c/2 + (d*x)/2) - 684684*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^23*b^19*sin(c/2 + (d*x)/2) + 196560*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^25*b^17*sin(c/2 + (d*x)/2) - 22932*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^27*b^15*sin(c/2 + (d*x)/2) - 6552*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^29*b^13*sin(c/2 + (d*x)/2) + 3348*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^31*b^11*sin(c/2 + (d*x)/2) - 576*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^33*b^9*sin(c/2 + (d*x)/2) + 36*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^2*a^35*b^7*sin(c/2 + (d*x)/2) + 1080*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 +

12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^3*b^40*sin(c/2 + (d*x)/2) - 6048*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^5*b^38*sin(c/2 + (d*x)/2) - 23625*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^7*b^36*sin(c/2 + (d*x)/2) + 361044*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^9*b^34*sin(c/2 + (d*x)/2) - 1757511*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^11*b^32*sin(c/2 + (d*x)/2) + 5066334*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^13*b^30*sin(c/2 + (d*x)/2) - 9830457*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^15*b^28*sin(c/2 + (d*x)/2) + 13374504*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^17*b^26*sin(c/2 + (d*x)/2) - 12675663*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^19*b^24*sin(c/2 + (d*x)/2) + 7729722*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^21*b^22*sin(c/2 + (d*x)/2) - 1942083*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^23*b^20*sin(c/2 + (d*x)/2) - 1366092*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^25*b^18*sin(c/2 + (d*x)/2) + 1796067*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^27*b^16*sin(c/2 + (d*x)/2) - 993006*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^29*b^14*sin(c/2 + (d*x)/2) + 318789*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^31*b^12*sin(c/2 + (d*x)/2) - 57456*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^33*b^10*sin(c/2 + (d*x)/2) + 4347*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^35*b^8*sin(c/2 + (d*x)/2) + 54*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*

b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^3*a^37*b^6*sin(c/2
+ (d*x)/2) + 648*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^4*a^3*b^41*sin(c/2 + (d*x)/2) - 7776*root(7290*a^10*b^4*d^6 -
 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^5*b^39*sin(c/2 + (d*x)/
2) + 35964*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^4*a^7*b^37*sin(c/2 + (d*x)/2) - 46656*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^9*b^35*sin(c/2 + (d*x)/2) -
311040*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645
*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a
^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d,
 k)^4*a^11*b^33*sin(c/2 + (d*x)/2) + 2068416*root(7290*a^10*b^4*d^6 - 7290*
a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729
*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a
^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^13*b^31*sin(c/2 + (d*x)/2) -
6722352*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 364
5*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*
a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d
, k)^4*a^15*b^29*sin(c/2 + (d*x)/2) + 14758848*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^17*b^27*sin(c/2 + (d*x)/2)
- 23907312*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 +
3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 36
45*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8
, d, k)^4*a^19*b^25*sin(c/2 + (d*x)/2) + 29652480*root(7290*a^10*b^4*d^6 -
7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6
+ 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 -
135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^21*b^23*sin(c/2 + (d*x)/
2) - 28633176*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6
 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
b^8, d, k)^4*a^23*b^21*sin(c/2 + (d*x)/2) + 21632832*root(7290*a^10*b^4*d^6
 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d
^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4
 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^25*b^19*sin(c/2 + (d*
x)/2) - 12737088*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*
d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^
4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2
 + b^8, d, k)^4*a^27*b^17*sin(c/2 + (d*x)/2) + 5769792*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^29*b^15*sin(c/2 + (
d*x)/2) - 1963440*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2
*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d
^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^
2 + b^8, d, k)^4*a^31*b^13*sin(c/2 + (d*x)/2) + 482112*root(7290*a^10*b^4*d
^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10
*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d
^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^33*b^11*sin(c/2 + (
d*x)/2) - 79704*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d

^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^35*b^9*sin(c/2 + (d*x)/2) + 7776*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^37*b^7*sin(c/2 + (d*x)/2) - 324*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^4*a^39*b^5*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^5*b^40*sin(c/2 + (d*x)/2) - 4374*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^7*b^38*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^9*b^36*sin(c/2 + (d*x)/2) - 198288*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^11*b^34*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^13*b^32*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^15*b^30*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^17*b^28*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^19*b^26*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^21*b^24*sin(c/2 + (d*x)/2) - 11814660*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^23*b^22*sin(c/2 + (d*x)/2) + 10633194*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^25*b^20*sin(c/2 + (d*x)/2) - 7733232*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^27*b^18*sin(c/2 + (d*x)/2) + 4511052*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^29*b^16*sin(c/2 + (d*x)/2) - 2082024*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^31*b^14*sin(c/2 + (d*x)/2) + 743580*root(7290*a^10*b^4*d^6 - 7290*a^8*b

^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^33*b^12*sin(c/2 + (d*x)/2) - 19828
8*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*
b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^
4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5
*a^35*b^10*sin(c/2 + (d*x)/2) + 37179*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6
*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d
^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*
d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^37*b^8*sin(c/2 + (d*x)/2) - 4374*roo
t(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d
^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4
 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)^5*a^39
*b^6*sin(c/2 + (d*x)/2) + 243*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3
645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 123
93*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 13
5*a^2*b^8*d^2 + b^8, d, k)^5*a^41*b^4*sin(c/2 + (d*x)/2) + 24*root(7290*a^1
0*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a
^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^
4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a*b^40*sin(c/2 +
 (d*x)/2) - 57*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^
6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4
+ 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 +
 b^8, d, k)*a^2*b^39*cos(c/2 + (d*x)/2) + 846*root(7290*a^10*b^4*d^6 - 7290
*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 72
9*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*
a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^4*b^37*cos(c/2 + (d*x)/2) - 58
59*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6
*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b
^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*
a^6*b^35*cos(c/2 + (d*x)/2) + 25116*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d
^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6
 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^
2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^8*b^33*cos(c/2 + (d*x)/2) - 74529*root(7
290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6
- 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 +
3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^10*b^31
*cos(c/2 + (d*x)/2) + 162162*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 36
45*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 1239
3*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135
*a^2*b^8*d^2 + b^8, d, k)*a^12*b^29*cos(c/2 + (d*x)/2) - 267267*root(7290*a
^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729
*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*
a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^14*b^27*cos(
c/2 + (d*x)/2) + 339768*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^
12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6
*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*
b^8*d^2 + b^8, d, k)*a^16*b^25*cos(c/2 + (d*x)/2) - 335907*root(7290*a^10*b
^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*
b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b
^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^18*b^23*cos(c/2 +
 (d*x)/2) + 258258*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^
2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*
d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d
^2 + b^8, d, k)*a^20*b^21*cos(c/2 + (d*x)/2) - 153153*root(7290*a^10*b^4*d^
6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*
d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^
4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^22*b^19*cos(c/2 + (d*x
)/2) + 68796*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6

```
+ 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 +
3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b
^8, d, k)*a^24*b^17*cos(c/2 + (d*x)/2) - 22659*root(7290*a^10*b^4*d^6 - 729
0*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 7
29*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135
*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^26*b^15*cos(c/2 + (d*x)/2) +
5166*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a
^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8
*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k
)*a^28*b^13*cos(c/2 + (d*x)/2) - 729*root(7290*a^10*b^4*d^6 - 7290*a^8*b^6*
d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14*d^
6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^6*d
^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^30*b^11*cos(c/2 + (d*x)/2) + 48*root(72
90*a^10*b^4*d^6 - 7290*a^8*b^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 -
 729*a^4*b^10*d^6 + 729*a^14*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3
645*a^4*b^8*d^4 - 135*a^4*b^6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k)*a^32*b^9*c
os(c/2 + (d*x)/2)))/cos(c/2 + (d*x)/2))*root(7290*a^10*b^4*d^6 - 7290*a^8*b
^6*d^6 - 3645*a^12*b^2*d^6 + 3645*a^6*b^8*d^6 - 729*a^4*b^10*d^6 + 729*a^14
*d^6 + 12393*a^6*b^6*d^4 + 3645*a^8*b^4*d^4 + 3645*a^4*b^8*d^4 - 135*a^4*b^
6*d^2 + 135*a^2*b^8*d^2 + b^8, d, k), k, 1, 6))/(a^4*d*cos(c/2 + (d*x)/2)^6
 + b^4*d*cos(c/2 + (d*x)/2)^6 - a^4*d*sin(c/2 + (d*x)/2)^6 - b^4*d*sin(c/2
+ (d*x)/2)^6 - 2*a^2*b^2*d*cos(c/2 + (d*x)/2)^6 + 2*a^2*b^2*d*sin(c/2 + (d*
x)/2)^6 + 3*a^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 - 3*a^4*d*cos(c
/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 + 3*b^4*d*cos(c/2 + (d*x)/2)^2*sin(c/2
 + (d*x)/2)^4 - 3*b^4*d*cos(c/2 + (d*x)/2)^4*sin(c/2 + (d*x)/2)^2 - 6*a^2*b
^2*d*cos(c/2 + (d*x)/2)^2*sin(c/2 + (d*x)/2)^4 + 6*a^2*b^2*d*cos(c/2 + (d*x
)/2)^4*sin(c/2 + (d*x)/2)^2)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3),x)
[Out] Timed out
```

3.393 $\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=288

$$-\frac{\sin(c+dx)\left(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)}-\frac{\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+d\right.}{9a^{5/3}b^{7/3}d}$$

```
[Out] 2/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^
(7/3)/d-1/9*(2*a^2-3*a^(4/3)*b^(2/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*
x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(7/3)/d-sin(d*x+c)/b^2/d-1/3*sin(d*x+c
)*(a^2-b^2+3*a*b*sin(d*x+c)+3*b^2*sin(d*x+c)^2)/a/b^2/d/(a+b*sin(d*x+c)^3)-
2/9*(2*a^2+3*a^(4/3)*b^(2/3)+b^2)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))
/a^(1/3)*3^(1/2))/a^(5/3)/b^(7/3)/d*3^(1/2)
```

Rubi [A] time = 0.34, antiderivative size = 288, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1858, 1887, 1860, 31, 634, 617, 204, 628}

$$-\frac{\sin(c+dx)\left(a^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)-b^2\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)}-\frac{\left(-3a^{4/3}b^{2/3}+2a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+d\right.}{9a^{5/3}b^{7/3}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(2*a^2 + 3*a^(4/3)*b^(2/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d
*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(7/3)*d) + (2*(2*a^2 - 3*a^(4
/3)*b^(2/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(7/3)*
d) - ((2*a^2 - 3*a^(4/3)*b^(2/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c
 + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5/3)*b^(7/3)*d) - Sin[c + d*x]/(b^
2*d) - (Sin[c + d*x]*(a^2 - b^2 + 3*a*b*Sin[c + d*x] + 3*b^2*Sin[c + d*x]^2
))/(3*a*b^2*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1887

```
Int[(Pq_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[ExpandIntegrand[Pq/(a
+ b*x^n), x], x] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IntegerQ[n]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x^2)^3}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-a^2-2b^2-6abx}{a+bx^3} \right.}{3}$$

$$= -\frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \left(3a - \frac{2(2a^2}{a}\right.\right.}{}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subs}}{}$$

$$= -\frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2-b^2+3ab\sin(c+dx)+3b^2\sin^2(c+dx)\right)}{3ab^2d\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subs}}{}$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\sin(c+dx)}{b^2d} - \frac{\sin(c+dx)\left(a^2\right.}{}$$

$$= \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{7/3}d} - \frac{\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(a^2\right.}{}$$

$$= -\frac{2\left(2a^2+3a^{4/3}b^{2/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{7/3}d} + \frac{2\left(2a^2-3a^{4/3}b^{2/3}+b^2\right)\log\left(\sqrt[3]{a}\right.}{9a^{5/3}b^{7/3}d}$$

Mathematica [C] time = 3.50, size = 402, normalized size = 1.40

$$\frac{\frac{6\sqrt[3]{-1}\left(2\sqrt[3]{-1}a^{2/3}+3b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}\,b^{7/3}} + \frac{6\left(2a^{2/3}-3b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{\sqrt[3]{a}\,b^{7/3}} - \frac{6\sqrt[3]{-1}\left(2a^{2/3}+3\sqrt[3]{-1}b^{2/3}\right)\log\left(\sqrt[3]{a}+(-1)^{2/3}\right.}{\sqrt[3]{a}\,b^{7/3}}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((6*(-1)^(1/3)*(2*(-1)^(1/3)*a^(2/3) + 3*b^(2/3))*Log[-((-1)^(2/3)*a^(1/3))
 - b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) + (6*(2*a^(2/3) - 3*b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(1/3)*b^(7/3)) - (6*(-1)^(1/3)*(2*a^(
2/3) + 3*(-1)^(1/3)*b^(2/3))*Log[a^(1/3) + (-1)^(2/3)*b^(1/3)*Sin[c + d*x]]
)/(a^(1/3)*b^(7/3)) + (2*(a^2 - b^2)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)
*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] +
 Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]))/(a^
(5/3)*b^(7/3)) - (18*Sin[c + d*x])/b^2 - (27*Hypergeometric2F1[2/3, 2, 5/3,
 -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 18/(b*(a + b*Sin[c + d*x]
^3)) + (6*(1 - a^2/b^2)*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

fricas [C] time = 178.22, size = 6415, normalized size = 22.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/36*(12*(7*a^2*b - b^3)*cos(d*x + c)^4 + 84*a^2*b - 12*b^3 - 6*sqrt(1/3)*(
a*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b^4*d*cos(d*x + c)^
2 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a^2*b^3*d)*sin(d*x
+ c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6
)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/
3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2)
)*arctan(1/64*sqrt(1/3)*((8*a^11*b^7 + 39*a^9*b^9 + 6*a^7*b^11 + a^5*b^13)*
(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/
3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2
*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7
*d^3))^(1/3)))^2*d^3 + 8*(16*a^11*b^5 + 86*a^9*b^7 + 51*a^7*b^9 + 8*a^5*b^1
1 + a^3*b^13)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(
1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a
^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 +
 b^6)/(a^5*b^7*d^3))^(1/3)))*d^2*sin(d*x + c) - 288*(8*a^10*b^4 + 39*a^8*b^
6 + 6*a^6*b^8 + a^4*b^10)*d*sin(d*x + c) + 48*(16*a^11*b^3 + 86*a^9*b^5 + 5
1*a^7*b^7 + 8*a^5*b^9 + a^3*b^11)*d + 2*sqrt(4096*a^12 + 28032*a^10*b^2 + 4
3920*a^8*b^4 + 14176*a^6*b^6 + 2784*a^4*b^8 + 288*a^2*b^10 + 16*b^12 - (3*(
8*a^11*b^5 + 39*a^9*b^7 + 6*a^7*b^9 + a^5*b^11)*d^2*sin(d*x + c) - (16*a^12
*b^4 + 86*a^10*b^6 + 51*a^8*b^8 + 8*a^6*b^10 + a^4*b^12)*d^2)*(4^(1/3)*(I*s
qrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6
- 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b
^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(
a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))
)^2 - 16*(64*a^12 + 624*a^10*b^2 + 1617*a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8
 + 12*a^2*b^10 + b^12)*cos(d*x + c)^2 + 4*((32*a^12*b^2 + 188*a^10*b^4 + 18
8*a^8*b^6 + 67*a^6*b^8 + 10*a^4*b^10 + a^2*b^12)*d*sin(d*x + c) + 9*(8*a^11
*b^3 + 39*a^9*b^5 + 6*a^7*b^7 + a^5*b^9)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^
6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a
^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) +
1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*
a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))) - 288*(16*a^11*b
 + 86*a^9*b^3 + 51*a^7*b^5 + 8*a^5*b^7 + a^3*b^9)*sin(d*x + c))*(36*a^4*b^4
*d - (2*a^5*b^5 + a^3*b^7)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 +
6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^
5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*(
(8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2
+ 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*d^2))*sqrt(((4^(1/3)*(I*sqrt(3) +
 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*
sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*
d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*
b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2))/(64*a^12 + 624*a^10*b^2 + 1617*
a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8 + 12*a^2*b^10 + b^12)) + 6*sqrt(1/3)*(a
*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b^4*d*cos(d*x + c)^2
 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a^2*b^3*d)*sin(d*x +
 c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)
/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3
) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*
b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^
6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2))
*arctan(-1/64*sqrt(1/3)*((8*a^11*b^7 + 39*a^9*b^9 + 6*a^7*b^11 + a^5*b^13)*
```

```
(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/
3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2
*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7
*d^3))^(1/3)))^2*d^3 + 8*(16*a^11*b^5 + 86*a^9*b^7 + 51*a^7*b^9 + 8*a^5*b^1
1 + a^3*b^13)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b
^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(
1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a
^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 +
 b^6)/(a^5*b^7*d^3))^(1/3)))*d^2*sin(d*x + c) - 288*(8*a^10*b^4 + 39*a^8*b^
6 + 6*a^6*b^8 + a^4*b^10)*d*sin(d*x + c) + 48*(16*a^11*b^3 + 86*a^9*b^5 + 5
1*a^7*b^7 + 8*a^5*b^9 + a^3*b^11)*d - 2*sqrt(4096*a^12 + 28032*a^10*b^2 + 4
3920*a^8*b^4 + 14176*a^6*b^6 + 2784*a^4*b^8 + 288*a^2*b^10 + 16*b^12 - (3*(
8*a^11*b^5 + 39*a^9*b^7 + 6*a^7*b^9 + a^5*b^11)*d^2*sin(d*x + c) - (16*a^12
*b^4 + 86*a^10*b^6 + 51*a^8*b^8 + 8*a^6*b^10 + a^4*b^12)*d^2)*(4^(1/3)*(I*s
qrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6
 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b
^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(
a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))
)^2 - 16*(64*a^12 + 624*a^10*b^2 + 1617*a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8
 + 12*a^2*b^10 + b^12)*cos(d*x + c)^2 + 4*((32*a^12*b^2 + 188*a^10*b^4 + 18
8*a^8*b^6 + 67*a^6*b^8 + 10*a^4*b^10 + a^2*b^12)*d*sin(d*x + c) + 9*(8*a^11
*b^3 + 39*a^9*b^5 + 6*a^7*b^7 + a^5*b^9)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^
6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a
^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) +
1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*
a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))) - 288*(16*a^11*b
 + 86*a^9*b^3 + 51*a^7*b^5 + 8*a^5*b^7 + a^3*b^9)*sin(d*x + c))*(36*a^4*b^4
*d - (2*a^5*b^5 + a^3*b^7)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 +
6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^
5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*(
(8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2
+ 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*d^2))*sqrt(((4^(1/3)*(I*sqrt(3) +
 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*
sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*
d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))^2*a^2*
b^4*d^2 + 384*a^2 + 192*b^2)/(a^2*b^4*d^2))/(64*a^12 + 624*a^10*b^2 + 1617*
a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8 + 12*a^2*b^10 + b^12)) - 12*(17*a^2*b -
 2*b^3)*cos(d*x + c)^2 + (a*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4
 + 3*a*b^4*d*cos(d*x + c)^2 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x +
c)^2 - a^2*b^3*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b
^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6
)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*
d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4
*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))*log(16384*a^12 + 112128*a^10
*b^2 + 175680*a^8*b^4 + 56704*a^6*b^6 + 11136*a^4*b^8 + 1152*a^2*b^10 + 64*
b^12 - 4*(3*(8*a^11*b^5 + 39*a^9*b^7 + 6*a^7*b^9 + a^5*b^11)*d^2*sin(d*x +
c) - (16*a^12*b^4 + 86*a^10*b^6 + 51*a^8*b^8 + 8*a^6*b^10 + a^4*b^12)*d^2)*
(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d
^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/
3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2
*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7
*d^3))^(1/3)))^2 - 64*(64*a^12 + 624*a^10*b^2 + 1617*a^8*b^4 + 484*a^6*b^6
+ 114*a^4*b^8 + 12*a^2*b^10 + b^12)*cos(d*x + c)^2 + 16*((32*a^12*b^2 + 188
*a^10*b^4 + 188*a^8*b^6 + 67*a^6*b^8 + 10*a^4*b^10 + a^2*b^12)*d*sin(d*x +
c) + 9*(8*a^11*b^3 + 39*a^9*b^5 + 6*a^7*b^7 + a^5*b^9)*d)*(4^(1/3)*(I*sqrt(
3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15
*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*
```

```
(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*
b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))) -
1152*(16*a^11*b + 86*a^9*b^3 + 51*a^7*b^5 + 8*a^5*b^7 + a^3*b^9)*sin(d*x +
c)) - (a*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b^4*d*cos(d*
x + c)^2 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a^2*b^3*d)*s
in(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^
6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1
/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^
4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 +
b^6)/(a^5*b^7*d^3))^(1/3)))*log(4096*a^12 + 28032*a^10*b^2 + 43920*a^8*b^4
+ 14176*a^6*b^6 + 2784*a^4*b^8 + 288*a^2*b^10 + 16*b^12 + (6*(8*a^11*b^5 +
39*a^9*b^7 + 6*a^7*b^9 + a^5*b^11)*d^2*sin(d*x + c) + (16*a^12*b^4 + 86*a^1
0*b^6 + 51*a^8*b^8 + 8*a^6*b^10 + a^4*b^12)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*(
(8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2
+ 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(
3) + 1)/(a^2*b^4*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3)
+ (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3)))^2 - 16*(64*
a^12 + 624*a^10*b^2 + 1617*a^8*b^4 + 484*a^6*b^6 + 114*a^4*b^8 + 12*a^2*b^1
0 + b^12)*cos(d*x + c)^2 - 4*(2*(32*a^12*b^2 + 188*a^10*b^4 + 188*a^8*b^6 +
 67*a^6*b^8 + 10*a^4*b^10 + a^2*b^12)*d*sin(d*x + c) - 9*(8*a^11*b^3 + 39*a
^9*b^5 + 6*a^7*b^7 + a^5*b^9)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((8*a^6 + 39*a^4*
b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^4*b^2 + 6*a^2*b^4 + b^
6)/(a^5*b^7*d^3))^(1/3) - 3*4^(2/3)*(2*a^2 + b^2)*(-I*sqrt(3) + 1)/(a^2*b^4
*d^2*((8*a^6 + 39*a^4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3) + (8*a^6 - 15*a^
4*b^2 + 6*a^2*b^4 + b^6)/(a^5*b^7*d^3))^(1/3))) + 576*(16*a^11*b + 86*a^9*b
^3 + 51*a^7*b^5 + 8*a^5*b^7 + a^3*b^9)*sin(d*x + c)) - 12*(3*a*b^2*cos(d*x
+ c)^6 - 12*a*b^2*cos(d*x + c)^4 + 12*a*b^2*cos(d*x + c)^2 - 4*a^3 - 2*a*b^
2)*sin(d*x + c))/(a*b^4*d*cos(d*x + c)^6 - 3*a*b^4*d*cos(d*x + c)^4 + 3*a*b
^4*d*cos(d*x + c)^2 - (a^3*b^2 + a*b^4)*d + 2*(a^2*b^3*d*cos(d*x + c)^2 - a
^2*b^3*d)*sin(d*x + c))
```

giac [**A**] time = 0.21, size = 277, normalized size = 0.96

$$-\frac{\frac{9\,\sin(dx+c)}{b^2}+\frac{2\left(3\,ab\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2+b^2\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b^2}+\frac{2\,\sqrt{3}\left(3\left(-ab^2\right)^{\frac{2}{3}}a-\left(-ab^2\right)^{\frac{1}{3}}\left(2\,a^2+b^2\right)\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,\sin(dx\cdots}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right.}{a^2b^3}}{9\,\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(9*sin(d*x + c)/b^2 + 2*(3*a*b*(-a/b)^(1/3) + 2*a^2 + b^2)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^2*b^2) + 2*sqrt(3)*(3*(-a*b^2)^
(2/3)*a - (-a*b^2)^(1/3)*(2*a^2 + b^2))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) +
2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) + 3*(3*a*b*sin(d*x + c)^2 + a^2*sin
(d*x + c) - b^2*sin(d*x + c) - 3*a*b)/((b*sin(d*x + c)^3 + a)*a*b^2) - (3*(
-a*b^2)^(2/3)*a + (-a*b^2)^(1/3)*(2*a^2 + b^2))*log(sin(d*x + c)^2 + (-a/b)
^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

maple [**B**] time = 0.93, size = 490, normalized size = 1.70

$$-\frac{\sin(dx+c)}{b^2d}-\frac{\sin^2(dx+c)}{db\left(a+b\left(\sin^3(dx+c)\right)\right)}-\frac{\sin(dx+c)\,a}{3d\,b^2\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\sin(dx+c)}{3ad\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\cdots}{db\left(a+b\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -sin(d*x+c)/b^2/d-1/d/b/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2-1/3/d/b^2/(a+b*sin(
d*x+c)^3)*sin(d*x+c)*a+1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+1/d/b/(a+b*sin
(d*x+c)^3)+4/9/d/b^3*a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+2/9/d/b/a/(a/
b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-2/9/d/b^3*a/(a/b)^(2/3)*ln(sin(d*x+c)^2
-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-
(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+4/9/d/b^3*a/(a/b)^(2/3)*3^(1/2)*arctan(
1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arc
tan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-2/3/d/b^2/(a/b)^(1/3)*ln(sin(
d*x+c)+(a/b)^(1/3))+1/3/d/b^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d
*x+c)+(a/b)^(2/3))+2/3/d/b^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*sin(d*x+c)-1))
```

maxima [A] time = 0.45, size = 263, normalized size = 0.91

$$-\frac{\frac{3\left(3\,ab\sin(dx+c)^2-3\,ab+\left(a^2-b^2\right)\sin(dx+c)\right)}{ab^3\sin(dx+c)^3+a^2b^2}+\frac{9\,\sin(dx+c)}{b^2}-\frac{2\sqrt{3}\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}+2\,a^2+b^2\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^3\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(3\,ab\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,a^2-b^2\right)\mathrm{l}\cdots}{\cdots}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(3*(3*a*b*sin(d*x + c)^2 - 3*a*b + (a^2 - b^2)*sin(d*x + c))/(a*b^3*si
n(d*x + c)^3 + a^2*b^2) + 9*sin(d*x + c)/b^2 - 2*sqrt(3)*(3*a*b*(a/b)^(1/3)
 + 2*a^2 + b^2)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1
/3))/(a*b^3*(a/b)^(2/3)) - (3*a*b*(a/b)^(1/3) - 2*a^2 - b^2)*log(sin(d*x +
c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^3*(a/b)^(2/3)) + 2*(3*a
*b*(a/b)^(1/3) - 2*a^2 - b^2)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^3*(a/b)^
(2/3)))/d
```

mupad [B] time = 0.42, size = 384, normalized size = 1.33

$$\sum_{k=1}^{3}\ln\left(\frac{8\,a^2+4\,b^2+\mathrm{root}\left(729\,a^5\,b^7\,d^3+648\,a^5\,b^3\,d+324\,a^3\,b^5\,d+120\,a^4\,b^2-48\,a^2\,b^4-8\,b^6-64\,a^6,d,k\right)^2\,a^2\,b^4\,27+12\,a\,b\,\sin(c+d\,x)+\mathrm{root}\left(729\,a^5\,b^7\,d^3+6\cdots\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log((8*a^2 + 4*b^2 + 27*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a
^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)^2*a^2*b^4 + 12*
a*b*sin(c + d*x) + 6*root(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d +
 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 64*a^6, d, k)*b^4*sin(c + d*x) + 12*roo
t(729*a^5*b^7*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^
4 - 8*b^6 - 64*a^6, d, k)*a^2*b^2*sin(c + d*x))/(3*a*b^2))*root(729*a^5*b^7
*d^3 + 648*a^5*b^3*d + 324*a^3*b^5*d + 120*a^4*b^2 - 48*a^2*b^4 - 8*b^6 - 6
4*a^6, d, k), k, 1, 3)/d - sin(c + d*x)/(b^2*d) - (b*sin(c + d*x)^2 - b + (
sin(c + d*x)*(a^2 - b^2))/(3*a))/(d*(a*b^2 + b^3*sin(c + d*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.394 $\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=238

$$\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/\right.}}{}$$

```
[Out] -2/9*(a^(4/3)-b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(5/3)/d+1/9
*(a^(4/3)-b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)
^2)/a^(5/3)/b^(5/3)/d+1/3*sin(d*x+c)*(b-a*sin(d*x+c)-2*b*sin(d*x+c)^2)/a/b/
d/(a+b*sin(d*x+c)^3)-2/9*(a^(4/3)+b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*si
n(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/b^(5/3)/d*3^(1/2)
```

Rubi [A] time = 0.22, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.348$, Rules used = {3223, 1858, 1860, 31, 634, 617, 204, 628}

$$\frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d}-\frac{2\left(a^{4/\right.}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*(a^(4/3) + b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*
a^(1/3))])/(3*Sqrt[3]*a^(5/3)*b^(5/3)*d) - (2*(a^(4/3) - b^(4/3))*Log[a^(1/
3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b^(5/3)*d) + ((a^(4/3) - b^(4/3))*Lo
g[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(9*a^(5
/3)*b^(5/3)*d) + (Sin[c + d*x]*(b - a*Sin[c + d*x] - 2*b*Sin[c + d*x]^2))/(
3*a*b*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1858

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_.))^(p_), x_Symbol] :> With[{q = Expon[Pq,
x]}, Module[{Q = PolynomialQuotient[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n,
 x], R = PolynomialRemainder[b^(Floor[(q - 1)/n] + 1)*Pq, a + b*x^n, x]}, D
ist[1/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), Int[(a + b*x^n)^(p + 1)*Expan
dToSum[a*n*(p + 1)*Q + n*(p + 1)*R + D[x*R, x], x], x], x] - Simp[(x*R*(a +
 b*x^n)^(p + 1))/(a*n*(p + 1)*b^(Floor[(q - 1)/n] + 1)), x]] /; GeQ[q, n]]
/; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+dx)\right)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{-2b^2-2abx}{a+bx^3}\,dx, x, \sin(\ldots\right.}{3ab^2d}$$

$$= \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2b\sin^2(c+dx)\right)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int \frac{\sqrt[3]{a}\left(-2a^{4/3}b-4b^{7/3}\right)+\sqrt[3]{b}\ldots}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x\ldots}\right.}{9a^5\ldots}$$

$$= -\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\sin(c+dx)\left(b-a\sin(c+dx)-2\ldots\right.}{3abd\left(a+b\sin^3(c+d\ldots\right.}$$

$$= -\frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}b^{5/3}d} + \frac{\left(a^{4/3}-b^{4/3}\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin\ldots\right.}{9a^{5/3}b^{5\ldots}}$$

$$= -\frac{2\left(a^{4/3}+b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}b^{5/3}d} - \frac{2\left(a^{4/3}-b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right.}{9a^{5/3}b^{5/3}d}$$

Mathematica [C] time = 1.09, size = 258, normalized size = 1.08

$$\frac{-\frac{2\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}+\frac{4\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}\sqrt[3]{b}}-\frac{4\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}}+\frac{9\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(}{a}\right.}{ab}}{18d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-4*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (4*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^(5/3)*b^(1/3
)) - (2*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2
])/(a^(5/3)*b^(1/3)) + (9*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^
3)/a)]*Sin[c + d*x]^2)/(a*b) - (9*Hypergeometric2F1[2/3, 2, 5/3, -((b*Sin[c
 + d*x]^3)/a)]*Sin[c + d*x]^2)/(a*b) + 12/(b*(a + b*Sin[c + d*x]^3)) + (6*S
in[c + d*x])/(a*(a + b*Sin[c + d*x]^3)))/(18*d)
```

fricas [C] time = 109.78, size = 3878, normalized size = 16.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] -1/36*(12*b^2*cos(d*x + c)^4 - 6*sqrt(1/3)*(a*b^3*d*cos(d*x + c)^6 - 3*a*b^
3*d*cos(d*x + c)^4 + 3*a*b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*
b^2*d*cos(d*x + c)^2 - a^2*b^2*d)*sin(d*x + c))*sqrt(((4^(1/3)*(I*sqrt(3) +
 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)
*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^
5*b^5*d^3))^(1/3)))^2*a^2*b^2*d^2 + 64)/(a^2*b^2*d^2))*arctan(-1/64*sqrt(1/
3)*((a^9*b^5 + a^5*b^9)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3)
 - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2
*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*d^3 + 8*
(a^7*b^5 + a^3*b^9)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (
a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a
^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*d^2*sin(d*x +
c) - 32*(a^8*b^2 + a^4*b^6)*d*sin(d*x + c) + 16*(a^7*b^3 + a^3*b^7)*d + 2*(
(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*
d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d
^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*a^3*b^5*d^2 - 4*a^4*b^2*d)*sqrt(16
*a^8 + 32*a^6*b^2 + 32*a^4*b^4 + 32*a^2*b^6 + 16*b^8 - ((a^9*b^3 + a^5*b^7)
*d^2*sin(d*x + c) - (a^8*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3)))^2 - 16*(a^8 + 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 4*((a^6*b^4 + a^
2*b^8)*d*sin(d*x + c) + (a^9*b + a^5*b^5)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3))) - 32*(a^7*b + a^3*b^5)*sin(d*x + c)))*sqrt(((4^(1/3)*(I*sqrt(3)
+ 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3
)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a
^5*b^5*d^3))^(1/3)))^2*a^2*b^2*d^2 + 64)/(a^2*b^2*d^2))/(a^8 + 2*a^4*b^4 +
b^8)) + 6*sqrt(1/3)*(a*b^3*d*cos(d*x + c)^6 - 3*a*b^3*d*cos(d*x + c)^4 + 3*
a*b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*b^2*d*cos(d*x + c)^2 -
a^2*b^2*d)*sin(d*x + c))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b
^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*
b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*a
^2*b^2*d^2 + 64)/(a^2*b^2*d^2))*arctan(1/64*sqrt(1/3)*((a^9*b^5 + a^5*b^9)*
```

```
(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*
d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d
^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*d^3 + 8*(a^7*b^5 + a^3*b^9)*(4^(
1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3)
)^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3)
- (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*d^2*sin(d*x + c) - 32*(a^8*b^2 + a^4*b
^6)*d*sin(d*x + c) + 16*(a^7*b^3 + a^3*b^7)*d - 2*((4^(1/3)*(I*sqrt(3) + 1)
*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-
I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b
^5*d^3))^(1/3)))*a^3*b^5*d^2 - 4*a^4*b^2*d)*sqrt(16*a^8 + 32*a^6*b^2 + 32*a
^4*b^4 + 32*a^2*b^6 + 16*b^8 - ((a^9*b^3 + a^5*b^7)*d^2*sin(d*x + c) - (a^8
*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) -
(a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((
a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2 - 16*(a^8 +
 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 4*((a^6*b^4 + a^2*b^8)*d*sin(d*x + c) +
(a^9*b + a^5*b^5)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) -
(a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((
a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3))) - 32*(a^7*b +
 a^3*b^5)*sin(d*x + c)))*sqrt(((4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b
^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*
b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2*a
^2*b^2*d^2 + 64)/(a^2*b^2*d^2))/(a^8 + 2*a^4*b^4 + b^8)) + 12*(a^2 - 2*b^2)
*cos(d*x + c)^2 - (a*b^3*d*cos(d*x + c)^6 - 3*a*b^3*d*cos(d*x + c)^4 + 3*a*
b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d + 2*(a^2*b^2*d*cos(d*x + c)^2 - a^
2*b^2*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3)
- (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*
((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))*log(64*a^8
+ 128*a^6*b^2 + 128*a^4*b^4 + 128*a^2*b^6 + 64*b^8 - 4*((a^9*b^3 + a^5*b^7)
*d^2*sin(d*x + c) - (a^8*b^4 + a^4*b^8)*d^2)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4
 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt
(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3
))^(1/3)))^2 - 64*(a^8 + 2*a^4*b^4 + b^8)*cos(d*x + c)^2 + 16*((a^6*b^4 + a
^2*b^8)*d*sin(d*x + c) + (a^9*b + a^5*b^5)*d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^
4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2/3)*(-I*sqr
t(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^
3))^(1/3))) - 128*(a^7*b + a^3*b^5)*sin(d*x + c)) + (a*b^3*d*cos(d*x + c)^6
 - 3*a*b^3*d*cos(d*x + c)^4 + 3*a*b^3*d*cos(d*x + c)^2 - (a^3*b + a*b^3)*d
+ 2*(a^2*b^2*d*cos(d*x + c)^2 - a^2*b^2*d)*sin(d*x + c))*(4^(1/3)*(I*sqrt(3
) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3) - 4^(2
/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/
(a^5*b^5*d^3))^(1/3)))*log(16*a^8 + 32*a^6*b^2 + 32*a^4*b^4 + 32*a^2*b^6 +
16*b^8 + (2*(a^9*b^3 + a^5*b^7)*d^2*sin(d*x + c) + (a^8*b^4 + a^4*b^8)*d^2)
*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b^5
*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^5*
d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3)))^2 - 16*(a^8 + 2*a^4*b^4 + b^8)*co
s(d*x + c)^2 - 4*(2*(a^6*b^4 + a^2*b^8)*d*sin(d*x + c) - (a^9*b + a^5*b^5)*
d)*(4^(1/3)*(I*sqrt(3) + 1)*((a^4 + b^4)/(a^5*b^5*d^3) - (a^4 - b^4)/(a^5*b
^5*d^3))^(1/3) - 4^(2/3)*(-I*sqrt(3) + 1)/(a^2*b^2*d^2*((a^4 + b^4)/(a^5*b^
5*d^3) - (a^4 - b^4)/(a^5*b^5*d^3))^(1/3))) + 64*(a^7*b + a^3*b^5)*sin(d*x
+ c)) + 12*a^2 + 12*b^2 - 12*(a*b*cos(d*x + c)^4 - 2*a*b)*sin(d*x + c))/(a*
b^3*d*cos(d*x + c)^6 - 3*a*b^3*d*cos(d*x + c)^4 + 3*a*b^3*d*cos(d*x + c)^2
- (a^3*b + a*b^3)*d + 2*(a^2*b^2*d*cos(d*x + c)^2 - a^2*b^2*d)*sin(d*x + c)
)
```

giac [A] time = 0.26, size = 228, normalized size = 0.96

$$-\frac{\frac{2\left(a\left(-\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2b}+\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{\left(b\sin(dx+c)^3+a\right)ab}-\frac{2\sqrt{3}\left(\left(-ab^2\right)^{\frac{1}{3}}b^2-\left(-ab^2\right)^{\frac{2}{3}}a\right)\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b^3}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(a*(-a/b)^(1/3) + b)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x +
 c)))/(a^2*b) + 3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/((b*sin(d*x + c
)^3 + a)*a*b) - 2*sqrt(3)*((-a*b^2)^(1/3)*b^2 - (-a*b^2)^(2/3)*a)*arctan(1/
3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b^3) - ((-a*b^
2)^(1/3)*b^2 + (-a*b^2)^(2/3)*a)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x
+ c) + (-a/b)^(2/3))/(a^2*b^3))/d
```

maple [A] time = 0.98, size = 327, normalized size = 1.37

$$-\frac{\sin^2(dx+c)}{3db\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{\sin(dx+c)}{3ad\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{2}{3db\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9dba\left(\frac{a}{b}\right)^{\frac{2}{3}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d/b/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2+1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)
^3)+2/3/d/b/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1
/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/
3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x
+c)-1))-2/9/d/b^2/(a/b)^(1/3)*ln(sin(d*x+c)+(a/b)^(1/3))+1/9/d/b^2/(a/b)^(1
/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/9/d/b^2*3^(1/2)/(
a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))
```

maxima [A] time = 0.45, size = 213, normalized size = 0.89

$$-\frac{\frac{3\left(a\sin(dx+c)^2-b\sin(dx+c)-2a\right)}{ab^2\sin(dx+c)^3+a^2b}-\frac{2\sqrt{3}\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}+b\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\left(a\left(\frac{a}{b}\right)^{\frac{1}{3}}-b\right)\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2}{}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/9*(3*(a*sin(d*x + c)^2 - b*sin(d*x + c) - 2*a)/(a*b^2*sin(d*x + c)^3 + a
^2*b) - 2*sqrt(3)*(a*(a/b)^(1/3) + b)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*
sin(d*x + c))/(a/b)^(1/3))/(a*b^2*(a/b)^(2/3)) - (a*(a/b)^(1/3) - b)*log(si
n(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b^2*(a/b)^(2/3))
```

```
+ 2*(a*(a/b)^(1/3) - b)*log((a/b)^(1/3) + sin(d*x + c))/(a*b^2*(a/b)^(2/3))
)/d
```

mupad [B] time = 14.98, size = 203, normalized size = 0.85

$$\frac{\sum_{k=1}^{3}\ln\left(\frac{4\,b+4\,a\,\sin(c+d\,x)+\mathrm{root}\left(729\,a^5\,b^5\,d^3+108\,a^3\,b^3\,d-8\,b^4+8\,a^4,d,k\right)^2\,a^2\,b^3\,81+\mathrm{root}\left(729\,a^5\,b^5\,d^3+108\,a^3\,b^3\,d-8\,b^4+8\,a^4,d,k\right)b^3\,\sin(c+d\,x)}{a\,b\,9}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log((4*b + 4*a*sin(c + d*x) + 81*root(729*a^5*b^5*d^3 + 108*a^3*b^3*
d - 8*b^4 + 8*a^4, d, k)^2*a^2*b^3 + 18*root(729*a^5*b^5*d^3 + 108*a^3*b^3*
d - 8*b^4 + 8*a^4, d, k)*b^3*sin(c + d*x))/(9*a*b))*root(729*a^5*b^5*d^3 +
108*a^3*b^3*d - 8*b^4 + 8*a^4, d, k), k, 1, 3)/d + (sin(c + d*x)/(3*a) + 2/
(3*b) - sin(c + d*x)^2/(3*b))/(d*(a + b*sin(c + d*x)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.395 $\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=183

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}+$$

```
[Out] 2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*(a+b*sin(d*
x+c))/a/b/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c)
)/a^(1/3)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 183, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.391, Rules used = {3223, 1854, 12, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}+$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + (a + b*Sin[c + d*x])/(3*a*b*d*(a + b*Sin[c +
d*x]^3))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
```

```
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} - \frac{\text{Subst}\left(\int -\frac{2}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\,\text{Subst}\left(\int -\frac{}{a^{2}\cdots}\right.}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} + \frac{a+b\sin(c+dx)}{3abd\left(a+b\sin^3(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+\cdots}\right.}{3a\cdots}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c\cdots\right.}{9a^{5}\cdots}$$

Mathematica [A] time = 0.79, size = 184, normalized size = 1.01

$$\frac{\frac{-\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{a^{5/3}}+\frac{2b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{a^{5/3}}+\frac{3}{a+b\sin^3(c+dx)}}{b} - \frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{a^{5/3}\sqrt[3]{b}} + \frac{3\sin(c+dx)}{a\left(a+b\sin^3(c+dx)\right)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
(a^(5/3)*b^(1/3)) + (3*Sin[c + d*x])/(a*(a + b*Sin[c + d*x]^3)) + ((2*b^(2/
3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/a^(5/3) - (b^(2/3)*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/a^(5/3) + 3/(a + b*Si
n[c + d*x]^3))/b)/(9*d)
```

fricas [A] time = 0.55, size = 665, normalized size = 3.63

$$\left[\frac{3a^2b\sin(dx+c)+3a^3+3\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos(dx+c)^2-ab^2\right)\sin(dx+c)\right)\sqrt{-\frac{(a^2b)^{\frac{1}{3}}}{b}}\,\log\left(-\frac{3(a^2b)^{\frac{1}{3}}a\sin(dx+c)+a^{2}\cdots}{\cdots}\right.}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*a^3 + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x +
c)^2 - a*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*
sin(d*x + c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2
/3)*sin(d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x
 + c)^2 - a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (
a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c
)^2 + a*b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)
*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(
2/3)))/(a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9
*(3*a^2*b*sin(d*x + c) + 3*a^3 + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2
 - a*b^2)*sin(d*x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(
2/3)*sin(d*x + c) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

giac [A] time = 0.24, size = 169, normalized size = 0.92

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(-\frac{a}{b}\right)^{\frac{2}{3}}\right)}{a^2b}-}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 2*sqrt(3)
*(-a*b^2)^(1/3)*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(
1/3))/(a^2*b) - (-a*b^2)^(1/3)*log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x +
c) + (-a/b)^(2/3))/(a^2*b) - 3*(b*sin(d*x + c) + a)/((b*sin(d*x + c)^3 + a)
*a*b))/d
```

maple [A] time = 0.93, size = 179, normalized size = 0.98

$$\frac{\sin(dx+c)}{3ad\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9dba\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9dba\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\ \mathrm{ar}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(
a/b)^(1/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a
/b)^(2/3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
sin(d*x+c)-1))+1/3/d/b/(a+b*sin(d*x+c)^3)
```

maxima [A] time = 0.47, size = 163, normalized size = 0.89

$$\frac{\frac{3\,(b\sin(dx+c)+a)}{ab^2\sin(dx+c)^3+a^2b}+\frac{2\,\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\,\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\,\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(3*(b*sin(d*x + c) + a)/(a*b^2*sin(d*x + c)^3 + a^2*b) + 2*sqrt(3)*arct
an(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2/3
)) - log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(a/b
)^(2/3)) + 2*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

mupad [B] time = 0.38, size = 172, normalized size = 0.94

$$\frac{\frac{\sin(c+d\,x)}{3\,a}+\frac{1}{3\,b}}{d\,\left(b\sin\left(c+d\,x\right)^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1i\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1i\right)}{9\,a^{5/3}\,b^{1/3}\,d}-\frac{\ln\left(\frac{2\,b^2\,\sin(c+}{a}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] (sin(c + d*x)/(3*a) + 1/(3*b))/(d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5/
3))/a^(2/3) + (2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^2*
sin(c + d*x))/a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/(9*
a^(5/3)*b^(1/3)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i + 1
))/a^(2/3))*(3^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.396 $\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=176

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}$$

```
[Out] 2/9*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/b^(1/3)/d-1/9*ln(a^(2/3)-a^(1/3)
*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/b^(1/3)/d+1/3*sin(d*x+c)/
a/d/(a+b*sin(d*x+c)^3)-2/9*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3
)*3^(1/2))/a^(5/3)/b^(1/3)/d*3^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.381, Rules used = {3223, 199, 200, 31, 634, 617, 204, 628}

$$-\frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}+\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}-\frac{2\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-2*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3
]*a^(5/3)*b^(1/3)*d) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*b
^(1/3)*d) - Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*
x]^2]/(9*a^(5/3)*b^(1/3)*d) + Sin[c + d*x]/(3*a*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(c+dx)\right)}{3ad}$$

$$= \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{2\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(c+dx)\right)}{9a^{5/3}d} + \frac{2\,\text{Subst}\left(\int \frac{}{a^{2/3}}\right.}{}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} + \frac{\sin(c+dx)}{3ad\left(a+b\sin^3(c+dx)\right)} + \frac{\text{Subst}\left(\int \frac{1}{a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,x+b^{2}}\right.}{3a^{4/}}$$

$$= \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d}$$

$$= -\frac{2\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\sqrt[3]{b}\,d} + \frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{9a^{5/3}\sqrt[3]{b}\,d} - \frac{\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c\right.}{9a^{5}}$$

Mathematica [A] time = 0.47, size = 152, normalized size = 0.86

$$\frac{\frac{2\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)-\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{\sqrt[3]{b}}+\frac{3a^{2/3}\sin(c+dx)}{a+b\sin^3(c+dx)}-\frac{2\sqrt{3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(c+dx)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt[3]{b}}}{9a^{5/3}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/
b^(1/3) + (2*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]] - Log[a^(2/3) - a^(1/3)*b^
(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/b^(1/3) + (3*a^(2/3)*Sin[c +
d*x])/(a + b*Sin[c + d*x]^3))/(9*a^(5/3)*d)
```

fricas [B] time = 0.53, size = 655, normalized size = 3.72

$$\left[\frac{3\,a^2b\sin(dx+c)+3\sqrt{\frac{1}{3}}\left(a^2b-\left(ab^2\cos(dx+c)^2-ab^2\right)\sin(dx+c)\right)\sqrt{-\frac{\left(a^2b\right)^{\frac{1}{3}}}{b}}\log\left(-\frac{3\left(a^2b\right)^{\frac{1}{3}}a\sin(dx+c)+a^2+3\,\cdots}{\cdots}\right)\cdots}{\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] [1/9*(3*a^2*b*sin(d*x + c) + 3*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a
*b^2)*sin(d*x + c))*sqrt(-(a^2*b)^(1/3)/b)*log(-(3*(a^2*b)^(1/3)*a*sin(d*x
+ c) + a^2 + 3*sqrt(1/3)*(2*a*b*cos(d*x + c)^2 - 2*a*b - (a^2*b)^(2/3)*sin(
d*x + c) + (a^2*b)^(1/3)*a)*sqrt(-(a^2*b)^(1/3)/b) + 2*(a*b*cos(d*x + c)^2
- a*b)*sin(d*x + c))/((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)) + (a^2*b)^(
2/3)*((b*cos(d*x + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*
b - (a^2*b)^(2/3)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos
(d*x + c)^2 - b)*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(
a^4*b*d - (a^3*b^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c)), 1/9*(3*a^2*
b*sin(d*x + c) + 6*sqrt(1/3)*(a^2*b - (a*b^2*cos(d*x + c)^2 - a*b^2)*sin(d*
x + c))*sqrt((a^2*b)^(1/3)/b)*arctan(sqrt(1/3)*(2*(a^2*b)^(2/3)*sin(d*x + c
) - (a^2*b)^(1/3)*a)*sqrt((a^2*b)^(1/3)/b)/a^2) + (a^2*b)^(2/3)*((b*cos(d*x
 + c)^2 - b)*sin(d*x + c) - a)*log(-a*b*cos(d*x + c)^2 + a*b - (a^2*b)^(2/3
)*sin(d*x + c) + (a^2*b)^(1/3)*a) - 2*(a^2*b)^(2/3)*((b*cos(d*x + c)^2 - b)
*sin(d*x + c) - a)*log(a*b*sin(d*x + c) + (a^2*b)^(2/3)))/(a^4*b*d - (a^3*b
^2*d*cos(d*x + c)^2 - a^3*b^2*d)*sin(d*x + c))]
```

giac [A] time = 0.22, size = 162, normalized size = 0.92

$$-\frac{\frac{2\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^2}-\frac{3\sin(dx+c)}{\left(b\sin(dx+c)^3+a\right)a}-\frac{2\sqrt{3}\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\sin(dx+c)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{a^2b}-\frac{\left(-ab^2\right)^{\frac{1}{3}}\log\left(\sin(dx+c)^2+\left(-\frac{a}{b}\right)^{\frac{1}{3}}\cdots\right)}{a^2b}}{9\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/9*(2*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/a^2 - 3*sin(d*x
 + c)/((b*sin(d*x + c)^3 + a)*a) - 2*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*sqrt
(3)*((-a/b)^(1/3) + 2*sin(d*x + c))/(-a/b)^(1/3))/(a^2*b) - (-a*b^2)^(1/3)*
log(sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^2*b))/d
```

maple [A] time = 0.53, size = 157, normalized size = 0.89

$$\frac{\sin(dx+c)}{3ad\left(a+b\left(\sin^3(dx+c)\right)\right)}+\frac{2\ln\left(\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{9dba\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\ln\left(\sin^2(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{9dba\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\sqrt{3}\ \text{arcta}}{9}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 1/3*sin(d*x+c)/a/d/(a+b*sin(d*x+c)^3)+2/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)+(
a/b)^(1/3))-1/9/d/b/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a
/b)^(2/3))+2/9/d/b/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*
sin(d*x+c)-1))
```

maxima [A] time = 0.43, size = 155, normalized size = 0.88

$$\frac{\frac{3\sin(dx+c)}{ab\sin(dx+c)^3+a^2}+\frac{2\sqrt{3}\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\log\left(\sin(dx+c)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{2\log\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right)}{ab\left(\frac{a}{b}\right)^{\frac{2}{3}}}}{9d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/9*(3*sin(d*x + c)/(a*b*sin(d*x + c)^3 + a^2) + 2*sqrt(3)*arctan(-1/3*sqrt
(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/3))/(a*b*(a/b)^(2/3)) - log(sin
(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a*b*(a/b)^(2/3)) + 2
*log((a/b)^(1/3) + sin(d*x + c))/(a*b*(a/b)^(2/3)))/d
```

mupad [B] time = 15.00, size = 165, normalized size = 0.94

$$\frac{\sin(c+d\,x)}{3\,a\,d\,\left(b\sin(c+d\,x)^3+a\right)}+\frac{2\,\ln\left(\frac{2\,b^{5/3}}{a^{2/3}}+\frac{2\,b^2\,\sin(c+d\,x)}{a}\right)}{9\,a^{5/3}\,b^{1/3}\,d}+\frac{\ln\left(\frac{2\,b^2\,\sin(c+d\,x)}{a}+\frac{b^{5/3}\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{a^{2/3}}\right)\,\left(-1+\sqrt{3}\,1\mathrm{i}\right)}{9\,a^{5/3}\,b^{1/3}\,d}-\frac{\ln\left(\frac{2\,b^2\,\mathrm{si}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] sin(c + d*x)/(3*a*d*(a + b*sin(c + d*x)^3)) + (2*log((2*b^(5/3))/a^(2/3) +
(2*b^2*sin(c + d*x))/a))/(9*a^(5/3)*b^(1/3)*d) + (log((2*b^2*sin(c + d*x))/
a + (b^(5/3)*(3^(1/2)*1i - 1))/a^(2/3))*(3^(1/2)*1i - 1))/(9*a^(5/3)*b^(1/3
)*d) - (log((2*b^2*sin(c + d*x))/a - (b^(5/3)*(3^(1/2)*1i + 1))/a^(2/3))*(3
^(1/2)*1i + 1))/(9*a^(5/3)*b^(1/3)*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.397 $\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=587

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3ad\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}-\frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^2}+\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\,\mathrm{si}\right.}{6\sqrt[3]{a}\,d\left(a^2-b^2\right)^2}$$

```
[Out] -1/2*ln(1-sin(d*x+c))/(a+b)^2/d+1/2*ln(1+sin(d*x+c))/(a-b)^2/d-1/9*b^(1/3)*
(a^(4/3)+2*b^(4/3))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(5/3)/(a^2-b^2)/d-1/3*
b^(1/3)*(a^2+2*a^(2/3)*b^(4/3)+b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/3)/
(a^2-b^2)^2/d+1/18*b^(1/3)*(a^(4/3)+2*b^(4/3))*ln(a^(2/3)-a^(1/3)*b^(1/3)*s
in(d*x+c)+b^(2/3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)/d+1/6*b^(1/3)*(a^2+2*a^(2
/3)*b^(4/3)+b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x+c)^2
)/a^(1/3)/(a^2-b^2)^2/d-2/3*a*b*ln(a+b*sin(d*x+c)^3)/(a^2-b^2)^2/d+1/3*b*(a
-sin(d*x+c)*(b-a*sin(d*x+c)))/a/(a^2-b^2)/d/(a+b*sin(d*x+c)^3)-1/9*b^(1/3)*
(a^(4/3)-2*b^(4/3))*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/
2))/a^(5/3)/(a^2-b^2)/d*3^(1/2)-1/3*b^(1/3)*(a^2-2*a^(2/3)*b^(4/3)+b^2)*arc
tan(1/3*(a^(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a^2-b^2)^2
/d*3^(1/2)
```

Rubi [A] time = 0.69, antiderivative size = 587, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.524, Rules used = {3223, 2074, 1854, 1860, 31, 634, 617, 204, 628, 1871, 260}

$$\frac{b(a-\sin(c+dx)(b-a\sin(c+dx)))}{3ad\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}+\frac{\sqrt[3]{b}\left(2a^{2/3}b^{4/3}+a^2+b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(c+dx)+b^{2/3}\sin^2(c+dx)\right)}{6\sqrt[3]{a}\,d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] -(b^(1/3)*(a^(4/3) - 2*b^(4/3))*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(
Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)*d) - (b^(1/3)*(a^2 - 2*a^
(2/3)*b^(4/3) + b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(
1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2)^2*d) - Log[1 - Sin[c + d*x]]/(2*(a + b
)^2*d) + Log[1 + Sin[c + d*x]]/(2*(a - b)^2*d) - (b^(1/3)*(a^(4/3) + 2*b^(4
/3))*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)*d) - (b^(1
/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(3
*a^(1/3)*(a^2 - b^2)^2*d) + (b^(1/3)*(a^(4/3) + 2*b^(4/3))*Log[a^(2/3) - a^
(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^
2)*d) + (b^(1/3)*(a^2 + 2*a^(2/3)*b^(4/3) + b^2)*Log[a^(2/3) - a^(1/3)*b^(1
/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^2*d) - (
2*a*b*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^2*d) + (b*(a - Sin[c + d*x]
*(b - a*Sin[c + d*x])))/(3*a*(a^2 - b^2)*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
```

```
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
 PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)\left(a+bx^3\right)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{2(a+b)^2(-1+x)} + \frac{1}{2(a-b)^2(1+x)} + \frac{b\left(b-ax+bx^2\right)}{\left(-a^2+b^2\right)\left(a+bx^3\right)^2} + \frac{b\left(-2ab+\left(a^2+b^2\right)x-2abx^2\right)}{\left(a^2-b^2\right)^2\left(a+bx^3\right)}\right) dx,\right.}{d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b\,\text{Subst}\left(\int \frac{-2ab+\left(a^2+b^2\right)x-2abx^2}{a+bx^3}\,dx, x\right.}{\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} + \frac{b(a-\sin(c+dx)(b-a\sin(c+dx)}{3a\left(a^2-b^2\right)d\left(a+b\sin^3(c+dx)\right.}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{2ab\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^2 d} + \frac{b(a}{3a}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\text{si}\right.}{9a^{5/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\log(1-\sin(c+dx))}{2(a+b)^2d} + \frac{\log(1+\sin(c+dx))}{2(a-b)^2d} - \frac{\sqrt[3]{b}\left(a^{4/3}+2b^{4/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\,\text{si}\right.}{9a^{5/3}\left(a^2-b^2\right)d}$$

$$= -\frac{\sqrt[3]{b}\left(a^{4/3}-2b^{4/3}\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)d} - \frac{\sqrt[3]{b}\left(a^2-2a^{2/3}b^{4/3}+b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(}{\sqrt[3]{a}}}{\sqrt{3}}\right.}{\sqrt{3}\sqrt[3]{a}\left(a^2-b^2\right)^2 d}$$

Mathematica [C] time = 4.30, size = 503, normalized size = 0.86

$$\frac{\frac{9b\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},2;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{a^3-ab^2} + \frac{9b\left(a^2+b^2\right)\sin^2(c+dx)\,{}_2F_1\left(\frac{2}{3},1;\frac{5}{3};-\frac{b\sin^3(c+dx)}{a}\right)}{a\left(a^2-b^2\right)^2} - \frac{6b^2\sin(c+dx)}{a\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)} + \frac{6b}{\left(a^2-b^2\right)\left(a+b\sin^3(c+dx)\right)}}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((-9*Log[1 - Sin[c + d*x]])/(a + b)^2 + (9*Log[1 + Sin[c + d*x]])/(a - b)^2
 - (12*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(a^2 - b^2)^2 +
 (6*a^(1/3)*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(S
qrt[3]*a^(1/3))] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin
```

```
[c + d*x]^2]))/(a^2 - b^2)^2 + (2*b^(5/3)*(2*Sqrt[3]*ArcTan[(a^(1/3) - 2*b^
(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] - 2*Log[a^(1/3) + b^(1/3)*Sin[c + d*
x]] + Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2])
)/(a^(5/3)*(a^2 - b^2)) - (12*a*b*Log[a + b*Sin[c + d*x]^3])/(a^2 - b^2)^2
+ (9*b*(a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin[c + d*x]^3)/a)]*
Sin[c + d*x]^2)/(a*(a^2 - b^2)^2) + (9*b*Hypergeometric2F1[2/3, 2, 5/3, -((
b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(a^3 - a*b^2) + (6*b)/((a^2 - b^2)*(a
 + b*Sin[c + d*x]^3)) - (6*b^2*Sin[c + d*x])/(a*(a^2 - b^2)*(a + b*Sin[c +
d*x]^3)))/(18*d)
```

fricas [C] time = 3.88, size = 10855, normalized size = 18.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] 1/324*(216*a^3*b - 216*a*b^3 - 108*(a^3*b - a*b^3)*cos(d*x + c)^2 - 2*((a^6
 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 -
 (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*
b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3)
 + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a
^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4
*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b
^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 +
 a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d
^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4
+ b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d -
2*a^2*b^2*d + b^4*d))*log(-56*a^5*b^2 + 20*a^3*b^4 + 1/324*(2*a^11 - 3*a^9*
b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^
2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^
2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1
/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5 + a^2*b^7)*(4*(9*a^2*b^2/(a^4*d - 2*a
^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt
(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*
d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(
8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*
a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^
3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^
2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^
9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b
^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d
 - 2*a^2*b^2*d + b^4*d))*d + 4*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*si
n(d*x + c)) - (324*a^3*b - ((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3
*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c
))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2
*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d +
 b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*
a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*
b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a
^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a
*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)
```

```
) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(
8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*
sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d)) + 3*sqrt(1/3)*((a^6 -
 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (
a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*sqrt((29808*a^4*b^2 + 10368*a^2
*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*(
9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 +
a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3
) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3)
)^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((
a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/7
29*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 +
 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3)
 + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*b
^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
- 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*
a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*
(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2
*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2
 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)
^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2
*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3
) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3)
)^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^10
- 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d^2)) - 324*(a^2*b^2*cos(d*x
 + c)^2 - a^2*b^2)*sin(d*x + c))*log(56*a^5*b^2 - 20*a^3*b^4 - 1/324*(2*a^1
1 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
- 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 +
a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))
^2*d^2 + 1/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5 + a^2*b^7)*(4*(9*a^2*b^2/(a
^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
)*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*
b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d))
 - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8
*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81
*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*
a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b
- b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2
- 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*
a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - 1/108*sqrt(1/3)*((2*a^11 - 3*a^9*b^2
 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a
^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d
^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 -
 b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3)
+ 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^
(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d^2 - 36*(6*
```

```
a^8*b - 13*a^6*b^3 + 8*a^4*b^5 - a^2*b^7)*d)*sqrt((29808*a^4*b^2 + 10368*a^
2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*(4*
(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 +
 a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/(
(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/
729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6
+ 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3
) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 + 216*(a^7*b - 2*a^5*
b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2
 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2
*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)
*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^
2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^
2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d
)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^
2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^
3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3
))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d)/((a^10
 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d^2)) + 8*(8*a^6*b + 28*a^4
*b^3 - 10*a^2*b^5 + b^7)*sin(d*x + c)) - (324*a^3*b - ((a^6 - 2*a^4*b^2 + a
^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*
b^3 + a*b^5)*d)*sin(d*x + c))*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2
 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3
*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2
 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9
*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^
4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a
^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d
^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 -
 b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^
4*d)) - 3*sqrt(1/3)*((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 +
a*b^5)*d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))*sqrt
((29808*a^4*b^2 + 10368*a^2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^4
- 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/
(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a
^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*
b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 -
2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6
)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d
 + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d -
2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^
5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4
*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))^2
*d^2 + 216*(a^7*b - 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d
 + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)
/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*
a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b
- b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2
- 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a
^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*
b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 -
2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6
)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2
*b^2*d + b^4*d))*d)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)*d
```

```
^2)) - 324*(a^2*b^2*cos(d*x + c)^2 - a^2*b^2)*sin(d*x + c))*log(-56*a^5*b^2
 + 20*a^3*b^4 + 1/324*(2*a^11 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d -
 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*
sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((
a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/7
29*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 +
 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/2
7*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^
2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)
/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a
^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a
^4*d - 2*a^2*b^2*d + b^4*d))^2*d^2 - 1/9*(12*a^8*b + 22*a^6*b^3 - 8*a^4*b^5
 + a^2*b^7)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^2/(a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/(a^4*d - 2*a^
2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a
^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d
^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 -
 b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3
 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d
 + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3)
+ 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^
(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))*d - 1/108*sq
rt(1/3)*((2*a^11 - 3*a^9*b^2 + a^5*b^6)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d
+ b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/
(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a
^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b -
 b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 -
 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^
4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b
^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2
*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)
*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*
b^2*d + b^4*d))*d^2 - 36*(6*a^8*b - 13*a^6*b^3 + 8*a^4*b^5 - a^2*b^7)*d)*sq
rt((29808*a^4*b^2 + 10368*a^2*b^4 - 5184*b^6 - (a^10 - 4*a^8*b^2 + 6*a^6*b^
4 - 4*a^4*b^6 + a^2*b^8)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2*d + b^4*d)^2 - b^
2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) + 1)/(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2
*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d
- 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 +
a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)
^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a^2*b^2*d + b^4*d))
^2*d^2 + 216*(a^7*b - 2*a^5*b^3 + a^3*b^5)*(4*(9*a^2*b^2/(a^4*d - 2*a^2*b^2
*d + b^4*d)^2 - b^2/(a^6*d^2 - 2*a^4*b^2*d^2 + a^2*b^4*d^2))*(-I*sqrt(3) +
1)/(-8/27*a^3*b^3/(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 -
2*a^4*b^2*d^2 + a^2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*
b - b^3)/(a^9*d^3 - 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^
2 - 10*a^2*b^4 + b^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3) + 81*(-8/27*a^3*b^3/
(a^4*d - 2*a^2*b^2*d + b^4*d)^3 + 4/81*a*b^3/((a^6*d^2 - 2*a^4*b^2*d^2 + a^
2*b^4*d^2)*(a^4*d - 2*a^2*b^2*d + b^4*d)) - 4/729*(8*a^2*b - b^3)/(a^9*d^3
- 2*a^7*b^2*d^3 + a^5*b^4*d^3) + 4/729*(8*a^6 + 28*a^4*b^2 - 10*a^2*b^4 + b
^6)*b/((a^2 - b^2)^4*a^5*d^3))^(1/3)*(I*sqrt(3) + 1) + 108*a*b/(a^4*d - 2*a
^2*b^2*d + b^4*d))*d)/((a^10 - 4*a^8*b^2 + 6*a^6*b^4 - 4*a^4*b^6 + a^2*b^8)
*d^2)) - 8*(8*a^6*b + 28*a^4*b^3 - 10*a^2*b^5 + b^7)*sin(d*x + c)) + 162*(a
^4 + 2*a^3*b + a^2*b^2 + (a^3*b + 2*a^2*b^2 + a*b^3 - (a^3*b + 2*a^2*b^2 +
a*b^3)*cos(d*x + c)^2)*sin(d*x + c))*log(sin(d*x + c) + 1) - 162*(a^4 - 2*a
^3*b + a^2*b^2 + (a^3*b - 2*a^2*b^2 + a*b^3 - (a^3*b - 2*a^2*b^2 + a*b^3)*c
```

```
os(d*x + c)^2)*sin(d*x + c))*log(-sin(d*x + c) + 1) - 108*(a^2*b^2 - b^4)*s
in(d*x + c))/((a^6 - 2*a^4*b^2 + a^2*b^4)*d - ((a^5*b - 2*a^3*b^3 + a*b^5)*
d*cos(d*x + c)^2 - (a^5*b - 2*a^3*b^3 + a*b^5)*d)*sin(d*x + c))
```

giac [A] time = 0.28, size = 566, normalized size = 0.96

$$-\frac{\frac{12\,ab\log\left(\left|b\sin(dx+c)^{3}+a\right|\right)}{a^{4}-2\,a^{2}b^{2}+b^{4}}+\frac{4\left(2\,a^{8}b^{2}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^{6}b^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+a^{2}b^{8}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-4\,a^{7}b^{3}+9\,a^{5}b^{5}-6\,a^{3}b^{7}+ab^{9}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(dx+c)\right|\right)}{a^{11}b-4\,a^{9}b^{3}+6\,a^{7}b^{5}-4\,a^{5}b^{7}+a^{3}b^{9}}+\frac{4\left(\cdots\right.}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] -1/18*(12*a*b*log(abs(b*sin(d*x + c)^3 + a))/(a^4 - 2*a^2*b^2 + b^4) + 4*(2
*a^8*b^2*(-a/b)^(1/3) - 3*a^6*b^4*(-a/b)^(1/3) + a^2*b^8*(-a/b)^(1/3) - 4*a
^7*b^3 + 9*a^5*b^5 - 6*a^3*b^7 + a*b^9)*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3)
+ sin(d*x + c)))/(a^11*b - 4*a^9*b^3 + 6*a^7*b^5 - 4*a^5*b^7 + a^3*b^9) + 4
*((2*sqrt(3)*a^3 + sqrt(3)*a*b^2)*(-a*b^2)^(2/3) + (4*sqrt(3)*a^2*b^2 - sqr
t(3)*b^4)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x + c)
)/(-a/b)^(1/3))/(a^6*b - 2*a^4*b^3 + a^2*b^5) - 2*((2*a^3 + a*b^2)*(-a*b^2)
^(2/3) - (4*a^2*b^2 - b^4)*(-a*b^2)^(1/3))*log(sin(d*x + c)^2 + (-a/b)^(1/3
)*sin(d*x + c) + (-a/b)^(2/3))/(a^6*b - 2*a^4*b^3 + a^2*b^5) - 9*log(abs(si
n(d*x + c) + 1))/(a^2 - 2*a*b + b^2) + 9*log(abs(sin(d*x + c) - 1))/(a^2 +
2*a*b + b^2) - 6*(2*a^2*b^2*sin(d*x + c)^3 + a^3*b*sin(d*x + c)^2 - a*b^3*s
in(d*x + c)^2 - a^2*b^2*sin(d*x + c) + b^4*sin(d*x + c) + 3*a^3*b - a*b^3)/
((a^5 - 2*a^3*b^2 + a*b^4)*(b*sin(d*x + c)^3 + a)))/d
```

maple [A] time = 0.96, size = 934, normalized size = 1.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/2/d/(a+b)^2*ln(sin(d*x+c)-1)+1/3/d*b/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*
sin(d*x+c)^2*a^2-1/3/d*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2-
1/3/d*b^2/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)*sin(d*x+c)*a+1/3/d*b^4/(a-b)^2
/(a+b)^2/(a+b*sin(d*x+c)^3)/a*sin(d*x+c)+1/3/d*b/(a-b)^2/(a+b)^2/(a+b*sin(d
*x+c)^3)*a^2-1/3/d*b^3/(a-b)^2/(a+b)^2/(a+b*sin(d*x+c)^3)-8/9/d*b/(a-b)^2/(
a+b)^2*a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+2/9/d*b^3/(a-b)^2/(a+b)^2/a
/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+4/9/d*b/(a-b)^2/(a+b)^2*a/(a/b)^(2/
3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-1/9/d*b^3/(a-b)^2/(a
+b)^2/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-8/9
/d*b/(a-b)^2/(a+b)^2*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3
)*sin(d*x+c)-1))+2/9/d*b^3/(a-b)^2/(a+b)^2/a/(a/b)^(2/3)*3^(1/2)*arctan(1/3
*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-4/9/d/(a-b)^2/(a+b)^2*a^2/(a/b)^(1/3
)*ln(sin(d*x+c)+(a/b)^(1/3))-2/9/d*b^2/(a-b)^2/(a+b)^2/(a/b)^(1/3)*ln(sin(d
*x+c)+(a/b)^(1/3))+2/9/d/(a-b)^2/(a+b)^2*a^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a
/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1/9/d*b^2/(a-b)^2/(a+b)^2/(a/b)^(1/3)*ln(
sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+4/9/d/(a-b)^2/(a+b)^2*a^2*
3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+2/9/d*
b^2/(a-b)^2/(a+b)^2*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*s
in(d*x+c)-1))-2/3/d*b/(a-b)^2/(a+b)^2*a*ln(a+b*sin(d*x+c)^3)+1/2*ln(1+sin(d
*x+c))/(a-b)^2/d
```

maxima [A] time = 0.45, size = 483, normalized size = 0.82

$$\frac{4\sqrt{3}\left(2a^3\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)-2a^2b\left(2\left(\frac{a}{b}\right)^{\frac{1}{3}}+\frac{a}{b}\right)+ab^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+b^3\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(dx+c)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{\left(a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)\left(\frac{a}{b}\right)^{\frac{1}{3}}}-\frac{2\left(2a^2b\left(3\left(\frac{a}{b}\right)^{\frac{2}{3}}-2\right)-2a^3\left(\frac{a}{b}\right)^{\frac{1}{3}}-ab^2\left(\frac{a}{b}\right)^{\frac{1}{3}}+b^3\right)\log\left(\sin(\right.}{a^5\left(\frac{a}{b}\right)^{\frac{2}{3}}-2a^3b^2\left(\frac{a}{b}\right)^{\frac{2}{3}}+ab^4\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] 1/18*(4*sqrt(3)*(2*a^3*((a/b)^(2/3) + 1) - 2*a^2*b*(2*(a/b)^(1/3) + a/b) +
a*b^2*(a/b)^(2/3) + b^3*(a/b)^(1/3))*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*s
in(d*x + c))/(a/b)^(1/3))/((a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*b^4
*(a/b)^(2/3))*(a/b)^(1/3)) - 2*(2*a^2*b*(3*(a/b)^(2/3) - 2) - 2*a^3*(a/b)^(
1/3) - a*b^2*(a/b)^(1/3) + b^3)*log(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x +
c) + (a/b)^(2/3))/(a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^(2/3) + a*b^4*(a/b)^(2
/3)) - 4*(a^2*b*(3*(a/b)^(2/3) + 4) + 2*a^3*(a/b)^(1/3) + a*b^2*(a/b)^(1/3)
 - b^3)*log((a/b)^(1/3) + sin(d*x + c))/(a^5*(a/b)^(2/3) - 2*a^3*b^2*(a/b)^
(2/3) + a*b^4*(a/b)^(2/3)) + 6*(a*b*sin(d*x + c)^2 - b^2*sin(d*x + c) + a*b
)/(a^4 - a^2*b^2 + (a^3*b - a*b^3)*sin(d*x + c)^3) + 9*log(sin(d*x + c) + 1
)/(a^2 - 2*a*b + b^2) - 9*log(sin(d*x + c) - 1)/(a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 15.17, size = 980, normalized size = 1.67

$$\sum_{k=1}^{3}\ln\left(\frac{\frac{8b^6}{27}-\frac{16a^2b^4}{27}}{a^7-2a^5b^2+a^3b^4}+\mathrm{root}\left(1458a^7b^2z^3-729a^5b^4z^3-729a^9z^3-1458a^6bz^2-108a^3b^2z-64a^2b+8b^3,\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(((8*b^6)/27 - (16*a^2*b^4)/27)/(a^7 + a^3*b^4 - 2*a^5*b^2) + roo
t(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a
^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(((32*a*b^7)/27 + (128*a^3*b^5)/27)/(a^7
 + a^3*b^4 - 2*a^5*b^2) - root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9
*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(root(1458*
a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*
z - 64*a^2*b + 8*b^3, z, k)*((16*a^3*b^9 - 77*a^5*b^7 + 34*a^7*b^5 + 27*a^9
*b^3)/(a^7 + a^3*b^4 - 2*a^5*b^2) + root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3
 - 729*a^9*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k)*(
(36*a^4*b^10 + 108*a^6*b^8 - 324*a^8*b^6 + 180*a^10*b^4)/(a^7 + a^3*b^4 - 2
*a^5*b^2) + (sin(c + d*x)*(4374*a^5*b^9 - 7290*a^7*b^7 + 1458*a^9*b^5 + 145
8*a^11*b^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*(216*a^2*b^1
0 - 864*a^4*b^8 - 1836*a^6*b^6 + 2484*a^8*b^4))/(27*(a^7 + a^3*b^4 - 2*a^5*
b^2))) + ((64*a^2*b^8)/9 - (353*a^4*b^6)/9 + (388*a^6*b^4)/9)/(a^7 + a^3*b^
4 - 2*a^5*b^2) + (sin(c + d*x)*(96*a*b^9 - 408*a^3*b^7 + 447*a^5*b^5))/(27*
(a^7 + a^3*b^4 - 2*a^5*b^2))) + (sin(c + d*x)*(16*b^8 + 134*a^2*b^6 - 236*a
^4*b^4))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2))) + (8*a*b^5*sin(c + d*x))/(9*(a^7
 + a^3*b^4 - 2*a^5*b^2)))*root(1458*a^7*b^2*z^3 - 729*a^5*b^4*z^3 - 729*a^9
*z^3 - 1458*a^6*b*z^2 - 108*a^3*b^2*z - 64*a^2*b + 8*b^3, z, k), k, 1, 3)/d
 - log(sin(c + d*x) - 1)/(d*(4*a*b + 2*a^2 + 2*b^2)) + log(sin(c + d*x) + 1
)/(d*(2*a^2 - 4*a*b + 2*b^2)) + (b/(3*(a^2 - b^2)) + (b*sin(c + d*x)^2)/(3*
(a^2 - b^2)) - (b^2*sin(c + d*x))/(3*a*(a^2 - b^2)))/(d*(a + b*sin(c + d*x)
^3))
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.398 $\int \frac{\sec^3(c+dx)}{(a+b\sin^3(c+dx))^2}\,dx$

Optimal. Leaf size=747

$$-\frac{b\left(a\left(a^2+2b^2\right)-b\sin(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)\right)}{3ad\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)}+\frac{2ab\left(a^2+5b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{3d\left(a^2-b^2\right)^3}-\frac{b^{5/3}\left(3a^{4/}\right.}{}$$

```
[Out] -1/4*(a+7*b)*ln(1-sin(d*x+c))/(a+b)^3/d+1/4*(a-7*b)*ln(1+sin(d*x+c))/(a-b)^
3/d+1/9*b^(5/3)*(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(1/3)+b^(1/3)*sin(d*x+
c))/a^(5/3)/(a^2-b^2)^2/d+1/3*b^(5/3)*(3*b^(2/3)*(3*a^2+b^2)+4*a^(2/3)*(a^2
+2*b^2))*ln(a^(1/3)+b^(1/3)*sin(d*x+c))/a^(1/3)/(a^2-b^2)^3/d-1/18*b^(5/3)*
(4*a^2+3*a^(4/3)*b^(2/3)+2*b^2)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/
3)*sin(d*x+c)^2)/a^(5/3)/(a^2-b^2)^2/d-1/6*b^(5/3)*(3*b^(2/3)*(3*a^2+b^2)+4
*a^(2/3)*(a^2+2*b^2))*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(d*x+c)+b^(2/3)*sin(d*x
+c)^2)/a^(1/3)/(a^2-b^2)^3/d+2/3*a*b*(a^2+5*b^2)*ln(a+b*sin(d*x+c)^3)/(a^2-
b^2)^3/d+1/4/(a+b)^2/d/(1-sin(d*x+c))-1/4/(a-b)^2/d/(1+sin(d*x+c))-1/3*b*(a
*(a^2+2*b^2)-b*sin(d*x+c)*(2*a^2+b^2-3*a*b*sin(d*x+c)))/a/(a^2-b^2)^2/d/(a+
b*sin(d*x+c)^3)-1/9*b^(5/3)*(4*a^2-3*a^(4/3)*b^(2/3)+2*b^2)*arctan(1/3*(a^(
1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(5/3)/(a^2-b^2)^2/d*3^(1/2)-1
/3*b^(5/3)*(4*a^(8/3)-9*a^2*b^(2/3)+8*a^(2/3)*b^2-3*b^(8/3))*arctan(1/3*(a^
(1/3)-2*b^(1/3)*sin(d*x+c))/a^(1/3)*3^(1/2))/a^(1/3)/(a^2-b^2)^3/d*3^(1/2)
```

Rubi [A] time = 1.02, antiderivative size = 747, normalized size of antiderivative = 1.00, number of steps used = 18, number of rules used = 11, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.478, Rules used = {3223, 2074, 1854, 1860, 31, 634, 617, 204, 628, 1871, 260}

$$-\frac{b\left(a\left(a^2+2b^2\right)-b\sin(c+dx)\left(2a^2-3ab\sin(c+dx)+b^2\right)\right)}{3ad\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)}-\frac{b^{5/3}\left(3a^{4/3}b^{2/3}+4a^2+2b^2\right)\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin\right.}{18a^{5/3}d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] -(b^(5/3)*(4*a^2 - 3*a^(4/3)*b^(2/3) + 2*b^2)*ArcTan[(a^(1/3) - 2*b^(1/3)*S
in[c + d*x])/(Sqrt[3]*a^(1/3))])/(3*Sqrt[3]*a^(5/3)*(a^2 - b^2)^2*d) - (b^(
5/3)*(4*a^(8/3) - 9*a^2*b^(2/3) + 8*a^(2/3)*b^2 - 3*b^(8/3))*ArcTan[(a^(1/3
) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a^(1/3)*(a^2 - b^2
)^3*d) - ((a + 7*b)*Log[1 - Sin[c + d*x]])/(4*(a + b)^3*d) + ((a - 7*b)*Log
[1 + Sin[c + d*x]])/(4*(a - b)^3*d) + (b^(5/3)*(4*a^2 + 3*a^(4/3)*b^(2/3) +
 2*b^2)*Log[a^(1/3) + b^(1/3)*Sin[c + d*x]])/(9*a^(5/3)*(a^2 - b^2)^2*d) +
(b^(5/3)*(3*b^(2/3)*(3*a^2 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(1/3) +
b^(1/3)*Sin[c + d*x]])/(3*a^(1/3)*(a^2 - b^2)^3*d) - (b^(5/3)*(4*a^2 + 3*a^
(4/3)*b^(2/3) + 2*b^2)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)
*Sin[c + d*x]^2])/(18*a^(5/3)*(a^2 - b^2)^2*d) - (b^(5/3)*(3*b^(2/3)*(3*a^2
 + b^2) + 4*a^(2/3)*(a^2 + 2*b^2))*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*
x] + b^(2/3)*Sin[c + d*x]^2])/(6*a^(1/3)*(a^2 - b^2)^3*d) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3*d) + 1/(4*(a + b)^2*d*(1
- Sin[c + d*x])) - 1/(4*(a - b)^2*d*(1 + Sin[c + d*x])) - (b*(a*(a^2 + 2*b^
2) - b*Sin[c + d*x]*(2*a^2 + b^2 - 3*a*b*Sin[c + d*x])))/(3*a*(a^2 - b^2)^2
*d*(a + b*Sin[c + d*x]^3))
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1854

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Module[{q = Expon[Pq,
 x], i}, Simp[((a*Coeff[Pq, x, q] - b*x*ExpandToSum[Pq - Coeff[Pq, x, q]*x^
q, x])*(a + b*x^n)^(p + 1))/(a*b*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int
[Sum[(n*(p + 1) + i + 1)*Coeff[Pq, x, i]*x^i, {i, 0, q - 1}]*(a + b*x^n)^(p
 + 1), x], x] /; q == n - 1] /; FreeQ[{a, b}, x] && PolyQ[Pq, x] && IGtQ[n,
 0] && LtQ[p, -1]
```

Rule 1860

```
Int[((A_) + (B_.)*(x_))/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{r = Numer
ator[Rt[a/b, 3]], s = Denominator[Rt[a/b, 3]]}, -Dist[(r*(B*r - A*s))/(3*a*
s), Int[1/(r + s*x), x], x] + Dist[r/(3*a*s), Int[(r*(B*r + 2*A*s) + s*(B*r
 - A*s)*x)/(r^2 - r*s*x + s^2*x^2), x], x]] /; FreeQ[{a, b, A, B}, x] && Ne
Q[a*B^3 - b*A^3, 0] && PosQ[a/b]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 2074

```
Int[(P_)^(p_)*(Q_)^(q_.), x_Symbol] :> With[{PP = Factor[P]}, Int[ExpandInt
egrand[PP^p*Q^q, x], x] /;  !SumQ[NonfreeFactors[PP, x]]] /; FreeQ[q, x] &&
```

```
PolyQ[P, x] && PolyQ[Q, x] && IntegerQ[p] && NeQ[P, x]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a+bx^3)^2}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a+b)^2(-1+x)^2} + \frac{-a-7b}{4(a+b)^3(-1+x)} + \frac{1}{4(a-b)^2(1+x)^2} + \frac{a-7b}{4(a-b)^3(1+x)} + \frac{b^2\left(2a^2+b^2-3abx+\left(\right.\right.}{\left(a^2-b^2\right)^2\left(a+\right.}\right.\right.}{d}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{1}{4(a+b)^2d(1-\sin}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{1}{4(a+b)^2d(1-\sin}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{2ab\left(a^2+5b^2\right)\log}{3\left(a^2\right.}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}\left(4a^2+3a^{4/3}b\right.}{}$$

$$= -\frac{(a+7b)\log(1-\sin(c+dx))}{4(a+b)^3d} + \frac{(a-7b)\log(1+\sin(c+dx))}{4(a-b)^3d} + \frac{b^{5/3}\left(4a^2+3a^{4/3}b\right.}{}$$

$$= -\frac{b^{5/3}\left(4a^2-3a^{4/3}b^{2/3}+2b^2\right)\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(c+dx)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{3\sqrt{3}\,a^{5/3}\left(a^2-b^2\right)^2d} - \frac{b^{5/3}\left(4a^{8/3}-9a^2b^{2/3}+8a^{2/3}b^2\right.}{\sqrt{3}\,\sqrt[3]{a}\,\left(a^2\right.}$$

Mathematica [C] time = 6.38, size = 657, normalized size = 0.88

$$\frac{ab^2\left(\frac{b^2}{a^2}+2\right)\sin(c+dx)}{3\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)} - \frac{b\left(a^2+2b^2\right)}{3\left(a^2-b^2\right)^2\left(a+b\sin^3(c+dx)\right)} + \frac{2ab\left(a^2+5b^2\right)\log\left(a+b\sin^3(c+dx)\right)}{3\left(a^2-b^2\right)^3} + \frac{4\sqrt[3]{a}\,b^{5/3}\left(a^2+2b^2\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(c+dx)\right)}{3\left(a^2-b^2\right)^3} - $$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (-1/4*((a + 7*b)*Log[1 - Sin[c + d*x]])/(a + b)^3 + ((a - 7*b)*Log[1 + Sin[
c + d*x]])/(4*(a - b)^3) + (4*a^(1/3)*b^(5/3)*(a^2 + 2*b^2)*Log[a^(1/3) + b
^(1/3)*Sin[c + d*x]])/(3*(a^2 - b^2)^3) - (2*a^(1/3)*(a^2 + 2*b^2)*(2*Sqrt[
3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b
^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[c + d*x] + b^(2/3)*Sin[c + d*x]^2]
))/(3*(a^2 - b^2)^3) + ((2 + b^2/a^2)*(2*a^(1/3)*b^(5/3)*Log[a^(1/3) + b^(1
/3)*Sin[c + d*x]] - a^(1/3)*(2*Sqrt[3]*b^(5/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*
Sin[c + d*x])/(Sqrt[3]*a^(1/3))] + b^(5/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Si
n[c + d*x] + b^(2/3)*Sin[c + d*x]^2])))/(9*(a^2 - b^2)^2) + (2*a*b*(a^2 + 5
*b^2)*Log[a + b*Sin[c + d*x]^3])/(3*(a^2 - b^2)^3) + 1/(4*(a + b)^2*(1 - Si
n[c + d*x])) - (3*b^3*(3*a^2 + b^2)*Hypergeometric2F1[2/3, 1, 5/3, -((b*Sin
[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)^3) - (3*b^3*Hypergeometri
c2F1[2/3, 2, 5/3, -((b*Sin[c + d*x]^3)/a)]*Sin[c + d*x]^2)/(2*a*(a^2 - b^2)
^2) - 1/(4*(a - b)^2*(1 + Sin[c + d*x])) - (b*(a^2 + 2*b^2))/(3*(a^2 - b^2)
^2*(a + b*Sin[c + d*x]^3)) + (a*b^2*(2 + b^2/a^2)*Sin[c + d*x])/(3*(a^2 - b
^2)^2*(a + b*Sin[c + d*x]^3)))/d
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.33, size = 790, normalized size = 1.06

$$\frac{\frac{8\left(15\,a^{10}b^{4}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-42\,a^{8}b^{6}\left(-\frac{a}{b}\right)^{\frac{1}{3}}+36\,a^{6}b^{8}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-6\,a^{4}b^{10}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-3\,a^{2}b^{12}\left(-\frac{a}{b}\right)^{\frac{1}{3}}-8\,a^{11}b^{3}+13\,a^{9}b^{5}+10\,a^{7}b^{7}-28\,a^{5}b^{9}+14\,a^{3}b^{11}-ab^{13}\right)\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\right.\right.}{a^{15}b-6\,a^{13}b^{3}+15\,a^{11}b^{5}-20\,a^{9}b^{7}+15\,a^{7}b^{9}-6\,a^{5}b^{11}+a^{3}b^{13}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] 1/36*(8*(15*a^10*b^4*(-a/b)^(1/3) - 42*a^8*b^6*(-a/b)^(1/3) + 36*a^6*b^8*(-
a/b)^(1/3) - 6*a^4*b^10*(-a/b)^(1/3) - 3*a^2*b^12*(-a/b)^(1/3) - 8*a^11*b^3
 + 13*a^9*b^5 + 10*a^7*b^7 - 28*a^5*b^9 + 14*a^3*b^11 - a*b^13)*(-a/b)^(1/3
)*log(abs(-(-a/b)^(1/3) + sin(d*x + c)))/(a^15*b - 6*a^13*b^3 + 15*a^11*b^5
 - 20*a^9*b^7 + 15*a^7*b^9 - 6*a^5*b^11 + a^3*b^13) + 8*(3*(5*sqrt(3)*a^3*b
 + sqrt(3)*a*b^3)*(-a*b^2)^(2/3) + (8*sqrt(3)*a^4*b + 11*sqrt(3)*a^2*b^3 -
sqrt(3)*b^5)*(-a*b^2)^(1/3))*arctan(1/3*sqrt(3)*((-a/b)^(1/3) + 2*sin(d*x +
 c))/(-a/b)^(1/3))/(a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6) - 4*(3*(5*a^3*b
+ a*b^3)*(-a*b^2)^(2/3) - (8*a^4*b + 11*a^2*b^3 - b^5)*(-a*b^2)^(1/3))*log(
sin(d*x + c)^2 + (-a/b)^(1/3)*sin(d*x + c) + (-a/b)^(2/3))/(a^8 - 3*a^6*b^2
 + 3*a^4*b^4 - a^2*b^6) + 24*(a^3*b + 5*a*b^3)*log(abs(b*sin(d*x + c)^3 + a
))/(a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6) + 9*(a - 7*b)*log(abs(sin(d*x + c) +
 1))/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) - 9*(a + 7*b)*log(abs(sin(d*x + c) - 1
))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(3*a^3*b*sin(d*x + c)^4 + 9*a*b^3*si
n(d*x + c)^4 - 10*a^2*b^2*sin(d*x + c)^3 - 2*b^4*sin(d*x + c)^3 + 2*a^3*b*s
in(d*x + c)^2 - 2*a*b^3*sin(d*x + c)^2 + 3*a^4*sin(d*x + c) + 7*a^2*b^2*sin
(d*x + c) + 2*b^4*sin(d*x + c) - 8*a^3*b - 4*a*b^3)/((b*sin(d*x + c)^5 - b*
sin(d*x + c)^3 + a*sin(d*x + c)^2 - a)*(a^5 - 2*a^3*b^2 + a*b^4)))/d
```

maple [B] time = 1.05, size = 1309, normalized size = 1.75

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 22/9/d*b^3/(a-b)^3/(a+b)^3*a/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b
)^(1/3)*sin(d*x+c)-1))-2/9/d*b^5/(a-b)^3/(a+b)^3/a/(a/b)^(2/3)*3^(1/2)*arct
an(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-10/3/d*b^2/(a-b)^3/(a+b)^3*a^2
*3^(1/2)/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))+16/9/
d*b/(a-b)^3/(a+b)^3*a^3/(a/b)^(2/3)*3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/
3)*sin(d*x+c)-1))-1/4/d/(a+b)^2/(sin(d*x+c)-1)-1/4/(a-b)^2/d/(1+sin(d*x+c))
+2/3/d*b^2/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a^3*sin(d*x+c)+16/9/d*b/(a-b)
^3/(a+b)^3*a^3/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))+22/9/d*b^3/(a-b)^3/(a
+b)^3*a/(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-2/9/d*b^5/(a-b)^3/(a+b)^3/a/
(a/b)^(2/3)*ln(sin(d*x+c)+(a/b)^(1/3))-8/9/d*b/(a-b)^3/(a+b)^3*a^3/(a/b)^(2
/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))-11/9/d*b^3/(a-b)^3/
(a+b)^3*a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+1
/9/d*b^5/(a-b)^3/(a+b)^3/a/(a/b)^(2/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+
c)+(a/b)^(2/3))-1/d*b^3/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*sin(d*x+c)^2*a^2
-1/3/d*b^4/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a*sin(d*x+c)-1/3/d*b^6/(a-b)^
3/(a+b)^3/(a+b*sin(d*x+c)^3)/a*sin(d*x+c)-2/3/d*b^4/(a-b)^3/(a+b)^3*3^(1/2)
/(a/b)^(1/3)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(d*x+c)-1))-5/3/d*b^2/(a-
b)^3/(a+b)^3*a^2/(a/b)^(1/3)*ln(sin(d*x+c)^2-(a/b)^(1/3)*sin(d*x+c)+(a/b)^(
2/3))+2/3/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)-7/4/d/(a-b)^3*ln(1+sin(d
*x+c))*b-1/4/d/(a+b)^3*ln(sin(d*x+c)-1)*a-7/4/d/(a+b)^3*ln(sin(d*x+c)-1)*b+
1/4*a*ln(1+sin(d*x+c))/(a-b)^3/d+10/3/d*b^2/(a-b)^3/(a+b)^3*a^2/(a/b)^(1/3)
*ln(sin(d*x+c)+(a/b)^(1/3))+1/d*b^5/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*sin(
d*x+c)^2-1/3/d*b/(a-b)^3/(a+b)^3/(a+b*sin(d*x+c)^3)*a^4-1/3/d*b^3/(a-b)^3/(
a+b)^3/(a+b*sin(d*x+c)^3)*a^2+2/3/d*b^4/(a-b)^3/(a+b)^3/(a/b)^(1/3)*ln(sin(
d*x+c)+(a/b)^(1/3))-1/3/d*b^4/(a-b)^3/(a+b)^3/(a/b)^(1/3)*ln(sin(d*x+c)^2-(
a/b)^(1/3)*sin(d*x+c)+(a/b)^(2/3))+2/3/d*b/(a-b)^3/(a+b)^3*a^3*ln(a+b*sin(d
*x+c)^3)+10/3/d*b^3/(a-b)^3/(a+b)^3*a*ln(a+b*sin(d*x+c)^3)
```

maxima [A] time = 0.45, size = 788, normalized size = 1.05

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] -1/36*(8*sqrt(3)*(5*a^3*b^2*(3*(a/b)^(2/3) + 2) - a^2*b^3*(11*(a/b)^(1/3) +
 10*a/b) - 2*a^4*b*(4*(a/b)^(1/3) + a/b) + 3*a*b^4*(a/b)^(2/3) + b^5*(a/b)^
(1/3) + 2*a^5)*arctan(-1/3*sqrt(3)*((a/b)^(1/3) - 2*sin(d*x + c))/(a/b)^(1/
3))/((a^7*(a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b
^6*(a/b)^(2/3))*(a/b)^(1/3)) - 4*(a^2*b^3*(30*(a/b)^(2/3) - 11) + 2*a^4*b*(
3*(a/b)^(2/3) - 4) - 15*a^3*b^2*(a/b)^(1/3) - 3*a*b^4*(a/b)^(1/3) + b^5)*lo
g(sin(d*x + c)^2 - (a/b)^(1/3)*sin(d*x + c) + (a/b)^(2/3))/(a^7*(a/b)^(2/3)
 - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b^6*(a/b)^(2/3)) - 8*(
a^2*b^3*(15*(a/b)^(2/3) + 11) + a^4*b*(3*(a/b)^(2/3) + 8) + 15*a^3*b^2*(a/b
)^(1/3) + 3*a*b^4*(a/b)^(1/3) - b^5)*log((a/b)^(1/3) + sin(d*x + c))/(a^7*(
a/b)^(2/3) - 3*a^5*b^2*(a/b)^(2/3) + 3*a^3*b^4*(a/b)^(2/3) - a*b^6*(a/b)^(2
/3)) - 9*(a - 7*b)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*b^2 - b^3) +
9*(a + 7*b)*log(sin(d*x + c) - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 6*(3*(a
^3*b + 3*a*b^3)*sin(d*x + c)^4 - 8*a^3*b - 4*a*b^3 - 2*(5*a^2*b^2 + b^4)*si
n(d*x + c)^3 + 2*(a^3*b - a*b^3)*sin(d*x + c)^2 + (3*a^4 + 7*a^2*b^2 + 2*b^
4)*sin(d*x + c))/(a^6 - 2*a^4*b^2 + a^2*b^4 - (a^5*b - 2*a^3*b^3 + a*b^5)*s
in(d*x + c)^5 + (a^5*b - 2*a^3*b^3 + a*b^5)*sin(d*x + c)^3 - (a^6 - 2*a^4*b
^2 + a^2*b^4)*sin(d*x + c)^2))/d
```

mupad [B] time = 15.75, size = 1605, normalized size = 2.15

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729
*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4
*z + 216*a^2*b^3 - 8*b^5, z, k)*(((32*a*b^11)/27 + (2173*a^3*b^9)/27 + (847
*a^5*b^7)/3 - 20*a^7*b^5)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b
^2) - root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^11
*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z +
216*a^2*b^3 - 8*b^5, z, k)*(((32*a^2*b^12)/3 - (1017*a^4*b^10)/4 + 325*a^6*
b^8 + (4153*a^8*b^6)/12 - (63*a^10*b^4)/2)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*
a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b
^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z +
 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z, k)*((16*a^3*b^13 - (563*a^5*b^11)/
2 + 303*a^7*b^9 + 188*a^9*b^7 - 239*a^11*b^5 + (27*a^13*b^3)/2)/(a^11 + a^3
*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + root(2187*a^9*b^2*z^3 - 2187*a^
7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^11*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*
z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z + 216*a^2*b^3 - 8*b^5, z, k)*((36*a^4*b
^14 + 36*a^6*b^12 - 504*a^8*b^10 + 936*a^10*b^8 - 684*a^12*b^6 + 180*a^14*b
^4)/(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (sin(c + d*x)*(1
7496*a^5*b^13 - 64152*a^7*b^11 + 81648*a^9*b^9 - 34992*a^11*b^7 - 5832*a^13
*b^5 + 5832*a^15*b^3))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9
*b^2))) - (sin(c + d*x)*(13824*a^4*b^12 - 864*a^2*b^14 + 30780*a^6*b^10 - 9
6660*a^8*b^8 + 50004*a^10*b^6 + 2916*a^12*b^4))/(108*(a^11 + a^3*b^8 - 4*a^
5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))) - (sin(c + d*x)*(7200*a^3*b^11 - 384*a*b^1
3 - 68247*a^5*b^9 + 31542*a^7*b^7 + 10449*a^9*b^5))/(108*(a^11 + a^3*b^8 -
4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2))) + (sin(c + d*x)*(64*b^12 + 4758*a^2*b^
10 - 29860*a^4*b^8 - 9234*a^6*b^6))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^
7*b^4 - 4*a^9*b^2))) + ((28*b^10)/27 + (122*a^2*b^8)/27 + (10*a^4*b^6)/3)/(
a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*b^2) + (sin(c + d*x)*(1080*a
*b^9 + 2568*a^3*b^7))/(108*(a^11 + a^3*b^8 - 4*a^5*b^6 + 6*a^7*b^4 - 4*a^9*
b^2)))*root(2187*a^9*b^2*z^3 - 2187*a^7*b^4*z^3 + 729*a^5*b^6*z^3 - 729*a^1
1*z^3 + 7290*a^6*b^3*z^2 + 1458*a^8*b*z^2 - 972*a^5*b^2*z + 324*a^3*b^4*z +
 216*a^2*b^3 - 8*b^5, z, k), k, 1, 3)/d + ((b*sin(c + d*x)^2)/(3*(a^2 - b^2
)) + (sin(c + d*x)^4*((a^2*b)/2 + (3*b^3)/2))/(a^4 + b^4 - 2*a^2*b^2) - (2*
b*(2*a^2 + b^2))/(3*(a^2 - b^2)^2) + (sin(c + d*x)*(3*a^4 + 2*b^4 + 7*a^2*b
^2))/(6*a*(a^4 + b^4 - 2*a^2*b^2)) - (sin(c + d*x)^3*(b^4/3 + (5*a^2*b^2)/3
))/(a*(a^4 + b^4 - 2*a^2*b^2)))/(d*(a - a*sin(c + d*x)^2 + b*sin(c + d*x)^3
 - b*sin(c + d*x)^5)) - (log(sin(c + d*x) - 1)*(a + 7*b))/(d*(12*a*b^2 + 12
*a^2*b + 4*a^3 + 4*b^3)) + (log(sin(c + d*x) + 1)*(a - 7*b))/(d*(12*a*b^2 -
 12*a^2*b + 4*a^3 - 4*b^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.399 $\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
[Out] Defer[Int][Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\cos^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.36, size = 394, normalized size = 15.15

$$\frac{24\cos(c+dx)(a+b\sin(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))}-i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-4i\#1^3a\tan^{-1}\left(\frac{\text{s}}{\text{co}}\right.}{}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b*Log[1 - 2*Cos[c + d*x
]*#1 + #1^2] + (4*I)*a*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*a*Lo
g[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 12*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1^2 - (6*I)*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*Arc
Tan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 2*a*Log[1 - 2*Cos[c + d*x]*#1
+ #1^2]*#1^3 + 2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b*Log[
1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^
5) & ] + (24*Cos[c + d*x]*(a + b*Sin[c + d*x]))/(4*a + 3*b*Sin[c + d*x] - b
*Sin[3*(c + d*x)]))/(18*a*b*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] Timed out
```

maple [A] time = 0.96, size = 550, normalized size = 21.15

$$-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a}+\frac{}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2
*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x
+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x
+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1
/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*ta
n(1/2*d*x+1/2*c)^3+4/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/b*tan(1/2*d*x+1/2*c)^2+
2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/
2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/
2*c)^2*a+a)/b+2/9/d/a/b*sum((_R^4*b+_R^3*a+_R*a+b)/(_R^5*a+2*_R^3*a+4*_R^2*
b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2
*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 15.99, size = 2431, normalized size = 93.50

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] 2/(3*d*(a*b + 8*b^2*tan(c/2 + (d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a
*b*tan(c/2 + (d*x)/2)^4 + a*b*tan(c/2 + (d*x)/2)^6)) + symsum(log((638976*a
^2*b^4 - 655360*b^6 - 8192*a^6 + 24576*a^4*b^2 - 2949120*root(531441*a^10*b
^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a
^6 - 64*b^6, d, k)*a^3*b^5 + 2138112*root(531441*a^10*b^8*d^6 + 59049*a^8*b
^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a
^5*b^3 - 9437184*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^
4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*b^8*tan(c/2 + (d*x)/2
) - 786432*a*b^5*tan(c/2 + (d*x)/2) + 98304*a^5*b*tan(c/2 + (d*x)/2) - 2123
```

```
3664*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a
^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^2*b^8 + 18579456*root(531441*
a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b
^2 + a^6 - 64*b^6, d, k)^2*a^4*b^6 + 2654208*root(531441*a^10*b^8*d^6 + 590
49*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6,
 d, k)^2*a^6*b^4 - 167215104*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 +
 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^5*b^7
 + 113467392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^
2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^7*b^5 - 107495424*roo
t(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 +
 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^6*b^8 + 107495424*root(531441*a^10*b^
8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^
6 - 64*b^6, d, k)^4*a^8*b^6 - 1934917632*root(531441*a^10*b^8*d^6 + 59049*a
^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d,
k)^5*a^7*b^9 + 1451188224*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 21
87*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^5*a^9*b^7 +
688128*a^3*b^3*tan(c/2 + (d*x)/2) - 1179648*root(531441*a^10*b^8*d^6 + 5904
9*a^8*b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6,
d, k)*a*b^7 + 12976128*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*
a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^2*b^6*tan(c/2
 + (d*x)/2) - 6266880*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a
^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^4*b^4*tan(c/2
+ (d*x)/2) + 737280*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6
*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)*a^6*b^2*tan(c/2 +
(d*x)/2) - 53084160*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a^6
*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^3*b^7*tan(c/2
+ (d*x)/2) + 50429952*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*a
^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^5*b^5*tan(c/
2 + (d*x)/2) + 2654208*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 2187*
a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^2*a^7*b^3*tan(c
/2 + (d*x)/2) - 59719680*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 218
7*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^6*b^6*tan
(c/2 + (d*x)/2) + 5971968*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 + 21
87*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^3*a^8*b^4*ta
n(c/2 + (d*x)/2) - 859963392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^4 +
 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^5*b^9
*tan(c/2 + (d*x)/2) + 859963392*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6*d^
4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^4*a^7*
b^7*tan(c/2 + (d*x)/2) - 483729408*root(531441*a^10*b^8*d^6 + 59049*a^8*b^6
*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k)^5*a
^8*b^8*tan(c/2 + (d*x)/2))/(a^3*b^4))*root(531441*a^10*b^8*d^6 + 59049*a^8*
b^6*d^4 + 2187*a^6*b^4*d^2 + 48*a^2*b^4 + 15*a^4*b^2 + a^6 - 64*b^6, d, k),
 k, 1, 6)/d + (8*tan(c/2 + (d*x)/2)^3)/(3*d*(3*a^2*tan(c/2 + (d*x)/2)^2 + 3
*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c/2
+ (d*x)/2)^3)) - (2*tan(c/2 + (d*x)/2)^5)/(3*d*(3*a^2*tan(c/2 + (d*x)/2)^2
+ 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*b*tan(c
/2 + (d*x)/2)^3)) + (4*tan(c/2 + (d*x)/2)^2)/(3*d*(a*b + 8*b^2*tan(c/2 + (d
*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a*b*tan(c/2 + (d*x)/2)^4 + a*b*ta
n(c/2 + (d*x)/2)^6)) + (2*tan(c/2 + (d*x)/2)^4)/(3*d*(a*b + 8*b^2*tan(c/2 +
 (d*x)/2)^3 + 3*a*b*tan(c/2 + (d*x)/2)^2 + 3*a*b*tan(c/2 + (d*x)/2)^4 + a*b
*tan(c/2 + (d*x)/2)^6)) + (2*tan(c/2 + (d*x)/2))/(3*d*(3*a^2*tan(c/2 + (d*x
)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 + 8*a*
b*tan(c/2 + (d*x)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.400 $\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
[Out] Defer[Int][Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{\cos^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.24, size = 273, normalized size = 10.50

$$\frac{\frac{12\sin(2(c+dx))}{4a+3b\sin(c+dx)-b\sin(3(c+dx))}-i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-6i\#1^2\log\left(\#1^2-2\#\right.}{}\right.}{18ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
[Out] ((-I)*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 &
, (2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2] + 12*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - (6*I)*Log[1 -
 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + 2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)
]*#1^4 - I*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2
*b*#1^3 + b*#1^5) & ] + (12*Sin[2*(c + d*x)])/(4*a + 3*b*Sin[c + d*x] - b*S
in[3*(c + d*x)]))/(18*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cos\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")

[Out] integrate(cos(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)

maple [A] time = 0.94, size = 236, normalized size = 9.08

$$-\frac{2\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a}+\frac{}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}\right.\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)

[Out] -2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5+2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)+2/9/d/a*sum((_R^4+1)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")

[Out] Timed out

mupad [A] time = 15.75, size = 1648, normalized size = 63.38

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(c + d*x)^2/(a + b*sin(c + d*x)^3)^2,x)

[Out] symsum(log(-((131072*b^2)/243 - (16384*a^2)/243 + (8192*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a^4*tan(c/2 + (d*x)/2))/27 + (1048576*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*b^4*tan(c/2 + (d*x)/2))/27 + (262144*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^2*b^4)/3 - (131072*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^4*b^2)/3 - 98304*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^3*a^5*b^3 + 442368*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^4*a^6*b^4 + 221184*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^4*a^8*b^2 + 7962624*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^5*a^7*b^5 - 5971968*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^5*a^9*b^3 + (131072*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a*b^3)/27 - (65536*root(531

```
441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^
2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)*a^3*b)/27 - (131072*root(531441*a^12*b
^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2
*b^2 + a^4 + 64*b^4, d, k)*a^2*b^2*tan(c/2 + (d*x)/2))/9 - (32768*root(5314
41*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2
 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^5*b*tan(c/2 + (d*x)/2))/3 - (131072
*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a
^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^2*a^3*b^3*tan(c/2 + (d*x)/2))
/3 + 245760*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683*a^8*b^4*
d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^3*a^6*b^2*tan(c/2
+ (d*x)/2) + 3538944*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^6 + 19683
*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^4*a^5*b^5
*tan(c/2 + (d*x)/2) - 2654208*root(531441*a^12*b^4*d^6 - 531441*a^10*b^6*d^
6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^4, d, k)^
4*a^7*b^3*tan(c/2 + (d*x)/2) + 1990656*root(531441*a^12*b^4*d^6 - 531441*a^
10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 + 64*b^
4, d, k)^5*a^8*b^4*tan(c/2 + (d*x)/2))/a^3)*root(531441*a^12*b^4*d^6 - 5314
41*a^10*b^6*d^6 + 19683*a^8*b^4*d^4 + 729*a^6*b^2*d^2 - 16*a^2*b^2 + a^4 +
64*b^4, d, k), k, 1, 6)/d - (2*tan(c/2 + (d*x)/2)^5)/(3*d*(3*a^2*tan(c/2 +
(d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 +
8*a*b*tan(c/2 + (d*x)/2)^3)) + (2*tan(c/2 + (d*x)/2))/(3*d*(3*a^2*tan(c/2 +
 (d*x)/2)^2 + 3*a^2*tan(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2 +
 8*a*b*tan(c/2 + (d*x)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.401 $\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\frac{1}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(1/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^3)^(-2),x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^3)^(-2), x]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx=\int \frac{1}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 0.48, size = 502, normalized size = 29.53

$$-\frac{12b\cos(c+dx)(a\cos(2(c+dx))-3a+2b\sin(c+dx))}{(a-b)(a+b)(4a+3b\sin(c+dx)-b\sin(3(c+dx)))}+\frac{i\text{RootSum}\left[i\#1^6b-3i\#1^4b+8\#1^3a+3i\#1^2b-ib\&,\frac{2\#1^4b^2\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)-4i\#1^3ab\tan^{-1}\left(\frac{\sin(}{\cos(c+}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^3)^(-2),x]
```

```
[Out] ((I*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6 & ,
(2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] - I*b^2*Log[1 - 2*Cos[c + d
*x]*#1 + #1^2] + (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 2*
a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 24*a^2*ArcTan[Sin[c + d*x]/(Cos[
c + d*x] - #1)]*#1^2 + 12*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2
 + (12*I)*a^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^2*Log[1 - 2*
Cos[c + d*x]*#1 + #1^2]*#1^2 - (4*I)*a*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^3 - 2*a*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 + 2*b^2*ArcTan[S
in[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - I*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #
1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a^2 - b^2) - (12
*b*Cos[c + d*x]*(-3*a + a*Cos[2*(c + d*x)] + 2*b*Sin[c + d*x]))/((a - b)*(a
 + b)*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)])))/(18*a*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

[Out] Timed out

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(dx+c\right)^{3}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^3 + a)^(-2), x)
```

maple [A] time = 0.65, size = 658, normalized size = 38.71

$$\frac{2b^{2}\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d\left(\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+3\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+8b\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+3\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+a\right)a\left(a^{2}-b^{2}\right)}-\frac{}{3d\left(\left(\tan\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] 2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^5-2/3/d
/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+
3*tan(1/2*d*x+1/2*c)^2*a+a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^4+8/3/d/(tan(1/2
*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2
*d*x+1/2*c)^2*a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)^3+8/3/d/(tan(1/2*d*x+
1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+
1/2*c)^2*a+a)*b/(a^2-b^2)*tan(1/2*d*x+1/2*c)^2-2/3/d/(tan(1/2*d*x+1/2*c)^6*
a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*
a+a)*b^2/a/(a^2-b^2)*tan(1/2*d*x+1/2*c)+2/3/d/(tan(1/2*d*x+1/2*c)^6*a+3*tan
(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*b/
(a^2-b^2)+1/9/d/a/(a^2-b^2)*sum(((3*a^2-2*b^2)*_R^4-2*_R^3*a*b+6*_R^2*a^2-2
*a*_R*b+3*a^2-2*b^2)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-
_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 17.89, size = 1567, normalized size = 92.18

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^3)^2,x)
```

```
[Out] symsum(log(- (8192*(80*b^6 - 270*a^2*b^4))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)
) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*b^6*d^6
- 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147*a^12*d^
4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a^4, d, k
)*((8192*(144*a*b^7 + 648*a^3*b^5 - 2187*a^5*b^3))/(243*(a^7 + a^3*b^4 - 2*
```

```
a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*
b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147
*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a
^4, d, k)*(root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 + 531441*a^10*b
^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4*d^4 - 177147*
a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 64*b^4 - 729*a^
4, d, k)*((8192*(26973*a^7*b^5 - 20412*a^5*b^7 + 39366*a^9*b^3))/(243*(a^7
+ a^3*b^4 - 2*a^5*b^2)) - root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6
+ 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^
4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 -
64*b^4 - 729*a^4, d, k)*(root(1594323*a^14*b^2*d^6 - 1594323*a^12*b^4*d^6 +
 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 + 59049*a^8*b^4
*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 432*a^2*b^2 - 6
4*b^4 - 729*a^4, d, k)*((8192*(236196*a^7*b^9 - 649539*a^9*b^7 + 590490*a^1
1*b^5 - 177147*a^13*b^3))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2
 + (d*x)/2)*(6561*a^8*b^8 - 13122*a^10*b^6 + 6561*a^12*b^4))/(27*(a^7 + a^3
*b^4 - 2*a^5*b^2))) + (8192*(13122*a^6*b^8 - 85293*a^8*b^6 + 72171*a^10*b^4
))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2 + (d*x)/2)*(11664*a^5*
b^9 - 40824*a^7*b^7 + 37908*a^9*b^5 - 8748*a^11*b^3))/(27*(a^7 + a^3*b^4 -
2*a^5*b^2))) + (8192*tan(c/2 + (d*x)/2)*(3078*a^6*b^6 - 8181*a^8*b^4))/(27*
(a^7 + a^3*b^4 - 2*a^5*b^2))) - (8192*(2592*a^2*b^8 - 11340*a^4*b^6 + 11664
*a^6*b^4))/(243*(a^7 + a^3*b^4 - 2*a^5*b^2)) + (8192*tan(c/2 + (d*x)/2)*(12
60*a^5*b^5 - 720*a^3*b^7 + 1944*a^7*b^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2)))
 + (8192*tan(c/2 + (d*x)/2)*(128*b^8 - 688*a^2*b^6 + 1053*a^4*b^4))/(27*(a^
7 + a^3*b^4 - 2*a^5*b^2))) - (8192*tan(c/2 + (d*x)/2)*(32*a*b^5 - 108*a^3*b
^3))/(27*(a^7 + a^3*b^4 - 2*a^5*b^2)))*root(1594323*a^14*b^2*d^6 - 1594323*
a^12*b^4*d^6 + 531441*a^10*b^6*d^6 - 531441*a^16*d^6 - 59049*a^10*b^2*d^4 +
 59049*a^8*b^4*d^4 - 177147*a^12*d^4 + 8019*a^6*b^2*d^2 - 19683*a^8*d^2 + 4
32*a^2*b^2 - 64*b^4 - 729*a^4, d, k), k, 1, 6)/d + ((2*b)/(3*(a^2 - b^2)) +
 (8*b*tan(c/2 + (d*x)/2)^2)/(3*(a^2 - b^2)) - (2*b*tan(c/2 + (d*x)/2)^4)/(3
*(a^2 - b^2)) - (2*b^2*tan(c/2 + (d*x)/2))/(3*a*(a^2 - b^2)) + (8*b^2*tan(c
/2 + (d*x)/2)^3)/(3*a*(a^2 - b^2)) + (2*b^2*tan(c/2 + (d*x)/2)^5)/(3*a*(a^2
 - b^2)))/(d*(a + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 + a*t
an(c/2 + (d*x)/2)^6 + 8*b*tan(c/2 + (d*x)/2)^3))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.402 $\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\sec^2(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 1.60, size = 845, normalized size = 32.50

$$-\frac{ib\text{RootSum}\left[ib\#1^6-3ib\#1^4+8a\#1^3+3ib\#1^2-ib\&,\frac{2b^3\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4+16a^2b\tan^{-1}\left(\frac{\sin(c+dx)}{\cos(c+dx)-\#1}\right)\#1^4-ib^3\log\left(\#1^2-2\cos(c+dx)\#1+1\right)\#1^4-8ia^2b\log\left(\#1^2-2\,\text{c}\right.}{}\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] (((-I)*b*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1^6
 & , (16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^3*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)] - (8*I)*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^
2] - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] + (20*I)*a^3*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1 + (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
 - #1)]*#1 + 10*a^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 + 8*a*b^2*Log[1 -
2*Cos[c + d*x]*#1 + #1^2]*#1 - 120*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x]
- #1)]*#1^2 + 12*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 + (60*I)
*a^2*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^3*Log[1 - 2*Cos[c +
 d*x]*#1 + #1^2]*#1^2 - (20*I)*a^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (16*I)*a*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 10*a^3
*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 8*a*b^2*Log[1 - 2*Cos[c + d*x]*#1
 + #1^2]*#1^3 + 16*a^2*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*
b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (8*I)*a^2*b*Log[1 - 2*C
os[c + d*x]*#1 + #1^2]*#1^4 - I*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)
/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b*#1^5) & ])/(a*(a^2 - b^2)^2) + (18*Sin
[(c + d*x)/2])/((a + b)^2*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (18*Sin[
(c + d*x)/2])/((a - b)^2*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (12*b*Cos
```

```
[c + d*x]*(-2*a^3 - 7*a*b^2 + 3*a*b^2*Cos[2*(c + d*x)] + 2*b*(2*a^2 + b^2)*
Sin[c + d*x]))/(a*(a - b)^2*(a + b)^2*(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c
+ d*x)]))))/(18*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec\left(dx+c\right)^2}{\left(b\sin\left(dx+c\right)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^2/(b*sin(d*x + c)^3 + a)^2, x)
```

maple [A] time = 0.96, size = 1276, normalized size = 49.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)-1/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)-4/3/
d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*
tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)^5-2/3
/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b
*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^5-2/
3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*
tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^4*a^2+8
/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^4-8/
3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*
b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)^3-1
6/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+
8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)^3
-4/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8
*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^2*a^
2-20/3/d*b^3/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4
*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*c)^
2+4/3/d*b^2/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*
a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1/2*c)
+2/3/d*b^4/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a
+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/2*c)-
2/3/d*b/(a-b)^2/(a+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*
b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2-4/3/d*b^3/(a-b)^2/(a
+b)^2/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x+1/2*
c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)-1/9/d*b/(a-b)^2/(a+b)^2/a*sum((b*(11*a^2-2
*b^2)*_R^4+2*a*(-5*a^2-4*b^2)*_R^3+54*_R^2*a^2*b+2*a*(-5*a^2-4*b^2)*_R+11*a
^2*b-2*b^3)/(_R^5*a+2*_R^3*a+4*_R^2*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=Ro
otOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^2*a+a))
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [A] time = 21.21, size = 3148, normalized size = 121.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(5479612416*a^8*b^36 - 180486144*a^6*b^38 - root(5314410*a^16*b^4
*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 -
 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a
^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^
2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c
/2 + (d*x)/2)*(764411904*a^6*b^40 - 27805483008*a^8*b^38 + 437297356800*a^1
0*b^36 - 3672461721600*a^12*b^34 + 19250011791360*a^14*b^32 - 6915063575347
2*a^16*b^30 + 180165872001024*a^18*b^28 - 352655758540800*a^20*b^26 + 52992
3028377600*a^22*b^24 - 618699706859520*a^24*b^22 + 563713761042432*a^26*b^2
0 - 399760062234624*a^28*b^18 + 218398602240000*a^30*b^16 - 90108039168000*
a^32*b^14 + 27130620764160*a^34*b^12 - 5617221156864*a^36*b^10 + 7135367086
08*a^38*b^8 - 41803776000*a^40*b^6) - root(5314410*a^16*b^4*d^6 - 5314410*a
^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 531441*a^10*b^1
0*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 10
62882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^
6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(root(5314410*a^16*b^4
*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 -
 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a
^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^
2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(tan(c
/2 + (d*x)/2)*(157695787008*a^12*b^38 - 4039140556800*a^14*b^36 + 391830495
06816*a^16*b^34 - 212750482120704*a^18*b^32 + 750889290203136*a^20*b^30 - 1
854140141887488*a^22*b^28 + 3327952874029056*a^24*b^26 - 4413464400863232*a
^26*b^24 + 4311710468702208*a^28*b^22 - 3009938035433472*a^30*b^20 + 135980
8836452352*a^32*b^18 - 238981192998912*a^34*b^16 - 150898421366784*a^36*b^1
4 + 136937506922496*a^38*b^12 - 52028967665664*a^40*b^10 + 10565134000128*a
^42*b^8 - 976165945344*a^44*b^6 + 12093235200*a^46*b^4) - root(5314410*a^16
*b^4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6 + 2657205*a^12*b^8*d
^6 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*a^12*b^4*d^4 + 20667
15*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^
4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k)*(t
an(c/2 + (d*x)/2)*(69657034752*a^11*b^41 - 1619526057984*a^13*b^39 + 164042
31684096*a^15*b^37 - 99052303417344*a^17*b^35 + 405403942256640*a^19*b^33 -
 1203882531618816*a^21*b^31 + 2700324609196032*a^23*b^29 - 4688893637296128
*a^25*b^27 + 6394933732442112*a^27*b^25 - 6897962008903680*a^29*b^23 + 5886
924977995776*a^31*b^21 - 3949971812646912*a^33*b^19 + 2053768012627968*a^35
*b^17 - 806001549115392*a^37*b^15 + 227778503639040*a^39*b^13 - 42212163059
712*a^41*b^11 + 3970450980864*a^43*b^9 + 52242776064*a^45*b^7 - 34828517376
*a^47*b^5) + 8707129344*a^12*b^40 - 470184984576*a^14*b^38 + 6308315209728*
a^16*b^36 - 44092902998016*a^18*b^34 + 197477693521920*a^20*b^32 - 62315183
2891392*a^22*b^30 + 1459506434899968*a^24*b^28 - 2616109254180864*a^26*b^26
 + 3653180601827328*a^28*b^24 - 4009284777738240*a^30*b^22 + 34626773189099
52*a^32*b^20 - 2339013569937408*a^34*b^18 + 1217047711186944*a^36*b^16 - 47
```

3946464452608*a^38*b^14 + 130868154040320*a^40*b^12 - 22777850363904*a^42*b
^10 + 1645647446016*a^44*b^8 + 156728328192*a^46*b^6 - 30474952704*a^48*b^4
 + root(5314410*a^16*b^4*d^6 - 5314410*a^14*b^6*d^6 - 2657205*a^18*b^2*d^6
+ 2657205*a^12*b^8*d^6 - 531441*a^10*b^10*d^6 + 531441*a^20*d^6 + 11514555*
a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 1062882*a^10*b^6*d^4 - 295245*a^8*b^8
*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^4*b^4 - 2000*a^2*b^
6 + 64*b^8, d, k)*(tan(c/2 + (d*x)/2)*(39182082048*a^14*b^40 - 705277476864
*a^16*b^38 + 5994858553344*a^18*b^36 - 31972578951168*a^20*b^34 + 119897171
066880*a^22*b^32 - 335712078987264*a^24*b^30 + 727376171139072*a^26*b^28 -
1246930579095552*a^28*b^26 + 1714529546256384*a^30*b^24 - 1905032829173760*
a^32*b^22 + 1714529546256384*a^34*b^20 - 1246930579095552*a^36*b^18 + 72737
6171139072*a^38*b^16 - 335712078987264*a^40*b^14 + 119897171066880*a^42*b^1
2 - 31972578951168*a^44*b^10 + 5994858553344*a^46*b^8 - 705277476864*a^48*b
^6 + 39182082048*a^50*b^4) + 156728328192*a^13*b^41 - 2938656153600*a^15*b^
39 + 26095266643968*a^17*b^37 - 145874891464704*a^19*b^35 + 575506421121024
*a^21*b^33 - 1702539829149696*a^23*b^31 + 3916640921518080*a^25*b^29 - 7169
850829799424*a^27*b^27 + 10598909922312192*a^29*b^25 - 12763719955464192*a^
31*b^23 + 12573216672546816*a^33*b^21 - 10131310955151360*a^35*b^19 + 66502
96421842944*a^37*b^17 - 3524976829366272*a^39*b^15 + 1486724921229312*a^41*
b^13 - 487581829005312*a^43*b^11 + 119897171066880*a^45*b^9 - 2080568556748
8*a^47*b^7 + 2272560758784*a^49*b^5 - 117546246144*a^51*b^3)) - 59982446592
*a^11*b^39 + 1080651497472*a^13*b^37 - 6860250464256*a^15*b^35 + 1648211211
8784*a^17*b^33 + 27170113388544*a^19*b^31 - 327284061511680*a^21*b^29 + 119
4949984370688*a^23*b^27 - 2698934854606848*a^25*b^25 + 4276847122808832*a^2
7*b^23 - 4968511002943488*a^29*b^21 + 4288329891495936*a^31*b^19 - 27309180
75604992*a^33*b^17 + 1245220111908864*a^35*b^15 - 377418744815616*a^37*b^13
 + 60571629010944*a^39*b^11 + 1483598094336*a^41*b^9 - 2465085063168*a^43*b
^7 + 316842762240*a^45*b^5) - 1719926784*a^8*b^40 + 52457766912*a^10*b^38 -
 657657004032*a^12*b^36 + 4778655326208*a^14*b^34 - 23130112868352*a^16*b^3
2 + 80237540597760*a^18*b^30 - 208280123670528*a^20*b^28 + 415493301510144*
a^22*b^26 - 647354535100416*a^24*b^24 + 794486155567104*a^26*b^22 - 7697297
98176768*a^28*b^20 + 586362545233920*a^30*b^18 - 347391134318592*a^32*b^16
+ 156884680286208*a^34*b^14 - 52204937674752*a^36*b^12 + 12071252385792*a^3
8*b^10 - 1732933730304*a^40*b^8 + 116363796480*a^42*b^6 - tan(c/2 + (d*x)/2
)*(19779158016*a^9*b^39 - 436216430592*a^11*b^37 + 3308494159872*a^13*b^35
- 11619395371008*a^15*b^33 + 12486453460992*a^17*b^31 + 61196714901504*a^19
*b^29 - 334332052733952*a^21*b^27 + 871706622099456*a^23*b^25 - 15073939263
65184*a^25*b^23 + 1878255074082816*a^27*b^21 - 1736372938899456*a^29*b^19 +
 1197522672353280*a^31*b^17 - 608856446435328*a^33*b^15 + 221032950792192*a
^35*b^13 - 53644731383808*a^37*b^11 + 7499310759936*a^39*b^9 - 345490292736
*a^41*b^7 - 26873856000*a^43*b^5)) + 95551488*a^7*b^39 + 6640828416*a^9*b^3
7 - 187507851264*a^11*b^35 + 1874314100736*a^13*b^33 - 10498349481984*a^15*
b^31 + 38554452099072*a^17*b^29 - 100273965023232*a^19*b^27 + 1928073517793
28*a^21*b^25 - 280858991542272*a^23*b^23 + 313783776903168*a^25*b^21 - 2696
40960196608*a^27*b^19 + 177127448150016*a^29*b^17 - 87483347288064*a^31*b^1
5 + 31483928641536*a^33*b^13 - 7801408733184*a^35*b^11 + 1191025410048*a^37
*b^9 - 84503347200*a^39*b^7) - 59837128704*a^10*b^34 + 363432738816*a^12*b^
32 - 1444185759744*a^14*b^30 + 4071882866688*a^16*b^28 - 8529191903232*a^18
*b^26 + 13638053265408*a^20*b^24 - 16903052255232*a^22*b^22 + 1634520607948
8*a^24*b^20 - 12319205842944*a^26*b^18 + 7172803362816*a^28*b^16 - 31669193
68704*a^30*b^14 + 1026022588416*a^32*b^12 - 230217375744*a^34*b^10 + 319832
06400*a^36*b^8 - 2073600000*a^38*b^6 - tan(c/2 + (d*x)/2)*(1911029760*a^7*b
^37 - 56614256640*a^9*b^35 + 591941468160*a^11*b^33 - 3412860272640*a^13*b^
31 + 12781922549760*a^15*b^29 - 33715581419520*a^17*b^27 + 65518222049280*a
^19*b^25 - 96227753656320*a^21*b^23 + 108217793249280*a^23*b^21 - 934949810
99520*a^25*b^19 + 61692340469760*a^27*b^17 - 30585314672640*a^29*b^15 + 110
42885468160*a^31*b^13 - 2743999856640*a^33*b^11 + 419948789760*a^35*b^9 - 2
9859840000*a^37*b^7))*root(5314410*a^16*b^4*d^6 - 5314410*a^14*b^6*d^6 - 26
57205*a^18*b^2*d^6 + 2657205*a^12*b^8*d^6 - 531441*a^10*b^10*d^6 + 531441*a

```
^20*d^6 + 11514555*a^12*b^4*d^4 + 2066715*a^14*b^2*d^4 + 1062882*a^10*b^6*d
^4 - 295245*a^8*b^8*d^4 + 984150*a^8*b^4*d^2 - 98415*a^6*b^6*d^2 + 15625*a^
4*b^4 - 2000*a^2*b^6 + 64*b^8, d, k), k, 1, 6)/d - ((2*(7*a^2*b + 2*b^3))/(
3*(a^2 - b^2)^2) + (2*tan(c/2 + (d*x)/2)^6*(5*a^2*b + 4*b^3))/(3*(a^2 - b^2
)^2) + (2*tan(c/2 + (d*x)/2)^2*(19*a^2*b + 8*b^3))/(3*(a^2 - b^2)^2) - (2*t
an(c/2 + (d*x)/2)^4*(7*a^2*b + 38*b^3))/(3*(a^2 - b^2)^2) + (6*tan(c/2 + (d
*x)/2)^3*(b^4 - a^4 + 5*a^2*b^2))/(a*(a^2 - b^2)^2) - (2*tan(c/2 + (d*x)/2)
^5*(9*a^4 + 7*b^4 + 11*a^2*b^2))/(3*a*(a^2 - b^2)^2) - (2*tan(c/2 + (d*x)/2
)^7*(3*a^4 + b^4 + 5*a^2*b^2))/(3*a*(a^4 + b^4 - 2*a^2*b^2)) - (2*tan(c/2 +
 (d*x)/2)*(3*a^4 + b^4 + 5*a^2*b^2))/(3*a*(a^2 - b^2)^2))/(d*(a + 2*a*tan(c
/2 + (d*x)/2)^2 - 2*a*tan(c/2 + (d*x)/2)^6 - a*tan(c/2 + (d*x)/2)^8 + 8*b*t
an(c/2 + (d*x)/2)^3 - 8*b*tan(c/2 + (d*x)/2)^5))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.403 $\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2},x\right)$$

```
[Out] Unintegrable(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] Defer[Int][Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2, x]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx = \int \frac{\sec^4(c+dx)}{\left(a+b\sin^3(c+dx)\right)^2}\,dx$$

Mathematica [A] time = 1.74, size = 1158, normalized size = 44.54

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4/(a + b*Sin[c + d*x]^3)^2,x]
```

```
[Out] ((4*I)*b^2*RootSum[(-I)*b + (3*I)*b*#1^2 + 8*a*#1^3 - (3*I)*b*#1^4 + I*b*#1
^6 & , (14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 74*a^2*b^2*ArcTan
[Sin[c + d*x]/(Cos[c + d*x] - #1)] + 2*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x
] - #1)] - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] - (37*I)*a^2*b^2*Log
[1 - 2*Cos[c + d*x]*#1 + #1^2] - I*b^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2] +
(144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + (36*I)*a*b^3*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1 + 72*a^3*b*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1 + 18*a*b^3*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1 - 180*a^4*Ar
cTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^2 - 372*a^2*b^2*ArcTan[Sin[c + d*
x]/(Cos[c + d*x] - #1)]*#1^2 + 12*b^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #
1)]*#1^2 + (90*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 + (186*I)*a^2*
b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^2 - (6*I)*b^4*Log[1 - 2*Cos[c + d*
x]*#1 + #1^2]*#1^2 - (144*I)*a^3*b*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]
*#1^3 - (36*I)*a*b^3*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^3 - 72*a^3
*b*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^3 - 18*a*b^3*Log[1 - 2*Cos[c + d*x]
*#1 + #1^2]*#1^3 + 14*a^4*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 7
4*a^2*b^2*ArcTan[Sin[c + d*x]/(Cos[c + d*x] - #1)]*#1^4 + 2*b^4*ArcTan[Sin[
c + d*x]/(Cos[c + d*x] - #1)]*#1^4 - (7*I)*a^4*Log[1 - 2*Cos[c + d*x]*#1 +
#1^2]*#1^4 - (37*I)*a^2*b^2*Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4 - I*b^4*
Log[1 - 2*Cos[c + d*x]*#1 + #1^2]*#1^4)/(b*#1 - (4*I)*a*#1^2 - 2*b*#1^3 + b
*#1^5) & ] + (3*Sec[c + d*x]^3*(48*a^5*b + 568*a^3*b^3 + 14*a*b^5 + (78*a^5
*b + 606*a^3*b^3 + 81*a*b^5)*Cos[2*(c + d*x)] + 18*a*b^3*(4*a^2 + b^2)*Cos[
```

```
4*(c + d*x)] + 2*a^5*b*Cos[6*(c + d*x)] - 30*a^3*b^3*Cos[6*(c + d*x)] - 17*
a*b^5*Cos[6*(c + d*x)] + 48*a^6*Sin[c + d*x] - 244*a^4*b^2*Sin[c + d*x] + 2
0*a^2*b^4*Sin[c + d*x] - 4*b^6*Sin[c + d*x] + 16*a^6*Sin[3*(c + d*x)] - 194
*a^4*b^2*Sin[3*(c + d*x)] - 86*a^2*b^4*Sin[3*(c + d*x)] - 6*b^6*Sin[3*(c +
d*x)] - 14*a^4*b^2*Sin[5*(c + d*x)] - 74*a^2*b^4*Sin[5*(c + d*x)] - 2*b^6*S
in[5*(c + d*x)]))/(4*a + 3*b*Sin[c + d*x] - b*Sin[3*(c + d*x)]))/(72*a*(a^2
 - b^2)^3*d)
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="fricas")
```

```
[Out] Timed out
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)^4}{\left(b\sin(dx+c)^3+a\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="giac")
```

```
[Out] integrate(sec(d*x + c)^4/(b*sin(d*x + c)^3 + a)^2, x)
```

maple [A] time = 1.18, size = 1549, normalized size = 59.58

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x)
```

```
[Out] -1/3/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1)^3-1/2/d/(a+b)^2/(tan(1/2*d*x+1/2*c)-1
)^2-1/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)*a-4/d/(a+b)^3/(tan(1/2*d*x+1/2*c)-1)
*b-1/3/d/(a-b)^2/(tan(1/2*d*x+1/2*c)+1)^3+1/2/d/(a-b)^2/(tan(1/2*d*x+1/2*c)
+1)^2-1/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+1)*a+4/d/(a-b)^3/(tan(1/2*d*x+1/2*c)+
1)*b+2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x+1
/2*c)^5+14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x
+1/2*c)^5+2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1
/2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*
x+1/2*c)^5-6/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1
/2*c)^4+16/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*
c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x+1
/2*c)^3+8/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c
)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+1/
2*c)^3+12/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c
)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/2*
c)^2*a^2+12/d*b^5/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2
*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*tan(1/2*d*x+1/
2*c)^2-2/3/d*b^2/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*
c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^3*tan(1/2*d*x
+1/2*c)-14/3/d*b^4/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/
2*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a*tan(1/2*d*x
```

```
+1/2*c)-2/3/d*b^6/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2
*c)^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)/a*tan(1/2*d*x+
1/2*c)+4/d*b^3/(a-b)^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)
^4*a+8*b*tan(1/2*d*x+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)*a^2+2/d*b^5/(a-b)
^3/(a+b)^3/(tan(1/2*d*x+1/2*c)^6*a+3*tan(1/2*d*x+1/2*c)^4*a+8*b*tan(1/2*d*x
+1/2*c)^3+3*tan(1/2*d*x+1/2*c)^2*a+a)+1/9/d*b^2/(a-b)^3/(a+b)^3/a*sum(((19*
a^4+28*a^2*b^2-2*b^4)*_R^4+18*a*b*(-4*a^2-b^2)*_R^3+6*a^2*(11*a^2+34*b^2)*_
R^2+18*a*b*(-4*a^2-b^2)*_R+19*a^4+28*a^2*b^2-2*b^4)/(_R^5*a+2*_R^3*a+4*_R^2
*b+_R*a)*ln(tan(1/2*d*x+1/2*c)-_R),_R=RootOf(_Z^6*a+3*_Z^4*a+8*_Z^3*b+3*_Z^
2*a+a))
```

maxima [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a+b*sin(d*x+c)^3)^2,x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [**A**] time = 25.44, size = 4657, normalized size = 179.12

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a + b*sin(c + d*x)^3)^2),x)
```

```
[Out] symsum(log(26838024192*a^8*b^54 - tan(c/2 + (d*x)/2)*(7962624000*a^7*b^55 -
 508612608000*a^9*b^53 + 8841498624000*a^11*b^51 - 82283765760000*a^13*b^49
 + 501714984960000*a^15*b^47 - 2205295497216000*a^17*b^45 + 737918163763200
0*a^19*b^43 - 19451488075776000*a^21*b^41 + 41318016122880000*a^23*b^39 - 7
1811432161280000*a^25*b^37 + 103155513237504000*a^27*b^35 - 123224906907648
000*a^29*b^33 + 122756816093184000*a^31*b^31 - 101967282708480000*a^33*b^29
 + 70396872007680000*a^35*b^27 - 40129785593856000*a^37*b^25 + 186876255928
32000*a^39*b^23 - 6994754113536000*a^41*b^21 + 2053854351360000*a^43*b^19 -
 455730831360000*a^45*b^17 + 71860690944000*a^47*b^15 - 7177310208000*a^49*
b^13 + 341397504000*a^51*b^11) - 392822784*a^6*b^56 - root(18600435*a^18*b^
6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 + 11160261*a^14*b^10*
d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531441*a^10*b^14*d^6 -
 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^12*b^8*d^4 - 232259
40*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^12*d^4 - 11565585*a^
8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5
488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(764411904*a^6*b^58 - 614
39606784*a^8*b^56 + 2110475575296*a^10*b^54 - 33643637121024*a^12*b^52 + 31
9697763065856*a^14*b^50 - 2067381036048384*a^16*b^48 + 9810082122817536*a^1
8*b^46 - 35797302942326784*a^20*b^44 + 103613766013034496*a^22*b^42 - 24300
4699498881024*a^24*b^40 + 468678655511248896*a^26*b^38 - 750973819695611904
*a^28*b^36 + 1006348379003928576*a^30*b^34 - 1132028278205497344*a^32*b^32
+ 1070100496146087936*a^34*b^30 - 848821864657895424*a^36*b^28 + 5626355927
01198336*a^38*b^26 - 309384400894377984*a^40*b^24 + 139566181489975296*a^42
*b^22 - 50807786761396224*a^44*b^20 + 14569217952178176*a^46*b^18 - 3172130
021597184*a^48*b^16 + 494158536400896*a^50*b^14 - 49418889191424*a^52*b^12
+ 2463538323456*a^54*b^10 - 14338695168*a^56*b^8) + 95551488*a^7*b^57 + 358
79583744*a^9*b^55 - 1812522147840*a^11*b^53 + 29896430247936*a^13*b^51 - 27
3690491977728*a^15*b^49 + 1665068560662528*a^17*b^47 - 7358934856605696*a^1
9*b^45 + 24887080515133440*a^21*b^43 - 66575487905316864*a^23*b^41 + 144045
035942510592*a^25*b^39 - 255939373888192512*a^27*b^37 + 377317716543258624*
a^29*b^35 - 464589495171809280*a^31*b^33 + 479470084160126976*a^33*b^31 - 4
15092174607761408*a^35*b^29 + 300910589340991488*a^37*b^27 - 18182304326703
5136*a^39*b^25 + 90863416678809600*a^41*b^23 - 37111903240495104*a^43*b^21
```

+ 12175612162301952*a^45*b^19 - 3127996467412992*a^47*b^17 + 60541899359846
4*a^49*b^15 - 82897275985920*a^51*b^13 + 7145262637056*a^53*b^11 - 29087067
3408*a^55*b^9 + root(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 111602
61*a^20*b^4*d^6 + 11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a
^12*b^12*d^6 + 531441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*
d^4 - 155830311*a^12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^
4 + 688905*a^8*b^12*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433
755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2
 + (d*x)/2)*(45578059776*a^9*b^57 - 1988020371456*a^11*b^55 + 2172525517209
6*a^13*b^53 - 78629462802432*a^15*b^51 - 330769869373440*a^17*b^49 + 533728
8405614592*a^19*b^47 - 32144913894998016*a^21*b^45 + 126404118900965376*a^2
3*b^43 - 367050326151462912*a^25*b^41 + 829818883454238720*a^27*b^39 - 1502
808604998893568*a^29*b^37 + 2216700870917750784*a^31*b^35 - 268852344938260
0704*a^33*b^33 + 2692902186903011328*a^35*b^31 - 2227622993351147520*a^37*b
^29 + 1515332894269243392*a^39*b^27 - 839694861496221696*a^41*b^25 + 372789
943915216896*a^43*b^23 - 128854679612424192*a^45*b^21 + 32863270985072640*a
^47*b^19 - 5445156193763328*a^49*b^17 + 316457498640384*a^51*b^15 + 9146398
6446336*a^53*b^13 - 25165538721792*a^55*b^11 + 2461645209600*a^57*b^9 - 737
41860864*a^59*b^7) + root(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 1
1160261*a^20*b^4*d^6 + 11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720
087*a^12*b^12*d^6 + 531441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14
*b^6*d^4 - 155830311*a^12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^
10*d^4 + 688905*a^8*b^12*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2
+ 433755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(ro
ot(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 +
11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531
441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^
12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^1
2*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 -
 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(69657
034752*a^11*b^59 - 2855938424832*a^13*b^57 + 46200028299264*a^15*b^55 - 432
918470983680*a^17*b^53 + 2732993758494720*a^19*b^51 - 12560556506480640*a^2
1*b^49 + 43925900257198080*a^23*b^47 - 119837962587340800*a^25*b^45 + 25765
1619562782720*a^27*b^43 - 433619569038458880*a^29*b^41 + 549558392034263040
*a^31*b^39 - 452796847276032000*a^33*b^37 + 36223747782082560*a^35*b^35 + 6
41677817854033920*a^37*b^33 - 1337691257381191680*a^39*b^31 + 1759439177986
867200*a^41*b^29 - 1756851767431004160*a^43*b^27 + 1404659530591764480*a^45
*b^25 - 917046791277281280*a^47*b^23 + 491599995054981120*a^49*b^21 - 21579
6448806174720*a^51*b^19 + 76837281894236160*a^53*b^17 - 21824767985909760*a
^55*b^15 + 4817480523448320*a^57*b^13 - 793393625825280*a^59*b^11 + 9118105
8490368*a^61*b^9 - 6460689973248*a^63*b^7 + 208971104256*a^65*b^5) + root(1
8600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^6 - 11160261*a^20*b^4*d^6 + 1116
0261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 - 3720087*a^12*b^12*d^6 + 531441*
a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622*a^14*b^6*d^4 - 155830311*a^12*b
^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^10*b^10*d^4 + 688905*a^8*b^12*d^
4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6*d^2 + 433755*a^6*b^10*d^2 - 117
649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k)*(tan(c/2 + (d*x)/2)*(391820820
48*a^14*b^58 - 1057916215296*a^16*b^56 + 13752910798848*a^18*b^54 - 1146075
89990400*a^20*b^52 + 687645539942400*a^22*b^50 - 3163169483735040*a^24*b^48
 + 11598288107028480*a^26*b^46 - 34794864321085440*a^28*b^44 + 869871608027
13600*a^30*b^42 - 183639561694617600*a^32*b^40 + 330551211050311680*a^34*b^
38 - 510851871623208960*a^36*b^36 + 681135828830945280*a^38*b^34 - 78592595
6343398400*a^40*b^32 + 785925956343398400*a^42*b^30 - 681135828830945280*a^
44*b^28 + 510851871623208960*a^46*b^26 - 330551211050311680*a^48*b^24 + 183
639561694617600*a^50*b^22 - 86987160802713600*a^52*b^20 + 34794864321085440
*a^54*b^18 - 11598288107028480*a^56*b^16 + 3163169483735040*a^58*b^14 - 687
645539942400*a^60*b^12 + 114607589990400*a^62*b^10 - 13752910798848*a^64*b^
8 + 1057916215296*a^66*b^6 - 39182082048*a^68*b^4) + 156728328192*a^13*b^59
 - 4349211107328*a^15*b^57 + 58185391841280*a^17*b^55 - 499689092358144*a^1

9*b^53 + 3094404929740800*a^21*b^51 - 14715614554767360*a^23*b^49 + 5588266 0879319040*a^25*b^47 - 173974321605427200*a^27*b^45 + 452333236174110720*a^ 29*b^43 - 995519729186611200*a^31*b^41 + 1873123529285099520*a^33*b^39 - 30 35061119643770880*a^35*b^37 + 4257098930193408000*a^37*b^35 - 5187111311866 429440*a^39*b^33 + 5501481694403788800*a^41*b^31 - 5082321184353976320*a^43 *b^29 + 4086814972985671680*a^45*b^27 - 2854760459070873600*a^47*b^25 + 172 6211879929405440*a^49*b^23 - 898867328294707200*a^51*b^21 + 400140939692482 560*a^53*b^19 - 150777745391370240*a^55*b^17 + 47447542256025600*a^57*b^15 - 12240090610974720*a^59*b^13 + 2521366979788800*a^61*b^11 - 39883441316659 2*a^63*b^9 + 45490397257728*a^65*b^7 - 3330476974080*a^67*b^5 + 11754624614 4*a^69*b^3) + 8707129344*a^12*b^58 - 1332190789632*a^14*b^56 + 286812840591 36*a^16*b^54 - 311301641871360*a^18*b^52 + 2177740120227840*a^20*b^50 - 109 22397191700480*a^22*b^48 + 41634880384204800*a^24*b^46 - 125003771820195840 *a^26*b^44 + 302447666790973440*a^28*b^42 - 598319665965711360*a^30*b^40 + 975644030336532480*a^32*b^38 - 1314242849218682880*a^34*b^36 + 145541843767 2960000*a^36*b^34 - 1304054920154972160*a^38*b^32 + 908181105107927040*a^40 *b^30 - 436625531301888000*a^42*b^28 + 66949248132956160*a^44*b^26 + 118659 409094983680*a^46*b^24 - 149422959601090560*a^48*b^22 + 105921118310768640* a^50*b^20 - 54125344499466240*a^52*b^18 + 21015701527265280*a^54*b^16 - 623 6220178759680*a^56*b^14 + 1388221166960640*a^58*b^12 - 222162405212160*a^60 *b^10 + 23587613392896*a^62*b^8 - 1410554953728*a^64*b^6 + 30474952704*a^66 *b^4) - tan(c/2 + (d*x)/2)*(505980960768*a^12*b^56 - 28050984640512*a^14*b^ 54 + 435764251090944*a^16*b^52 - 3575718109347840*a^18*b^50 + 1873026485909 9136*a^20*b^48 - 67896173119315968*a^22*b^46 + 175151109969174528*a^24*b^44 - 313178493592682496*a^26*b^42 + 322543721316925440*a^28*b^40 + 8781790172 4942336*a^30*b^38 - 1141107740572336128*a^32*b^36 + 2683287241504063488*a^3 4*b^34 - 4099946394045874176*a^36*b^32 + 4680202272693534720*a^38*b^30 - 41 59807137221197824*a^40*b^28 + 2907691359083200512*a^42*b^26 - 1583635567837 888512*a^44*b^24 + 650291463103832064*a^46*b^22 - 184497987902054400*a^48*b ^20 + 25459845498372096*a^50*b^18 + 4948055537467392*a^52*b^16 - 3746991697 108992*a^54*b^14 + 988831432433664*a^56*b^12 - 136164991057920*a^58*b^10 + 8069573984256*a^60*b^8 + 13544423424*a^62*b^6) + 137379151872*a^11*b^57 - 4 254400143360*a^13*b^55 + 29689859874816*a^15*b^53 + 87020018122752*a^17*b^5 1 - 2614627107274752*a^19*b^49 + 20133104812498944*a^21*b^47 - 940057649259 72480*a^23*b^45 + 309275227789295616*a^25*b^43 - 759972938071523328*a^27*b^ 41 + 1428994663615807488*a^29*b^39 - 2057877923764617216*a^31*b^37 + 219990 8326418841600*a^33*b^35 - 1543980376177311744*a^35*b^33 + 26007819686269747 2*a^37*b^31 + 1033592707257090048*a^39*b^29 - 1728050263069556736*a^41*b^27 + 1665648670228807680*a^43*b^25 - 1148576443783962624*a^45*b^23 + 59309889 9751084032*a^47*b^21 - 228687912023703552*a^49*b^19 + 63216104157609984*a^5 1*b^17 - 11132817065533440*a^53*b^15 + 707704347303936*a^55*b^13 + 17592464 6019072*a^57*b^11 - 46657636319232*a^59*b^9 + 3600881713152*a^61*b^7) + 171 9926784*a^8*b^58 - 109860323328*a^10*b^56 + 2586984873984*a^12*b^54 - 35812 476739584*a^14*b^52 + 329722810195968*a^16*b^50 - 2157051013447680*a^18*b^4 8 + 10507597396918272*a^20*b^46 - 39457190948069376*a^22*b^44 + 11717768641 9562496*a^24*b^42 - 280405445559386112*a^26*b^40 + 547971334969098240*a^28* b^38 - 882457306853326848*a^30*b^36 + 1177391139070132224*a^32*b^34 - 13039 49437690281984*a^34*b^32 + 1196629258750230528*a^36*b^30 - 9044258529787084 80*a^38*b^28 + 556165530870792192*a^40*b^26 - 272082763494752256*a^42*b^24 + 101333478214434816*a^44*b^22 - 25813305663086592*a^46*b^20 + 275617165307 9040*a^48*b^18 + 957737252339712*a^50*b^16 - 557094927384576*a^52*b^14 + 13 5955536224256*a^54*b^12 - 17862568353792*a^56*b^10 + 1032386052096*a^58*b^8)) - 547736297472*a^10*b^52 + 5998567809024*a^12*b^50 - 42798845214720*a^14 *b^48 + 218837397897216*a^16*b^46 - 847734439845888*a^18*b^44 + 25781072509 25568*a^20*b^42 - 6304715180015616*a^22*b^40 + 12605115522908160*a^24*b^38 - 20839646107090944*a^26*b^36 + 28704537977536512*a^28*b^34 - 3308333250900 7872*a^30*b^32 + 31955047610056704*a^32*b^30 - 25837736359772160*a^34*b^28 + 17420116682981376*a^36*b^26 - 9723722502832128*a^38*b^24 + 44438937496289 28*a^40*b^22 - 1635506216902656*a^42*b^20 + 472961442078720*a^44*b^18 - 103

```
502089764864*a^46*b^16 + 16115525517312*a^48*b^14 - 1591065649152*a^50*b^12
 + 74879852544*a^52*b^10)*root(18600435*a^18*b^6*d^6 - 18600435*a^16*b^8*d^
6 - 11160261*a^20*b^4*d^6 + 11160261*a^14*b^10*d^6 + 3720087*a^22*b^2*d^6 -
 3720087*a^12*b^12*d^6 + 531441*a^10*b^14*d^6 - 531441*a^24*d^6 - 173879622
*a^14*b^6*d^4 - 155830311*a^12*b^8*d^4 - 23225940*a^16*b^4*d^4 - 6475707*a^
10*b^10*d^4 + 688905*a^8*b^12*d^4 - 11565585*a^8*b^8*d^2 + 3750705*a^10*b^6
*d^2 + 433755*a^6*b^10*d^2 - 117649*a^4*b^8 + 5488*a^2*b^10 - 64*b^12, d, k
), k, 1, 6)/d + ((2*(4*a^4*b + 3*b^5 + 38*a^2*b^3))/(3*(a^2 - b^2)*(a^4 + b
^4 - 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^8*(47*b^5 - 4*a^4*b + 62*a^2*b^3))
/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (4*tan(c/2 + (d*x)/2)^6*(119*b^5 -
 24*a^4*b + 220*a^2*b^3))/(3*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (6*tan(
c/2 + (d*x)/2)^2*(b^5 + 4*a^2*b^3))/((a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) -
 (100*tan(c/2 + (d*x)/2)^4*(b^5 + 2*a^2*b^3))/((a^2 - b^2)*(a^4 + b^4 - 2*a
^2*b^2)) + (6*tan(c/2 + (d*x)/2)^10*(b^5 + 4*a^2*b^3))/((a^2 - b^2)*(a^4 +
b^4 - 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)^11*(b^6 - 3*a^6 + 19*a^2*b^4 + 28
*a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2
)^9*(9*b^6 - 7*a^6 + 19*a^2*b^4 + 24*a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4
- 2*a^2*b^2)) + (4*tan(c/2 + (d*x)/2)^7*(3*a^6 + 17*b^6 + 179*a^2*b^4 + 26*
a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) + (2*tan(c/2 + (d*x)/2)
^3*(7*a^6 + 15*b^6 + 285*a^2*b^4 + 8*a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4
- 2*a^2*b^2)) - (4*tan(c/2 + (d*x)/2)^5*(19*b^6 - 3*a^6 + 277*a^2*b^4 + 22*
a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4 - 2*a^2*b^2)) - (2*tan(c/2 + (d*x)/2)
*(b^6 - 3*a^6 + 19*a^2*b^4 + 28*a^4*b^2))/(3*a*(a^2 - b^2)*(a^4 + b^4 - 2*a
^2*b^2)))/(d*(a - 3*a*tan(c/2 + (d*x)/2)^4 + 3*a*tan(c/2 + (d*x)/2)^8 - a*t
an(c/2 + (d*x)/2)^12 + 8*b*tan(c/2 + (d*x)/2)^3 - 24*b*tan(c/2 + (d*x)/2)^5
 + 24*b*tan(c/2 + (d*x)/2)^7 - 8*b*tan(c/2 + (d*x)/2)^9))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a+b*sin(d*x+c)**3)**2,x)
```

```
[Out] Timed out
```

3.404 $\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}+\frac{\sin^3(c+dx)}{3bd}-\frac{3\sin(c+dx)}{bd}$$

```
[Out] -3*sin(d*x+c)/b/d+1/3*sin(d*x+c)^3/b/d-1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/
4))*(a^(1/2)-b^(1/2))^3/a^(3/4)/b^(7/4)/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(
1/4))*(a^(1/2)+b^(1/2))^3/a^(3/4)/b^(7/4)/d
```

Rubi [A] time = 0.19, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3223, 1171, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}+\frac{\sin^3(c+dx)}{3bd}-\frac{3\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^3*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(7/4)*d) - ((Sqrt[a] - Sqrt[b])^3*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])
/(2*a^(3/4)*b^(7/4)*d) - (3*Sin[c + d*x])/(b*d) + Sin[c + d*x]^3/(3*b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^7(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^3}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{3}{b}+\frac{x^2}{b}+\frac{3a+b-(a+3b)x^2}{b\left(a-bx^4\right)}\right)dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}+\frac{\text{Subst}\left(\int \frac{3a+b+(-a-3b)x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd}$$

$$= -\frac{3\sin(c+dx)}{bd}+\frac{\sin^3(c+dx)}{3bd}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\,bd} -$$

$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{7/4}d}-\frac{3\sin(c+dx)}{bd}$$

Mathematica [C] time = 0.26, size = 207, normalized size = 1.58

$$\frac{4a^{3/4}b^{3/4}\sin^3(c+dx)-36a^{3/4}b^{3/4}\sin(c+dx)+3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)-3\left(\sqrt{a}-\sqrt{b}\right)^3\log\left(\right.}{12a^{3/}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (3*(Sqrt[a] - Sqrt[b])^3*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + (3*I)*(Sqrt[
a] + Sqrt[b])^3*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (3*I)*(Sqrt[a] + Sq
rt[b])^3*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - 3*(Sqrt[a] - Sqrt[b])^3*Lo
g[a^(1/4) + b^(1/4)*Sin[c + d*x]] - 36*a^(3/4)*b^(3/4)*Sin[c + d*x] + 4*a^(
3/4)*b^(3/4)*Sin[c + d*x]^3)/(12*a^(3/4)*b^(7/4)*d)
```

fricas [B] time = 0.74, size = 1429, normalized size = 10.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/12*(3*b*d*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b
^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)
/(a*b^3*d^2))*log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5
- b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) -
 (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3
*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*
b^5 + b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) - 3*b*d*s
qrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b
^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*
log(1/2*(a^6 + 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x
 + c) + 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 +
46*a^4*b^3 + 60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6
+ 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3
*b^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))) - 3*b*d*sqrt(-(a*b^3*d^2
```

```
*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5
+ b^6)/(a^3*b^7*d^4)) + 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6
+ 12*a^5*b - 27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*(
(a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3
+ 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - (3*a^5*b^2 + 46*a^4*b^3 +
60*a^3*b^4 + 18*a^2*b^5 + a*b^6)*d)*sqrt(-(a*b^3*d^2*sqrt((a^6 + 30*a^5*b +
 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) +
 6*a^2 + 20*a*b + 6*b^2)/(a*b^3*d^2))) + 3*b*d*sqrt((a*b^3*d^2*sqrt((a^6 +
30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b
^7*d^4)) - 6*a^2 - 20*a*b - 6*b^2)/(a*b^3*d^2))*log(-1/2*(a^6 + 12*a^5*b -
27*a^4*b^2 + 27*a^2*b^4 - 12*a*b^5 - b^6)*sin(d*x + c) + 1/2*((a^4*b^5 + 3*
a^3*b^6)*d^3*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 + 452*a^3*b^3 + 255*a^2*b^4
 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) + (3*a^5*b^2 + 46*a^4*b^3 + 60*a^3*b^4 +
18*a^2*b^5 + a*b^6)*d)*sqrt((a*b^3*d^2*sqrt((a^6 + 30*a^5*b + 255*a^4*b^2 +
 452*a^3*b^3 + 255*a^2*b^4 + 30*a*b^5 + b^6)/(a^3*b^7*d^4)) - 6*a^2 - 20*a*
b - 6*b^2)/(a*b^3*d^2))) - 4*(cos(d*x + c)^2 + 8)*sin(d*x + c))/(b*d)
```

giac [B] time = 0.92, size = 360, normalized size = 2.75

$$\frac{\frac{8\left(b^{2}\sin(dx+c)^{3}-9\,b^{2}\sin(dx+c)\right)}{b^{3}}-\frac{6\,\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{3}{4}}(a+3\,b)-\left(-ab^{3}\right)^{\frac{1}{4}}\left(3\,ab^{2}+b^{3}\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\,\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^{4}}-\frac{6\,\sqrt{2}\left(\left(-ab^{3}\right)^{\frac{3}{4}}(a+3\,b)-\left(-\ldots\right.\right.}{\ldots}}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/24*(8*(b^2*sin(d*x + c)^3 - 9*b^2*sin(d*x + c))/b^3 - 6*sqrt(2)*((-a*b^3)
^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(1/2*sqrt(2)*(sqrt
(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) - 6*sqrt(2)*((-a*b
^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^4) + 3*sqrt(2)*((
-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^
2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4) - 3*sqrt(2)*((-
a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2
 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^4))/d
```

maple [B] time = 0.63, size = 350, normalized size = 2.67

$$\frac{\sin^{3}(dx+c)}{3bd}-\frac{3\sin(dx+c)}{bd}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2db}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2da}+\frac{3\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4db}+\frac{\left(\frac{a}{b}\right.\ldots}{\ldots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/3*sin(d*x+c)^3/b/d-3*sin(d*x+c)/b/d+3/2/d/b*(a/b)^(1/4)*arctan(sin(d*x+c)
/(a/b)^(1/4))+1/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+3/4/d/b*(a
/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/4/d*(a/b)
^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/2/d/b^2/(a
/b)^(1/4)*a*arctan(sin(d*x+c)/(a/b)^(1/4))+3/2/d/b/(a/b)^(1/4)*arctan(sin(d
*x+c)/(a/b)^(1/4))-1/4/d/b^2/(a/b)^(1/4)*a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin
(d*x+c)-(a/b)^(1/4)))-3/4/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(
d*x+c)-(a/b)^(1/4)))
```

maxima [A] time = 0.64, size = 177, normalized size = 1.35

$$\frac{\frac{4\left(\sin(dx+c)^3-9\sin(dx+c)\right)}{b}+\frac{3\left(\frac{2\left(b\left(3\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(3\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{b}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/12*(4*(sin(d*x + c)^3 - 9*sin(d*x + c))/b + 3*(2*(b*(3*sqrt(a) + sqrt(b))
 + a^(3/2) + 3*a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b))
)/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(3*sqrt(a) - sqrt(b)) + a^(3
/2) - 3*a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt
(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*s
qrt(b)))/b)/d
```

mupad [B] time = 0.77, size = 1931, normalized size = 14.74

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*
b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))/(
16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*8i)/(92*a*b + (120*(a^3*b
^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)
^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5
 + (92*a*(a^3*b^7)^(1/2))/b^4) + (b^3*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b
^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^
6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2
)*8i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b +
(2*a^4)/b^2 + (36*(a^3*b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) +
(6*a^2*(a^3*b^7)^(1/2))/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4) + (a*b^2*sin(c +
d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) -
(15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) - (a^3*
b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b + (120*(a^3*b^7)^(1/2))/b^3 +
120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 + (36*(a^3*b^7)^(1/2))/(a*b^2) +
 (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b^7)^(1/2))/b^5 + (92*a*(a^3*b^7
)^(1/2))/b^4) + (a^2*b*sin(c + d*x)*(- (a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*
b^3) - 5/(4*b^2) - 3/(8*a*b) - (15*(a^3*b^7)^(1/2))/(16*a*b^6) - (15*(a^3*b
^7)^(1/2))/(16*a^2*b^5) - (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b
 + (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 +
 (36*(a^3*b^7)^(1/2))/(a*b^2) + (2*(a^3*b^7)^(1/2))/(a^2*b) + (6*a^2*(a^3*b
^7)^(1/2))/b^5 + (92*a*(a^3*b^7)^(1/2))/b^4))*(-(a^3*(a^3*b^7)^(1/2) + b^3*
(a^3*b^7)^(1/2) + 6*a^2*b^6 + 20*a^3*b^5 + 6*a^4*b^4 + 15*a*b^2*(a^3*b^7)^(
1/2) + 15*a^2*b*(a^3*b^7)^(1/2))/(16*a^3*b^7))^(1/2)*2i)/d - (3*sin(c + d*x
))/(b*d) + (atan((a^3*sin(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3
) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^3*b^7)
^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*8i)/(92*a*b - (1
20*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 - (36*
(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^3*b^7)^(
1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4) + (b^3*sin(c + d*x)*((a^3*b^7)^(1/2
)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(
16*a*b^6) + (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4
))^(1/2)*8i)/(92*a*b - (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^
```

```
3)/b + (2*a^4)/b^2 - (36*(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^
2*b) - (6*a^2*(a^3*b^7)^(1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4) + (a*b^2*s
in(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/(8*b^3) - 5/(4*b^2) - 3/(8*a*
b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^3*b^7)^(1/2))/(16*a^2*b^5) +
(a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*a*b - (120*(a^3*b^7)^(1/2))/b
^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^2 - (36*(a^3*b^7)^(1/2))/(a*b
^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^3*b^7)^(1/2))/b^5 - (92*a*(a^
3*b^7)^(1/2))/b^4) + (a^2*b*sin(c + d*x)*((a^3*b^7)^(1/2)/(16*b^7) - (3*a)/
(8*b^3) - 5/(4*b^2) - 3/(8*a*b) + (15*(a^3*b^7)^(1/2))/(16*a*b^6) + (15*(a^
3*b^7)^(1/2))/(16*a^2*b^5) + (a^3*b^7)^(1/2)/(16*a^3*b^4))^(1/2)*120i)/(92*
a*b - (120*(a^3*b^7)^(1/2))/b^3 + 120*a^2 + 6*b^2 + (36*a^3)/b + (2*a^4)/b^
2 - (36*(a^3*b^7)^(1/2))/(a*b^2) - (2*(a^3*b^7)^(1/2))/(a^2*b) - (6*a^2*(a^
3*b^7)^(1/2))/b^5 - (92*a*(a^3*b^7)^(1/2))/b^4))*((a^3*(a^3*b^7)^(1/2) + b^
3*(a^3*b^7)^(1/2) - 6*a^2*b^6 - 20*a^3*b^5 - 6*a^4*b^4 + 15*a*b^2*(a^3*b^7)
^(1/2) + 15*a^2*b*(a^3*b^7)^(1/2))/(16*a^3*b^7))^(1/2)*2i)/d + sin(c + d*x)
^3/(3*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.405 $\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=113

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

```
[Out] -sin(d*x+c)/b/d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^2/
a^(3/4)/b^(5/4)/d+1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a+b-2*a^(1/2)*b^
(1/2))/a^(3/4)/b^(5/4)/d
```

Rubi [A] time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3223, 1171, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])^2*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b
^(5/4)*d) + ((a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(
1/4)])/(2*a^(3/4)*b^(5/4)*d) - Sin[c + d*x]/(b*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b}+\frac{a+b-2bx^2}{b\left(a-bx^4\right)}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{\sin(c+dx)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b-2bx^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{bd}$$
$$= -\frac{\sin(c+dx)}{bd}-\frac{\left(2\sqrt{b}-\frac{a+b}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{b}\,d}-\frac{\left(2\sqrt{b}+\frac{a+b}{\sqrt{a}}\right)}{}$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/4}d}-\frac{\sin(c+dx}{bd}$$

Mathematica [C] time = 0.20, size = 189, normalized size = 1.67

$$\frac{-4a^{3/4}\sqrt[4]{b}\sin(c+dx)+\left(\sqrt{a}-\sqrt{b}\right)^2\left(-\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)\right)+i\left(-i\left(\sqrt{a}-\sqrt{b}\right)^2\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c\right.\right.}{4a^{3/4}b^{5/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-((Sqrt[a] - Sqrt[b])^2*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]]) + I*((Sqrt[a]
 + Sqrt[b])^2*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] + Sqrt[b])^2
*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] - Sqrt[b])^2*Log[a^(1/4
) + b^(1/4)*Sin[c + d*x]]) - 4*a^(3/4)*b^(1/4)*Sin[c + d*x])/(4*a^(3/4)*b^(
5/4)*d)
```

fricas [B] time = 0.62, size = 1041, normalized size = 9.21

$$-\frac{bd\sqrt{-\frac{ab^2d^2\sqrt{\frac{a^4+12\,a^3b+38\,a^2b^2+12\,ab^3+b^4}{a^3b^5d^4}}+4\,a+4\,b}{ab^2d^2}}\log\left(\frac{1}{2}\left(a^4+4\,a^3b-10\,a^2b^2+4\,ab^3+b^4\right)\sin\left(dx+c\right)+\frac{1}{2}\left(2\,a^3b^4d^3\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*(b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b
^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^
2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3
*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a
^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*
a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))) - b*d*sqrt((a*b^2*d^
2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a
- 4*b)/(a*b^2*d^2))*log(1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*si
n(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^
3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt((
a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)
) - 4*a - 4*b)/(a*b^2*d^2))) - b*d*sqrt(-(a*b^2*d^2*sqrt((a^4 + 12*a^3*b +
```

```
38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d^2))*log(-
1/2*(a^4 + 4*a^3*b - 10*a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*
b^4*d^3*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4))
- (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-(a*b^2*d^2*sqrt((a^4 + 1
2*a^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + 4*a + 4*b)/(a*b^2*d
^2))) + b*d*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12*a*b^3 +
b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))*log(-1/2*(a^4 + 4*a^3*b - 10*
a^2*b^2 + 4*a*b^3 + b^4)*sin(d*x + c) + 1/2*(2*a^3*b^4*d^3*sqrt((a^4 + 12*a
^3*b + 38*a^2*b^2 + 12*a*b^3 + b^4)/(a^3*b^5*d^4)) + (a^4*b + 7*a^3*b^2 + 7
*a^2*b^3 + a*b^4)*d)*sqrt((a*b^2*d^2*sqrt((a^4 + 12*a^3*b + 38*a^2*b^2 + 12
*a*b^3 + b^4)/(a^3*b^5*d^4)) - 4*a - 4*b)/(a*b^2*d^2))) + 4*sin(d*x + c))/(
b*d)
```

giac [B] time = 0.89, size = 311, normalized size = 2.75

$$-\frac{\frac{8\sin(dx+c)}{b} - \frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3} - \frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)-2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right) \cdots}{2\left(\cdots\right.}\right.}{ab^3}\cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(8*sin(d*x + c)/b - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)
^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(
1/4))/(a*b^3) - 2*sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) - 2*(-a*b^3)^(3/4))*a
rctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a
*b^3) - sqrt(2)*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3) + sqrt(2)
*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sqrt(
2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

maple [B] time = 0.61, size = 252, normalized size = 2.23

$$-\frac{\sin(dx+c)}{bd}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2db}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2da}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4db}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -sin(d*x+c)/b/d+1/2/d/b*(a/b)^(1/4)*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d*(a
/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/4/d/b*(a/b)^(1/4)*ln((sin(d*x+
c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c
)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/d/b/(a/b)^(1/4)*arctan(sin(d*x+c
)/(a/b)^(1/4))-1/2/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-
(a/b)^(1/4)))
```

maxima [A] time = 0.54, size = 158, normalized size = 1.40

$$\frac{\frac{\frac{2\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{b}-\frac{4\sin(dx+c)}{b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*((2*(b*(2*sqrt(a) + sqrt(b)) + a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/s
qrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*sqrt(
a) - sqrt(b)) - a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)
))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sq
rt(b))*sqrt(b)))/b - 4*sin(d*x + c)/b)/d
```

mupad [B] time = 15.78, size = 1097, normalized size = 9.71

$$\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{1}{4\,a\,b}-\frac{1}{4\,b^2}+\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}+\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{\frac{2\,\sqrt{a^3\,b^5}}{a^2}-24\,a\,b+\frac{14\,\sqrt{a^3\,b^5}}{b^2}-4\,a^2-4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}+\frac{48\,a\,b^2\,\sin(c+d\,x)\,\sqrt{\frac{\sqrt{a^3\,b^5}}{16\,a\,b^5}-\frac{1}{4\,a\,b}-\frac{1}{4\,b^2}+\frac{3\,\sqrt{a^3\,b^5}}{8\,a^2\,b^4}+\frac{\sqrt{a^3\,b^5}}{16\,a^3\,b^3}}}{\frac{2\,\sqrt{a^3\,b^5}}{a^2}-24\,a\,b+\frac{14\,\sqrt{a^3\,b^5}}{b^2}-4\,a^2-4\,b^2+\frac{14\,\sqrt{a^3\,b^5}}{a\,b}+\frac{2\,a\,\sqrt{a^3\,b^5}}{b^3}}+\frac{8\,a^2 \cdots}{\frac{2\,\sqrt{a^3 \cdots}}{a^2} \cdots}\right.}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (2*atanh((8*b^3*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4
*b^2) + (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/
2))/((2*(a^3*b^5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 -
4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (48*a*b^2
*sin(c + d*x)*((a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3*(a^3
*b^5)^(1/2))/(8*a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(a^3*b^
5)^(1/2))/a^2 - 24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 + (14*(a^
3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (8*a^2*b*sin(c + d*x)*((
a^3*b^5)^(1/2)/(16*a*b^5) - 1/(4*a*b) - 1/(4*b^2) + (3*(a^3*b^5)^(1/2))/(8*
a^2*b^4) + (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/((2*(a^3*b^5)^(1/2))/a^2 -
24*a*b + (14*(a^3*b^5)^(1/2))/b^2 - 4*a^2 - 4*b^2 + (14*(a^3*b^5)^(1/2))/(a
*b) + (2*a*(a^3*b^5)^(1/2))/b^3))*((a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/
2) - 4*a^2*b^4 - 4*a^3*b^3 + 6*a*b*(a^3*b^5)^(1/2))/(16*a^3*b^5))^(1/2))/d
- (2*atanh((8*b^3*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(
16*a*b^5) - (3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))
^(1/2))/(24*a*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^
2 + 4*b^2 + (14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (48*a
*b^2*sin(c + d*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(16*a*b^5) - (
3*(a^3*b^5)^(1/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/(24*a
*b + (2*(a^3*b^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^2 + 4*b^2 + (
14*(a^3*b^5)^(1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3) + (8*a^2*b*sin(c + d
*x)*(- 1/(4*b^2) - 1/(4*a*b) - (a^3*b^5)^(1/2)/(16*a*b^5) - (3*(a^3*b^5)^(1
/2))/(8*a^2*b^4) - (a^3*b^5)^(1/2)/(16*a^3*b^3))^(1/2))/(24*a*b + (2*(a^3*b
^5)^(1/2))/a^2 + (14*(a^3*b^5)^(1/2))/b^2 + 4*a^2 + 4*b^2 + (14*(a^3*b^5)^(
1/2))/(a*b) + (2*a*(a^3*b^5)^(1/2))/b^3))*(-(a^2*(a^3*b^5)^(1/2) + b^2*(a^3
*b^5)^(1/2) + 4*a^2*b^4 + 4*a^3*b^3 + 6*a*b*(a^3*b^5)^(1/2))/(16*a^3*b^5))^
(1/2))/d - sin(c + d*x)/(b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.406 $\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=95

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

```
[Out] -1/2*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))/a^(3/4)/b^(3/4)/
d+1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))/a^(3/4)/b^(3/4)/
d
```

Rubi [A] time = 0.10, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3223, 1167, 205, 208}

$$\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}-\frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] + Sqrt[b])*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(
3/4)*d) - ((Sqrt[a] - Sqrt[b])*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^(3/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1-x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= -\frac{\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2d} - \frac{\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}-bx^2}\,d\right.}{2d}$$
$$= \frac{\left(\sqrt{a}+\sqrt{b}\right)\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d} - \frac{\left(\sqrt{a}-\sqrt{b}\right)\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/4}d}$$

Mathematica [C] time = 0.08, size = 160, normalized size = 1.68

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)+i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)-i\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}+\right.}{4a^{3/4}b^{3/4}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - b^(1/4)*Sin[c + d*x]] + I*(Sqrt[a] + Sqr
t[b])*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]] - I*(Sqrt[a] + Sqrt[b])*Log[a^(
1/4) + I*b^(1/4)*Sin[c + d*x]] - (Sqrt[a] - Sqrt[b])*Log[a^(1/4) + b^(1/4)*
Sin[c + d*x]])/(4*a^(3/4)*b^(3/4)*d)
```

fricas [B] time = 0.56, size = 631, normalized size = 6.64

$$\frac{1}{4}\sqrt{-\frac{abd^2\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}+2}{abd^2}}\log\left(\frac{1}{2}\left(a^2-b^2\right)\sin\left(dx+c\right)+\frac{1}{2}\left(a^3b^2d^3\sqrt{\frac{a^2+2ab+b^2}{a^3b^3d^4}}-\left(a^2b+ab^2\right)d\right)\sqrt{-\frac{abd}{}}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/4*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*
log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2
)/(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^
2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) - 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b +
b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*
(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*s
qrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))) - 1/4
*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + 2)/(a*b*d^2))*log
(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/
(a^3*b^3*d^4)) - (a^2*b + a*b^2)*d)*sqrt(-(a*b*d^2*sqrt((a^2 + 2*a*b + b^2)
/(a^3*b^3*d^4)) + 2)/(a*b*d^2))) + 1/4*sqrt((a*b*d^2*sqrt((a^2 + 2*a*b + b^
2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2))*log(-1/2*(a^2 - b^2)*sin(d*x + c) + 1/2*(
a^3*b^2*d^3*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) + (a^2*b + a*b^2)*d)*sq
rt((a*b*d^2*sqrt((a^2 + 2*a*b + b^2)/(a^3*b^3*d^4)) - 2)/(a*b*d^2)))
```

giac [B] time = 0.77, size = 280, normalized size = 2.95

$$\frac{\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{2\sqrt{2}\left(\left(-ab^3\right)^{\frac{1}{4}}b^2-\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab^3}+\frac{\sqrt{2}\left((-ab\right.}{}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*((-a*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sq
rt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + 2*sqrt(2)*((-a
*b^3)^(1/4)*b^2 - (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4)
 - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b^3) + sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-
a*b^3)^(3/4))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt
(-a/b))/(a*b^3) - sqrt(2)*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*log(sin(d*x
 + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b^3))/d
```

maple [B] time = 0.61, size = 160, normalized size = 1.68

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4da}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2da}+\frac{\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2db\left(\frac{a}{b}\right)^{\frac{1}{4}}}-\frac{\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4db\left(\frac{a}{b}\right)^{\frac{1}{4}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1
/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d/b/(a/b)^(1/4)*arcta
n(sin(d*x+c)/(a/b)^(1/4))-1/4/d/b/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(
sin(d*x+c)-(a/b)^(1/4)))
```

maxima [A] time = 0.54, size = 121, normalized size = 1.27

$$\frac{\frac{2\left(\sqrt{a}+\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}-\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(2*(sqrt(a) + sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)
))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) - sqrt(b))*log((sqrt(
b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(
a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/d
```

mupad [B] time = 15.81, size = 489, normalized size = 5.15

$$-\frac{2\,\mathrm{atanh}\left(\frac{8\,b^3\,\sin(c+d\,x)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}+\frac{8\,a\,b^2\,\sin(c+d\,x)\,\sqrt{-\frac{1}{8\,a\,b}-\frac{\sqrt{a^3\,b^3}}{16\,a^2\,b^3}-\frac{\sqrt{a^3\,b^3}}{16\,a^3\,b^2}}}{2\,a\,b+\frac{2\,\sqrt{a^3\,b^3}}{a}+2\,b^2+\frac{2\,b\,\sqrt{a^3\,b^3}}{a^2}}\right)\sqrt{-\frac{a\,\sqrt{a^3\,b^3}+b\,\sqrt{a^3\,b^3}+2\,a^2\,b^2}{16\,a^3\,b^3}}}{d}-\frac{2\,\mathrm{ata}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3/(a - b*sin(c + d*x)^4),x)
```

```
[Out] - (2*atanh((8*b^3*sin(c + d*x)*(- 1/(8*a*b) - (a^3*b^3)^(1/2)/(16*a^2*b^3)
- (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b + (2*(a^3*b^3)^(1/2))/a + 2*b
^2 + (2*b*(a^3*b^3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*(- 1/(8*a*b) - (a^3
*b^3)^(1/2)/(16*a^2*b^3) - (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b + (2
*(a^3*b^3)^(1/2))/a + 2*b^2 + (2*b*(a^3*b^3)^(1/2))/a^2))*(-(a*(a^3*b^3)^(1
/2) + b*(a^3*b^3)^(1/2) + 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d - (2*atanh((8*b
```

```
^3*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^3) - 1/(8*a*b) + (a^3*b^3)^(1/2)
/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3)^(1/2))/a + 2*b^2 - (2*b*(a^3*b^
3)^(1/2))/a^2) + (8*a*b^2*sin(c + d*x)*((a^3*b^3)^(1/2)/(16*a^2*b^3) - 1/(8
*a*b) + (a^3*b^3)^(1/2)/(16*a^3*b^2))^(1/2))/(2*a*b - (2*(a^3*b^3)^(1/2))/a
 + 2*b^2 - (2*b*(a^3*b^3)^(1/2))/a^2))*((a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1
/2) - 2*a^2*b^2)/(16*a^3*b^3))^(1/2))/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.407 $\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

```
[Out] 1/2*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d+1/2*arctanh(b^(1/4
)*sin(d*x+c)/a^(1/4))/a^(3/4)/b^(1/4)/d
```

Rubi [A] time = 0.07, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3223, 212, 208, 205}

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}+\frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d) + ArcTanh[(b^(
1/4)*Sin[c + d*x])/a^(1/4)]/(2*a^(3/4)*b^(1/4)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^4)^(-1), x_Symbol] :> With[{r = Numerator[Rt[-(a/b),
 2]], s = Denominator[Rt[-(a/b), 2]]}, Dist[r/(2*a), Int[1/(r - s*x^2), x],
 x] + Dist[r/(2*a), Int[1/(r + s*x^2), x], x]] /; FreeQ[{a, b}, x] &&  !GtQ
[a/b, 0]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\cos(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}-\sqrt{b}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\,d} + \frac{\text{Subst}\left(\int \frac{1}{\sqrt{a}+\sqrt{b}x^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\,d}$$
$$= \frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d} + \frac{\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Mathematica [A] time = 0.02, size = 54, normalized size = 0.76

$$\frac{\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right) + \tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt[4]{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)] + ArcTanh[(b^(1/4)*Sin[c + d*x])/a^
(1/4)])/(2*a^(3/4)*b^(1/4)*d)
```

fricas [B] time = 110.36, size = 330, normalized size = 4.65

$$\frac{1}{2}\left(\frac{1}{a^3bd^4}\right)^{\frac{1}{4}}\arctan\left(a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\sin(dx+c) + \sqrt{a^2d^2\sqrt{\frac{1}{a^3bd^4}} - \cos(dx+c)^2 + 1}\,a^2bd^3\left(\frac{1}{a^3bd^4}\right)^{\frac{3}{4}}\right) - \frac{1}{2}\left(\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/2*(1/(a^3*b*d^4))^(1/4)*arctan(a^2*b*d^3*(1/(a^3*b*d^4))^(3/4)*sin(d*x +
c) + sqrt(a^2*d^2*sqrt(1/(a^3*b*d^4)) - cos(d*x + c)^2 + 1)*a^2*b*d^3*(1/(a
^3*b*d^4))^(3/4)) - 1/2*(1/(a^3*b*d^4))^(1/4)*arctan(-a^2*b*d^3*(1/(a^3*b*d
^4))^(3/4)*sin(d*x + c) + sqrt(a^2*d^2*sqrt(1/(a^3*b*d^4)) - cos(d*x + c)^2
 + 1)*a^2*b*d^3*(1/(a^3*b*d^4))^(3/4)) + 1/8*(1/(a^3*b*d^4))^(1/4)*log(1/4*
a^2*d^2*sqrt(1/(a^3*b*d^4)) + 1/2*a*d*(1/(a^3*b*d^4))^(1/4)*sin(d*x + c) -
1/4*cos(d*x + c)^2 + 1/4) - 1/8*(1/(a^3*b*d^4))^(1/4)*log(1/4*a^2*d^2*sqrt(
1/(a^3*b*d^4)) - 1/2*a*d*(1/(a^3*b*d^4))^(1/4)*sin(d*x + c) - 1/4*cos(d*x +
 c)^2 + 1/4)
```

giac [B] time = 0.75, size = 224, normalized size = 3.15

$$\frac{\frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{2\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{ab} + \frac{\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}\log\left(\sin(dx+c)^2+\sqrt{2}\left(-\frac{a}{b}\right)\right.}{ab}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(2*sqrt(2)*(-a*b^3)^(1/4)*arctan(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*
sin(d*x + c))/(-a/b)^(1/4))/(a*b) + 2*sqrt(2)*(-a*b^3)^(1/4)*arctan(-1/2*sq
```

```
rt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(a*b) + sqrt(2)
*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sq
rt(-a/b))/(a*b) - sqrt(2)*(-a*b^3)^(1/4)*log(sin(d*x + c)^2 - sqrt(2)*(-a/b
)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a*b))/d
```

maple [A] time = 0.23, size = 81, normalized size = 1.14

$$\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4da}+\frac{\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/4/d*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))+1
/2/d*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))
```

maxima [A] time = 0.65, size = 100, normalized size = 1.41

$$\frac{\frac{2\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}-\frac{\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(2*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqr
t(a)*sqrt(b))) - log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b
)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))))/d
```

mupad [B] time = 0.11, size = 40, normalized size = 0.56

$$\frac{\mathrm{atan}\left(\frac{b^{1/4}\sin(c+d\,x)}{a^{1/4}}\right)+\mathrm{atanh}\left(\frac{b^{1/4}\sin(c+d\,x)}{a^{1/4}}\right)}{2\,a^{3/4}\,b^{1/4}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((b^(1/4)*sin(c + d*x))/a^(1/4)) + atanh((b^(1/4)*sin(c + d*x))/a^(1/4
)))/(2*a^(3/4)*b^(1/4)*d)
```

sympy [A] time = 8.90, size = 155, normalized size = 2.18

$$\begin{cases} \frac{\tilde{\infty}x\cos(c)}{\sin^4(c)} & \text{for } a=0\wedge b=0\wedge d=0 \\ \frac{1}{3bd\sin^3(c+dx)} & \text{for } a=0 \\ \frac{\sin(c+dx)}{ad} & \text{for } b=0 \\ \frac{x\cos(c)}{a-b\sin^4(c)} & \text{for } d=0 \\ -\frac{\sqrt[4]{\frac{1}{b}}\log\left(-\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sin(c+dx)\right)}{4a^{\frac{3}{4}}d}+\frac{\sqrt[4]{\frac{1}{b}}\log\left(\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}+\sin(c+dx)\right)}{4a^{\frac{3}{4}}d}+\frac{\sqrt[4]{\frac{1}{b}}\operatorname{atan}\left(\frac{\sin(c+dx)}{\sqrt[4]{a}\sqrt[4]{\frac{1}{b}}}\right)}{2a^{\frac{3}{4}}d} & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Piecewise((zoo*x*cos(c)/sin(c)**4, Eq(a, 0) & Eq(b, 0) & Eq(d, 0)), (1/(3*b
*d*sin(c + d*x)**3), Eq(a, 0)), (sin(c + d*x)/(a*d), Eq(b, 0)), (x*cos(c)/(
a - b*sin(c)**4), Eq(d, 0)), (-(1/b)**(1/4)*log(-a**(1/4)*(1/b)**(1/4) + si
n(c + d*x))/(4*a**(3/4)*d) + (1/b)**(1/4)*log(a**(1/4)*(1/b)**(1/4) + sin(c
 + d*x))/(4*a**(3/4)*d) + (1/b)**(1/4)*atan(sin(c + d*x)/(a**(1/4)*(1/b)**(
1/4)))/(2*a**(3/4)*d), True))
```

3.408 $\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{d(a-b)}$$

```
[Out] arctanh(sin(d*x+c))/(a-b)/d-1/2*b^(1/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))
/a^(3/4)/d/(a^(1/2)-b^(1/2))+1/2*b^(1/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))
/a^(3/4)/d/(a^(1/2)+b^(1/2))
```

Rubi [A] time = 0.15, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 22, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.273, Rules used = {3223, 1171, 207, 1167, 205, 208}

$$\frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)}-\frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)}+\frac{\tanh^{-1}(\sin(c+dx))}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(1/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])*d) + ArcTanh[Sin[c + d*x]]/((a - b)*d) - (b^(1/4)*ArcTanh[(b^(1/4)*Sin
[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a-b)(-1+x^2)} - \frac{b(1+x^2)}{(a-b)(a-bx^4)}\right) dx, x, \sin(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx, x, \sin(c+dx)\right)}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1+x^2}{a-bx^4}\,dx, x, \sin(c+dx)\right)}{(a-b)d}$$

$$= \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt{a}\sqrt{b}-bx^2}\,dx, x, \sin(c+dx)\right)}{2\sqrt{a}\left(\sqrt{a}-\sqrt{b}\right)d} - \frac{b\,\text{Subst}\left(\int \frac{1}{-\sqrt{a}\sqrt{b}\ldots}\right.}{2\sqrt{a}\left(\ldots\right.}$$

$$= \frac{\sqrt[4]{b}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)d} + \frac{\tanh^{-1}(\sin(c+dx))}{(a-b)d} - \frac{\sqrt[4]{b}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)d}$$

Mathematica [C] time = 0.18, size = 184, normalized size = 1.57

$$\frac{4a^{3/4}\tanh^{-1}(\sin(c+dx)) + \sqrt[4]{b}\left(\left(\sqrt{a}+\sqrt{b}\right)\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right) + i\left(\left(\sqrt{a}-\sqrt{b}\right)\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c\ldots\right.\right.\right.}{4a^{3/4}d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (4*a^(3/4)*ArcTanh[Sin[c + d*x]] + b^(1/4)*((Sqrt[a] + Sqrt[b])*Log[a^(1/4)
 - b^(1/4)*Sin[c + d*x]] + I*((Sqrt[a] - Sqrt[b])*Log[a^(1/4) - I*b^(1/4)*S
in[c + d*x]] + (-Sqrt[a] + Sqrt[b])*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]] +
 I*(Sqrt[a] + Sqrt[b])*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])))/(4*a^(3/4)*(a
 - b)*d)
```

fricas [B] time = 0.68, size = 1329, normalized size = 11.36

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a - b)*d*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^
4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2
 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a
*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*
b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqr
t(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*
```

```
b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*
d^2))*log(1/2*(a*b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3
*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3
*b^4)*d^4)) + (a^2*b + a*b^2)*d)*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a
^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^
4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))) - (a - b)*d*sqrt(((a^3 - 2*a^2*b
 + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*
a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*
b + b^2)*sin(d*x + c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*
a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - (a^
2*b + a*b^2)*d)*sqrt(((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b
^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + 2*b)/((a^3 -
 2*a^2*b + a*b^2)*d^2))) + (a - b)*d*sqrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqr
t((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4
)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2)*d^2))*log(-1/2*(a*b + b^2)*sin(d*x
+ c) + 1/2*((a^5 - 2*a^4*b + a^3*b^2)*d^3*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^
7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) + (a^2*b + a*b^2)*d)*s
qrt(-((a^3 - 2*a^2*b + a*b^2)*d^2*sqrt((a^2*b + 2*a*b^2 + b^3)/((a^7 - 4*a^
6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^4)) - 2*b)/((a^3 - 2*a^2*b + a*b^2
)*d^2))) - 2*log(sin(d*x + c) + 1) + 2*log(-sin(d*x + c) + 1))/((a - b)*d)
```

giac [B] time = 0.79, size = 370, normalized size = 3.16

$$-\frac{\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2-\sqrt{2}\,ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}b^2+\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^2b^2-\sqrt{2}\,ab^3}+\frac{\left(\sqrt{2}\left(-ab^3\right)^{\frac{1}{4}}b^2-\sqrt{}\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/8*(4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + 4*((-a*b^3)^(1/4)*b^2 + (-a*b^3)^(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(
-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^2*b^2 - sqrt(2)*a*b^
3) + (sqrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x + c)
^2 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3) - (s
qrt(2)*(-a*b^3)^(1/4)*b^2 - sqrt(2)*(-a*b^3)^(3/4))*log(sin(d*x + c)^2 - sq
rt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^2*b^2 - a*b^3) - 4*log(abs
(sin(d*x + c) + 1))/(a - b) + 4*log(abs(sin(d*x + c) - 1))/(a - b))/d
```

maple [B] time = 0.61, size = 229, normalized size = 1.96

$$-\frac{\ln\left(\sin\left(dx+c\right)-1\right)}{d\left(2a-2b\right)}+\frac{\ln\left(1+\sin\left(dx+c\right)\right)}{d\left(2a-2b\right)}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\ln\left(\frac{\sin(dx+c)+\left(\frac{a}{b}\right)^{\frac{1}{4}}}{\sin(dx+c)-\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{4d\left(a-b\right)a}-\frac{b\left(\frac{a}{b}\right)^{\frac{1}{4}}\arctan\left(\frac{\sin(dx+c)}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{2d\left(a-b\right)a}+\frac{\arctan\left(\frac{\sin(dx+c}{\left(\frac{a}{b}\right)^{\frac{1}{4}}}\right.}{2d\left(a-b\right)\left(\frac{a}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/(2*a-2*b)*ln(sin(d*x+c)-1)+1/d/(2*a-2*b)*ln(1+sin(d*x+c))-1/4/d*b/(a-b
)*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))-1/2/d
*b/(a-b)*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a/b)^(1/4))+1/2/d/(a-b)/(a/b)^(1/
4)*arctan(sin(d*x+c)/(a/b)^(1/4))-1/4/d/(a-b)/(a/b)^(1/4)*ln((sin(d*x+c)+(a
/b)^(1/4))/(sin(d*x+c)-(a/b)^(1/4)))
```

maxima [A] time = 0.61, size = 167, normalized size = 1.43

$$\frac{\frac{b\left(\frac{2\left(\sqrt{a}-\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin(dx+c)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(\sqrt{a}+\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin(dx+c)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin(dx+c)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a-b}+\frac{2\log(\sin(dx+c)+1)}{a-b}-\frac{2\log(\sin(dx+c)-1)}{a-b}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/4*(b*(2*(sqrt(a) - sqrt(b))*arctan(sqrt(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt
(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (sqrt(a) + sqrt(b))*log((sq
rt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sq
rt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a - b) + 2*log(s
in(d*x + c) + 1)/(a - b) - 2*log(sin(d*x + c) - 1)/(a - b))/d
```

mupad [B] time = 17.91, size = 3891, normalized size = 33.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((b^5*sin(c + d*x)*3i + ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128
*a^4*b^4 - (sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5
*b^4))/(4*(a - b)))/(2*(a - b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^
2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b) + (b^5*
sin(c + d*x)*3i - ((((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 + (
sin(c + d*x)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a
 - b)))/(2*(a - b)) - (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(
2*(a - b)) - 10*a*b^5 + 2*b^6)*1i)/(2*(a - b)))/(a - b))/((3*b^5*sin(c + d*
x) + (((32*a*b^7 + 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 - (sin(c + d*x)*(
512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b)))/(2*(a
- b)) + (sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(a - b)) - 1
0*a*b^5 + 2*b^6)/(2*(a - b)))/(a - b) - (3*b^5*sin(c + d*x) - (((32*a*b^7 +
 64*a^2*b^6 - 224*a^3*b^5 + 128*a^4*b^4 + (sin(c + d*x)*(512*a^2*b^7 - 512*
a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4))/(4*(a - b)))/(2*(a - b)) - (sin(c + d
*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))/2)/(2*(a - b)) - 10*a*b^5 + 2*b^6)/(
2*(a - b)))/(a - b)))*1i)/(d*(a - b)) - (atan((((((((2*a^2*b + a*(a^3*b)^(1/
2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128
*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(
1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7
- 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c + d*x)*(32*a*b^6 - 16*b
^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5
- 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2
) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 6*b^5*sin(c +
d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a
^3*b^2)))^(1/2)*1i - (((((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*
(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 2
56*a^4*b^4 - sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(1
6*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^
5 + 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2
*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/
2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(
a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b
)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*1i)/((((((
2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2))
```

```
)^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*
((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2
)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c
+ d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(
a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2
*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))
^(1/2) - 6*b^5*sin(c + d*x))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))
/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + (((((2*a^2*b + a*(a^3*b)^(1/2) + b
*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b
^6 - 448*a^3*b^5 + 256*a^4*b^4 - sin(c + d*x)*((2*a^2*b + a*(a^3*b)^(1/2) +
 b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*
a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 2
40*a^2*b^5))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^
4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*((2*a^2*b + a*(a^3*b)^(1/2) + b*
(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c + d*x))*
((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2
)))^(1/2)))*((2*a^2*b + a*(a^3*b)^(1/2) + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4
*b + a^3*b^2)))^(1/2)*2i)/d - (atan((((((-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a
^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6
- 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b
*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^
3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240
*a^2*b^5))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4
*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*
(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 6*b^5*sin(c + d*x))*
(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^
2)))^(1/2)*1i - ((((-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5
 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a
^4*b^4 - sin(c + d*x)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(
a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 +
 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*(-(a*(a^3*
b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)
 - 20*a*b^5 + 4*b^6)*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a
^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c + d*x))*(-(a*(a^3*b)^(1/2) -
2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*1i)/(((((-
(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)
))^(1/2)*(64*a*b^7 + 128*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 + sin(c + d*x)
*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b
^2)))^(1/2)*(512*a^2*b^7 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) - sin(
c + d*x)*(32*a*b^6 - 16*b^7 + 240*a^2*b^5))*(-(a*(a^3*b)^(1/2) - 2*a^2*b +
b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*
(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^
2)))^(1/2) - 6*b^5*sin(c + d*x))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(
1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + ((((-(a*(a^3*b)^(1/2) - 2*a^2
*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(64*a*b^7 + 128
*a^2*b^6 - 448*a^3*b^5 + 256*a^4*b^4 - sin(c + d*x)*(-(a*(a^3*b)^(1/2) - 2*
a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*(512*a^2*b^7
 - 512*a^3*b^6 - 512*a^4*b^5 + 512*a^5*b^4)) + sin(c + d*x)*(32*a*b^6 - 16*
b^7 + 240*a^2*b^5))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^
5 - 2*a^4*b + a^3*b^2)))^(1/2) - 20*a*b^5 + 4*b^6)*(-(a*(a^3*b)^(1/2) - 2*a
^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b + a^3*b^2)))^(1/2) + 6*b^5*sin(c
 + d*x))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(a^5 - 2*a^4*b
 + a^3*b^2)))^(1/2)))*(-(a*(a^3*b)^(1/2) - 2*a^2*b + b*(a^3*b)^(1/2))/(16*(
a^5 - 2*a^4*b + a^3*b^2)))^(1/2)*2i)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.409 $\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{1}{4d(a-b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}+\frac{(a-}{}$$

```
[Out] 1/2*(a-5*b)*arctanh(sin(d*x+c))/(a-b)^2/d+1/4/(a-b)/d/(1-sin(d*x+c))-1/4/(a
-b)/d/(1+sin(d*x+c))+1/2*b^(3/4)*arctanh(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3/4
)/d/(a^(1/2)-b^(1/2))^2+1/2*b^(3/4)*arctan(b^(1/4)*sin(d*x+c)/a^(1/4))/a^(3
/4)/d/(a^(1/2)+b^(1/2))^2
```

Rubi [A] time = 0.21, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.250$, Rules used = $\{3223, 1171, 207, 1167, 205, 208\}$

$$\frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{1}{4d(a-b)(1-\sin(c+dx))}-\frac{1}{4d(a-b)(\sin(c+dx)+1)}+\frac{(a-}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(3/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^2*d) + ((a - 5*b)*ArcTanh[Sin[c + d*x]])/(2*(a - b)^2*d) + (b^(3/4)*Ar
cTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^2*d)
+ 1/(4*(a - b)*d*(1 - Sin[c + d*x])) - 1/(4*(a - b)*d*(1 + Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^3(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x^2)^2(a-bx^4)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{4(a-b)(-1+x)^2}+\frac{1}{4(a-b)(1+x)^2}+\frac{-a+5b}{2(a-b)^2(-1+x^2)}+\frac{b(a+b+2bx^2)}{(a-b)^2(a-bx^4)}\right) dx, x, \sin(c+d$$

$$= \frac{1}{4(a-b)d(1-\sin(c+dx))}-\frac{1}{4(a-b)d(1+\sin(c+dx))}-\frac{(a-5b)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,d\right.}{2(a-b)^2}$$

$$= \frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d}+\frac{1}{4(a-b)d(1-\sin(c+dx))}-\frac{1}{4(a-b)d(1+\sin(c+$$

$$= \frac{b^{3/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2d}+\frac{(a-5b)\tanh^{-1}(\sin(c+dx))}{2(a-b)^2d}+\frac{b^{3/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2d}$$

Mathematica [C] time = 1.02, size = 255, normalized size = 1.46

$$-\frac{\frac{b^{3/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2}-\frac{ib^{3/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2}+\frac{ib^{3/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^2}-\frac{b^{3/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^2}+\frac{}{(a-b)(\sin}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/4*((-2*(a - 5*b)*ArcTanh[Sin[c + d*x]])/(a - b)^2 + (b^(3/4)*Log[a^(1/4)
 - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^2) - (I*b^(3/4)*Log[
a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2) + (I*b^(
3/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[b])^2)
 - (b^(3/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b
])^2) + 1/((a - b)*(-1 + Sin[c + d*x])) + 1/((a - b)*(1 + Sin[c + d*x])))/d
```

fricas [B] time = 1.23, size = 2529, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/4*((a^2 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 +
a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11
 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^
6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b
```

```
^2 - 4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b
^4)*sin(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)
*d^3*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a
^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*
a^4*b^7 + a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(
((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3
*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8
*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) +
4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) -
(a^2 - 2*a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4
)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*
a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8
*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 -
4*a^2*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(1/2*(a^2*b^2 + 6*a*b^3 + b^4)*s
in(d*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*
sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b
 + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b
^7 + a^3*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^
5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4
 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3
 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*
b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a^2
 - 2*a*b + b^2)*d*sqrt(((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2
*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*
b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*
b^7 + a^3*b^8)*d^4)) + 4*a*b^2 + 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2
*b^3 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d
*x + c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt
((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 2
8*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 +
 a^3*b^8)*d^4)) - (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(((a^5 - 4
*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38
*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70
*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) + 4*a*b^2 +
 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) + (a^2 - 2*
a*b + b^2)*d*sqrt(-((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqr
t((a^4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b +
28*a^9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7
+ a^3*b^8)*d^4)) - 4*a*b^2 - 4*b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3
 + a*b^4)*d^2))*cos(d*x + c)^2*log(-1/2*(a^2*b^2 + 6*a*b^3 + b^4)*sin(d*x +
 c) + 1/2*(2*(a^7 - 4*a^6*b + 6*a^5*b^2 - 4*a^4*b^3 + a^3*b^4)*d^3*sqrt((a^
4*b^3 + 12*a^3*b^4 + 38*a^2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^
9*b^2 - 56*a^8*b^3 + 70*a^7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3
*b^8)*d^4)) + (a^4*b + 7*a^3*b^2 + 7*a^2*b^3 + a*b^4)*d)*sqrt(-((a^5 - 4*a^
4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2*sqrt((a^4*b^3 + 12*a^3*b^4 + 38*a^
2*b^5 + 12*a*b^6 + b^7)/((a^11 - 8*a^10*b + 28*a^9*b^2 - 56*a^8*b^3 + 70*a^
7*b^4 - 56*a^6*b^5 + 28*a^5*b^6 - 8*a^4*b^7 + a^3*b^8)*d^4)) - 4*a*b^2 - 4*
b^3)/((a^5 - 4*a^4*b + 6*a^3*b^2 - 4*a^2*b^3 + a*b^4)*d^2))) - (a - 5*b)*co
s(d*x + c)^2*log(sin(d*x + c) + 1) + (a - 5*b)*cos(d*x + c)^2*log(-sin(d*x
+ c) + 1) - 2*(a - b)*sin(d*x + c))/((a^2 - 2*a*b + b^2)*d*cos(d*x + c)^2)
```

giac [B] time = 0.80, size = 475, normalized size = 2.71

$$\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^3b-2\sqrt{2}\,a^2b^2+\sqrt{2}\,ab^3}+\frac{4\left(\left(-ab^3\right)^{\frac{1}{4}}\left(ab+b^2\right)+2\left(-ab^3\right)^{\frac{3}{4}}\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}-2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}\,a^3b-2\sqrt{2}\,a^2b^2+\sqrt{2}\,ab^3}-\frac{\left(2\sqrt{2}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^(3/4))*arctan(1/2*sqrt(2)*(
sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^3*b - 2*sqr
t(2)*a^2*b^2 + sqrt(2)*a*b^3) + 4*((-a*b^3)^(1/4)*(a*b + b^2) + 2*(-a*b^3)^
(3/4))*arctan(-1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(
1/4))/(sqrt(2)*a^3*b - 2*sqrt(2)*a^2*b^2 + sqrt(2)*a*b^3) - (2*sqrt(2)*(-a*
b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqrt(2)*b^2))*log(sin(d*x + c)^2
 + sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^3*b - 2*a^2*b^2 + a*b
^3) + (2*sqrt(2)*(-a*b^3)^(3/4) - (-a*b^3)^(1/4)*(sqrt(2)*a*b + sqrt(2)*b^2
))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) + sqrt(-a/b))/(a^
3*b - 2*a^2*b^2 + a*b^3) + 2*(a - 5*b)*log(abs(sin(d*x + c) + 1))/(a^2 - 2*
a*b + b^2) - 2*(a - 5*b)*log(abs(sin(d*x + c) - 1))/(a^2 - 2*a*b + b^2) - 4
*sin(d*x + c)/((sin(d*x + c)^2 - 1)*(a - b)))/d
```

maple [B] time = 0.70, size = 415, normalized size = 2.37

$$-\frac{1}{d\left(4a-4b\right)\left(\sin\left(dx+c\right)-1\right)}-\frac{\ln\left(\sin\left(dx+c\right)-1\right)a}{4d\left(a-b\right)^{2}}+\frac{5\ln\left(\sin\left(dx+c\right)-1\right)b}{4d\left(a-b\right)^{2}}-\frac{1}{d\left(4a-4b\right)\left(1+\sin\left(dx+c\right)\right)}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/(4*a-4*b)/(sin(d*x+c)-1)-1/4/d/(a-b)^2*ln(sin(d*x+c)-1)*a+5/4/d/(a-b)^
2*ln(sin(d*x+c)-1)*b-1/d/(4*a-4*b)/(1+sin(d*x+c))+1/4*a*ln(1+sin(d*x+c))/(a
-b)^2/d-5/4*b*ln(1+sin(d*x+c))/(a-b)^2/d+1/2/d*b/(a-b)^2*(a/b)^(1/4)*arctan
(sin(d*x+c)/(a/b)^(1/4))+1/2/d*b^2/(a-b)^2*(a/b)^(1/4)/a*arctan(sin(d*x+c)/
(a/b)^(1/4))+1/4/d*b/(a-b)^2*(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))+1/4/d*b^2/(a-b)^2*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1
/4))/(sin(d*x+c)-(a/b)^(1/4)))-1/d*b/(a-b)^2/(a/b)^(1/4)*arctan(sin(d*x+c)/
(a/b)^(1/4))+1/2/d*b/(a-b)^2/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))
```

maxima [A] time = 0.71, size = 244, normalized size = 1.39

$$-\frac{\frac{b\left(\frac{2\left(b\left(2\sqrt{a}-\sqrt{b}\right)-a\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\sin\left(dx+c\right)}{\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}+\frac{\left(b\left(2\sqrt{a}+\sqrt{b}\right)+a\sqrt{b}\right)\log\left(\frac{\sqrt{b}\sin\left(dx+c\right)-\sqrt{\sqrt{a}\sqrt{b}}}{\sqrt{b}\sin\left(dx+c\right)+\sqrt{\sqrt{a}\sqrt{b}}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}}\sqrt{b}}\right)}{a^{2}-2\,ab+b^{2}}-\frac{\left(a-5\,b\right)\log\left(\sin\left(dx+c\right)+1\right)}{a^{2}-2\,ab+b^{2}}+\frac{\left(a-5\,b\right)\log\left(\sin\left(dx+c\right)\right.}{a^{2}-2\,ab+b^{2}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4*(b*(2*(b*(2*sqrt(a) - sqrt(b)) - a*sqrt(b))*arctan(sqrt(b)*sin(d*x + c
)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)) + (b*(2*sq
rt(a) + sqrt(b)) + a*sqrt(b))*log((sqrt(b)*sin(d*x + c) - sqrt(sqrt(a)*sqrt
(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt(b))))/(sqrt(a)*sqrt(sqrt(a)
*sqrt(b))*sqrt(b)))/(a^2 - 2*a*b + b^2) - (a - 5*b)*log(sin(d*x + c) + 1)/(
a^2 - 2*a*b + b^2) + (a - 5*b)*log(sin(d*x + c) - 1)/(a^2 - 2*a*b + b^2) +
2*sin(d*x + c)/((a - b)*sin(d*x + c)^2 - a + b))/d
```

mupad [B] time = 19.19, size = 7758, normalized size = 44.33

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^3*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880
*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^
3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*
a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b
^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*
b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^
3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3
)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (s
in(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b
^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)
)*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*
a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2))
)^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/
(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3*b^3)^(1/2) + b
^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^
7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(11*
a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b
^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^
5*b^2)))^(1/2)*1i - (((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^
4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (sin(c + d*x)*(-(a^2*(a^3*b^3)^(1
/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/
(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11
 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^
6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2
))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6
*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)
))^(1/2) - (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8
 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4
 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4
*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
+ 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6
+ 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3*b
^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(
1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(
c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*
b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a
^4*b^3 + 6*a^5*b^2)))^(1/2)*1i)/((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*
b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*
b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*(-(a^2
*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3
*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*
(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7
+ 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^
4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 -
4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3
 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 -
 2208*a^3*b^8 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b -
4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) -
4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4
 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7
+ 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*
(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*
```

b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^
(1/2) + (sin(c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a
^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(
1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b +
a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (((((128*a*b^11 + 256*a^2*b^10 -
 3456*a^3*b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5
+ 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)) + (sin(c + d
x)(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 +
6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2
)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 256
0*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*
a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*
a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 -
 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(c + d*x)*(48*a*b^10 - 16*b^11 + 1024
*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 -
4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3
)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b
 + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 480*a^2*b^8 - 78
4*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a
^2*b^2)))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*
b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a
^5*b^2)))^(1/2) - (sin(c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/
(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(
a^3*b^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 -
4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (a*b^7 - 5*b^8)/(a^4 -
 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(-(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b
^3)^(1/2) - 4*a^2*b^3 - 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6
*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*2i)/d + (atan((((((128*a*b^1
1 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 6912*a^6*b
^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*
b^2)) - (sin(c + d*x)*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b
^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^
4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4*b^9 - 2
560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a^9*b^4))/(a
^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3
*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a
^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(48*a*b^10
- 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b^7 - 144*a^5*b^6 + 32*a
^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^3)^(1/2)
 + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16
*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (200*a*b^9 + 4
80*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*b - 4*a*
b^3 + b^4 + 6*a^2*b^2)))*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^
2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4
*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(11*a*b^8 + 27*b^9 - 7*a^2*b^
7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^3)^
(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2)
)/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*1i - (((((1
28*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880*a^5*b^7 + 69
12*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^3 + b^4
+ 6*a^2*b^2)) + (sin(c + d*x)*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) +
 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^
4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^10 + 4608*a^4
*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b^5 + 512*a^9*
b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^3)^(1/2) +
b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a
^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(c + d*x)*(48
*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b^7 - 144*a^5*b^
6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^

```
3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1
/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (200*a
*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4 - 4*a^3*
b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2
) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3
*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(c + d*x)*(11*a*b^8 + 27*b^9 -
7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a
^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^
3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*1i)
/((((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880*a^5*
b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4*a*b^
3 + b^4 + 6*a^2*b^2)) - (sin(c + d*x)*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)
^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b
+ a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^10 +
4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b^5 +
512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*b^3)^
(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2)
)/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c +
d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b^7 - 14
4*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2
*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3
*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)
- (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/(2*(a^4
- 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b
^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6
*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) + (sin(c + d*x)*(11*a*b^8 + 2
7*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*
((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b
*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(
1/2) + (((((128*a*b^11 + 256*a^2*b^10 - 3456*a^3*b^9 + 8960*a^4*b^8 - 10880
*a^5*b^7 + 6912*a^6*b^6 - 2176*a^7*b^5 + 256*a^8*b^4)/(2*(a^4 - 4*a^3*b - 4
*a*b^3 + b^4 + 6*a^2*b^2)) + (sin(c + d*x)*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3
*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a
^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2)*(512*a^2*b^11 - 2560*a^3*b^
10 + 4608*a^4*b^9 - 2560*a^5*b^8 - 2560*a^6*b^7 + 4608*a^7*b^6 - 2560*a^8*b
^5 + 512*a^9*b^4))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*((a^2*(a^3*
b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^
(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin
(c + d*x)*(48*a*b^10 - 16*b^11 + 1024*a^2*b^9 - 2208*a^3*b^8 + 1264*a^4*b^7
 - 144*a^5*b^6 + 32*a^6*b^5))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2))*
((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b
*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(
1/2) - (200*a*b^9 + 480*a^2*b^8 - 784*a^3*b^7 + 96*a^4*b^6 + 8*a^5*b^5)/(2*
(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*((a^2*(a^3*b^3)^(1/2) + b^2*(
a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 -
4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1/2) - (sin(c + d*x)*(11*a*b^
8 + 27*b^9 - 7*a^2*b^7 + a^3*b^6))/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b
^2))*((a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 +
6*a*b*(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2
)))^(1/2) - (a*b^7 - 5*b^8)/(a^4 - 4*a^3*b - 4*a*b^3 + b^4 + 6*a^2*b^2)))*(
(a^2*(a^3*b^3)^(1/2) + b^2*(a^3*b^3)^(1/2) + 4*a^2*b^3 + 4*a^3*b^2 + 6*a*b*
(a^3*b^3)^(1/2))/(16*(a^7 - 4*a^6*b + a^3*b^4 - 4*a^4*b^3 + 6*a^5*b^2)))^(1
/2)*2i)/d + (log(sin(c + d*x) - 1)*(b/(a - b)^2 - 1/(4*(a - b))))/d + sin(c
 + d*x)/(2*d*cos(c + d*x)^2*(a - b)) + (log(sin(c + d*x) + 1)*(a - 5*b))/(4
*d*(a - b)^2)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3/(a-b*sin(d*x+c)**4),x)
[Out] Timed out
```

3.410 $\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=249

$$\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8d(a-b)^3}+\frac{3a-11b}{16d(a-b)^2(1-\sin}$$

```
[Out] 1/8*(3*a^2-6*a*b+35*b^2)*arctanh(sin(d*x+c))/(a-b)^3/d+1/16/(a-b)/d/(1-sin(
d*x+c))^2+1/16*(3*a-11*b)/(a-b)^2/d/(1-sin(d*x+c))-1/16/(a-b)/d/(1+sin(d*x+
c))^2+1/16*(-3*a+11*b)/(a-b)^2/d/(1+sin(d*x+c))-1/2*b^(5/4)*arctanh(b^(1/4)
*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b^(1/2))^3+1/2*b^(5/4)*arctan(b^(1/
4)*sin(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^3
```

Rubi [A] time = 0.30, antiderivative size = 249, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3223, 1171, 207, 1167, 205, 208}

$$\frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^3}-\frac{b^{5/4}\tanh^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^3}+\frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8d(a-b)^3}+\frac{3a-11b}{16d(a-b)^2(1-\sin}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b^(5/4)*ArcTan[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt
[b])^3*d) + ((3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(8*(a - b)^3*d
) - (b^(5/4)*ArcTanh[(b^(1/4)*Sin[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] -
 Sqrt[b])^3*d) + 1/(16*(a - b)*d*(1 - Sin[c + d*x])^2) + (3*a - 11*b)/(16*(
a - b)^2*d*(1 - Sin[c + d*x])) - 1/(16*(a - b)*d*(1 + Sin[c + d*x])^2) - (3
*a - 11*b)/(16*(a - b)^2*d*(1 + Sin[c + d*x]))
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 1167

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[
-(a*c), 2]}, Dist[e/2 + (c*d)/(2*q), Int[1/(-q + c*x^2), x], x] + Dist[e/2
- (c*d)/(2*q), Int[1/(q + c*x^2), x], x]] /; FreeQ[{a, c, d, e}, x] && NeQ[
c*d^2 - a*e^2, 0] && PosQ[-(a*c)]
```

Rule 1171

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (c_.)*(x_)^4), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)^q/(a + c*x^4), x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0] && IntegerQ[q]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \frac{\sec^5(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1-x^2\right)^3\left(a-bx^4\right)}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{8(a-b)(-1+x)^3} + \frac{3a-11b}{16(a-b)^2(-1+x)^2} + \frac{1}{8(a-b)(1+x)^3} + \frac{3a-11b}{16(a-b)^2(1+x)^2} + \frac{-3a^2+6ab-35b}{8(a-b)^3\left(-1+x^2\right.}\right.\right.}{d}$$

$$= \frac{1}{16(a-b)d(1-\sin(c+dx))^2} + \frac{3a-11b}{16(a-b)^2d(1-\sin(c+dx))} - \frac{1}{16(a-b)d(1+\sin(c}$$

$$= \frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d} + \frac{1}{16(a-b)d(1-\sin(c+dx))^2} + \frac{}{16(a-b)}$$

$$= \frac{b^{5/4}\tan^{-1}\left(\frac{\sqrt[4]{b}\sin(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3d} + \frac{\left(3a^2-6ab+35b^2\right)\tanh^{-1}(\sin(c+dx))}{8(a-b)^3d} - \frac{b^{5/4}\tanh^{-1}\left(\right.}{2a^{3/4}\left(\sqrt{a}\right.}$$

Mathematica [C] time = 5.60, size = 317, normalized size = 1.27

$$\frac{\frac{4b^{5/4}\log\left(\sqrt[4]{a}-\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3} + \frac{4ib^{5/4}\log\left(\sqrt[4]{a}-i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3} - \frac{4ib^{5/4}\log\left(\sqrt[4]{a}+i\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^3} - \frac{4b^{5/4}\log\left(\sqrt[4]{a}+\sqrt[4]{b}\sin(c+dx)\right)}{a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^3} + \frac{2(3a^2}{}}{16d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^5/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((2*(3*a^2 - 6*a*b + 35*b^2)*ArcTanh[Sin[c + d*x]])/(a - b)^3 + (4*b^(5/4)*
Log[a^(1/4) - b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] - Sqrt[b])^3) + ((4*
I)*b^(5/4)*Log[a^(1/4) - I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sqrt[a] + Sqrt[
b])^3) - ((4*I)*b^(5/4)*Log[a^(1/4) + I*b^(1/4)*Sin[c + d*x]])/(a^(3/4)*(Sq
rt[a] + Sqrt[b])^3) - (4*b^(5/4)*Log[a^(1/4) + b^(1/4)*Sin[c + d*x]])/(a^(3
/4)*(Sqrt[a] - Sqrt[b])^3) + 1/((a - b)*(-1 + Sin[c + d*x])^2) + (-3*a + 11
*b)/((a - b)^2*(-1 + Sin[c + d*x])) - 1/((a - b)*(1 + Sin[c + d*x])^2) + (-
3*a + 11*b)/((a - b)^2*(1 + Sin[c + d*x])))/(16*d)
```

fricas [B] time = 2.59, size = 3703, normalized size = 14.87

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="fricas")

[Out] -1/16*(4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) + (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - 4*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1/2*(a^3*b^4 + 15*a^2*b^5 + 15*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11 + a^3*b^12)*d^4)) - (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 + (a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b

^10 - 12*a^4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*
b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) + 4*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5*b^2 -
20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*a^5*b^6
 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^15 - 12*
a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 + 924*a^9
*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^4*b^11
+ a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^3*b^4 -
 6*a^2*b^5 + a*b^6)*d^2))*cos(d*x + c)^4*log(-1/2*(a^3*b^4 + 15*a^2*b^5 + 1
5*a*b^6 + b^7)*sin(d*x + c) + 1/2*((a^10 - 3*a^9*b - 3*a^8*b^2 + 25*a^7*b^3
 - 45*a^6*b^4 + 39*a^5*b^5 - 17*a^4*b^6 + 3*a^3*b^7)*d^3*sqrt((a^6*b^5 + 30
*a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^
15 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 +
 924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a
^4*b^11 + a^3*b^12)*d^4)) + (3*a^5*b^3 + 46*a^4*b^4 + 60*a^3*b^5 + 18*a^2*b
^6 + a*b^7)*d)*sqrt((6*a^2*b^3 + 20*a*b^4 + 6*b^5 - (a^7 - 6*a^6*b + 15*a^5
*b^2 - 20*a^4*b^3 + 15*a^3*b^4 - 6*a^2*b^5 + a*b^6)*d^2*sqrt((a^6*b^5 + 30*
a^5*b^6 + 255*a^4*b^7 + 452*a^3*b^8 + 255*a^2*b^9 + 30*a*b^10 + b^11)/((a^1
5 - 12*a^14*b + 66*a^13*b^2 - 220*a^12*b^3 + 495*a^11*b^4 - 792*a^10*b^5 +
924*a^9*b^6 - 792*a^8*b^7 + 495*a^7*b^8 - 220*a^6*b^9 + 66*a^5*b^10 - 12*a^
4*b^11 + a^3*b^12)*d^4)))/((a^7 - 6*a^6*b + 15*a^5*b^2 - 20*a^4*b^3 + 15*a^
3*b^4 - 6*a^2*b^5 + a*b^6)*d^2))) - (3*a^2 - 6*a*b + 35*b^2)*cos(d*x + c)^4
*log(sin(d*x + c) + 1) + (3*a^2 - 6*a*b + 35*b^2)*cos(d*x + c)^4*log(-sin(d
x + c) + 1) - 2((3*a^2 - 14*a*b + 11*b^2)*cos(d*x + c)^2 + 2*a^2 - 4*a*b
+ 2*b^2)*sin(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(d*x + c)^4)

giac [B] time = 0.83, size = 630, normalized size = 2.53

$$-\frac{\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\arctan\left(\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}+2\sin(dx+c)\right)}{2\left(-\frac{a}{b}\right)^{\frac{1}{4}}}\right)}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}+\frac{8\left(\left(-ab^3\right)^{\frac{3}{4}}(a+3b)+\left(-ab^3\right)^{\frac{1}{4}}\left(3ab^2+b^3\right)\right)\arctan\left(-\frac{\sqrt{2}\left(\sqrt{2}\left(-\frac{a}{b}\right)^{\frac{1}{4}}\cdots\right.}{2\left(-\frac{a}{b}\cdots\right.}\right.}{\sqrt{2}a^4b-3\sqrt{2}a^3b^2+3\sqrt{2}a^2b^3-\sqrt{2}ab^4}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="giac")

[Out] -1/16*(8*((-a*b^3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan
(1/2*sqrt(2)*(sqrt(2)*(-a/b)^(1/4) + 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)
*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) + 8*((-a*b^
3)^(3/4)*(a + 3*b) + (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*arctan(-1/2*sqrt(2)*(s
qrt(2)*(-a/b)^(1/4) - 2*sin(d*x + c))/(-a/b)^(1/4))/(sqrt(2)*a^4*b - 3*sqrt
(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2)*a*b^4) - 4*((-a*b^3)^(3/4)*(a + 3
*b) - (-a*b^3)^(1/4)*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 + sqrt(2)*(-a/b)^(
1/4)*sin(d*x + c) + sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt
(2)*a^2*b^3 - sqrt(2)*a*b^4) + 4*((-a*b^3)^(3/4)*(a + 3*b) - (-a*b^3)^(1/4)
*(3*a*b^2 + b^3))*log(sin(d*x + c)^2 - sqrt(2)*(-a/b)^(1/4)*sin(d*x + c) +
sqrt(-a/b))/(sqrt(2)*a^4*b - 3*sqrt(2)*a^3*b^2 + 3*sqrt(2)*a^2*b^3 - sqrt(2
)*a*b^4) - (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) + 1))/(a^3 - 3*a^2
*b + 3*a*b^2 - b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(abs(sin(d*x + c) - 1))/(
a^3 - 3*a^2*b + 3*a*b^2 - b^3) + 2*(3*a*sin(d*x + c)^3 - 11*b*sin(d*x + c)^
3 - 5*a*sin(d*x + c) + 13*b*sin(d*x + c))/((a^2 - 2*a*b + b^2)*(sin(d*x + c
)^2 - 1)^2))/d

maple [B] time = 0.71, size = 660, normalized size = 2.65

$$\frac{1}{2d\left(8a-8b\right)\left(\sin\left(dx+c\right)-1\right)^{2}}-\frac{3a}{16d\left(a-b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}+\frac{11b}{16d\left(a-b\right)^{2}\left(\sin\left(dx+c\right)-1\right)}-\frac{3\ln\left(\sin\left(dx+c\right)-\right.}{16d\left(a-b\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d/(8*a-8*b)/(sin(d*x+c)-1)^2-3/16/d/(a-b)^2/(sin(d*x+c)-1)*a+11/16/d/(a
-b)^2/(sin(d*x+c)-1)*b-3/16/d/(a-b)^3*ln(sin(d*x+c)-1)*a^2+3/8/d/(a-b)^3*ln
(sin(d*x+c)-1)*a*b-35/16/d/(a-b)^3*ln(sin(d*x+c)-1)*b^2-1/2/d/(8*a-8*b)/(1+
sin(d*x+c))^2-3/16/d/(a-b)^2/(1+sin(d*x+c))*a+11/16/d/(a-b)^2/(1+sin(d*x+c)
)*b+3/16/d/(a-b)^3*ln(1+sin(d*x+c))*a^2-3/8/d/(a-b)^3*ln(1+sin(d*x+c))*a*b+
35/16/d/(a-b)^3*ln(1+sin(d*x+c))*b^2-3/2/d*b^2/(a-b)^3*(a/b)^(1/4)*arctan(s
in(d*x+c)/(a/b)^(1/4))-1/2/d*b^3/(a-b)^3*(a/b)^(1/4)/a*arctan(sin(d*x+c)/(a
/b)^(1/4))-3/4/d*b^2/(a-b)^3*(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d
*x+c)-(a/b)^(1/4)))-1/4/d*b^3/(a-b)^3*(a/b)^(1/4)/a*ln((sin(d*x+c)+(a/b)^(1
/4))/(sin(d*x+c)-(a/b)^(1/4)))+1/2/d*b/(a-b)^3/(a/b)^(1/4)*a*arctan(sin(d*x
+c)/(a/b)^(1/4))+3/2/d*b^2/(a-b)^3/(a/b)^(1/4)*arctan(sin(d*x+c)/(a/b)^(1/4
))-1/4/d*b/(a-b)^3/(a/b)^(1/4)*a*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(d*x+c)-(a
/b)^(1/4)))-3/4/d*b^2/(a-b)^3/(a/b)^(1/4)*ln((sin(d*x+c)+(a/b)^(1/4))/(sin(
d*x+c)-(a/b)^(1/4)))
```

maxima [A] time = 1.50, size = 363, normalized size = 1.46

$$\frac{\frac{4\,b^{2}\left(\frac{2\left(b\left(3\,\sqrt{a}-\sqrt{b}\right)+a^{\frac{3}{2}}-3\,a\,\sqrt{b}\right)\arctan\left(\frac{\sqrt{b}\,\sin(dx+c)}{\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}+\frac{\left(b\left(3\,\sqrt{a}+\sqrt{b}\right)+a^{\frac{3}{2}}+3\,a\,\sqrt{b}\right)\log\left(\frac{\sqrt{b}\,\sin(dx+c)-\sqrt{\sqrt{a}\,\sqrt{b}}}{\sqrt{b}\,\sin(dx+c)+\sqrt{\sqrt{a}\,\sqrt{b}}}\right)}{\sqrt{a}\,\sqrt{\sqrt{a}\,\sqrt{b}}\,\sqrt{b}}\right)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}+\frac{\left(3\,a^{2}-6\,ab+35\,b^{2}\right)\log(\sin(dx+c)+1)}{a^{3}-3\,a^{2}b+3\,ab^{2}-b^{3}}-\frac{(3}{}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/16*(4*b^2*(2*(b*(3*sqrt(a) - sqrt(b)) + a^(3/2) - 3*a*sqrt(b))*arctan(sqr
t(b)*sin(d*x + c)/sqrt(sqrt(a)*sqrt(b)))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqr
t(b)) + (b*(3*sqrt(a) + sqrt(b)) + a^(3/2) + 3*a*sqrt(b))*log((sqrt(b)*sin(
d*x + c) - sqrt(sqrt(a)*sqrt(b)))/(sqrt(b)*sin(d*x + c) + sqrt(sqrt(a)*sqrt
(b))))/(sqrt(a)*sqrt(sqrt(a)*sqrt(b))*sqrt(b)))/(a^3 - 3*a^2*b + 3*a*b^2 -
b^3) + (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) + 1)/(a^3 - 3*a^2*b + 3*a*
b^2 - b^3) - (3*a^2 - 6*a*b + 35*b^2)*log(sin(d*x + c) - 1)/(a^3 - 3*a^2*b
+ 3*a*b^2 - b^3) - 2*((3*a - 11*b)*sin(d*x + c)^3 - (5*a - 13*b)*sin(d*x +
c))/((a^2 - 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 - 2*a*b + b^2)*sin(d*x + c
)^2 + a^2 - 2*a*b + b^2))/d
```

mupad [B] time = 20.57, size = 12217, normalized size = 49.06

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^5*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116
624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/
```

(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (sin(c + d*x)*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*1i - (((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (sin(c + d*x)*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6

*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a
^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8
*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3
+ 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 -
20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2
))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*
a^7*b^2)))^(1/2)*1i)/((((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888
*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7
- 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5
 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14
 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10
 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 5
5296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*
b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (sin(c + d*x)*(
-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^
4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a
^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*
(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 73728
0*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^1
0*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7
*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28
*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a
^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(
16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*
b^2)))^(1/2) - (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 -
 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096
*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8
 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3
*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3
 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b
+ a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*(-(a
^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b
^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*
b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (
sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^
4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^
2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^
5)^(1/2) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*
b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^
6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (((18064*a*
b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116624*a^4*b^10 + 288
48*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b
 - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a
^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*
b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*
b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(6
4*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 5
6*a^5*b^3 + 28*a^6*b^2)) + (sin(c + d*x)*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*
b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2)
+ 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a
^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 +
 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 34
4064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a
^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 -
 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2
) + b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^
3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a
^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(256
*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^1

0 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^1
0*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a
^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^
(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*
a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^
4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5
)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 1
5*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*
b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(6802*a*b^12 + 1257*
b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/
(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 -
56*a^5*b^3 + 28*a^6*b^2)))*(-(a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) -
6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a
^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a
^6*b^3 + 15*a^7*b^2)))^(1/2) - (1505*b^12 - 748*a*b^11 + 318*a^2*b^10 - 60*
a^3*b^9 + 9*a^4*b^8)/(32*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a
^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2))))*(-(a^3*(a^3*b^5)^(1/2) +
b^3*(a^3*b^5)^(1/2) - 6*a^2*b^5 - 20*a^3*b^4 - 6*a^4*b^3 + 15*a*b^2*(a^3*b^
5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b
^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*2i)/d - (log(sin(c + d*x
) - 1)*((2*b^2)/(a - b)^3 + 3/(16*(a - b))))/d + (atan(((((18064*a*b^13 + 2
56*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b
^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b
^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2))
 - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2
457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 87
2448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 -
 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^
3 + 28*a^6*b^2)) - (sin(c + d*x)*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2
) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*
b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 -
20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a
^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*
b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 +
 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b
^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a
^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/
2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 1
5*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(256*b^15 - 50
464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296
*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(1
6*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 5
6*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a
^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*
b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*
b^3 + 15*a^7*b^2)))^(1/2))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*
a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3
b^5)^(1/2))/(16(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6
*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a
^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8
*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3
+ 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 2
0*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2)
)/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a
^7*b^2)))^(1/2)*1i - (((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*
a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 -
 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5
+ 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14
- 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10

- 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (sin(c + d*x)*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*1i)/((((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 275888*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^14 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^10 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 - 55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (sin(c + d*x)*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 737280*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^10*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 - 280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (sin(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5

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*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^
5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^
(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 +
 15*a^7*b^2)))^(1/2) + (((18064*a*b^13 + 256*b^14 + 119760*a^2*b^12 - 27588
8*a^3*b^11 + 116624*a^4*b^10 + 28848*a^5*b^9 - 13712*a^6*b^8 + 6768*a^7*b^7
 - 720*a^8*b^6)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^
5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) - (((4096*a*b^15 + 12288*a^2*b^1
4 - 251904*a^3*b^13 + 1087488*a^4*b^12 - 2457600*a^5*b^11 + 3440640*a^6*b^1
0 - 3182592*a^7*b^9 + 2002944*a^8*b^8 - 872448*a^9*b^7 + 266240*a^10*b^6 -
55296*a^11*b^5 + 6144*a^12*b^4)/(64*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2
*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)) + (sin(c + d*x)*
((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^
4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a
^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2)*
(8192*a^2*b^15 - 73728*a^3*b^14 + 286720*a^4*b^13 - 614400*a^5*b^12 + 73728
0*a^6*b^11 - 344064*a^7*b^10 - 344064*a^8*b^9 + 737280*a^9*b^8 - 614400*a^1
0*b^7 + 286720*a^11*b^6 - 73728*a^12*b^5 + 8192*a^13*b^4))/(16*(a^8 - 8*a^7
*b - 8*a*b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28
*a^6*b^2)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^
3*b^4 + 6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(1
6*(a^9 - 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b
^2)))^(1/2) + (sin(c + d*x)*(256*b^15 - 50464*a^2*b^13 + 190720*a^3*b^12 -
280960*a^4*b^11 + 212736*a^5*b^10 - 111296*a^6*b^9 + 57088*a^7*b^8 - 20096*
a^8*b^7 + 2304*a^9*b^6 - 288*a^10*b^5))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8
+ 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(
a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 +
 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b +
a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2))*((a^3*
(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3
+ 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b +
 a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) + (sin
(c + d*x)*(6802*a*b^12 + 1257*b^13 - 857*a^2*b^11 + 892*a^3*b^10 + 71*a^4*b
^9 + 18*a^5*b^8 + 9*a^6*b^7))/(16*(a^8 - 8*a^7*b - 8*a*b^7 + b^8 + 28*a^2*b
^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)))*((a^3*(a^3*b^5)^(
1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 + 6*a^4*b^3 + 15*a*b^2*
(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 - 6*a^8*b + a^3*b^6 -
6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1/2) - (1505*b^12 - 74
8*a*b^11 + 318*a^2*b^10 - 60*a^3*b^9 + 9*a^4*b^8)/(32*(a^8 - 8*a^7*b - 8*a*
b^7 + b^8 + 28*a^2*b^6 - 56*a^3*b^5 + 70*a^4*b^4 - 56*a^5*b^3 + 28*a^6*b^2)
)))*((a^3*(a^3*b^5)^(1/2) + b^3*(a^3*b^5)^(1/2) + 6*a^2*b^5 + 20*a^3*b^4 +
6*a^4*b^3 + 15*a*b^2*(a^3*b^5)^(1/2) + 15*a^2*b*(a^3*b^5)^(1/2))/(16*(a^9 -
 6*a^8*b + a^3*b^6 - 6*a^4*b^5 + 15*a^5*b^4 - 20*a^6*b^3 + 15*a^7*b^2)))^(1
/2)*2i)/d + ((sin(c + d*x)*(5*a - 13*b))/(8*(a^2 - 2*a*b + b^2)) - (sin(c +
 d*x)^3*(3*a - 11*b))/(8*(a^2 - 2*a*b + b^2)))/(d*(cos(c + d*x)^2 - sin(c +
 d*x)^2 + sin(c + d*x)^4)) + (log(sin(c + d*x) + 1)*(3*a^2 - 6*a*b + 35*b^2
))/(16*d*(a - b)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.411 $\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=252

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}-\frac{(a+3b)\sin(c+dx)\cos(c+}{2b^2d}$$

```
[Out] -17/16*x/b-4*(a+b)*x/b^2-1/2*(a+3*b)*x/b^2-17/16*cos(d*x+c)*sin(d*x+c)/b/d-
1/2*(a+3*b)*cos(d*x+c)*sin(d*x+c)/b^2/d-17/24*cos(d*x+c)^3*sin(d*x+c)/b/d-1
/6*cos(d*x+c)^5*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c
)/a^(1/4))*(a^(1/2)-b^(1/2))^(9/2)/a^(3/4)/b^(5/2)/d+1/2*arctan((a^(1/2)+b^
(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(9/2)/a^(3/4)/b^(5/2)/d
```

Rubi [A] time = 0.44, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d}-\frac{(a+3b)\sin(c+dx)\cos(c+}{2b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-17*x)/(16*b) - (4*(a + b)*x)/b^2 - ((a + 3*b)*x)/(2*b^2) - ((Sqrt[a] - Sq
rt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(
3/4)*b^(5/2)*d) + ((Sqrt[a] + Sqrt[b])^(9/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]
]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(5/2)*d) - (17*Cos[c + d*x]*Sin[c +
d*x])/(16*b*d) - ((a + 3*b)*Cos[c + d*x]*Sin[c + d*x])/(2*b^2*d) - (17*Cos[
c + d*x]^3*Sin[c + d*x])/(24*b*d) - (Cos[c + d*x]^5*Sin[c + d*x])/(6*b*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
```

```
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^{10}(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)^4} - \frac{2}{b(1+x^2)^3} + \frac{-a-3b}{b^2(1+x^2)^2} - \frac{4(a+b)}{b^2(1+x^2)} + \frac{5a^2+10ab+b^2+4(a^2-b^2)x^2}{b^2(a+2ax^2+(a-b)x^4)}\right) dx, x, \text{t}$$

$$= \frac{\text{Subst}\left(\int \frac{5a^2+10ab+b^2+4(a^2-b^2)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2 d} - \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)^4}\,dx, x, \tan(c+\right.}{bd}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)\cos(c+dx)\sin(c+dx)}{2b^2d} - \frac{\cos^3(c+dx)\sin(c+dx)}{2bd} - \frac{\cos^5(c}{}$$

$$= -\frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{9/2}}{}$$

$$= -\frac{3x}{4b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\right.}{}$$

$$= -\frac{17x}{16b} - \frac{4(a+b)x}{b^2} - \frac{(a+3b)x}{2b^2} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{9/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{5/2}d} + \frac{\left(\sqrt{a}+\right.}{}$$

Mathematica [A] time = 0.91, size = 233, normalized size = 0.92

$$-\frac{36b(24a+35b)(c+dx)+3b(16a+95b)\sin(2(c+dx)) - \frac{96\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^5\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} - \frac{96\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^5 \text{t}}{\sqrt{a}\,}}{192b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^10/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/192*(36*b*(24*a + 35*b)*(c + d*x) - (96*(Sqrt[a] + Sqrt[b])^5*Sqrt[b]*Ar
cTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a
]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (96*(Sqrt[a] - Sqrt[b])^5*Sqrt[b]*ArcTanh[((
Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt
[-a + Sqrt[a]*Sqrt[b]]) + 3*b*(16*a + 95*b)*Sin[2*(c + d*x)] + 21*b^2*Sin[4
*(c + d*x)] + b^2*Sin[6*(c + d*x)])/(b^3*d)
```

fricas [B] time = 1.96, size = 2948, normalized size = 11.70

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/48*(6*b^2*d*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21
816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)
/(a^3*b^9*d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d
^2))*log(9/4*a^8 + 12*a^7*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a
^2*b^6 + 8*a*b^7 + 1/4*b^8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*
b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a
^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*
b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^
9*d^4))*cos(d*x + c)*sin(d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4
+ 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x +
c))*sqrt((a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4)) - a^4 - 36*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4
*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c
)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81
*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3
*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 6*b^2*d*sqrt((a*b^5
*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b
^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 3
6*a^3*b - 126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2))*log(9/4*a^8 + 12*a^7
*b - 39*a^6*b^2 + 143/2*a^4*b^4 - 52*a^3*b^5 - 3*a^2*b^6 + 8*a*b^7 + 1/4*b^
8 - 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^
2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqr
t((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 924
0*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(
d*x + c) + (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b
^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^5*d^2*sqrt(
(81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*
a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) - a^4 - 36*a^3*b -
126*a^2*b^2 - 84*a*b^3 - 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5
+ 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^
5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*
a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*
a*b^7 + b^8)/(a^3*b^9*d^4))) + 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512
*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548
*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 +
84*a*b^3 + 9*b^4)/(a*b^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2
*a^4*b^4 + 52*a^3*b^5 + 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7
*b - 156*a^6*b^2 + 286*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)
*cos(d*x + c)^2 + 1/2*(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b
+ 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^
6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 +
 138*a^6*b^3 + 639*a^5*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8
)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b +
 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6
 + 72*a*b^7 + b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3
```

```
 + 9*b^4)/(a*b^5*d^2)) + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^
7 + a^2*b^8)*d^2*cos(d*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*
b^7 + a^2*b^8)*d^2)*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^
3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*
d^4))) - 6*b^2*d*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2
+ 21816*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 +
b^8)/(a^3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b
^5*d^2))*log(-9/4*a^8 - 12*a^7*b + 39*a^6*b^2 - 143/2*a^4*b^4 + 52*a^3*b^5
+ 3*a^2*b^6 - 8*a*b^7 - 1/4*b^8 + 1/4*(9*a^8 + 48*a^7*b - 156*a^6*b^2 + 286
*a^4*b^4 - 208*a^3*b^5 - 12*a^2*b^6 + 32*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*
(4*(a^4*b^7 + a^3*b^8)*d^3*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816
*a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a
^3*b^9*d^4))*cos(d*x + c)*sin(d*x + c) - (9*a^7*b^2 + 138*a^6*b^3 + 639*a^5
*b^4 + 876*a^4*b^5 + 343*a^3*b^6 + 42*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d
*x + c))*sqrt(-(a*b^5*d^2*sqrt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*
a^5*b^3 + 21942*a^4*b^4 + 9240*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^
3*b^9*d^4)) + a^4 + 36*a^3*b + 126*a^2*b^2 + 84*a*b^3 + 9*b^4)/(a*b^5*d^2))
 + 1/4*(2*(a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2*cos(d
*x + c)^2 - (a^6*b^4 - 4*a^5*b^5 + 6*a^4*b^6 - 4*a^3*b^7 + a^2*b^8)*d^2)*sq
rt((81*a^8 + 1512*a^7*b + 9324*a^6*b^2 + 21816*a^5*b^3 + 21942*a^4*b^4 + 92
40*a^3*b^5 + 1548*a^2*b^6 + 72*a*b^7 + b^8)/(a^3*b^9*d^4))) - 9*(24*a + 35*
b)*d*x - (8*b*cos(d*x + c)^5 + 34*b*cos(d*x + c)^3 + 3*(8*a + 41*b)*cos(d*x
 + c))*sin(d*x + c))/(b^2*d)
```

giac [B] time = 1.16, size = 896, normalized size = 3.56

$$24\left(15\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^4b-62\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b^3-16\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^4-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^5-3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^4-24\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/48*(24*(15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b
+ sqrt(a*b)*(a - b))*a^2*b^3 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4
 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^5 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*a^4 - 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b + 46*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 + 40*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 + 5*sqrt(a^2 - a*b + sqrt(a*b)
*(a - b))*sqrt(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c
)/sqrt((a*b^2 + sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-
a + b)/(3*a^5*b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) + 24*(15*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 62*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^2*b^3 - 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 -
a*b - sqrt(a*b)*(a - b))*b^5 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^4 + 24*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 46*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 40*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*b^4)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2
- sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*
b^3 - 12*a^4*b^4 + 14*a^3*b^5 - 4*a^2*b^6 - a*b^7) - 9*(d*x + c)*(24*a + 35
*b)/b^2 - (24*a*tan(d*x + c)^5 + 123*b*tan(d*x + c)^5 + 48*a*tan(d*x + c)^3
 + 280*b*tan(d*x + c)^3 + 24*a*tan(d*x + c) + 165*b*tan(d*x + c))/((tan(d*x
 + c)^2 + 1)^3*b^2))/d
```

maple [B] time = 0.67, size = 880, normalized size = 3.49

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)^5*a-41/16/d/b/(tan(d*x+c)^2+1)^3*t
an(d*x+c)^5-1/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)^3*a-35/6/d/b/(tan(d*x+c)^
2+1)^3*tan(d*x+c)^3-1/2/d/b^2/(tan(d*x+c)^2+1)^3*tan(d*x+c)*a-55/16/d/b/(ta
n(d*x+c)^2+1)^3*tan(d*x+c)-9/2/d/b^2*arctan(tan(d*x+c))*a-105/16/d/b*arctan
(tan(d*x+c))+2/d/b^2/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c
)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+1/2/d/b^2/(a*b)^(1/2)/(((a*b)^(1/2)-a)
*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^3+
5/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)
/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-5/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a
-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+2/d/b^2
/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-
b))^(1/2))*a^2-1/2/d/b^2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((
a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^3-5/2/d/b/(a*b)^(1/2)/(((a
*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(
1/2))*a^2+5/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*ta
n(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-2/d/(((a*b)^(1/2)-a)*(a-b))^(1/2)*a
rctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)
/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(
a-b))^(1/2))-2/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a
*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2
)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^10/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/192*(192*b^2*d*integrate(-4*(4*(a^2*b + 10*a*b^2 + 5*b^3)*cos(6*d*x + 6*
c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(a^2
*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c)^2 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*si
n(6*d*x + 6*c)^2 + 4*(72*a^3 + 53*a^2*b - 54*a*b^2 + 9*b^3)*sin(4*d*x + 4*c
)^2 + 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c)^2 - ((a^2*b + 10*a*b
^2 + 5*b^3)*cos(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 - 3*b^3)*cos(4*d*x + 4
*c) + (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a^2*
b + 10*a*b^2 + 5*b^3 - 2*(8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos(4*d*x
+ 4*c) - 8*(a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) -
2*(9*a^2*b + 10*a*b^2 - 3*b^3 - (8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*cos
(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a^2*b + 10*a*b^2 + 5*b^3)*cos(2*d*x + 2*
c) - ((a^2*b + 10*a*b^2 + 5*b^3)*sin(6*d*x + 6*c) + 2*(9*a^2*b + 10*a*b^2 -
 3*b^3)*sin(4*d*x + 4*c) + (a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin
(8*d*x + 8*c) + 2*((8*a^3 + 113*a^2*b + 50*a*b^2 - 27*b^3)*sin(4*d*x + 4*c)
 + 4*(a^2*b + 10*a*b^2 + 5*b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/(b^4*co
s(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*c)^2 +
b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*x + 2*c
)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*cos(
4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2 + 16*
(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6*d*x +
6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x + 4*c))
*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*c
os(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3 - 3*b^4
)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2*b^4*si
n(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c) + 16*
(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*d*x + 6
```

```
*c)), x) + 36*(24*a + 35*b)*d*x + b*sin(6*d*x + 6*c) + 21*b*sin(4*d*x + 4*c
) + 3*(16*a + 95*b)*sin(2*d*x + 2*c))/(b^2*d)
```

mupad [B] time = 18.80, size = 10319, normalized size = 40.95

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^10/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan(((((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11
 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534
*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) + (3*
(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (
240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^
7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 -
2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 - (3*tan(c
 + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 +
 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) - (tan(c + d*x)*(61
7264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9
- 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6
))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i)*3i
)/(32*b^2) + (((tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 1115
3*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 -
387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6)
 - (3*(((9873*a*b^13)/16 + 8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)
/8 - (240165*a^4*b^10)/8 + (68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*
a^7*b^7 - 2766*a^8*b^6 - 152*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b
^14 - 2064*a^3*b^13 - 480*a^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 + (3
*tan(c + d*x)*(a*24i + b*35i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*
b^11 + 49152*a^5*b^10))/(2048*b^8))*(a*24i + b*35i))/(32*b^2) + (tan(c + d*
x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^
4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(
64*b^6))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*3
5i)*3i)/(32*b^2))/(((92769*a*b^11)/64 - (39*a^11*b)/8 + 9*a^12 - (11865*b^1
2)/64 - (76467*a^2*b^10)/16 + (133839*a^3*b^9)/16 - (243927*a^4*b^8)/32 + (
58743*a^5*b^7)/32 + (50967*a^6*b^6)/16 - (52227*a^7*b^5)/16 + (61119*a^8*b^
4)/64 + (12729*a^9*b^3)/64 - (1137*a^10*b^2)/8)/b^8 - (3*((tan(c + d*x)*(12
3962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a
^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4
+ 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) + (3*(((9873*a*b^13)/16 + 8*b^1
4 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (240165*a^4*b^10)/8 + (6880
5*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^7 - 2766*a^8*b^6 - 152*a^9
*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 - 2064*a^3*b^13 - 480*a^4*b^
12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 - (3*tan(c + d*x)*(a*24i + b*35i)*(4
9152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(2048*b^
8))*(a*24i + b*35i))/(32*b^2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13
- 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*
a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(a*24i + b*35i))/(32*b^
2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2) + (3*((tan(c + d*x
)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2
370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7
*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6) - (3*(((9873*a*b^13)/16 +
8*b^14 + (198963*a^2*b^12)/16 - (13467*a^3*b^11)/8 - (240165*a^4*b^10)/8 +
(68805*a^5*b^9)/16 + (307047*a^6*b^8)/16 - 1929*a^7*b^7 - 2766*a^8*b^6 - 15
2*a^9*b^5)/b^8 + (3*((3*((64*a*b^15 + 1216*a^2*b^14 - 2064*a^3*b^13 - 480*a
^4*b^12 + 1968*a^5*b^11 - 704*a^6*b^10)/b^8 + (3*tan(c + d*x)*(a*24i + b*35
i)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(20
48*b^8))*(a*24i + b*35i))/(32*b^2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024*
b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 49
```

8944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(a*24i + b*35i))/(
32*b^2))*(a*24i + b*35i))/(32*b^2))*(a*24i + b*35i))/(32*b^2)))*(a*24i + b*
35i)*3i)/(16*b^2*d) - (atan(((((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12
 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 -
246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15
 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 9
0112*a^6*b^10)/(128*b^8) - (tan(c + d*x)*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^
3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5
 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3
*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152
*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*
b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 +
 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b
^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^
13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 4989
44*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*((9*a^4*(a^3*b^11)^(
1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5
*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) +
 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*((9*a^4*(a^3*b^11)^(1/2)
+ b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6
- a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a
^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(123962*a*b^10
- 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780
960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8
*b^3 - 74000*a^9*b^2))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^
(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a
^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(
1/2))/(16*a^3*b^10))^(1/2)*1i - (((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b
^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8
 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b
^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11
- 90112*a^6*b^10)/(128*b^8) + (tan(c + d*x)*((9*a^4*(a^3*b^11)^(1/2) + b^4*
(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*
b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(
a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49
152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a
^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^
5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^
3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024
*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 4
98944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*((9*a^4*(a^3*b^11
)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*
a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2
) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*((9*a^4*(a^3*b^11)^(1/
2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b
^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 8
4*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(123962*a*b^
10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 +
780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*
a^8*b^3 - 74000*a^9*b^2))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^1
1)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 12
6*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11
)^(1/2))/(16*a^3*b^10))^(1/2)*1i)/((((78984*a*b^13 + 1024*b^14 + 1591704*a^
2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*
b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*
a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^
11 - 90112*a^6*b^10)/(128*b^8) - (tan(c + d*x)*((9*a^4*(a^3*b^11)^(1/2) + b
^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a
^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*

b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/(128*b^8) + (tan(c + d*x)*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (92769*a*b^11 - 312*a^11*b + 576*a^12 - 11865*b^12 - 305868*a^2*b^10 + 535356*a^3*b^9 - 487854*a^4*b^8 + 117486*a^5*b^7 + 203868*a^6*b^6 - 208908*a^7*b^5 + 61119*a^8*b^4 + 12729*a^9*b^3 - 9096*a^10*b^2)/(64*b^8)))*((9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) - 9*a^2*b^9 - 84*a^3*b^8 - 126*a^4*b^7 - 36*a^5*b^6 - a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*2i)/d - ((tan(c + d*x)*(8*a + 55*b))/(16*b^2) + (tan(c + d*x)^3*(6*a + 35*b))/(6*b^2) + (tan(c + d*x)^5*(8*a + 41*b))/(16*b^2))/(d*(3*tan(c + d*x)^2 + 3*tan(c + d*x)^4 + tan(c + d*x)^6 + 1)) - (atan(((((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/(128*b^8) - (tan(c + d*x)*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*

(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*1i - (((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/(128*b^8) + (tan(c + d*x)*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9*b^2))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*1i)/((((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7 - 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/(128*b^8) - (tan(c + d*x)*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49152*a^5*b^10))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 + 232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^

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11)^(1/2))/(16*a^3*b^10))^(1/2))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)
^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*
a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^
(1/2))/(16*a^3*b^10))^(1/2) + (tan(c + d*x)*(123962*a*b^10 - 3776*a^10*b -
128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7 - 44
4642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 74000*a^9
*b^2))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b
^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^1
1)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b
^10))^(1/2) + (((78984*a*b^13 + 1024*b^14 + 1591704*a^2*b^12 - 215472*a^3*b
^11 - 3842640*a^4*b^10 + 550440*a^5*b^9 + 2456376*a^6*b^8 - 246912*a^7*b^7
- 354048*a^8*b^6 - 19456*a^9*b^5)/(128*b^8) + (((8192*a*b^15 + 155648*a^2*b
^14 - 264192*a^3*b^13 - 61440*a^4*b^12 + 251904*a^5*b^11 - 90112*a^6*b^10)/
(128*b^8) + (tan(c + d*x)*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2)
+ 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2
*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/
(16*a^3*b^10))^(1/2)*(49152*a^2*b^13 - 49152*a^3*b^12 - 49152*a^4*b^11 + 49
152*a^5*b^10))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) +
 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*
(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(
16*a^3*b^10))^(1/2) + (tan(c + d*x)*(617264*a^2*b^11 - 1024*b^13 - 10240*a*
b^12 + 46512*a^3*b^10 - 919536*a^4*b^9 - 469488*a^5*b^8 + 498944*a^6*b^7 +
232448*a^7*b^6 + 5120*a^8*b^5))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(
a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b
^5 + 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a
^3*b^11)^(1/2))/(16*a^3*b^10))^(1/2))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*
b^11)^(1/2) + 9*a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 +
 126*a^2*b^2*(a^3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b
^11)^(1/2))/(16*a^3*b^10))^(1/2) - (tan(c + d*x)*(123962*a*b^10 - 3776*a^10
*b - 128*a^11 + 11153*b^11 - 387826*a^2*b^9 + 2370*a^3*b^8 + 780960*a^4*b^7
 - 444642*a^5*b^6 - 387534*a^6*b^5 + 261366*a^7*b^4 + 118095*a^8*b^3 - 7400
0*a^9*b^2))/(64*b^6))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*
a^2*b^9 + 84*a^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^
3*b^11)^(1/2) + 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*
a^3*b^10))^(1/2) - (92769*a*b^11 - 312*a^11*b + 576*a^12 - 11865*b^12 - 305
868*a^2*b^10 + 535356*a^3*b^9 - 487854*a^4*b^8 + 117486*a^5*b^7 + 203868*a^
6*b^6 - 208908*a^7*b^5 + 61119*a^8*b^4 + 12729*a^9*b^3 - 9096*a^10*b^2)/(64
*b^8)))*(-(9*a^4*(a^3*b^11)^(1/2) + b^4*(a^3*b^11)^(1/2) + 9*a^2*b^9 + 84*a
^3*b^8 + 126*a^4*b^7 + 36*a^5*b^6 + a^6*b^5 + 126*a^2*b^2*(a^3*b^11)^(1/2)
+ 36*a*b^3*(a^3*b^11)^(1/2) + 84*a^3*b*(a^3*b^11)^(1/2))/(16*a^3*b^10))^(1/
2)*2i)/d
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**10/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.412 $\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=186

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{x(a+3b)}{b^2}-\frac{\sin(c+dx)\cos^3(c}{4bd}$$

```
[Out] -11/8*x/b-(a+3*b)*x/b^2-11/8*cos(d*x+c)*sin(d*x+c)/b/d-1/4*cos(d*x+c)^3*sin
(d*x+c)/b/d+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)
-b^(1/2))^(7/2)/a^(3/4)/b^2/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)
/a^(1/4))*(a^(1/2)+b^(1/2))^(7/2)/a^(3/4)/b^2/d
```

Rubi [A] time = 0.33, antiderivative size = 186, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d}-\frac{x(a+3b)}{b^2}-\frac{\sin(c+dx)\cos^3(c}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-11*x)/(8*b) - ((a + 3*b)*x)/b^2 + ((Sqrt[a] - Sqrt[b])^(7/2)*ArcTan[(Sqrt
[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^2*d) + ((Sqrt[a] +
 Sqrt[b])^(7/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*
a^(3/4)*b^2*d) - (11*Cos[c + d*x]*Sin[c + d*x])/(8*b*d) - (Cos[c + d*x]^3*S
in[c + d*x])/(4*b*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^8(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b\left(1+x^2\right)^3}-\frac{2}{b\left(1+x^2\right)^2}+\frac{-a-3b}{b^2\left(1+x^2\right)}+\frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{b^2\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \frac{a^2+6ab+b^2+(a-b)(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{b^2d} - \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^3}\,dx, x, \tan(c+\right.}{bd}$$

$$= -\frac{(a+3b)x}{b^2} - \frac{\cos(c+dx)\sin(c+dx)}{bd} - \frac{\cos^3(c+dx)\sin(c+dx)}{4bd} + \frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^4\right.}{}$$

$$= -\frac{x}{b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-1}\left(\right.}{2a^{3/4}b}$$

$$= -\frac{11x}{8b} - \frac{(a+3b)x}{b^2} + \frac{\left(\sqrt{a}-\sqrt{b}\right)^{7/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^2d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{7/2}\tan^{-}}{2a^{3/}}$$

Mathematica [A] time = 0.68, size = 200, normalized size = 1.08

$$-\frac{4(8a+35b)(c+dx)-\frac{16\left(\sqrt{a}+\sqrt{b}\right)^4\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{16\left(\sqrt{a}-\sqrt{b}\right)^4\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+24b\sin(2(c+dx)}{32b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^8/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -1/32*(4*(8*a + 35*b)*(c + d*x) - (16*(Sqrt[a] + Sqrt[b])^4*ArcTan[((Sqrt[a
] + Sqrt[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sq
rt[a]*Sqrt[b]]) + (16*(Sqrt[a] - Sqrt[b])^4*ArcTanh[((Sqrt[a] - Sqrt[b])*Ta
n[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]
]) + 24*b*Sin[2*(c + d*x)] + b*Sin[4*(c + d*x)])/(b^2*d)
```

fricas [B] time = 1.28, size = 2433, normalized size = 13.08

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*
a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 3
5*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^
3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4
*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4
*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^
3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c)
- (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x +
c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 +
 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2
*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b
^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b
^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b
^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt(-(a*b^4*d^2*sqrt((49*a^6
+ 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(
a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2))*log(7/4*a^6
 + 7/2*a^5*b - 63/4*a^4*b^2 + 9*a^3*b^3 + 25/4*a^2*b^4 - 9/2*a*b^5 - 1/4*b^
6 - 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5
 - b^6)*cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*
a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^
7*d^4))*cos(d*x + c)*sin(d*x + c) - (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4
+ 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^4*d^2*sqrt((4
9*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 +
b^6)/(a^3*b^7*d^4)) + a^3 + 21*a^2*b + 35*a*b^2 + 7*b^3)/(a*b^4*d^2)) - 1/4
*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b
^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*
a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) + b^
2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3
+ 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2
- 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 -
 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*(7*a^6 + 14*a^5*b - 63*a^4*b^2 +
36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*cos(d*x + c)^2 + 1/2*((a^4*b^5 +
3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511
*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*cos(d*x + c)*sin(d*x + c) + (21*a
^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(
d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^
3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4)) - a^3 - 21*a^2*b - 35*
a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2
*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)
*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*
a*b^5 + b^6)/(a^3*b^7*d^4))) - b^2*d*sqrt((a*b^4*d^2*sqrt((49*a^6 + 490*a^5
*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d
^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2))*log(-7/4*a^6 - 7/2*a
^5*b + 63/4*a^4*b^2 - 9*a^3*b^3 - 25/4*a^2*b^4 + 9/2*a*b^5 + 1/4*b^6 + 1/4*
(7*a^6 + 14*a^5*b - 63*a^4*b^2 + 36*a^3*b^3 + 25*a^2*b^4 - 18*a*b^5 - b^6)*
cos(d*x + c)^2 - 1/2*((a^4*b^5 + 3*a^3*b^6)*d^3*sqrt((49*a^6 + 490*a^5*b +
1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))*
cos(d*x + c)*sin(d*x + c) + (21*a^5*b^2 + 112*a^4*b^3 + 98*a^3*b^4 + 24*a^2
*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^4*d^2*sqrt((49*a^6 + 4
90*a^5*b + 1519*a^4*b^2 + 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3
*b^7*d^4)) - a^3 - 21*a^2*b - 35*a*b^2 - 7*b^3)/(a*b^4*d^2)) - 1/4*(2*(a^5*
b^3 - 3*a^4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^5*b^3 - 3*a^
4*b^4 + 3*a^3*b^5 - a^2*b^6)*d^2)*sqrt((49*a^6 + 490*a^5*b + 1519*a^4*b^2 +
 1484*a^3*b^3 + 511*a^2*b^4 + 42*a*b^5 + b^6)/(a^3*b^7*d^4))) - (8*a + 35*b
)*d*x - (2*b*cos(d*x + c)^3 + 11*b*cos(d*x + c))*sin(d*x + c))/(b^2*d)
```

giac [B] time = 1.10, size = 836, normalized size = 4.49

$$\frac{4\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^4+12\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^3b-34\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^2b^2-12\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,ab^3-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,b^4-12\sqrt{a^2-\cdots}\right.}{3a\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/8*(4*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b + sqr
t(a*b)*(a - b))*a^3*b - 34*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 12
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*b^4 - 12*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 + 12*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b + 28*sqrt(a^2 - a*b + sqrt(a
*b)*(a - b))*sqrt(a*b)*a*b^2 + 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*b^3)*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 +
sqrt(a^2*b^4 - (a*b^2 - b^3)*a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*b^
2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6) + 4*(3*sqrt(a^2 - a*b - sq
rt(a*b)*(a - b))*a^4 + 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b - 34*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4 + 12*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3 - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a - b)
)*sqrt(a*b)*a^2*b - 28*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^2
- 4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(pi*floor((d*x + c)/
pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b^2 - sqrt(a^2*b^4 - (a*b^2 - b^3)*
a*b^2))/(a*b^2 - b^3))))*abs(-a + b)/(3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 -
 4*a^2*b^5 - a*b^6) - (d*x + c)*(8*a + 35*b)/b^2 - (11*tan(d*x + c)^3 + 13*
tan(d*x + c))/((tan(d*x + c)^2 + 1)^2*b))/d
```

maple [B] time = 0.68, size = 750, normalized size = 4.03

$$-\frac{11\left(\tan^3(dx+c)\right)}{8db\left(\tan^2(dx+c)+1\right)^2}-\frac{13\tan(dx+c)}{8db\left(\tan^2(dx+c)+1\right)^2}-\frac{35\arctan(\tan(dx+c))}{8db}-\frac{\arctan(\tan(dx+c))\,a}{d\,b^2}+\frac{\operatorname{arctan}\cdots}{2d\,b^2\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -11/8/d/b/(tan(d*x+c)^2+1)^2*tan(d*x+c)^3-13/8/d/b/(tan(d*x+c)^2+1)^2*tan(d
*x+c)-35/8/d/b*arctan(tan(d*x+c))-1/d/b^2*arctan(tan(d*x+c))*a+1/2/d/b^2/((
(a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b
))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+3/2/d/b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b
))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2-1/d*a
/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b
)^(1/2)-a)*(a-b))^(1/2))+1/2/d/b^2/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)+a)*(
a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/2/d/b/
(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))*a^2+1/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*a
rctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-3/2/d/(((a*b)^(1/2)-a
)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2
/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))-3/2/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
```

```
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1
/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^8/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/32*(32*b^2*d*integrate(-16*(4*(a*b^2 + b^3)*cos(6*d*x + 6*c)^2 + 2*(8*a^
3 + 29*a^2*b - 20*a*b^2 + 3*b^3)*cos(4*d*x + 4*c)^2 + 4*(a*b^2 + b^3)*cos(2
*d*x + 2*c)^2 + 4*(a*b^2 + b^3)*sin(6*d*x + 6*c)^2 + 2*(8*a^3 + 29*a^2*b -
20*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)^2 + 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*sin(
4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c)^2 - ((a*b^
2 + b^3)*cos(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*cos(4*d*x + 4*c) + (a*b
^2 + b^3)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^2 + b^3 - 2*(10*a^2*b +
 13*a*b^2 - 5*b^3)*cos(4*d*x + 4*c) - 8*(a*b^2 + b^3)*cos(2*d*x + 2*c))*cos
(6*d*x + 6*c) - (a^2*b + 4*a*b^2 - b^3 - 2*(10*a^2*b + 13*a*b^2 - 5*b^3)*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^2 + b^3)*cos(2*d*x + 2*c) - ((a*b^2
 + b^3)*sin(6*d*x + 6*c) + (a^2*b + 4*a*b^2 - b^3)*sin(4*d*x + 4*c) + (a*b^
2 + b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((10*a^2*b + 13*a*b^2 - 5*b
^3)*sin(4*d*x + 4*c) + 4*(a*b^2 + b^3)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))/
(b^4*cos(8*d*x + 8*c)^2 + 16*b^4*cos(6*d*x + 6*c)^2 + 16*b^4*cos(2*d*x + 2*
c)^2 + b^4*sin(8*d*x + 8*c)^2 + 16*b^4*sin(6*d*x + 6*c)^2 + 16*b^4*sin(2*d*
x + 2*c)^2 - 8*b^4*cos(2*d*x + 2*c) + b^4 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^
4)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b^2 - 48*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^4*cos(6
*d*x + 6*c) + 4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3*b^4)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^4*cos(2*d*x + 2*c) - b^4 + 2*(8*a*b^3 - 3
*b^4)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^3 - 3*b^4 - 4*(8*a*b^3
- 3*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^4*sin(6*d*x + 6*c) + 2
*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^4*sin(2*d*x + 2*c) + (8*a*b^3 - 3*b^4)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 4*(8*a + 35*b)*d*x + b*sin(4*d*x + 4*c) + 24*b*sin(2*d*x
+ 2*c))/(b^2*d)
```

mupad [B] time = 17.46, size = 8773, normalized size = 47.17

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^8/(a - b*sin(c + d*x)^4),x)
```

```
[Out] - (atan(((((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 352
96*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/
(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^
9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-(7*a^3*(a^3*b
^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5
*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(
1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16
*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*
b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^
(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a
^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 23
04*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a
^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*
a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*
(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*
(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c +
```

d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*1i - (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*1i)/((11696*a*b^8 + 1247*a^8*b - 344*a^9 - 1505*b^9 - 39388*a^2*b^7 + 74648*a^3*b^6 - 86086*a^4*b^5 + 60200*a^5*b^4 - 22876*a^6*b^3 + 2408*a^7*b^2)/(32*b^5) + (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^

```
3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9
)^(1/2))/(16*a^3*b^8))^(1/2))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2
) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2
) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(336*a^8*
b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*
a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*(-(7*a^3*(
a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35*a^3*b^6 + 21*a^4*b^5
 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^
8))^(1/2)))*(-(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) + 7*a^2*b^7 + 35
*a^3*b^6 + 21*a^4*b^5 + a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*
b^9)^(1/2))/(16*a^3*b^8))^(1/2)*2i)/d - (atan(((((373728*a^3*b^8 - 256*b^11
 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a
^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2
*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(6
4*b^5) - (tan(c + d*x)*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^
2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a
^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10
 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(
a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(
a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d
*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7
 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*((7*a^3*(a^3*b^9
)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b
^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/
2))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6
 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2
))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1
257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 92
23*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^
9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^
9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*1i - (((373728*a^3
*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*a^4*b^7 - 240464*a^
5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64*b^5) - (((4096*a*b
^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 + 14336*a^5*b^8 - 12
288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^
9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^
9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*(12288*a^2*b^11 -
12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4))*((7*a^3*(a^3*b^9
)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b
^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/
2) - (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 +
 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(
(7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*
a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(1
6*a^3*b^8))^(1/2))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^
7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b
*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(336*a^8*b - 1497*a*b
^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 30
77*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2
) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 2
1*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*1i)
/((11696*a*b^8 + 1247*a^8*b - 344*a^9 - 1505*b^9 - 39388*a^2*b^7 + 74648*a^
3*b^6 - 86086*a^4*b^5 + 60200*a^5*b^4 - 22876*a^6*b^3 + 2408*a^7*b^2)/(32*b
^5) + (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b^10 + 35296*
a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*a^8*b^3)/(64
*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 69632*a^4*b^9 +
 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) - (tan(c + d*x)*((7*a^3*(a^3*b^9)^
(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4
 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)
```

```
*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(16*b^4
))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 -
 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2)
)/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^
9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^
7*b^4))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7
 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*
(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^
9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^
9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) + (tan(c + d*x)*(3
36*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 +
 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*((7
*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^
4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*
a^3*b^8))^(1/2) + (((373728*a^3*b^8 - 256*b^11 - 208208*a^2*b^9 - 17552*a*b
^10 + 35296*a^4*b^7 - 240464*a^5*b^6 + 29040*a^6*b^5 + 27648*a^7*b^4 + 768*
a^8*b^3)/(64*b^5) - (((4096*a*b^12 + 53248*a^2*b^11 - 129024*a^3*b^10 + 696
32*a^4*b^9 + 14336*a^5*b^8 - 12288*a^6*b^7)/(64*b^5) + (tan(c + d*x)*((7*a^
3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b
^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3
*b^8))^(1/2)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b
^8))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 -
35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^
3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan(c + d*x)*(256*a*b^10 + 256*b^11 -
70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b
^5 - 2304*a^7*b^4))/(16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2)
 - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2)
 + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2))*((7*a^3*(a^3*b^9)^(1/2) +
 b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a
*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2) - (tan
(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 418
61*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(
16*b^4))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2) - 7*a^2*b^7 - 35*a^3
*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2) + 35*a^2*b*(a^3*b^9)
^(1/2))/(16*a^3*b^8))^(1/2)))*((7*a^3*(a^3*b^9)^(1/2) + b^3*(a^3*b^9)^(1/2)
 - 7*a^2*b^7 - 35*a^3*b^6 - 21*a^4*b^5 - a^5*b^4 + 21*a*b^2*(a^3*b^9)^(1/2)
 + 35*a^2*b*(a^3*b^9)^(1/2))/(16*a^3*b^8))^(1/2)*2i)/d - ((13*tan(c + d*x))
/(8*b) + (11*tan(c + d*x)^3)/(8*b))/(d*(2*tan(c + d*x)^2 + tan(c + d*x)^4 +
 1)) - (atan((((a*8i + b*35i)*(((((11679*a^3*b^8)/2 - 4*b^11 - (13013*a^2*b
^9)/4 - (1097*a*b^10)/4 + (1103*a^4*b^7)/2 - (15029*a^5*b^6)/4 + (1815*a^6*
b^5)/4 + 432*a^7*b^4 + 12*a^8*b^3)/b^5 - (((((64*a*b^12 + 832*a^2*b^11 - 20
16*a^3*b^10 + 1088*a^4*b^9 + 224*a^5*b^8 - 192*a^6*b^7)/b^5 - (tan(c + d*x)
*(a*8i + b*35i)*(12288*a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^
5*b^8))/(256*b^6))*(a*8i + b*35i))/(16*b^2) + (tan(c + d*x)*(256*a*b^10 + 2
56*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 2
9696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(a*8i + b*35i))/(16*b^2))*(a*8i + b
*35i))/(16*b^2) + (tan(c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9
 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*
b^3 + 1721*a^7*b^2))/(16*b^4))*1i)/(16*b^2) - ((a*8i + b*35i)*(((((11679*a^
3*b^8)/2 - 4*b^11 - (13013*a^2*b^9)/4 - (1097*a*b^10)/4 + (1103*a^4*b^7)/2
- (15029*a^5*b^6)/4 + (1815*a^6*b^5)/4 + 432*a^7*b^4 + 12*a^8*b^3)/b^5 - ((
(((64*a*b^12 + 832*a^2*b^11 - 2016*a^3*b^10 + 1088*a^4*b^9 + 224*a^5*b^8 -
192*a^6*b^7)/b^5 + (tan(c + d*x)*(a*8i + b*35i)*(12288*a^2*b^11 - 12288*a^3
*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i + b*35i))/(16*b^2)
 - (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 5
3616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(a*
8i + b*35i))/(16*b^2))*(a*8i + b*35i))/(16*b^2) - (tan(c + d*x)*(336*a^8*b
 - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^
4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4))*1i)/(16*b^2)
```

```
)/(((731*a*b^8)/2 + (1247*a^8*b)/32 - (43*a^9)/4 - (1505*b^9)/32 - (9847*a^
2*b^7)/8 + (9331*a^3*b^6)/4 - (43043*a^4*b^5)/16 + (7525*a^5*b^4)/4 - (5719
*a^6*b^3)/8 + (301*a^7*b^2)/4)/b^5 + ((a*8i + b*35i)*(((((11679*a^3*b^8)/2
- 4*b^11 - (13013*a^2*b^9)/4 - (1097*a*b^10)/4 + (1103*a^4*b^7)/2 - (15029*
a^5*b^6)/4 + (1815*a^6*b^5)/4 + 432*a^7*b^4 + 12*a^8*b^3)/b^5 - (((((64*a*b
^12 + 832*a^2*b^11 - 2016*a^3*b^10 + 1088*a^4*b^9 + 224*a^5*b^8 - 192*a^6*b
^7)/b^5 - (tan(c + d*x)*(a*8i + b*35i)*(12288*a^2*b^11 - 12288*a^3*b^10 - 1
2288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8i + b*35i))/(16*b^2) + (tan(c
 + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9 + 61136*a^3*b^8 + 53616*a^4*
b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7*b^4))/(16*b^4))*(a*8i + b*35
i))/(16*b^2))*(a*8i + b*35i))/(16*b^2) + (tan(c + d*x)*(336*a^8*b - 1497*a*
b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 41861*a^3*b^6 + 27109*a^4*b^5 + 3
077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(16*b^4)))/(16*b^2) + ((a*8i +
b*35i)*(((((11679*a^3*b^8)/2 - 4*b^11 - (13013*a^2*b^9)/4 - (1097*a*b^10)/4
 + (1103*a^4*b^7)/2 - (15029*a^5*b^6)/4 + (1815*a^6*b^5)/4 + 432*a^7*b^4 +
12*a^8*b^3)/b^5 - (((((64*a*b^12 + 832*a^2*b^11 - 2016*a^3*b^10 + 1088*a^4*
b^9 + 224*a^5*b^8 - 192*a^6*b^7)/b^5 + (tan(c + d*x)*(a*8i + b*35i)*(12288*
a^2*b^11 - 12288*a^3*b^10 - 12288*a^4*b^9 + 12288*a^5*b^8))/(256*b^6))*(a*8
i + b*35i))/(16*b^2) - (tan(c + d*x)*(256*a*b^10 + 256*b^11 - 70832*a^2*b^9
 + 61136*a^3*b^8 + 53616*a^4*b^7 - 12432*a^5*b^6 - 29696*a^6*b^5 - 2304*a^7
*b^4))/(16*b^4))*(a*8i + b*35i))/(16*b^2))*(a*8i + b*35i))/(16*b^2) - (tan(
c + d*x)*(336*a^8*b - 1497*a*b^8 + 96*a^9 - 1257*b^9 + 21499*a^2*b^7 - 4186
1*a^3*b^6 + 27109*a^4*b^5 + 3077*a^5*b^4 - 9223*a^6*b^3 + 1721*a^7*b^2))/(1
6*b^4)))/(16*b^2)))*(a*8i + b*35i)*1i)/(8*b^2*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**8/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.413 $\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=155

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}-$$

```
[Out] -5/2*x/b-1/2*cos(d*x+c)*sin(d*x+c)/b/d-1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*t
an(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(5/2)/a^(3/4)/b^(3/2)/d+1/2*arctan((a^
(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)+b^(1/2))^(5/2)/a^(3/4)/b^
(3/2)/d
```

Rubi [A] time = 0.29, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3224, 1170, 199, 203, 1166, 205}

$$-\frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d}-\frac{\sin(c+dx)\cos(c+dx)}{2bd}-$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-5*x)/(2*b) - ((Sqrt[a] - Sqrt[b])^(5/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d) + ((Sqrt[a] + Sqrt[b])^(5/2)*A
rcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b^(3/2)*d
) - (Cos[c + d*x]*Sin[c + d*x])/(2*b*d)
```

Rule 199

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Simp[(x*(a + b*x^n)^(p + 1
))/(a*n*(p + 1)), x] + Dist[(n*(p + 1) + 1)/(a*n*(p + 1)), Int[(a + b*x^n)^
(p + 1), x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && LtQ[p, -1] && (Integer
Q[2*p] || (n == 2 && IntegerQ[4*p]) || (n == 2 && IntegerQ[3*p]) || Denomin
ator[p + 1/n] < Denominator[p])
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b\left(1+x^2\right)^2} - \frac{2}{b\left(1+x^2\right)} + \frac{3a+b+2(a-b)x^2}{b\left(a+2ax^2+(a-b)x^4\right)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{\left(1+x^2\right)^2}\,dx, x, \tan(c+dx)\right)}{bd} + \frac{\text{Subst}\left(\int \frac{3a+b+2(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd} - \cdots 2$$

$$= -\frac{2x}{b} - \frac{\cos(c+dx)\sin(c+dx)}{2bd} - \frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{2bd} + \frac{\left(2a-2b+\frac{(a-b)(\cdots}{\sqrt{a}\cdots}\right.}{\cdots}$$

$$= -\frac{5x}{2b} - \frac{\left(\sqrt{a}-\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}b^{3/2}d} + \frac{\left(\sqrt{a}+\sqrt{b}\right)^{5/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+\cdots}{\sqrt[4]{a}}\right.}{2a^{3/4}b^{3/2}d}$$

Mathematica [A] time = 0.48, size = 194, normalized size = 1.25

$$\frac{\frac{2\sqrt{b}\left(\sqrt{a}+\sqrt{b}\right)^3\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\,\sqrt{b}+a}} + \frac{2\sqrt{b}\left(\sqrt{a}-\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\,\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\,\sqrt{b}-a}} - 10b(c+dx) - b\sin(2(c+dx))}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-10*b*(c + d*x) + (2*(Sqrt[a] + Sqrt[b])^3*Sqrt[b]*ArcTan[((Sqrt[a] + Sqrt
[b])*Tan[c + d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sq
rt[b]]) + (2*(Sqrt[a] - Sqrt[b])^3*Sqrt[b]*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan
[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]
) - b*Sin[2*(c + d*x)])/(4*b^2*d)
```

fricas [B] time = 0.92, size = 1751, normalized size = 11.30

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

[Out] 1/8*(b*d*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3
+ b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/
2*a^2*b^2 + 2*a*b^3 + 1/4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*co
s(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 +
20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^
3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sq
rt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2
 - 10*a*b - 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^
2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*
a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - b*d*sqrt((a*b^3*d^2
*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) -
a^2 - 10*a*b - 5*b^2)/(a*b^3*d^2))*log(5/4*a^4 - 7/2*a^2*b^2 + 2*a*b^3 + 1/
4*b^4 - 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 - 1/2*(2*a^
3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5
*d^4))*cos(d*x + c)*sin(d*x + c) + (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b
^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b +
 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) - a^2 - 10*a*b - 5*b^2)/(a*b^
3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*
b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20
*a*b^3 + b^4)/(a^3*b^5*d^4))) + b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3
*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(
a*b^3*d^2))*log(-5/4*a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 1
4*a^2*b^2 + 8*a*b^3 + b^4)*cos(d*x + c)^2 + 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4
 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*si
n(d*x + c) - (5*a^4*b + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin
(d*x + c))*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b
^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4
*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2
*b^4)*d^2)*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^
5*d^4))) - b*d*sqrt(-(a*b^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20
*a*b^3 + b^4)/(a^3*b^5*d^4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2))*log(-5/4*
a^4 + 7/2*a^2*b^2 - 2*a*b^3 - 1/4*b^4 + 1/4*(5*a^4 - 14*a^2*b^2 + 8*a*b^3 +
 b^4)*cos(d*x + c)^2 - 1/2*(2*a^3*b^4*d^3*sqrt((25*a^4 + 100*a^3*b + 110*a^
2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))*cos(d*x + c)*sin(d*x + c) - (5*a^4*b
 + 15*a^3*b^2 + 11*a^2*b^3 + a*b^4)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b
^3*d^2*sqrt((25*a^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^
4)) + a^2 + 10*a*b + 5*b^2)/(a*b^3*d^2)) + 1/4*(2*(a^4*b^2 - 2*a^3*b^3 + a^
2*b^4)*d^2*cos(d*x + c)^2 - (a^4*b^2 - 2*a^3*b^3 + a^2*b^4)*d^2)*sqrt((25*a
^4 + 100*a^3*b + 110*a^2*b^2 + 20*a*b^3 + b^4)/(a^3*b^5*d^4))) - 20*d*x - 4
*cos(d*x + c)*sin(d*x + c))/(b*d)

giac [B] time = 1.00, size = 995, normalized size = 6.42

$$-\frac{\frac{5\,(dx+c)}{b}+\frac{\left(2\left(3\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2-6\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)b^2|-a+b|-\left(9\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^3b-\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")

[Out] -1/2*(5*(d*x + c)/b + (2*(3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a
^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (9*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*a^3*b - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 9
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a
- b))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt
(a*b)*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 7*s

```
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + a
rctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2)
)))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(b)) - (2
*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*b^2)*b^2*abs(-a + b) + (9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b -
 15*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 9*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4)*abs(-a + b)*a
bs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b - 3*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 7*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*
b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt(
(a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4
*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(b)) + tan(d*x + c)/((tan(d*x + c
)^2 + 1)*b))/d
```

maple [B] time = 0.66, size = 483, normalized size = 3.12

$$-\frac{\tan(dx+c)}{2db\left(\tan^2(dx+c)+1\right)}-\frac{5\arctan(\tan(dx+c))}{2db}+\frac{\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)a^2}{2db\sqrt{ab}\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{a\operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{db\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{a}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/2/d/b*tan(d*x+c)/(tan(d*x+c)^2+1)-5/2/d/b*arctan(tan(d*x+c))+1/2/d/b/(a*
b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1
/2)-a)*(a-b))^(1/2))*a^2+1/d/b*a/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+
b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/b/(a*b)^(1/2)/(((a*b)^(1
/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*
a^2+1/d/b*a/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))-1/d/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(
d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*
(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/d/(
((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b)
)^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan
(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/4*(4*b*d*integrate(-4*(4*(a*b + 3*b^2)*cos(6*d*x + 6*c)^2 + 4*(40*a^2 -
23*a*b + 3*b^2)*cos(4*d*x + 4*c)^2 + 4*(a*b + 3*b^2)*cos(2*d*x + 2*c)^2 + 4
*(a*b + 3*b^2)*sin(6*d*x + 6*c)^2 + 4*(40*a^2 - 23*a*b + 3*b^2)*sin(4*d*x +
 4*c)^2 + 2*(8*a^2 + 41*a*b - 13*b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*(a*b + 3*b^2)*sin(2*d*x + 2*c)^2 - ((a*b + 3*b^2)*cos(6*d*x + 6*c) + 2*(5*
a*b - b^2)*cos(4*d*x + 4*c) + (a*b + 3*b^2)*cos(2*d*x + 2*c))*cos(8*d*x + 8
*c) - (a*b + 3*b^2 - 2*(8*a^2 + 41*a*b - 13*b^2)*cos(4*d*x + 4*c) - 8*(a*b
+ 3*b^2)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 2*(5*a*b - b^2 - (8*a^2 + 41*
a*b - 13*b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b + 3*b^2)*cos(2*d*x
+ 2*c) - ((a*b + 3*b^2)*sin(6*d*x + 6*c) + 2*(5*a*b - b^2)*sin(4*d*x + 4*c)
 + (a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2 + 41*a*b -
13*b^2)*sin(4*d*x + 4*c) + 4*(a*b + 3*b^2)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
```

```
c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos(2*d*x
+ 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b^3*sin(
2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b^2 + 9*
b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x + 4*c)^
2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^3*cos(6
*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos(4*d*x
+ 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3
*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8*a*b^2
- 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6*c) + 2
*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*x + 8*c
) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(6*
d*x + 6*c)), x) + 10*d*x + sin(2*d*x + 2*c))/(b*d)
```

mupad [B] time = 18.04, size = 3088, normalized size = 19.92

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan((a^4*b^8*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2)
+ 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^
(1/2)*240i - a^3*b^9*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^
(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*
b^6))^(1/2)*108i - a^5*b^7*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3
*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(1
6*a^3*b^6))^(1/2)*80i - a^6*b^6*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2
*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2
))/(16*a^3*b^6))^(1/2)*120i + a^7*b^5*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2)
 + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7
)^(1/2))/(16*a^3*b^6))^(1/2)*60i + a^8*b^4*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^
(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^
3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*8i - a^3*b^11*sin(c + d*x)*(-(5*a^2*(a^3*
b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*
b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*64i + a^4*b^10*sin(c + d*x)*(-(5*a^2
*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 +
 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*128i + a^5*b^9*sin(c + d*x)*(-
(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4
*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*6080i + a^6*b^8*sin(c +
d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^
4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*4032i + a^7*b^7*s
in(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10
*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*320i + a^5
*b^11*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b
^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(5/2)*307
2i + a^6*b^10*sin(c + d*x)*(-(5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) +
 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(
5/2)*3072i)/(55*a^2*b^9*cos(c + d*x) + 540*a^3*b^8*cos(c + d*x) + 1035*a^4*
b^7*cos(c + d*x) + 45*a^5*b^6*cos(c + d*x) + a^6*b^5*cos(c + d*x) + 110*a^7
*b^4*cos(c + d*x) + 5*a^8*b^3*cos(c + d*x) + 50*a^6*cos(c + d*x)*(a^3*b^7)^
(1/2) + 10*b^6*cos(c + d*x)*(a^3*b^7)^(1/2) + a*b^10*cos(c + d*x) + 195*a*b
^5*cos(c + d*x)*(a^3*b^7)^(1/2) + 75*a^5*b*cos(c + d*x)*(a^3*b^7)^(1/2) + 1
002*a^2*b^4*cos(c + d*x)*(a^3*b^7)^(1/2) + 490*a^3*b^3*cos(c + d*x)*(a^3*b^
7)^(1/2) - 30*a^4*b^2*cos(c + d*x)*(a^3*b^7)^(1/2)))*(-(5*a^2*(a^3*b^7)^(1/
2) + b^2*(a^3*b^7)^(1/2) + 5*a^2*b^5 + 10*a^3*b^4 + a^4*b^3 + 10*a*b*(a^3*b
^7)^(1/2))/(16*a^3*b^6))^(1/2)*2i)/d + (atan((a^4*b^8*sin(c + d*x)*((5*a^2*
(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 +
10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*240i - a^3*b^9*sin(c + d*x)*((5
*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b
^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*108i - a^5*b^7*sin(c + d*x
```

```
)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 -
a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*80i - a^6*b^6*sin(c +
 d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^
4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*120i + a^7*b^5*si
n(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a
^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*60i + a^8*b^
4*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 -
10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*8i - a^3
*b^11*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^
5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)*64i
+ a^4*b^10*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a
^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(3/2)
*128i + a^5*b^9*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2)
- 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^
(3/2)*6080i + a^6*b^8*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^
(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*
b^6))^(3/2)*4032i + a^7*b^7*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3
*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(1
6*a^3*b^6))^(3/2)*320i + a^5*b^11*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/2) + b^
2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b^7)^(1/
2))/(16*a^3*b^6))^(5/2)*3072i + a^6*b^10*sin(c + d*x)*((5*a^2*(a^3*b^7)^(1/
2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4 - a^4*b^3 + 10*a*b*(a^3*b
^7)^(1/2))/(16*a^3*b^6))^(5/2)*3072i)/(55*a^2*b^9*cos(c + d*x) + 540*a^3*b^
8*cos(c + d*x) + 1035*a^4*b^7*cos(c + d*x) + 45*a^5*b^6*cos(c + d*x) + a^6*
b^5*cos(c + d*x) + 110*a^7*b^4*cos(c + d*x) + 5*a^8*b^3*cos(c + d*x) - 50*a
^6*cos(c + d*x)*(a^3*b^7)^(1/2) - 10*b^6*cos(c + d*x)*(a^3*b^7)^(1/2) + a*b
^10*cos(c + d*x) - 195*a*b^5*cos(c + d*x)*(a^3*b^7)^(1/2) - 75*a^5*b*cos(c
+ d*x)*(a^3*b^7)^(1/2) - 1002*a^2*b^4*cos(c + d*x)*(a^3*b^7)^(1/2) - 490*a^
3*b^3*cos(c + d*x)*(a^3*b^7)^(1/2) + 30*a^4*b^2*cos(c + d*x)*(a^3*b^7)^(1/2
)))*((5*a^2*(a^3*b^7)^(1/2) + b^2*(a^3*b^7)^(1/2) - 5*a^2*b^5 - 10*a^3*b^4
- a^4*b^3 + 10*a*b*(a^3*b^7)^(1/2))/(16*a^3*b^6))^(1/2)*2i)/d - sin(2*c + 2
*d*x)/(4*b*d) - (5*atan(sin(c + d*x)/cos(c + d*x)))/(2*b*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.414 $\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=127

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}-\frac{x}{b}$$

```
[Out] -x/b+1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2
))^(3/2)/a^(3/4)/b/d+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))
*(a^(1/2)+b^(1/2))^(3/2)/a^(3/4)/b/d
```

Rubi [A] time = 0.24, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3224, 1170, 203, 1166, 205}

$$\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}-\frac{x}{b}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(x/b) + ((Sqrt[a] - Sqrt[b])^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c +
 d*x])/a^(1/4)])/(2*a^(3/4)*b*d) + ((Sqrt[a] + Sqrt[b])^(3/2)*ArcTan[(Sqrt[
Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*b*d)
```

Rule 203

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTan[(Rt[b, 2]*x)/Rt
[a, 2]])/(Rt[a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
```

```
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\left(a+2ax^2+(a-b)x^4\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{1}{b(1+x^2)}+\frac{a+b+(a-b)x^2}{b\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\text{Subst}\left(\int \frac{1}{1+x^2}\,dx, x, \tan(c+dx)\right)}{bd}+\frac{\text{Subst}\left(\int \frac{a+b+(a-b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{bd}$$

$$= -\frac{x}{b}+\frac{\left(\left(1-\frac{\sqrt{b}}{\sqrt{a}}\right)(a-b)\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2bd}+\frac{\left(\left(1+\frac{\sqrt{b}}{\sqrt{a}}\right)(a-\right.}{}$$

$$= -\frac{x}{b}+\frac{\left(\sqrt{a}-\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}bd}+\frac{\left(\sqrt{a}+\sqrt{b}\right)^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+d}{\sqrt[4]{a}}\right.}{2a^{3/4}bd}$$

Mathematica [A] time = 0.25, size = 171, normalized size = 1.35

$$\frac{\frac{\left(\sqrt{a}+\sqrt{b}\right)^2\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}-\frac{\left(\sqrt{a}-\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}-2(c+dx)}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (-2*(c + d*x) + ((Sqrt[a] + Sqrt[b])^2*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c +
d*x])/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - ((S
qrt[a] - Sqrt[b])^2*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sq
rt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]))/(2*b*d)
```

fricas [B] time = 0.68, size = 1197, normalized size = 9.43

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*(b*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)
/(a*b^2*d^2))*log(1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*
a*b + 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*
cos(d*x + c)*sin(d*x + c) + (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*
sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2
*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^
2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt((a*b^2*d^2*sqrt((9*a
^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2))*log(1/4*(3*a^2 - 2
*a*b - b^2)*cos(d*x + c)^2 - 3/4*a^2 + 1/2*a*b + 1/4*b^2 - 1/2*(a^3*b^2*d^3
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a
^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt((a*b^2*d^2*sqrt((9*a^2 + 6*
```

```
a*b + b^2)/(a^3*b^3*d^4)) - a - 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2
)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))) + b*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)
) + a + 3*b)/(a*b^2*d^2))*log(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3
/4*a^2 - 1/2*a*b - 1/4*b^2 + 1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a
^3*b^3*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a*b^2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a
 + 3*b)/(a*b^2*d^2)) - 1/4*(2*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b
 - a^2*b^2)*d^2)*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))) - b*sqrt(-(a*b^
2*d^2*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2))*log
(-1/4*(3*a^2 - 2*a*b - b^2)*cos(d*x + c)^2 + 3/4*a^2 - 1/2*a*b - 1/4*b^2 -
1/2*(a^3*b^2*d^3*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4))*cos(d*x + c)*sin
(d*x + c) - (3*a^2*b + a*b^2)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a*b^2*d^2
*sqrt((9*a^2 + 6*a*b + b^2)/(a^3*b^3*d^4)) + a + 3*b)/(a*b^2*d^2)) - 1/4*(2
*(a^3*b - a^2*b^2)*d^2*cos(d*x + c)^2 - (a^3*b - a^2*b^2)*d^2)*sqrt((9*a^2
+ 6*a*b + b^2)/(a^3*b^3*d^4))) - 8*x)/b
```

giac [B] time = 0.97, size = 906, normalized size = 7.13

$$-\frac{\frac{2\,(dx+c)}{b}+\frac{\left(\left(3\,\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,a^2-6\,\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,ab-\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,\sqrt{ab}\,b^2\right)b^2|-a+b|-\left(3\,\sqrt{a^2-ab+\sqrt{ab}\,(a-b)}\,a^3b-3\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*(2*(d*x + c)/b + ((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b + s
qrt(a*b)*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a^3*b - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^2 - 7*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*b^4)*abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^2*b^2 - 6*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a
^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x +
c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a*b + sqrt(a^2*b^2 - (a*b - b^2)*a
*b))/(a*b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b
^6)*abs(b)) - ((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)
*(a - b))*sqrt(a*b)*b^2)*b^2*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a
 - b))*a^3*b - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^2 - 7*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a*b^3 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^4)*
abs(-a + b)*abs(b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b
^2 - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*abs(-a + b))*(pi*floor((d*x + c)/pi +
1/2) + arctan(tan(d*x + c)/sqrt((a*b - sqrt(a^2*b^2 - (a*b - b^2)*a*b))/(a*
b - b^2))))/((3*a^5*b^2 - 12*a^4*b^3 + 14*a^3*b^4 - 4*a^2*b^5 - a*b^6)*abs(
b)))/d
```

maple [B] time = 0.62, size = 449, normalized size = 3.54

$$-\frac{\arctan\left(\tan\left(dx+c\right)\right)}{db}+\frac{a\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2db\sqrt{\left(\sqrt{ab}\,-a\right)(a-b)}}+\frac{a\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}\,-a\right)(a-b)}}+\frac{a\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2db\sqrt{\left(\sqrt{ab}\,+a\right)(a-b)}}-$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] -1/d/b*arctan(tan(d*x+c))+1/2/d/b*a/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((
-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*a/(a*b)^(1/2)/(((a*b)
^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1
/2))+1/2/d/b*a/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b
)^(1/2)+a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d/(((a*b)^(1/2)-
a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/
2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(
((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a
-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(
1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -(b*integrate(-8*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^2*cos(2*d*x + 2*c)^2 + 4*b
^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2 + 4*(8*a^2 - 3*a*b)*cos(4*
d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) + 4*(8*a^2 - 3*a*b)*sin(4*d*x + 4*c)^2
+ 6*(4*a*b - b^2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - (b^2*cos(6*d*x + 6*c)
 + 2*a*b*cos(4*d*x + 4*c) + b^2*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + (8*b^2
*cos(2*d*x + 2*c) - b^2 + 6*(4*a*b - b^2)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c
) - 2*(a*b - 3*(4*a*b - b^2)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(
6*d*x + 6*c) + 2*a*b*sin(4*d*x + 4*c) + b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8
*c) + 2*(4*b^2*sin(2*d*x + 2*c) + 3*(4*a*b - b^2)*sin(4*d*x + 4*c))*sin(6*d
*x + 6*c))/(b^3*cos(8*d*x + 8*c)^2 + 16*b^3*cos(6*d*x + 6*c)^2 + 16*b^3*cos
(2*d*x + 2*c)^2 + b^3*sin(8*d*x + 8*c)^2 + 16*b^3*sin(6*d*x + 6*c)^2 + 16*b
^3*sin(2*d*x + 2*c)^2 - 8*b^3*cos(2*d*x + 2*c) + b^3 + 4*(64*a^2*b - 48*a*b
^2 + 9*b^3)*cos(4*d*x + 4*c)^2 + 4*(64*a^2*b - 48*a*b^2 + 9*b^3)*sin(4*d*x
+ 4*c)^2 + 16*(8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) - 2*(4*b^
3*cos(6*d*x + 6*c) + 4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*b^2 - 3*b^3)*cos
(4*d*x + 4*c))*cos(8*d*x + 8*c) + 8*(4*b^3*cos(2*d*x + 2*c) - b^3 + 2*(8*a*
b^2 - 3*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) - 4*(8*a*b^2 - 3*b^3 - 4*(8
*a*b^2 - 3*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(2*b^3*sin(6*d*x + 6
*c) + 2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))*sin(8*d*
x + 8*c) + 16*(2*b^3*sin(2*d*x + 2*c) + (8*a*b^2 - 3*b^3)*sin(4*d*x + 4*c))
*sin(6*d*x + 6*c)), x) + x)/b
```

mupad [B] time = 16.40, size = 4299, normalized size = 33.85

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4/(a - b*sin(c + d*x)^4),x)
```

```
[Out] atan((90*a^4*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*
b^2 + (18*a^5)/b) - (18*a^5*tan(c + d*x))/(10*a*b^4 - 90*a^4*b + 18*a^5 - 2
*b^5 - 68*a^2*b^3 + 132*a^3*b^2) + (2*b^4*tan(c + d*x))/(10*a*b^3 + 132*a^3
*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) + (68*a^2*b^2*tan(c + d*x))/
(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*a^5)/b) - (10*a*b
^3*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 - 68*a^2*b^2 + (18*
a^5)/b) - (132*a^3*b*tan(c + d*x))/(10*a*b^3 + 132*a^3*b - 90*a^4 - 2*b^4 -
 68*a^2*b^2 + (18*a^5)/b))/(b*d) + (atan(((tan(c + d*x)*(30*a*b^4 - 30*a^4*
b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + (-(3*a*(a^3*b^5)^(1/2) + b*(
a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^
```

```
5*b - 4*b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3
*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b
^4 - 192*a^5*b^3 + tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2)
+ 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768
*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 -
480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^
5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b
^3 + 12*a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 +
a^3*b^2)/(16*a^3*b^4))^(1/2)*1i + (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^
5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)
^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*
b^6 + ((-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(1
6*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192
*a^5*b^3 - tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*
b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5
 + 768*a^5*b^4)) - tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*
b^4 + 48*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2)
 + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*
a^4*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)
/(16*a^3*b^4))^(1/2)*1i)/((tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^
5 - 60*a^2*b^3 + 60*a^3*b^2) + (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) +
 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + ((
-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^
4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3
 + tan(c + d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^
3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a
^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48
*a^4*b^3 + 144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2
*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2)
)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3
*b^4))^(1/2) - (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*
b^3 + 60*a^3*b^2) - (-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3
+ a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + ((-(3*a*(a^3*
b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2)*(
64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 - tan(c +
d*x)*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*
a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) -
tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 +
144*a^5*b^2))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*
b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*(-(3*a*(a
^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a^3*b^4))^(1/2
)))*(-(3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) + 3*a^2*b^3 + a^3*b^2)/(16*a
^3*b^4))^(1/2)*2i)/d + (atan(((tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 -
6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) + ((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2
) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 +
 (((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*
b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b
^3 + tan(c + d*x)*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a
^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*
a^5*b^4)) + tan(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 4
8*a^4*b^3 + 144*a^5*b^2))*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2
*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2)
)*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*
b^4))^(1/2)*1i + (tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^
2*b^3 + 60*a^3*b^2) - ((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3
 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + (((3*a*(a^3*
b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(
64*a*b^7 + 256*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 - tan(c +
d*x)*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a
```

```
^3*b^4))^(1/2)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) - t
an(c + d*x)*(80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 1
44*a^5*b^2))*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^
2)/(16*a^3*b^4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*((3*a*(a^3*
b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*1
i)/((tan(c + d*x)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^
3*b^2) + ((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(
16*a^3*b^4))^(1/2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + (((3*a*(a^3*b^5)^(1/2) +
b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 25
6*a^2*b^6 - 896*a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 + tan(c + d*x)*((3*a*(a
^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2
)*(768*a^2*b^7 - 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) + tan(c + d*x)*(
80*a*b^6 - 16*b^7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*
((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^
4))^(1/2) - 72*a^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*((3*a*(a^3*b^5)^(1/2) +
b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2) - (tan(c + d*x
)*(30*a*b^4 - 30*a^4*b + 6*a^5 - 6*b^5 - 60*a^2*b^3 + 60*a^3*b^2) - ((3*a*(
a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/
2)*(36*a*b^5 - 12*a^5*b - 4*b^6 + (((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2
) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(64*a*b^7 + 256*a^2*b^6 - 896*
a^3*b^5 + 768*a^4*b^4 - 192*a^5*b^3 - tan(c + d*x)*((3*a*(a^3*b^5)^(1/2) +
b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*(768*a^2*b^7 -
 768*a^3*b^6 - 768*a^4*b^5 + 768*a^5*b^4)) - tan(c + d*x)*(80*a*b^6 - 16*b^
7 + 224*a^2*b^5 - 480*a^3*b^4 + 48*a^4*b^3 + 144*a^5*b^2))*((3*a*(a^3*b^5)^
(1/2) + b*(a^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2) - 72*a
^2*b^4 + 40*a^3*b^3 + 12*a^4*b^2))*((3*a*(a^3*b^5)^(1/2) + b*(a^3*b^5)^(1/2
) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)))*((3*a*(a^3*b^5)^(1/2) + b*(a
^3*b^5)^(1/2) - 3*a^2*b^3 - a^3*b^2)/(16*a^3*b^4))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.415 $\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}$$

```
[Out] -1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*(a^(1/2)-b^(1/2))^(
1/2)/a^(3/4)/d/b^(1/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4
))*(a^(1/2)+b^(1/2))^(1/2)/a^(3/4)/d/b^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3224, 1093, 205}

$$\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}-\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[Sqrt[a] - Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(
1/4)])/(2*a^(3/4)*Sqrt[b]*d) + (Sqrt[Sqrt[a] + Sqrt[b]]*ArcTan[(Sqrt[Sqrt[a
] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*Sqrt[b]*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1093

```
Int[((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(-1), x_Symbol] :> With[{q = Rt[b^
2 - 4*a*c, 2]}, Dist[c/q, Int[1/(b/2 - q/2 + c*x^2), x], x] - Dist[c/q, Int
[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c,
0] && PosQ[b^2 - 4*a*c]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2\sqrt{a}\sqrt{b}\,d} - \frac{(a-b)\,\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\right.}{2\sqrt{a}\sqrt{b}\,d}$$
$$= -\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d} + \frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\sqrt{b}\,d}$$

Mathematica [A] time = 0.28, size = 158, normalized size = 1.26

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}+b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(\sqrt{a}\sqrt{b}-b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{\sqrt{a}\sqrt{b}-a}}}{2\sqrt{a}\,bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/Sqrt[a + Sqrt[a]*Sqrt[b]] + ((Sqrt[a]*Sqrt[b] - b)*ArcTa
nh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]])/Sqrt[-a
+ Sqrt[a]*Sqrt[b]])/(2*Sqrt[a]*b*d)
```

fricas [B] time = 0.56, size = 541, normalized size = 4.33

$$-\frac{1}{8}\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\log\left(\frac{1}{2}ad\sqrt{-\frac{abd^2\sqrt{\frac{1}{a^3bd^4}}+1}{abd^2}}\cos(dx+c)\sin(dx+c)+\frac{1}{4}\cos(dx+c)^2+\frac{1}{4}\left(2a^2d^2\cos\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] -1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(1/2*a*d*sqrt(-(
a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d*x + c) + 1/4
*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^
4)) - 1/4) + 1/8*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*log(-1/
2*a*d*sqrt(-(a*b*d^2*sqrt(1/(a^3*b*d^4)) + 1)/(a*b*d^2))*cos(d*x + c)*sin(d
*x + c) + 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^2*d^2)*sqr
t(1/(a^3*b*d^4)) - 1/4) + 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d
^2))*log(1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2))*cos(d*x
+ c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x + c)^2 - a^
2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4) - 1/8*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) -
 1)/(a*b*d^2))*log(-1/2*a*d*sqrt((a*b*d^2*sqrt(1/(a^3*b*d^4)) - 1)/(a*b*d^2
))*cos(d*x + c)*sin(d*x + c) - 1/4*cos(d*x + c)^2 + 1/4*(2*a^2*d^2*cos(d*x
+ c)^2 - a^2*d^2)*sqrt(1/(a^3*b*d^4)) + 1/4)
```

giac [B] time = 0.93, size = 559, normalized size = 4.47

$$\frac{\left(3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,a^2b-6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,ab^2-\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,b^3-3\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,\sqrt{ab}\,a^2+6\sqrt{a^2-ab+\sqrt{ab}(a-b)}\,\sqrt{ab}\,ab+\sqrt{a^2-ab+\right.}{3a^5b-12a^4b^2+14a^3b^3-4a^2b^4-ab^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/2*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b - 6*sqrt(a^2 - a*b + sqrt
(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^3 - 3*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 + 6*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*sqrt(a*b)*a*b + sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^2)*(
pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sqrt((4*a + sqrt(-16*(
a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12*a^4*b^2 + 14*a^3*b^
3 - 4*a^2*b^4 - a*b^5) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b - 6*s
qrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*b^3 + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2 - 6*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b - sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*b^2)*(pi*floor((d*x + c)/pi + 1/2) + arctan(2*tan(d*x + c)/sq
rt((4*a - sqrt(-16*(a - b)*a + 16*a^2))/(a - b))))*abs(a - b)/(3*a^5*b - 12
*a^4*b^2 + 14*a^3*b^3 - 4*a^2*b^4 - a*b^5))/d
```

maple [B] time = 0.58, size = 226, normalized size = 1.81

$$\frac{a \operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{a\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}-\frac{b \operatorname{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{b\arctan\left(\frac{(a-b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}\right)}{2d\sqrt{ab}\,\sqrt{\left(\sqrt{ab}+a\right)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)
/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*a/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))
^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b/(a*b)
^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2
)-a)*(a-b))^(1/2))+1/2/d*b/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan
((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$-\int \frac{\cos\left(dx+c\right)^{2}}{b\sin\left(dx+c\right)^{4}-a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -integrate(cos(d*x + c)^2/(b*sin(d*x + c)^4 - a), x)
```

mupad [B] time = 15.66, size = 1409, normalized size = 11.27

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2/(a - b*sin(c + d*x)^4),x)
```

```
[Out] (atan(((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^(
1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c
 + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(
16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i + (
tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2) + a
^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*
(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b
```

```
^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)*1i)/((tan(c +
d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (-((a^3*b^3)^(1/2) + a^2*b)/(1
6*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan(c + d*x)*(64*a^4*
b + 64*a^2*b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/
2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2) - (tan(c + d*x)*(12*a*
b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))
^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*(64*a^4*b + 64*a^2*
b^3 - 128*a^3*b^2)*(-((a^3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)))*(-((a^
3*b^3)^(1/2) + a^2*b)/(16*a^3*b^2))^(1/2)))*(-((a^3*b^3)^(1/2) + a^2*b)/(16
*a^3*b^2))^(1/2)*2i)/d + (atan(((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3
- 4*b^3) + (((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^
3*b - 32*a^2*b^2 + tan(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^
3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*
a^3*b^2))^(1/2)*1i + (tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) -
(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^
2*b^2 - tan(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2
) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(
1/2)*1i)/((tan(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) + (((a^3*b^3)
^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 + tan
(c + d*x)*(64*a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2) - a^2*b)/
(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2) - (tan
(c + d*x)*(12*a*b^2 - 12*a^2*b + 4*a^3 - 4*b^3) - (((a^3*b^3)^(1/2) - a^2*b
)/(16*a^3*b^2))^(1/2)*(16*a*b^3 + 16*a^3*b - 32*a^2*b^2 - tan(c + d*x)*(64*
a^4*b + 64*a^2*b^3 - 128*a^3*b^2)*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^
(1/2)))*(((a^3*b^3)^(1/2) - a^2*b)/(16*a^3*b^2))^(1/2)))*(((a^3*b^3)^(1/2)
- a^2*b)/(16*a^3*b^2))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.416 $\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=142

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{d(a-b)}$$

```
[Out] -1/2*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))*b^(1/2)/a^(3/4)/d/(
a^(1/2)-b^(1/2))^(3/2)+1/2*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4
))*b^(1/2)/a^(3/4)/d/(a^(1/2)+b^(1/2))^(3/2)+tan(d*x+c)/(a-b)/d
```

Rubi [A] time = 0.23, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$-\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{3/2}}+\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{3/2}}+\frac{\tan(c+dx)}{d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4
)*(Sqrt[a] - Sqrt[b])^(3/2)*d) + (Sqrt[b]*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*T
an[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(3/2)*d) + Tan[c + d*
x]/((a - b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{1}{a-b} - \frac{b\left(1+2x^2\right)}{(a-b)\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\tan(c+dx)}{(a-b)d} - \frac{b\,\text{Subst}\left(\int \frac{1+2x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{(a-b)d}$$

$$= \frac{\tan(c+dx)}{(a-b)d} - \frac{\left(\left(\sqrt{a}+\sqrt{b}\right)^2\sqrt{b}\right)\text{Subst}\left(\int \frac{1}{a+\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x, \tan(c+dx)\right)}{2\sqrt{a}\,(a-b)d} - \frac{\left(b\left(2\right.\right.}{}$$

$$= -\frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{3/2}d} + \frac{\sqrt{b}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{3/2}d} + \frac{\tan(c+dx)}{(a-b)d}$$

Mathematica [A] time = 0.49, size = 175, normalized size = 1.23

$$\frac{\frac{\left(\sqrt{a}\sqrt{b}-b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}} + \frac{\left(\sqrt{a}\sqrt{b}+b\right)\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}} + 2\tan(c+dx)}{2d(a-b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (((Sqrt[a]*Sqrt[b] - b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])/Sqrt[a +
Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + ((Sqrt[a]*Sqrt[b]
+ b)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqrt[a]*Sqrt[b]]]
)/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*Tan[c + d*x])/(2*(a - b)*d)
```

fricas [B] time = 0.89, size = 2589, normalized size = 18.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/8*((a - b)*d*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2 + 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d
*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^
2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 -
a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 -
20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 -
 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a
^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqr
t((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 1
```

```
5*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(((a^4 - 3*a^3*b +
3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 1
5*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) - a*b - 3*
b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(3/4*a*b^2
+ 1/4*b^3 - 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b + 3*
a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 1
5*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x +
c)*sin(d*x + c) + (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c)
)*sqrt(((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 +
 b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 +
a^3*b^6)*d^4)) - a*b - 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) -
1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*
a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 -
6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))
) + (a - b)*d*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^
3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 -
 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3)*d^2))*cos(d*x + c)*log(-3/4*a*b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(
d*x + c)^2 + 1/2*(2*(a^6 - 3*a^5*b + 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b
^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4
- 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b
^2 + a*b^3)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2
- a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2
- 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4
 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 -
 a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*s
qrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 +
 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))) - (a - b)*d*sqrt(-((a^4 - 3*a^3*b
 + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b
+ 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4)) + a*b +
 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2))*cos(d*x + c)*log(-3/4*a*
b^2 - 1/4*b^3 + 1/4*(3*a*b^2 + b^3)*cos(d*x + c)^2 - 1/2*(2*(a^6 - 3*a^5*b
+ 3*a^4*b^2 - a^3*b^3)*d^3*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a^9 - 6*a^8*b
 + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*d^4))*cos(d*
x + c)*sin(d*x + c) - (3*a^3*b + 4*a^2*b^2 + a*b^3)*d*cos(d*x + c)*sin(d*x
+ c))*sqrt(-((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2*sqrt((9*a^2*b^3 + 6*a*
b^4 + b^5)/((a^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b
^5 + a^3*b^6)*d^4)) + a*b + 3*b^2)/((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*d^2
)) - 1/4*(2*(a^5 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2*cos(d*x + c)^2 - (a^5
 - 3*a^4*b + 3*a^3*b^2 - a^2*b^3)*d^2)*sqrt((9*a^2*b^3 + 6*a*b^4 + b^5)/((a
^9 - 6*a^8*b + 15*a^7*b^2 - 20*a^6*b^3 + 15*a^5*b^4 - 6*a^4*b^5 + a^3*b^6)*
d^4))) + 8*sin(d*x + c))/((a - b)*d*cos(d*x + c))
```

giac [B] time = 1.00, size = 1211, normalized size = 8.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] -1/2*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5 - 9*sqrt(a^2 - a
*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2 - a*b - sqrt(a*b)*(a -
 b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
^2*b^3 - 5*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(a^2 - a*b - sqrt(a*b)*(
a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a
*b^2 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^3)*(a - b)^2 + (3*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*a^3*b^2 + 14*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^3 - 4*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^5
```

```
)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^
2 - a*b + sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*a*b + b^2)))/(a^2 - 2*a
*b + b^2))))/(3*a^8 - 21*a^7*b + 59*a^6*b^2 - 85*a^5*b^3 + 65*a^4*b^4 - 23*
a^3*b^5 + a^2*b^6 + a*b^7) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^5 - 9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b + 2*sqrt(a^2
 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 + 10*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^2*b^3 - 5*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5 - 2*(3*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b - 6*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a*b^2 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)
*b^3)*(a - b)^2 - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b - 12*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*a^3*b^2 + 14*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*a^2*b^3 - 4*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^4 - sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*b^5)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + arcta
n(tan(d*x + c)/sqrt((a^2 - a*b - sqrt((a^2 - a*b)^2 - (a^2 - a*b)*(a^2 - 2*
a*b + b^2)))/(a^2 - 2*a*b + b^2))))/(3*a^8 - 21*a^7*b + 59*a^6*b^2 - 85*a^5
*b^3 + 65*a^4*b^4 - 23*a^3*b^5 + a^2*b^6 + a*b^7) - 2*tan(d*x + c)/(a - b))
/d
```

maple [B] time = 0.70, size = 393, normalized size = 2.77

$$\frac{\tan(dx+c)}{(a-b)\,d}+\frac{b\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)a}{2d\sqrt{ab}\,\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)b^{2}}{2d\sqrt{ab}\,\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}-\frac{b\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)}{d\,(a-b)\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x)
```

```
[Out] tan(d*x+c)/(a-b)/d+1/2/d*b/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*
arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a+1/2/d/(a*b)^(1/2
)/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/
2)-a)*(a-b))^(1/2))*b^2-1/d*b/(a-b)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((
-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))-1/2/d*b/(a*b)^(1/2)/(a-b)/(
((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b)
)^(1/2))*a-1/2/d/(a*b)^(1/2)/(a-b)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-
b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*b^2-1/d*b/(a-b)/(((a*b)^(1/2)+
a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] (((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d*sin(2*d*x + 2*c)^2 + 2*(a - b)*d
*cos(2*d*x + 2*c) + (a - b)*d)*integrate(4*(4*b^2*cos(6*d*x + 6*c)^2 + 4*b^
2*cos(2*d*x + 2*c)^2 + 4*b^2*sin(6*d*x + 6*c)^2 + 4*b^2*sin(2*d*x + 2*c)^2
- 12*(8*a*b - 3*b^2)*cos(4*d*x + 4*c)^2 - b^2*cos(2*d*x + 2*c) - 12*(8*a*b
- 3*b^2)*sin(4*d*x + 4*c)^2 + 2*(8*a*b - 15*b^2)*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) - (b^2*cos(6*d*x + 6*c) - 6*b^2*cos(4*d*x + 4*c) + b^2*cos(2*d*x +
2*c))*cos(8*d*x + 8*c) + (8*b^2*cos(2*d*x + 2*c) - b^2 + 2*(8*a*b - 15*b^2)
*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(3*b^2 + (8*a*b - 15*b^2)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - (b^2*sin(6*d*x + 6*c) - 6*b^2*sin(4*d*x + 4*c)
+ b^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*(4*b^2*sin(2*d*x + 2*c) + (8*a
*b - 15*b^2)*sin(4*d*x + 4*c))*sin(6*d*x + 6*c))/(a*b^2 - b^3 + (a*b^2 - b^
3)*cos(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*cos(6*d*x + 6*c)^2 + 4*(64*a^3 - 1
12*a^2*b + 57*a*b^2 - 9*b^3)*cos(4*d*x + 4*c)^2 + 16*(a*b^2 - b^3)*cos(2*d*
```

```
x + 2*c)^2 + (a*b^2 - b^3)*sin(8*d*x + 8*c)^2 + 16*(a*b^2 - b^3)*sin(6*d*x
+ 6*c)^2 + 4*(64*a^3 - 112*a^2*b + 57*a*b^2 - 9*b^3)*sin(4*d*x + 4*c)^2 + 1
6*(8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a*b^
2 - b^3)*sin(2*d*x + 2*c)^2 + 2*(a*b^2 - b^3 - 4*(a*b^2 - b^3)*cos(6*d*x +
6*c) - 2*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*co
s(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a*b^2 - b^3 - 2*(8*a^2*b - 11*a*b^2 +
 3*b^3)*cos(4*d*x + 4*c) - 4*(a*b^2 - b^3)*cos(2*d*x + 2*c))*cos(6*d*x + 6*
c) - 4*(8*a^2*b - 11*a*b^2 + 3*b^3 - 4*(8*a^2*b - 11*a*b^2 + 3*b^3)*cos(2*d
*x + 2*c))*cos(4*d*x + 4*c) - 8*(a*b^2 - b^3)*cos(2*d*x + 2*c) - 4*(2*(a*b^
2 - b^3)*sin(6*d*x + 6*c) + (8*a^2*b - 11*a*b^2 + 3*b^3)*sin(4*d*x + 4*c) +
 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*((8*a^2*b - 11*a*b
^2 + 3*b^3)*sin(4*d*x + 4*c) + 2*(a*b^2 - b^3)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c)), x) + 2*sin(2*d*x + 2*c))/((a - b)*d*cos(2*d*x + 2*c)^2 + (a - b)*d
*sin(2*d*x + 2*c)^2 + 2*(a - b)*d*cos(2*d*x + 2*c) + (a - b)*d)
```

mupad [B] time = 16.94, size = 2832, normalized size = 19.94

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^2*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] tan(c + d*x)/(d*(a - b)) + (atan(((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i - (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) + (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2)*1i)/((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/
(a - b) - (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b
 + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b -
 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*((3*a*(a^
3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 +
a^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*b^3)/(a - b) + (((2*(8*a*b^4 - 16*a^2*b^3
 + 8*a^3*b^2))/(a - b) + (4*tan(c + d*x)*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1
/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*((3*a*(a^3*
b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^
3*b^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a
- b))*((3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3
*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)))*((3*a*(a^3*b^3)^(1/2) + b*(a^
3*b^3)^(1/2) + a^3*b + 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)
))^(1/2)*2i)/d + (atan(((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b) -
(4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*
b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^2*b
^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1
/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^3)^
(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3
 - 3*a^4*b^2)))^(1/2)*1i - (((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a - b)
```

```
 + (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a
^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16*a^
2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*
b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))
^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3*b^
3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*
b^3 - 3*a^4*b^2)))^(1/2)*1i)/((((2*(8*a*b^4 - 16*a^2*b^3 + 8*a^3*b^2))/(a -
 b) - (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b -
3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)*(16*a^5*b - 16
*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a
^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2
)))^(1/2) - (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a - b))*(-(3*a*(a^3
*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a
^3*b^3 - 3*a^4*b^2)))^(1/2) - (4*b^3)/(a - b) + (((2*(8*a*b^4 - 16*a^2*b^3
+ 8*a^3*b^2))/(a - b) + (4*tan(c + d*x)*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3
)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1
/2)*(16*a^5*b - 16*a^2*b^4 + 48*a^3*b^3 - 48*a^4*b^2))/(a - b))*(-(3*a*(a^3
*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a
^3*b^3 - 3*a^4*b^2)))^(1/2) + (4*tan(c + d*x)*(6*a*b^3 + b^4 + a^2*b^2))/(a
 - b))*(-(3*a*(a^3*b^3)^(1/2) + b*(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*
(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b^2)))^(1/2)))*(-(3*a*(a^3*b^3)^(1/2) + b*
(a^3*b^3)^(1/2) - a^3*b - 3*a^2*b^2)/(16*(3*a^5*b - a^6 + a^3*b^3 - 3*a^4*b
^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.417 $\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=161

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a-b)}+\frac{(a-3b)\tan(c+dx)}{d(a-b)^2}$$

```
[Out] 1/2*b*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)
-b^(1/2))^(5/2)+1/2*b*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^
(3/4)/d/(a^(1/2)+b^(1/2))^(5/2)+(a-3*b)*tan(d*x+c)/(a-b)^2/d+1/3*tan(d*x+c)
^3/(a-b)/d
```

Rubi [A] time = 0.35, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{5/2}}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a-b)}+\frac{(a-3b)\tan(c+dx)}{d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] (b*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt
[a] - Sqrt[b])^(5/2)*d) + (b*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/
a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(5/2)*d) + ((a - 3*b)*Tan[c + d*x]
)/((a - b)^2*d) + Tan[c + d*x]^3/(3*(a - b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :
> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2
 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2
+ c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && Ne
Q[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symb
ol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; Fre
eQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2,
 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(
p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Sub
st[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p +
1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
 && IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^3}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{a-3b}{(a-b)^2}+\frac{x^2}{a-b}+\frac{b(a+b)+b(a+3b)x^2}{(a-b)^2\left(a+2ax^2+(a-b)x^4\right)}\right) dx, x, \tan(c+dx)\right)}{d}$$
$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\text{Subst}\left(\int \frac{b(a+b)+b(a+3b)x^2}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{(a-b)^2d}$$
$$= \frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan^3(c+dx)}{3(a-b)d}+\frac{\left(\left(\sqrt{a}-\sqrt{b}\right)^3b\right)\text{Subst}\left(\int \frac{1}{a-\sqrt{a}\sqrt{b}+(a-b)x^2}\,dx, x,\right.}{2\sqrt{a}\,(a-b)^2d}$$
$$= \frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{5/2}d}+\frac{b\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{5/2}d}+\frac{(a-3b)\tan(c+dx)}{(a-b)^2d}+\frac{\tan}{3}$$

Mathematica [A] time = 1.01, size = 205, normalized size = 1.27

$$\frac{\frac{3b\left(-2\sqrt{a}\sqrt{b}+a+b\right)\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+4(a-4b)\tan(c+dx)-\frac{3b\left(\sqrt{a}+\sqrt{b}\right)^2\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+2(a-b)\tan(c+}{6d(a-b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((3*b*(a - 2*Sqrt[a]*Sqrt[b] + b)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x])
/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) - (3*(Sqrt
[a] + Sqrt[b])^2*b*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[-a + Sqr
t[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 4*(a - 4*b)*Tan[c +
d*x] + 2*(a - b)*Sec[c + d*x]^2*Tan[c + d*x])/(6*(a - b)^2*d)
```

fricas [B] time = 1.18, size = 4113, normalized size = 25.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/24*(3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*
a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 +
 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b
^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 +
 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 -
10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(5/4*a^2*b^4 + 5/2*
a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a
^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6
)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x
+ c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*
```

```
x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*
a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10
*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^
3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 12
0*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5
*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) - 3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b
^2 + 10*a*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^
4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^
9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*
b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))
)/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(
d*x + c)^3*log(5/4*a^2*b^4 + 5/2*a*b^5 + 1/4*b^6 - 1/4*(5*a^2*b^4 + 10*a*b^
5 + b^6)*cos(d*x + c)^2 - 1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25
*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110
*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
+ 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a
^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) + (15*a^4*b^3 + 35*a^3*b
^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a
*b^3 + 5*b^4 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5
)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13
 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210
*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 -
 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7
*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x
 + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^
6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^
13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 2
10*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) + 3*(
a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 1
0*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*
b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*
a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b
^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^
3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)^3*log(-5/4*a^2*b^4 - 5/2*a*b^5 -
1/4*b^6 + 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6)*cos(d*x + c)^2 + 1/2*((a^9 - 2*a
^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sq
rt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^
12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6
 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))*cos(d*x + c)*sin
(d*x + c) - (15*a^4*b^3 + 35*a^3*b^4 + 13*a^2*b^5 + a*b^6)*d*cos(d*x + c)*s
in(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^
2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 11
0*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3
 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*
a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^
2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4
+ 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3
 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)*sqrt((25*a^4*b^5 + 100*a^3*b^6 +
110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b
^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 1
0*a^4*b^9 + a^3*b^10)*d^4))) - 3*(a^2 - 2*a*b + b^2)*d*sqrt(-(a^2*b^2 + 10*
a*b^3 + 5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^
5)*d^2*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^1
3 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 21
```

```
0*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6
- 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2))*cos(d*x + c)
^3*log(-5/4*a^2*b^4 - 5/2*a*b^5 - 1/4*b^6 + 1/4*(5*a^2*b^4 + 10*a*b^5 + b^6
)*cos(d*x + c)^2 - 1/2*((a^9 - 2*a^8*b - 5*a^7*b^2 + 20*a^6*b^3 - 25*a^5*b^
4 + 14*a^4*b^5 - 3*a^3*b^6)*d^3*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^
7 + 20*a*b^8 + b^9)/((a^13 - 10*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a
^9*b^4 - 252*a^8*b^5 + 210*a^7*b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9
+ a^3*b^10)*d^4))*cos(d*x + c)*sin(d*x + c) - (15*a^4*b^3 + 35*a^3*b^4 + 13
*a^2*b^5 + a*b^6)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^2*b^2 + 10*a*b^3 +
5*b^4 + (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2*s
qrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10*a
^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*b^
6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4)))/((a^6 - 5*a^5*
b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*d^2)) - 1/4*(2*(a^7*b - 5*
a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2*cos(d*x + c)^2
 - (a^7*b - 5*a^6*b^2 + 10*a^5*b^3 - 10*a^4*b^4 + 5*a^3*b^5 - a^2*b^6)*d^2)
*sqrt((25*a^4*b^5 + 100*a^3*b^6 + 110*a^2*b^7 + 20*a*b^8 + b^9)/((a^13 - 10
*a^12*b + 45*a^11*b^2 - 120*a^10*b^3 + 210*a^9*b^4 - 252*a^8*b^5 + 210*a^7*
b^6 - 120*a^6*b^7 + 45*a^5*b^8 - 10*a^4*b^9 + a^3*b^10)*d^4))) + 8*(2*(a -
4*b)*cos(d*x + c)^2 + a - b)*sin(d*x + c))/((a^2 - 2*a*b + b^2)*d*cos(d*x +
 c)^3)
```

giac [**B**] time = 1.29, size = 2183, normalized size = 13.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/6*(2*(a^2*tan(d*x + c)^3 - 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 + 3*
a^2*tan(d*x + c) - 12*a*b*tan(d*x + c) + 9*b^2*tan(d*x + c))/(a^3 - 3*a^2*b
 + 3*a*b^2 - b^3) - 3*((3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3
*b + 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^2 - 19*sqrt(a^2
- a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(
a - b))*sqrt(a*b)*b^4)*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2*abs(-a + b) - (3*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b - 15*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^6*b^2 + 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^5*b^3 - 3*sqrt(a^
2 - a*b + sqrt(a*b)*(a - b))*a^4*b^4 - 23*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*a^3*b^5 + 19*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^2*b^6 - 3*sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*a*b^7 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*b^8)*a
bs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*abs(-a + b) - (9*sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*a^9*b - 69*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a
*b)*a^8*b^2 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^3 - 3
52*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4 + 306*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 114*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*sqrt(a*b)*a^4*b^6 - 16*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b^7 + 24*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^8 - 3*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^9 - sqrt(a^2 - a*b + sqrt(
a*b)*(a - b))*sqrt(a*b)*b^10)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) +
arctan(tan(d*x + c)/sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3 + sqrt((a^4 - 3
*a^3*b + 3*a^2*b^2 - a*b^3)^2 - (a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)*(a^4 -
4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))/(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3
 + b^4))))/((3*a^11 - 30*a^10*b + 131*a^9*b^2 - 328*a^8*b^3 + 518*a^7*b^4 -
 532*a^6*b^5 + 350*a^5*b^6 - 136*a^4*b^7 + 23*a^3*b^8 + 2*a^2*b^9 - a*b^10)
*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)) + 3*((3*sqrt(a^2 - a*b - sqrt(a*b)*(a
- b))*sqrt(a*b)*a^3*b + 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2
*b^2 - 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^3 - 3*sqrt(a^2
- a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^4)*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)^2
*abs(-a + b) + (3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^7*b - 15*sqrt(a^2 -
 a*b - sqrt(a*b)*(a - b))*a^6*b^2 + 23*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*
```

```
a^5*b^3 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^4*b^4 - 23*sqrt(a^2 - a*b
 - sqrt(a*b)*(a - b))*a^3*b^5 + 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*
b^6 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a*b^7 - sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*b^8)*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*abs(-a + b) - (9*sqrt(
a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b - 69*sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*a^8*b^2 + 216*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sq
rt(a*b)*a^7*b^3 - 352*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^4
 + 306*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^5 - 114*sqrt(a^2
 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^6 - 16*sqrt(a^2 - a*b - sqrt(a*
b)*(a - b))*sqrt(a*b)*a^3*b^7 + 24*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt
(a*b)*a^2*b^8 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^9 - sqr
t(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^10)*abs(-a + b))*(pi*floor((d*
x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*
b^3 - sqrt((a^4 - 3*a^3*b + 3*a^2*b^2 - a*b^3)^2 - (a^4 - 3*a^3*b + 3*a^2*b
^2 - a*b^3)*(a^4 - 4*a^3*b + 6*a^2*b^2 - 4*a*b^3 + b^4)))/(a^4 - 4*a^3*b +
6*a^2*b^2 - 4*a*b^3 + b^4))))/((3*a^11 - 30*a^10*b + 131*a^9*b^2 - 328*a^8*
b^3 + 518*a^7*b^4 - 532*a^6*b^5 + 350*a^5*b^6 - 136*a^4*b^7 + 23*a^3*b^8 +
2*a^2*b^9 - a*b^10)*abs(a^3 - 3*a^2*b + 3*a*b^2 - b^3)))/d
```

maple [B] time = 0.76, size = 581, normalized size = 3.61

$$\frac{\left(\tan^3(dx+c)\right)a}{3d(a-b)^2}-\frac{\left(\tan^3(dx+c)\right)b}{3d(a-b)^2}+\frac{\tan(dx+c)\,a}{d(a-b)^2}-\frac{3\tan(dx+c)\,b}{d(a-b)^2}+\frac{b\,\mathrm{arctanh}\left(\frac{(-a+b)\tan(dx+c)}{\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}\right)a}{2d(a-b)^2\sqrt{\left(\sqrt{ab}-a\right)(a-b)}}+\frac{3b^2\,a}{2d(a-}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/3/d/(a-b)^2*tan(d*x+c)^3*a-1/3/d/(a-b)^2*tan(d*x+c)^3*b+1/d/(a-b)^2*tan(d
*x+c)*a-3/d/(a-b)^2*tan(d*x+c)*b+1/2/d*b/(a-b)^2/(((a*b)^(1/2)-a)*(a-b))^(1
/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a+3/2/d*b^2/(a
-b)^2/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)
-a)*(a-b))^(1/2))-3/2/d*b^2/(a-b)^2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/
2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a-1/2/d*b^3/(a-
b)^2/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/((
(a*b)^(1/2)-a)*(a-b))^(1/2))+1/2/d*b/(a-b)^2/(((a*b)^(1/2)+a)*(a-b))^(1/2)*
arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a+3/2/d*b^2/(a-b)^2/
(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b
))^(1/2))+3/2/d*b^2/(a-b)^2/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arcta
n((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a+1/2/d*b^3/(a-b)^2/(a*b)
^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+
a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] -1/3*(36*(a - 2*b)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 12*(b*sin(4*d*x + 4*
c) - (a - 3*b)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) - 3*((a^2 - 2*a*b + b^2)*
d*cos(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2
- 2*a*b + b^2)*d*cos(2*d*x + 2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c
)^2 + 9*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*
c)^2 + 6*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2
*(3*(a^2 - 2*a*b + b^2)*d*cos(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*
```

```
d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b +
b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2
 - 2*a*b + b^2)*d*sin(4*d*x + 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)
)*sin(6*d*x + 6*c))*integrate(-8*(4*b^3*cos(6*d*x + 6*c)^2 + 4*b^3*cos(2*d*
x + 2*c)^2 + 4*b^3*sin(6*d*x + 6*c)^2 + 4*b^3*sin(2*d*x + 2*c)^2 - b^3*cos(
2*d*x + 2*c) - 4*(8*a^2*b + 13*a*b^2 - 6*b^3)*cos(4*d*x + 4*c)^2 - 4*(8*a^2
*b + 13*a*b^2 - 6*b^3)*sin(4*d*x + 4*c)^2 + 2*(4*a*b^2 - 11*b^3)*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) - (b^3*cos(6*d*x + 6*c) + b^3*cos(2*d*x + 2*c) - 2*
(a*b^2 + 2*b^3)*cos(4*d*x + 4*c))*cos(8*d*x + 8*c) + (8*b^3*cos(2*d*x + 2*c
) - b^3 + 2*(4*a*b^2 - 11*b^3)*cos(4*d*x + 4*c))*cos(6*d*x + 6*c) + 2*(a*b^
2 + 2*b^3 + (4*a*b^2 - 11*b^3)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (b^3*si
n(6*d*x + 6*c) + b^3*sin(2*d*x + 2*c) - 2*(a*b^2 + 2*b^3)*sin(4*d*x + 4*c))
*sin(8*d*x + 8*c) + 2*(4*b^3*sin(2*d*x + 2*c) + (4*a*b^2 - 11*b^3)*sin(4*d*
x + 4*c))*sin(6*d*x + 6*c))/(a^2*b^2 - 2*a*b^3 + b^4 + (a^2*b^2 - 2*a*b^3 +
 b^4)*cos(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c)^2
+ 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*cos(4*d*x + 4*c)^
2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c)^2 + (a^2*b^2 - 2*a*b^3 +
b^4)*sin(8*d*x + 8*c)^2 + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c)^2 +
 4*(64*a^4 - 176*a^3*b + 169*a^2*b^2 - 66*a*b^3 + 9*b^4)*sin(4*d*x + 4*c)^2
 + 16*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x + 4*c)*sin(2*d*x
+ 2*c) + 16*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c)^2 + 2*(a^2*b^2 - 2*a
*b^3 + b^4 - 4*(a^2*b^2 - 2*a*b^3 + b^4)*cos(6*d*x + 6*c) - 2*(8*a^3*b - 19
*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3 + b^4)
*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4 - 2*(8*a^3
*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(4*d*x + 4*c) - 4*(a^2*b^2 - 2*a*b^3
 + b^4)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) - 4*(8*a^3*b - 19*a^2*b^2 + 14*a
*b^3 - 3*b^4 - 4*(8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*cos(2*d*x + 2*c)
)*cos(4*d*x + 4*c) - 8*(a^2*b^2 - 2*a*b^3 + b^4)*cos(2*d*x + 2*c) - 4*(2*(a
^2*b^2 - 2*a*b^3 + b^4)*sin(6*d*x + 6*c) + (8*a^3*b - 19*a^2*b^2 + 14*a*b^3
 - 3*b^4)*sin(4*d*x + 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) + 16*((8*a^3*b - 19*a^2*b^2 + 14*a*b^3 - 3*b^4)*sin(4*d*x
+ 4*c) + 2*(a^2*b^2 - 2*a*b^3 + b^4)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c)), x
) + 4*(3*b*cos(4*d*x + 4*c) - 3*(a - 3*b)*cos(2*d*x + 2*c) - a + 4*b)*sin(6
*d*x + 6*c) - 12*(3*(a - 2*b)*cos(2*d*x + 2*c) + a - 3*b)*sin(4*d*x + 4*c)
+ 12*b*sin(2*d*x + 2*c))/((a^2 - 2*a*b + b^2)*d*cos(6*d*x + 6*c)^2 + 9*(a^2
 - 2*a*b + b^2)*d*cos(4*d*x + 4*c)^2 + 9*(a^2 - 2*a*b + b^2)*d*cos(2*d*x +
2*c)^2 + (a^2 - 2*a*b + b^2)*d*sin(6*d*x + 6*c)^2 + 9*(a^2 - 2*a*b + b^2)*d
*sin(4*d*x + 4*c)^2 + 18*(a^2 - 2*a*b + b^2)*d*sin(4*d*x + 4*c)*sin(2*d*x +
 2*c) + 9*(a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c)^2 + 6*(a^2 - 2*a*b + b^2)*
d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b^2)*d + 2*(3*(a^2 - 2*a*b + b^2)*d*cos
(4*d*x + 4*c) + 3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^2 - 2*a*b + b
^2)*d)*cos(6*d*x + 6*c) + 6*(3*(a^2 - 2*a*b + b^2)*d*cos(2*d*x + 2*c) + (a^
2 - 2*a*b + b^2)*d)*cos(4*d*x + 4*c) + 6*((a^2 - 2*a*b + b^2)*d*sin(4*d*x +
 4*c) + (a^2 - 2*a*b + b^2)*d*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))
```

mupad [B] time = 17.74, size = 4664, normalized size = 28.97

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^4*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] tan(c + d*x)^3/(3*d*(a - b)) - (tan(c + d*x)*((2*a)/(a - b)^2 - 3/(a - b)))
/d + (atan(((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 -
3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b
^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*
(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16
*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))
/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(
1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^
```

7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i - (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i)/((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (2*(a*b^4 + 3*b^5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)))*((5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) + 5*a^2*b^4 + 10*a^3*b^3 + a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*2i)/d + (atan(((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*1i - (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3

```
 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^
3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*
a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^
6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b
^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b
*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 1
0*a^6*b^2)))^(1/2)*1i)/((((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3 - 16*a^5*b^2)
/(3*a*b^2 - 3*a^2*b + a^3 - b^3) - (4*tan(c + d*x)*(-(5*a^2*(a^3*b^5)^(1/2)
 + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5
)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2
)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 + 160*a^5*b^3 -
 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b
^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1
/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^
(1/2) - (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^3))/(3*a*b^2 -
 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a
^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8
+ a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2) - (2*(a*b^4 + 3*b^
5))/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (((16*a*b^6 - 32*a^2*b^5 + 32*a^4*b^3
 - 16*a^5*b^2)/(3*a*b^2 - 3*a^2*b + a^3 - b^3) + (4*tan(c + d*x)*(-(5*a^2*(
a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 1
0*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^
3 - 10*a^6*b^2)))^(1/2)*(16*a^7*b - 16*a^2*b^6 + 80*a^3*b^5 - 160*a^4*b^4 +
 160*a^5*b^3 - 80*a^6*b^2))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*
b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*
b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 -
10*a^6*b^2)))^(1/2) + (4*tan(c + d*x)*(15*a*b^5 + b^6 + 15*a^2*b^4 + a^3*b^
3))/(3*a*b^2 - 3*a^2*b + a^3 - b^3))*(-(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^
5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(
5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 10*a^5*b^3 - 10*a^6*b^2)))^(1/2)))*(-
(5*a^2*(a^3*b^5)^(1/2) + b^2*(a^3*b^5)^(1/2) - 5*a^2*b^4 - 10*a^3*b^3 - a^4
*b^2 + 10*a*b*(a^3*b^5)^(1/2))/(16*(5*a^7*b - a^8 + a^3*b^5 - 5*a^4*b^4 + 1
0*a^5*b^3 - 10*a^6*b^2)))^(1/2)*2i)/d
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.418 $\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=204

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{d(a-b)^3}+\frac{\tan^5(c+dx)}{5d(a-b)}+$$

[Out] -1/2*b^(3/2)*arctan((a^(1/2)-b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)-b^(1/2))^(7/2)+1/2*b^(3/2)*arctan((a^(1/2)+b^(1/2))^(1/2)*tan(d*x+c)/a^(1/4))/a^(3/4)/d/(a^(1/2)+b^(1/2))^(7/2)+(a^2-3*a*b+6*b^2)*tan(d*x+c)/(a-b)^3/d+2/3*(a-2*b)*tan(d*x+c)^3/(a-b)^2/d+1/5*tan(d*x+c)^5/(a-b)/d

Rubi [A] time = 0.38, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3224, 1170, 1166, 205}

$$-\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}-\sqrt{b}\right)^{7/2}}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}d\left(\sqrt{a}+\sqrt{b}\right)^{7/2}}+\frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{d(a-b)^3}+\frac{\tan^5(c+dx)}{5d(a-b)}+$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] -(b^(3/2)*ArcTan[(Sqrt[Sqrt[a] - Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] - Sqrt[b])^(7/2)*d) + (b^(3/2)*ArcTan[(Sqrt[Sqrt[a] + Sqrt[b]]*Tan[c + d*x])/a^(1/4)])/(2*a^(3/4)*(Sqrt[a] + Sqrt[b])^(7/2)*d) + ((a^2 - 3*a*b + 6*b^2)*Tan[c + d*x])/((a - b)^3*d) + (2*(a - 2*b)*Tan[c + d*x]^3)/(3*(a - b)^2*d) + Tan[c + d*x]^5/(5*(a - b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 1166

```
Int[((d_) + (e_.)*(x_)^2)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> With[{q = Rt[b^2 - 4*a*c, 2]}, Dist[e/2 + (2*c*d - b*e)/(2*q), Int[1/(b/2 - q/2 + c*x^2), x], x] + Dist[e/2 - (2*c*d - b*e)/(2*q), Int[1/(b/2 + q/2 + c*x^2), x], x]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - a*e^2, 0] && PosQ[b^2 - 4*a*c]
```

Rule 1170

```
Int[((d_) + (e_.)*(x_)^2)^(q_)/((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4), x_Symbol] :> Int[ExpandIntegrand[(d + e*x^2)^q/(a + b*x^2 + c*x^4), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 - 4*a*c, 0] && NeQ[c*d^2 - b*d*e + a*e^2, 0] && IntegerQ[q]
```

Rule 3224

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_.), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[ff/f, Subst[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(m/2 + 2*p + 1), x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f}, x] && IntegerQ[m/2]
```

```
&& IntegerQ[p]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)}{a-b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\left(1+x^2\right)^4}{a+2ax^2+(a-b)x^4}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2-3ab+6b^2}{(a-b)^3}+\frac{2(a-2b)x^2}{(a-b)^2}+\frac{x^4}{a-b}-\frac{b^2(3a+b)+4b^2(a+b)x^2}{(a-b)^3\left(a+2ax^2+(a-b)x^4\right)}\right)dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{(a-b)^3d}+\frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d}+\frac{\tan^5(c+dx)}{5(a-b)d}-\frac{\text{Subst}\left(\int \frac{b^2(3}{a\cdot}\right.}{}$$

$$= \frac{\left(a^2-3ab+6b^2\right)\tan(c+dx)}{(a-b)^3d}+\frac{2(a-2b)\tan^3(c+dx)}{3(a-b)^2d}+\frac{\tan^5(c+dx)}{5(a-b)d}+\frac{\left(\left(\sqrt{a}-\sqrt{b}\right)\right.}{}$$

$$= -\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}-\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}-\sqrt{b}\right)^{7/2}d}+\frac{b^{3/2}\tan^{-1}\left(\frac{\sqrt{\sqrt{a}+\sqrt{b}}\,\tan(c+dx)}{\sqrt[4]{a}}\right)}{2a^{3/4}\left(\sqrt{a}+\sqrt{b}\right)^{7/2}d}+\frac{\left(a^2-3ab+6b^2\right)\tan}{(a-b)^3d}$$

Mathematica [A] time = 1.32, size = 253, normalized size = 1.24

$$\frac{2\left(8a^2-21ab+73b^2\right)\tan(c+dx)+\frac{15b^{3/2}\left(\sqrt{a}-\sqrt{b}\right)^3\tan^{-1}\left(\frac{\left(\sqrt{a}+\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}+a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}+a}}+\frac{15b^{3/2}\left(\sqrt{a}+\sqrt{b}\right)^3\tanh^{-1}\left(\frac{\left(\sqrt{a}-\sqrt{b}\right)\tan(c+dx)}{\sqrt{\sqrt{a}\sqrt{b}-a}}\right)}{\sqrt{a}\sqrt{\sqrt{a}\sqrt{b}-a}}+6(}{30d(a-b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^6/(a - b*Sin[c + d*x]^4),x]
```

```
[Out] ((15*(Sqrt[a] - Sqrt[b])^3*b^(3/2)*ArcTan[((Sqrt[a] + Sqrt[b])*Tan[c + d*x]
)/Sqrt[a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[a + Sqrt[a]*Sqrt[b]]) + (15*(Sq
rt[a] + Sqrt[b])^3*b^(3/2)*ArcTanh[((Sqrt[a] - Sqrt[b])*Tan[c + d*x])/Sqrt[
-a + Sqrt[a]*Sqrt[b]]])/(Sqrt[a]*Sqrt[-a + Sqrt[a]*Sqrt[b]]) + 2*(8*a^2 - 2
1*a*b + 73*b^2)*Tan[c + d*x] + 4*(2*a - 7*b)*(a - b)*Sec[c + d*x]^2*Tan[c +
 d*x] + 6*(a - b)^2*Sec[c + d*x]^4*Tan[c + d*x])/(30*(a - b)^3*d)
```

fricas [B] time = 1.96, size = 5587, normalized size = 27.39

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="fricas")
```

```
[Out] 1/120*(15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 3
5*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 2
1*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^
4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b
 + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b
^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*
b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^
6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos
(d*x + c)^5*log(7/4*a^3*b^5 + 35/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*
```

a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) - 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos(d*x + c)^5*log(7/4*a^3*b^5 + 35/4*a^2*b^6 + 21/4*a*b^7 + 1/4*b^8 - 1/4*(7*a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) + (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 - (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) + 15*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5

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+ 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^
10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*
a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b
^7)*d^2))*cos(d*x + c)^5*log(-7/4*a^3*b^5 - 35/4*a^2*b^6 - 21/4*a*b^7 - 1/4
*b^8 + 1/4*(7*a^3*b^5 + 35*a^2*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 + 1/2*(
4*(a^11 - 6*a^10*b + 14*a^9*b^2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*
a^4*b^7 - a^3*b^8)*d^3*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484
*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b
^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^
10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^
5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))*cos(d*x + c)*sin(d*x + c) - (7*a^6*b
^3 + 77*a^5*b^4 + 238*a^4*b^5 + 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x
 + c)*sin(d*x + c))*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 -
 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 -
a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 +
511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^1
4*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 300
3*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*
a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*
a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2
 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)
*d^2*cos(d*x + c)^2 - (a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5
*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^
8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17
- 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 +
3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10
 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4))) - 15*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*sqrt(-(a^3*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 +
 (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2
*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*
b^10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 -
364*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^
7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^1
2 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^
3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2))*cos(d*x + c)^5*log(-
7/4*a^3*b^5 - 35/4*a^2*b^6 - 21/4*a*b^7 - 1/4*b^8 + 1/4*(7*a^3*b^5 + 35*a^2
*b^6 + 21*a*b^7 + b^8)*cos(d*x + c)^2 - 1/2*(4*(a^11 - 6*a^10*b + 14*a^9*b^
2 - 14*a^8*b^3 + 14*a^6*b^5 - 14*a^5*b^6 + 6*a^4*b^7 - a^3*b^8)*d^3*sqrt((4
9*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*
a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*
b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a
^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^1
4)*d^4))*cos(d*x + c)*sin(d*x + c) - (7*a^6*b^3 + 77*a^5*b^4 + 238*a^4*b^5
+ 162*a^3*b^6 + 27*a^2*b^7 + a*b^8)*d*cos(d*x + c)*sin(d*x + c))*sqrt(-(a^3
*b^3 + 21*a^2*b^4 + 35*a*b^5 + 7*b^6 + (a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5
*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b^6 - a*b^7)*d^2*sqrt((49*a^6*b^7 +
490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^10 + 511*a^2*b^11 + 42*a*b^12 + b^1
3)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 364*a^14*b^3 + 1001*a^13*b^4 - 2002*a
^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7 + 3003*a^9*b^8 - 2002*a^8*b^9 + 100
1*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12 - 14*a^4*b^13 + a^3*b^14)*d^4)))/((
a^8 - 7*a^7*b + 21*a^6*b^2 - 35*a^5*b^3 + 35*a^4*b^4 - 21*a^3*b^5 + 7*a^2*b
^6 - a*b^7)*d^2)) - 1/4*(2*(a^9*b - 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 3
5*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7 - a^2*b^8)*d^2*cos(d*x + c)^2 - (a^9*b -
 7*a^8*b^2 + 21*a^7*b^3 - 35*a^6*b^4 + 35*a^5*b^5 - 21*a^4*b^6 + 7*a^3*b^7
- a^2*b^8)*d^2)*sqrt((49*a^6*b^7 + 490*a^5*b^8 + 1519*a^4*b^9 + 1484*a^3*b^
10 + 511*a^2*b^11 + 42*a*b^12 + b^13)/((a^17 - 14*a^16*b + 91*a^15*b^2 - 36
4*a^14*b^3 + 1001*a^13*b^4 - 2002*a^12*b^5 + 3003*a^11*b^6 - 3432*a^10*b^7
+ 3003*a^9*b^8 - 2002*a^8*b^9 + 1001*a^7*b^10 - 364*a^6*b^11 + 91*a^5*b^12
- 14*a^4*b^13 + a^3*b^14)*d^4))) + 8*((8*a^2 - 21*a*b + 73*b^2)*cos(d*x + c
```

```
)^4 + 2*(2*a^2 - 9*a*b + 7*b^2)*cos(d*x + c)^2 + 3*a^2 - 6*a*b + 3*b^2)*sin
(d*x + c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(d*x + c)^5)
```

giac [B] time = 1.67, size = 3106, normalized size = 15.23

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="giac")
```

```
[Out] 1/30*(2*(3*a^4*tan(d*x + c)^5 - 12*a^3*b*tan(d*x + c)^5 + 18*a^2*b^2*tan(d*
x + c)^5 - 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 + 10*a^4*tan(d*x
+ c)^3 - 50*a^3*b*tan(d*x + c)^3 + 90*a^2*b^2*tan(d*x + c)^3 - 70*a*b^3*tan
(d*x + c)^3 + 20*b^4*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) - 75*a^3*b*tan(d*
x + c) + 195*a^2*b^2*tan(d*x + c) - 225*a*b^3*tan(d*x + c) + 90*b^4*tan(d*x
 + c))/(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5) + 15*(4*(3
*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^2 - 3*sqrt(a^2 - a*b +
 sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 7*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*b^5)*(a^
5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)^2*abs(-a + b) - (9*s
qrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^9*b^2 - 69*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^8*b^3 + 216*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^7*b^4 - 352*sq
rt(a^2 - a*b + sqrt(a*b)*(a - b))*a^6*b^5 + 306*sqrt(a^2 - a*b + sqrt(a*b)*
(a - b))*a^5*b^6 - 114*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a^4*b^7 - 16*sqr
t(a^2 - a*b + sqrt(a*b)*(a - b))*a^3*b^8 + 24*sqrt(a^2 - a*b + sqrt(a*b)*(a
 - b))*a^2*b^9 - 3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*a*b^10 - sqrt(a^2 -
a*b + sqrt(a*b)*(a - b))*b^11)*abs(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3
+ 5*a*b^4 - b^5)*abs(-a + b) - (3*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(
a*b)*a^14*b - 18*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^13*b^2 - 1
9*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^12*b^3 + 508*sqrt(a^2 - a
*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 - 2221*sqrt(a^2 - a*b + sqrt(a*b
)*(a - b))*sqrt(a*b)*a^10*b^5 + 5314*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sq
rt(a*b)*a^9*b^6 - 8139*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^8*b^
7 + 8328*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^7*b^8 - 5631*sqrt(
a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^6*b^9 + 2322*sqrt(a^2 - a*b + sq
rt(a*b)*(a - b))*sqrt(a*b)*a^5*b^10 - 417*sqrt(a^2 - a*b + sqrt(a*b)*(a - b
))*sqrt(a*b)*a^4*b^11 - 68*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^
3*b^12 + 41*sqrt(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^13 - 2*sqrt
(a^2 - a*b + sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^14 - sqrt(a^2 - a*b + sqrt(a*
b)*(a - b))*sqrt(a*b)*b^15)*abs(-a + b))*(pi*floor((d*x + c)/pi + 1/2) + ar
ctan(tan(d*x + c)/sqrt((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4
 - a*b^5 + sqrt((a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^
5)^2 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)*(a^6 -
 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6)))/(a^6 - 6
*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^4 - 6*a*b^5 + b^6))))/((3*a^14
- 39*a^13*b + 230*a^12*b^2 - 814*a^11*b^3 + 1925*a^10*b^4 - 3201*a^9*b^5 +
3828*a^8*b^6 - 3300*a^7*b^7 + 2013*a^6*b^8 - 825*a^5*b^9 + 198*a^4*b^10 - 1
4*a^3*b^11 - 5*a^2*b^12 + a*b^13)*abs(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b
^3 + 5*a*b^4 - b^5)) - 15*(4*(3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*
b)*a^3*b^2 - 3*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^2*b^3 - 7*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*b^4 - sqrt(a^2 - a*b - sqrt(a
*b)*(a - b))*sqrt(a*b)*b^5)*(a^5 - 5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*
b^4 - b^5)^2*abs(-a + b) + (9*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^9*b^2 -
 69*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^8*b^3 + 216*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^7*b^4 - 352*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^6*b^5 +
306*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^5*b^6 - 114*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*a^4*b^7 - 16*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^3*b^8 + 2
4*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*a^2*b^9 - 3*sqrt(a^2 - a*b - sqrt(a*b
)*(a - b))*a*b^10 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*b^11)*abs(a^5 - 5*a
^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)*abs(-a + b) - (3*sqrt(a^2 -
```

```
a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^14*b - 18*sqrt(a^2 - a*b - sqrt(a*b)*
(a - b))*sqrt(a*b)*a^13*b^2 - 19*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a
*b)*a^12*b^3 + 508*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^11*b^4 -
 2221*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^10*b^5 + 5314*sqrt(a^
2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^9*b^6 - 8139*sqrt(a^2 - a*b - sqrt
(a*b)*(a - b))*sqrt(a*b)*a^8*b^7 + 8328*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))
*sqrt(a*b)*a^7*b^8 - 5631*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^6
*b^9 + 2322*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^5*b^10 - 417*sq
rt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a^4*b^11 - 68*sqrt(a^2 - a*b -
sqrt(a*b)*(a - b))*sqrt(a*b)*a^3*b^12 + 41*sqrt(a^2 - a*b - sqrt(a*b)*(a -
b))*sqrt(a*b)*a^2*b^13 - 2*sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*a*
b^14 - sqrt(a^2 - a*b - sqrt(a*b)*(a - b))*sqrt(a*b)*b^15)*abs(-a + b))*(pi
*floor((d*x + c)/pi + 1/2) + arctan(tan(d*x + c)/sqrt((a^6 - 5*a^5*b + 10*a
^4*b^2 - 10*a^3*b^3 + 5*a^2*b^4 - a*b^5 - sqrt((a^6 - 5*a^5*b + 10*a^4*b^2
- 10*a^3*b^3 + 5*a^2*b^4 - a*b^5)^2 - (a^6 - 5*a^5*b + 10*a^4*b^2 - 10*a^3*
b^3 + 5*a^2*b^4 - a*b^5)*(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*
b^4 - 6*a*b^5 + b^6)))/(a^6 - 6*a^5*b + 15*a^4*b^2 - 20*a^3*b^3 + 15*a^2*b^
4 - 6*a*b^5 + b^6))))/((3*a^14 - 39*a^13*b + 230*a^12*b^2 - 814*a^11*b^3 +
1925*a^10*b^4 - 3201*a^9*b^5 + 3828*a^8*b^6 - 3300*a^7*b^7 + 2013*a^6*b^8 -
 825*a^5*b^9 + 198*a^4*b^10 - 14*a^3*b^11 - 5*a^2*b^12 + a*b^13)*abs(a^5 -
5*a^4*b + 10*a^3*b^2 - 10*a^2*b^3 + 5*a*b^4 - b^5)))/d
```

maple [B] time = 0.76, size = 839, normalized size = 4.11

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x)
```

```
[Out] 1/5/d/(a-b)^3*tan(d*x+c)^5*a^2-2/5/d/(a-b)^3*tan(d*x+c)^5*a*b+1/5/d/(a-b)^3
*tan(d*x+c)^5*b^2+2/3/d/(a-b)^3*tan(d*x+c)^3*a^2-2/d/(a-b)^3*tan(d*x+c)^3*b
*a+4/3/d/(a-b)^3*tan(d*x+c)^3*b^2+1/d/(a-b)^3*a^2*tan(d*x+c)-3/d/(a-b)^3*a*
b*tan(d*x+c)+6/d/(a-b)^3*b^2*tan(d*x+c)-2/d*b^2/(a-b)^3/(((a*b)^(1/2)-a)*(a
-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a-2/d*b
^3/(a-b)^3/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^
(1/2)-a)*(a-b))^(1/2))+1/2/d*b^2/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b)
)^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a^2+3/d*b^
3/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+
c)/(((a*b)^(1/2)-a)*(a-b))^(1/2))*a+1/2/d*b^4/(a-b)^3/(a*b)^(1/2)/(((a*b)^(
1/2)-a)*(a-b))^(1/2)*arctanh((-a+b)*tan(d*x+c)/(((a*b)^(1/2)-a)*(a-b))^(1/2
))-2/d*b^2/(a-b)^3/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/((
(a*b)^(1/2)+a)*(a-b))^(1/2))*a-2/d*b^3/(a-b)^3/(((a*b)^(1/2)+a)*(a-b))^(1/2
)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))-1/2/d*b^2/(a-b)^3/
(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(
1/2)+a)*(a-b))^(1/2))*a^2-3/d*b^3/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b
))^(1/2)*arctan((a-b)*tan(d*x+c)/(((a*b)^(1/2)+a)*(a-b))^(1/2))*a-1/2/d*b^4
/(a-b)^3/(a*b)^(1/2)/(((a*b)^(1/2)+a)*(a-b))^(1/2)*arctan((a-b)*tan(d*x+c)/
(((a*b)^(1/2)+a)*(a-b))^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6/(a-b*sin(d*x+c)^4),x, algorithm="maxima")
```

```
[Out] 1/15*(300*(a*b - 5*b^2)*cos(4*d*x + 4*c)*sin(2*d*x + 2*c) - 10*(48*b^2*sin(
6*d*x + 6*c) + 3*(a*b + 3*b^2)*sin(8*d*x + 8*c) + 2*(8*a^2 - 21*a*b + 49*b^
2)*sin(4*d*x + 4*c) + 8*(a^2 - 3*a*b + 8*b^2)*sin(2*d*x + 2*c))*cos(10*d*x
+ 10*c) + 50*(6*(a*b - 5*b^2)*sin(6*d*x + 6*c) - 16*(a^2 - 3*a*b + 5*b^2)*s
```

```
in(4*d*x + 4*c) - (8*a^2 - 27*a*b + 55*b^2)*sin(2*d*x + 2*c))*cos(8*d*x + 8
*c) - 200*((8*a^2 - 21*a*b + 25*b^2)*sin(4*d*x + 4*c) + 4*(a^2 - 3*a*b + 5*
b^2)*sin(2*d*x + 2*c))*cos(6*d*x + 6*c) + 15*((a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*cos(10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x
+ 8*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 100*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3*a^2*b + 3
*a*b^2 - b^3)*d*cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(
10*d*x + 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2
+ 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 100*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b
^3)*d*sin(2*d*x + 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x +
 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*cos(8*d*x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x +
 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)
*d)*cos(10*d*x + 10*c) + 10*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x
 + 6*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 -
3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d)*cos(8*d*x + 8*c) + 20*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x
 + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d)*cos(6*d*x + 6*c) + 20*(5*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(4*d*x +
4*c) + 10*((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c) + 2*(a^3 - 3*
a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^
3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))
*sin(10*d*x + 10*c) + 50*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6
*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b
 + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*(a^3 - 3*a^
2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d
*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*integrate(4*(4*(a*b^3 + 3*b^4)*cos(6*d
*x + 6*c)^2 - 4*(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*cos(4*d*x + 4*c)^2 + 4*(a*
b^3 + 3*b^4)*cos(2*d*x + 2*c)^2 + 4*(a*b^3 + 3*b^4)*sin(6*d*x + 6*c)^2 - 4*
(56*a^2*b^2 + 19*a*b^3 - 15*b^4)*sin(4*d*x + 4*c)^2 + 2*(8*a^2*b^2 - 7*a*b^
3 - 29*b^4)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x
 + 2*c)^2 - ((a*b^3 + 3*b^4)*cos(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*cos(4*d
*x + 4*c) + (a*b^3 + 3*b^4)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - (a*b^3 + 3
*b^4 - 2*(8*a^2*b^2 - 7*a*b^3 - 29*b^4)*cos(4*d*x + 4*c) - 8*(a*b^3 + 3*b^4
)*cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 2*(7*a*b^3 + 5*b^4 + (8*a^2*b^2 - 7*
a*b^3 - 29*b^4)*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - (a*b^3 + 3*b^4)*cos(2*
d*x + 2*c) - ((a*b^3 + 3*b^4)*sin(6*d*x + 6*c) - 2*(7*a*b^3 + 5*b^4)*sin(4*
d*x + 4*c) + (a*b^3 + 3*b^4)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 2*((8*a^2
*b^2 - 7*a*b^3 - 29*b^4)*sin(4*d*x + 4*c) + 4*(a*b^3 + 3*b^4)*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c))/(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 + (a^3*b^2 - 3*
a^2*b^3 + 3*a*b^4 - b^5)*cos(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a
*b^4 - b^5)*cos(6*d*x + 6*c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*
a^2*b^3 + 75*a*b^4 - 9*b^5)*cos(4*d*x + 4*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 +
3*a*b^4 - b^5)*cos(2*d*x + 2*c)^2 + (a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*s
in(8*d*x + 8*c)^2 + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*
c)^2 + 4*(64*a^5 - 240*a^4*b + 345*a^3*b^2 - 235*a^2*b^3 + 75*a*b^4 - 9*b^5
)*sin(4*d*x + 4*c)^2 + 16*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3
*b^5)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4
 - b^5)*sin(2*d*x + 2*c)^2 + 2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5 - 4*(a^
3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(6*d*x + 6*c) - 2*(8*a^4*b - 27*a^3*b
^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b
^3 + 3*a*b^4 - b^5)*cos(2*d*x + 2*c))*cos(8*d*x + 8*c) - 8*(a^3*b^2 - 3*a^2
*b^3 + 3*a*b^4 - b^5 - 2*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*
b^5)*cos(4*d*x + 4*c) - 4*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d*x +
 2*c))*cos(6*d*x + 6*c) - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 +
```

```
3*b^5 - 4*(8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*cos(2*d*x
 + 2*c))*cos(4*d*x + 4*c) - 8*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*cos(2*d
*x + 2*c) - 4*(2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(6*d*x + 6*c) + (
8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*sin(4*d*x + 4*c) + 2*
(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
16*((8*a^4*b - 27*a^3*b^2 + 33*a^2*b^3 - 17*a*b^4 + 3*b^5)*sin(4*d*x + 4*c)
 + 2*(a^3*b^2 - 3*a^2*b^3 + 3*a*b^4 - b^5)*sin(2*d*x + 2*c))*sin(6*d*x + 6*
c)), x) + 2*(240*b^2*cos(6*d*x + 6*c) + 8*a^2 - 21*a*b + 73*b^2 + 15*(a*b +
 3*b^2)*cos(8*d*x + 8*c) + 10*(8*a^2 - 21*a*b + 49*b^2)*cos(4*d*x + 4*c) +
40*(a^2 - 3*a*b + 8*b^2)*cos(2*d*x + 2*c))*sin(10*d*x + 10*c) + 10*(8*a^2 -
 24*a*b + 64*b^2 - 30*(a*b - 5*b^2)*cos(6*d*x + 6*c) + 80*(a^2 - 3*a*b + 5*
b^2)*cos(4*d*x + 4*c) + 5*(8*a^2 - 27*a*b + 55*b^2)*cos(2*d*x + 2*c))*sin(8
*d*x + 8*c) + 20*(8*a^2 - 21*a*b + 49*b^2 + 10*(8*a^2 - 21*a*b + 25*b^2)*co
s(4*d*x + 4*c) + 40*(a^2 - 3*a*b + 5*b^2)*cos(2*d*x + 2*c))*sin(6*d*x + 6*c
) + 60*(8*b^2 - 5*(a*b - 5*b^2)*cos(2*d*x + 2*c))*sin(4*d*x + 4*c) + 30*(a*
b + 3*b^2)*sin(2*d*x + 2*c))/((a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(10*d*x
+ 10*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*x + 8*c)^2 + 100*(
a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c)^2 + 100*(a^3 - 3*a^2*b +
3*a*b^2 - b^3)*d*cos(4*d*x + 4*c)^2 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*
cos(2*d*x + 2*c)^2 + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(10*d*x + 10*c)^2
 + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c)^2 + 100*(a^3 - 3*a
^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d*sin(4*d*x + 4*c)^2 + 100*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*
x + 4*c)*sin(2*d*x + 2*c) + 25*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x
+ 2*c)^2 + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d + 2*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(8*d*
x + 8*c) + 10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(10*d*x +
 10*c) + 10*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(6*d*x + 6*c) + 10*(a^
3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3 - 3*a^2*b + 3*a*b^
2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(8*d*x
+ 8*c) + 20*(10*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(4*d*x + 4*c) + 5*(a^3
 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 -
 b^3)*d)*cos(6*d*x + 6*c) + 20*(5*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*cos(2*d
*x + 2*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d)*cos(4*d*x + 4*c) + 10*((a^3
- 3*a^2*b + 3*a*b^2 - b^3)*d*sin(8*d*x + 8*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2
- b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x +
 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c))*sin(10*d*x + 10
*c) + 50*(2*(a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(6*d*x + 6*c) + 2*(a^3 - 3
*a^2*b + 3*a*b^2 - b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3
)*d*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*(2*(a^3 - 3*a^2*b + 3*a*b^2 -
b^3)*d*sin(4*d*x + 4*c) + (a^3 - 3*a^2*b + 3*a*b^2 - b^3)*d*sin(2*d*x + 2*c
))*sin(6*d*x + 6*c))
```

mupad [B] time = 18.23, size = 6534, normalized size = 32.03

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(cos(c + d*x)^6*(a - b*sin(c + d*x)^4)),x)
```

```
[Out] (atan(((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 1
2*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) - (4*
tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35
*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*
b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6
*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^
7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(
5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)
```

```
^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^
3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^1
0 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2
)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4
*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(
a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4
+ a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9
*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21
*a^8*b^2)))^(1/2)*1i - (((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5
- 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 1
0*a^3*b^2) + (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2)
+ 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2)
+ 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*
a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*
b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 -
 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*(
(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*
a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(1
6*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*
b^3 - 21*a^8*b^2)))^(1/2) + (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 +
28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3
*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b
^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(
1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 +
 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i)/((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b
^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5
- 10*a^2*b^3 + 10*a^3*b^2) - (4*tan(c + d*x)*((7*a^3*(a^3*b^7)^(1/2) + b^3*
(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*
(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 -
 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16
*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4
 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3
+ 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 +
35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^
3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a
^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8
 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*
a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^
2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a
^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^
5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (((4*(4*a*b^8 - 4*a^2*b
^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b
 + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) + (4*tan(c + d*x)*((7*a^3*(a^3*b^7)
^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^
3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^1
0 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2
)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5
- 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5
- 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) +
 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) +
 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a
^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (4*tan(c + d*x)*(2
8*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^
5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*((7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)
^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)
^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^
6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (8*(a*b^6
+ b^7))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2)))*((7*a^3
*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) + 7*a^2*b^6 + 35*a^3*b^5 + 21*a^4*b^
```

4 + a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a
^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 -
21*a^8*b^2)))^(1/2)*2i)/d + (atan(((((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 +
56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10
*a^2*b^3 + 10*a^3*b^2) - (4*tan(c + d*x)*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^
3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^
3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*
a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^
9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 +
336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 1
0*a^3*b^2))*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35
*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*
b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6
*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 +
70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^
2*b^3 + 10*a^3*b^2))*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2
*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^
2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5
- 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i - (((4*(4*a*b^8 - 4*a^2
*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4
*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) + (4*tan(c + d*x)*(-(7*a^3*(a^3*b
^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5
*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b -
a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*
b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b
^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b
^5 - 10*a^2*b^3 + 10*a^3*b^2))*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/
2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/
2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 +
21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2) + (4*tan(c + d*x
)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b
+ a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3
*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3
*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a
^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*1i)/(((
(4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*b^4 + 12*a^6*b^3
))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2) - (4*tan(c + d
x)(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5
- 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/
2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 3
5*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 112*a^3*b^7 - 336*
a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8*b^2))/(5*a*b^4
- 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*(-(7*a^3*(a^3*b^7)^(1/2)
+ b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*
a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3
*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/
2) - (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*b^5 + a^4*b^4))/
(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*(-(7*a^3*(a^3*b^
7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*
b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a
^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b
^2)))^(1/2) + (((4*(4*a*b^8 - 4*a^2*b^7 - 24*a^3*b^6 + 56*a^4*b^5 - 44*a^5*
b^4 + 12*a^6*b^3))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2
) + (4*tan(c + d*x)*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*
b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2
b(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5
- 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*(16*a^9*b - 16*a^2*b^8 + 11
2*a^3*b^7 - 336*a^4*b^6 + 560*a^5*b^5 - 560*a^6*b^4 + 336*a^7*b^3 - 112*a^8
*b^2))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*(-(7*a^3*

```
(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4
 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^
9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 2
1*a^8*b^2)))^(1/2) + (4*tan(c + d*x)*(28*a*b^7 + b^8 + 70*a^2*b^6 + 28*a^3*
b^5 + a^4*b^4))/(5*a*b^4 - 5*a^4*b + a^5 - b^5 - 10*a^2*b^3 + 10*a^3*b^2))*
(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 2
1*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/
(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6 + 21*a^5*b^5 - 35*a^6*b^4 + 35*a^
7*b^3 - 21*a^8*b^2)))^(1/2) - (8*(a*b^6 + b^7))/(5*a*b^4 - 5*a^4*b + a^5 -
b^5 - 10*a^2*b^3 + 10*a^3*b^2)))*(-(7*a^3*(a^3*b^7)^(1/2) + b^3*(a^3*b^7)^(
1/2) - 7*a^2*b^6 - 35*a^3*b^5 - 21*a^4*b^4 - a^5*b^3 + 21*a*b^2*(a^3*b^7)^(
1/2) + 35*a^2*b*(a^3*b^7)^(1/2))/(16*(7*a^9*b - a^10 + a^3*b^7 - 7*a^4*b^6
+ 21*a^5*b^5 - 35*a^6*b^4 + 35*a^7*b^3 - 21*a^8*b^2)))^(1/2)*2i)/d - (tan(c
 + d*x)^3*((2*a)/(3*(a - b)^2) - 4/(3*(a - b))))/d + (tan(c + d*x)*(6/(a -
b) - a/(a - b)^2 + (2*a*((2*a)/(a - b)^2 - 4/(a - b)))/(a - b)))/d + tan(c
+ d*x)^5/(5*d*(a - b))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6/(a-b*sin(d*x+c)**4),x)
```

```
[Out] Timed out
```

3.419 $\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 8.54, size = 0, normalized size = 0.00

$$\int \cos^m(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.95, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^m,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

maple [A] time = 4.55, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\cos\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^{m}\,\left(b\sin\left(e+f\,x\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.420 $\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=197

$$-\frac{(a-b(4p+5))\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{bf(4p+5)}-\frac{2\sin^3(e+fx)\left(a\right.}{}$$

[Out] sin(f*x+e)*(a+b*sin(f*x+e)^4)^(1+p)/b/f/(5+4*p)-(a-b*(5+4*p))*hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^p/b/f/(5+4*p)/((1+b*sin(f*x+e)^4/a)^p)-2/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)

Rubi [A] time = 0.22, antiderivative size = 191, normalized size of antiderivative = 0.97, number of steps used = 8, number of rules used = 7, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.304, Rules used = {3223, 1207, 1204, 246, 245, 365, 364}

$$-\frac{2\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\left(1-\frac{a}{4bp+5b}\right)\sin(e+fx)\left(a+\right.}{}$$

Antiderivative was successfully verified.

[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]

[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^(1 + p))/(b*f*(5 + 4*p)) + ((1 - a/(5*b + 4*b*p))*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (2*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
NeQ[c*d^2 + a*e^2, 0]
```

Rule 1207

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Sim
p[(e^q*x^(2*q - 3)*(a + c*x^4)^(p + 1))/(c*(4*p + 2*q + 1)), x] + Dist[1/(c
*(4*p + 2*q + 1)), Int[(a + c*x^4)^p*ExpandToSum[c*(4*p + 2*q + 1)*(d + e*x
^2)^q - a*(2*q - 3)*e^q*x^(2*q - 4) - c*(4*p + 2*q + 1)*e^q*x^(2*q), x], x]
, x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] && IGtQ[q, 1]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)^2\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} + \frac{\text{Subst}\left(\int\left(-a+b(5+4p)-\right.\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} + \frac{\text{Subst}\left(\int\left(-a\left(1-\frac{b(5+4p)}{a}\right)\right.\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{2\,\text{Subst}\left(\int x^2\left(a+bx^4\right)^p\,dx,\right.}{f}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{\left(2\left(a+b\sin^4(e+fx)\right)^p\left(1+\right.\right.}{}$$

$$= \frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^{1+p}}{bf(5+4p)} - \frac{(a-b(5+4p))\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};\right.}{}$$

Mathematica [A] time = 0.14, size = 141, normalized size = 0.72

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\left(3\sin^4(e+fx)\,{}_2F_1\left(\frac{5}{4},-p;\frac{9}{4};-\frac{b\sin^4(e+fx)}{a}\right)+15\,{}_2F_1\left(\frac{1}{4},-p;\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Sin[e + f*x]*(a + b*Sin[e + f*x]^4)^p*(15*Hypergeometric2F1[1/4, -p, 5/4,
-((b*Sin[e + f*x]^4)/a)] - 10*Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e +
f*x]^4)/a)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/4, -p, 9/4, -((b*Sin[e +
 f*x]^4)/a)]*Sin[e + f*x]^4))/(15*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^{4}-2\,b\cos\left(fx+e\right)^{2}+a+b\right)^{p}\cos\left(fx+e\right)^{5},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^5,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

maple [F] time = 5.24, size = 0, normalized size = 0.00

$$\int\left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cos\left(e+f\,x\right)^{5}\,\left(b\,\sin\left(e+f\,x\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.421 $\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=140

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^4(e+f\right.}{}$$

```
[Out] hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^
p/f/((1+b*sin(f*x+e)^4/a)^p)-1/3*hypergeom([3/4, -p],[7/4],-b*sin(f*x+e)^4/
a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1204, 246, 245, 365, 364}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^4(e+f\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) - (Hypergeometric2F1
[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*(a + b*Sin[e + f*x]^
4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1204

```
Int[((d_) + (e_.)*(x_)^2)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int[Expa
ndIntegrand[(d + e*x^2)*(a + c*x^4)^p, x], x] /; FreeQ[{a, c, d, e}, x] &&
```

```
NeQ[c*d^2 + a*e^2, 0]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(1-x^2\right)\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int\left(\left(a+bx^4\right)^p-x^2\left(a+bx^4\right)^p\right)dx,x,\sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}-\frac{\text{Subst}\left(\int x^2\left(a+bx^4\right)^p\,dx,\right.}{f}$$
$$= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p\,dx,x,\text{s}\right.}{f}$$
$$= \frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4}{}\right.}{f}$$

Mathematica [A] time = 0.05, size = 106, normalized size = 0.76

$$-\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}\left(\sin^2(e+fx)\,{}_2F_1\left(\frac{3}{4},-p;\frac{7}{4};-\frac{b\sin^4(e+fx)}{a}\right)-3\,{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] -1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)
/a)] + Hypergeometric2F1[3/4, -p, 7/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x
]^2)*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

maple [F] time = 10.18, size = 0, normalized size = 0.00

$$\int \left(\cos^3\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^3\,\left(b\,\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.422 $\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=67

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

```
[Out] hypergeom([1/4, -p],[5/4],-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f*x+e)^4)^
p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A] time = 0.04, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3223, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int\left(a+bx^4\right)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^4}{a}\right)^p\,dx,x,\text{si}\right.}{f}$$

$$=\frac{{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(\right.}{a}}{f}$$

Mathematica [A] time = 0.02, size = 67, normalized size = 1.00

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{4},-p;\frac{5}{4};-\frac{b\sin^4(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
[Out] (Hypergeometric2F1[1/4, -p, 5/4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]*(a +
 b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

maple [F] time = 4.07, size = 0, normalized size = 0.00

$$\int\cos\left(fx+e\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e), x)
```

mupad [B] time = 15.62, size = 64, normalized size = 0.96

$$\frac{\sin\left(e+f\,x\right)\left(b\sin\left(e+f\,x\right)^4+a\right)^p{}_2\mathrm{F}_1\left(\frac{1}{4},-p;\ \frac{5}{4};\ -\frac{b\sin\left(e+f\,x\right)^4}{a}\right)}{f\left(\frac{b\sin\left(e+f\,x\right)^4}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^4)^p,x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^4)^p*hypergeom([1/4, -p], 5/4, -(b*sin(e
+ f*x)^4)/a))/(f*((b*sin(e + f*x)^4)/a + 1)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

3.423 $\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=158

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{f}+\frac{\sin^3(e+fx)\left(a\right.}{}$$

```
[Out] AppellF1(1/4,1,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+1/3*AppellF1(3/4,1,-p,7/4,sin(f*x+e)^
4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4
/a)^p)
```

Rubi [A] time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.286, Rules used = {3223, 1240, 430, 429, 511, 510}

$$\frac{\sin^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{3}{4};1,-p;\frac{7}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{3f}+\frac{\sin(e+fx)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (AppellF1[1/4, 1, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF
1[3/4, 1, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^3*
(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{1-x^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a+bx^4\right)^p}{1-x^4} - \frac{x^2\left(a+bx^4\right)^p}{-1+x^4}\right)dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+bx^4\right)^p}{1-x^4}\,dx, x, \sin(e+fx)\right)}{f} - \frac{\text{Subst}\left(\int \frac{x^2\left(a+bx^4\right)^p}{-1+x^4}\,dx, x, \sin(e+\right.}{f}$$
$$= \frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int \frac{\left(1+\frac{bx^4}{a}\right)^p}{1-x^4}\,dx, x, \sin(e\right.}{f}$$
$$= \frac{F_1\left(\frac{1}{4};1,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e+fx\right.}{f}$$

Mathematica [F] time = 5.93, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4 - 2\,b\cos\left(fx+e\right)^2 + a + b\right)^p\sec\left(fx+e\right), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

maple [F] time = 2.94, size = 0, normalized size = 0.00

$$\int \sec\left(fx+e\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(b\sin\left(e+f\,x\right)^{4}+a\right)^{p}}{\cos\left(e+f\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.424 $\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=239

$$\frac{\sin(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{1}{4};2,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{f}+\frac{\sin^5(e+fx)\left(a+b\right.}{}$$

```
[Out] AppellF1(1/4,2,-p,5/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)*(a+b*sin(f
*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)+2/3*AppellF1(3/4,2,-p,7/4,sin(f*x+e)^
4,-b*sin(f*x+e)^4/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4
/a)^p)+1/5*AppellF1(5/4,2,-p,9/4,sin(f*x+e)^4,-b*sin(f*x+e)^4/a)*sin(f*x+e)
^5*(a+b*sin(f*x+e)^4)^p/f/((1+b*sin(f*x+e)^4/a)^p)
```

Rubi [A] time = 0.22, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3223, 1240, 430, 429, 511, 510}

$$\frac{\sin^5(e+fx)\left(a+b\sin^4(e+fx)\right)^p\left(\frac{b\sin^4(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{5}{4};2,-p;\frac{9}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)}{5f}+\frac{2\sin^3(e+fx)\left(a\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
```

```
[Out] (AppellF1[1/4, 2, -p, 5/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^4)^p)/(f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (2*Appel
lF1[3/4, 2, -p, 7/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^
3*(a + b*Sin[e + f*x]^4)^p)/(3*f*(1 + (b*Sin[e + f*x]^4)/a)^p) + (AppellF1[
5/4, 2, -p, 9/4, Sin[e + f*x]^4, -((b*Sin[e + f*x]^4)/a)]*Sin[e + f*x]^5*(a
 + b*Sin[e + f*x]^4)^p)/(5*f*(1 + (b*Sin[e + f*x]^4)/a)^p)
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
```

```
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 1240

```
Int[((d_) + (e_.)*(x_)^2)^(q_)*((a_) + (c_.)*(x_)^4)^(p_), x_Symbol] :> Int
[ExpandIntegrand[(a + c*x^4)^p, (d/(d^2 - e^2*x^4) - (e*x^2)/(d^2 - e^2*x^4
))^(-q), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !
IntegerQ[p] && ILtQ[q, 0]
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{\left(a+bx^4\right)^p}{\left(1-x^2\right)^2}\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\left(\frac{\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{2x^2\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}+\frac{x^4\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\right)dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\text{Subst}\left(\int\frac{\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\,dx,x,\sin(e+fx)\right)}{f}+\frac{\text{Subst}\left(\int\frac{x^4\left(a+bx^4\right)^p}{\left(-1+x^4\right)^2}\,dx,x,\sin\right.}{f}$$

$$=\frac{\left(\left(a+b\sin^4(e+fx)\right)^p\left(1+\frac{b\sin^4(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\frac{\left(1+\frac{bx^4}{a}\right)^p}{\left(-1+x^4\right)^2}\,dx,x,s\right.}{f}$$

$$=\frac{F_1\left(\frac{1}{4};2,-p;\frac{5}{4};\sin^4(e+fx),-\frac{b\sin^4(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^4(e\right.}{f}$$

Mathematica [F] time = 9.24, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{4}+a\right)^{p} \sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

maple [F] time = 2.92, size = 0, normalized size = 0.00

$$\int \left(\sec^{3}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{4}+a\right)^{p} \sec\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(b \sin\left(e+f\,x\right)^{4}+a\right)^{p}}{\cos\left(e+f\,x\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3,x)
```

```
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

3.425 $\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 5.31, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^4,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

maple [A] time = 8.28, size = 0, normalized size = 0.00

$$\int \left(\cos^4\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^4\,\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.426 $\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 17.24, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*cos(f*x + e)^2,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

maple [A] time = 9.68, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*cos(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\left(b\sin\left(e+f\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.427 $\int \left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left(\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx=\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 1.13, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

maple [A] time = 2.14, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx + e\right)^{4} + a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(b \sin \left(e + f\,x\right)^{4} + a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p,x)
```

```
[Out] int((a + b*sin(e + f*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

3.428 $\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 6.00, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^2,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

maple [**A**] time = 2.36, size = 0, normalized size = 0.00

$$\int \left(\sec^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{4}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

```
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{4}+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^2, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\sin\left(e+f\,x\right)^{4}+a\right)^{p}}{\cos\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2,x)
```

```
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^2, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**4)**p,x)
```

```
[Out] Timed out
```

3.429 $\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Mathematica [A] time = 9.08, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\left(a+b\sin^4(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p,x]
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^4)^p, x]
```

fricas [A] time = 0.59, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(fx+e\right)^4-2\,b\cos\left(fx+e\right)^2+a+b\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="fricas")
[Out] integral((b*cos(f*x + e)^4 - 2*b*cos(f*x + e)^2 + a + b)^p*sec(f*x + e)^4,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

maple [**A**] time = 1.84, size = 0, normalized size = 0.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sin^4\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x)
```

maxima [**A**] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^4+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^4 + a)^p*sec(f*x + e)^4, x)
```

mupad [**A**] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(b\sin\left(e+f\,x\right)^4+a\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4,x)
[Out] int((a + b*sin(e + f*x)^4)^p/cos(e + f*x)^4, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**4)**p,x)
[Out] Timed out
```

3.430 $\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 4.59, size = 0, normalized size = 0.00

$$\int \cos^m(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Cos[e + f*x]^m*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

maple [A] time = 2.61, size = 0, normalized size = 0.00

$$\int \left(\cos^{m}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^m*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^{m}\,\left(a+b\sin\left(e+f\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^m*(a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**m*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.431 $\int \cos^5(e + fx) \left(a + b\sin^n(e + fx)\right)^p dx$

Optimal. Leaf size=226

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}+\frac{\sin^5(e+fx)\left(a+b\sin^n(e\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)-2/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f*x
+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)+1/5*h
ypergeom([-p, 5/n],[(5+n)/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)^5*(a+b*sin(f*x+e
)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A] time = 0.17, antiderivative size = 226, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1893, 246, 245, 365, 364}

$$\frac{\sin^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{5}{n},-p;\frac{n+5}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{5f}-\frac{2\sin^3(e+fx)\left(a+b\sin^n(\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (2*Hyper
geometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a
 + b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p) + (Hypergeometri
c2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^5*(a + b*Sin
[e + f*x]^n)^p)/(5*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p
, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] &&  !IGtQ[p, 0] &&  !IntegerQ[1/n] &&  !ILtQ[Sim
plify[1/n + p], 0] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^5(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx = \frac{\text{Subst}\left(\int \left(1-x^2\right)^2 (a+bx^n)^p\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left((a+bx^n)^p - 2x^2(a+bx^n)^p + x^4(a+bx^n)^p\right)dx, x, \sin(e+f\right.}{f}$$

$$= \frac{\text{Subst}\left(\int (a+bx^n)^p\,dx, x, \sin(e+fx)\right)}{f} + \frac{\text{Subst}\left(\int x^4(a+bx^n)^p\,dx,\right.}{f}$$

$$= \frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p dx, x,\right.}{f}$$

$$= \frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1+\right.}{f}$$

Mathematica [A] time = 0.22, size = 155, normalized size = 0.69

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\left(15\,{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)+3\sin^4(e+fx)\,{}_2F_1\left(\frac{5}{n},-\right.\right.}{15f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^5*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Sin[e + f*x]*(15*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*
x]^n)/a)] - 10*Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a
)]*Sin[e + f*x]^2 + 3*Hypergeometric2F1[5/n, -p, (5 + n)/n, -((b*Sin[e + f*
x]^n)/a)]*Sin[e + f*x]^4)*(a + b*Sin[e + f*x]^n)^p)/(15*f*(1 + (b*Sin[e + f
*x]^n)/a)^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n + a\right)^p\cos\left(fx+e\right)^5, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

maple [F] time = 1.69, size = 0, normalized size = 0.00

$$\int \left(\cos^{5}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{5}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^5*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^5, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(e+f\,x\right)}^5\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^5*(a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**5*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.432 $\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=148

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^n(e+\right.}{}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)-1/3*hypergeom([-p, 3/n],[(3+n)/n],-b*sin(f*x
+e)^n/a)*sin(f*x+e)^3*(a+b*sin(f*x+e)^n)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3223, 1893, 246, 245, 365, 364}

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}-\frac{\sin^3(e+fx)\left(a+b\sin^n(e+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p) - (Hyperge
ometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)/a)]*Sin[e + f*x]^3*(a +
 b*Sin[e + f*x]^n)^p)/(3*f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1893

```
Int[(Pq_)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[
Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, n, p}, x] && (PolyQ[Pq, x] || Poly
```

```
Q[Pq, x^n])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int\left(1-x^2\right)(a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int\left((a+bx^n)^p-x^2(a+bx^n)^p\right)dx,x,\sin(e+fx)\right)}{f}$$
$$=\frac{\text{Subst}\left(\int(a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}-\frac{\text{Subst}\left(\int x^2(a+bx^n)^p\,d\right.}{f}$$
$$=\frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p dx,\right.}{f}$$
$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1\right.}{f}$$

Mathematica [A] time = 0.09, size = 114, normalized size = 0.77

$$-\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}\left(\sin^2(e+fx)\,{}_2F_1\left(\frac{3}{n},-p;\frac{n+3}{n};-\frac{b\sin^n(e+fx)}{a}\right)-3\,{}_2F_1\left(\frac{1}{n},-p\right.\right.}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
[Out] -1/3*(Sin[e + f*x]*(-3*Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e
 + f*x]^n)/a)] + Hypergeometric2F1[3/n, -p, (3 + n)/n, -((b*Sin[e + f*x]^n)
/a)]*Sin[e + f*x]^2)*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a
)^p)
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

maple [F] time = 1.72, size = 0, normalized size = 0.00

$$\int \left(\cos^{3}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(e+f\,x\right)^{3}\,{\left(a+b\,{\sin\left(e+f\,x\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] int(cos(e + f*x)^3*(a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

3.433 $\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=69

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

```
[Out] hypergeom([-p, 1/n],[1+1/n],-b*sin(f*x+e)^n/a)*sin(f*x+e)*(a+b*sin(f*x+e)^n
)^p/f/((1+b*sin(f*x+e)^n/a)^p)
```

Rubi [A] time = 0.05, antiderivative size = 69, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3223, 246, 245}

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3223

```
Int[cos[(e_.) + (f_.)*(x_)]^(m_.)*((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x
_)])^(n_))^(p_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff/f, Subst[Int[(1 - ff^2*x^2)^((m - 1)/2)*(a + b*(c*ff*x)^n)^p, x], x,
Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && IntegerQ[(m - 1
)/2] && (EqQ[n, 4] || GtQ[m, 0] || IGtQ[p, 0] || IntegersQ[m, p])
```

Rubi steps

$$\int \cos(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\frac{\text{Subst}\left(\int (a+bx^n)^p\,dx,x,\sin(e+fx)\right)}{f}$$

$$=\frac{\left(\left(a+b\sin^n(e+fx)\right)^p\left(1+\frac{b\sin^n(e+fx)}{a}\right)^{-p}\right)\text{Subst}\left(\int\left(1+\frac{bx^n}{a}\right)^p\,dx,x\right.}{f}$$

$$=\frac{{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(1\right.}{f}$$

Mathematica [A] time = 0.02, size = 69, normalized size = 1.00

$$\frac{\sin(e+fx)\left(a+b\sin^n(e+fx)\right)^p\left(\frac{b\sin^n(e+fx)}{a}+1\right)^{-p}{}_2F_1\left(\frac{1}{n},-p;1+\frac{1}{n};-\frac{b\sin^n(e+fx)}{a}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[n^(-1), -p, 1 + n^(-1), -((b*Sin[e + f*x]^n)/a)]*Sin[e +
 f*x]*(a + b*Sin[e + f*x]^n)^p)/(f*(1 + (b*Sin[e + f*x]^n)/a)^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

maple [F] time = 1.03, size = 0, normalized size = 0.00

$$\int\cos\left(fx+e\right)\left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e), x)
```

mupad [B] time = 15.91, size = 70, normalized size = 1.01

$$\frac{\sin\left(e+f\,x\right)\,\left(a+b\,\sin\left(e+f\,x\right)^n\right)^p\,{}_2\mathrm{F}_1\left(\frac{1}{n},-p;\ \frac{1}{n}+1;\ -\frac{b\,\sin\left(e+f\,x\right)^n}{a}\right)}{f\,\left(\frac{b\,\sin\left(e+f\,x\right)^n}{a}+1\right)^p}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)*(a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] (sin(e + f*x)*(a + b*sin(e + f*x)^n)^p*hypergeom([1/n, -p], 1/n + 1, -(b*si
n(e + f*x)^n)/a))/(f*((b*sin(e + f*x)^n)/a + 1)^p)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

3.434 $\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=24

$$\text{Int}\left(\sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 2.85, size = 0, normalized size = 0.00

$$\int \sec(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

maple [A] time = 0.85, size = 0, normalized size = 0.00

$$\int \sec \left(fx+e\right) \left(a+b \left(\sin^{n} \left(fx+e\right)\right)\right)^{p} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^{n} + a\right)^{p} \sec \left(fx+e\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b \sin \left(e+f\,x\right)^{n}\right)^{p}}{\cos \left(e+f\,x\right)} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x),x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.435 $\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 8.15, size = 0, normalized size = 0.00

$$\int \sec^3(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^3*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

maple [A] time = 0.86, size = 0, normalized size = 0.00

$$\int \left(\sec^3\left(fx+e\right)\right)\left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^3*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\sin\left(e+f\,x\right)^n\right)^p}{\cos\left(e+f\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3,x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**3*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.436 $\int \cos^4(e+fx) \left(a+b\sin^n(e+fx)\right)^p \, dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx = \int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$$

Mathematica [A] time = 19.66, size = 0, normalized size = 0.00

$$\int \cos^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p \, dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Cos[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^4 \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

maple [A] time = 1.59, size = 0, normalized size = 0.00

$$\int \left(\cos^{4}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\cos\left(fx+e\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^{4}\,\left(a+b\,\sin\left(e+f\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^4*(a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.437 $\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 11.71, size = 0, normalized size = 0.00

$$\int \cos^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Cos[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

maple [A] time = 1.31, size = 0, normalized size = 0.00

$$\int \left(\cos^2\left(fx+e\right)\right)\left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\cos\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*cos(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cos\left(e+f\,x\right)^2\,{\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p,x)
[Out] int(cos(e + f*x)^2*(a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.438 $\int \left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^n(e+fx)\right)^p\,dx=\int \left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 1.57, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

maple [A] time = 0.62, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int((a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(f x + e\right)^{n} + a\right)^{p} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int {\left(a+b\,{\sin\left(e+f\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p,x)
```

```
[Out] int((a + b*sin(e + f*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

3.439 $\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 4.38, size = 0, normalized size = 0.00

$$\int \sec^2(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^2*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

maple [A] time = 0.82, size = 0, normalized size = 0.00

$$\int \left(\sec^{2}\left(fx+e\right)\right)\left(a+b\left(\sin^{n}\left(fx+e\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
[Out] int(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{n}+a\right)^{p}\sec\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^2*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\sin\left(e+f\,x\right)^{n}\right)^{p}}{\cos\left(e+f\,x\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2,x)
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**2*(a+b*sin(f*x+e)**n)**p,x)
[Out] Timed out
```

3.440 $\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p,x\right)$$

```
[Out] Unintegrable(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Defer[Int][Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

Rubi steps

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx=\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Mathematica [A] time = 8.06, size = 0, normalized size = 0.00

$$\int \sec^4(e+fx)\left(a+b\sin^n(e+fx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p,x]
[Out] Integrate[Sec[e + f*x]^4*(a + b*Sin[e + f*x]^n)^p, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p\sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

maple [A] time = 0.88, size = 0, normalized size = 0.00

$$\int \left(\sec^4\left(fx+e\right)\right)\left(a+b\left(\sin^n\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

```
[Out] int(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^n+a\right)^p \sec\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)^4*(a+b*sin(f*x+e)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^n + a)^p*sec(f*x + e)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\left(a+b\sin\left(e+f\,x\right)^n\right)^p}{\cos\left(e+f\,x\right)^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4,x)
```

```
[Out] int((a + b*sin(e + f*x)^n)^p/cos(e + f*x)^4, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)**4*(a+b*sin(f*x+e)**n)**p,x)
```

```
[Out] Timed out
```

3.441 $\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=128

$$-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^4}+\frac{a^3\log(\cos(c+dx))}{d(a+b)^4}+\frac{\left(3a^2+3ab+b^2\right)\sec^2(c+dx)}{2d(a+b)^3}+\frac{\sec^6(c+dx)}{6d(a+b)}-\frac{(3a+2b)\sec^4(c+dx)}{4d(a+b)^2}$$

```
[Out] a^3*ln(cos(d*x+c))/(a+b)^4/d-1/2*a^3*ln(a+b*sin(d*x+c)^2)/(a+b)^4/d+1/2*(3*
a^2+3*a*b+b^2)*sec(d*x+c)^2/(a+b)^3/d-1/4*(3*a+2*b)*sec(d*x+c)^4/(a+b)^2/d+
1/6*sec(d*x+c)^6/(a+b)/d
```

Rubi [A] time = 0.13, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3194, 88}

$$\frac{\left(3a^2+3ab+b^2\right)\sec^2(c+dx)}{2d(a+b)^3}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^4}+\frac{a^3\log(\cos(c+dx))}{d(a+b)^4}+\frac{\sec^6(c+dx)}{6d(a+b)}-\frac{(3a+2b)\sec^4(c+dx)}{4d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^3*Log[Cos[c + d*x]])/((a + b)^4*d) - (a^3*Log[a + b*Sin[c + d*x]^2])/(2*
(a + b)^4*d) + ((3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(2*(a + b)^3*d) - ((3
*a + 2*b)*Sec[c + d*x]^4)/(4*(a + b)^2*d) + Sec[c + d*x]^6/(6*(a + b)*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^7(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^3}{(1-x)^4(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^4}+\frac{3a+2b}{(a+b)^2(-1+x)^3}+\frac{3a^2+3ab+b^2}{(a+b)^3(-1+x)^2}+\frac{a^3}{(a+b)^4(-1+x)}-\frac{a^3b}{(a+b)^4(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}\\
&= \frac{a^3\log(\cos(c+dx))}{(a+b)^4d}-\frac{a^3\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^4d}+\frac{\left(3a^2+3ab+b^2\right)\sec^2(c+dx)}{2(a+b)^3d}-\frac{(3a+2b)\sec^4(c+dx)}{4(a+b)^2d}+\frac{\sec^6(c+dx)}{6(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.28, size = 113, normalized size = 0.88

$$\frac{-\frac{6a^3\log\left(a+b\sin^2(c+dx)\right)}{(a+b)^4}+\frac{12a^3\log(\cos(c+dx))}{(a+b)^4}+\frac{6\left(3a^2+3ab+b^2\right)\sec^2(c+dx)}{(a+b)^3}+\frac{2\sec^6(c+dx)}{a+b}-\frac{3(3a+2b)\sec^4(c+dx)}{(a+b)^2}}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((12*a^3*Log[Cos[c + d*x]])/(a + b)^4 - (6*a^3*Log[a + b*Sin[c + d*x]^2])/(
a + b)^4 + (6*(3*a^2 + 3*a*b + b^2)*Sec[c + d*x]^2)/(a + b)^3 - (3*(3*a + 2
*b)*Sec[c + d*x]^4)/(a + b)^2 + (2*Sec[c + d*x]^6)/(a + b))/(12*d)
```

fricas [A] time = 0.83, size = 179, normalized size = 1.40

$$-\frac{6\,a^3\cos\left(dx+c\right)^6\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-12\,a^3\cos\left(dx+c\right)^6\log\left(-\cos\left(dx+c\right)\right)-6\left(3\,a^3+6\,a^2b+\cdots\right.}{12\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3\cdots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/12*(6*a^3*cos(d*x + c)^6*log(-b*cos(d*x + c)^2 + a + b) - 12*a^3*cos(d*x
 + c)^6*log(-cos(d*x + c)) - 6*(3*a^3 + 6*a^2*b + 4*a*b^2 + b^3)*cos(d*x +
c)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + 3*(3*a^3 + 8*a^2*b + 7*a*b^2 + 2
*b^3)*cos(d*x + c)^2)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*d*cos(d*
x + c)^6)
```

giac [B] time = 7.07, size = 603, normalized size = 4.71

$$-\frac{\frac{30\,a^3\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{60\,a^3\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{147\,a^3+\frac{1002\,a^3(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{120\,a^2b(\cos(dx+c)-1)}{\cos(dx+c)+1}\cdots}{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/60*(30*a^3*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(
d*x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1
)^2)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4) - 60*a^3*log(abs(-(cos(d*x
 + c) - 1)/(cos(d*x + c) + 1) - 1))/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4) + (147*a^3 + 1002*a^3*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 120*a^2*
b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 2925*a^3*(cos(d*x + c) - 1)^2/(co
s(d*x + c) + 1)^2 + 960*a^2*b*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 2
40*a*b^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 4780*a^3*(cos(d*x + c)
 - 1)^3/(cos(d*x + c) + 1)^3 + 3600*a^2*b*(cos(d*x + c) - 1)^3/(cos(d*x + c
) + 1)^3 + 2400*a*b^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 640*b^3*(
cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 2925*a^3*(cos(d*x + c) - 1)^4/(c
os(d*x + c) + 1)^4 + 960*a^2*b*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 +
240*a*b^2*(cos(d*x + c) - 1)^4/(cos(d*x + c) + 1)^4 + 1002*a^3*(cos(d*x + c
) - 1)^5/(cos(d*x + c) + 1)^5 + 120*a^2*b*(cos(d*x + c) - 1)^5/(cos(d*x + c
) + 1)^5 + 147*a^3*(cos(d*x + c) - 1)^6/(cos(d*x + c) + 1)^6)/((a^4 + 4*a^3
*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*((cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 1)
^6))/d
```

maple [A] time = 0.53, size = 170, normalized size = 1.33

$$-\frac{a^3\ln\left(b\left(\cos^2\left(dx+c\right)\right)-a-b\right)}{2d\left(a+b\right)^4}+\frac{a^3\ln\left(\cos\left(dx+c\right)\right)}{\left(a+b\right)^4d}-\frac{3a}{4d\left(a+b\right)^2\cos\left(dx+c\right)^4}-\frac{b}{2d\left(a+b\right)^2\cos\left(dx+c\right)^4}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x)
```

[Out] -1/2/d*a^3/(a+b)^4*ln(b*cos(d*x+c)^2-a-b)+a^3*ln(cos(d*x+c))/(a+b)^4/d-3/4/
d/(a+b)^2/cos(d*x+c)^4*a-1/2/d/(a+b)^2/cos(d*x+c)^4*b+3/2/d/(a+b)^3/cos(d*x
+c)^2*a^2+3/2/d/(a+b)^3/cos(d*x+c)^2*a*b+1/2/d/(a+b)^3/cos(d*x+c)^2*b^2+1/6
/d/(a+b)/cos(d*x+c)^6

maxima [B] time = 0.34, size = 273, normalized size = 2.13

$$-\frac{\frac{6\,a^3\log\left(b\sin(dx+c)^2+a\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}-\frac{6\,a^3\log\left(\sin(dx+c)^2-1\right)}{a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4}+\frac{6\left(3\,a^2+3\,ab+b^2\right)\sin(dx+c)^4-3\left(9\,a^2+7\,ab+2\,b^2\right)\sin(dx+c)^2+\cdots}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sin(dx+c)^6-3\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sin(dx+c)^4-a^3-3\,a^2b-3\,ab^2\cdots}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")

[Out] -1/12*(6*a^3*log(b*sin(d*x + c)^2 + a)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3
 + b^4) - 6*a^3*log(sin(d*x + c)^2 - 1)/(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^
3 + b^4) + (6*(3*a^2 + 3*a*b + b^2)*sin(d*x + c)^4 - 3*(9*a^2 + 7*a*b + 2*b
^2)*sin(d*x + c)^2 + 11*a^2 + 7*a*b + 2*b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sin(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^4 - a^3
- 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sin(d*x + c)^
2))/d

mupad [B] time = 14.34, size = 115, normalized size = 0.90

$$\frac{\tan\left(c+d\,x\right)^6}{6\,d\,\left(a+b\right)}+\frac{a^2\tan\left(c+d\,x\right)^2}{2\,d\,{\left(a+b\right)}^3}-\frac{a^3\,\ln\left(\left(a+b\right)\,\tan\left(c+d\,x\right)^2+a\right)}{d\,\left(2\,a^4+8\,a^3\,b+12\,a^2\,b^2+8\,a\,b^3+2\,b^4\right)}-\frac{a\tan\left(c+d\,x\right)^4}{4\,d\,{\left(a+b\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)^7/(a + b*sin(c + d*x)^2),x)

[Out] tan(c + d*x)^6/(6*d*(a + b)) + (a^2*tan(c + d*x)^2)/(2*d*(a + b)^3) - (a^3*
log(a + tan(c + d*x)^2*(a + b)))/(d*(8*a*b^3 + 8*a^3*b + 2*a^4 + 2*b^4 + 12
*a^2*b^2)) - (a*tan(c + d*x)^4)/(4*d*(a + b)^2)

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)**7/(a+b*sin(d*x+c)**2),x)

[Out] Timed out

3.442 $\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)} dx$

Optimal. Leaf size=94

$$\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^3}-\frac{a^2\log(\cos(c+dx))}{d(a+b)^3}+\frac{\sec^4(c+dx)}{4d(a+b)}-\frac{(2a+b)\sec^2(c+dx)}{2d(a+b)^2}$$

```
[Out] -a^2*ln(cos(d*x+c))/(a+b)^3/d+1/2*a^2*ln(a+b*sin(d*x+c)^2)/(a+b)^3/d-1/2*(2
*a+b)*sec(d*x+c)^2/(a+b)^2/d+1/4*sec(d*x+c)^4/(a+b)/d
```

Rubi [A] time = 0.10, antiderivative size = 94, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 88}

$$\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^3}-\frac{a^2\log(\cos(c+dx))}{d(a+b)^3}+\frac{\sec^4(c+dx)}{4d(a+b)}-\frac{(2a+b)\sec^2(c+dx)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^2*Log[Cos[c + d*x]])/((a + b)^3*d)) + (a^2*Log[a + b*Sin[c + d*x]^2])/
(2*(a + b)^3*d) - ((2*a + b)*Sec[c + d*x]^2)/(2*(a + b)^2*d) + Sec[c + d*x]
^4/(4*(a + b)*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(-\frac{1}{(a+b)(-1+x)^3}+\frac{-2a-b}{(a+b)^2(-1+x)^2}-\frac{a^2}{(a+b)^3(-1+x)}+\frac{a^2b}{(a+b)^3(a+bx)}\right)dx,x,\sin^2(c+d\right.}{2d}\\ &= -\frac{a^2\log(\cos(c+dx))}{(a+b)^3d}+\frac{a^2\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^3d}-\frac{(2a+b)\sec^2(c+dx)}{2(a+b)^2d}+\frac{\sec^4(}{4(a}\end{aligned}$$

Mathematica [A] time = 0.26, size = 78, normalized size = 0.83

$$\frac{-2\left(2a^2+3ab+b^2\right)\sec^2(c+dx)+2a^2\left(\log\left(a+b\sin^2(c+dx)\right)-2\log(\cos(c+dx))\right)+(a+b)^2\sec^4(c+dx)}{4d(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (2*a^2*(-2*Log[Cos[c + d*x]] + Log[a + b*Sin[c + d*x]^2]) - 2*(2*a^2 + 3*a*
b + b^2)*Sec[c + d*x]^2 + (a + b)^2*Sec[c + d*x]^4)/(4*(a + b)^3*d)
```

fricas [A] time = 0.64, size = 118, normalized size = 1.26

$$\frac{2\,a^2\cos\left(dx+c\right)^4\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-4\,a^2\cos\left(dx+c\right)^4\log\left(-\cos\left(dx+c\right)\right)-2\left(2\,a^2+3\,ab+b^2\right)\cos}{4\left(a^3+3\,a^2b+3\,ab^2+b^3\right)d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/4*(2*a^2*cos(d*x + c)^4*log(-b*cos(d*x + c)^2 + a + b) - 4*a^2*cos(d*x +
c)^4*log(-cos(d*x + c)) - 2*(2*a^2 + 3*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*
a*b + b^2)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^4)
```

giac [B] time = 2.75, size = 393, normalized size = 4.18

$$\frac{\frac{6\,a^2\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{12\,a^2\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{25\,a^2+\frac{124\,a^2(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{24\,ab(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{246\,a^2(\cos}{(\cos(dx}}{}}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/12*(6*a^2*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*
x + c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^
2)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 12*a^2*log(abs(-(cos(d*x + c) - 1)/(co
s(d*x + c) + 1) - 1))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (25*a^2 + 124*a^2*(
cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 24*a*b*(cos(d*x + c) - 1)/(cos(d*x +
 c) + 1) + 246*a^2*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 144*a*b*(cos
(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2 + 48*b^2*(cos(d*x + c) - 1)^2/(cos(d*
x + c) + 1)^2 + 124*a^2*(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 24*a*b*
(cos(d*x + c) - 1)^3/(cos(d*x + c) + 1)^3 + 25*a^2*(cos(d*x + c) - 1)^4/(co
s(d*x + c) + 1)^4)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((cos(d*x + c) - 1)/(co
s(d*x + c) + 1) + 1)^4))/d
```

maple [A] time = 0.46, size = 109, normalized size = 1.16

$$\frac{a^2\ln\left(b\left(\cos^2\left(dx+c\right)\right)-a-b\right)}{2d\left(a+b\right)^3}-\frac{a}{d\left(a+b\right)^2\cos\left(dx+c\right)^2}-\frac{b}{2d\left(a+b\right)^2\cos\left(dx+c\right)^2}+\frac{1}{4d\left(a+b\right)\cos\left(dx+c\right)^4}-\frac{a^2\,\mathrm{l}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/2/d*a^2/(a+b)^3*ln(b*cos(d*x+c)^2-a-b)-1/d/(a+b)^2/cos(d*x+c)^2*a-1/2/d/(
a+b)^2/cos(d*x+c)^2*b+1/4/d/(a+b)/cos(d*x+c)^4-a^2*ln(cos(d*x+c))/(a+b)^3/d
```

maxima [A] time = 0.34, size = 159, normalized size = 1.69

$$\frac{\frac{2\,a^2\log\left(b\sin(dx+c)^2+a\right)}{a^3+3\,a^2b+3\,ab^2+b^3}-\frac{2\,a^2\log\left(\sin(dx+c)^2-1\right)}{a^3+3\,a^2b+3\,ab^2+b^3}+\frac{2\,(2\,a+b)\sin(dx+c)^2-3\,a-b}{\left(a^2+2\,ab+b^2\right)\sin(dx+c)^4-2\left(a^2+2\,ab+b^2\right)\sin(dx+c)^2+a^2+2\,ab+b^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/4*(2*a^2*log(b*sin(d*x + c)^2 + a)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) - 2*a^
2*log(sin(d*x + c)^2 - 1)/(a^3 + 3*a^2*b + 3*a*b^2 + b^3) + (2*(2*a + b)*si
n(d*x + c)^2 - 3*a - b)/((a^2 + 2*a*b + b^2)*sin(d*x + c)^4 - 2*(a^2 + 2*a*
b + b^2)*sin(d*x + c)^2 + a^2 + 2*a*b + b^2))/d
```

mupad [B] time = 14.35, size = 90, normalized size = 0.96

$$\frac{a^2\left(\frac{\ln\left((a+b)\tan(c+d\,x)^2+a\right)}{2}-\frac{\tan(c+d\,x)^2}{2}+\frac{\tan(c+d\,x)^4}{4}\right)+\frac{b^2\tan(c+d\,x)^4}{4}-a\,b\left(\frac{\tan(c+d\,x)^2}{2}-\frac{\tan(c+d\,x)^4}{2}\right)}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (a^2*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2 + tan(c + d*x)^4
/4) + (b^2*tan(c + d*x)^4)/4 - a*b*(tan(c + d*x)^2/2 - tan(c + d*x)^4/2))/(
d*(a + b)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

3.443 $\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=64

$$\frac{\sec^2(c+dx)}{2d(a+b)}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^2}+\frac{a\log(\cos(c+dx))}{d(a+b)^2}$$

```
[Out] a*ln(cos(d*x+c))/(a+b)^2/d-1/2*a*ln(a+b*sin(d*x+c)^2)/(a+b)^2/d+1/2*sec(d*x
+c)^2/(a+b)/d
```

Rubi [A] time = 0.08, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 77}

$$\frac{\sec^2(c+dx)}{2d(a+b)}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)^2}+\frac{a\log(\cos(c+dx))}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*Log[Cos[c + d*x]])/((a + b)^2*d) - (a*Log[a + b*Sin[c + d*x]^2])/(2*(a +
 b)^2*d) + Sec[c + d*x]^2/(2*(a + b)*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{(a+b)(-1+x)^2}+\frac{a}{(a+b)^2(-1+x)}-\frac{ab}{(a+b)^2(a+bx)}\right)dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{a\log(\cos(c+dx))}{(a+b)^2d}-\frac{a\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)^2d}+\frac{\sec^2(c+dx)}{2(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.09, size = 52, normalized size = 0.81

$$\frac{(a+b)\sec^2(c+dx)+a\left(2\log(\cos(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)}{2d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a*(2*Log[Cos[c + d*x]] - Log[a + b*Sin[c + d*x]^2]) + (a + b)*Sec[c + d*x]
^2)/(2*(a + b)^2*d)
```

fricas [A] time = 0.53, size = 78, normalized size = 1.22

$$-\frac{a\cos(dx+c)^2\log\left(-b\cos(dx+c)^2+a+b\right)-2\,a\cos(dx+c)^2\log(-\cos(dx+c))-a-b}{2\left(a^2+2\,ab+b^2\right)d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/2*(a*cos(d*x + c)^2*log(-b*cos(d*x + c)^2 + a + b) - 2*a*cos(d*x + c)^2*
log(-cos(d*x + c)) - a - b)/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)
```

giac [B] time = 0.81, size = 234, normalized size = 3.66

$$-\frac{\frac{a\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}\right)}{a^2+2\,ab+b^2}-\frac{2\,a\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a^2+2\,ab+b^2}+\frac{3\,a+\frac{10\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{3\,a(\cos(dx+c)-1)^2}{(\cos(dx+c)+1)^2}}{\left(a^2+2\,ab+b^2\right)\left(\frac{\cos(dx+c)-1}{\cos(dx+c)+1}+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/2*(a*log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x +
c) - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(
a^2 + 2*a*b + b^2) - 2*a*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1
))/(a^2 + 2*a*b + b^2) + (3*a + 10*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1)
+ 4*b*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) + 3*a*(cos(d*x + c) - 1)^2/(cos
(d*x + c) + 1)^2)/((a^2 + 2*a*b + b^2)*((cos(d*x + c) - 1)/(cos(d*x + c) +
1) + 1)^2))/d
```

maple [A] time = 0.49, size = 66, normalized size = 1.03

$$-\frac{a\ln\left(b\left(\cos^2(dx+c)\right)-a-b\right)}{2d\,(a+b)^2}+\frac{a\ln(\cos(dx+c))}{(a+b)^2\,d}+\frac{1}{2d\,(a+b)\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/2/d*a/(a+b)^2*ln(b*cos(d*x+c)^2-a-b)+a*ln(cos(d*x+c))/(a+b)^2/d+1/2/d/(a
+b)/cos(d*x+c)^2
```

maxima [A] time = 0.32, size = 82, normalized size = 1.28

$$-\frac{\frac{a\log\left(b\sin(dx+c)^2+a\right)}{a^2+2\,ab+b^2}-\frac{a\log\left(\sin(dx+c)^2-1\right)}{a^2+2\,ab+b^2}+\frac{1}{(a+b)\sin(dx+c)^2-a-b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/2*(a*log(b*sin(d*x + c)^2 + a)/(a^2 + 2*a*b + b^2) - a*log(sin(d*x + c)^
2 - 1)/(a^2 + 2*a*b + b^2) + 1/((a + b)*sin(d*x + c)^2 - a - b))/d
```

mupad [B] time = 14.28, size = 52, normalized size = 0.81

$$-\frac{a\left(\frac{\ln\left((a+b)\tan(c+d\,x)^2+a\right)}{2}-\frac{\tan(c+d\,x)^2}{2}\right)-\frac{b\tan(c+d\,x)^2}{2}}{d\,(a+b)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -(a*(log(a + tan(c + d*x)^2*(a + b))/2 - tan(c + d*x)^2/2) - (b*tan(c + d*x
)^2)/2)/(d*(a + b)^2)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.444 $\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)}-\frac{\log(\cos(c+dx))}{d(a+b)}$$

```
[Out] -ln(cos(d*x+c))/(a+b)/d+1/2*ln(a+b*sin(d*x+c)^2)/(a+b)/d
```

Rubi [A] time = 0.04, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3194, 36, 31}

$$\frac{\log\left(a+b\sin^2(c+dx)\right)}{2d(a+b)}-\frac{\log(\cos(c+dx))}{d(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(Log[Cos[c + d*x]]/((a + b)*d)) + Log[a + b*Sin[c + d*x]^2]/(2*(a + b)*d)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2(a+b)d}+\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,\sin^2(c+dx)\right)}{2(a+b)d}\\
&= -\frac{\log(\cos(c+dx))}{(a+b)d}+\frac{\log\left(a+b\sin^2(c+dx)\right)}{2(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.03, size = 37, normalized size = 0.86

$$\frac{\log\left(a-b\cos^2(c+dx)+b\right)-2\log(\cos(c+dx))}{2ad+2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-2*Log[Cos[c + d*x]] + Log[a + b - b*Cos[c + d*x]^2])/(2*a*d + 2*b*d)
```

fricas [A] time = 0.50, size = 37, normalized size = 0.86

$$\frac{\log\left(-b\cos\left(dx+c\right)^{2}+a+b\right)-2\,\log\left(-\cos\left(dx+c\right)\right)}{2\,(a+b)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(-cos(d*x + c)))/((a + b)*d)
```

giac [B] time = 0.23, size = 110, normalized size = 2.56

$$\frac{\frac{\log\left(a-\frac{2\,a(\cos(dx+c)-1)}{\cos(dx+c)+1}-\frac{4\,b(\cos(dx+c)-1)}{\cos(dx+c)+1}+\frac{a(\cos(dx+c)-1)^{2}}{(\cos(dx+c)+1)^{2}}\right)}{a+b}-\frac{2\,\log\left(\left|-\frac{\cos(dx+c)-1}{\cos(dx+c)+1}-1\right|\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/2*(log(a - 2*a*(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 4*b*(cos(d*x + c)
 - 1)/(cos(d*x + c) + 1) + a*(cos(d*x + c) - 1)^2/(cos(d*x + c) + 1)^2)/(a +
 b) - 2*log(abs(-(cos(d*x + c) - 1)/(cos(d*x + c) + 1) - 1))/(a + b))/d
```

maple [A] time = 0.44, size = 47, normalized size = 1.09

$$\frac{\ln\left(b\left(\cos^{2}\left(dx+c\right)\right)-a-b\right)}{2d\left(a+b\right)}-\frac{\ln\left(\cos\left(dx+c\right)\right)}{\left(a+b\right)d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/2/d/(a+b)*ln(b*cos(d*x+c)^2-a-b)-ln(cos(d*x+c))/(a+b)/d
```

maxima [A] time = 0.32, size = 43, normalized size = 1.00

$$\frac{\frac{\log\left(b\sin(dx+c)^{2}+a\right)}{a+b}-\frac{\log\left(\sin(dx+c)^{2}-1\right)}{a+b}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(b*sin(d*x + c)^2 + a)/(a + b) - log(sin(d*x + c)^2 - 1)/(a + b))/d
```

mupad [B] time = 14.51, size = 28, normalized size = 0.65

$$\frac{\ln\left(\left(a+b\right)\,\tan\left(c+d\,x\right)^2+a\right)}{d\,\left(2\,a+2\,b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

```
[Out] log(a + tan(c + d*x)^2*(a + b))/(d*(2*a + 2*b))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.445 $\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=38

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

```
[Out] ln(sin(d*x+c))/a/d-1/2*ln(a+b*sin(d*x+c)^2)/a/d
```

Rubi [A] time = 0.04, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.190, Rules used = {3194, 36, 29, 31}

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

Rule 29

```
Int[(x_)^(-1), x_Symbol] :> Simp[Log[x], x]
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 36

```
Int[1/(((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))), x_Symbol] :> Dist[b/(b*c
 - a*d), Int[1/(a + b*x), x], x] - Dist[d/(b*c - a*d), Int[1/(c + d*x), x],
 x] /; FreeQ[{a, b, c, d}, x] && NeQ[b*c - a*d, 0]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \frac{1}{x}\,dx,x,\sin^2(c+dx)\right)}{2ad}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx}\,dx,x,\sin^2(c+dx)\right)}{2ad}\\
&= \frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}
\end{aligned}$$

Mathematica [A] time = 0.02, size = 38, normalized size = 1.00

$$\frac{\log(\sin(c+dx))}{ad}-\frac{\log\left(a+b\sin^2(c+dx)\right)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/(a + b*Sin[c + d*x]^2),x]
[Out] Log[Sin[c + d*x]]/(a*d) - Log[a + b*Sin[c + d*x]^2]/(2*a*d)
```

fricas [A] time = 0.47, size = 35, normalized size = 0.92

$$-\frac{\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\,\log\left(\frac{1}{2}\,\sin\left(dx+c\right)\right)}{2\,ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
[Out] -1/2*(log(-b*cos(d*x + c)^2 + a + b) - 2*log(1/2*sin(d*x + c)))/(a*d)
```

giac [A] time = 0.17, size = 38, normalized size = 1.00

$$\frac{\frac{\log\left(\sin(dx+c)^2\right)}{a}-\frac{\log\left(\left|b\sin(dx+c)^2+a\right|\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="giac")
[Out] 1/2*(log(sin(d*x + c)^2)/a - log(abs(b*sin(d*x + c)^2 + a))/a)/d
```

maple [A] time = 0.30, size = 37, normalized size = 0.97

$$\frac{\ln\left(\sin\left(dx+c\right)\right)}{ad}-\frac{\ln\left(a+b\left(\sin^2\left(dx+c\right)\right)\right)}{2da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)^2),x)
[Out] ln(sin(d*x+c))/a/d-1/2*ln(a+b*sin(d*x+c)^2)/d/a
```

maxima [A] time = 0.33, size = 37, normalized size = 0.97

$$-\frac{\frac{\log\left(b\sin(dx+c)^2+a\right)}{a}-\frac{\log\left(\sin(dx+c)^2\right)}{a}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
[Out] -1/2*(log(b*sin(d*x + c)^2 + a)/a - log(sin(d*x + c)^2)/a)/d
```

mupad [B] time = 14.43, size = 41, normalized size = 1.08

$$-\frac{\ln\left(a+a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)^2\right)-2\,\ln\left(\tan\left(c+d\,x\right)\right)}{2\,a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)^2),x)
```

```
[Out] -(log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2) - 2*log(tan(c + d*x)))/(2*a*
d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)/(a + b*sin(c + d*x)**2), x)
```

3.446 $\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=63

$$\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

```
[Out] -1/2*csc(d*x+c)^2/a/d-(a+b)*ln(sin(d*x+c))/a^2/d+1/2*(a+b)*ln(a+b*sin(d*x+c
)^2)/a^2/d
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {3194, 77}

$$\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}-\frac{\csc^2(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -Csc[c + d*x]^2/(2*a*d) - ((a + b)*Log[Sin[c + d*x]])/(a^2*d) + ((a + b)*Lo
g[a + b*Sin[c + d*x]^2])/(2*a^2*d)
```

Rule 77

```
Int[((a_.) + (b_.)*(x_))*((c_) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p_
.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)*(c + d*x)^n*(e + f*x)^p, x],
 x] /; FreeQ[{a, b, c, d, e, f, n}, x] && NeQ[b*c - a*d, 0] && ((ILtQ[n, 0]
 && ILtQ[p, 0]) || EqQ[p, 1] || (IGtQ[p, 0] && ( !IntegerQ[n] || LeQ[9*p +
5*(n + 2), 0] || GeQ[n + p + 1, 0] || (GeQ[n + p + 2, 0] && RationalQ[a, b,
 c, d, e, f]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\
&= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^2}+\frac{-a-b}{a^2x}+\frac{b(a+b)}{a^2(a+bx)}\right)dx,x,\sin^2(c+dx)\right)}{2d}\\
&= -\frac{\csc^2(c+dx)}{2ad}-\frac{(a+b)\log(\sin(c+dx))}{a^2d}+\frac{(a+b)\log\left(a+b\sin^2(c+dx)\right)}{2a^2d}
\end{aligned}$$

Mathematica [A] time = 0.15, size = 50, normalized size = 0.79

$$-\frac{(a+b)\left(2\log(\sin(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)+a\csc^2(c+dx)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/2*(a*Csc[c + d*x]^2 + (a + b)*(2*Log[Sin[c + d*x]] - Log[a + b*Sin[c + d
*x]^2]))/(a^2*d)
```

fricas [A] time = 0.49, size = 91, normalized size = 1.44

$$\frac{\left((a+b)\cos\left(dx+c\right)^2-a-b\right)\log\left(-b\cos\left(dx+c\right)^2+a+b\right)-2\left((a+b)\cos\left(dx+c\right)^2-a-b\right)\log\left(\frac{1}{2}\sin\left(dx+c\right.\right.}{2\left(a^2d\cos\left(dx+c\right)^2-a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/2*(((a + b)*cos(d*x + c)^2 - a - b)*log(-b*cos(d*x + c)^2 + a + b) - 2*((
a + b)*cos(d*x + c)^2 - a - b)*log(1/2*sin(d*x + c)) + a)/(a^2*d*cos(d*x +
c)^2 - a^2*d)
```

giac [A] time = 0.23, size = 108, normalized size = 1.71

$$\frac{\frac{\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}}{a}+\frac{4\,(a+b)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+2\,a+4\,b\right|\right)}{a^2}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/8*(((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x +
 c) + 1))/a + 4*(a + b)*log(abs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 2*a + 4*b))/a^2)/d
```

maple [B] time = 0.66, size = 161, normalized size = 2.56

$$\frac{1}{4da\left(\cos\left(dx+c\right)-1\right)}-\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2da}-\frac{\ln\left(\cos\left(dx+c\right)-1\right)b}{2d\,a^2}+\frac{\ln\left(b\left(\cos^2\left(dx+c\right)\right)-a-b\right)}{2da}+\frac{\ln\left(b\left(\cos^2\left(d\right.\right.\right.}{2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/4/d/a/(cos(d*x+c)-1)-1/2/d/a*ln(cos(d*x+c)-1)-1/2/d/a^2*ln(cos(d*x+c)-1)*
b+1/2/d/a*ln(b*cos(d*x+c)^2-a-b)+1/2/d/a^2*ln(b*cos(d*x+c)^2-a-b)*b-1/4/a/d
/(1+cos(d*x+c))-1/2/d/a*ln(1+cos(d*x+c))-1/2/d/a^2*ln(1+cos(d*x+c))*b
```

maxima [A] time = 0.32, size = 56, normalized size = 0.89

$$\frac{\frac{(a+b)\log\left(b\sin(dx+c)^2+a\right)}{a^2}-\frac{(a+b)\log\left(\sin(dx+c)^2\right)}{a^2}-\frac{1}{a\sin(dx+c)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/2*((a + b)*log(b*sin(d*x + c)^2 + a)/a^2 - (a + b)*log(sin(d*x + c)^2)/a^
2 - 1/(a*sin(d*x + c)^2))/d
```

mupad [B] time = 14.46, size = 69, normalized size = 1.10

$$\frac{\ln\left(a+a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)^2\right)\,\left(a+b\right)}{2\,a^2\,d}-\frac{\cot\left(c+d\,x\right)^2}{2\,a\,d}-\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,\left(a+b\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(a + b))/(2*a^2*d) - cot(c +
d*x)^2/(2*a*d) - (log(tan(c + d*x))*(a + b))/(a^2*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**3/(a + b*sin(c + d*x)**2), x)
```

3.447 $\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=89

$$-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}+\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{\csc^4(c+dx)}{4ad}$$

```
[Out] 1/2*(2*a+b)*csc(d*x+c)^2/a^2/d-1/4*csc(d*x+c)^4/a/d+(a+b)^2*ln(sin(d*x+c))/
a^3/d-1/2*(a+b)^2*ln(a+b*sin(d*x+c)^2)/a^3/d
```

Rubi [A] time = 0.09, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3194, 88}

$$\frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{(a+b)^2\log\left(a+b\sin^2(c+dx)\right)}{2a^3d}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{\csc^4(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((2*a + b)*Csc[c + d*x]^2)/(2*a^2*d) - Csc[c + d*x]^4/(4*a*d) + ((a + b)^2*
Log[Sin[c + d*x]])/(a^3*d) - ((a + b)^2*Log[a + b*Sin[c + d*x]^2])/(2*a^3*d
)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)}\,dx, x, \sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^3}+\frac{-2a-b}{a^2x^2}+\frac{(a+b)^2}{a^3x}-\frac{b(a+b)^2}{a^3(a+bx)}\right)dx, x, \sin^2(c+dx)\right)}{2d}\\ &= \frac{(2a+b)\csc^2(c+dx)}{2a^2d}-\frac{\csc^4(c+dx)}{4ad}+\frac{(a+b)^2\log(\sin(c+dx))}{a^3d}-\frac{(a+b)^2\log\left(a+b\right.}{2a^3d}\end{aligned}$$

Mathematica [A] time = 0.52, size = 72, normalized size = 0.81

$$\frac{-a^2\csc^4(c+dx)+2a(2a+b)\csc^2(c+dx)+2(a+b)^2\left(2\log(\sin(c+dx))-\log\left(a+b\sin^2(c+dx)\right)\right)}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (2*a*(2*a + b)*Csc[c + d*x]^2 - a^2*Csc[c + d*x]^4 + 2*(a + b)^2*(2*Log[Sin
[c + d*x]] - Log[a + b*Sin[c + d*x]^2]))/(4*a^3*d)
```

fricas [B] time = 0.55, size = 198, normalized size = 2.22

$$-\frac{2\left(2\,a^2+ab\right)\cos\left(dx+c\right)^2-3\,a^2-2\,ab+2\left(\left(a^2+2\,ab+b^2\right)\cos\left(dx+c\right)^4-2\left(a^2+2\,ab+b^2\right)\cos\left(dx+c\right)^2\right.}{4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] -1/4*(2*(2*a^2 + a*b)*cos(d*x + c)^2 - 3*a^2 - 2*a*b + 2*((a^2 + 2*a*b + b^
2)*cos(d*x + c)^4 - 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^
2)*log(-b*cos(d*x + c)^2 + a + b) - 4*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 -
 2*(a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*log(1/2*sin(d*x
+ c)))/(a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)
```

giac [B] time = 0.28, size = 205, normalized size = 2.30

$$-\frac{\frac{a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+12\,a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+8\,b\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)}{a^2}+\frac{32\left(a^2+2\,ab+b^2\right)\log\left(\left|-a\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-}{\cos(dx+c)+}\right.\right.\right.}{a^3}}{64\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/64*((a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(
d*x + c) + 1))^2 + 12*a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x +
 c) - 1)/(cos(d*x + c) + 1)) + 8*b*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) +
 (cos(d*x + c) - 1)/(cos(d*x + c) + 1)))/a^2 + 32*(a^2 + 2*a*b + b^2)*log(a
bs(-a*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x
+ c) + 1)) + 2*a + 4*b))/a^3)/d
```

maple [B] time = 0.64, size = 302, normalized size = 3.39

$$-\frac{1}{16da\left(\cos\left(dx+c\right)-1\right)^2}-\frac{7}{16da\left(\cos\left(dx+c\right)-1\right)}-\frac{b}{4d\,a^2\left(\cos\left(dx+c\right)-1\right)}+\frac{\ln\left(\cos\left(dx+c\right)-1\right)}{2da}+\frac{\ln\left(\cos\left(dx\right.\right.}{d\,a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/16/d/a/(cos(d*x+c)-1)^2-7/16/d/a/(cos(d*x+c)-1)-1/4/d/a^2/(cos(d*x+c)-1)
*b+1/2/d/a*ln(cos(d*x+c)-1)+1/d/a^2*ln(cos(d*x+c)-1)*b+1/2/d/a^3*ln(cos(d*x
+c)-1)*b^2-1/2/d/a*ln(b*cos(d*x+c)^2-a-b)-1/d/a^2*ln(b*cos(d*x+c)^2-a-b)*b-
1/2/d/a^3*ln(b*cos(d*x+c)^2-a-b)*b^2-1/16/a/d/(1+cos(d*x+c))^2+7/16/a/d/(1+
cos(d*x+c))+1/4/d/a^2/(1+cos(d*x+c))*b+1/2/d/a*ln(1+cos(d*x+c))+1/d/a^2*ln(
1+cos(d*x+c))*b+1/2/d/a^3*ln(1+cos(d*x+c))*b^2
```

maxima [A] time = 0.33, size = 92, normalized size = 1.03

$$-\frac{\frac{2\left(a^2+2\,ab+b^2\right)\log\left(b\sin(dx+c)^2+a\right)}{a^3}-\frac{2\left(a^2+2\,ab+b^2\right)\log\left(\sin(dx+c)^2\right)}{a^3}-\frac{2\,(2\,a+b)\sin(dx+c)^2-a}{a^2\sin(dx+c)^4}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*(a^2 + 2*a*b + b^2)*log(b*sin(d*x + c)^2 + a)/a^3 - 2*(a^2 + 2*a*b
+ b^2)*log(sin(d*x + c)^2)/a^3 - (2*(2*a + b)*sin(d*x + c)^2 - a)/(a^2*sin(
d*x + c)^4))/d
```

mupad [B] time = 14.50, size = 103, normalized size = 1.16

$$\frac{\ln\left(\tan\left(c+d\,x\right)\right)\,\left(a^2+2\,a\,b+b^2\right)}{a^3\,d}-\frac{\ln\left(a+a\,\tan\left(c+d\,x\right)^2+b\,\tan\left(c+d\,x\right)^2\right)\,\left(a^2+2\,a\,b+b^2\right)}{2\,a^3\,d}-\frac{\frac{1}{4\,a}-\frac{\tan(c+d\,x)^2\,(a+}{2\,a^2}}{d\,\tan\left(c+d\,x\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(tan(c + d*x))*(2*a*b + a^2 + b^2))/(a^3*d) - (log(a + a*tan(c + d*x)^2
 + b*tan(c + d*x)^2)*(2*a*b + a^2 + b^2))/(2*a^3*d) - (1/(4*a) - (tan(c + d
*x)^2*(a + b))/(2*a^2))/(d*tan(c + d*x)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**5/(a + b*sin(c + d*x)**2), x)
```

3.448 $\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=121

$$\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}-\frac{(a+b)^3\log(\sin(c+dx))}{a^4d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+\cdots}{2a^3d}$$

```
[Out] -1/2*(3*a^2+3*a*b+b^2)*csc(d*x+c)^2/a^3/d+1/4*(3*a+b)*csc(d*x+c)^4/a^2/d-1/
6*csc(d*x+c)^6/a/d-(a+b)^3*ln(sin(d*x+c))/a^4/d+1/2*(a+b)^3*ln(a+b*sin(d*x+
c)^2)/a^4/d
```

Rubi [A] time = 0.11, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.087$, Rules used = {3194, 88}

$$-\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}+\frac{(a+b)^3\log\left(a+b\sin^2(c+dx)\right)}{2a^4d}-\frac{(a+b)^3\log(\sin(c+\cdots}{a^4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2)/(2*a^3*d) + ((3*a + b)*Csc[c + d*x]
^4)/(4*a^2*d) - Csc[c + d*x]^6/(6*a*d) - ((a + b)^3*Log[Sin[c + d*x]])/(a^4
*d) + ((a + b)^3*Log[a + b*Sin[c + d*x]^2])/(2*a^4*d)
```

Rule 88

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x
_))^(p_.), x_Symbol] :> Int[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n*(e + f*
x)^p, x], x] /; FreeQ[{a, b, c, d, e, f, p}, x] && IntegersQ[m, n] && (Inte
gerQ[p] || (GtQ[m, 0] && GeQ[n, -1]))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \frac{\cot^7(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{(1-x)^3}{x^4(a+bx)}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \left(\frac{1}{ax^4}+\frac{-3a-b}{a^2x^3}+\frac{3a^2+3ab+b^2}{a^3x^2}-\frac{(a+b)^3}{a^4x}+\frac{b(a+b)^3}{a^4(a+bx)}\right)dx,x,\sin^2(c+dx)\right)}{2d}\\ &= -\frac{\left(3a^2+3ab+b^2\right)\csc^2(c+dx)}{2a^3d}+\frac{(3a+b)\csc^4(c+dx)}{4a^2d}-\frac{\csc^6(c+dx)}{6ad}-\frac{(a+b)^3\,\mathrm{l}\cdots}{\cdots}\end{aligned}$$

Mathematica [A] time = 0.26, size = 100, normalized size = 0.83

$$-\frac{2a^3\csc^6(c+dx)+6a\left(3a^2+3ab+b^2\right)\csc^2(c+dx)-3a^2(3a+b)\csc^4(c+dx)-6(a+b)^3\log\left(a+b\sin^2(c+\cdots\right.}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^7/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -1/12*(6*a*(3*a^2 + 3*a*b + b^2)*Csc[c + d*x]^2 - 3*a^2*(3*a + b)*Csc[c + d
*x]^4 + 2*a^3*Csc[c + d*x]^6 + 12*(a + b)^3*Log[Sin[c + d*x]] - 6*(a + b)^3
*Log[a + b*Sin[c + d*x]^2])/(a^4*d)
```

fricas [B] time = 0.60, size = 371, normalized size = 3.07

$$\frac{6\left(3a^3+3a^2b+ab^2\right)\cos\left(dx+c\right)^4+11a^3+15a^2b+6ab^2-3\left(9a^3+11a^2b+4ab^2\right)\cos\left(dx+c\right)^2+6\left(\left(a^3+3a^2b\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] 1/12*(6*(3*a^3 + 3*a^2*b + a*b^2)*cos(d*x + c)^4 + 11*a^3 + 15*a^2*b + 6*a*
b^2 - 3*(9*a^3 + 11*a^2*b + 4*a*b^2)*cos(d*x + c)^2 + 6*((a^3 + 3*a^2*b + 3
*a*b^2 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*co
s(d*x + c)^2)*log(-b*cos(d*x + c)^2 + a + b) - 12*((a^3 + 3*a^2*b + 3*a*b^2
 + b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 -
 a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x
+ c)^2)*log(1/2*sin(d*x + c)))/(a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*x + c)
^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)
```

giac [B] time = 0.32, size = 353, normalized size = 2.92

$$\frac{\frac{a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^3+12a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+6ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)^2+84a^2\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}+\frac{\cos(dx+c)-1}{\cos(dx+c)+1}\right)+120ab\left(\frac{\cos(dx+c)+1}{\cos(dx+c)-1}\right.}{a^3}}{384d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/384*((a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(co
s(d*x + c) + 1))^3 + 12*a^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d
*x + c) - 1)/(cos(d*x + c) + 1))^2 + 6*a*b*((cos(d*x + c) + 1)/(cos(d*x + c
) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1))^2 + 84*a^2*((cos(d*x + c) +
 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)) + 120*a*b*(
(cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) +
1)) + 48*b^2*((cos(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(c
os(d*x + c) + 1)))/a^3 + 192*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(abs(-a*((c
os(d*x + c) + 1)/(cos(d*x + c) - 1) + (cos(d*x + c) - 1)/(cos(d*x + c) + 1)
) + 2*a + 4*b))/a^4)/d
```

maple [B] time = 0.64, size = 489, normalized size = 4.04

$$\frac{1}{48da\left(\cos\left(dx+c\right)-1\right)^3}+\frac{5}{32da\left(\cos\left(dx+c\right)-1\right)^2}+\frac{b}{16d\,a^2\left(\cos\left(dx+c\right)-1\right)^2}+\frac{19}{32da\left(\cos\left(dx+c\right)-1\right)}+\frac{}{16d\,a^2\left(\cos\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/48/d/a/(cos(d*x+c)-1)^3+5/32/d/a/(cos(d*x+c)-1)^2+1/16/d/a^2/(cos(d*x+c)-
1)^2*b+19/32/d/a/(cos(d*x+c)-1)+11/16/d/a^2/(cos(d*x+c)-1)*b+1/4/d/a^3/(cos
(d*x+c)-1)*b^2-1/2/d/a*ln(cos(d*x+c)-1)-3/2/d/a^2*ln(cos(d*x+c)-1)*b-3/2/d/
a^3*ln(cos(d*x+c)-1)*b^2-1/2/d/a^4*ln(cos(d*x+c)-1)*b^3+1/2/d/a*ln(b*cos(d*
```

x+c)^2-a-b)+3/2/d/a^2*ln(b*cos(d*x+c)^2-a-b)*b+3/2/d/a^3*ln(b*cos(d*x+c)^2-a-b)*b^2+1/2/d/a^4*ln(b*cos(d*x+c)^2-a-b)*b^3-1/48/d/a/(1+cos(d*x+c))^3+5/32/a/d/(1+cos(d*x+c))^2+1/16/d/a^2/(1+cos(d*x+c))^2*b-19/32/a/d/(1+cos(d*x+c))-11/16/d/a^2/(1+cos(d*x+c))*b-1/4/d/a^3/(1+cos(d*x+c))*b^2-1/2/d/a*ln(1+cos(d*x+c))-3/2/d/a^2*ln(1+cos(d*x+c))*b-3/2/d/a^3*ln(1+cos(d*x+c))*b^2-1/2/d/a^4*ln(1+cos(d*x+c))*b^3

maxima [A] time = 0.32, size = 137, normalized size = 1.13

$$\frac{\frac{6\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(b\sin(dx+c)^2+a\right)}{a^4}-\frac{6\left(a^3+3a^2b+3ab^2+b^3\right)\log\left(\sin(dx+c)^2\right)}{a^4}-\frac{6\left(3a^2+3ab+b^2\right)\sin(dx+c)^4-3\left(3a^2+ab\right)\sin(dx+c)^2+2}{a^3\sin(dx+c)^6}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^7/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/12*(6*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(b*sin(d*x + c)^2 + a)/a^4 - 6*(
a^3 + 3*a^2*b + 3*a*b^2 + b^3)*log(sin(d*x + c)^2)/a^4 - (6*(3*a^2 + 3*a*b
+ b^2)*sin(d*x + c)^4 - 3*(3*a^2 + a*b)*sin(d*x + c)^2 + 2*a^2)/(a^3*sin(d*
x + c)^6))/d
```

mupad [B] time = 15.35, size = 138, normalized size = 1.14

$$\frac{\ln\left(a+a\tan\left(c+d\,x\right)^2+b\tan\left(c+d\,x\right)^2\right)\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,a^4\,d}-\frac{\frac{1}{6\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{4\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{2\,a^3}}{d\tan\left(c+d\,x\right)^6}-\ln$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^7/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (log(a + a*tan(c + d*x)^2 + b*tan(c + d*x)^2)*(3*a*b^2 + 3*a^2*b + a^3 + b^
3))/(2*a^4*d) - (1/(6*a) - (tan(c + d*x)^2*(a + b))/(4*a^2) + (tan(c + d*x)
^4*(a + b)^2)/(2*a^3))/(d*tan(c + d*x)^6) - (log(tan(c + d*x))*(3*a*b^2 + 3
*a^2*b + a^3 + b^3))/(a^4*d)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**7/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.449 $\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=120

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{d(a+b)^4}+\frac{a^2\tan^3(c+dx)}{3d(a+b)^3}+\frac{\tan^7(c+dx)}{7d(a+b)}-\frac{a\tan^5(c+dx)}{5d(a+b)^2}$$

```
[Out] a^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(9/2)/d-a^3*tan(d*x+c)
/(a+b)^4/d+1/3*a^2*tan(d*x+c)^3/(a+b)^3/d-1/5*a*tan(d*x+c)^5/(a+b)^2/d+1/7*
tan(d*x+c)^7/(a+b)/d
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 302, 205}

$$\frac{a^2\tan^3(c+dx)}{3d(a+b)^3}+\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}}-\frac{a^3\tan(c+dx)}{d(a+b)^4}+\frac{\tan^7(c+dx)}{7d(a+b)}-\frac{a\tan^5(c+dx)}{5d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) - (a
^3*Tan[c + d*x])/((a + b)^4*d) + (a^2*Tan[c + d*x]^3)/(3*(a + b)^3*d) - (a*
Tan[c + d*x]^5)/(5*(a + b)^2*d) + Tan[c + d*x]^7/(7*(a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\tan^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^8}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(-\frac{a^3}{(a+b)^4} + \frac{a^2x^2}{(a+b)^3} - \frac{ax^4}{(a+b)^2} + \frac{x^6}{a+b} + \frac{a^4}{(a+b)^4\left(a+(a+b)x^2\right)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{a^3\tan(c+dx)}{(a+b)^4d} + \frac{a^2\tan^3(c+dx)}{3(a+b)^3d} - \frac{a\tan^5(c+dx)}{5(a+b)^2d} + \frac{\tan^7(c+dx)}{7(a+b)d} + \frac{a^4\,\text{Subst}\left(\int\right.}{}$$

$$= \frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{9/2}d} - \frac{a^3\tan(c+dx)}{(a+b)^4d} + \frac{a^2\tan^3(c+dx)}{3(a+b)^3d} - \frac{a\tan^5(c+dx)}{5(a+b)^2d} + \frac{\text{t}}{}$$

Mathematica [A] time = 2.39, size = 147, normalized size = 1.22

$$\frac{a^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{9/2}} + \frac{\tan(c+dx)\left(-176a^3 - 122a^2b + \left(122a^3 + 254a^2b + 177ab^2 + 45b^3\right)\sec^2(c+dx) - \right.}{105d(a+}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(9/2)*d) + ((
-176*a^3 - 122*a^2*b - 66*a*b^2 - 15*b^3 + (122*a^3 + 254*a^2*b + 177*a*b^2
 + 45*b^3)*Sec[c + d*x]^2 - 3*(a + b)^2*(22*a + 15*b)*Sec[c + d*x]^4 + 15*(
a + b)^3*Sec[c + d*x]^6)*Tan[c + d*x])/(105*(a + b)^4*d)
```

fricas [B] time = 0.56, size = 602, normalized size = 5.02

$$\left[\frac{105\,a^3\sqrt{-\frac{a}{a+b}}\,\cos(dx+c)^7\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right.\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(105*a^3*sqrt(-a/(a + b))*cos(d*x + c)^7*log(((8*a^2 + 8*a*b + b^2)*
cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b
 + b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))
*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(
d*x + c)^2 + a^2 + 2*a*b + b^2)) - 4*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*
b^3)*cos(d*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x +
c)^4 - 15*a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*
b^2 + 15*b^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4
*a*b^3 + b^4)*d*cos(d*x + c)^7), -1/210*(105*a^3*sqrt(a/(a + b))*arctan(1/2
*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*sin(d*x
 + c)))*cos(d*x + c)^7 + 2*((176*a^3 + 122*a^2*b + 66*a*b^2 + 15*b^3)*cos(d
*x + c)^6 - (122*a^3 + 254*a^2*b + 177*a*b^2 + 45*b^3)*cos(d*x + c)^4 - 15*
a^3 - 45*a^2*b - 45*a*b^2 - 15*b^3 + 3*(22*a^3 + 59*a^2*b + 52*a*b^2 + 15*b
^3)*cos(d*x + c)^2)*sin(d*x + c))/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b
^4)*d*cos(d*x + c)^7)]
```

giac [B] time = 10.51, size = 472, normalized size = 3.93

$$\frac{105\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^4}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{a^2+ab}}+\frac{15a^6\tan(dx+c)^7+90a^5b\tan(dx+c)^7+225a^4b^2\tan(dx+c)^7+300a^3b^3\tan(dx+c)^7+\cdots}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
 + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^4/((a^4 + 4*a^3*b + 6*a^2*b^2 +
 4*a*b^3 + b^4)*sqrt(a^2 + a*b)) + (15*a^6*tan(d*x + c)^7 + 90*a^5*b*tan(d*
x + c)^7 + 225*a^4*b^2*tan(d*x + c)^7 + 300*a^3*b^3*tan(d*x + c)^7 + 225*a^
2*b^4*tan(d*x + c)^7 + 90*a*b^5*tan(d*x + c)^7 + 15*b^6*tan(d*x + c)^7 - 21
*a^6*tan(d*x + c)^5 - 105*a^5*b*tan(d*x + c)^5 - 210*a^4*b^2*tan(d*x + c)^5
 - 210*a^3*b^3*tan(d*x + c)^5 - 105*a^2*b^4*tan(d*x + c)^5 - 21*a*b^5*tan(d
*x + c)^5 + 35*a^6*tan(d*x + c)^3 + 140*a^5*b*tan(d*x + c)^3 + 210*a^4*b^2*
tan(d*x + c)^3 + 140*a^3*b^3*tan(d*x + c)^3 + 35*a^2*b^4*tan(d*x + c)^3 - 1
05*a^6*tan(d*x + c) - 315*a^5*b*tan(d*x + c) - 315*a^4*b^2*tan(d*x + c) - 1
05*a^3*b^3*tan(d*x + c))/(a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*
b^4 + 21*a^2*b^5 + 7*a*b^6 + b^7))/d
```

maple [B] time = 0.55, size = 252, normalized size = 2.10

$$\frac{\left(\tan^7(dx+c)\right)a^3}{7d(a+b)^4}+\frac{3\left(\tan^7(dx+c)\right)a^2b}{7d(a+b)^4}+\frac{3\left(\tan^7(dx+c)\right)ab^2}{7d(a+b)^4}+\frac{\left(\tan^7(dx+c)\right)b^3}{7d(a+b)^4}-\frac{a^3\left(\tan^5(dx+c)\right)}{5d(a+b)^4}-\frac{2a^2\left(\tan\cdots\right.}{5d(\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/7/d/(a+b)^4*tan(d*x+c)^7*a^3+3/7/d/(a+b)^4*tan(d*x+c)^7*a^2*b+3/7/d/(a+b)
^4*tan(d*x+c)^7*a*b^2+1/7/d/(a+b)^4*tan(d*x+c)^7*b^3-1/5/d/(a+b)^4*a^3*tan(
d*x+c)^5-2/5/d/(a+b)^4*a^2*tan(d*x+c)^5*b-1/5/d/(a+b)^4*tan(d*x+c)^5*a*b^2+
1/3/d/(a+b)^4*tan(d*x+c)^3*a^3+1/3/d/(a+b)^4*tan(d*x+c)^3*a^2*b-a^3*tan(d*x
+c)/(a+b)^4/d+1/d*a^4/(a+b)^4/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a
+b))^(1/2))
```

maxima [A] time = 0.42, size = 180, normalized size = 1.50

$$\frac{\frac{105a^4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{(a^4+4a^3b+6a^2b^2+4ab^3+b^4)\sqrt{(a+b)a}}+\frac{15(a^3+3a^2b+3ab^2+b^3)\tan(dx+c)^7-21(a^3+2a^2b+ab^2)\tan(dx+c)^5-105a^3\tan(dx+c)+35(a^3+a^2b)\tan(dx+\cdots}{a^4+4a^3b+6a^2b^2+4ab^3+b^4}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*a^4*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^4 + 4*a^3*b
 + 6*a^2*b^2 + 4*a*b^3 + b^4)*sqrt((a + b)*a)) + (15*(a^3 + 3*a^2*b + 3*a*b
^2 + b^3)*tan(d*x + c)^7 - 21*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^5 - 105*
a^3*tan(d*x + c) + 35*(a^3 + a^2*b)*tan(d*x + c)^3)/(a^4 + 4*a^3*b + 6*a^2*
b^2 + 4*a*b^3 + b^4))/d
```

mupad [B] time = 14.78, size = 141, normalized size = 1.18

$$\frac{\tan(c+dx)^7}{7d(a+b)}+\frac{a^2\tan(c+dx)^3}{3d(a+b)^3}+\frac{a^{7/2}\,\mathrm{atan}\left(\frac{\tan(c+dx)(2a+2b)(a^4+4a^3b+6a^2b^2+4ab^3+b^4)}{2\sqrt{a}(a+b)^{9/2}}\right)}{d(a+b)^{9/2}}-\frac{a\tan(c+dx)^5}{5d(a+b)^2}-\frac{a^3\tan(c+\cdots}{d(a+b\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^7/(7*d*(a + b)) + (a^2*tan(c + d*x)^3)/(3*d*(a + b)^3) + (a^(7
/2)*atan((tan(c + d*x)*(2*a + 2*b)*(4*a*b^3 + 4*a^3*b + a^4 + b^4 + 6*a^2*b
^2))/(2*a^(1/2)*(a + b)^(9/2))))/(d*(a + b)^(9/2)) - (a*tan(c + d*x)^5)/(5*
d*(a + b)^2) - (a^3*tan(c + d*x))/(d*(a + b)^4)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.450 $\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=97

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{d(a+b)^3}+\frac{\tan^5(c+dx)}{5d(a+b)}-\frac{a\tan^3(c+dx)}{3d(a+b)^2}$$

```
[Out] -a^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(7/2)/d+a^2*tan(d*x+c
)/(a+b)^3/d-1/3*a*tan(d*x+c)^3/(a+b)^2/d+1/5*tan(d*x+c)^5/(a+b)/d
```

Rubi [A] time = 0.11, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3195, 302, 205}

$$-\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{7/2}}+\frac{a^2\tan(c+dx)}{d(a+b)^3}+\frac{\tan^5(c+dx)}{5d(a+b)}-\frac{a\tan^3(c+dx)}{3d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(7/2)*d)) +
 (a^2*Tan[c + d*x])/((a + b)^3*d) - (a*Tan[c + d*x]^3)/(3*(a + b)^2*d) + Ta
n[c + d*x]^5/(5*(a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\tan^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{x^6}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{a^2}{(a+b)^3} - \frac{ax^2}{(a+b)^2} + \frac{x^4}{a+b} - \frac{a^3}{(a+b)^3(a+(a+b)x^2)}\right) dx, x, \tan(c+dx)\right)}{d}$$

$$= \frac{a^2\tan(c+dx)}{(a+b)^3 d} - \frac{a\tan^3(c+dx)}{3(a+b)^2 d} + \frac{\tan^5(c+dx)}{5(a+b)d} - \frac{a^3\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c\right.}{(a+b)^3 d}$$

$$= -\frac{a^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{7/2}d} + \frac{a^2\tan(c+dx)}{(a+b)^3 d} - \frac{a\tan^3(c+dx)}{3(a+b)^2 d} + \frac{\tan^5(c+dx)}{5(a+b)d}$$

Mathematica [A] time = 0.75, size = 111, normalized size = 1.14

$$\frac{\sqrt{a+b}\,\tan(c+dx)\left(-\left(11a^2+17ab+6b^2\right)\sec^2(c+dx)+23a^2+3(a+b)^2\sec^4(c+dx)+11ab+3b^2\right)-15a^{5/}}{15d(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*a^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(23*a
^2 + 11*a*b + 3*b^2 - (11*a^2 + 17*a*b + 6*b^2)*Sec[c + d*x]^2 + 3*(a + b)^
2*Sec[c + d*x]^4)*Tan[c + d*x])/(15*(a + b)^(7/2)*d)
```

fricas [B] time = 0.51, size = 472, normalized size = 4.87

$$\left[\frac{15\,a^2\sqrt{-\frac{a}{a+b}}\cos(dx+c)^5\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+3ab+b^2\right)\cos(dx+c)^3-\left(a^2+2ab+b^2\right)\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{60\,(}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/60*(15*a^2*sqrt(-a/(a + b))*cos(d*x + c)^5*log(((8*a^2 + 8*a*b + b^2)*co
s(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b +
 b^2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*s
in(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*
x + c)^2 + a^2 + 2*a*b + b^2)) + 4*((23*a^2 + 11*a*b + 3*b^2)*cos(d*x + c)^
4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a*b + 3*b^2)*sin(d
*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5), 1/30*(15*a^2*s
qrt(a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b)
)/(a*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c)^5 + 2*((23*a^2 + 11*a*b + 3*b
^2)*cos(d*x + c)^4 - (11*a^2 + 17*a*b + 6*b^2)*cos(d*x + c)^2 + 3*a^2 + 6*a
*b + 3*b^2)*sin(d*x + c))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*d*cos(d*x + c)^5
)]
```

giac [B] time = 4.32, size = 296, normalized size = 3.05

$$-\frac{\frac{15\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^3}{\left(a^3+3a^2b+3ab^2+b^3\right)\sqrt{a^2+ab}} - \frac{3a^4\tan(dx+c)^5+12a^3b\tan(dx+c)^5+18a^2b^2\tan(dx+c)^5+12ab^3\tan(dx+c)^5+}{}}{15}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x
+ c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^3/((a^3 + 3*a^2*b + 3*a*b^2 + b^
3)*sqrt(a^2 + a*b)) - (3*a^4*tan(d*x + c)^5 + 12*a^3*b*tan(d*x + c)^5 + 18*
a^2*b^2*tan(d*x + c)^5 + 12*a*b^3*tan(d*x + c)^5 + 3*b^4*tan(d*x + c)^5 - 5
*a^4*tan(d*x + c)^3 - 15*a^3*b*tan(d*x + c)^3 - 15*a^2*b^2*tan(d*x + c)^3 -
 5*a*b^3*tan(d*x + c)^3 + 15*a^4*tan(d*x + c) + 30*a^3*b*tan(d*x + c) + 15*
a^2*b^2*tan(d*x + c))/(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 +
b^5))/d
```

maple [A] time = 0.54, size = 161, normalized size = 1.66

$$\frac{\left(\tan^5(dx+c)\right)a^2}{5d(a+b)^3}+\frac{2\left(\tan^5(dx+c)\right)ab}{5d(a+b)^3}+\frac{\left(\tan^5(dx+c)\right)b^2}{5d(a+b)^3}-\frac{a^2\left(\tan^3(dx+c)\right)}{3(a+b)^3 d}-\frac{\left(\tan^3(dx+c)\right)ba}{3d(a+b)^3}+\frac{a^2\tan(dx+}{(a+b)^3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/5/d/(a+b)^3*tan(d*x+c)^5*a^2+2/5/d/(a+b)^3*tan(d*x+c)^5*a*b+1/5/d/(a+b)^3
*tan(d*x+c)^5*b^2-1/3*a^2*tan(d*x+c)^3/(a+b)^3/d-1/3/d/(a+b)^3*tan(d*x+c)^3
*b*a+a^2*tan(d*x+c)/(a+b)^3/d-1/d*a^3/(a+b)^3/(a*(a+b))^(1/2)*arctan((a+b)*
tan(d*x+c)/(a*(a+b))^(1/2))
```

maxima [A] time = 0.42, size = 130, normalized size = 1.34

$$-\frac{\frac{15\,a^3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{(a+b)a}}-\frac{3\left(a^2+2\,ab+b^2\right)\tan(dx+c)^5-5\left(a^2+ab\right)\tan(dx+c)^3+15\,a^2\tan(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*a^3*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/((a^3 + 3*a^2*b
+ 3*a*b^2 + b^3)*sqrt((a + b)*a)) - (3*(a^2 + 2*a*b + b^2)*tan(d*x + c)^5 -
 5*(a^2 + a*b)*tan(d*x + c)^3 + 15*a^2*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b
^2 + b^3))/d
```

mupad [B] time = 15.11, size = 112, normalized size = 1.15

$$\frac{\tan(c+d\,x)^5}{5\,d\,(a+b)}-\frac{a^{5/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,\left(a^3+3\,a^2\,b+3\,a\,b^2+b^3\right)}{2\,\sqrt{a}\,(a+b)^{7/2}}\right)}{d\,(a+b)^{7/2}}-\frac{a\tan(c+d\,x)^3}{3\,d\,(a+b)^2}+\frac{a^2\tan(c+d\,x)}{d\,(a+b)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^5/(5*d*(a + b)) - (a^(5/2)*atan((tan(c + d*x)*(2*a + 2*b)*(3*a
*b^2 + 3*a^2*b + a^3 + b^3))/(2*a^(1/2)*(a + b)^(7/2))))/(d*(a + b)^(7/2))
- (a*tan(c + d*x)^3)/(3*d*(a + b)^2) + (a^2*tan(c + d*x))/(d*(a + b)^3)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.451 $\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=74

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a+b)}-\frac{a\tan(c+dx)}{d(a+b)^2}$$

```
[Out] a^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/(a+b)^(5/2)/d-a*tan(d*x+c)/(
a+b)^2/d+1/3*tan(d*x+c)^3/(a+b)/d
```

Rubi [A] time = 0.10, antiderivative size = 74, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 302, 205}

$$\frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{5/2}}+\frac{\tan^3(c+dx)}{3d(a+b)}-\frac{a\tan(c+dx)}{d(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(5/2)*d) - (a
*Tan[c + d*x])/((a + b)^2*d) + Tan[c + d*x]^3/(3*(a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 302

```
Int[(x_)^(m_)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[PolynomialDivide[x
^m, a + b*x^n, x], x] /; FreeQ[{a, b}, x] && IGtQ[m, 0] && IGtQ[n, 0] && Gt
Q[m, 2*n - 1]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\tan^4(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^4}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{d}\\
&= \frac{\text{Subst}\left(\int \left(-\frac{a}{(a+b)^2}+\frac{x^2}{a+b}+\frac{a^2}{(a+b)^2\left(a+(a+b)x^2\right)}\right)dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{a\tan(c+dx)}{(a+b)^2d}+\frac{\tan^3(c+dx)}{3(a+b)d}+\frac{a^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{(a+b)^2d}\\
&= \frac{a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{5/2}d}-\frac{a\tan(c+dx)}{(a+b)^2d}+\frac{\tan^3(c+dx)}{3(a+b)d}
\end{aligned}$$

Mathematica [A] time = 0.30, size = 75, normalized size = 1.01

$$\frac{3a^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)+\sqrt{a+b}\tan(c+dx)\left((a+b)\sec^2(c+dx)-4a-b\right)}{3d(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (3*a^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a + b]*(-4*a -
 b + (a + b)*Sec[c + d*x]^2)*Tan[c + d*x])/(3*(a + b)^(5/2)*d)
```

fricas [A] time = 0.49, size = 366, normalized size = 4.95

$$\left[\frac{3\,a\sqrt{-\frac{a}{a+b}}\cos\left(dx+c\right)^3\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4-2\left(4\,a^2+5\,ab+b^2\right)\cos(dx+c)^2-4\left(\left(2\,a^2+3\,ab+b^2\right)\cos(dx+c)^3-\left(a^2+2\,ab+b^2\right)\cos(dx\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2\,ab+b^2}\right.}{12\left(a^2+2\,ab+b^2\right)d\cos\left(dx+c\right)^3}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*a*sqrt(-a/(a + b))*cos(d*x + c)^3*log(((8*a^2 + 8*a*b + b^2)*cos(d
*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + 3*a*b + b^
2)*cos(d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(
d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x +
 c)^2 + a^2 + 2*a*b + b^2)) - 4*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*x
+ c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3), -1/6*(3*a*sqrt(a/(a + b))*arc
tan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a*cos(d*x + c)*
sin(d*x + c)))*cos(d*x + c)^3 + 2*((4*a + b)*cos(d*x + c)^2 - a - b)*sin(d*
x + c))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^3)]
```

giac [B] time = 1.35, size = 164, normalized size = 2.22

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a^2}{\left(a^2+2\,ab+b^2\right)\sqrt{a^2+ab}}+\frac{a^2\tan(dx+c)^3+2\,ab\tan(dx+c)^3+b^2\tan(dx+c)^3-3\,a^2\tan(dx+c)-3\,ab\tan(dx+c)}{a^3+3\,a^2b+3\,ab^2+b^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*a^2/((a^2 + 2*a*b + b^2)*sqrt(a^2 + a
*b)) + (a^2*tan(d*x + c)^3 + 2*a*b*tan(d*x + c)^3 + b^2*tan(d*x + c)^3 - 3*
a^2*tan(d*x + c) - 3*a*b*tan(d*x + c))/(a^3 + 3*a^2*b + 3*a*b^2 + b^3))/d
```

maple [A] time = 0.49, size = 94, normalized size = 1.27

$$\frac{a\left(\tan^3\left(dx+c\right)\right)}{3\left(a+b\right)^2d}+\frac{\left(\tan^3\left(dx+c\right)\right)b}{3d\left(a+b\right)^2}-\frac{a\tan\left(dx+c\right)}{\left(a+b\right)^2d}+\frac{a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\left(a+b\right)^2\sqrt{a\left(a+b\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/3*a*tan(d*x+c)^3/(a+b)^2/d+1/3/d/(a+b)^2*tan(d*x+c)^3*b-a*tan(d*x+c)/(a+b
)^2/d+1/d*a^2/(a+b)^2/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))
```

maxima [A] time = 0.42, size = 85, normalized size = 1.15

$$\frac{\frac{3\,a^2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a^2+2\,ab+b^2)}+\frac{(a+b)\tan(dx+c)^3-3\,a\tan(dx+c)}{a^2+2\,ab+b^2}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*a^2*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a
^2 + 2*a*b + b^2)) + ((a + b)*tan(d*x + c)^3 - 3*a*tan(d*x + c))/(a^2 + 2*a
*b + b^2))/d
```

mupad [B] time = 15.18, size = 83, normalized size = 1.12

$$\frac{\tan\left(c+d\,x\right)^3}{3\,d\,\left(a+b\right)}-\frac{a\tan\left(c+d\,x\right)}{d\,{\left(a+b\right)}^2}+\frac{a^{3/2}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)\,\left(a^2+2\,a\,b+b^2\right)}{2\,\sqrt{a}\,{(a+b)}^{5/2}}\right)}{d\,{\left(a+b\right)}^{5/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)^3/(3*d*(a + b)) - (a*tan(c + d*x))/(d*(a + b)^2) + (a^(3/2)*at
an((tan(c + d*x)*(2*a + 2*b)*(2*a*b + a^2 + b^2))/(2*a^(1/2)*(a + b)^(5/2))
))/(d*(a + b)^(5/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\tan^4\left(c+dx\right)}{a+b\sin^2\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.452 $\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=53

$$\frac{\tan(c+dx)}{d(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

```
[Out] -arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*a^(1/2)/(a+b)^(3/2)/d+tan(d*x+c)/(a
+b)/d
```

Rubi [A] time = 0.08, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3195, 321, 205}

$$\frac{\tan(c+dx)}{d(a+b)}-\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{x^2}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{d}\\ &= \frac{\tan(c+dx)}{(a+b)d}-\frac{a\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{(a+b)d}\\ &= -\frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{(a+b)^{3/2}d}+\frac{\tan(c+dx)}{(a+b)d}\end{aligned}$$

Mathematica [A] time = 0.13, size = 53, normalized size = 1.00

$$\frac{\tan(c+dx)}{d(a+b)} - \frac{\sqrt{a}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/((a + b)^(3/2)*d)) +
 Tan[c + d*x]/((a + b)*d)
```

fricas [B] time = 0.48, size = 300, normalized size = 5.66

$$\left[\frac{\sqrt{-\frac{a}{a+b}}\cos(dx+c)\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+3ab+b^2\right)\cos(dx+c)^3-\left(a^2+2ab+b^2\right)\cos(dx+\right.}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right.}{4(a+b)d\cos(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-a/(a + b))*cos(d*x + c)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)
^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + 3*a*b + b^2)*cos(
d*x + c)^3 - (a^2 + 2*a*b + b^2)*cos(d*x + c))*sqrt(-a/(a + b))*sin(d*x + c
) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 +
 a^2 + 2*a*b + b^2)) + 4*sin(d*x + c))/((a + b)*d*cos(d*x + c)), 1/2*(sqrt(
a/(a + b))*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt(a/(a + b))/(a
*cos(d*x + c)*sin(d*x + c)))*cos(d*x + c) + 2*sin(d*x + c))/((a + b)*d*cos(
d*x + c))]
```

giac [A] time = 0.49, size = 86, normalized size = 1.62

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)a}{\sqrt{a^2+ab}\,(a+b)}-\frac{\tan(dx+c)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*a/(sqrt(a^2 + a*b)*(a + b)) - tan(d*x + c)
/(a + b))/d
```

maple [A] time = 0.41, size = 53, normalized size = 1.00

$$\frac{\tan(dx+c)}{(a+b)\,d} - \frac{a\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\,(a+b)\sqrt{a\,(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x)
```

```
[Out] tan(d*x+c)/(a+b)/d-1/d*a/(a+b)/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(
a+b))^(1/2))
```

maxima [A] time = 0.42, size = 51, normalized size = 0.96

$$-\frac{\frac{a \arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,(a+b)} - \frac{\tan(dx+c)}{a+b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -(a*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*(a + b))
- tan(d*x + c)/(a + b))/d
```

mupad [B] time = 14.92, size = 53, normalized size = 1.00

$$\frac{\tan\left(c+d\,x\right)}{d\,\left(a+b\right)}-\frac{\sqrt{a}\,\mathrm{atan}\left(\frac{\tan(c+d\,x)\,(2\,a+2\,b)}{2\,\sqrt{a}\,\sqrt{a+b}}\right)}{d\,{\left(a+b\right)}^{3/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] tan(c + d*x)/(d*(a + b)) - (a^(1/2)*atan((tan(c + d*x)*(2*a + 2*b))/(2*a^(1
/2)*(a + b)^(1/2))))/(d*(a + b)^(3/2))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(tan(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.453 $\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=52

$$-\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

```
[Out] -cot(d*x+c)/a/d-arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))*(a+b)^(1/2)/a^(3/2)/
d
```

Rubi [A] time = 0.07, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 325, 205}

$$-\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -((Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(3/2)*d)) - C
ot[c + d*x]/(a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx &= \frac{\text{Subst}\left(\int \frac{1}{x^2\left(a+(a+b)x^2\right)}\,dx,x,\tan(c+dx)\right)}{d}\\
&= -\frac{\cot(c+dx)}{ad}-\frac{(a+b)\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx,x,\tan(c+dx)\right)}{ad}\\
&= -\frac{\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{3/2}d}-\frac{\cot(c+dx)}{ad}
\end{aligned}$$

Mathematica [A] time = 0.18, size = 52, normalized size = 1.00

$$\frac{-\sqrt{a+b}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)-\sqrt{a}\cot(c+dx)}{a^{3/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-(Sqrt[a + b]*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]]) - Sqrt[a]*Cot[c
+ d*x])/(a^(3/2)*d)
```

fricas [B] time = 0.48, size = 290, normalized size = 5.58

$$\left[\frac{\sqrt{-\frac{a+b}{a}}\log\left(\frac{\left(8a^2+8ab+b^2\right)\cos(dx+c)^4-2\left(4a^2+5ab+b^2\right)\cos(dx+c)^2+4\left(\left(2a^2+ab\right)\cos(dx+c)^3-\left(a^2+ab\right)\cos(dx+c)\right)\sqrt{-\frac{a+b}{a}}\sin(dx+c)+a^2+2ab\cdots}{b^2\cos(dx+c)^4-2\left(ab+b^2\right)\cos(dx+c)^2+a^2+2ab+b^2}\right)}{4ad\sin(dx+c)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
 + 5*a*b + b^2)*cos(d*x + c)^2 + 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a
*b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*c
os(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x
+ c) - 4*cos(d*x + c))/(a*d*sin(d*x + c)), 1/2*(sqrt((a + b)/a)*arctan(1/2*
((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x + c)*si
n(d*x + c)))*sin(d*x + c) - 2*cos(d*x + c))/(a*d*sin(d*x + c))]
```

giac [A] time = 0.20, size = 85, normalized size = 1.63

$$-\frac{\frac{\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)(a+b)}{\sqrt{a^2+ab}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -((pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b
*tan(d*x + c))/sqrt(a^2 + a*b)))*(a + b)/(sqrt(a^2 + a*b)*a) + 1/(a*tan(d*x
 + c)))/d
```

maple [A] time = 0.47, size = 82, normalized size = 1.58

$$-\frac{1}{da\tan(dx+c)}-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{da\sqrt{a(a+b)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d/a/tan(d*x+c)-1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/
2))-1/d/a/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b
```

maxima [A] time = 0.43, size = 50, normalized size = 0.96

$$-\frac{\frac{(a+b)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a}+\frac{1}{a\tan(dx+c)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -((a + b)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a)
+ 1/(a*tan(d*x + c)))/d
```

mupad [B] time = 14.71, size = 44, normalized size = 0.85

$$-\frac{\cot(c+d\,x)}{a\,d}-\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,\sqrt{a+b}}{a^{3/2}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^2),x)
```

```
[Out] - cot(c + d*x)/(a*d) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^
(1/2))/(a^(3/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^2(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**2/(a + b*sin(c + d*x)**2), x)
```

3.454 $\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=71

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

```
[Out] (a+b)^(3/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(5/2)/d+(a+b)*cot(d*x+
c)/a^2/d-1/3*cot(d*x+c)^3/a/d
```

Rubi [A] time = 0.08, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 325, 205}

$$\frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d}+\frac{(a+b)\cot(c+dx)}{a^2d}-\frac{\cot^3(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(5/2)*d) + ((
a + b)*Cot[c + d*x])/(a^2*d) - Cot[c + d*x]^3/(3*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$
$$= -\frac{\cot^3(c+dx)}{3ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$
$$= \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{a+(a+b)x^2}\,dx, x, \tan(c+dx)\right)}{a^2d}$$
$$= \frac{(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{5/2}d} + \frac{(a+b)\cot(c+dx)}{a^2d} - \frac{\cot^3(c+dx)}{3ad}$$

Mathematica [A] time = 0.29, size = 72, normalized size = 1.01

$$\frac{3(a+b)^{3/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right) + \sqrt{a}\cot(c+dx)\left(-a\csc^2(c+dx)+4a+3b\right)}{3a^{5/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (3*(a + b)^(3/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] + Sqrt[a]*Cot[c
 + d*x]*(4*a + 3*b - a*Csc[c + d*x]^2))/(3*a^(5/2)*d)
```

fricas [B] time = 0.49, size = 402, normalized size = 5.66

$$\left[\frac{4\,(4\,a+3\,b)\cos(dx+c)^3 + 3\left((a+b)\cos(dx+c)^2 - a - b\right)\sqrt{-\frac{a+b}{a}}\,\log\left(\frac{\left(8\,a^2+8\,ab+b^2\right)\cos(dx+c)^4 - 2\left(4\,a^2+5\,ab+b^2\right)\cos\cdots}{b^2\cos\cdots}\right.}{12\left(a^2d\cos(dx+c)^2 - \cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/12*(4*(4*a + 3*b)*cos(d*x + c)^3 + 3*((a + b)*cos(d*x + c)^2 - a - b)*sq
rt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b
 + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(
d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x +
 c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) - 1
2*(a + b)*cos(d*x + c))/((a^2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c)), 1/6*
(2*(4*a + 3*b)*cos(d*x + c)^3 - 3*((a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
+ b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a +
 b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 6*(a + b)*cos(d*x + c))/((a^
2*d*cos(d*x + c)^2 - a^2*d)*sin(d*x + c))]
```

giac [A] time = 0.22, size = 120, normalized size = 1.69

$$\frac{\frac{3\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\text{sgn}(2\,a+2\,b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)\left(a^2+2\,ab+b^2\right)}{\sqrt{a^2+ab}\,a^2} + \frac{3\,a\tan(dx+c)^2+3\,b\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(pi*floor((d*x + c)/pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c
) + b*tan(d*x + c))/sqrt(a^2 + a*b)))*(a^2 + 2*a*b + b^2)/(sqrt(a^2 + a*b)*
a^2) + (3*a*tan(d*x + c)^2 + 3*b*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))/
d
```

maple [B] time = 0.55, size = 147, normalized size = 2.07

$$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}+\frac{2\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{da\sqrt{a(a+b)}}+\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b^2}{d\,a^2\sqrt{a(a+b)}}-\frac{1}{3da\tan(dx+c)^3}+\frac{1}{da\tan(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+2/d/a/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+1/d/a^2/(a*(a+b))^(1/2)*
arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2-1/3/d/a/tan(d*x+c)^3+1/d/a/tan
(d*x+c)+1/d/a^2/tan(d*x+c)*b
```

maxima [A] time = 0.44, size = 76, normalized size = 1.07

$$\frac{\frac{3\left(a^2+2\,ab+b^2\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^2}+\frac{3\,(a+b)\tan(dx+c)^2-a}{a^2\tan(dx+c)^3}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a^2 + 2*a*b + b^2)*arctan((a + b)*tan(d*x + c)/sqrt((a + b)*a))/(sq
rt((a + b)*a)*a^2) + (3*(a + b)*tan(d*x + c)^2 - a)/(a^2*tan(d*x + c)^3))/d
```

mupad [B] time = 15.09, size = 64, normalized size = 0.90

$$\frac{\mathrm{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)(a+b)^{3/2}}{a^{5/2}\,d}-\frac{\frac{1}{3\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{a^2}}{d\tan(c+d\,x)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(3/2))/(a^(5/2)*d) - (1
/(3*a) - (tan(c + d*x)^2*(a + b))/a^2)/(d*tan(c + d*x)^3)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot^4(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**4/(a + b*sin(c + d*x)**2), x)
```

3.455 $\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=96

$$-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{\cot^5(c+dx)}{5ad}$$

```
[Out] -(a+b)^(5/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(7/2)/d-(a+b)^2*cot(d
*x+c)/a^3/d+1/3*(a+b)*cot(d*x+c)^3/a^2/d-1/5*cot(d*x+c)^5/a/d
```

Rubi [A] time = 0.10, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 325, 205}

$$-\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d}+\frac{(a+b)\cot^3(c+dx)}{3a^2d}-\frac{(a+b)^2\cot(c+dx)}{a^3d}-\frac{\cot^5(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] -(((a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(7/2)*d)) -
 ((a + b)^2*Cot[c + d*x])/(a^3*d) + ((a + b)*Cot[c + d*x]^3)/(3*a^2*d) - Co
t[c + d*x]^5/(5*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^6(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^4(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{x^2(a+(a+b)x^2)}\,dx, x, \tan(c+dx)\right)}{a^2d}$$

$$= -\frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(c+dx)}{5ad} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{a+(a+b}\right.}{a^3}$$

$$= -\frac{(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{7/2}d} - \frac{(a+b)^2\cot(c+dx)}{a^3d} + \frac{(a+b)\cot^3(c+dx)}{3a^2d} - \frac{\cot^5(}{5}$$

Mathematica [A] time = 0.86, size = 101, normalized size = 1.05

$$\frac{-\sqrt{a}\,\cot(c+dx)\left(3a^2\csc^4(c+dx)+23a^2-a(11a+5b)\csc^2(c+dx)+35ab+15b^2\right)-15(a+b)^{5/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\text{ta}}{\sqrt{}}\right.}{15a^{7/2}d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^6/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] (-15*(a + b)^(5/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]] - Sqrt[a]*Cot
[c + d*x]*(23*a^2 + 35*a*b + 15*b^2 - a*(11*a + 5*b)*Csc[c + d*x]^2 + 3*a^2
*Csc[c + d*x]^4))/(15*a^(7/2)*d)
```

fricas [B] time = 0.48, size = 576, normalized size = 6.00

$$\left[-\frac{4\left(23a^2+35ab+15b^2\right)\cos(dx+c)^5-20\left(7a^2+13ab+6b^2\right)\cos(dx+c)^3-15\left(\left(a^2+2ab+b^2\right)\cos(dx+\right.\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [-1/60*(4*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 20*(7*a^2 + 13*a*b +
6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(a^2 + 2
*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt(-(a + b)/a)*log(((8*a^
2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2 + 5*a*b + b^2)*cos(d*x + c)^2 +
4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*b)*cos(d*x + c))*sqrt(-(a + b)/a
)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*cos(d*x + c)^4 - 2*(a*b + b^2)*cos
(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x + c) + 60*(a^2 + 2*a*b + b^2)*cos
(d*x + c))/((a^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x
 + c)), -1/30*(2*(23*a^2 + 35*a*b + 15*b^2)*cos(d*x + c)^5 - 10*(7*a^2 + 13
*a*b + 6*b^2)*cos(d*x + c)^3 - 15*((a^2 + 2*a*b + b^2)*cos(d*x + c)^4 - 2*(
a^2 + 2*a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2)*sqrt((a + b)/a)*arct
an(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a + b)/a)/((a + b)*cos(d*x
+ c)*sin(d*x + c)))*sin(d*x + c) + 30*(a^2 + 2*a*b + b^2)*cos(d*x + c))/((a
^3*d*cos(d*x + c)^4 - 2*a^3*d*cos(d*x + c)^2 + a^3*d)*sin(d*x + c))]
```

giac [B] time = 0.26, size = 171, normalized size = 1.78

$$-\frac{\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,a^3}+\frac{15a^2\tan(dx+c)^4+30ab\tan(dx+c)^4+15b^2\tan(dx+c)^4-5a}{a^3\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] -1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(pi*floor((d*x + c)/pi + 1/2)*sgn
(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^2 + a*b)))/(s
qrt(a^2 + a*b)*a^3) + (15*a^2*tan(d*x + c)^4 + 30*a*b*tan(d*x + c)^4 + 15*b
^2*tan(d*x + c)^4 - 5*a^2*tan(d*x + c)^2 - 5*a*b*tan(d*x + c)^2 + 3*a^2)/(a
^3*tan(d*x + c)^5))/d
```

maple [B] time = 0.67, size = 239, normalized size = 2.49

$$-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a(a+b)}}-\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{da\sqrt{a(a+b)}}-\frac{3\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b^2}{d\,a^2\sqrt{a(a+b)}}-\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b^3}{d\,a^3\sqrt{a(a+b)}}-\frac{}{da}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x)
```

```
[Out] -1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))-3/d/a/(a*(a+b
))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b-3/d/a^2/(a*(a+b))^(1/2)
*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2-1/d/a^3/(a*(a+b))^(1/2)*arcta
n((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^3-1/d/a/tan(d*x+c)-2/d/a^2/tan(d*x+c)
*b-1/d/a^3/tan(d*x+c)*b^2-1/5/d/a/tan(d*x+c)^5+1/3/d/a/tan(d*x+c)^3+1/3/d/a
^2/tan(d*x+c)^3*b
```

maxima [A] time = 0.42, size = 111, normalized size = 1.16

$$-\frac{\frac{15\left(a^3+3a^2b+3ab^2+b^3\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^3}+\frac{15\left(a^2+2ab+b^2\right)\tan(dx+c)^4-5\left(a^2+ab\right)\tan(dx+c)^2+3a^2}{a^3\tan(dx+c)^5}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^6/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] -1/15*(15*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*arctan((a + b)*tan(d*x + c)/sqrt(
(a + b)*a))/(sqrt((a + b)*a)*a^3) + (15*(a^2 + 2*a*b + b^2)*tan(d*x + c)^4
- 5*(a^2 + a*b)*tan(d*x + c)^2 + 3*a^2)/(a^3*tan(d*x + c)^5))/d
```

mupad [B] time = 16.15, size = 82, normalized size = 0.85

$$-\frac{\frac{1}{5a}-\frac{\tan(c+dx)^2(a+b)}{3a^2}+\frac{\tan(c+dx)^4(a+b)^2}{a^3}}{d\tan(c+dx)^5}-\frac{\mathrm{atan}\left(\frac{\tan(c+dx)\sqrt{a+b}}{\sqrt{a}}\right)(a+b)^{5/2}}{a^{7/2}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^6/(a + b*sin(c + d*x)^2),x)
```

```
[Out] - (1/(5*a) - (tan(c + d*x)^2*(a + b))/(3*a^2) + (tan(c + d*x)^4*(a + b)^2)/
a^3)/(d*tan(c + d*x)^5) - (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a +
b)^(5/2))/(a^(7/2)*d)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(c+dx)}{a+b\sin^2(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**6/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Integral(cot(c + d*x)**6/(a + b*sin(c + d*x)**2), x)
```

3.456 $\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx$

Optimal. Leaf size=117

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{\cot^7(c+dx)}{7ad}$$

```
[Out] (a+b)^(7/2)*arctan((a+b)^(1/2)*tan(d*x+c)/a^(1/2))/a^(9/2)/d+(a+b)^3*cot(d*
x+c)/a^4/d-1/3*(a+b)^2*cot(d*x+c)^3/a^3/d+1/5*(a+b)*cot(d*x+c)^5/a^2/d-1/7*
cot(d*x+c)^7/a/d
```

Rubi [A] time = 0.11, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3195, 325, 205}

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\,\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d}+\frac{(a+b)\cot^5(c+dx)}{5a^2d}-\frac{(a+b)^2\cot^3(c+dx)}{3a^3d}+\frac{(a+b)^3\cot(c+dx)}{a^4d}-\frac{\cot^7(c+dx)}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + ((
a + b)^3*Cot[c + d*x])/(a^4*d) - ((a + b)^2*Cot[c + d*x]^3)/(3*a^3*d) + ((a
 + b)*Cot[c + d*x]^5)/(5*a^2*d) - Cot[c + d*x]^7/(7*a*d)
```

Rule 205

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]*ArcTan[x/Rt[a
/b, 2]])/a, x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 325

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[((c*
x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*c*(m + 1)), x] - Dist[(b*(m + n*(p + 1)
+ 1))/(a*c^n*(m + 1)), Int[(c*x)^(m + n)*(a + b*x^n)^p, x], x] /; FreeQ[{a,
 b, c, p}, x] && IGtQ[n, 0] && LtQ[m, -1] && IntBinomialQ[a, b, c, n, m, p,
 x]
```

Rule 3195

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[f
f/f, Subst[Int[((d*ff*x)^m*(a + (a + b)*ff^2*x^2)^p)/(1 + ff^2*x^2)^(p + 1)
, x], x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[
p]
```

Rubi steps

$$\int \frac{\cot^8(c+dx)}{a+b\sin^2(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x^8\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{d}$$

$$= -\frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{x^6\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{ad}$$

$$= \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} + \frac{(a+b)^2\,\text{Subst}\left(\int \frac{1}{x^4\left(a+(a+b)x^2\right)}\,dx, x, \tan(c+dx)\right)}{a^2d}$$

$$= -\frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} - \frac{(a+b)^3\,\text{Subst}\left(\int \frac{1}{x^2(a+\cdots}\right.}{}$$

$$= \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+b)\cot^5(c+dx)}{5a^2d} - \frac{\cot^7(c+dx)}{7ad} + \cdots$$

$$= \frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{(a+b)^3\cot(c+dx)}{a^4d} - \frac{(a+b)^2\cot^3(c+dx)}{3a^3d} + \frac{(a+\cdots}{}$$

Mathematica [A] time = 1.08, size = 135, normalized size = 1.15

$$\frac{(a+b)^{7/2}\tan^{-1}\left(\frac{\sqrt{a+b}\tan(c+dx)}{\sqrt{a}}\right)}{a^{9/2}d} + \frac{\cot(c+dx)\left(-15a^3\csc^6(c+dx)+176a^3-a\left(122a^2+112ab+35b^2\right)\csc^2(c+dx\cdots\right.}{105a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^8/(a + b*Sin[c + d*x]^2),x]
```

```
[Out] ((a + b)^(7/2)*ArcTan[(Sqrt[a + b]*Tan[c + d*x])/Sqrt[a]])/(a^(9/2)*d) + (C
ot[c + d*x]*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3 - a*(122*a^2 + 112*a
*b + 35*b^2)*Csc[c + d*x]^2 + 3*a^2*(22*a + 7*b)*Csc[c + d*x]^4 - 15*a^3*Cs
c[c + d*x]^6))/(105*a^4*d)
```

fricas [B] time = 0.49, size = 834, normalized size = 7.13

$$\left[\frac{4\left(176\,a^3+406\,a^2b+350\,ab^2+105\,b^3\right)\cos\left(dx+c\right)^7-28\left(58\,a^3+158\,a^2b+145\,ab^2+45\,b^3\right)\cos\left(dx+c\right)^5+14\cdots}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="fricas")
```

```
[Out] [1/420*(4*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 28*(
58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 140*(10*a^3 + 29*
a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 + 105*((a^3 + 3*a^2*b + 3*a*b^2 +
b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a^
3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c
)^2)*sqrt(-(a + b)/a)*log(((8*a^2 + 8*a*b + b^2)*cos(d*x + c)^4 - 2*(4*a^2
+ 5*a*b + b^2)*cos(d*x + c)^2 - 4*((2*a^2 + a*b)*cos(d*x + c)^3 - (a^2 + a*
b)*cos(d*x + c))*sqrt(-(a + b)/a)*sin(d*x + c) + a^2 + 2*a*b + b^2)/(b^2*co
s(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))*sin(d*x +
 c) - 420*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c
)^6 - 3*a^4*d*cos(d*x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c)
```

```
), 1/210*(2*(176*a^3 + 406*a^2*b + 350*a*b^2 + 105*b^3)*cos(d*x + c)^7 - 14
*(58*a^3 + 158*a^2*b + 145*a*b^2 + 45*b^3)*cos(d*x + c)^5 + 70*(10*a^3 + 29
*a^2*b + 28*a*b^2 + 9*b^3)*cos(d*x + c)^3 - 105*((a^3 + 3*a^2*b + 3*a*b^2 +
 b^3)*cos(d*x + c)^6 - 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x + c)^4 - a
^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^3 + 3*a^2*b + 3*a*b^2 + b^3)*cos(d*x +
c)^2)*sqrt((a + b)/a)*arctan(1/2*((2*a + b)*cos(d*x + c)^2 - a - b)*sqrt((a
 + b)/a)/((a + b)*cos(d*x + c)*sin(d*x + c)))*sin(d*x + c) - 210*(a^3 + 3*a
^2*b + 3*a*b^2 + b^3)*cos(d*x + c))/((a^4*d*cos(d*x + c)^6 - 3*a^4*d*cos(d*
x + c)^4 + 3*a^4*d*cos(d*x + c)^2 - a^4*d)*sin(d*x + c))]
```

giac [**B**] time = 0.33, size = 238, normalized size = 2.03

$$\frac{\frac{105\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\left(\pi\left\lfloor\frac{dx+c}{\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a+2b)+\arctan\left(\frac{a\tan(dx+c)+b\tan(dx+c)}{\sqrt{a^2+ab}}\right)\right)}{\sqrt{a^2+ab}\,a^4}+\frac{105a^3\tan(dx+c)^6+315a^2b\tan(dx+c)^6+315ab^2\tan\cdots}{\cdots}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="giac")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(pi*floor((d*x + c)/
pi + 1/2)*sgn(2*a + 2*b) + arctan((a*tan(d*x + c) + b*tan(d*x + c))/sqrt(a^
2 + a*b)))/(sqrt(a^2 + a*b)*a^4) + (105*a^3*tan(d*x + c)^6 + 315*a^2*b*tan(
d*x + c)^6 + 315*a*b^2*tan(d*x + c)^6 + 105*b^3*tan(d*x + c)^6 - 35*a^3*tan
(d*x + c)^4 - 70*a^2*b*tan(d*x + c)^4 - 35*a*b^2*tan(d*x + c)^4 + 21*a^3*ta
n(d*x + c)^2 + 21*a^2*b*tan(d*x + c)^2 - 15*a^3)/(a^4*tan(d*x + c)^7))/d
```

maple [**B**] time = 0.68, size = 342, normalized size = 2.92

$$\frac{\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)}{d\sqrt{a\,(a+b)}}+\frac{4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b}{da\sqrt{a\,(a+b)}}+\frac{6\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b^2}{d\,a^2\sqrt{a\,(a+b)}}+\frac{4\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{a(a+b)}}\right)b^3}{d\,a^3\sqrt{a\,(a+b)}}+\cdots$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x)
```

```
[Out] 1/d/(a*(a+b))^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))+4/d/a/(a*(a+b)
)^(1/2)*arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b+6/d/a^2/(a*(a+b))^(1/2)*
arctan((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^2+4/d/a^3/(a*(a+b))^(1/2)*arctan
((a+b)*tan(d*x+c)/(a*(a+b))^(1/2))*b^3+1/d/a^4/(a*(a+b))^(1/2)*arctan((a+b)
*tan(d*x+c)/(a*(a+b))^(1/2))*b^4+1/d/a/tan(d*x+c)+3/d/a^2/tan(d*x+c)*b+3/d/
a^3/tan(d*x+c)*b^2+1/d/a^4/tan(d*x+c)*b^3+1/5/d/a/tan(d*x+c)^5+1/5/d/a^2/ta
n(d*x+c)^5*b-1/7/d/a/tan(d*x+c)^7-1/3/d/a/tan(d*x+c)^3-2/3/d/a^2/tan(d*x+c)
^3*b-1/3/d/a^3/tan(d*x+c)^3*b^2
```

maxima [**A**] time = 0.48, size = 154, normalized size = 1.32

$$\frac{\frac{105\left(a^4+4a^3b+6a^2b^2+4ab^3+b^4\right)\arctan\left(\frac{(a+b)\tan(dx+c)}{\sqrt{(a+b)a}}\right)}{\sqrt{(a+b)a}\,a^4}+\frac{105\left(a^3+3a^2b+3ab^2+b^3\right)\tan(dx+c)^6-35\left(a^3+2a^2b+ab^2\right)\tan(dx+c)^4-15a^3+21\left(a^3+a\cdots\right)}{a^4\tan(dx+c)^7}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^8/(a+b*sin(d*x+c)^2),x, algorithm="maxima")
```

```
[Out] 1/105*(105*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*arctan((a + b)*tan(d
*x + c)/sqrt((a + b)*a))/(sqrt((a + b)*a)*a^4) + (105*(a^3 + 3*a^2*b + 3*a*
b^2 + b^3)*tan(d*x + c)^6 - 35*(a^3 + 2*a^2*b + a*b^2)*tan(d*x + c)^4 - 15*
a^3 + 21*(a^3 + a^2*b)*tan(d*x + c)^2)/(a^4*tan(d*x + c)^7))/d
```

mupad [B] time = 18.91, size = 100, normalized size = 0.85

$$\frac{\operatorname{atan}\left(\frac{\tan(c+d\,x)\,\sqrt{a+b}}{\sqrt{a}}\right)\,(a+b)^{7/2}}{a^{9/2}\,d}-\frac{\frac{1}{7\,a}-\frac{\tan(c+d\,x)^2\,(a+b)}{5\,a^2}+\frac{\tan(c+d\,x)^4\,(a+b)^2}{3\,a^3}-\frac{\tan(c+d\,x)^6\,(a+b)^3}{a^4}}{d\,\tan\left(c+d\,x\right)^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^8/(a + b*sin(c + d*x)^2),x)
```

```
[Out] (atan((tan(c + d*x)*(a + b)^(1/2))/a^(1/2))*(a + b)^(7/2))/(a^(9/2)*d) - (1
/(7*a) - (tan(c + d*x)^2*(a + b))/(5*a^2) + (tan(c + d*x)^4*(a + b)^2)/(3*a
^3) - (tan(c + d*x)^6*(a + b)^3)/a^4)/(d*tan(c + d*x)^7)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**8/(a+b*sin(d*x+c)**2),x)
```

```
[Out] Timed out
```

3.457 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^5(e+fx)\,dx$

Optimal. Leaf size=64

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] 1/3*a^2/f/(a*cos(f*x+e)^2)^(3/2)-2*a/f/(a*cos(f*x+e)^2)^(1/2)-(a*cos(f*x+e)
^2)^(1/2)/f
```

Rubi [A] time = 0.11, antiderivative size = 64, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{2a}{f\sqrt{a\cos^2(e+fx)}}-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] a^2/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - (2*a)/(f*Sqrt[a*Cos[e + f*x]^2]) - Sqr
t[a*Cos[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\,\tan^5(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\,\tan^5(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{ax}}{x^3}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{2}{a(ax)^{3/2}} + \frac{1}{a^2\sqrt{ax}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a^2}{3f\left(a\cos^2(e+fx)\right)^{3/2}} - \frac{2a}{f\sqrt{a\cos^2(e+fx)}} - \frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A] time = 0.09, size = 51, normalized size = 0.80

$$-\frac{\left(3\cos^4(e+fx)+6\cos^2(e+fx)-1\right)\sec^4(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] -1/3*(Sqrt[a*Cos[e + f*x]^2]*(-1 + 6*Cos[e + f*x]^2 + 3*Cos[e + f*x]^4)*Sec
[e + f*x]^4)/f
```

fricas [A] time = 0.45, size = 47, normalized size = 0.73

$$-\frac{\left(3\cos\left(fx+e\right)^4+6\cos\left(fx+e\right)^2-1\right)\sqrt{a\cos\left(fx+e\right)^2}}{3f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(f*x + e)^4 + 6*cos(f*x + e)^2 - 1)*sqrt(a*cos(f*x + e)^2)/(f*co
s(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a)*(1/3*(12*tan((f*x+exp(1))/2)^2*sign(tan((f
*x+exp(1))/2)^4-1)-3*tan((f*x+exp(1))/2)^4*sign(tan((f*x+exp(1))/2)^4-1)-5*
sign(tan((f*x+exp(1))/2)^4-1))/(tan((f*x+exp(1))/2)^2-1)^3+sign(tan((f*x+ex
p(1))/2)^4-1)/(tan((f*x+exp(1))/2)^2+1))
```

maple [A] time = 2.51, size = 48, normalized size = 0.75

$$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(3\left(\cos^4\left(fx+e\right)\right)+6\left(\cos^2\left(fx+e\right)\right)-1\right)}{3\cos\left(fx+e\right)^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] -1/3/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^4+6*cos(f*x+e)^2-1)/
f
```

maxima [A] time = 0.35, size = 69, normalized size = 1.08

$$-\frac{3\sqrt{-a\sin\left(fx+e\right)^2+a}\,a^3-\frac{6\left(a\sin\left(fx+e\right)^2-a\right)a^4+a^5}{\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}}{3\,a^3 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/3*(3*sqrt(-a*sin(f*x + e)^2 + a)*a^3 - (6*(a*sin(f*x + e)^2 - a)*a^4 + a
^5)/(-a*sin(f*x + e)^2 + a)^(3/2))/(a^3*f)
```

mupad [B] time = 19.71, size = 326, normalized size = 5.09

$$-\frac{\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f}-\frac{8\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}}}{}\right.}}{3\,f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2\left(\mathrm{e}^{e\,1}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*x*1i)
+ exp(e*3i + f*x*3i))) - (8*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i
)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(f*(exp(e*2i + f*x*2i) + 1)*
(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)/f - (16*exp(e*3i + f*x*3i)*(a -
 a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*f
*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**5, x)
```

3.458 $\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx$

Optimal. Leaf size=38

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] a/f/(a*cos(f*x+e)^2)^(1/2)+(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.10, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] a/(f*Sqrt[a*Cos[e + f*x]^2]) + Sqrt[a*Cos[e + f*x]^2]/f
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\tan^3(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\tan^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{ax}}{x^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{3/2}}-\frac{1}{a\sqrt{ax}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{f\sqrt{a\cos^2(e+fx)}}+\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [**A**] time = 0.08, size = 29, normalized size = 0.76

$$\frac{a\left(\cos^2(e+fx)+1\right)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
[Out] (a*(1 + Cos[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [**A**] time = 0.43, size = 34, normalized size = 0.89

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2+1\right)}{f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 + 1)/(f*cos(f*x + e)^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*sqrt(a)*sign(tan((f*x+exp(1))/
2)^4-1)/(tan((f*x+exp(1))/2)^4-1)
```

maple [**A**] time = 2.10, size = 35, normalized size = 0.92

$$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(\cos^2\left(fx+e\right)+1\right)}{\cos\left(fx+e\right)^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/cos(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2+1)/f
```

maxima [A] time = 0.35, size = 46, normalized size = 1.21

$$\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}\,a^2+\frac{a^3}{\sqrt{-a\sin\left(fx+e\right)^2+a}}}{a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] (sqrt(-a*sin(f*x + e)^2 + a)*a^2 + a^3/sqrt(-a*sin(f*x + e)^2 + a))/(a^2*f)
```

mupad [B] time = 0.72, size = 69, normalized size = 1.82

$$\frac{\sqrt{2}\,\sqrt{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}\,\left(8\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+7\right)}{2\,f\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (2^(1/2)*(a*(cos(2*e + 2*f*x) + 1))^(1/2)*(8*cos(2*e + 2*f*x) + cos(4*e + 4
*f*x) + 7))/(2*f*(4*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**3, x)
```

3.459 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan(e+fx)\,dx$

Optimal. Leaf size=19

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] -(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.06, antiderivative size = 19, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3176, 3205, 16, 32}

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}
\int \sqrt{a-a\sin^2(e+fx)}\,\tan(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\,\tan(e+fx)\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{x}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{a\,\text{Subst}\left(\int \frac{1}{\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}}{f}
\end{aligned}$$

Mathematica [A] time = 0.04, size = 19, normalized size = 1.00

$$-\frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] -(Sqrt[a*Cos[e + f*x]^2]/f)
```

fricas [A] time = 0.43, size = 17, normalized size = 0.89

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] -sqrt(a*cos(f*x + e)^2)/f
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2*sqrt(a)/2*sign(tan((f*x+exp(1)
)/2)^4-1)/(tan((f*x+exp(1))/2)^2+1)
```

maple [A] time = 0.17, size = 21, normalized size = 1.11

$$-\frac{\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] -1/f*(a-a*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.43, size = 20, normalized size = 1.05

$$-\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -sqrt(-a*sin(f*x + e)^2 + a)/f
```

mupad [B] time = 15.25, size = 20, normalized size = 1.05

$$-\frac{\sqrt{a-a\sin\left(e+f\,x\right)^2}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] -(a - a*sin(e + f*x)^2)^(1/2)/f
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x), x)
```

3.460 $\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=50

$$\frac{\sqrt{a\cos^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 50, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3176, 3205, 50, 63, 206}

$$\frac{\sqrt{a\cos^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a*Cos[e + f*x
]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
```

```
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}}{f} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{f}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a\cos^2(e+fx)}}{f}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 1.10

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(\cos(e+fx)+\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (Sqrt[a*Cos[e + f*x]^2]*(Cos[e + f*x] - Log[Cos[(e + f*x)/2]] + Log[Sin[(e
+ f*x)/2]])*Sec[e + f*x])/f
```

fricas [A] time = 0.48, size = 57, normalized size = 1.14

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(2\cos\left(fx+e\right)-\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)\right)}{2f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] 1/2*sqrt(a*cos(f*x + e)^2)*(2*cos(f*x + e) - log(-(cos(f*x + e) + 1)/(cos(f
*x + e) - 1)))/(f*cos(f*x + e))
```

giac [A] time = 0.14, size = 55, normalized size = 1.10

$$\frac{a\left(\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{a}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] a*(arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(-a*sin(f*x
+ e)^2 + a)/a)/f
```

maple [A] time = 1.64, size = 55, normalized size = 1.10

$$-\frac{\sqrt{a}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{f}+\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*a^(1/2)*ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a))+(a*cos(f*x
+e)^2)^(1/2)/f
```

maxima [A] time = 0.42, size = 70, normalized size = 1.40

$$-\frac{\sqrt{a}\,\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)-\sqrt{-a\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a
/abs(sin(f*x + e))) - sqrt(-a*sin(f*x + e)^2 + a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)*(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x), x)
```

3.461 $\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=87

$$-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{\csc^2(e+fx)\left(a\cos^2(e+fx)\right)^{3/2}}{2af}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

```
[Out] -1/2*(a*cos(f*x+e)^2)^(3/2)*csc(f*x+e)^2/a/f+3/2*arctanh((a*cos(f*x+e)^2)^(
1/2)/a^(1/2))*a^(1/2)/f-3/2*(a*cos(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.12, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.269$, Rules used = {3176, 3205, 16, 47, 50, 63, 206}

$$-\frac{3\sqrt{a\cos^2(e+fx)}}{2f}-\frac{\csc^2(e+fx)\left(a\cos^2(e+fx)\right)^{3/2}}{2af}+\frac{3\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*Sqrt[a]*ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]])/(2*f) - (3*Sqrt[a*Cos[e
 + f*x]^2])/(2*f) - ((a*Cos[e + f*x]^2)^(3/2)*Csc[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
```

```
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^3(e+fx)\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{x\sqrt{ax}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{(ax)^{3/2}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2af}$$
$$= -\frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{3\,\text{Subst}\left(\int \frac{\sqrt{ax}}{1-x}\,dx, x, \cos^2(e+fx)\right)}{4f}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{(3a)\,\text{Subst}\left(\int \right.}{}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\csc^2(e+fx)}{2af} + \frac{3\,\text{Subst}\left(\int \frac{1}{1-}\right.}{}$$
$$= \frac{3\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2f} - \frac{3\sqrt{a\cos^2(e+fx)}}{2f} - \frac{\left(a\cos^2(e+fx)\right)^{3/2}\text{c}}{2af}$$

Mathematica [A] time = 0.42, size = 88, normalized size = 1.01

$$-\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\left(8\cos(e+fx)+\csc^2\left(\frac{1}{2}(e+fx)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)+12\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-1\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/8*(Sqrt[a*Cos[e + f*x]^2]*(8*Cos[e + f*x] + Csc[(e + f*x)/2]^2 - 12*Log[
Cos[(e + f*x)/2]] + 12*Log[Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2)*Sec[e +
f*x])/f
```

fricas [A] time = 0.46, size = 88, normalized size = 1.01

$$-\frac{\sqrt{a\cos\left(fx+e\right)^{2}}\left(4\cos\left(fx+e\right)^{3}+3\left(\cos\left(fx+e\right)^{2}-1\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-6\cos\left(fx+e\right)\right)}{4\left(f\cos\left(fx+e\right)^{3}-f\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(4*cos(f*x + e)^3 + 3*(cos(f*x + e)^2 - 1)*log(
-(cos(f*x + e) - 1)/(cos(f*x + e) + 1)) - 6*cos(f*x + e))/(f*cos(f*x + e)^3
 - f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a)*(-1/16*tan((f*x+exp(1))/2)^2*
sign(tan((f*x+exp(1))/2)^4-1)+1/16*(14*tan((f*x+exp(1))/2)^2*sign(tan((f*x+
exp(1))/2)^4-1)-3*tan((f*x+exp(1))/2)^4*sign(tan((f*x+exp(1))/2)^4-1)+sign(
tan((f*x+exp(1))/2)^4-1))/(tan((f*x+exp(1))/2)^4+tan((f*x+exp(1))/2)^2)+3/8
*sign(tan((f*x+exp(1))/2)^4-1)*ln(tan((f*x+exp(1))/2)^2))
```

maple [A] time = 1.85, size = 83, normalized size = 0.95

$$-\frac{\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}}{f}+\frac{3\sqrt{a}\,\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{2f}-\frac{\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}}{2f\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -(a*cos(f*x+e)^2)^(1/2)/f+3/2/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^
(1/2))/sin(f*x+e))-1/2/f/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.42, size = 99, normalized size = 1.14

$$\frac{3\sqrt{a}\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)-3\sqrt{-a\sin\left(fx+e\right)^{2}+a}-\frac{\left(-a\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)^{2}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(3*sqrt(a)*log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e))
+ 2*a/abs(sin(f*x + e))) - 3*sqrt(-a*sin(f*x + e)^2 + a) - (-a*sin(f*x + e)
^2 + a)^(3/2)/(a*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^3\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**3, x)
```

3.462 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^6(e+fx)\,dx$

Optimal. Leaf size=120

$$\frac{\tan^5(e+fx)\sqrt{a\cos^2(e+fx)}}{4f}-\frac{5\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}-\frac{15\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}+\frac{15\sec(e+f\ldots}{}$$

```
[Out] 15/8*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-15/8*(a*cos(f*
x+e)^2)^(1/2)*tan(f*x+e)/f-5/8*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f+1/4*(a
*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^5/f
```

Rubi [A] time = 0.13, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 2592, 288, 321, 206}

$$\frac{\tan^5(e+fx)\sqrt{a\cos^2(e+fx)}}{4f}-\frac{5\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}-\frac{15\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{8f}+\frac{15\sec(e+f\ldots}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] (15*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(8*f) - (15*
Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(8*f) - (5*Sqrt[a*Cos[e + f*x]^2]*Tan[
e + f*x]^3)/(8*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x]^5)/(4*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\,\tan^6(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\,\tan^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan^5(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^6}{\left(1-x^2\right)^3}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}\,\tan^5(e+fx)}{4f} - \frac{\left(5\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}}{4f}$$
$$= -\frac{5\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{8f} + \frac{\sqrt{a\cos^2(e+fx)}\,\tan^5(e+fx)}{4f} + \frac{(1}{}$$
$$= -\frac{15\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{8f} - \frac{5\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{8f} + $$
$$= \frac{15\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{8f} - \frac{15\sqrt{a\cos^2(e+f}}{8}$$

Mathematica [A] time = 0.33, size = 75, normalized size = 0.62

$$\frac{\sec^5(e+fx)\sqrt{a\cos^2(e+fx)}\,\left(-5\sin(e+fx)-15\sin(3(e+fx))-2\sin(5(e+fx))+60\cos^4(e+fx)\tanh^{-1}(\sin}{32f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^6,x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]^5*(60*ArcTanh[Sin[e + f*x]]*Cos[e + f*
x]^4 - 5*Sin[e + f*x] - 15*Sin[3*(e + f*x)] - 2*Sin[5*(e + f*x)]))/(32*f)
```

fricas [A] time = 0.47, size = 87, normalized size = 0.72

$$-\frac{\left(15\cos\left(fx+e\right)^4\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\left(8\cos\left(fx+e\right)^4+9\cos\left(fx+e\right)^2-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^2}}{16f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="fricas")
```

```
[Out] -1/16*(15*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(8
*cos(f*x + e)^4 + 9*cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2
)/(f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x
/2)2/f*sqrt(a)*(-1/8*(-7*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^3*sign
(tan((f*x+exp(1))/2)^4-1)+36*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))*si
gn(tan((f*x+exp(1))/2)^4-1))/((tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^2
-4)^2-15/32*sign(tan((f*x+exp(1))/2)^4-1)*ln(abs(tan((f*x+exp(1))/2)+2+1/ta
n((f*x+exp(1))/2)))+15/32*sign(tan((f*x+exp(1))/2)^4-1)*ln(abs(tan((f*x+exp
(1))/2)-2+1/tan((f*x+exp(1))/2)))-sign(tan((f*x+exp(1))/2)^4-1)/(-tan((f*x+
exp(1))/2)-1/tan((f*x+exp(1))/2)))
```

maple [A] time = 1.67, size = 120, normalized size = 1.00

$$\frac{a\left(16\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)+18\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-4\sin\left(fx+e\right)+\left(-15\ln\left(1+\sin\left(fx+e\right.\right.\right.\right.}{16\left(1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x)
```

```
[Out] 1/16*a*(16*cos(f*x+e)^4*sin(f*x+e)+18*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+
(-15*ln(1+sin(f*x+e))+15*ln(sin(f*x+e)-1))*cos(f*x+e)^4)/(1+sin(f*x+e))/(si
n(f*x+e)-1)/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 1.50, size = 1955, normalized size = 16.29

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^6,x, algorithm="maxima")
```

```
[Out] 1/16*(8*(sin(9*f*x + 9*e) + 4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin
(3*f*x + 3*e) + sin(f*x + e))*cos(10*f*x + 10*e) - 20*(3*sin(8*f*x + 8*e) +
 sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin(2*f*x + 2*e))*cos(9*f*x + 9*e)
 + 60*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f
*x + e))*cos(8*f*x + 8*e) - 80*(sin(6*f*x + 6*e) - sin(4*f*x + 4*e) - 3*sin
(2*f*x + 2*e))*cos(7*f*x + 7*e) + 20*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*
e) + sin(f*x + e))*cos(6*f*x + 6*e) + 120*(sin(4*f*x + 4*e) + 3*sin(2*f*x +
 2*e))*cos(5*f*x + 5*e) - 20*(4*sin(3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x
+ 4*e) + 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e
) + cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x +
5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x +
 7*e)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(
5*f*x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) +
cos(f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x +
 3*e) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*
```

```
x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f
*x + 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*
sin(5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e
) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) +
1) - 15*(2*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(9*f*x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e)
 + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e
)^2 + 12*(4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*
x + 5*e)^2 + 16*cos(3*f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(
f*x + e)^2 + 2*(4*sin(7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e
) + sin(f*x + e))*sin(9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x +
5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x +
 7*e)^2 + 12*(4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(
5*f*x + 5*e)^2 + 16*sin(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) +
sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) -
 8*(cos(9*f*x + 9*e) + 4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*
x + 3*e) + cos(f*x + e))*sin(10*f*x + 10*e) + 4*(15*cos(8*f*x + 8*e) + 5*co
s(6*f*x + 6*e) - 5*cos(4*f*x + 4*e) - 15*cos(2*f*x + 2*e) - 2)*sin(9*f*x +
9*e) - 60*(4*cos(7*f*x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + c
os(f*x + e))*sin(8*f*x + 8*e) + 16*(5*cos(6*f*x + 6*e) - 5*cos(4*f*x + 4*e)
 - 15*cos(2*f*x + 2*e) - 2)*sin(7*f*x + 7*e) - 20*(6*cos(5*f*x + 5*e) + 4*c
os(3*f*x + 3*e) + cos(f*x + e))*sin(6*f*x + 6*e) - 24*(5*cos(4*f*x + 4*e) +
 15*cos(2*f*x + 2*e) + 2)*sin(5*f*x + 5*e) + 20*(4*cos(3*f*x + 3*e) + cos(f
*x + e))*sin(4*f*x + 4*e) - 16*(15*cos(2*f*x + 2*e) + 2)*sin(3*f*x + 3*e) +
 240*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 60*cos(f*x + e)*sin(2*f*x + 2*e) -
 60*cos(2*f*x + 2*e)*sin(f*x + e) - 8*sin(f*x + e))*sqrt(a)/((2*(4*cos(7*f*
x + 7*e) + 6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e) + cos(f*x + e))*cos(9*f*
x + 9*e) + cos(9*f*x + 9*e)^2 + 8*(6*cos(5*f*x + 5*e) + 4*cos(3*f*x + 3*e)
+ cos(f*x + e))*cos(7*f*x + 7*e) + 16*cos(7*f*x + 7*e)^2 + 12*(4*cos(3*f*x
+ 3*e) + cos(f*x + e))*cos(5*f*x + 5*e) + 36*cos(5*f*x + 5*e)^2 + 16*cos(3*
f*x + 3*e)^2 + 8*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(4*sin(
7*f*x + 7*e) + 6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3*e) + sin(f*x + e))*sin(
9*f*x + 9*e) + sin(9*f*x + 9*e)^2 + 8*(6*sin(5*f*x + 5*e) + 4*sin(3*f*x + 3
*e) + sin(f*x + e))*sin(7*f*x + 7*e) + 16*sin(7*f*x + 7*e)^2 + 12*(4*sin(3*
f*x + 3*e) + sin(f*x + e))*sin(5*f*x + 5*e) + 36*sin(5*f*x + 5*e)^2 + 16*si
n(3*f*x + 3*e)^2 + 8*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+fx\right)^6 \sqrt{a-a\sin\left(e+fx\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**6,x)
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**6, x)
```

3.463 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=91

$$\frac{\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}+\frac{3\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}-\frac{3\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+}{2f}$$

```
[Out] -3/2*arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+3/2*(a*cos(f*x
+e)^2)^(1/2)*tan(f*x+e)/f+1/2*(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3207, 2592, 288, 321, 206}

$$\frac{\tan^3(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}+\frac{3\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{2f}-\frac{3\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+}{2f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (-3*ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/(2*f) + (3*S
qrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/(2*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
 f*x]^3)/(2*f)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 288

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^
(n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*n*(p + 1)), x] - Dist[(c^
n*(m - n + 1))/(b*n*(p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^(p + 1), x], x]
 /; FreeQ[{a, b, c}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m + 1, n] &&  !I
LtQ[(m + n*(p + 1) + 1)/n, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \sqrt{a-a\sin^2(e+fx)}\,\tan^4(e+fx)\,dx = \int \sqrt{a\cos^2(e+fx)}\,\tan^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan^3(e+fx)\,dx$$
$$= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^2}\,dx, x, \sin(e+fx)\right)}{f}$$
$$= \frac{\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{2f} - \frac{\left(3\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}}{2f}$$
$$= \frac{3\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{2f} + \frac{\sqrt{a\cos^2(e+fx)}\,\tan^3(e+fx)}{2f} - \frac{\left(3\sqrt{\right.}}{}$$
$$= -\frac{3\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{2f} + \frac{3\sqrt{a\cos^2(e+f}}{2}$$

Mathematica [A] time = 0.19, size = 55, normalized size = 0.60

$$\frac{a\left((\cos(2(e+fx))+2)\tan(e+fx)-3\cos(e+fx)\tanh^{-1}(\sin(e+fx))\right)}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (a*(-3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + (2 + Cos[2*(e + f*x)])*Tan[e +
f*x]))/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.47, size = 77, normalized size = 0.85

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\cos\left(fx+e\right)^2\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)-2\left(2\cos\left(fx+e\right)^2+1\right)\sin\left(fx+e\right)\right)}{4f\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(
f*x + e) - 1)) - 2*(2*cos(f*x + e)^2 + 1)*sin(f*x + e))/(f*cos(f*x + e)^3)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to
check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a)*(-1/2*(3*(tan((f*x+exp(1))/2)
+1/tan((f*x+exp(1))/2))^2*sign(tan((f*x+exp(1))/2)^4-1)-8*sign(tan((f*x+exp
(1))/2)^4-1))/((tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^3-4*(tan((f*x+ex
p(1))/2)+1/tan((f*x+exp(1))/2)))+3/8*sign(tan((f*x+exp(1))/2)^4-1)*ln(abs(t
an((f*x+exp(1))/2)+2+1/tan((f*x+exp(1))/2)))-3/8*sign(tan((f*x+exp(1))/2)^4
-1)*ln(abs(tan((f*x+exp(1))/2)-2+1/tan((f*x+exp(1))/2))))
```

maple [A] time = 1.85, size = 84, normalized size = 0.92

$$-\frac{a\left(-4\left(\cos^{2}\left(fx+e\right)\right)\sin\left(fx+e\right)-2\sin\left(fx+e\right)+\left(-3\ln\left(\sin\left(fx+e\right)-1\right)+3\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos\right.}{4\cos\left(fx+e\right)\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/4*a*(-4*cos(f*x+e)^2*sin(f*x+e)-2*sin(f*x+e)+(-3*ln(sin(f*x+e)-1)+3*ln(1
+sin(f*x+e)))*cos(f*x+e)^2)/cos(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.54, size = 827, normalized size = 9.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] -1/4*(2*(sin(5*f*x + 5*e) + 2*sin(3*f*x + 3*e) + sin(f*x + e))*cos(6*f*x +
6*e) - 6*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 6*(2*sin(
3*f*x + 3*e) + sin(f*x + e))*cos(4*f*x + 4*e) + 3*(2*(2*cos(3*f*x + 3*e) +
cos(f*x + e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2
+ 4*cos(3*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e)
+ sin(f*x + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^
2 + 4*sin(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 +
sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - 3*(2*(2*cos(3*f*x + 3*e) + cos(f*x +
 e))*cos(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3
*f*x + 3*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x
 + e))*sin(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin
(3*f*x + 3*e)*sin(f*x + e) + sin(f*x + e)^2)*log(cos(f*x + e)^2 + sin(f*x +
 e)^2 - 2*sin(f*x + e) + 1) - 2*(cos(5*f*x + 5*e) + 2*cos(3*f*x + 3*e) + co
s(f*x + e))*sin(6*f*x + 6*e) + 2*(3*cos(4*f*x + 4*e) - 3*cos(2*f*x + 2*e) -
 1)*sin(5*f*x + 5*e) - 6*(2*cos(3*f*x + 3*e) + cos(f*x + e))*sin(4*f*x + 4*
e) - 4*(3*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) + 12*cos(3*f*x + 3*e)*sin(
2*f*x + 2*e) + 6*cos(f*x + e)*sin(2*f*x + 2*e) - 6*cos(2*f*x + 2*e)*sin(f*x
 + e) - 2*sin(f*x + e))*sqrt(a)/((2*(2*cos(3*f*x + 3*e) + cos(f*x + e))*cos
(5*f*x + 5*e) + cos(5*f*x + 5*e)^2 + 4*cos(3*f*x + 3*e)^2 + 4*cos(3*f*x + 3
*e)*cos(f*x + e) + cos(f*x + e)^2 + 2*(2*sin(3*f*x + 3*e) + sin(f*x + e))*s
in(5*f*x + 5*e) + sin(5*f*x + 5*e)^2 + 4*sin(3*f*x + 3*e)^2 + 4*sin(3*f*x +
 3*e)*sin(f*x + e) + sin(f*x + e)^2)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^4\,\sqrt{a-a\,\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\tan^{4}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**4, x)
```

3.464 $\int \sqrt{a-a\sin^2(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=57

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+fx))}{f}-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] arctanh(sin(f*x+e))*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(1
/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.10, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2592, 321, 206}

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\tanh^{-1}(\sin(e+fx))}{f}-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x])/f - (Sqrt[a*Cos
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2592

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_
Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[ff/f, Subst[Int[(
ff*x)^(m + n)/(a^2 - ff^2*x^2)^((n + 1)/2), x], x, (a*Sin[e + f*x])/ff], x]
] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n + 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\begin{aligned}\int \sqrt{a-a\sin^2(e+fx)}\,\tan^2(e+fx)\,dx &= \int \sqrt{a\cos^2(e+fx)}\,\tan^2(e+fx)\,dx\\ &= \left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\int \sin(e+fx)\tan(e+fx)\,dx\\ &= \frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{1-x^2}\,dx,x,\sin(e+fx)\right)}{f}\\ &= -\frac{\sqrt{a\cos^2(e+fx)}\,\tan(e+fx)}{f}+\frac{\left(\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}}{f}\\ &= \frac{\tanh^{-1}(\sin(e+fx))\sqrt{a\cos^2(e+fx)}\,\sec(e+fx)}{f}-\frac{\sqrt{a\cos^2(e+fx)}\,\text{ta}}{f}\end{aligned}$$

Mathematica [A] time = 0.05, size = 40, normalized size = 0.70

$$\frac{\sec(e+fx)\sqrt{a\cos^2(e+fx)}\,\left(\tanh^{-1}(\sin(e+fx))-\sin(e+fx)\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a - a*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
[Out] (Sqrt[a*Cos[e + f*x]^2]*Sec[e + f*x]*(ArcTanh[Sin[e + f*x]] - Sin[e + f*x])
)/f
```

fricas [A] time = 0.46, size = 55, normalized size = 0.96

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\,\sin\left(fx+e\right)\right)}{2\,f\cos\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) +
2*sin(f*x + e))/(f*cos(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a
)*(-1/4*sign(tan((f*x+exp(1))/2)^4-1)*ln(abs(tan((f*x+exp(1))/2)+2+1/tan((f
*x+exp(1))/2)))+1/4*sign(tan((f*x+exp(1))/2)^4-1)*ln(abs(tan((f*x+exp(1))/2
)-2+1/tan((f*x+exp(1))/2)))-sign(tan((f*x+exp(1))/2)^4-1)/(-tan((f*x+exp(1)
)/2)-1/tan((f*x+exp(1))/2)))
```

maple [A] time = 1.44, size = 54, normalized size = 0.95

$$-\frac{a\cos\left(fx+e\right)\left(2\sin\left(fx+e\right)+\ln\left(\sin\left(fx+e\right)-1\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)}{2\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] -1/2*a*cos(f*x+e)*(2*sin(f*x+e)+ln(sin(f*x+e)-1)-ln(1+sin(f*x+e)))/(a*cos(f
*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.48, size = 73, normalized size = 1.28

$$\frac{\sqrt{a}\left(\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\,\sin\left(fx+e\right)+1\right)-\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}-2\,\sin\left(fx\right.\right.}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] 1/2*sqrt(a)*(log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - lo
g(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 2*sin(f*x + e))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)^2\,\sqrt{a-a\sin\left(e+f\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\,\tan^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a-a*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*tan(e + f*x)**2, x)
```

3.465 $\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=57

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] -csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-(a*cos(f*x+e)^2)^(1/2)*tan(
f*x+e)/f
```

Rubi [A] time = 0.11, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3176, 3207, 2590, 14}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f) - (Sqrt[a*Cos[e + f
*x]^2]*Tan[e + f*x])/f
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^2(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^2(e+fx)\,dx$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{1-x^2}{x^2}\,dx, x, -\sin(e+fx)\right)}{f}$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^2}\right)dx, x, -\sin(e+\right.}{f}$$
$$= -\frac{\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\tan(e}{f}$$

Mathematica [A] time = 0.07, size = 35, normalized size = 0.61

$$-\frac{\tan(e+fx)\left(\csc^2(e+fx)+1\right)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -((Sqrt[a*Cos[e + f*x]^2]*(1 + Csc[e + f*x]^2)*Tan[e + f*x])/f)
```

fricas [A] time = 0.44, size = 42, normalized size = 0.74

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2-2\right)}{f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2 - 2)/(f*cos(f*x + e)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*sqrt(a)*(1/4*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))*sign(t
an((f*x+exp(1))/2)^4-1)+sign(tan((f*x+exp(1))/2)^4-1)/(tan((f*x+exp(1))/2)+
1/tan((f*x+exp(1))/2)))
```

maple [A] time = 0.93, size = 43, normalized size = 0.75

$$-\frac{\cos\left(fx+e\right)a\left(1+\sin^2\left(fx+e\right)\right)}{\sin\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -cos(f*x+e)*a*(1+sin(f*x+e)^2)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.42, size = 42, normalized size = 0.74

$$-\frac{2\sqrt{a}\tan\left(fx+e\right)^{2}+\sqrt{a}}{\sqrt{\tan\left(fx+e\right)^{2}+1}\,f\tan\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(2*sqrt(a)*tan(f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x +
 e))
```

mupad [B] time = 18.53, size = 88, normalized size = 1.54

$$\frac{\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^{2}}\,\left(-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,6\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,1\mathrm{i}+1\mathrm{i}\right)}{f\,\left(\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] ((a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*
(exp(e*4i + f*x*4i)*1i - exp(e*2i + f*x*2i)*6i + 1i))/(f*(exp(e*4i + f*x*4i
) - 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^{2}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**2, x)
```

3.466 $\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=91

$$\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}+\frac{2\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

```
[Out] 2*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f-1/3*csc(f*x+e)^3*sec(f*x+e
)*(a*cos(f*x+e)^2)^(1/2)/f+(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3176, 3207, 2590, 270}

$$\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{3f}+\frac{2\csc(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (2*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/(3*f) + (Sqrt[a*Cos[e + f*x]^2]*Tan[e +
f*x])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^4(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^4(e+fx)\,dx$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^2}{x^4}\,dx, x, -\sin(e+fx)\right)}{f}$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(1+\frac{1}{x^4}-\frac{2}{x^2}\right)dx, x, -\sin(e+$$
$$= \frac{2\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} - \frac{\sqrt{a\cos^2(e+fx)}\csc^3(e+$$

Mathematica [A] time = 0.07, size = 47, normalized size = 0.52

$$-\frac{\tan(e+fx)\left(\csc^4(e+fx)-6\csc^2(e+fx)-3\right)\sqrt{a\cos^2(e+fx)}}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Sqrt[a*Cos[e + f*x]^2]*(-3 - 6*Csc[e + f*x]^2 + Csc[e + f*x]^4)*Tan[e
 + f*x])/f
```

fricas [A] time = 0.45, size = 66, normalized size = 0.73

$$-\frac{\left(3\cos\left(fx+e\right)^4-12\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+e\right)^2}}{3\left(f\cos\left(fx+e\right)^3-f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/3*(3*cos(f*x + e)^4 - 12*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((f*
cos(f*x + e)^3 - f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a)*(-sign(tan(
(f*x+exp(1))/2)^4-1)/(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))+1/4096*(25
6/3*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^3*sign(tan((f*x+exp(1))/2)^
4-1)-2048*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))*sign(tan((f*x+exp(1))
/2)^4-1)))
```

maple [A] time = 0.90, size = 55, normalized size = 0.60

$$\frac{\cos\left(fx+e\right)a\left(3\left(\sin^4\left(fx+e\right)\right)+6\left(\sin^2\left(fx+e\right)\right)-1\right)}{3\sin\left(fx+e\right)^3\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cos(f*x+e)*a*(3*sin(f*x+e)^4+6*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+
e)^2)^(1/2)/f
```

maxima [A] time = 0.42, size = 57, normalized size = 0.63

$$\frac{8\sqrt{a}\tan\left(fx+e\right)^4+4\sqrt{a}\tan\left(fx+e\right)^2-\sqrt{a}}{3\sqrt{\tan\left(fx+e\right)^2+1}\,f\tan\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(8*sqrt(a)*tan(f*x + e)^4 + 4*sqrt(a)*tan(f*x + e)^2 - sqrt(a))/(sqrt(t
an(f*x + e)^2 + 1)*f*tan(f*x + e)^3)
```

mupad [B] time = 18.42, size = 364, normalized size = 4.00

$$\frac{\left(\frac{1i}{f}-\frac{e^{e\,2i+f\,x\,2i}\,1i}{f}\right)\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}}{e^{e\,2i+f\,x\,2i}+1}+\frac{e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}\,8i}{f\left(e^{e\,2i+f\,x\,2i}-1\right)\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}}{3\,f\left(e^{e\,2}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] ((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (
exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) + (exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)
^(1/2)*8i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3i + f*x
*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i
+ f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
- f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(3*f*(exp(e*2i +
 f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**4, x)
```

3.467 $\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx$

Optimal. Leaf size=124

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{5f}+\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+f}}{f}$$

```
[Out] -3*csc(f*x+e)*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/f+csc(f*x+e)^3*sec(f*x+e)*(
a*cos(f*x+e)^2)^(1/2)/f-1/5*csc(f*x+e)^5*sec(f*x+e)*(a*cos(f*x+e)^2)^(1/2)/
f-(a*cos(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2590, 270}

$$-\frac{\tan(e+fx)\sqrt{a\cos^2(e+fx)}}{f}-\frac{\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{5f}+\frac{\csc^3(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+f}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-3*Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x]*Sec[e + f*x])/f + (Sqrt[a*Cos[e + f
*x]^2]*Csc[e + f*x]^3*Sec[e + f*x])/f - (Sqrt[a*Cos[e + f*x]^2]*Csc[e + f*x
]^5*Sec[e + f*x])/(5*f) - (Sqrt[a*Cos[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2590

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*tan[(e_.) + (f_.)*(x_)]^(n_.), x_Symbol]
:> -Dist[f^(-1), Subst[Int[(1 - x^2)^((m + n - 1)/2)/x^n, x], x, Cos[e + f*
x]], x] /; FreeQ[{e, f}, x] && IntegersQ[m, n, (m + n - 1)/2]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \cot^6(e+fx)\sqrt{a-a\sin^2(e+fx)}\,dx = \int \sqrt{a\cos^2(e+fx)}\cot^6(e+fx)\,dx$$
$$= \left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\int \cos(e+fx)\cot^6(e+fx)\,dx$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{(1-x^2)^3}{x^6}dx, x, -\sin(e+fx)\right)}{f}$$
$$= -\frac{\left(\sqrt{a\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \left(-1+\frac{1}{x^6}-\frac{3}{x^4}+\frac{3}{x^2}\right)dx, x,\right.}{f}$$
$$= -\frac{3\sqrt{a\cos^2(e+fx)}\csc(e+fx)\sec(e+fx)}{f} + \frac{\sqrt{a\cos^2(e+fx)}\csc^3(}{f}$$

Mathematica [A] time = 0.18, size = 67, normalized size = 0.54

$$\frac{(235\cos(2(e+fx))-90\cos(4(e+fx))+5\cos(6(e+fx))-182)\csc^5(e+fx)\sec(e+fx)\sqrt{a\cos^2(e+fx)}}{160f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6*Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[a*Cos[e + f*x]^2]*(-182 + 235*Cos[2*(e + f*x)] - 90*Cos[4*(e + f*x)]
+ 5*Cos[6*(e + f*x)])*Csc[e + f*x]^5*Sec[e + f*x])/(160*f)
```

fricas [A] time = 0.46, size = 86, normalized size = 0.69

$$\frac{\left(5\cos\left(fx+e\right)^6-30\cos\left(fx+e\right)^4+40\cos\left(fx+e\right)^2-16\right)\sqrt{a\cos\left(fx+e\right)^2}}{5\left(f\cos\left(fx+e\right)^5-2f\cos\left(fx+e\right)^3+f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/5*(5*cos(f*x + e)^6 - 30*cos(f*x + e)^4 + 40*cos(f*x + e)^2 - 16)*sqrt(a*
cos(f*x + e)^2)/((f*cos(f*x + e)^5 - 2*f*cos(f*x + e)^3 + f*cos(f*x + e))*s
in(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*sqrt(a)*(sign(tan((f*x+exp(1))/2)^4-1)/(tan((f*x+e
xp(1))/2)+1/tan((f*x+exp(1))/2))+1/1073741824*(-67108864*(tan((f*x+exp(1))/
2)+1/tan((f*x+exp(1))/2))^3*sign(tan((f*x+exp(1))/2)^4-1)+16777216/5*(tan((
f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^5*sign(tan((f*x+exp(1))/2)^4-1)+80530
6368*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))*sign(tan((f*x+exp(1))/2)^4
-1)))
```

maple [A] time = 1.10, size = 65, normalized size = 0.52

$$-\frac{\cos\left(fx+e\right)a\left(5\left(\sin^6\left(fx+e\right)\right)+15\left(\sin^4\left(fx+e\right)\right)-5\left(\sin^2\left(fx+e\right)\right)+1\right)}{5\sin\left(fx+e\right)^5\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/5*cos(f*x+e)*a*(5*sin(f*x+e)^6+15*sin(f*x+e)^4-5*sin(f*x+e)^2+1)/sin(f*x
+e)^5/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.42, size = 68, normalized size = 0.55

$$-\frac{16\sqrt{a}\tan\left(fx+e\right)^6+8\sqrt{a}\tan\left(fx+e\right)^4-2\sqrt{a}\tan\left(fx+e\right)^2+\sqrt{a}}{5\sqrt{\tan\left(fx+e\right)^2+1}\,f\tan\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6*(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/5*(16*sqrt(a)*tan(f*x + e)^6 + 8*sqrt(a)*tan(f*x + e)^4 - 2*sqrt(a)*tan(
f*x + e)^2 + sqrt(a))/(sqrt(tan(f*x + e)^2 + 1)*f*tan(f*x + e)^5)
```

mupad [B] time = 26.64, size = 555, normalized size = 4.48

$$-\frac{\left(\frac{1\mathrm{i}}{f}-\frac{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,1\mathrm{i}}{f}\right)\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}\,12\mathrm{i}}{f\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}}{f\left(\mathrm{e}^{e\,2\mathrm{i}}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6*(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] - ((1i/f - (exp(e*2i + f*x*2i)*1i)/f)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
 (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(exp(e*2i + f*x*2i) + 1) - (exp(e*3i
+ f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^
2)^(1/2)*12i)/(f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i) + exp(e*3i +
f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(
e*1i + f*x*1i)*1i)/2)^2)^(1/2)*16i)/(f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i
 + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i
 - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*144i)/(5*f*(exp(e*2i
 + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i +
f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)
^(1/2)*128i)/(5*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp(e*3i
 + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (e
xp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*f*(exp(e*2i + f*x*2i) - 1)^5*(exp
(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}\cot^6\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6*(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))*cot(e + f*x)**6, x)
```

3.468 $\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}} dx$

Optimal. Leaf size=65

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/5*a^2/f/(a*cos(f*x+e)^2)^(5/2)-2/3*a/f/(a*cos(f*x+e)^2)^(3/2)+1/f/(a*cos(
f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}+\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a^2/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - (2*a)/(3*f*(a*Cos[e + f*x]^2)^(3/2)) +
 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^5(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$

$$= -\frac{\operatorname{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{a^3\operatorname{Subst}\left(\int \frac{(1-x)^2}{(ax)^{7/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{a^3\operatorname{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{2}{a(ax)^{5/2}}+\frac{1}{a^2(ax)^{3/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= \frac{a^2}{5f\left(a\cos^2(e+fx)\right)^{5/2}} - \frac{2a}{3f\left(a\cos^2(e+fx)\right)^{3/2}} + \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.08, size = 43, normalized size = 0.66

$$\frac{3\sec^4(e+fx)-10\sec^2(e+fx)+15}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (15 - 10*Sec[e + f*x]^2 + 3*Sec[e + f*x]^4)/(15*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 50, normalized size = 0.77

$$\frac{\left(15\cos\left(fx+e\right)^4-10\cos\left(fx+e\right)^2+3\right)\sqrt{a\cos\left(fx+e\right)^2}}{15\,af\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] 1/15*(15*cos(f*x + e)^4 - 10*cos(f*x + e)^2 + 3)*sqrt(a*cos(f*x + e)^2)/(a*
f*cos(f*x + e)^6)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(a)/15*(80*tan((f*x+exp(1))/
2)^4-40*tan((f*x+exp(1))/2)^2+8)/(tan((f*x+exp(1))/2)^2-1)^5/sign(tan((f*x+
exp(1))/2)^4-1)
```

maple [A] time = 2.11, size = 51, normalized size = 0.78

$$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(15\left(\cos^4\left(fx+e\right)\right)-10\left(\cos^2\left(fx+e\right)\right)+3\right)}{15a\cos\left(fx+e\right)^6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/15/a/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(15*cos(f*x+e)^4-10*cos(f*x+e)^2
+3)/f
```

maxima [A] time = 0.35, size = 69, normalized size = 1.06

$$\frac{15\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+10\left(a\sin\left(fx+e\right)^2-a\right)a^4+3\,a^5}{15\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/15*(15*(a*sin(f*x + e)^2 - a)^2*a^3 + 10*(a*sin(f*x + e)^2 - a)*a^4 + 3*a
^5)/((-a*sin(f*x + e)^2 + a)^(5/2)*a^3*f)
```

mupad [B] time = 23.61, size = 486, normalized size = 7.48

$$\frac{4\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{a\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{32\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^2}}{3\,a\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^2\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{352\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{\dots}}{15\,a\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\dots}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (4*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2))/(a*f*(exp(e*2i + f*x*2i) + 1)*(exp(e*1i + f*x*1i) + e
xp(e*3i + f*x*3i))) - (32*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a*f*(exp(e*2i + f*x*2i) + 1
)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (352*exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(1
5*a*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
 - (128*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
+ f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*
x*1i) + exp(e*3i + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a*f*(exp(e*2i + f*
x*2i) + 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.469 $\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=42

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/3*a/f/(a*cos(f*x+e)^2)^(3/2)-1/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.11, antiderivative size = 42, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}}-\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] a/(3*f*(a*Cos[e + f*x]^2)^(3/2)) - 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{5/2}} - \frac{1}{a(ax)^{3/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{3f\left(a\cos^2(e+fx)\right)^{3/2}} - \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.06, size = 31, normalized size = 0.74

$$\frac{\sec^2(e+fx)-3}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] (-3 + Sec[e + f*x]^2)/(3*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.44, size = 40, normalized size = 0.95

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\cos\left(fx+e\right)^2-1\right)}{3\,af\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 1)/(a*f*cos(f*x + e)^4)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(a)/3*(6*tan((f*x+exp(1))/2)
^2-2)/(tan((f*x+exp(1))/2)^2-1)^3/sign(tan((f*x+exp(1))/2)^4-1)
```

maple [A] time = 2.14, size = 41, normalized size = 0.98

$$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(3\left(\cos^2\left(fx+e\right)\right)-1\right)}{3a\cos\left(fx+e\right)^4 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3/a/cos(f*x+e)^4*(a*cos(f*x+e)^2)^(1/2)*(3*cos(f*x+e)^2-1)/f
```

maxima [A] time = 0.34, size = 46, normalized size = 1.10

$$\frac{3\left(a\sin\left(fx+e\right)^2-a\right)a^2+a^3}{3\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*(3*(a*sin(f*x + e)^2 - a)*a^2 + a^3)/((-a*sin(f*x + e)^2 + a)^(3/2)*a^2
*f)
```

mupad [B] time = 19.50, size = 100, normalized size = 2.38

$$-\frac{4\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}\,\left(2\,{\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+3\,{\mathrm{e}}^{e\,4{}\mathrm{i}+f\,x\,4{}\mathrm{i}}+3\right)}{3\,a\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] -(4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2)*(2*exp(e*2i + f*x*2i) + 3*exp(e*4i + f*x*4i) + 3))/(3
*a*f*(exp(e*2i + f*x*2i) + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}{\left(e+fx\right)}}{\sqrt{-a\left(\sin{\left(e+fx\right)}-1\right)\left(\sin{\left(e+fx\right)}+1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.470 $\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=18

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 18, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 16, 32}

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [**A**] time = 0.03, size = 18, normalized size = 1.00

$$\frac{1}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] 1/(f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [**A**] time = 0.45, size = 27, normalized size = 1.50

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{af\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^2)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f*2/sqrt(a)/2/(tan((f*x+exp(1))/2)
^2-1)/sign(tan((f*x+exp(1))/2)^4-1)
```

maple [**A**] time = 0.14, size = 20, normalized size = 1.11

$$\frac{1}{f\sqrt{a-a\left(\sin^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x)
[Out] 1/f/(a-a*sin(f*x+e)^2)^(1/2)
```

maxima [B] time = 0.44, size = 65, normalized size = 3.61

$$\frac{\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{a\sin\left(fx+e\right)+a}-\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{a\sin\left(fx+e\right)-a}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e) + a) - sqrt(-a*sin(f*x + e
)^2 + a)/(a*sin(f*x + e) - a))/f
```

mupad [B] time = 0.39, size = 61, normalized size = 3.39

$$\frac{2\,\sqrt{2}\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)\,\sqrt{a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}}{a\,f\,\left(4\,\cos\left(2\,e+2\,f\,x\right)+\cos\left(4\,e+4\,f\,x\right)+3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (2*2^(1/2)*(cos(2*e + 2*f*x) + 1)*(a*(cos(2*e + 2*f*x) + 1))^(1/2))/(a*f*(4
*cos(2*e + 2*f*x) + cos(4*e + 4*f*x) + 3))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.471 $\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=31

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)

Rubi [A] time = 0.08, antiderivative size = 31, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.167, Rules used = {3176, 3205, 63, 206}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{af}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A] time = 0.04, size = 49, normalized size = 1.58

$$\frac{\cos(e+fx)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(f*Sqrt[a*C
os[e + f*x]^2])
```

fricas [A] time = 0.46, size = 84, normalized size = 2.71

$$\left[-\frac{\sqrt{a\cos\left(fx+e\right)^2}\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)}{2\,af\cos\left(fx+e\right)}, \frac{\sqrt{-a}\arctan\left(\frac{\sqrt{a\cos\left(fx+e\right)^2}\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*sqrt(a*cos(f*x + e)^2)*log(-(cos(f*x + e) + 1)/(cos(f*x + e) - 1))/(a
*f*cos(f*x + e)), sqrt(-a)*arctan(sqrt(a*cos(f*x + e)^2)*sqrt(-a)/a)/(a*f)]
```

giac [A] time = 0.15, size = 32, normalized size = 1.03

$$\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

maple [A] time = 1.07, size = 40, normalized size = 1.29

$$-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

maxima [B] time = 0.31, size = 51, normalized size = 1.65

$$-\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin(
f*x + e)))/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot\left(e+f\,x\right)}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.472 $\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=66

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2af}$$

```
[Out] 1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*csc(f*x+e)^2*(a*c
os(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.11, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3205, 16, 47, 63, 206}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*Sqrt[a]*f) - (Sqrt[a*Cos[e + f*x
]^2]*Csc[e + f*x]^2)/(2*a*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 47

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + 1)), x] - Dist[(d*n)/(b*(m + 1)), I
nt[(a + b*x)^(m + 1)*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b, c, d}, x] &&
 NeQ[b*c - a*d, 0] && GtQ[n, 0] && LtQ[m, -1] &&  !(IntegerQ[n] &&  !Intege
rQ[m]) &&  !(ILeQ[m + n + 2, 0] && (FractionQ[m] || GeQ[2*n + m + 1, 0])) &
& IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
```

```
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^3(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$

$$= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$

$$= -\frac{\text{Subst}\left(\int \frac{\sqrt{ax}}{(1-x)^2}\,dx, x, \cos^2(e+fx)\right)}{2af}$$

$$= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{4f}$$

$$= -\frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af} + \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{2af}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f} - \frac{\sqrt{a\cos^2(e+fx)}\,\csc^2(e+fx)}{2af}$$

Mathematica [A] time = 0.17, size = 80, normalized size = 1.21

$$\frac{\cos(e+fx)\left(-\csc^2\left(\frac{1}{2}(e+fx)\right)+\sec^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (Cos[e + f*x]*(-Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[Sin[(e
 + f*x)/2]] + Sec[(e + f*x)/2]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 79, normalized size = 1.20

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{\cos\left(fx+e\right)-1}{\cos\left(fx+e\right)+1}\right)-2\cos\left(fx+e\right)\right)}{4\left(af\cos\left(fx+e\right)^3-af\cos\left(fx+e\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) - 1)/(
cos(f*x + e) + 1)) - 2*cos(f*x + e))/(a*f*cos(f*x + e)^3 - a*f*cos(f*x + e)
)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(a)*(-1/16*tan((f*x+exp(1))/2)^2/sign(tan((f*x
+exp(1))/2)^4-1)+1/16*(-2*tan((f*x+exp(1))/2)^2+1)/tan((f*x+exp(1))/2)^2/si
gn(tan((f*x+exp(1))/2)^4-1)+1/8*ln(tan((f*x+exp(1))/2)^2)/sign(tan((f*x+exp
(1))/2)^4-1))
```

maple [A] time = 1.72, size = 69, normalized size = 1.05

$$\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{2\sqrt{a}\,f}-\frac{\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}}{2fa\sin\left(fx+e\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/a^(1/2)*ln((2*a+2*a^(1/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))/f-1/2/f/a
/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.41, size = 81, normalized size = 1.23

$$\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}-\frac{\sqrt{-a\sin\left(fx+e\right)^{2}+a}}{a\sin\left(fx+e\right)^{2}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(
sin(f*x + e)))/sqrt(a) - sqrt(-a*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot\left(e+f\,x\right)^{3}}{\sqrt{a-a\sin\left(e+f\,x\right)^{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.473 $\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=91

$$\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 3/8*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-3/8*tan(f*x+e)/
f/(a*cos(f*x+e)^2)^(1/2)+1/4*tan(f*x+e)^3/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}-\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}}+\frac{3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) - (3*Ta
n[e + f*x])/(8*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e + f*x]^3/(4*f*Sqrt[a*Cos[e
 + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\tan^4(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} - \frac{(3\cos(e+fx))\int \sec(e+fx)\tan^2(e+fx)\,dx}{4\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}} + \frac{(3\cos(e+fx))\int \sec(e+fx)\,dx}{8\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{3\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} - \frac{3\tan(e+fx)}{8f\sqrt{a\cos^2(e+fx)}} + \frac{\tan^3(e+fx)}{4f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.13, size = 66, normalized size = 0.73

$$\frac{\tan(e+fx)\left(8\tan^2(e+fx)-6\sec^2(e+fx)+3\right)+3\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (3*ArcTanh[Sin[e + f*x]]*Cos[e + f*x] + Tan[e + f*x]*(3 - 6*Sec[e + f*x]^2
+ 8*Tan[e + f*x]^2))/(8*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.46, size = 80, normalized size = 0.88

$$-\frac{\left(3\cos\left(fx+e\right)^4\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)+2\left(5\cos\left(fx+e\right)^2-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^2}}{16\,af\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/16*(3*cos(f*x + e)^4*log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1)) + 2*(5*
cos(f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)^
5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(a
)*(-1/8*(-3*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^3+20*(tan((f*x+exp(
1))/2)+1/tan((f*x+exp(1))/2)))/((tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))
^2-4)^2/sign(tan((f*x+exp(1))/2)^4-1)-3/32*ln(abs(tan((f*x+exp(1))/2)+2+1/t
an((f*x+exp(1))/2)))/sign(tan((f*x+exp(1))/2)^4-1)+3/32*ln(abs(tan((f*x+exp
(1))/2-2+1/tan((f*x+exp(1))/2)))/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 1.60, size = 103, normalized size = 1.13

$$\frac{10\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)-4\sin\left(fx+e\right)+\left(3\ln\left(\sin\left(fx+e\right)-1\right)-3\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos^4\left(fx+e\right)\right)}{16\left(1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*(10*cos(f*x+e)^2*sin(f*x+e)-4*sin(f*x+e)+(3*ln(sin(f*x+e)-1)-3*ln(1+si
n(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*
x+e)^2)^(1/2)/f
```

maxima [B] time = 0.73, size = 1518, normalized size = 16.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(5*sin(7*f*x + 7*e) - 3*sin(5*f*x + 5*e) + 3*sin(3*f*x + 3*e) - 5*
sin(f*x + e))*cos(8*f*x + 8*e) - 40*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e
) + 2*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(3*sin(5*f*x + 5*e) - 3*sin(3
*f*x + 3*e) + 5*sin(f*x + e))*cos(6*f*x + 6*e) + 24*(3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 24*(3*sin(3*f*x + 3*e) - 5*sin(f*x +
e))*cos(4*f*x + 4*e) - 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x
+ 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 +
 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*
cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f
*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) +
 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*
x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 +
 8*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x +
e) + 1) + 3*(2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*co
s(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*
f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2
*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*si
n(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x
+ 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 +
48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x
+ 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(
5*cos(7*f*x + 7*e) - 3*cos(5*f*x + 5*e) + 3*cos(3*f*x + 3*e) - 5*cos(f*x +
e))*sin(8*f*x + 8*e) + 20*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(
2*f*x + 2*e) + 1)*sin(7*f*x + 7*e) + 16*(3*cos(5*f*x + 5*e) - 3*cos(3*f*x +
 3*e) + 5*cos(f*x + e))*sin(6*f*x + 6*e) - 12*(6*cos(4*f*x + 4*e) + 4*cos(2
*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 24*(3*cos(3*f*x + 3*e) - 5*cos(f*x + e)
)*sin(4*f*x + 4*e) + 12*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 48*cos(
3*f*x + 3*e)*sin(2*f*x + 2*e) + 80*cos(f*x + e)*sin(2*f*x + 2*e) - 80*cos(2
*f*x + 2*e)*sin(f*x + e) - 20*sin(f*x + e))/((2*(4*cos(6*f*x + 6*e) + 6*cos
(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)
^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*
cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(
4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*
x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(
3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x +
6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*
sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^4}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
[Out] int(tan(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(tan(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.474 $\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=62

$$\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/2*arctanh(sin(f*x+e))*cos(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+1/2*tan(f*x+e)
/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2611, 3770}

$$\frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(2*f*Sqrt[a*Cos[e + f*x]^2]) + Tan[e
+ f*x]/(2*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\tan^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \sec(e+fx)\tan^2(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} - \frac{\cos(e+fx)\int \sec(e+fx)\,dx}{2\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{2f\sqrt{a\cos^2(e+fx)}} + \frac{\tan(e+fx)}{2f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [**A**] time = 0.05, size = 43, normalized size = 0.69

$$\frac{\tan(e+fx)-\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{2f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + Tan[e + f*x])/(2*f*Sqrt[a*Cos[e +
f*x]^2])
```

fricas [**A**] time = 0.47, size = 67, normalized size = 1.08

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)^2\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)-2\,\sin\left(fx+e\right)\right)}{4\,af\cos\left(fx+e\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)^2*log(-(sin(f*x + e) + 1)/(sin(f*
x + e) - 1)) - 2*sin(f*x + e))/(a*f*cos(f*x + e)^3)
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)2/f/sqrt(a
)*(-1/2*(tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))/((tan((f*x+exp(1))/2)+1
/tan((f*x+exp(1))/2))^2-4)/sign(tan((f*x+exp(1))/2)^4-1)+1/8*ln(abs(tan((f*
x+exp(1))/2)+2+1/tan((f*x+exp(1))/2)))/sign(tan((f*x+exp(1))/2)^4-1)-1/8*ln
(abs(tan((f*x+exp(1))/2)-2+1/tan((f*x+exp(1))/2)))/sign(tan((f*x+exp(1))/2)
^4-1))
```

maple [**A**] time = 1.50, size = 65, normalized size = 1.05

$$\frac{\frac{\sin\left(fx+e\right)}{2}+\frac{\left(\ln\left(\sin\left(fx+e\right)-1\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos^2\left(fx+e\right)\right)}{4}}{\cos\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (1/2*sin(f*x+e)+1/4*(ln(sin(f*x+e)-1)-ln(1+sin(f*x+e)))*cos(f*x+e)^2)/cos(f
*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.50, size = 527, normalized size = 8.50

$$\frac{4\left(\sin\left(3\,fx+3\,e\right)-\sin\left(fx+e\right)\right)\cos\left(4\,fx+4\,e\right)-\left(2\left(2\,\cos\left(2\,fx+2\,e\right)+1\right)\cos\left(4\,fx+4\,e\right)+\cos\left(4\,fx+\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(sin(3*f*x + 3*e) - sin(f*x + e))*cos(4*f*x + 4*e) - (2*(2*cos(2*f*x
 + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 +
 sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2
*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin
(f*x + e) + 1) + (2*(2*cos(2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + cos(4*f*x +
 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x + 4*e)^2 + 4*sin(4*f*x + 4*e)*si
n(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*cos(2*f*x + 2*e) + 1)*log(cos(f*x
 + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(3*f*x + 3*e) - cos(
f*x + e))*sin(4*f*x + 4*e) + 4*(2*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) -
8*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) + 8*cos(f*x + e)*sin(2*f*x + 2*e) - 8*c
os(2*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((2*(2*cos(2*f*x + 2*e) + 1)
*cos(4*f*x + 4*e) + cos(4*f*x + 4*e)^2 + 4*cos(2*f*x + 2*e)^2 + sin(4*f*x +
 4*e)^2 + 4*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 4*sin(2*f*x + 2*e)^2 + 4*co
s(2*f*x + 2*e) + 1)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan\left(e+f\,x\right)^2}{\sqrt{a-a\sin\left(e+f\,x\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.475 $\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=25

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.10, antiderivative size = 25, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 8}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^2(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\mathrm{Subst}(\int 1\,dx, x, \csc(e+fx))}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [**A**] time = 0.03, size = 25, normalized size = 1.00

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a - a*Sin[e + f*x]^2],x]
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]))
```

fricas [**A**] time = 0.51, size = 36, normalized size = 1.44

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}}{af\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] -sqrt(a*cos(f*x + e)^2)/(a*f*cos(f*x + e)*sin(f*x + e))
```

giac [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/sqrt(a)*(1/4*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)^4-1)+
1/4/tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [**A**] time = 0.37, size = 32, normalized size = 1.28

$$-\frac{\cos\left(fx+e\right)}{\sin\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x)
[Out] -cos(f*x+e)/sin(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.48, size = 90, normalized size = 3.60

$$-\frac{2\left(\cos\left(fx+e\right)\sin\left(2\,fx+2\,e\right)-\cos\left(2\,fx+2\,e\right)\sin\left(fx+e\right)+\sin\left(fx+e\right)\right)\sqrt{a}}{\left(a\cos\left(2\,fx+2\,e\right)^{2}+a\sin\left(2\,fx+2\,e\right)^{2}-2\,a\cos\left(2\,fx+2\,e\right)+a\right)f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2*(cos(f*x + e)*sin(2*f*x + 2*e) - cos(2*f*x + 2*e)*sin(f*x + e) + sin(f*x
 + e))*sqrt(a)/((a*cos(2*f*x + 2*e)^2 + a*sin(2*f*x + 2*e)^2 - 2*a*cos(2*f*
x + 2*e) + a)*f)
```

mupad [B] time = 15.06, size = 37, normalized size = 1.48

$$-\frac{\sqrt{2\,a\,\left(\cos\left(2\,e+2\,f\,x\right)+1\right)}}{a\,f\,\sin\left(2\,e+2\,f\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] -(2*a*(cos(2*e + 2*f*x) + 1))^(1/2)/(a*f*sin(2*e + 2*f*x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.476 $\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=60

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)-1/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(f*
x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.115$, Rules used = {3176, 3207, 2606}

$$\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^2)/(3*
f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^4(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-1+x^2\right)dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.06, size = 37, normalized size = 0.62

$$-\frac{\cot(e+fx)\left(\csc^2(e+fx)-3\right)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/3*(Cot[e + f*x]*(-3 + Csc[e + f*x]^2))/(f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.49, size = 58, normalized size = 0.97

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(3\cos\left(fx+e\right)^2-2\right)}{3\left(af\cos\left(fx+e\right)^3-af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)*(3*cos(f*x + e)^2 - 2)/((a*f*cos(f*x + e)^3 - a*
f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/sqrt(a)*(1/4096*(256/3*tan((f*x+exp(1))/2)^3-768*tan((f*x+exp(1
))/2))/sign(tan((f*x+exp(1))/2)^4-1)+1/48*(-9*tan((f*x+exp(1))/2)^2+1)/tan(
(f*x+exp(1))/2)^3/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 0.93, size = 44, normalized size = 0.73

$$\frac{\cos\left(fx+e\right)\left(3\left(\sin^2\left(fx+e\right)\right)-1\right)}{3\sin\left(fx+e\right)^3\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/3*cos(f*x+e)*(3*sin(f*x+e)^2-1)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.50, size = 525, normalized size = 8.75

$$-\frac{2\left(\left(3\sin\left(5fx+5e\right)-2\sin\left(3fx+3e\right)+3\sin\left(fx+e\right)\right)\cos\left(6fx+6e\right)+9\left(\sin\left(4fx+4e\right)-\sin\left(2fx\right.\right.}{3\left(a\cos\left(6fx+\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -2/3*((3*sin(5*f*x + 5*e) - 2*sin(3*f*x + 3*e) + 3*sin(f*x + e))*cos(6*f*x
+ 6*e) + 9*(sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 3*(2*si
n(3*f*x + 3*e) - 3*sin(f*x + e))*cos(4*f*x + 4*e) - (3*cos(5*f*x + 5*e) - 2
*cos(3*f*x + 3*e) + 3*cos(f*x + e))*sin(6*f*x + 6*e) - 3*(3*cos(4*f*x + 4*e
) - 3*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e) - 3*(2*cos(3*f*x + 3*e) - 3*co
s(f*x + e))*sin(4*f*x + 4*e) - 2*(3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e)
+ 6*cos(3*f*x + 3*e)*sin(2*f*x + 2*e) - 9*cos(f*x + e)*sin(2*f*x + 2*e) + 9
*cos(2*f*x + 2*e)*sin(f*x + e) - 3*sin(f*x + e))*sqrt(a)/((a*cos(6*f*x + 6*
e)^2 + 9*a*cos(4*f*x + 4*e)^2 + 9*a*cos(2*f*x + 2*e)^2 + a*sin(6*f*x + 6*e)
^2 + 9*a*sin(4*f*x + 4*e)^2 - 18*a*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a*
sin(2*f*x + 2*e)^2 - 2*(3*a*cos(4*f*x + 4*e) - 3*a*cos(2*f*x + 2*e) + a)*co
s(6*f*x + 6*e) - 6*(3*a*cos(2*f*x + 2*e) - a)*cos(4*f*x + 4*e) - 6*a*cos(2*
f*x + 2*e) - 6*(a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) +
 a)*f)
```

mupad [B] time = 19.19, size = 118, normalized size = 1.97

$$\frac{4\,\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,\left(-\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}\,2\mathrm{i}+\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,3\mathrm{i}+3\mathrm{i}\right)}{3\,a\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] (4*exp(e*2i + f*x*2i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*(exp(e*4i + f*x*4i)*3i - exp(e*2i + f*x*2i)*2i + 3i))/
(3*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x*2i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}{\left(e + fx \right)}}{\sqrt{- a \left(\sin{\left(e + fx \right)} - 1\right) \left(\sin{\left(e + fx \right)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.477 $\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=96

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -cot(f*x+e)/f/(a*cos(f*x+e)^2)^(1/2)+2/3*cot(f*x+e)*csc(f*x+e)^2/f/(a*cos(f
*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(f*x+e)^4/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 194}

$$-\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -(Cot[e + f*x]/(f*Sqrt[a*Cos[e + f*x]^2])) + (2*Cot[e + f*x]*Csc[e + f*x]^2
)/(3*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e + f*x]^4)/(5*f*Sqrt[a*
Cos[e + f*x]^2])
```

Rule 194

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Int[ExpandIntegrand[(a + b*
x^n)^p, x], x] /; FreeQ[{a, b}, x] && IGtQ[n, 0] && IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\sqrt{a-a\sin^2(e+fx)}}\,dx = \int \frac{\cot^6(e+fx)}{\sqrt{a\cos^2(e+fx)}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc(e+fx)\,dx}{\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(1-2x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{f\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)}{f\sqrt{a\cos^2(e+fx)}} + \frac{2\cot(e+fx)\csc^2(e+fx)}{3f\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^4(e+fx)}{5f\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.07, size = 49, normalized size = 0.51

$$-\frac{\cot(e+fx)\left(3\csc^4(e+fx)-10\csc^2(e+fx)+15\right)}{15f\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/Sqrt[a - a*Sin[e + f*x]^2],x]
```

```
[Out] -1/15*(Cot[e + f*x]*(15 - 10*Csc[e + f*x]^2 + 3*Csc[e + f*x]^4))/(f*Sqrt[a*
Cos[e + f*x]^2])
```

fricas [A] time = 0.54, size = 79, normalized size = 0.82

$$-\frac{\left(15\cos\left(fx+e\right)^4-20\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+e\right)^2}}{15\left(af\cos\left(fx+e\right)^5-2\,af\cos\left(fx+e\right)^3+af\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] -1/15*(15*cos(f*x + e)^4 - 20*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a*f*cos(f*x + e)^5 - 2*a*f*cos(f*x + e)^3 + a*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f/sqrt(a)*(1/960*(150*tan((f*x+exp(1))/2)^4-25*tan((f*x+exp(1))/2
)^2+3)/tan((f*x+exp(1))/2)^5/sign(tan((f*x+exp(1))/2)^4-1)+1/1073741824*(16
777216/5*tan((f*x+exp(1))/2)^5-83886080/3*tan((f*x+exp(1))/2)^3+167772160*t
an((f*x+exp(1))/2))/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 0.97, size = 54, normalized size = 0.56

$$-\frac{\cos\left(fx+e\right)\left(15\left(\sin^4\left(fx+e\right)\right)-10\left(\sin^2\left(fx+e\right)\right)+3\right)}{15\sin\left(fx+e\right)^5\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/15*cos(f*x+e)*(15*sin(f*x+e)^4-10*sin(f*x+e)^2+3)/sin(f*x+e)^5/(a*cos(f*
x+e)^2)^(1/2)/f
```

maxima [B] time = 0.51, size = 1236, normalized size = 12.88

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 2/15*((15*sin(9*f*x + 9*e) - 20*sin(7*f*x + 7*e) + 58*sin(5*f*x + 5*e) - 20
*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(10*f*x + 10*e) + 75*(sin(8*f*x + 8
*e) - 2*sin(6*f*x + 6*e) + 2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(9*f*x
 + 9*e) + 5*(20*sin(7*f*x + 7*e) - 58*sin(5*f*x + 5*e) + 20*sin(3*f*x + 3*e
) - 15*sin(f*x + e))*cos(8*f*x + 8*e) + 100*(2*sin(6*f*x + 6*e) - 2*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(58*sin(5*f*x + 5*e) -
20*sin(3*f*x + 3*e) + 15*sin(f*x + e))*cos(6*f*x + 6*e) + 290*(2*sin(4*f*x
+ 4*e) - sin(2*f*x + 2*e))*cos(5*f*x + 5*e) + 50*(4*sin(3*f*x + 3*e) - 3*si
n(f*x + e))*cos(4*f*x + 4*e) - (15*cos(9*f*x + 9*e) - 20*cos(7*f*x + 7*e) +
 58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x + e))*sin(10*f*x +
10*e) - 15*(5*cos(8*f*x + 8*e) - 10*cos(6*f*x + 6*e) + 10*cos(4*f*x + 4*e)
- 5*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) - 5*(20*cos(7*f*x + 7*e) - 58*co
s(5*f*x + 5*e) + 20*cos(3*f*x + 3*e) - 15*cos(f*x + e))*sin(8*f*x + 8*e) -
20*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*cos(2*f*x + 2*e) - 1)*sin
(7*f*x + 7*e) - 10*(58*cos(5*f*x + 5*e) - 20*cos(3*f*x + 3*e) + 15*cos(f*x
+ e))*sin(6*f*x + 6*e) - 58*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) + 1)*
sin(5*f*x + 5*e) - 50*(4*cos(3*f*x + 3*e) - 3*cos(f*x + e))*sin(4*f*x + 4*e
) - 20*(5*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) + 100*cos(3*f*x + 3*e)*sin
(2*f*x + 2*e) - 75*cos(f*x + e)*sin(2*f*x + 2*e) + 75*cos(2*f*x + 2*e)*sin(
f*x + e) - 15*sin(f*x + e))*sqrt(a)/((a*cos(10*f*x + 10*e)^2 + 25*a*cos(8*f
*x + 8*e)^2 + 100*a*cos(6*f*x + 6*e)^2 + 100*a*cos(4*f*x + 4*e)^2 + 25*a*co
s(2*f*x + 2*e)^2 + a*sin(10*f*x + 10*e)^2 + 25*a*sin(8*f*x + 8*e)^2 + 100*a
*sin(6*f*x + 6*e)^2 + 100*a*sin(4*f*x + 4*e)^2 - 100*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 25*a*sin(2*f*x + 2*e)^2 - 2*(5*a*cos(8*f*x + 8*e) - 10*a*co
s(6*f*x + 6*e) + 10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*x + 2*e) + a)*cos(10*f
*x + 10*e) - 10*(10*a*cos(6*f*x + 6*e) - 10*a*cos(4*f*x + 4*e) + 5*a*cos(2*
f*x + 2*e) - a)*cos(8*f*x + 8*e) - 20*(10*a*cos(4*f*x + 4*e) - 5*a*cos(2*f*
x + 2*e) + a)*cos(6*f*x + 6*e) - 20*(5*a*cos(2*f*x + 2*e) - a)*cos(4*f*x +
4*e) - 10*a*cos(2*f*x + 2*e) - 10*(a*sin(8*f*x + 8*e) - 2*a*sin(6*f*x + 6*e
) + 2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a
*sin(6*f*x + 6*e) - 2*a*sin(4*f*x + 4*e) + a*sin(2*f*x + 2*e))*sin(8*f*x +
8*e) - 100*(2*a*sin(4*f*x + 4*e) - a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a
)*f)
```

mupad [B] time = 22.47, size = 491, normalized size = 5.11

$$-\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a-a\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,4i}{a\,f\,\left(e^{e\,2i+f\,x\,2i}-1\right)\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}-\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a-a\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,32i}{3\,a\,f\,{\left(e^{e\,2i+f\,x\,2i}-1\right)}^2\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}-\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a-}}{15\,a\,f\,\left(e^{e\,2i+f}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*4i)/(a*f*(exp(e*2i + f*x*2i) - 1)*(exp(e*1i + f*x*1i)
```

```
+ exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*
1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*32i)/(3*a*f*(exp(e*2i + f*x*2i)
 - 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*352
i)/(15*a*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*
3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2)^2)^(1/2)*128i)/(5*a*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1
i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1
i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*a*f*(exp(e*
2i + f*x*2i) - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^6(e + fx)}{\sqrt{-a(\sin(e + fx) - 1)(\sin(e + fx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/sqrt(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1)), x)
```

3.478 $\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=68

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/7*a^2/f/(a*cos(f*x+e)^2)^(7/2)-2/5*a/f/(a*cos(f*x+e)^2)^(5/2)+1/3/f/(a*co
s(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.13, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a^2/(7*f*(a*Cos[e + f*x]^2)^(7/2)) - (2*a)/(5*f*(a*Cos[e + f*x]^2)^(5/2)) +
 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^5(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \frac{(1-x)^2}{(ax)^{9/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^3\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{9/2}}-\frac{2}{a(ax)^{7/2}}+\frac{1}{a^2(ax)^{5/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a^2}{7f\left(a\cos^2(e+fx)\right)^{7/2}}-\frac{2a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}+\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.10, size = 51, normalized size = 0.75

$$\frac{\left(35\cos^4(e+fx)-42\cos^2(e+fx)+15\right)\sec^4(e+fx)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((15 - 42*Cos[e + f*x]^2 + 35*Cos[e + f*x]^4)*Sec[e + f*x]^4)/(105*f*(a*Cos
[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.50, size = 50, normalized size = 0.74

$$\frac{\left(35\cos\left(fx+e\right)^4-42\cos\left(fx+e\right)^2+15\right)\sqrt{a\cos\left(fx+e\right)^2}}{105\,a^2f\cos\left(fx+e\right)^8}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 42*cos(f*x + e)^2 + 15)*sqrt(a*cos(f*x + e)^2)/(
a^2*f*cos(f*x + e)^8)
```

giac [B] time = 0.77, size = 151, normalized size = 2.22

$$\frac{\frac{7\left(3\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}-10\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a+15\sqrt{a\tan\left(fx+e\right)^2+a}\,a^2\right)}{a^2}+\frac{3\left(5\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{7}{2}}-21\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}a+35\left(a\tan\left(fx+\cdots\right.\right.}{a^3}}{105\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/105*(7*(3*(a*tan(f*x + e)^2 + a)^(5/2) - 10*(a*tan(f*x + e)^2 + a)^(3/2)*
a + 15*sqrt(a*tan(f*x + e)^2 + a)*a^2)/a^2 + 3*(5*(a*tan(f*x + e)^2 + a)^(7
/2) - 21*(a*tan(f*x + e)^2 + a)^(5/2)*a + 35*(a*tan(f*x + e)^2 + a)^(3/2)*a
^2 - 35*sqrt(a*tan(f*x + e)^2 + a)*a^3)/a^3)/(a^2*f)
```

maple [A] time = 2.13, size = 51, normalized size = 0.75

$$\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\left(35\left(\cos^4\left(fx+e\right)\right)-42\left(\cos^2\left(fx+e\right)\right)+15\right)}{105a^2\cos\left(fx+e\right)^8 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/105/a^2/cos(f*x+e)^8*(a*cos(f*x+e)^2)^(1/2)*(35*cos(f*x+e)^4-42*cos(f*x+e
)^2+15)/f
```

maxima [A] time = 0.33, size = 69, normalized size = 1.01

$$\frac{35\left(a\sin\left(fx+e\right)^2-a\right)^2a^3+42\left(a\sin\left(fx+e\right)^2-a\right)a^4+15\,a^5}{105\left(-a\sin\left(fx+e\right)^2+a\right)^{\frac{7}{2}}a^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/105*(35*(a*sin(f*x + e)^2 - a)^2*a^3 + 42*(a*sin(f*x + e)^2 - a)*a^4 + 15
*a^5)/((-a*sin(f*x + e)^2 + a)^(7/2)*a^3*f)
```

mupad [B] time = 33.91, size = 583, normalized size = 8.57

$$\frac{16\,e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}}{3\,a^2\,f\left(e^{e\,2i+f\,x\,2i}+1\right)^2\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}-\frac{464\,e^{e\,3i+f\,x\,3i}\sqrt{a-a\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)^2}}{15\,a^2\,f\left(e^{e\,2i+f\,x\,2i}+1\right)^3\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{3072\,e^{e\,3i+f\,x\,3i}\,\ldots}{35\,a^2\,f\left(e^{e\,2i+f}\ldots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^2*(exp(e*1i + f*x*
1i) + exp(e*3i + f*x*3i))) - (464*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2i +
f*x*2i) + 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (3072*exp(e*3i
+ f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^
2)^(1/2))/(35*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) + exp(e*
3i + f*x*3i))) - (4736*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)
/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(35*a^2*f*(exp(e*2i + f*x*2i) + 1
)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (768*exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(7
*a^2*f*(exp(e*2i + f*x*2i) + 1)^6*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))
) - (256*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i
 + f*x*1i)*1i)/2)^2)^(1/2))/(7*a^2*f*(exp(e*2i + f*x*2i) + 1)^7*(exp(e*1i +
 f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.479 $\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=44

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/5*a/f/(a*cos(f*x+e)^2)^(5/2)-1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.12, antiderivative size = 44, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3205, 16, 43}

$$\frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}}-\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] a/(5*f*(a*Cos[e + f*x]^2)^(5/2)) - 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 43

```
Int[((a_.) + (b_.)*(x_))^(m_.)*((c_.) + (d_.)*(x_))^(n_.), x_Symbol] :> Int
[ExpandIntegrand[(a + b*x)^m*(c + d*x)^n, x], x] /; FreeQ[{a, b, c, d, n},
x] && NeQ[b*c - a*d, 0] && IGtQ[m, 0] && ( !IntegerQ[n] || (EqQ[c, 0] && Le
Q[7*m + 4*n + 4, 0]) || LtQ[9*m + 5*(n + 1), 0] || GtQ[m + n + 2, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1-x}{x^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \frac{1-x}{(ax)^{7/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a^2\,\text{Subst}\left(\int \left(\frac{1}{(ax)^{7/2}}-\frac{1}{a(ax)^{5/2}}\right)dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{a}{5f\left(a\cos^2(e+fx)\right)^{5/2}} - \frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.11, size = 34, normalized size = 0.77

$$\frac{a\left(3-5\cos^2(e+fx)\right)}{15f\left(a\cos^2(e+fx)\right)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
[Out] (a*(3 - 5*Cos[e + f*x]^2))/(15*f*(a*Cos[e + f*x]^2)^(5/2))
```

fricas [A] time = 0.45, size = 40, normalized size = 0.91

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(5\,\cos\left(fx+e\right)^2-3\right)}{15\,a^2 f\cos\left(fx+e\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 3)/(a^2*f*cos(f*x + e)^6)
```

giac [B] time = 0.67, size = 108, normalized size = 2.45

$$\frac{\frac{5\left(\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-3\sqrt{a\tan\left(fx+e\right)^2+a}\,a\right)}{a}+\frac{3\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{5}{2}}-10\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a+15\sqrt{a\tan\left(fx+e\right)^2+a}\,a^2}{a^2}}{15\,a^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] 1/15*(5*((a*tan(f*x + e)^2 + a)^(3/2) - 3*sqrt(a*tan(f*x + e)^2 + a)*a)/a +
 (3*(a*tan(f*x + e)^2 + a)^(5/2) - 10*(a*tan(f*x + e)^2 + a)^(3/2)*a + 15*s
qrt(a*tan(f*x + e)^2 + a)*a^2)/a^2)/(a^2*f)
```

maple [A] time = 2.17, size = 41, normalized size = 0.93

$$-\frac{\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,\left(5\left(\cos^2\left(fx+e\right)\right)-3\right)}{15a^2\cos\left(fx+e\right)^6 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15/a^2/cos(f*x+e)^6*(a*cos(f*x+e)^2)^(1/2)*(5*cos(f*x+e)^2-3)/f
```

maxima [A] time = 0.35, size = 48, normalized size = 1.09

$$\frac{5\left(a\sin\left(fx+e\right)^{2}-a\right)a^{2}+3\,a^{3}}{15\left(-a\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/15*(5*(a*sin(f*x + e)^2 - a)*a^2 + 3*a^3)/((-a*sin(f*x + e)^2 + a)^(5/2)*
a^2*f)
```

mupad [B] time = 20.21, size = 389, normalized size = 8.84

$$-\frac{16\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^{2}}}{3\,a^{2}\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{2}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}+\frac{272\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\sqrt{a-a\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)^{2}}}{15\,a^{2}\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)^{3}\,\left(\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}+\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\right)}-\frac{128\,\mathrm{e}^{e\,3\mathrm{i}+f\,x\,3\mathrm{i}}\,\dots}{5\,a^{2}\,f\,\left(\mathrm{e}^{e\,2\mathrm{i}+f}\dots\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (272*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f
*x*1i)*1i)/2)^2)^(1/2))/(15*a^2*f*(exp(e*2i + f*x*2i) + 1)^3*(exp(e*1i + f*
x*1i) + exp(e*3i + f*x*3i))) - (16*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i -
 f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i +
f*x*2i) + 1)^2*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (128*exp(e*3i +
 f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i) + 1)^4*(exp(e*1i + f*x*1i) + exp(e*3i
 + f*x*3i))) + (64*exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 -
 (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2))/(5*a^2*f*(exp(e*2i + f*x*2i) + 1)^5*(
exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.480 $\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=21

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

```
[Out] 1/3/f/(a*cos(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.07, antiderivative size = 21, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}} = 0.167$, Rules used = {3176, 3205, 16, 32}

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 32

```
Int[((a_.) + (b_.)*(x_))^(m_), x_Symbol] :> Simp[(a + b*x)^(m + 1)/(b*(m +
1)), x] /; FreeQ[{a, b, m}, x] && NeQ[m, -1]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\tan(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{x(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= -\frac{a\,\text{Subst}\left(\int \frac{1}{(ax)^{5/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Mathematica [A] time = 0.03, size = 21, normalized size = 1.00

$$\frac{1}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] 1/(3*f*(a*Cos[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.45, size = 28, normalized size = 1.33

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\,a^2f\cos\left(fx+e\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^4)
```

giac [B] time = 0.55, size = 64, normalized size = 3.05

$$\frac{3\sqrt{a\tan\left(fx+e\right)^2+a} + \frac{\left(a\tan\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-3\sqrt{a\tan\left(fx+e\right)^2+a}\,a}{a}}{3\,a^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*sqrt(a*tan(f*x + e)^2 + a) + ((a*tan(f*x + e)^2 + a)^(3/2) - 3*sqrt(
a*tan(f*x + e)^2 + a)*a)/a)/(a^2*f)
```

maple [A] time = 0.11, size = 21, normalized size = 1.00

$$\frac{1}{3f\left(a-a\left(\sin^2\left(fx+e\right)\right)\right)^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f/(a-a*sin(f*x+e)^2)^(3/2)
```

maxima [B] time = 0.44, size = 95, normalized size = 4.52

$$\frac{\frac{1}{\sqrt{-a\sin(fx+e)^2+a}\,a\sin(fx+e)+\sqrt{-a\sin(fx+e)^2+a}\,a}-\frac{1}{\sqrt{-a\sin(fx+e)^2+a}\,a\sin(fx+e)-\sqrt{-a\sin(fx+e)^2+a}\,a}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) + sqrt(-a*sin(f*x + e)^2
 + a)*a) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a*sin(f*x + e) - sqrt(-a*sin(f*x
+ e)^2 + a)*a))/f
```

mupad [B] time = 18.31, size = 72, normalized size = 3.43

$$\frac{16\,\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (16*exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*
x*1i)*1i)/2)^2)^(1/2))/(3*a^2*f*(exp(e*2i + f*x*2i) + 1)^4)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)
```

3.481 $\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=53

$$\frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a*cos(f*x+e)^2)^(
1/2)
```

Rubi [A] time = 0.09, antiderivative size = 53, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 24, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.208, Rules used = {3176, 3205, 51, 63, 206}

$$\frac{1}{af\sqrt{a\cos^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a*Cos[
e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
```

```
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}$$
$$= \frac{1}{af\sqrt{a\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af}$$
$$= \frac{1}{af\sqrt{a\cos^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{a^2f}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.07, size = 55, normalized size = 1.04

$$\frac{\cos(e+fx)\left(\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)+1}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (1 + Cos[e + f*x]*(-Log[Cos[(e + f*x)/2]] + Log[Sin[(e + f*x)/2]]))/(a*f*Sq
rt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.45, size = 58, normalized size = 1.09

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\cos\left(fx+e\right)\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)-2\right)}{2\,a^2f\cos\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(cos(f*x + e)*log(-(cos(f*x + e) + 1)/(cos(f*x
+ e) - 1)) - 2)/(a^2*f*cos(f*x + e)^2)
```

giac [A] time = 0.16, size = 59, normalized size = 1.11

$$\frac{\arctan\left(\frac{\sqrt{-a\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,af} + \frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(-a*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(-a*si
n(f*x + e)^2 + a)*a*f)
```

maple [A] time = 2.96, size = 75, normalized size = 1.42

$$\frac{-\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^2\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)a^2\left(\cos^2\left(fx+e\right)\right)+\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,a^{\frac{3}{2}}}{a^{\frac{7}{2}}\cos\left(fx+e\right)^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a^(7/2)/cos(f*x+e)^2*(-ln(2/sin(f*x+e)*(a^(1/2)*(a*cos(f*x+e)^2)^(1/2)+a)
)*a^2*cos(f*x+e)^2+(a*cos(f*x+e)^2)^(1/2)*a^(3/2))/f
```

maxima [A] time = 0.33, size = 73, normalized size = 1.38

$$-\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^2+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin\left(fx+e\right)^2+a}\,a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs(sin
(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot\left(e+f\,x\right)}{{\left(a-a\,{\sin\left(e+f\,x\right)}^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)/(a - a*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2), x)
```

3.482 $\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=66

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2a^2f}$$

```
[Out] -1/2*arctanh((a*cos(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*x+e)^2*(a*
cos(f*x+e)^2)^(1/2)/a^2/f
```

Rubi [A] time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.231, Rules used = {3176, 3205, 16, 51, 63, 206}

$$-\frac{\csc^2(e+fx)\sqrt{a\cos^2(e+fx)}}{2a^2f}-\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -ArcTanh[Sqrt[a*Cos[e + f*x]^2]/Sqrt[a]]/(2*a^(3/2)*f) - (Sqrt[a*Cos[e + f*
x]^2]*Csc[e + f*x]^2)/(2*a^2*f)
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3205

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_)]^(m_.
), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m + 1
)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(b*ff^(n/2)*x^(n/2))^p)/(1 - ff*x)^((m
 + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{b, e, f, p}, x] && Integ
erQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cot^3(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\cot^3(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\
&= -\frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(ax)^{3/2}}\,dx, x, \cos^2(e+fx)\right)}{2f}\\
&= -\frac{\text{Subst}\left(\int \frac{1}{(1-x)^2\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{2af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f}-\frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{ax}}\,dx, x, \cos^2(e+fx)\right)}{4af}\\
&= -\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f}-\frac{\text{Subst}\left(\int \frac{1}{1-\frac{x^2}{a}}\,dx, x, \sqrt{a\cos^2(e+fx)}\right)}{2a^2f}\\
&= -\frac{\tanh^{-1}\left(\frac{\sqrt{a\cos^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\sqrt{a\cos^2(e+fx)}\csc^2(e+fx)}{2a^2f}
\end{aligned}
$$

Mathematica [A] time = 0.17, size = 82, normalized size = 1.24

$$
-\frac{\cos^3(e+fx)\left(\csc^2\left(\frac{1}{2}(e+fx)\right)-\sec^2\left(\frac{1}{2}(e+fx)\right)-4\log\left(\sin\left(\frac{1}{2}(e+fx)\right)\right)+4\log\left(\cos\left(\frac{1}{2}(e+fx)\right)\right)\right)}{8f\left(a\cos^2(e+fx)\right)^{3/2}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/8*(Cos[e + f*x]^3*(Csc[(e + f*x)/2]^2 + 4*Log[Cos[(e + f*x)/2]] - 4*Log[
Sin[(e + f*x)/2]] - Sec[(e + f*x)/2]^2))/(f*(a*Cos[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.46, size = 83, normalized size = 1.26

$$
-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\left(\cos\left(fx+e\right)^2-1\right)\log\left(-\frac{\cos\left(fx+e\right)+1}{\cos\left(fx+e\right)-1}\right)-2\cos\left(fx+e\right)\right)}{4\left(a^2f\cos\left(fx+e\right)^3-a^2f\cos\left(fx+e\right)\right)}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/4*sqrt(a*cos(f*x + e)^2)*((cos(f*x + e)^2 - 1)*log(-(cos(f*x + e) + 1)/(
cos(f*x + e) - 1)) - 2*cos(f*x + e))/(a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x
+ e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (
4*pi/x/2)>(-4*pi/x/2)2/f*(1/16*(2*sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a))/a^
2/tan((f*x+exp(1))/2)^2/sign(tan((f*x+exp(1))/2)^4-1)-1/8*ln(tan((f*x+exp(1
))/2)^2)/sqrt(a)/a/sign(tan((f*x+exp(1))/2)^4-1)-1/16/sqrt(a)/a*tan((f*x+ex
p(1))/2)^2/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 1.60, size = 69, normalized size = 1.05

$$-\frac{\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}}{2fa^{2}\sin\left(fx+e\right)^{2}}-\frac{\ln\left(\frac{2\sqrt{a}\,\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}+2a}{\sin\left(fx+e\right)}\right)}{2fa^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/f/a^2/sin(f*x+e)^2*(a*cos(f*x+e)^2)^(1/2)-1/2/f/a^(3/2)*ln((2*a+2*a^(1
/2)*(a*cos(f*x+e)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.33, size = 100, normalized size = 1.52

$$-\frac{\frac{\log\left(\frac{2\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,\sqrt{a}}{\left|\sin\left(fx+e\right)\right|}+\frac{2a}{\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,a}+\frac{1}{\sqrt{-a\sin\left(fx+e\right)^{2}+a}\,a\sin\left(fx+e\right)^{2}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(log(2*sqrt(-a*sin(f*x + e)^2 + a)*sqrt(a)/abs(sin(f*x + e)) + 2*a/abs
(sin(f*x + e)))/a^(3/2) - 1/(sqrt(-a*sin(f*x + e)^2 + a)*a) + 1/(sqrt(-a*si
n(f*x + e)^2 + a)*a*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot\left(e+fx\right)^{3}}{\left(a-a\sin\left(e+fx\right)^{2}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a - a*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.483 $\int \frac{\tan^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=106

$$-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/8*arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-1/8*tan(f*x+
e)/a/f/(a*cos(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^2*tan(f*x+e)/a/f/(a*cos(f*x+e)
^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2611, 3768, 3770}

$$-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) - Tan[
e + f*x]/(8*a*f*Sqrt[a*Cos[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(4*
a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 2611

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Simp[(b*(a*Sec[e + f*x])^m*(b*Tan[e + f*x])^(n - 1))/(f*(
m + n - 1)), x] - Dist[(b^2*(n - 1))/(m + n - 1), Int[(a*Sec[e + f*x])^m*(b
*Tan[e + f*x])^(n - 2), x], x] /; FreeQ[{a, b, e, f, m}, x] && GtQ[n, 1] &&
 NeQ[m + n - 1, 0] && IntegersQ[2*m, 2*n]
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rule 3768

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> -Simp[(b*Cos[c + d*x
]*(b*Csc[c + d*x])^(n - 1))/(d*(n - 1)), x] + Dist[(b^2*(n - 2))/(n - 1), I
nt[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &&
IntegerQ[2*n]
```

Rule 3770

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> -Simp[ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\tan^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx &= \int \frac{\tan^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx\\ &= \frac{\cos(e+fx)\int \sec^3(e+fx)\tan^2(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}\\ &= \frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\int \sec^3(e+fx)\,dx}{4a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\int \sec(e+fx)\,dx}{8a\sqrt{a\cos^2(e+fx)}}\\ &= -\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}-\frac{\tan(e+fx)}{8af\sqrt{a\cos^2(e+fx)}}+\frac{\sec^2(e+fx)\tan(e+fx)}{4af\sqrt{a\cos^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.09, size = 59, normalized size = 0.56

$$\frac{\tan(e+fx)\left(2\sec^2(e+fx)-1\right)-\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{8af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(ArcTanh[Sin[e + f*x]]*Cos[e + f*x]) + (-1 + 2*Sec[e + f*x]^2)*Tan[e + f*
x])/(8*a*f*Sqrt[a*Cos[e + f*x]^2])
```

fricas [A] time = 0.47, size = 77, normalized size = 0.73

$$-\frac{\left(\cos\left(fx+e\right)^4\log\left(-\frac{\sin\left(fx+e\right)+1}{\sin\left(fx+e\right)-1}\right)+2\left(\cos\left(fx+e\right)^2-2\right)\sin\left(fx+e\right)\right)\sqrt{a\cos\left(fx+e\right)^2}}{16\,a^2f\cos\left(fx+e\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/16*(cos(f*x + e)^4*log(-(sin(f*x + e) + 1)/(sin(f*x + e) - 1)) + 2*(cos(
f*x + e)^2 - 2)*sin(f*x + e))*sqrt(a*cos(f*x + e)^2)/(a^2*f*cos(f*x + e)^5)
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)-2/f/32768*(4096*sqrt(a)*(tan((f*x+e
xp(1))/2)+1/tan((f*x+exp(1))/2))^3+16384*sqrt(a)*(tan((f*x+exp(1))/2)+1/tan
((f*x+exp(1))/2)))/a^2/((tan((f*x+exp(1))/2)+1/tan((f*x+exp(1))/2))^2-4)^2/
sign(tan((f*x+exp(1))/2)^4-1)
```

maple [A] time = 1.40, size = 104, normalized size = 0.98

$$-\frac{-2\left(\cos^2\left(fx+e\right)\right)\sin\left(fx+e\right)+4\sin\left(fx+e\right)+\left(\ln\left(\sin\left(fx+e\right)-1\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)\left(\cos^4\left(fx+\right.\right.}{16a\left(1+\sin\left(fx+e\right)\right)\left(\sin\left(fx+e\right)-1\right)\cos\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16/a*(-2*cos(f*x+e)^2*sin(f*x+e)+4*sin(f*x+e)+(ln(sin(f*x+e)-1)-ln(1+sin
(f*x+e)))*cos(f*x+e)^4)/(1+sin(f*x+e))/(sin(f*x+e)-1)/cos(f*x+e)/(a*cos(f*x
+e)^2)^(1/2)/f
```

maxima [B] time = 0.73, size = 1532, normalized size = 14.45

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*(4*(sin(7*f*x + 7*e) - 7*sin(5*f*x + 5*e) + 7*sin(3*f*x + 3*e) - sin(
f*x + e))*cos(8*f*x + 8*e) - 8*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2
*sin(2*f*x + 2*e))*cos(7*f*x + 7*e) - 16*(7*sin(5*f*x + 5*e) - 7*sin(3*f*x
+ 3*e) + sin(f*x + e))*cos(6*f*x + 6*e) + 56*(3*sin(4*f*x + 4*e) + 2*sin(2*
f*x + 2*e))*cos(5*f*x + 5*e) + 24*(7*sin(3*f*x + 3*e) - sin(f*x + e))*cos(4
*f*x + 4*e) + (2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2
*e) + 1)*cos(8*f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*
cos(2*f*x + 2*e) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(
2*f*x + 2*e) + 1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x +
 2*e)^2 + 4*(2*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*
sin(8*f*x + 8*e) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*
x + 2*e))*sin(6*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2
+ 48*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*
x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (
2*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*cos(8*
f*x + 8*e) + cos(8*f*x + 8*e)^2 + 8*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e
) + 1)*cos(6*f*x + 6*e) + 16*cos(6*f*x + 6*e)^2 + 12*(4*cos(2*f*x + 2*e) +
1)*cos(4*f*x + 4*e) + 36*cos(4*f*x + 4*e)^2 + 16*cos(2*f*x + 2*e)^2 + 4*(2*
sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e
) + sin(8*f*x + 8*e)^2 + 16*(3*sin(4*f*x + 4*e) + 2*sin(2*f*x + 2*e))*sin(6
*f*x + 6*e) + 16*sin(6*f*x + 6*e)^2 + 36*sin(4*f*x + 4*e)^2 + 48*sin(4*f*x
+ 4*e)*sin(2*f*x + 2*e) + 16*sin(2*f*x + 2*e)^2 + 8*cos(2*f*x + 2*e) + 1)*l
og(cos(f*x + e)^2 + sin(f*x + e)^2 - 2*sin(f*x + e) + 1) - 4*(cos(7*f*x + 7
*e) - 7*cos(5*f*x + 5*e) + 7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(8*f*x + 8
*e) + 4*(4*cos(6*f*x + 6*e) + 6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*
sin(7*f*x + 7*e) + 16*(7*cos(5*f*x + 5*e) - 7*cos(3*f*x + 3*e) + cos(f*x +
e))*sin(6*f*x + 6*e) - 28*(6*cos(4*f*x + 4*e) + 4*cos(2*f*x + 2*e) + 1)*sin
(5*f*x + 5*e) - 24*(7*cos(3*f*x + 3*e) - cos(f*x + e))*sin(4*f*x + 4*e) + 2
8*(4*cos(2*f*x + 2*e) + 1)*sin(3*f*x + 3*e) - 112*cos(3*f*x + 3*e)*sin(2*f*
x + 2*e) + 16*cos(f*x + e)*sin(2*f*x + 2*e) - 16*cos(2*f*x + 2*e)*sin(f*x +
 e) - 4*sin(f*x + e))/((a*cos(8*f*x + 8*e)^2 + 16*a*cos(6*f*x + 6*e)^2 + 36
*a*cos(4*f*x + 4*e)^2 + 16*a*cos(2*f*x + 2*e)^2 + a*sin(8*f*x + 8*e)^2 + 16
*a*sin(6*f*x + 6*e)^2 + 36*a*sin(4*f*x + 4*e)^2 + 48*a*sin(4*f*x + 4*e)*sin
(2*f*x + 2*e) + 16*a*sin(2*f*x + 2*e)^2 + 2*(4*a*cos(6*f*x + 6*e) + 6*a*cos
(4*f*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(8*f*x + 8*e) + 8*(6*a*cos(4*f
*x + 4*e) + 4*a*cos(2*f*x + 2*e) + a)*cos(6*f*x + 6*e) + 12*(4*a*cos(2*f*x
+ 2*e) + a)*cos(4*f*x + 4*e) + 8*a*cos(2*f*x + 2*e) + 4*(2*a*sin(6*f*x + 6*
e) + 3*a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) + 16*(3*
a*sin(4*f*x + 4*e) + 2*a*sin(2*f*x + 2*e))*sin(6*f*x + 6*e) + a)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{{\left(a-a\,\sin\left(e+f\,x\right)^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.484 $\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] arctanh(sin(f*x+e))*cos(f*x+e)/a/f/(a*cos(f*x+e)^2)^(1/2)-cot(f*x+e)/a/f/(a
*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.192, Rules used = {3176, 3207, 2621, 321, 207}

$$\frac{\cos(e+fx)\tanh^{-1}(\sin(e+fx))}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (ArcTanh[Sin[e + f*x]]*Cos[e + f*x])/(a*f*Sqrt[a*Cos[e + f*x]^2]) - Cot[e +
 f*x]/(a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 207

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTanh[(Rt[b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (LtQ[a
, 0] || GtQ[b, 0])
```

Rule 321

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(c^(
n - 1)*(c*x)^(m - n + 1)*(a + b*x^n)^(p + 1))/(b*(m + n*p + 1)), x] - Dist[
(a*c^n*(m - n + 1))/(b*(m + n*p + 1)), Int[(c*x)^(m - n)*(a + b*x^n)^p, x],
 x] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && GtQ[m, n - 1] && NeQ[m + n*p
 + 1, 0] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 2621

```
Int[(csc[(e_.) + (f_.)*(x_)]*(a_.))^(m_)*sec[(e_.) + (f_.)*(x_)]^(n_.), x_S
ymbol] :> -Dist[(f*a^n)^(-1), Subst[Int[x^(m + n - 1)/(-1 + x^2/a^2)^((n +
1)/2), x], x, a*Csc[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n
+ 1)/2] &&  !(IntegerQ[(m + 1)/2] && LtQ[0, m, n])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
```

```
FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx=\int \frac{\cot^2(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$=\frac{\cos(e+fx)\int \csc^2(e+fx)\sec(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$=-\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{x^2}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$=-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cos(e+fx)\,\text{Subst}\left(\int \frac{1}{-1+x^2}\,dx,x,\csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$=\frac{\tanh^{-1}(\sin(e+fx))\cos(e+fx)}{af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)}{af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [C] time = 0.07, size = 44, normalized size = 0.70

$$-\frac{\cot(e+fx)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\sin^2(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((Cot[e + f*x]*Hypergeometric2F1[-1/2, 1, 1/2, Sin[e + f*x]^2])/(a*f*Sqrt[
a*Cos[e + f*x]^2]))
```

fricas [A] time = 0.47, size = 66, normalized size = 1.05

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(\log\left(-\frac{\sin\left(fx+e\right)-1}{\sin\left(fx+e\right)+1}\right)\sin\left(fx+e\right)+2\right)}{2\,a^2f\cos\left(fx+e\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/2*sqrt(a*cos(f*x + e)^2)*(log(-(sin(f*x + e) - 1)/(sin(f*x + e) + 1))*si
n(f*x + e) + 2)/(a^2*f*cos(f*x + e)*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(1/4/sqrt(a)/a*tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)^4-1
)+1/4/sqrt(a)/a/tan((f*x+exp(1))/2)/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 1.36, size = 65, normalized size = 1.03

$$-\frac{\cos\left(fx+e\right)\left(2+\sin\left(fx+e\right)\left(\ln\left(\sin\left(fx+e\right)-1\right)-\ln\left(1+\sin\left(fx+e\right)\right)\right)\right)}{2a\sin\left(fx+e\right)\sqrt{a\left(\cos^{2}\left(fx+e\right)\right)}\, f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/2/a*cos(f*x+e)*(2+sin(f*x+e)*(ln(sin(f*x+e)-1)-ln(1+sin(f*x+e))))/sin(f*
x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.50, size = 220, normalized size = 3.49

$$\frac{\left(\cos\left(2\,fx+2\,e\right)^{2}+\sin\left(2\,fx+2\,e\right)^{2}-2\,\cos\left(2\,fx+2\,e\right)+1\right)\log\left(\cos\left(fx+e\right)^{2}+\sin\left(fx+e\right)^{2}+2\,\sin\left(fx\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*((cos(2*f*x + 2*e)^2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log
(cos(f*x + e)^2 + sin(f*x + e)^2 + 2*sin(f*x + e) + 1) - (cos(2*f*x + 2*e)^
2 + sin(2*f*x + 2*e)^2 - 2*cos(2*f*x + 2*e) + 1)*log(cos(f*x + e)^2 + sin(f
*x + e)^2 - 2*sin(f*x + e) + 1) - 4*cos(f*x + e)*sin(2*f*x + 2*e) + 4*cos(2
*f*x + 2*e)*sin(f*x + e) - 4*sin(f*x + e))/((a*cos(2*f*x + 2*e)^2 + a*sin(2
*f*x + 2*e)^2 - 2*a*cos(2*f*x + 2*e) + a)*sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot\left(e+f\,x\right)^2}{{\left(a-a\,\sin\left(e+f\,x\right)^2\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a - a*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.485 $\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=38

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.13, antiderivative size = 38, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 30}

$$-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 30

```
Int[(x_)^(m_.), x_Symbol] :> Simp[x^(m + 1)/(m + 1), x] /; FreeQ[m, x] && N
eQ[m, -1]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot^4(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\mathrm{Subst}\left(\int x^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.03, size = 29, normalized size = 0.76

$$-\frac{\cot^3(e+fx)}{3f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/3*Cot[e + f*x]^3/(f*(a*Cos[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.43, size = 50, normalized size = 1.32

$$\frac{\sqrt{a\cos\left(fx+e\right)^2}}{3\left(a^2f\cos\left(fx+e\right)^3 - a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*sqrt(a*cos(f*x + e)^2)/((a^2*f*cos(f*x + e)^3 - a^2*f*cos(f*x + e))*sin
(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(1/48*(3*sqrt(a)*tan((f*x+exp(1))/2)^2+sqrt(a))/a^2/tan((f*x+ex
p(1))/2)^3/sign(tan((f*x+exp(1))/2)^4-1)+1/4096*(256/3*sqrt(a)*a^4*tan((f*x
+exp(1))/2)^3+256*sqrt(a)*a^4*tan((f*x+exp(1))/2))/a^6/sign(tan((f*x+exp(1)
)/2)^4-1))
```

maple [A] time = 0.63, size = 35, normalized size = 0.92

$$-\frac{\cos\left(fx+e\right)}{3a\sin\left(fx+e\right)^3\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/a*cos(f*x+e)/sin(f*x+e)^3/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.49, size = 382, normalized size = 10.05

$$\frac{8\left(\cos\left(3\,fx+3\,e\right)\sin\left(6\,fx+6\right.\right.}{3\left(a^2\cos\left(6\,fx+6\,e\right)^2+9\,a^2\cos\left(4\,fx+4\,e\right)^2+9\,a^2\cos\left(2\,fx+2\,e\right)^2+a^2\sin\left(6\,fx+6\,e\right)^2+9\,a^2\sin\left(4\,fx+4\,e\right.\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/3*(cos(3*f*x + 3*e)*sin(6*f*x + 6*e) - 3*cos(3*f*x + 3*e)*sin(4*f*x + 4*e
) - (3*cos(2*f*x + 2*e) - 1)*sin(3*f*x + 3*e) - cos(6*f*x + 6*e)*sin(3*f*x
+ 3*e) + 3*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) + 3*cos(3*f*x + 3*e)*sin(2*f*x
 + 2*e))*sqrt(a)/((a^2*cos(6*f*x + 6*e)^2 + 9*a^2*cos(4*f*x + 4*e)^2 + 9*a^
2*cos(2*f*x + 2*e)^2 + a^2*sin(6*f*x + 6*e)^2 + 9*a^2*sin(4*f*x + 4*e)^2 -
18*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 9*a^2*sin(2*f*x + 2*e)^2 - 6*a^2
*cos(2*f*x + 2*e) + a^2 - 2*(3*a^2*cos(4*f*x + 4*e) - 3*a^2*cos(2*f*x + 2*e
) + a^2)*cos(6*f*x + 6*e) - 6*(3*a^2*cos(2*f*x + 2*e) - a^2)*cos(4*f*x + 4*
e) - 6*(a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

mupad [B] time = 18.69, size = 88, normalized size = 2.32

$$\frac{\mathrm{e}^{e\,4\mathrm{i}+f\,x\,4\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{\mathrm{e}^{-e\,1\mathrm{i}-f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}-\frac{\mathrm{e}^{e\,1\mathrm{i}+f\,x\,1\mathrm{i}}\,1\mathrm{i}}{2}\right)}^2}\,16\mathrm{i}}{3\,a^2\,f\,{\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}-1\right)}^3\,\left(\mathrm{e}^{e\,2\mathrm{i}+f\,x\,2\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (exp(e*4i + f*x*4i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^3*(exp(e*2i + f*x
*2i) + 1))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}{\left(e+fx\right)}}{\left(-a\left(\sin{\left(e+fx\right)}-1\right)\left(\sin{\left(e+fx\right)}+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.486 $\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=77

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] 1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)-1/5*cot(f*x+e)*csc(f
*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 14}

$$\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x
]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 14

```
Int[(u_)*((c_.)*(x_))^(m_.), x_Symbol] :> Int[ExpandIntegrand[(c*x)^m*u, x]
, x] /; FreeQ[{c, m}, x] && SumQ[u] && !LinearQ[u, x] && !MatchQ[u, (a_)
+ (b_.)*(v_) /; FreeQ[{a, b}, x] && InverseFunctionQ[v]]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 && !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] && !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^6(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot^6(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^3(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int x^2\left(-1+x^2\right)\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(-x^2+x^4\right)\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= \frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.09, size = 41, normalized size = 0.53

$$-\frac{\cot^3(e+fx)\left(3\csc^2(e+fx)-5\right)}{15f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^6/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/15*(Cot[e + f*x]^3*(-5 + 3*Csc[e + f*x]^2))/(f*(a*Cos[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.44, size = 75, normalized size = 0.97

$$-\frac{\sqrt{a\cos\left(fx+e\right)^2}\left(5\cos\left(fx+e\right)^2-2\right)}{15\left(a^2f\cos\left(fx+e\right)^5-2a^2f\cos\left(fx+e\right)^3+a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] -1/15*sqrt(a*cos(f*x + e)^2)*(5*cos(f*x + e)^2 - 2)/((a^2*f*cos(f*x + e)^5
- 2*a^2*f*cos(f*x + e)^3 + a^2*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(1/1073741824*(16777216/5*sqrt(a)*a^8*tan((f*x+exp(1))/2)^5-167
77216/3*sqrt(a)*a^8*tan((f*x+exp(1))/2)^3-33554432*sqrt(a)*a^8*tan((f*x+exp
(1))/2))/a^10/sign(tan((f*x+exp(1))/2)^4-1)+1/960*(-30*sqrt(a)*tan((f*x+exp
(1))/2)^4-5*sqrt(a)*tan((f*x+exp(1))/2)^2+3*sqrt(a))/a^2/tan((f*x+exp(1))/2
)^5/sign(tan((f*x+exp(1))/2)^4-1))
```

maple [A] time = 1.33, size = 67, normalized size = 0.87

$$-\frac{\cos\left(fx+e\right)\left(5\left(\cos^2\left(fx+e\right)\right)-2\right)}{15\left(\cos\left(fx+e\right)+1\right)^2\left(-1+\cos\left(fx+e\right)\right)^2 a\sin\left(fx+e\right)\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/15*cos(f*x+e)*(5*cos(f*x+e)^2-2)/(cos(f*x+e)+1)^2/(-1+cos(f*x+e))^2/a/si
n(f*x+e)/(a*cos(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.51, size = 1063, normalized size = 13.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^6/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 8/15*((5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e))*cos(10
*f*x + 10*e) - 5*(5*sin(7*f*x + 7*e) + 2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3
*e))*cos(8*f*x + 8*e) - 25*(2*sin(6*f*x + 6*e) - 2*sin(4*f*x + 4*e) + sin(2
*f*x + 2*e))*cos(7*f*x + 7*e) + 10*(2*sin(5*f*x + 5*e) + 5*sin(3*f*x + 3*e)
)*cos(6*f*x + 6*e) + 10*(2*sin(4*f*x + 4*e) - sin(2*f*x + 2*e))*cos(5*f*x +
 5*e) - (5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(
10*f*x + 10*e) + 5*(5*cos(7*f*x + 7*e) + 2*cos(5*f*x + 5*e) + 5*cos(3*f*x +
 3*e))*sin(8*f*x + 8*e) + 5*(10*cos(6*f*x + 6*e) - 10*cos(4*f*x + 4*e) + 5*
cos(2*f*x + 2*e) - 1)*sin(7*f*x + 7*e) - 10*(2*cos(5*f*x + 5*e) + 5*cos(3*f
*x + 3*e))*sin(6*f*x + 6*e) - 2*(10*cos(4*f*x + 4*e) - 5*cos(2*f*x + 2*e) +
 1)*sin(5*f*x + 5*e) + 50*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 5*(5*cos(2*f*
x + 2*e) - 1)*sin(3*f*x + 3*e) - 50*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 25*
cos(3*f*x + 3*e)*sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(10*f*x + 10*e)^2 + 25*
a^2*cos(8*f*x + 8*e)^2 + 100*a^2*cos(6*f*x + 6*e)^2 + 100*a^2*cos(4*f*x + 4
*e)^2 + 25*a^2*cos(2*f*x + 2*e)^2 + a^2*sin(10*f*x + 10*e)^2 + 25*a^2*sin(8
*f*x + 8*e)^2 + 100*a^2*sin(6*f*x + 6*e)^2 + 100*a^2*sin(4*f*x + 4*e)^2 - 1
00*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e) + 25*a^2*sin(2*f*x + 2*e)^2 - 10*a
^2*cos(2*f*x + 2*e) + a^2 - 2*(5*a^2*cos(8*f*x + 8*e) - 10*a^2*cos(6*f*x +
6*e) + 10*a^2*cos(4*f*x + 4*e) - 5*a^2*cos(2*f*x + 2*e) + a^2)*cos(10*f*x +
 10*e) - 10*(10*a^2*cos(6*f*x + 6*e) - 10*a^2*cos(4*f*x + 4*e) + 5*a^2*cos(
2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 20*(10*a^2*cos(4*f*x + 4*e) - 5*a^2*
cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 20*(5*a^2*cos(2*f*x + 2*e) - a^2
)*cos(4*f*x + 4*e) - 10*(a^2*sin(8*f*x + 8*e) - 2*a^2*sin(6*f*x + 6*e) + 2*
a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(10*f*x + 10*e) - 50*(2*a^2
*sin(6*f*x + 6*e) - 2*a^2*sin(4*f*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*
x + 8*e) - 100*(2*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(6*f*x +
6*e))*f)
```

mupad [B] time = 20.61, size = 393, normalized size = 5.10

$$-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}\,16{}\mathrm{i}}{3\,a^2\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}-1\right)}^2\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\,\sqrt{a-a\,{\left(\frac{{\mathrm{e}}^{-e\,1{}\mathrm{i}-f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}-\frac{{\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}\,1{}\mathrm{i}}{2}\right)}^2}\,272{}\mathrm{i}}{15\,a^2\,f\,{\left({\mathrm{e}}^{e\,2{}\mathrm{i}+f\,x\,2{}\mathrm{i}}-1\right)}^3\,\left({\mathrm{e}}^{e\,1{}\mathrm{i}+f\,x\,1{}\mathrm{i}}+{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}\right)}-\frac{{\mathrm{e}}^{e\,3{}\mathrm{i}+f\,x\,3{}\mathrm{i}}}{5\,a^2\,f\,\left(\mathrm{e}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^6/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x
*1i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i + f
*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f
*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*272i)/(15*a^2*f*(exp(e*2
i + f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) - (exp(e*3i +
 f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2
)^(1/2)*128i)/(5*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp
(e*3i + f*x*3i))) - (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2
```

```
 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*64i)/(5*a^2*f*(exp(e*2i + f*x*2i) -
1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{6}\left(e+fx\right)}{\left(-a\left(\sin\left(e+fx\right)-1\right)\left(\sin\left(e+fx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**6/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**6/(-a*(sin(e + f*x) - 1)*(sin(e + f*x) + 1))**(3/2),
 x)
```

3.487 $\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=115

$$-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

```
[Out] -1/3*cot(f*x+e)*csc(f*x+e)^2/a/f/(a*cos(f*x+e)^2)^(1/2)+2/5*cot(f*x+e)*csc(
f*x+e)^4/a/f/(a*cos(f*x+e)^2)^(1/2)-1/7*cot(f*x+e)*csc(f*x+e)^6/a/f/(a*cos(
f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.154, Rules used = {3176, 3207, 2606, 270}

$$-\frac{\cot(e+fx)\csc^6(e+fx)}{7af\sqrt{a\cos^2(e+fx)}}+\frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}}-\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*f*Sqrt[a*Cos[e + f*x]^2]) + (2*Cot[e +
f*x]*Csc[e + f*x]^4)/(5*a*f*Sqrt[a*Cos[e + f*x]^2]) - (Cot[e + f*x]*Csc[e +
 f*x]^6)/(7*a*f*Sqrt[a*Cos[e + f*x]^2])
```

Rule 270

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> Int[Exp
andIntegrand[(c*x)^m*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n}, x] &&
IGtQ[p, 0]
```

Rule 2606

```
Int[((a_.)*sec[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_.), x_Symbol] :> Dist[a/f, Subst[Int[(a*x)^(m - 1)*(-1 + x^2)^((n - 1)/2)
, x], x, Sec[e + f*x]], x] /; FreeQ[{a, e, f, m}, x] && IntegerQ[(n - 1)/2]
 &&  !(IntegerQ[m/2] && LtQ[0, m, n + 1])
```

Rule 3176

```
Int[(u_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> Int[A
ctivateTrig[u*(a*cos[e + f*x]^2)^p], x] /; FreeQ[{a, b, e, f, p}, x] && EqQ
[a + b, 0]
```

Rule 3207

```
Int[(u_.)*((b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_), x_Symbol] :> With[{ff
= FreeFactors[Sin[e + f*x], x]}, Dist[((b*ff^n)^IntPart[p]*(b*Sin[e + f*x]^
n)^FracPart[p])/(Sin[e + f*x]/ff)^(n*FracPart[p]), Int[ActivateTrig[u]*(Sin
[e + f*x]/ff)^(n*p), x], x]] /; FreeQ[{b, e, f, n, p}, x] &&  !IntegerQ[p]
&& IntegerQ[n] && (EqQ[u, 1] || MatchQ[u, ((d_.)*(trig_)[e + f*x])^(m_.) /;
 FreeQ[{d, m}, x] && MemberQ[{sin, cos, tan, cot, sec, csc}, trig]])
```

Rubi steps

$$\int \frac{\cot^8(e+fx)}{\left(a-a\sin^2(e+fx)\right)^{3/2}}\,dx = \int \frac{\cot^8(e+fx)}{\left(a\cos^2(e+fx)\right)^{3/2}}\,dx$$
$$= \frac{\cos(e+fx)\int \cot^5(e+fx)\csc^3(e+fx)\,dx}{a\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int x^2\left(-1+x^2\right)^2\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cos(e+fx)\,\text{Subst}\left(\int \left(x^2-2x^4+x^6\right)\,dx, x, \csc(e+fx)\right)}{af\sqrt{a\cos^2(e+fx)}}$$
$$= -\frac{\cot(e+fx)\csc^2(e+fx)}{3af\sqrt{a\cos^2(e+fx)}} + \frac{2\cot(e+fx)\csc^4(e+fx)}{5af\sqrt{a\cos^2(e+fx)}} - \frac{\cot(e+fx)\csc^6(e+fx}{7af\sqrt{a\cos^2(e+fx)}}$$

Mathematica [A] time = 0.13, size = 51, normalized size = 0.44

$$-\frac{\cot^3(e+fx)\left(15\csc^4(e+fx)-42\csc^2(e+fx)+35\right)}{105f\left(a\cos^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^8/(a - a*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -1/105*(Cot[e + f*x]^3*(35 - 42*Csc[e + f*x]^2 + 15*Csc[e + f*x]^4))/(f*(a*
Cos[e + f*x]^2)^(3/2))
```

fricas [A] time = 0.51, size = 100, normalized size = 0.87

$$\frac{\left(35\cos\left(fx+e\right)^4-28\cos\left(fx+e\right)^2+8\right)\sqrt{a\cos\left(fx+e\right)^2}}{105\left(a^2f\cos\left(fx+e\right)^7-3\,a^2f\cos\left(fx+e\right)^5+3\,a^2f\cos\left(fx+e\right)^3-a^2f\cos\left(fx+e\right)\right)\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] 1/105*(35*cos(f*x + e)^4 - 28*cos(f*x + e)^2 + 8)*sqrt(a*cos(f*x + e)^2)/((
a^2*f*cos(f*x + e)^7 - 3*a^2*f*cos(f*x + e)^5 + 3*a^2*f*cos(f*x + e)^3 - a^
2*f*cos(f*x + e))*sin(f*x + e))
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: NotImplementedError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: NotImplementedError >> Unable to parse Giac output: Unabl
e to check sign: (4*pi/x/2)>(-4*pi/x/2)Unable to check sign: (4*pi/x/2)>(-4
*pi/x/2)2/f*(1/26880*(525*sqrt(a)*tan((f*x+exp(1))/2)^6+35*sqrt(a)*tan((f*x
+exp(1))/2)^4-63*sqrt(a)*tan((f*x+exp(1))/2)^2+15*sqrt(a))/a^2/tan((f*x+exp
(1))/2)^7/sign(tan((f*x+exp(1))/2)^4-1)+1/72057594037927936*(28147497671065
6/7*sqrt(a)*a^12*tan((f*x+exp(1))/2)^7-844424930131968/5*sqrt(a)*a^12*tan((
f*x+exp(1))/2)^5+281474976710656/3*sqrt(a)*a^12*tan((f*x+exp(1))/2)^3+14073
```

```
74883553280*sqrt(a)*a^12*tan((f*x+exp(1))/2))/a^14/sign(tan((f*x+exp(1))/2)
^4-1))
```

maple [A] time = 1.12, size = 57, normalized size = 0.50

$$-\frac{\cos\left(fx+e\right)\left(35\left(\cos^4\left(fx+e\right)\right)-28\left(\cos^2\left(fx+e\right)\right)+8\right)}{105a\sin\left(fx+e\right)^7\sqrt{a\left(\cos^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/105/a*cos(f*x+e)*(35*cos(f*x+e)^4-28*cos(f*x+e)^2+8)/sin(f*x+e)^7/(a*cos
(f*x+e)^2)^(1/2)/f
```

maxima [B] time = 0.54, size = 2026, normalized size = 17.62

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^8/(a-a*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -8/105*((35*sin(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e)
 + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(14*f*x + 14*e) - 7*(35*si
n(11*f*x + 11*e) + 28*sin(9*f*x + 9*e) + 114*sin(7*f*x + 7*e) + 28*sin(5*f*
x + 5*e) + 35*sin(3*f*x + 3*e))*cos(12*f*x + 12*e) - 245*(3*sin(10*f*x + 10
*e) - 5*sin(8*f*x + 8*e) + 5*sin(6*f*x + 6*e) - 3*sin(4*f*x + 4*e) + sin(2*
f*x + 2*e))*cos(11*f*x + 11*e) + 21*(28*sin(9*f*x + 9*e) + 114*sin(7*f*x +
7*e) + 28*sin(5*f*x + 5*e) + 35*sin(3*f*x + 3*e))*cos(10*f*x + 10*e) + 196*
(5*sin(8*f*x + 8*e) - 5*sin(6*f*x + 6*e) + 3*sin(4*f*x + 4*e) - sin(2*f*x +
 2*e))*cos(9*f*x + 9*e) - 35*(114*sin(7*f*x + 7*e) + 28*sin(5*f*x + 5*e) +
35*sin(3*f*x + 3*e))*cos(8*f*x + 8*e) - 798*(5*sin(6*f*x + 6*e) - 3*sin(4*f
*x + 4*e) + sin(2*f*x + 2*e))*cos(7*f*x + 7*e) + 245*(4*sin(5*f*x + 5*e) +
5*sin(3*f*x + 3*e))*cos(6*f*x + 6*e) + 196*(3*sin(4*f*x + 4*e) - sin(2*f*x
+ 2*e))*cos(5*f*x + 5*e) - (35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 1
14*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(14*f*x
 + 14*e) + 7*(35*cos(11*f*x + 11*e) + 28*cos(9*f*x + 9*e) + 114*cos(7*f*x +
 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(12*f*x + 12*e) + 35*
(21*cos(10*f*x + 10*e) - 35*cos(8*f*x + 8*e) + 35*cos(6*f*x + 6*e) - 21*cos
(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*sin(11*f*x + 11*e) - 21*(28*cos(9*f
*x + 9*e) + 114*cos(7*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e
))*sin(10*f*x + 10*e) - 28*(35*cos(8*f*x + 8*e) - 35*cos(6*f*x + 6*e) + 21*
cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(9*f*x + 9*e) + 35*(114*cos(7
*f*x + 7*e) + 28*cos(5*f*x + 5*e) + 35*cos(3*f*x + 3*e))*sin(8*f*x + 8*e) +
 114*(35*cos(6*f*x + 6*e) - 21*cos(4*f*x + 4*e) + 7*cos(2*f*x + 2*e) - 1)*s
in(7*f*x + 7*e) - 245*(4*cos(5*f*x + 5*e) + 5*cos(3*f*x + 3*e))*sin(6*f*x +
 6*e) - 28*(21*cos(4*f*x + 4*e) - 7*cos(2*f*x + 2*e) + 1)*sin(5*f*x + 5*e)
+ 735*cos(3*f*x + 3*e)*sin(4*f*x + 4*e) + 35*(7*cos(2*f*x + 2*e) - 1)*sin(3
*f*x + 3*e) - 735*cos(4*f*x + 4*e)*sin(3*f*x + 3*e) - 245*cos(3*f*x + 3*e)*
sin(2*f*x + 2*e))*sqrt(a)/((a^2*cos(14*f*x + 14*e)^2 + 49*a^2*cos(12*f*x +
12*e)^2 + 441*a^2*cos(10*f*x + 10*e)^2 + 1225*a^2*cos(8*f*x + 8*e)^2 + 1225
*a^2*cos(6*f*x + 6*e)^2 + 441*a^2*cos(4*f*x + 4*e)^2 + 49*a^2*cos(2*f*x + 2
*e)^2 + a^2*sin(14*f*x + 14*e)^2 + 49*a^2*sin(12*f*x + 12*e)^2 + 441*a^2*si
n(10*f*x + 10*e)^2 + 1225*a^2*sin(8*f*x + 8*e)^2 + 1225*a^2*sin(6*f*x + 6*e
)^2 + 441*a^2*sin(4*f*x + 4*e)^2 - 294*a^2*sin(4*f*x + 4*e)*sin(2*f*x + 2*e
) + 49*a^2*sin(2*f*x + 2*e)^2 - 14*a^2*cos(2*f*x + 2*e) + a^2 - 2*(7*a^2*co
s(12*f*x + 12*e) - 21*a^2*cos(10*f*x + 10*e) + 35*a^2*cos(8*f*x + 8*e) - 35
*a^2*cos(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) +
a^2)*cos(14*f*x + 14*e) - 14*(21*a^2*cos(10*f*x + 10*e) - 35*a^2*cos(8*f*x
```

```
+ 8*e) + 35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) + 7*a^2*cos(2*f*
x + 2*e) - a^2)*cos(12*f*x + 12*e) - 42*(35*a^2*cos(8*f*x + 8*e) - 35*a^2*c
os(6*f*x + 6*e) + 21*a^2*cos(4*f*x + 4*e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*c
os(10*f*x + 10*e) - 70*(35*a^2*cos(6*f*x + 6*e) - 21*a^2*cos(4*f*x + 4*e) +
 7*a^2*cos(2*f*x + 2*e) - a^2)*cos(8*f*x + 8*e) - 70*(21*a^2*cos(4*f*x + 4*
e) - 7*a^2*cos(2*f*x + 2*e) + a^2)*cos(6*f*x + 6*e) - 42*(7*a^2*cos(2*f*x +
 2*e) - a^2)*cos(4*f*x + 4*e) - 14*(a^2*sin(12*f*x + 12*e) - 3*a^2*sin(10*f
*x + 10*e) + 5*a^2*sin(8*f*x + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*
f*x + 4*e) - a^2*sin(2*f*x + 2*e))*sin(14*f*x + 14*e) - 98*(3*a^2*sin(10*f*
x + 10*e) - 5*a^2*sin(8*f*x + 8*e) + 5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f
*x + 4*e) + a^2*sin(2*f*x + 2*e))*sin(12*f*x + 12*e) - 294*(5*a^2*sin(8*f*x
 + 8*e) - 5*a^2*sin(6*f*x + 6*e) + 3*a^2*sin(4*f*x + 4*e) - a^2*sin(2*f*x +
 2*e))*sin(10*f*x + 10*e) - 490*(5*a^2*sin(6*f*x + 6*e) - 3*a^2*sin(4*f*x +
 4*e) + a^2*sin(2*f*x + 2*e))*sin(8*f*x + 8*e) - 490*(3*a^2*sin(4*f*x + 4*e
) - a^2*sin(2*f*x + 2*e))*sin(6*f*x + 6*e))*f)
```

mupad [B] time = 33.01, size = 589, normalized size = 5.12

$$\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a-a\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,16i}{3\,a^2\,f\,{\left(e^{e\,2i+f\,x\,2i}-1\right)}^2\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a-a\,{\left(\frac{e^{-e\,1i-f\,x\,1i}\,1i}{2}-\frac{e^{e\,1i+f\,x\,1i}\,1i}{2}\right)}^2}\,464i}{15\,a^2\,f\,{\left(e^{e\,2i+f\,x\,2i}-1\right)}^3\,\left(e^{e\,1i+f\,x\,1i}+e^{e\,3i+f\,x\,3i}\right)}+\frac{e^{e\,3i+f\,x\,3i}\,\sqrt{a}}{35\,a^2\,f\,\left(e^{e\,2i}\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^8/(a - a*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1
i)*1i)/2)^2)^(1/2)*16i)/(3*a^2*f*(exp(e*2i + f*x*2i) - 1)^2*(exp(e*1i + f*x
*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x
*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*464i)/(15*a^2*f*(exp(e*2i
+ f*x*2i) - 1)^3*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f
*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^
(1/2)*3072i)/(35*a^2*f*(exp(e*2i + f*x*2i) - 1)^4*(exp(e*1i + f*x*1i) + exp
(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2
 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*4736i)/(35*a^2*f*(exp(e*2i + f*x*2i)
 - 1)^5*(exp(e*1i + f*x*1i) + exp(e*3i + f*x*3i))) + (exp(e*3i + f*x*3i)*(a
 - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1i + f*x*1i)*1i)/2)^2)^(1/2)*768
i)/(7*a^2*f*(exp(e*2i + f*x*2i) - 1)^6*(exp(e*1i + f*x*1i) + exp(e*3i + f*x
*3i))) + (exp(e*3i + f*x*3i)*(a - a*((exp(- e*1i - f*x*1i)*1i)/2 - (exp(e*1
i + f*x*1i)*1i)/2)^2)^(1/2)*256i)/(7*a^2*f*(exp(e*2i + f*x*2i) - 1)^7*(exp(
e*1i + f*x*1i) + exp(e*3i + f*x*3i)))
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**8/(a-a*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.488 $\int \sqrt{a+b\sin^2(e+fx)}\tan^5(e+fx)\,dx$

Optimal. Leaf size=177

$$-\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}+\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}+\frac{\sec^4(e+fx)\,(a+}{4f(a}$$

```
[Out] 1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+
b)^(3/2)/f-1/8*(8*a+7*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f+1/
4*sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/8*(8*a^2+24*a*b+15*b^2)*(
a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f
```

Rubi [A] time = 0.21, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$-\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)^2}+\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{3/2}}+\frac{\sec^4(e+fx)\,(a+}{4f(a}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] ((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*(a + b)^(3/2)*f) - ((8*a^2 + 24*a*b + 15*b^2)*Sqrt[a + b*Sin[e + f*x]^2
])/(8*(a + b)^2*f) - ((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/
2))/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*(a +
 b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^5(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x^2\sqrt{a+bx}}{(1-x)^3}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}\left(\frac{1}{2}(4a+3b)+2(a+b)x\right)}{(1-x)^2}\right.}{4(a+b)f}$$

$$= -\frac{(8a+7b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\left(a+b\,\text{s}\right.}{4(a+b)}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+fx)\left(a\right.}{8(a+b}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} - \frac{(8a+7b)\sec^2(e+fx)\left(a\right.}{8(a+b}$$

$$= \frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{3/2}f} - \frac{\left(8a^2+24ab+15b^2\right)\sqrt{}}{8(a+b}$$

Mathematica [A] time = 0.56, size = 143, normalized size = 0.81

$$\frac{\left(8a^2+24ab+15b^2\right)\left(\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)-\sqrt{a+b\sin^2(e+fx)}\right)+2(a+b)\sec^4(e+fx)\left(a+b\sin^2\right.}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^5,x]
```

```
[Out] (-((8*a + 7*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2)) + 2*(a + b)*Sec
[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2) + (8*a^2 + 24*a*b + 15*b^2)*(Sqrt[
a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a + b*Sin[e +
 f*x]^2]))/(8*(a + b)^2*f)
```

fricas [A] time = 1.36, size = 354, normalized size = 2.00

$$\left[\frac{\left(8\,a^2+24\,ab+15\,b^2\right)\sqrt{a+b}\,\cos\left(fx+e\right)^4\log\left(\frac{b\cos\left(fx+e\right)^2-2\,\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(8\left(a^2+2\,ab\right.\right.}{16\left(a^2+2\,ab+b^2\right)f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 24*a*b + 15*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x
+ e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x
 + e)^2) - 2*(8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b^
2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))
/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4), -1/8*((8*a^2 + 24*a*b + 15*b^2)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos
(f*x + e)^4 + (8*(a^2 + 2*a*b + b^2)*cos(f*x + e)^4 + (8*a^2 + 17*a*b + 9*b
^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^4)]
```

giac [B] time = 3.63, size = 2790, normalized size = 15.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/4*((8*a^2 + 24*a*b + 15*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 16*(
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b) - 2*(8*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))^7*a*b + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
^6*a^(5/2) - 80*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3
/2)*b - 17*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b
^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 464*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 425*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
```

```
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 60*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) + 144*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^4*a^(5/2)*b - 425*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^4*a^(3/2)*b^2 - 468*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))^3*a^4 + 1520*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)^3*a^3*b + 2093*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2
*b^2 + 712*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 2
40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 592*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 2165*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 2808*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 + 1232*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 1072*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a))*a^4*b - 1675*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*a^3*b^2 - 652*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*a^2*b^3 + 624*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
4 + 448*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 - 104*a^(1
1/2) - 656*a^(9/2)*b - 1723*a^(7/2)*b^2 - 2340*a^(5/2)*b^3 - 1616*a^(3/2)*b
^4 - 448*sqrt(a)*b^5)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
)^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a
- 4*b)^4*(a + b)))/f
```

maple [B] time = 5.29, size = 721, normalized size = 4.07

$$\left(-16\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\ (a+b)^{\frac{3}{2}}\,a^2-48\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\ (a+b)^{\frac{3}{2}}\,ab-30b^2\sqrt{a+b-b\left(\cos^2\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/16*((-16*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)*a^2-48*(a+b-b*cos(f*x+e)^
2)^(1/2)*(a+b)^(3/2)*a*b-30*b^2*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(3/2)+8*ln
```

```
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a^4+40*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*a^3*b+71*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a^2*b^2+54*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+15*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4+8*ln(2/(1+sin(f*x+e))*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4+40*ln(2/(1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b+71*ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2
*b^2+54*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f
*x+e)+a))*a*b^3+15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)-b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(
3/2)*(8*a+7*b)*cos(f*x+e)^2+4*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2)*a+4*b*
(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(3/2))/(a+b)^(3/2)/cos(f*x+e)^4/(a^2+2*a*b
+b^2)/f
```

maxima [A] time = 0.44, size = 230, normalized size = 1.30

$$-\frac{16\sqrt{b\sin\left(fx+e\right)^2+a}\,b^3+\frac{\left(8a^2b^3+24ab^4+15b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}-\frac{2\left(\left(8ab^4+9b^5\right)\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-\left(8a^2b^4+15a\right.\right.}{\left(b\sin\left(fx+e\right)^2+a\right)^2(a+b)+a^3+3a^2b+3ab^2+b^3-2\left(\right.}}{16b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/16*(16*sqrt(b*sin(f*x + e)^2 + a)*b^3 + (8*a^2*b^3 + 24*a*b^4 + 15*b^5)*
log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a)
+ sqrt(a + b)))/(a + b)^(3/2) - 2*((8*a*b^4 + 9*b^5)*(b*sin(f*x + e)^2 + a)
^(3/2) - (8*a^2*b^4 + 15*a*b^5 + 7*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b*sin
(f*x + e)^2 + a)^2*(a + b) + a^3 + 3*a^2*b + 3*a*b^2 + b^3 - 2*(b*sin(f*x +
 e)^2 + a)*(a^2 + 2*a*b + b^2)))/(b^3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^5\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\tan^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**5,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**5, x)
```

3.489 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)\,dx$

Optimal. Leaf size=118

$$\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f(a+b)}$$

```
[Out] 1/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f-1/2*(2*a+3*b)*arctanh((a+
b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)+1/2*(2*a+3*b)*(a+b*sin(f*x
+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.11, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f\sqrt{a+b}}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] -((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*Sqrt[a +
b]*f) + ((2*a + 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f) + (Sec[e + f
*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(2*(a + b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{x\sqrt{a+bx}}{(1-x)^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2(a+b)f} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx,\right.}{4(a+b)f}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/}}{2(a+b)f}$$

$$= \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} + \frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/}}{2(a+b)f}$$

$$= -\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2\sqrt{a+b}\,f} + \frac{(2a+3b)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}$$

Mathematica [A] time = 0.39, size = 84, normalized size = 0.71

$$\frac{(\cos(2(e+fx))+2)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)} - \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3,x]
```

```
[Out] (-(((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/Sqrt[a + b
]) + (2 + Cos[2*(e + f*x)])*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*f
)
```

fricas [A] time = 0.86, size = 234, normalized size = 1.98

$$\left[\frac{(2a+3b)\sqrt{a+b}\,\cos\left(fx+e\right)^2\log\left(\frac{b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\left(2(a+b)\cos\left(fx+e\right)^2+\right.}{4(a+b)f\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + 3*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(2
```

```
*(a + b)*cos(f*x + e)^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*
f*cos(f*x + e)^2), 1/2*((2*a + 3*b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2 + (2*(a + b)*cos(f*x + e)
^2 + a + b)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^2)]
```

giac [B] time = 1.04, size = 1011, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] ((2*a + 3*b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 4*((sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
*sqrt(a) + a + 4*b) - 2*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/
2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
+ a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/((sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2)/f
```

maple [B] time = 4.80, size = 403, normalized size = 3.42

$$-\frac{\left(-4\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\sqrt{a+b}\,a-6b\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}\,\sqrt{a+b}+2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right.}}{\sin\left(fx+e\right)\cdots}\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/4*(-(-4*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)*a-6*b*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(1/2)+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)+b*sin(f*x+e)+a))*a^2+5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+5*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+3*ln(2/(1+sin(f*
x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x
+e)^2+2*(a+b-b*cos(f*x+e)^2)^(3/2)*(a+b)^(1/2))/(a+b)^(3/2)/cos(f*x+e)^2/f
```

maxima [A] time = 0.44, size = 127, normalized size = 1.08

$$\frac{4\sqrt{b\sin\left(fx+e\right)^{2}+a}\,b^{2}-\frac{2\sqrt{b\sin\left(fx+e\right)^{2}+a}\,b^{3}}{b\sin\left(fx+e\right)^{2}-b}+\frac{\left(2\,ab^{2}+3\,b^{3}\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}}{4\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/4*(4*sqrt(b*sin(f*x + e)^2 + a)*b^2 - 2*sqrt(b*sin(f*x + e)^2 + a)*b^3/(b
*sin(f*x + e)^2 - b) + (2*a*b^2 + 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a) -
sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a + b))/(b^2*
f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^{3}\,\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{2}\left(e+fx\right)}\,\tan^{3}\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**3,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**3, x)
```

3.490 $\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx$

Optimal. Leaf size=58

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f-(a+b*sin(f*x+e)
^2)^(1/2)/f
```

Rubi **[A]** time = 0.06, antiderivative size = 58, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}-\frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - Sqrt[a +
b*Sin[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+f$$
$$= -\frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(}$$
$$= \frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Mathematica [A] time = 0.06, size = 60, normalized size = 1.03

$$\frac{\sqrt{a+b}\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right) - \sqrt{a-b\cos^2(e+fx)+b}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x],x]
```

```
[Out] (Sqrt[a + b]*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] - Sqrt[a +
 b - b*Cos[e + f*x]^2])/f
```

fricas [A] time = 0.67, size = 145, normalized size = 2.50

$$\left[\frac{\sqrt{a+b}\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2f}, -\frac{\sqrt{-a-b}\arctan\left(\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*sqrt(-b*cos(f*x + e)^2 + a +
b))/f, -(sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(
a + b)) + sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

giac [B] time = 0.32, size = 329, normalized size = 5.67

$$-\frac{2\left(\frac{(a+b)\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{\sqrt{-a-b}} - \frac{}{\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)}\right.}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] -2*((a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*
x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
 - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))*b - sqrt(a)*b)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*
sqrt(a) + a + 4*b))/f
```

maple [B] time = 4.14, size = 134, normalized size = 2.31

$$\frac{\sqrt{a+b}\;\ln\left(\frac{2\sqrt{a+b}\;\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)}{2f}+\frac{\sqrt{a+b}\;\ln\left(\frac{2\sqrt{a+b}\;\sqrt{a+b-b\left(\cos^{2}\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)}{2f}-\frac{\sqrt{a+b}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x)
```

```
[Out] 1/2/f*(a+b)^(1/2)*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))+1/2/f*(a+b)^(1/2)*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))-1/f*(a+b-b*cos(f*x+e)^2)^(1/2)
```

maxima [B] time = 0.44, size = 122, normalized size = 2.10

$$-\frac{\sqrt{a+b}\;\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)-\sqrt{a+b}\;\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)+2\sqrt{}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/2*(sqrt(a + b)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a
/(sqrt(a*b)*(sin(f*x + e) + 1))) - sqrt(a + b)*arcsinh(-b*sin(f*x + e)/(sqr
t(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) + 2*sqrt(b*s
in(f*x + e)^2 + a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \tan\left(e+f\,x\right)\,\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{2}\left(e+fx\right)}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x), x)
```

3.491 $\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=54

$$\frac{\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^(1/2)/f+(a+b*sin(f*x+e)^2)^(1/
2)/f
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$\frac{\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + Sqrt[a + b*Sin
[e + f*x]^2]/f
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= -\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{\sqrt{a+b\sin^2(e+fx)}}{f}$$

Mathematica [A] time = 0.05, size = 53, normalized size = 0.98

$$-\frac{\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a+b\sin^2(e+fx)}}{f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a + b*Sin[e +
 f*x]^2])/f)
```

fricas [A] time = 1.13, size = 135, normalized size = 2.50

$$\left[\frac{\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{2f}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+}}{a}\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/2*(sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
)/f, (sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + sqrt(-b
*cos(f*x + e)^2 + a + b))/f]
```

giac [A] time = 0.13, size = 49, normalized size = 0.91

$$\frac{\frac{a\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] (a*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(f*x +
e)^2 + a))/f
```

maple [A] time = 1.46, size = 61, normalized size = 1.13

$$-\frac{\sqrt{a}\ \ln\left(\frac{2a+2\sqrt{a}\ \sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}+\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+(a+b*s
in(f*x+e)^2)^(1/2)/f
```

maxima [A] time = 0.34, size = 43, normalized size = 0.80

$$-\frac{\sqrt{a}\ \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-\sqrt{b\sin\left(fx+e\right)^2+a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -(sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - sqrt(b*sin(f*x + e)^2
+ a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(e+f\,x\right)\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\cot\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x), x)
```

3.492 $\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=110

$$-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}+\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

```
[Out] -1/2*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a/f+1/2*(2*a-b)*arctanh((a+b*sin
(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)-1/2*(2*a-b)*(a+b*sin(f*x+e)^2)^(1/2)/a/
f
```

Rubi [A] time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3194, 78, 50, 63, 208}

$$-\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af}+\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,f}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*Sqrt[a]*f) - ((2
*a - b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e
+ f*x]^2)^(3/2))/(2*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)\sqrt{a+bx}}{x^2}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, \right.}{4af}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

$$= -\frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{2af}$$

$$= \frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2\sqrt{a}f} - \frac{(2a-b)\sqrt{a+b\sin^2(e+fx)}}{2af} -$$

Mathematica [A] time = 0.20, size = 77, normalized size = 0.70

$$\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right) - \sqrt{a}\left(\csc^2(e+fx)+2\right)\sqrt{a+b\sin^2(e+fx)}}{2\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] - Sqrt[a]*(2 + Csc[e
 + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])/(2*Sqrt[a]*f)
```

fricas [A] time = 1.50, size = 239, normalized size = 2.17

$$\left[-\frac{\left((2a-b)\cos\left(fx+e\right)^2-2a+b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\left(2a\cos\left(fx+e\right)^2\right.}{4\left(af\cos\left(fx+e\right)^2-af\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1
```

```
)) + 2*(2*a*cos(f*x + e)^2 - 3*a)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos
(f*x + e)^2 - a*f), -1/2*(((2*a - b)*cos(f*x + e)^2 - 2*a + b)*sqrt(-a)*arc
tan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) + (2*a*cos(f*x + e)^2 - 3*a
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a*f*cos(f*x + e)^2 - a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to divide, perhaps due to rounding error%%%{4096,[8,
8]%%%}+%%%{%%%{16384,[1]%%%},[8,7]%%%}+%%%{%%%{24576,[2]%%%},[8,6]%%%}+%%%{
%%%{16384,[3]%%%},[8,5]%%%}+%%%{%%%{4096,[4]%%%},[8,4]%%%}+%%%{%%{[-16384,0
]:[1,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[%%%{-65536,[1]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[7,7]%%%}+%%%{%%{[%%%{-98304,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[7,6]%%%}+%%%{%%{[%%%{-65536,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,5]%%
%}+%%%{%%{[%%%{-16384,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,4]%%%}+%%%{3276
8,[6,9]%%%}+%%%{%%%{147456,[1]%%%},[6,8]%%%}+%%%{%%%{262144,[2]%%%},[6,7]%%
%}+%%%{%%%{229376,[3]%%%},[6,6]%%%}+%%%{%%%{98304,[4]%%%},[6,5]%%%}+%%%{%%%
{16384,[5]%%%},[6,4]%%%}+%%%{%%{[-65536,0]:[1,0,%%%{-1,[1]%%%}]%%},[5,9]%%%
}+%%%{%%{[%%%{-245760,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+%%%{%%{[
%%%{-327680,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,7]%%%}+%%%{%%{[%%%{-16384
0,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,6]%%%}+%%%{%%{[%%%{16384,[5]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[5,4]%%%}+%%%{65536,[4,10]%%%}+%%%{%%%{229376,[1]%
%%},[4,9]%%%}+%%%{%%%{221184,[2]%%%},[4,8]%%%}+%%%{%%%{-98304,[3]%%%},[4,7]
%%%}+%%%{%%%{-311296,[4]%%%},[4,6]%%%}+%%%{%%%{-196608,[5]%%%},[4,5]%%%}+%%
%{%%%{-40960,[6]%%%},[4,4]%%%}+%%%{%%{[%%%{131072,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[3,9]%%%}+%%%{%%{[%%%{540672,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
3,8]%%%}+%%%{%%{[%%%{851968,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,7]%%%}+%%
%{%%{[%%%{622592,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%%}+%%%{%%{[%%%{1
96608,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,5]%%%}+%%%{%%{[%%%{16384,[6]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,4]%%%}+%%%{%%%{-131072,[1]%%%},[2,10]%%%}+%
%%{%%%{-557056,[2]%%%},[2,9]%%%}+%%%{%%%{-901120,[3]%%%},[2,8]%%%}+%%%{%%%{
-655360,[4]%%%},[2,7]%%%}+%%%{%%%{-163840,[5]%%%},[2,6]%%%}+%%%{%%%{32768,[
6]%%%},[2,5]%%%}+%%%{%%%{16384,[7]%%%},[2,4]%%%}+%%%{%%{[%%%{-65536,[2]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,9]%%%}+%%%{%%{[%%%{-278528,[3]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[1,8]%%%}+%%%{%%{[%%%{-458752,[4]%%%},0]:[1,0,%%%{-1,[1]%
%%}]%%},[1,7]%%%}+%%%{%%{[%%%{-360448,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
,6]%%%}+%%%{%%{[%%%{-131072,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,5]%%%}+%%
%{%%{[%%%{-16384,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,4]%%%}+%%%{%%%{65536
,[2]%%%},[0,10]%%%}+%%%{%%%{294912,[3]%%%},[0,9]%%%}+%%%{%%%{528384,[4]%%%}
,[0,8]%%%}+%%%{%%%{475136,[5]%%%},[0,7]%%%}+%%%{%%%{221184,[6]%%%},[0,6]%%%
}+%%%{%%%{49152,[7]%%%},[0,5]%%%}+%%%{%%%{4096,[8]%%%},[0,4]%%%} / %%%{%%%{
1,[1]%%%},[8,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[7,0]%%%}+%%%{%%%{8,[1]%%%},[6,1]%%%}+%%%{%%%{4,[2]%%%},[6,0]%%%}+%%%{%%{po
ly1[%%%{-16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1]%%%}+%%%{%%{poly1[%%%{4
,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,0]%%%}+%%%{%%%{16,[1]%%%},[4,2]%%%}+
%%%{%%%{-8,[2]%%%},[4,1]%%%}+%%%{%%%{-10,[3]%%%},[4,0]%%%}+%%%{%%{poly1[%%%
{32,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,1]%%%}+%%%{%%{poly1[%%%{4,[3]%%%}
,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0]%%%}+%%%{%%%{-32,[2]%%%},[2,2]%%%}+%%%{%%%
```

```
{-8,[3]%%%},[2,1]%%%}+%%%{%%%{4,[4]%%%},[2,0]%%%}+%%%{%%{poly1[%%%{-16,[3]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%{poly1[%%%{-4,[4]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[1,0]%%%}+%%%{%%%{16,[3]%%%},[0,2]%%%}+%%%{%%%{8,[4]%%
%},[0,1]%%%}+%%%{%%%{1,[5]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 1.54, size = 130, normalized size = 1.18

$$-\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{f}+\frac{\sqrt{a}\,\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)b}{2f\sqrt{a}}-\frac{\sqrt{a+b\left(\sin^2\left(fx+\right.\right.}}{2f\sin\left(fx+e\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -(a+b*sin(f*x+e)^2)^(1/2)/f+1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e))-1/2/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/
2))/sin(f*x+e))*b-1/2/f/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.35, size = 113, normalized size = 1.03

$$\frac{2\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-\frac{b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}-2\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{\sqrt{b\sin\left(fx+e\right)^2+a}\,b}{a}-\frac{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}{a\sin\left(fx+e\right)^2}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - b*arcsinh(a/(sqrt
(a*b)*abs(sin(f*x + e))))/sqrt(a) - 2*sqrt(b*sin(f*x + e)^2 + a) + sqrt(b*s
in(f*x + e)^2 + a)*b/a - (b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^3\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\cot^3\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**3, x)
```

3.493 $\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=165

$$\frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}+\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f}-\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\sqrt{\cdot}\right.}{8a^{3/2}f}$$

```
[Out] -1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+
1/8*(8*a+b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f-1/4*csc(f*x+e)^4*(a
+b*sin(f*x+e)^2)^(3/2)/a/f+1/8*(8*a^2-8*a*b-b^2)*(a+b*sin(f*x+e)^2)^(1/2)/a
^2/f
```

Rubi [A] time = 0.15, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f}+\frac{(8a+b)\csc^2(e+fx)\left(a+b\sin\right.}{8a^2f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a^(
3/2)*f) + ((8*a^2 - 8*a*b - b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*f) + ((
8*a + b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2))/(8*a^2*f) - (Csc[e +
f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2))/(4*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2\sqrt{a+bx}}{x^3}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{4af} + \frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-8a-b)+2ax\right)\sqrt{a+bx}}{x^2}\right.}{4af}$$

$$= \frac{(8a+b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{8a^2f} - \frac{\csc^4(e+fx)\left(a+b\sin\right.}{4af}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\left(a+b\right.}{8a^2f}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} + \frac{(8a+b)\csc^2(e+fx)\left(a+b\right.}{8a^2f}$$

$$= -\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{3/2}f} + \frac{\left(8a^2-8ab-b^2\right)\sqrt{a+\right.}{8a^2f}$$

Mathematica [A] time = 0.57, size = 103, normalized size = 0.62

$$\frac{\left(-8a^2+8ab+b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a}\sqrt{a+b\sin^2(e+fx)}\left((8a-b)\csc^2(e+fx)-2a\csc^4(e+fx)\right.}{8a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((-8*a^2 + 8*a*b + b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[
a]*(8*a + (8*a - b)*Csc[e + f*x]^2 - 2*a*Csc[e + f*x]^4)*Sqrt[a + b*Sin[e +
 f*x]^2])/(8*a^(3/2)*f)
```

fricas [A] time = 2.06, size = 415, normalized size = 2.52

$$\left[-\frac{\left(\left(8\,a^2-8\,ab-b^2\right)\cos\left(fx+e\right)^4-2\left(8\,a^2-8\,ab-b^2\right)\cos\left(fx+e\right)^2+8\,a^2-8\,ab-b^2\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos\left(fx+e\right)^2-2\right.}{\cdots}\right.}{16\left(a^2f\cos\left(fx+e\right)^4\right.\cdots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2)*cos
(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(8
*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x + e)^2 + 14*a^2 - a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^4 - 2*a^2*f*cos(f*x + e)^2 +
a^2*f), 1/8*(((8*a^2 - 8*a*b - b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 8*a*b - b^2
)*cos(f*x + e)^2 + 8*a^2 - 8*a*b - b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(-a)/a) + (8*a^2*cos(f*x + e)^4 - (24*a^2 - a*b)*cos(f*x +
 e)^2 + 14*a^2 - a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^2*f*cos(f*x + e)^
4 - 2*a^2*f*cos(f*x + e)^2 + a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)E
valuation time: 0.85Unable to divide, perhaps due to rounding error%%%{1,[4
,0]%%%}+%%%{%%{[-4,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0]%%%}+%%%{8,[2,1]%%%}+%%%
{%%%{6,[1]%%%},[2,0]%%%}+%%%{%%{[-16,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%
%%{%%{[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0]%%%}+%%%{16,[0,2]%%%}
+%%%{%%%{8,[1]%%%},[0,1]%%%}+%%%{%%%{1,[2]%%%},[0,0]%%%} / %%%{%%%{1,[1]%%%
},[4,0]%%%}+%%%{%%{poly1[%%%{-4,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0]%%%
}+%%%{%%%{8,[1]%%%},[2,1]%%%}+%%%{%%%{6,[2]%%%},[2,0]%%%}+%%%{%%{poly1[%%%{
-16,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,1]%%%}+%%%{%%{poly1[%%%{-4,[2]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0]%%%}+%%%{%%%{16,[1]%%%},[0,2]%%%}+%%%{%%%
{8,[2]%%%},[0,1]%%%}+%%%{%%%{1,[3]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 1.87, size = 230, normalized size = 1.39

$$\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{f}-\frac{\sqrt{a}\;\ln\left(\frac{2a+2\sqrt{a}\;\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}+\frac{\ln\left(\frac{2a+2\sqrt{a}\;\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)b}{f\sqrt{a}}-\frac{b\sqrt{a+b\left(\sin^2\left(fx+e\right)\right.}}{8fa\sin\left(fx+e\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (a+b*sin(f*x+e)^2)^(1/2)/f-1/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)
^(1/2))/sin(f*x+e))+1/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))
/sin(f*x+e))*b-1/8/f*b/a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/8/f*b^2/a^
(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))+1/f/sin(f*x+e
)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/4/f/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.38, size = 215, normalized size = 1.30

$$-\frac{8\sqrt{a}\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-\frac{8\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}-\frac{b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-8\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{8\sqrt{b\sin\cdots}}{\cdots}}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*sqrt(a)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 8*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/sqrt(a) - b^2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x
 + e))))/a^(3/2) - 8*sqrt(b*sin(f*x + e)^2 + a) + 8*sqrt(b*sin(f*x + e)^2 +
 a)*b/a + sqrt(b*sin(f*x + e)^2 + a)*b^2/a^2 - 8*(b*sin(f*x + e)^2 + a)^(3/
2)/(a*sin(f*x + e)^2) - (b*sin(f*x + e)^2 + a)^(3/2)*b/(a^2*sin(f*x + e)^2)
 + 2*(b*sin(f*x + e)^2 + a)^(3/2)/(a*sin(f*x + e)^4))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^5\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\cot^5\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**5, x)
```

3.494 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx$

Optimal. Leaf size=234

$$\frac{\tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{4a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{bs}{}}}{3f\sqrt{a+}}$$

```
[Out] 1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*Ell
ipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*
x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(3*a+4*b)*(a+b*sin(f*x+e)^2)
^(1/2)*tan(f*x+e)/(a+b)/f+1/3*(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.28, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3196, 467, 578, 524, 426, 424, 421, 419}

$$\frac{\tan^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(3a+4b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{4a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{bs}{}}}{3f\sqrt{a+}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] ((7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*
x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x
]^2]) - ((3*a + 4*b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a + b)*f)
 + (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^3)/(3*f)
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

]

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 578

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(g^(n - 1)*(b*e - a*f)*
(g*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)
*(p + 1)), x] - Dist[g^n/(b*n*(b*c - a*d)*(p + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f)*(m - n + 1) + (d*(b*e - a*f
)*(m + n*q + 1) - b*n*(c*f - d*e)*(p + 1))*x^n, x], x], x] /; FreeQ[{a, b,
c, d, e, f, g, q}, x] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[m - n + 1, 0]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\sqrt{a+bx^2}}{(1-x^2)^{5/2}}dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan^3(e+fx)}{3f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subs}}{}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan}{3f}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan}{3f}$$

$$= -\frac{(3a+4b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} + \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan}{3f}$$

$$= \frac{(7a+8b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b}}{3(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 1.99, size = 198, normalized size = 0.85

$$-\frac{\frac{\tan(e+fx)\sec^2(e+fx)\left(4\left(4a^2+6ab+b^2\right)\cos(2(e+fx))+8a^2-b(4a+5b)\cos(4(e+fx))+12ab+b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\right|}{6f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^4,x]
```

```
[Out] (2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
-(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
f*x, -(b/a)] - ((8*a^2 + 12*a*b + b^2 + 4*(4*a^2 + 6*a*b + b^2)*Cos[2*(e +
f*x)] - b*(4*a + 5*b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqr
t[2]))/(6*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.52, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

maple [A] time = 3.00, size = 380, normalized size = 1.62

$$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b\,(4a+5b)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-2\sqrt{-b\left(\cos^4\left(fx+e\right.\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(4*a+5*b)*sin(f*x+e)*cos
(f*x+e)^4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*a^2+5*a*b+3*b^2)*
cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a
*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(
-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b
)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),
(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(
-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)/(sin(f*x+e)-
1)/(1+sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^4\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\tan^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**4,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**4, x)
```

3.495 $\int \sqrt{a+b\sin^2(e+fx)}\,\tan^2(e+fx)\,dx$

Optimal. Leaf size=171

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\text{co}}}{}$$

```
[Out] -2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+a*EllipticF(sin(f*x+e),(-b
/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.16, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 467, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{a\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}-\frac{2\sqrt{\text{co}}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (a*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2]) + (Sqrt[a + b*Sin
[e + f*x]^2]*Tan[e + f*x])/f
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \sqrt{a+b\sin^2(e+fx)}\,\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\sqrt{a+bx^2}}{\left(1-x^2\right)^{3/2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{S}}{}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{}\right.}{}$$

$$= -\frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.47, size = 140, normalized size = 0.82

$$\frac{\tan(e+fx)(2a-b\cos(2(e+fx))+b)+\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(\right.}{\sqrt{2}\,f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x]^2,x]
```

```
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x,
-(b/a)] + (2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*f*Sqrt[2*a
+ b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

maple [A] time = 2.53, size = 294, normalized size = 1.72

$$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)+a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))-2*a*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2)))/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin
(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2)*tan(f*x+e)^2,x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(e+f\,x\right)^2\,\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,\tan^2\left(e + fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2)*tan(f*x+e)**2,x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*tan(e + f*x)**2, x)
```

3.496 $\int \sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3178, 3177}

$$\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(f*Sqrt[1 + (b*Sin[
e + f*x]^2)/a])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+b\sin^2(e+fx)}\,dx &= \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\\ &= \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 61, normalized size = 1.20

$$\frac{a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sq
rt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 0.91, size = 71, normalized size = 1.39

$$\frac{a\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\;\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)}{\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-
1/a*b)^(1/2))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\begin{cases}\frac{\sqrt{a}\,\mathrm{E}\left(e+f\,x\middle|-\frac{b}{a}\right)}{f} & \text{if } 0<a\\ \int\sqrt{b\sin\left(e+f\,x\right)^2+a}\,dx & \text{if } \neg 0<a\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] piecewise(0 < a, (a^(1/2)*ellipticE(e + f*x, -b/a))/f, ~0 < a, int((a + b*s
in(e + f*x)^2)^(1/2), x))
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b\sin^2\left(e + fx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2), x)
```

3.497 $\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=174

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right|-}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)
^2/a)^(1/2)+(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)
^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 174, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.280, Rules used = {3196, 473, 524, 426, 424, 421, 419}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{(a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin^{-1}(\sin(e+fx))\right|-}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f) - (2*Sqrt[Cos[e + f*x]^2]*El
lipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^
2])/(f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + ((a + b)*Sqrt[Cos[e + f*x]^2]*Elli
pticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2
)/a])/(f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\,\sqrt{a+bx^2}}{x^2}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Sub}}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Sub}}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+}\right.}{}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+f\right.}{f\sqrt{1}}$$

Mathematica [A] time = 0.55, size = 143, normalized size = 0.82

$$\frac{-(\cot(e+fx)(2a-b\cos(2(e+fx))+b))+\sqrt{2}\,(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-2\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))}{a}}}{\sqrt{2}\,f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((2*a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x]) - 2*Sqrt[2]*a*Sqrt[(2*a + b
 - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*(a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*f*S
qrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\cot\left(fx+e\right)^{2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\,\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

maple [A] time = 1.40, size = 156, normalized size = 0.90

$$\frac{\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+\mathrm{EllipticF}\left(\sin\left(fx+\right.\right.\right.}{\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*(Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-2*El
lipticE(sin(f*x+e),(-1/a*b)^(1/2))*a)+b*cos(f*x+e)^4+(-a-b)*cos(f*x+e)^2)/s
in(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sin\left(fx+e\right)^{2}+a}\,\cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\cot\left(e+f\,x\right)^{2}\,\sqrt{b\sin\left(e+f\,x\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + b \sin^2\left(e + fx\right)} \cot^2\left(e + fx\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**2, x)
```

3.498 $\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx$

Optimal. Leaf size=232

$$-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\sec(e+}{3f}$$

```
[Out] 1/3*(3*a-b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-1/3*cot(f*x+e)^3*(a+b*s
in(f*x+e)^2)^(1/2)/f+1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x
+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+e)^2/a)^
(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 232, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 473, 580, 524, 426, 424, 421, 419}

$$-\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af}-\frac{4(a+b)\sqrt{\cos^2(e+fx)}\sec(e+}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((3*a - b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) - (Cot[e + f*x]
^3*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Elli
pticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2]
)/(3*a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*
EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*
x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

```
]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(e+fx)\sqrt{a+b\sin^2(e+fx)}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}}{x^4}dx, x, \sin(e+f}{f}$$

$$= -\frac{\cot^3(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)}{}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(}}{3f}$$

$$= \frac{(3a-b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} - \frac{\cot^3(e+fx)\sqrt{a+b\sin^2(}}{3f}$$

Mathematica [A] time = 3.16, size = 197, normalized size = 0.85

$$\frac{-\frac{\cot(e+fx)\csc^2(e+fx)\left(4\left(4a^2+2ab-b^2\right)\cos(2(e+fx))-8a^2+b(b-4a)\cos(4(e+fx))-4ab+3b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}F\left(e+fx\right.}{6af\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-1/2*((-8*a^2 - 4*a*b + 3*b^2 + 4*(4*a^2 + 2*a*b - b^2)*Cos[2*(e + f*x)] +
 b*(-4*a + b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 2*a*
(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)]
 - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(
b/a)])/(6*a*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

maple [A] time = 1.63, size = 351, normalized size = 1.51

$$-\frac{4\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\ \mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)+4b\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\cdots}}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*sin(f*x+e)^3+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6-b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-6*a*b*sin(f*x+e)^4+b^2*sin(f*x+e)^4-5*a^2*
sin(f*x+e)^2+2*a*b*sin(f*x+e)^2+a^2)/a/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{b\sin\left(fx+e\right)^2+a}\,\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt(b*sin(f*x + e)^2 + a)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^4\,\sqrt{b\,\sin\left(e+f\,x\right)^2+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^2\left(e+fx\right)}\,\cot^4\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(e + f*x)**2)*cot(e + f*x)**4, x)
```

3.499 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx$

Optimal. Leaf size=220

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)^2}-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}+\frac{\left(8a^2+40ab+35b^2\right.}{8f}$$

```
[Out] -1/24*(8*a^2+40*a*b+35*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)^2/f-1/8*(8*a+9*b
)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/(a+b)^2/f+1/4*sec(f*x+e)^4*(a+b*sin
(f*x+e)^2)^(5/2)/(a+b)/f+1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+e)^
2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)-1/8*(8*a^2+40*a*b+35*b^2)*(a+b*sin(f*x+
e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.26, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3194, 89, 78, 50, 63, 208}

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24f(a+b)^2}-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8f(a+b)}+\frac{\left(8a^2+40ab+35b^2\right.}{8f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] ((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])
/(8*Sqrt[a + b]*f) - ((8*a^2 + 40*a*b + 35*b^2)*Sqrt[a + b*Sin[e + f*x]^2])
/(8*(a + b)*f) - ((8*a^2 + 40*a*b + 35*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(
24*(a + b)^2*f) - ((8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))
/(8*(a + b)^2*f) + (Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*(a + b)
*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^5(e+fx)\,dx=\frac{\text{Subst}\left(\int \frac{x^2(a+bx)^{3/2}}{(1-x)^3}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=\frac{\sec^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4(a+b)f}-\frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}\left(\frac{1}{2}(4a+5b)+2(a+\right.}{(1-x)^2}\right.}{4(a+b)}$$

$$=-\frac{(8a+9b)\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8(a+b)^2f}+\frac{\sec^4(e+fx)\left(a+b\right.}{4(a+}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24(a+b)^2f}-\frac{(8a+9b)\sec^2(e+f}{8(a}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}-\frac{\left(8a^2+40ab+35b^2\right)\left(\right.}{24(a+}$$

$$=-\frac{\left(8a^2+40ab+35b^2\right)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)f}-\frac{\left(8a^2+40ab+35b^2\right)\left(\right.}{24(a+}$$

$$=\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8\sqrt{a+b}\,f}-\frac{\left(8a^2+40ab+35b^2\right)}{8(a+}$$

Mathematica [A] time = 2.01, size = 160, normalized size = 0.73

$$-\frac{\left(8a^2+40ab+35b^2\right)\left(\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)+3b\right)-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)-6}{24f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^5,x]
```

```
[Out] -1/24*(3*(8*a + 9*b)*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) - 6*(a + b
)*Sec[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(5/2) + (8*a^2 + 40*a*b + 35*b^2)*(
-3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a +
 b*Sin[e + f*x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/((a + b)^2*f)
```

fricas [A] time = 1.05, size = 385, normalized size = 1.75

$$\left[\frac{3\left(8\,a^{2}+40\,ab+35\,b^{2}\right)\sqrt{a+b}\,\cos\left(fx+e\right)^{4}\log\left(\frac{b\cos\left(fx+e\right)^{2}-2\,\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^{2}}\right)+2\left(8\left(ab+b^{2}\right)\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="fricas")
```

```
[Out] [1/48*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*
x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f
*x + e)^2) + 2*(8*(a*b + b^2)*cos(f*x + e)^6 - 16*(2*a^2 + 7*a*b + 5*b^2)*c
os(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)*cos(f*x + e)^2 + 6*a^2 + 12*a*b
 + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a + b)*f*cos(f*x + e)^4), -1/2
4*(3*(8*a^2 + 40*a*b + 35*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 +
 a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 - (8*(a*b + b^2)*cos(f*x + e)^
6 - 16*(2*a^2 + 7*a*b + 5*b^2)*cos(f*x + e)^4 - 3*(8*a^2 + 21*a*b + 13*b^2)
*cos(f*x + e)^2 + 6*a^2 + 12*a*b + 6*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/
((a + b)*f*cos(f*x + e)^4)]
```

giac [B] time = 5.52, size = 3781, normalized size = 17.19

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="giac")
```

```
[Out] -1/12*(3*(8*a^2 + 40*a*b + 35*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 16*(6*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 9*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b + 39*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^4*sqrt(a)*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*a^2*b + 66*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^3*a*b^2 + 88*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3
*b^3 - 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b
+ 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^2 + 72
```

*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^3 - 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b - 75*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^2 - 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^3 + 144*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^4 - 6*a^(7/2)*b - 45*a^(5/2)*b^2 - 136*a^(3/2)*b^3 - 144*sqrt(a)*b^4)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b)^3 - 6*(8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b + 11*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 104*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b - 13*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 - 520*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 581*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 76*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) + 248*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 357*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 644*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 + 1736*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 2841*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 1320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 - 304*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 584*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 2465*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 3736*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 + 1936*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4

```
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b
^4 - 232*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 1240*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2
*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*b - 2271*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b^2 - 1244*(sqrt(a)*tan(1/2*f
*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
+ 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^3 + 688*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*a*b^4 + 704*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))*b^5 - 104*a^(11/2) - 728*a^(9/2)*b - 2095*a^(7/2)*b^2 - 30
92*a^(5/2)*b^3 - 2320*a^(3/2)*b^4 - 704*sqrt(a)*b^5)/((sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^4)/f
```

maple [B] time = 4.04, size = 711, normalized size = 3.23

$$\frac{16\,(a+b)^{\frac{5}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\;b\left(\cos^6\left(fx+e\right)\right)+\left(-64\,(a+b)^{\frac{5}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\;a-160\,(a\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x)
```

```
[Out] 1/48*(16*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b*cos(f*x+e)^6+(-64*(a+b)^(
5/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a-160*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)*b+24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a))*a^4+168*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)-b*sin(f*x+e)+a))*a^3*b+369*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+330*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+105*ln(2/(1+sin(f*x+e))*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+24*ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+168*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^3*b+369*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2*b^2+330*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+105*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-6*(a+b)^(5/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)*(8*a+13*b)*cos(f*x+e)^2+12*(a+b)^(5/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)*a+12*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b)/(a+b)^(5/2)/co
s(f*x+e)^4/f
```

maxima [A] time = 0.43, size = 238, normalized size = 1.08

$$-\frac{16\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^3+48\left(ab^3+3\,b^4\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(8\,a^2b^3+40\,ab^4+35\,b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right.}{\sqrt{a+b}}}{48\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^5,x, algorithm="maxima")
```

```
[Out] -1/48*(16*(b*sin(f*x + e)^2 + a)^(3/2)*b^3 + 48*(a*b^3 + 3*b^4)*sqrt(b*sin(
f*x + e)^2 + a) + 3*(8*a^2*b^3 + 40*a*b^4 + 35*b^5)*log((sqrt(b*sin(f*x + e
)^2 + a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a
+ b) - 6*((8*a*b^4 + 13*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2*b^4 + 19
*a*b^5 + 11*b^6)*sqrt(b*sin(f*x + e)^2 + a))/((b*sin(f*x + e)^2 + a)^2 - 2*
(b*sin(f*x + e)^2 + a)*(a + b) + a^2 + 2*a*b + b^2))/(b^3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^5\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**5,x)
```

```
[Out] Timed out
```

3.500 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx$

Optimal. Leaf size=148

$$\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6f(a+b)}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\cdots$$

[Out] 1/6*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(3/2)/(a+b)/f+1/2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/(a+b)/f-1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))*(a+b)^(1/2)/f+1/2*(2*a+5*b)*(a+b*sin(f*x+e)^2)^(1/2)/f

Rubi [A] time = 0.14, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 50, 63, 208}

$$\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6f(a+b)}+\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\cdots$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] -(Sqrt[a + b]*(2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(2*f) + ((2*a + 5*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) + ((2*a + 5*b)*(a +
b*Sin[e + f*x]^2)^(3/2))/(6*(a + b)*f) + (Sec[e + f*x]^2*(a + b*Sin[e + f*x
]^2)^(5/2))/(2*(a + b)*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)\,dx=\frac{\operatorname{Subst}\left(\int \frac{x(a+bx)^{3/2}}{(1-x)^2}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f}-\frac{(2a+5b)\operatorname{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx\right.}{4(a+b)f}$$

$$=\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f}+\frac{\sec^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2(a+b)f}$$

$$=\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f}+$$

$$=\frac{(2a+5b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{(2a+5b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6(a+b)f}+$$

$$=-\frac{\sqrt{a+b}\,(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f}+\frac{(2a+5b)\sqrt{a+b\sin^2(e}}{2f}$$

Mathematica [A] time = 0.51, size = 116, normalized size = 0.78

$$\frac{(2a+5b)\left(\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)+3b\right)-3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)\right)+3\sec^2(e+fx}{6f(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3,x]
```

```
[Out] (3*Sec[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2) + (2*a + 5*b)*(-3*(a + b)^(3
/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] + Sqrt[a + b*Sin[e + f*
x]^2]*(4*a + 3*b + b*Sin[e + f*x]^2)))/(6*(a + b)*f)
```

fricas [A] time = 0.82, size = 265, normalized size = 1.79

$$\left[\frac{3\,(2\,a+5\,b)\sqrt{a+b}\,\cos\left(fx+e\right)^2\log\left(\frac{b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)-2\left(2\,b\cos\left(fx+e\right)^4-2\,(4\,a\right.}{12\,f\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="fricas")
```

```
[Out] [1/12*(3*(2*a + 5*b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2
*(2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*co
s(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2), 1/6*(3*(2*a + 5*b)*sqrt(-a - b)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^2
 - (2*b*cos(f*x + e)^4 - 2*(4*a + 7*b)*cos(f*x + e)^2 - 3*a - 3*b)*sqrt(-b*
cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2)]
```

giac [B] time = 2.38, size = 2185, normalized size = 14.76

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a^2 + 7*a*b + 5*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
- sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*
f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 6*(2*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 3*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*b^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^2*a^(5/2) + 7*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a
^(3/2)*b + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)
*b^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 3*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b + 3*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2 + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*b^3 - 2*a^(7/2) - 7*a^(5/2)*b - 9*a^(3/2)*b^2 - 4*s
qrt(a)*b^3)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2 -
8*(3*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 3*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 9*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b + 15*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^2 + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^3*a^2*b + 30*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^3*a*b^2 + 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*b^3 - 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(
5/2)*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b
^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^3
```

```
- 9*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b - 33*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^2 + 48*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^4 - 3*a^(7/2)*b - 21*a^(5/2)*b^2 - 5
6*a^(3/2)*b^3 - 48*sqrt(a)*b^4)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))^2 + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(
a) + a + 4*b)^3)/f
```

maple [B] time = 3.89, size = 567, normalized size = 3.83

$$\frac{-4\,(a+b)^{\frac{3}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\;b\left(\cos^4\left(fx+e\right)\right)-\left(-16\,(a+b)^{\frac{3}{2}}\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\;a-28\,(a+b\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x)
```

```
[Out] 1/12*(-4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b*cos(f*x+e)^4-(-16*(a+b)^(
3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a-28*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
*b+6*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a^3+27*a^2*b*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+b*sin(f*x+e)+a))+36*a*b^2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+15*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+6*ln(2/(1+sin(f*x+e))*((a+b)^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+27*a^2*b*ln(2/(1+sin(f*x+
e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+36*a*b^2*ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+15*
b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e
)+a)))*cos(f*x+e)^2+6*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a+6*(a+b)^(3/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)*b)/(a+b)^(3/2)/cos(f*x+e)^2/f
```

maxima [A] time = 0.42, size = 169, normalized size = 1.14

$$\frac{4\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b^2+12\left(ab^2+2\,b^3\right)\sqrt{b\sin\left(fx+e\right)^2+a}+\frac{3\left(2\,a^2b^2+7\,ab^3+5\,b^4\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\sqrt{a+b}}-\frac{6\,(ab}{}}{12\,b^2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3,x, algorithm="maxima")
```

```
[Out] 1/12*(4*(b*sin(f*x + e)^2 + a)^(3/2)*b^2 + 12*(a*b^2 + 2*b^3)*sqrt(b*sin(f*
x + e)^2 + a) + 3*(2*a^2*b^2 + 7*a*b^3 + 5*b^4)*log((sqrt(b*sin(f*x + e)^2
+ a) - sqrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/sqrt(a + b)
 - 6*(a*b^3 + b^4)*sqrt(b*sin(f*x + e)^2 + a)/(b*sin(f*x + e)^2 - b))/(b^2*
f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)^3\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**3,x)
[Out] Timed out
```

3.501 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx$

Optimal. Leaf size=84

$$-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

```
[Out] (a+b)^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f-1/3*(a+b*sin(f*
x+e)^2)^(3/2)/f-(a+b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 50, 63, 208}

$$-\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] ((a + b)^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/f - ((a + b
)*Sqrt[a + b*Sin[e + f*x]^2])/f - (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{1-x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= -\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{(a+b)\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{1-x}\,dx, x, \sin^2(e+f\right.}{2f}$$
$$= -\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{(a+b)^2\,\text{S}}{}$$
$$= -\frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{(a+b)^2\,\text{S}}{}$$
$$= \frac{(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f} - \frac{(a+b)\sqrt{a+b\sin^2(e+fx)}}{f} -$$

Mathematica [A] time = 0.15, size = 79, normalized size = 0.94

$$\frac{\sqrt{a-b\cos^2(e+fx)+b}\left(b\cos^2(e+fx)-4(a+b)\right)+3(a+b)^{3/2}\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x],x]
```

```
[Out] (3*(a + b)^(3/2)*ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]] + Sqrt
[a + b - b*Cos[e + f*x]^2]*(-4*(a + b) + b*Cos[e + f*x]^2))/(3*f)
```

fricas [A] time = 0.81, size = 186, normalized size = 2.21

$$\left[\frac{3\,(a+b)^{\frac{3}{2}}\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)+2\left(b\cos\left(fx+e\right)^2-4\,a-4\,b\right)\sqrt{-b\cos\left(fx+e\right)^2}}{6\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(a + b)^(3/2)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(b*cos(f*x + e)^2 - 4*a -
 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f, -1/3*(3*(a + b)*sqrt(-a - b)*arct
an(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (b*cos(f*x + e)^
2 - 4*a - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/f]
```

giac [B] time = 0.88, size = 1338, normalized size = 15.93

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="giac")
```

```
[Out] -2/3*(3*(a^2 + 2*a*b + b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/sqrt(-a - b) - 2*(6*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b + 3*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^5*b^2 + 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^4*a^(3/2)*b + 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))^4*sqrt(a)*b^2 + 12*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))^3*a^2*b + 54*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
3*a*b^2 + 40*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^3 - 1
2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b + 18*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^2 + 24*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^3 - 18*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b - 57*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))*a*b^3 + 48*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))*b^4 - 6*a^(7/2)*b - 39*a^(5/2)*b^2 - 88*a^(3/2)*b^3 - 48*sqrt(
a)*b^4)/((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 + 2*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) + a + 4*b)^3)/f
```

maple [B] time = 3.21, size = 423, normalized size = 5.04

$$\frac{\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\, b\left(\cos^2\left(fx+e\right)\right)}{3f}-\frac{4\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}\, a}{3f}-\frac{4b\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}{3f}+\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x)
```

```
[Out] 1/3/f*(a+b-b*cos(f*x+e)^2)^(1/2)*b*cos(f*x+e)^2-4/3/f*(a+b-b*cos(f*x+e)^2)^
(1/2)*a-4/3/f*b*(a+b-b*cos(f*x+e)^2)^(1/2)+1/2/(a+b)^(1/2)/f*ln(2/(sin(f*x+
e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2+1/(a+b)^
(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*a*b+1/2/(a+b)^(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^2+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+1/(a+b)^(1/2)
/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)
+a))*a*b+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2
```

maxima [B] time = 0.46, size = 157, normalized size = 1.87

$$-\frac{3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)-3\,(a+b)^{\frac{3}{2}}\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)-}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e),x, algorithm="maxima")
```

```
[Out] -1/6*(3*(a + b)^(3/2)*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1))
 - a/(sqrt(a*b)*(sin(f*x + e) + 1))) - 3*(a + b)^(3/2)*arcsinh(-b*sin(f*x +
 e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1))) + 2*
(b*sin(f*x + e)^2 + a)^(3/2) + 6*sqrt(b*sin(f*x + e)^2 + a)*a + 6*sqrt(b*si
n(f*x + e)^2 + a)*b)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}\tan\left(e+fx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*tan(e + f*x), x)
```

3.502 $\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=78

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

```
[Out] -a^(3/2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f+1/3*(a+b*sin(f*x+e)^2)
^(3/2)/f+a*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.08, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3194, 50, 63, 208}

$$-\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f}+\frac{a\sqrt{a+b\sin^2(e+fx)}}{f}+\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/f) + (a*Sqrt[a + b*
Sin[e + f*x]^2])/f + (a + b*Sin[e + f*x]^2)^(3/2)/(3*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\text{Subst}\left(\int \frac{(a+bx)^{3/2}}{x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a\,\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{x\sqrt{a}\cdots}\right.}{\cdots}$$

$$= \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{a^2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\cdots}\right.}{\cdots}$$

$$= -\frac{a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{f} + \frac{a\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{\left(a+b\sin\cdots\right.}{\cdots}$$

Mathematica [A] time = 0.13, size = 69, normalized size = 0.88

$$\frac{\sqrt{a+b\sin^2(e+fx)}\left(4a+b\sin^2(e+fx)\right)-3a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{3f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + Sqrt[a + b*Sin[e
+ f*x]^2]*(4*a + b*Sin[e + f*x]^2))/(3*f)
```

fricas [A] time = 1.15, size = 175, normalized size = 2.24

$$\left[\frac{3\,a^{\frac{3}{2}}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\left(b\cos\left(fx+e\right)^2-4\,a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{6\,f}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*a^(3/2)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(b*cos(f*x + e)^2 - 4*a - b)*
sqrt(-b*cos(f*x + e)^2 + a + b))/f, 1/3*(3*sqrt(-a)*a*arctan(sqrt(-b*cos(f*
x + e)^2 + a + b)*sqrt(-a)/a) - (b*cos(f*x + e)^2 - 4*a - b)*sqrt(-b*cos(f*
x + e)^2 + a + b))/f]
```

giac [A] time = 0.14, size = 71, normalized size = 0.91

$$\frac{\frac{3\,a^2\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}}+\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}+3\sqrt{b\sin\left(fx+e\right)^2+a}\,a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*a^2*arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/sqrt(-a) + (b*sin(f*
x + e)^2 + a)^(3/2) + 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

maple [A] time = 1.52, size = 91, normalized size = 1.17

$$\frac{b\left(\sin^2\left(fx+e\right)\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}+\frac{4a\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)/f-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.31, size = 61, normalized size = 0.78

$$-\frac{3\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-3\sqrt{b\sin\left(fx+e\right)^2+a}\,a}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - (b*sin(f*x + e)^
2 + a)^(3/2) - 3*sqrt(b*sin(f*x + e)^2 + a)*a)/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}} \cot{\left(e + f x \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x), x)
```

3.503 $\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=140

$$-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\csc^2$$

```
[Out] -1/6*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(3/2)/a/f-1/2*csc(f*x+e)^2*(a+b*sin(f*x+e
)^2)^(5/2)/a/f+1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))*a^(1
/2)/f-1/2*(2*a-3*b)*(a+b*sin(f*x+e)^2)^(1/2)/f
```

Rubi [A] time = 0.13, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3194, 78, 50, 63, 208}

$$-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}+\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\csc^2$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*f) - (
(2*a - 3*b)*Sqrt[a + b*Sin[e + f*x]^2])/(2*f) - ((2*a - 3*b)*(a + b*Sin[e +
 f*x]^2)^(3/2))/(6*a*f) - (Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(2*
a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx=\frac{\operatorname{Subst}\left(\int\frac{(1-x)(a+bx)^{3/2}}{x^2}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{2af}-\frac{(2a-3b)\operatorname{Subst}\left(\int\frac{(a+bx)^{3/2}}{x}\,d\right.}{4af}$$

$$=-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-\frac{\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right.}{2af}$$

$$=-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-$$

$$=-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx)}}{2f}-\frac{(2a-3b)\left(a+b\sin^2(e+fx)\right)^{3/2}}{6af}-$$

$$=\frac{\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2f}-\frac{(2a-3b)\sqrt{a+b\sin^2(e+fx}}{2f}$$

Mathematica [A] time = 0.46, size = 90, normalized size = 0.64

$$\frac{3\sqrt{a}\,(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)+\sqrt{a+b\sin^2(e+fx)}\left(-3a\csc^2(e+fx)-8a+b\cos(2(e+fx))+5b\right)}{6f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (3*Sqrt[a]*(2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] + (-8*a
+ 5*b + b*Cos[2*(e + f*x)] - 3*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2]
)/(6*f)
```

fricas [A] time = 1.59, size = 282, normalized size = 2.01

$$\left[-\frac{3\left((2a-3b)\cos\left(fx+e\right)^2-2a+3b\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\left(2b\cos\left(fx+e\right)\right.}{12\left(f\cos\left(fx+e\right)^2-f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a - 3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(a)*log(2*(b*cos(f*x
 + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e
)^2 - 1)) - 2*(2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a - 4*b
)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f), -1/6*(3*((2*a -
3*b)*cos(f*x + e)^2 - 2*a + 3*b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
 + b)*sqrt(-a)/a) - (2*b*cos(f*x + e)^4 - 2*(4*a - b)*cos(f*x + e)^2 + 11*a
 - 4*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(f*cos(f*x + e)^2 - f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Evaluation time: 1.8Unable to divide, perhaps due to roundi
ng error%%%{65536,[8,11]%%%}+%%%{%%%{393216,[1]%%%},[8,10]%%%}+%%%{%%%{9830
40,[2]%%%},[8,9]%%%}+%%%{%%%{1310720,[3]%%%},[8,8]%%%}+%%%{%%%{983040,[4]%%
%},[8,7]%%%}+%%%{%%%{393216,[5]%%%},[8,6]%%%}+%%%{%%%{65536,[6]%%%},[8,5]%%
%}+%%%{%%{[-524288,0]:[1,0,%%%{-1,[1]%%%}]%%},[7,11]%%%}+%%%{%%{[%%%{-31457
28,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,10]%%%}+%%%{%%{[%%%{-7864320,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,9]%%%}+%%%{%%{[%%%{-10485760,[3]%%%},0]:[1
,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[%%%{-7864320,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[7,7]%%%}+%%%{%%{[%%%{-3145728,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[7,6]%%%}+%%%{%%{[%%%{-524288,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,5]%
%%}+%%%{1048576,[6,12]%%%}+%%%{%%%{8126464,[1]%%%},[6,11]%%%}+%%%{%%%{26738
688,[2]%%%},[6,10]%%%}+%%%{%%%{48496640,[3]%%%},[6,9]%%%}+%%%{%%%{52428800,
[4]%%%},[6,8]%%%}+%%%{%%%{33816576,[5]%%%},[6,7]%%%}+%%%{%%%{12058624,[6]%%
%},[6,6]%%%}+%%%{%%%{1835008,[7]%%%},[6,5]%%%}+%%%{%%{[-6291456,0]:[1,0,%%%
{-1,[1]%%%}]%%},[5,12]%%%}+%%%{%%{[%%%{-41418752,[1]%%%},0]:[1,0,%%%{-1,[1]
%%%}]%%},[5,11]%%%}+%%%{%%{[%%%{-116391936,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%
%},[5,10]%%%}+%%%{%%{[%%%{-180879360,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,
9]%%%}+%%%{%%{[%%%{-167772160,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+
%%%{%%{[%%%{-92798976,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,7]%%%}+%%%{%%{[
%%%{-28311552,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,6]%%%}+%%%{%%{[%%%{-367
0016,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,5]%%%}+%%%{6291456,[4,13]%%%}+%%
%{%%%{53477376,[1]%%%},[4,12]%%%}+%%%{%%%{193331200,[2]%%%},[4,11]%%%}+%%%{
%%%{389283840,[3]%%%},[4,10]%%%}+%%%{%%%{477757440,[4]%%%},[4,9]%%%}+%%%{%%
%{365428736,[5]%%%},[4,8]%%%}+%%%{%%%{169476096,[6]%%%},[4,7]%%%}+%%%{%%%{4
3253760,[7]%%%},[4,6]%%%}+%%%{%%%{4587520,[8]%%%},[4,5]%%%}+%%%{%%{[-251658
24,0]:[1,0,%%%{-1,[1]%%%}]%%},[3,13]%%%}+%%%{%%{[%%%{-171966464,[1]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[3,12]%%%}+%%%{%%{[%%%{-506986496,[2]%%%},0]:[1,0,%
%%{-1,[1]%%%}]%%},[3,11]%%%}+%%%{%%{[%%%{-839909376,[3]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[3,10]%%%}+%%%{%%{[%%%{-851968000,[4]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[3,9]%%%}+%%%{%%{[%%%{-538968064,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
3,8]%%%}+%%%{%%{[%%%{-206045184,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,7]%%%
}+%%%{%%{[%%%{-42991616,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%%}+%%%{%%
{[%%%{-3670016,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,5]%%%}+%%%{16777216,[2
,14]%%%}+%%%{%%%{138412032,[1]%%%},[2,13]%%%}+%%%{%%%{493879296,[2]%%%},[2,
12]%%%}+%%%{%%%{997982208,[3]%%%},[2,11]%%%}+%%%{%%%{1253572608,[4]%%%},[2,
```

```
10]%%%}+%%%{%%%{1008992256,[5]%%%},[2,9]%%%}+%%%{%%%{515899392,[6]%%%},[2,8
]%%%}+%%%{%%%{159645696,[7]%%%},[2,7]%%%}+%%%{%%%{26738688,[8]%%%},[2,6]%%%
}+%%%{%%%{1835008,[9]%%%},[2,5]%%%}+%%%{%%{[-33554432,0]:[1,0,%%%{-1,[1]%%%
}]%%},[1,14]%%%}+%%%{%%{[%%%{-226492416,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[1,13]%%%}+%%%{%%{[%%%{-660602880,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,12]
%%%}+%%%{%%{[%%%{-1086849024,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,11]%%%}+
%%%{%%{[%%%{-1104150528,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,10]%%%}+%%%{%
%{[%%%{-712507392,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,9]%%%}+%%%{%%{[%%%{
-289406976,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,8]%%%}+%%%{%%{[%%%{-707788
80,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,7]%%%}+%%%{%%{[%%%{-9437184,[8]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,6]%%%}+%%%{%%{[%%%{-524288,[9]%%%},0]:[1,0,
%%%{-1,[1]%%%}]%%},[1,5]%%%}+%%%{16777216,[0,15]%%%}+%%%{%%%{117440512,[1]%
%%},[0,14]%%%}+%%%{%%%{358612992,[2]%%%},[0,13]%%%}+%%%{%%%{625999872,[3]%%
%},[0,12]%%%}+%%%{%%%{687931392,[4]%%%},[0,11]%%%}+%%%{%%%{494272512,[5]%%%
},[0,10]%%%}+%%%{%%%{233766912,[6]%%%},[0,9]%%%}+%%%{%%%{71565312,[7]%%%},[
0,8]%%%}+%%%{%%%{13565952,[8]%%%},[0,7]%%%}+%%%{%%%{1441792,[9]%%%},[0,6]%%
%}+%%%{%%%{65536,[10]%%%},[0,5]%%%} / %%%{%%%{1,[2]%%%},[8,0]%%%}+%%%{%%{po
ly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0]%%%}+%%%{%%%{16,[2]%%%},
[6,1]%%%}+%%%{%%%{28,[3]%%%},[6,0]%%%}+%%%{%%{[%%%{-96,[2]%%%},0]:[1,0,%%%{
-1,[1]%%%}]%%},[5,1]%%%}+%%%{%%{poly1[%%%{-56,[3]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[5,0]%%%}+%%%{%%%{96,[2]%%%},[4,2]%%%}+%%%{%%%{240,[3]%%%},[4,1]%%%}+
%%%{%%%{70,[4]%%%},[4,0]%%%}+%%%{%%{poly1[%%%{-384,[2]%%%},0]:[1,0,%%%{-1,[
1]%%%}]%%},[3,2]%%%}+%%%{%%{[%%%{-320,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3
,1]%%%}+%%%{%%{poly1[%%%{-56,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,0]%%%}+%
%%{%%%{256,[2]%%%},[2,3]%%%}+%%%{%%%{576,[3]%%%},[2,2]%%%}+%%%{%%%{240,[4]%
%%},[2,1]%%%}+%%%{%%%{28,[5]%%%},[2,0]%%%}+%%%{%%{[%%%{-512,[2]%%%},0]:[1,0
,%%%{-1,[1]%%%}]%%},[1,3]%%%}+%%%{%%{poly1[%%%{-384,[3]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[1,2]%%%}+%%%{%%{[%%%{-96,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1
,1]%%%}+%%%{%%{poly1[%%%{-8,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,0]%%%}+%%
%{%%%{256,[2]%%%},[0,4]%%%}+%%%{%%%{256,[3]%%%},[0,3]%%%}+%%%{%%%{96,[4]%%%
},[0,2]%%%}+%%%{%%%{16,[5]%%%},[0,1]%%%}+%%%{%%%{1,[6]%%%},[0,0]%%%} Error:
 Bad Argument Value
```

maple [A] time = 1.82, size = 179, normalized size = 1.28

$$-\frac{b\left(\sin^2\left(fx+e\right)\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}-\frac{4a\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}+\frac{b\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{f}+\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\ \cdots}{\cdots}\right.}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*(a+b*sin(f*x+e)^2)^(1/
2)/f+1/f*b*(a+b*sin(f*x+e)^2)^(1/2)+1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2))/sin(f*x+e))-3/2/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))*b-1/2/f*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.33, size = 148, normalized size = 1.06

$$\frac{6\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-9\,\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-2\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-6\sqrt{b\sin\left(fx+e\right)^2+a}\,a\ \cdots}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 9*sqrt(a)*b*arcsi
nh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 2*(b*sin(f*x + e)^2 + a)^(3/2) - 6*sq
```

```
rt(b*sin(f*x + e)^2 + a)*a + 9*sqrt(b*sin(f*x + e)^2 + a)*b + 3*(b*sin(f*x
+ e)^2 + a)^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^2))/
f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(e+f\,x\right)^3\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^3*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3*(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.504 $\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=208

$$\frac{(8a^2-24ab+3b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{(8a^2-24ab+3b^2)\sqrt{a+b\sin^2(e+fx)}}{8af}-\frac{(8a^2-24ab+3b^2)\tanh^{-1}}{8\sqrt{a}\,f}$$

```
[Out] 1/24*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(3/2)/a^2/f+1/8*(8*a-b)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(5/2)/a^2/f-1/4*csc(f*x+e)^4*(a+b*sin(f*x+e)^2)^(
5/2)/a/f-1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))
/f/a^(1/2)+1/8*(8*a^2-24*a*b+3*b^2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.19, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 50, 63, 208}

$$\frac{(8a^2-24ab+3b^2)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{(8a^2-24ab+3b^2)\sqrt{a+b\sin^2(e+fx)}}{8af}-\frac{(8a^2-24ab+3b^2)\tanh^{-1}}{8\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*
Sqrt[a]*f) + ((8*a^2 - 24*a*b + 3*b^2)*Sqrt[a + b*Sin[e + f*x]^2])/(8*a*f)
+ ((8*a^2 - 24*a*b + 3*b^2)*(a + b*Sin[e + f*x]^2)^(3/2))/(24*a^2*f) + ((8*
a - b)*Csc[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(5/2))/(8*a^2*f) - (Csc[e + f*
x]^4*(a + b*Sin[e + f*x]^2)^(5/2))/(4*a*f)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^5(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx=\frac{\text{Subst}\left(\int \frac{(1-x)^2(a+bx)^{3/2}}{x^3}dx,x,\sin^2(e+fx)\right)}{2f}$$

$$=-\frac{\csc^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{4af}+\frac{\text{Subst}\left(\int \frac{\left(\frac{1}{2}(-8a+b)+2ax\right)(a+}{x^2}\right.}{4a}$$

$$=\frac{(8a-b)\csc^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{5/2}}{8a^2f}-\frac{\csc^4(e+fx)\left(a+b\right.}{4a}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\left(a+b\sin^2(e+fx)\right)^{3/2}}{24a^2f}+\frac{(8a-b)\csc^2(e+fx)}{8}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\sqrt{a+b\sin^2(e+fx)}}{8af}+\frac{\left(8a^2-3(8a-b)b\right)\left(a+\right.}{24a^2}$$

$$=\frac{\left(8a^2-3(8a-b)b\right)\sqrt{a+b\sin^2(e+fx)}}{8af}+\frac{\left(8a^2-3(8a-b)b\right)\left(a+\right.}{24a^2}$$

$$=-\frac{\left(8a^2-3(8a-b)b\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8\sqrt{a}f}+\frac{\left(8a^2-3(8a-b)b\right)}{8}$$

Mathematica [A] time = 0.80, size = 123, normalized size = 0.59

$$\frac{\sqrt{a}\sqrt{a+b\sin^2(e+fx)}\left(8\left(4a+b\sin^2(e+fx)-6b\right)+3(8a-5b)\csc^2(e+fx)-6a\csc^4(e+fx)\right)-3\left(8a^2-\right.}{24\sqrt{a}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-3*(8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]] +
Sqrt[a]*Sqrt[a + b*Sin[e + f*x]^2]*(3*(8*a - 5*b)*Csc[e + f*x]^2 - 6*a*Csc[
e + f*x]^4 + 8*(4*a - 6*b + b*Sin[e + f*x]^2)))/(24*Sqrt[a]*f)
```

fricas [A] time = 2.16, size = 442, normalized size = 2.12

$$\left[\frac{3\left(\left(8a^2-24ab+3b^2\right)\cos\left(fx+e\right)^4-2\left(8a^2-24ab+3b^2\right)\cos\left(fx+e\right)^2+8a^2-24ab+3b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\ldots\right.}{\ldots}\right.}{\ldots}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b
^2)*cos(f*x + e)^2 + 8*a^2 - 24*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^
2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 -
1)) - 2*(8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
- 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b)
)/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f), 1/24*(3*((8*a^2 - 24*a
*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 - 24*a*b + 3*b^2)*cos(f*x + e)^2 + 8*
a^2 - 24*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
-a)/a) - (8*a*b*cos(f*x + e)^6 - 8*(4*a^2 - 3*a*b)*cos(f*x + e)^4 + (88*a^2
 - 87*a*b)*cos(f*x + e)^2 - 50*a^2 + 55*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b
))/(a*f*cos(f*x + e)^4 - 2*a*f*cos(f*x + e)^2 + a*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Unable to check sign: (2*pi/t_nostep/2)>(-2*
pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Una
ble to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign
: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*
pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2
*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to c
heck sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/t_nostep/2)>(-
2*pi/t_nostep/2)Unable to check sign: (4*pi/t_nostep/2)>(-4*pi/t_nostep/2)E
valuation time: 2.41Unable to divide, perhaps due to rounding error%%%{5242
88,[8,12]%%%}+%%%{%%%{3670016,[1]%%%},[8,11]%%%}+%%%{%%%{11010048,[2]%%%},[
8,10]%%%}+%%%{%%%{18350080,[3]%%%},[8,9]%%%}+%%%{%%%{18350080,[4]%%%},[8,8]
%%%}+%%%{%%%{11010048,[5]%%%},[8,7]%%%}+%%%{%%%{3670016,[6]%%%},[8,6]%%%}+%
%%{%%%{524288,[7]%%%},[8,5]%%%}+%%%{%%{[-4194304,0]:[1,0,%%%{-1,[1]%%%}]%%}
,[7,12]%%%}+%%%{%%{[%%%{-29360128,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,11]
%%%}+%%%{%%{[%%%{-88080384,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,10]%%%}+%%
%{%%{[%%%{-146800640,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,9]%%%}+%%%{%%{[%
%%{-146800640,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,8]%%%}+%%%{%%{[%%%{-880
80384,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,7]%%%}+%%%{%%{[%%%{-29360128,[6
]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,6]%%%}+%%%{%%{[%%%{-4194304,[7]%%%},0]:
[1,0,%%%{-1,[1]%%%}]%%},[7,5]%%%}+%%%{8388608,[6,13]%%%}+%%%{%%%{73400320,[
```

1]%%%},[6,12]%%%}+%%%{%%%{278921216,[2]%%%},[6,11]%%%}+%%%{%%%{601882624,[3
]%%%},[6,10]%%%}+%%%{%%%{807403520,[4]%%%},[6,9]%%%}+%%%{%%%{689963008,[5]%
%%},[6,8]%%%}+%%%{%%%{367001600,[6]%%%},[6,7]%%%}+%%%{%%%{111149056,[7]%%%}
,[6,6]%%%}+%%%{%%%{14680064,[8]%%%},[6,5]%%%}+%%%{%%{[-50331648,0]:[1,0,%%%
{-1,[1]%%%}]%%},[5,13]%%%}+%%%{%%{[%%%{-381681664,[1]%%%},0]:[1,0,%%%{-1,[1
]%%%}]%%},[5,12]%%%}+%%%{%%{[%%%{-1262485504,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}
]%%},[5,11]%%%}+%%%{%%{[%%%{-2378170368,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[5,10]%%%}+%%%{%%{[%%%{-2789212160,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,9]
%%%}+%%%{%%{[%%%{-2084569088,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,8]%%%}+%
%%{%%{[%%%{-968884224,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,7]%%%}+%%%{%%{[
%%%{-255852544,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,6]%%%}+%%%{%%{[%%%{-29
360128,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,5]%%%}+%%%{50331648,[4,14]%%%}
+%%%{%%%{478150656,[1]%%%},[4,13]%%%}+%%%{%%%{1974468608,[2]%%%},[4,12]%%%}
+%%%{%%%{4660920320,[3]%%%},[4,11]%%%}+%%%{%%%{6936330240,[4]%%%},[4,10]%%%
}+%%%{%%%{6745489408,[5]%%%},[4,9]%%%}+%%%{%%%{4279238656,[6]%%%},[4,8]%%%}
+%%%{%%%{1701838848,[7]%%%},[4,7]%%%}+%%%{%%%{382730240,[8]%%%},[4,6]%%%}+%
%%{%%%{36700160,[9]%%%},[4,5]%%%}+%%%{%%{[-201326592,0]:[1,0,%%%{-1,[1]%%%}
]%%},[3,14]%%%}+%%%{%%{[%%%{-1577058304,[1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},
[3,13]%%%}+%%%{%%{[%%%{-5431623680,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,12
]%%%}+%%%{%%{[%%%{-10775166976,[3]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,11]%%%
}+%%%{%%{[%%%{-13535019008,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,10]%%%}+%%
%{%%{[%%%{-11127488512,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,9]%%%}+%%%{%%{
[%%%{-5960105984,[6]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,8]%%%}+%%%{%%{[%%%{-
1992294400,[7]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,7]%%%}+%%%{%%{[%%%{-373293
056,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,6]%%%}+%%%{%%{[%%%{-29360128,[9]%
%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[3,5]%%%}+%%%{134217728,[2,15]%%%}+%%%{%%%{1
241513984,[1]%%%},[2,14]%%%}+%%%{%%%{5058330624,[2]%%%},[2,13]%%%}+%%%{%%%{
11934892032,[3]%%%},[2,12]%%%}+%%%{%%%{18012438528,[4]%%%},[2,11]%%%}+%%%{%
%%{18100518912,[5]%%%},[2,10]%%%}+%%%{%%%{12199133184,[6]%%%},[2,9]%%%}+%%%
{%%%{5404360704,[7]%%%},[2,8]%%%}+%%%{%%%{1491075072,[8]%%%},[2,7]%%%}+%%%{
%%%{228589568,[9]%%%},[2,6]%%%}+%%%{%%%{14680064,[10]%%%},[2,5]%%%}+%%%{%%{
[-268435456,0]:[1,0,%%%{-1,[1]%%%}]%%},[1,15]%%%}+%%%{%%{[%%%{-2080374784,[
1]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,14]%%%}+%%%{%%{[%%%{-7096762368,[2]%%%
},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,13]%%%}+%%%{%%{[%%%{-13979615232,[3]%%%},0]
:[1,0,%%%{-1,[1]%%%}]%%},[1,12]%%%}+%%%{%%{[%%%{-17527996416,[4]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[1,11]%%%}+%%%{%%{[%%%{-14533263360,[5]%%%},0]:[1,0,%%
%{-1,[1]%%%}]%%},[1,10]%%%}+%%%{%%{[%%%{-8015314944,[6]%%%},0]:[1,0,%%%{-1,
[1]%%%}]%%},[1,9]%%%}+%%%{%%{[%%%{-2881486848,[7]%%%},0]:[1,0,%%%{-1,[1]%%%
}]%%},[1,8]%%%}+%%%{%%{[%%%{-641728512,[8]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[
1,7]%%%}+%%%{%%{[%%%{-79691776,[9]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,6]%%%}
+%%%{%%{[%%%{-4194304,[10]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,5]%%%}+%%%{134
217728,[0,16]%%%}+%%%{%%%{1073741824,[1]%%%},[0,15]%%%}+%%%{%%%{3808428032,
[2]%%%},[0,14]%%%}+%%%{%%%{7876902912,[3]%%%},[0,13]%%%}+%%%{%%%{1051145011
2,[4]%%%},[0,12]%%%}+%%%{%%%{9457631232,[5]%%%},[0,11]%%%}+%%%{%%%{58243153
92,[6]%%%},[0,10]%%%}+%%%{%%%{2442657792,[7]%%%},[0,9]%%%}+%%%{%%%{68105011
2,[8]%%%},[0,8]%%%}+%%%{%%%{120061952,[9]%%%},[0,7]%%%}+%%%{%%%{12058624,[1
0]%%%},[0,6]%%%}+%%%{%%%{524288,[11]%%%},[0,5]%%%} / %%%{%%%{1,[2]%%%},[8,0
]%%%}+%%%{%%{poly1[%%%{-8,[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[7,0]%%%}+%%%{
%%%{16,[2]%%%},[6,1]%%%}+%%%{%%%{28,[3]%%%},[6,0]%%%}+%%%{%%{[%%%{-96,[2]%%
%},0]:[1,0,%%%{-1,[1]%%%}]%%},[5,1]%%%}+%%%{%%{poly1[%%%{-56,[3]%%%},0]:[1,
0,%%%{-1,[1]%%%}]%%},[5,0]%%%}+%%%{%%%{96,[2]%%%},[4,2]%%%}+%%%{%%%{240,[3]
%%%},[4,1]%%%}+%%%{%%%{70,[4]%%%},[4,0]%%%}+%%%{%%{poly1[%%%{-384,[2]%%%},0
]:[1,0,%%%{-1,[1]%%%}]%%},[3,2]%%%}+%%%{%%{[%%%{-320,[3]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[3,1]%%%}+%%%{%%{poly1[%%%{-56,[4]%%%},0]:[1,0,%%%{-1,[1]%%%}]
%%},[3,0]%%%}+%%%{%%%{256,[2]%%%},[2,3]%%%}+%%%{%%%{576,[3]%%%},[2,2]%%%}+%
%%{%%%{240,[4]%%%},[2,1]%%%}+%%%{%%%{28,[5]%%%},[2,0]%%%}+%%%{%%{[%%%{-512,
[2]%%%},0]:[1,0,%%%{-1,[1]%%%}]%%},[1,3]%%%}+%%%{%%{poly1[%%%{-384,[3]%%%},
0]:[1,0,%%%{-1,[1]%%%}]%%},[1,2]%%%}+%%%{%%{[%%%{-96,[4]%%%},0]:[1,0,%%%{-1
,[1]%%%}]%%},[1,1]%%%}+%%%{%%{poly1[%%%{-8,[5]%%%},0]:[1,0,%%%{-1,[1]%%%}]%

```
%},[1,0]%%%}+%%%{%%%{256,[2]%%%},[0,4]%%%}+%%%{%%%{256,[3]%%%},[0,3]%%%}+%%
%{%%%{96,[4]%%%},[0,2]%%%}+%%%{%%%{16,[5]%%%},[0,1]%%%}+%%%{%%%{1,[6]%%%},[
0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 1.85, size = 280, normalized size = 1.35

$$\frac{b\left(\sin^2\left(fx+e\right)\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}+\frac{4a\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{3f}-\frac{2b\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{f}-\frac{a^{\frac{3}{2}}\ln\left(\frac{2a+2\sqrt{a}\sqrt{}}{\sin}\right.}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3/f*b*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+4/3*a*(a+b*sin(f*x+e)^2)^(1/2
)/f-2/f*b*(a+b*sin(f*x+e)^2)^(1/2)-1/f*a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2))/sin(f*x+e))+3/f*a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e))*b-3/8/f/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(
1/2))/sin(f*x+e))*b^2+1/f*a/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-5/8/f*b/s
in(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/4/f*a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2
)^(1/2)
```

maxima [A] time = 0.33, size = 272, normalized size = 1.31

$$-\frac{24\,a^{\frac{3}{2}}\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)-72\,\sqrt{a}\,b\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)+\frac{9\,b^2\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}-8\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/24*(24*a^(3/2)*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) - 72*sqrt(a)*b*a
rcsinh(a/(sqrt(a*b)*abs(sin(f*x + e)))) + 9*b^2*arcsinh(a/(sqrt(a*b)*abs(si
n(f*x + e))))/sqrt(a) - 8*(b*sin(f*x + e)^2 + a)^(3/2) - 24*sqrt(b*sin(f*x
+ e)^2 + a)*a + 72*sqrt(b*sin(f*x + e)^2 + a)*b + 24*(b*sin(f*x + e)^2 + a)
^(3/2)*b/a - 3*(b*sin(f*x + e)^2 + a)^(3/2)*b^2/a^2 - 9*sqrt(b*sin(f*x + e)
^2 + a)*b^2/a - 24*(b*sin(f*x + e)^2 + a)^(5/2)/(a*sin(f*x + e)^2) + 3*(b*s
in(f*x + e)^2 + a)^(5/2)*b/(a^2*sin(f*x + e)^2) + 6*(b*sin(f*x + e)^2 + a)^
(5/2)/(a*sin(f*x + e)^4))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^5\,\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^5*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.505 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx$

Optimal. Leaf size=275

$$\frac{\tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(3a+8b)\sin(e+\cdots}{}$$

```
[Out] -1/3*(3*a+8*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+8/3*(a+2*b)
*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*si
n(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(5*a+8*b)*EllipticF(si
n(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)
^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)-(a+2*b)*sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(
1/2)*tan(f*x+e)/f+1/3*(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^3/f
```

Rubi [A] time = 0.37, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3196, 467, 577, 582, 524, 426, 424, 421, 419}

$$\frac{\tan^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}-\frac{(a+2b)\sin^2(e+fx)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f}-\frac{(3a+8b)\sin(e+\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] -((3*a + 8*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) +
 (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*
Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a
]) - (a*(5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(
b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((a + 2*b)*Sin[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x
])/f + ((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^3)/(3*f)
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 577

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
 + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*b*g*n*(p + 1)), x] + Dist[1/(a*
b*n*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(b
*e*n*(p + 1) + (b*e - a*f)*(m + 1)) + d*(b*e*n*(p + 1) + (b*e - a*f)*(m + n
*q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, m}, x] && IGtQ[n, 0
] && LtQ[p, -1] && GtQ[q, 0] &&  !(EqQ[q, 1] && SimplerQ[b*c - a*d, b*e - a
*f])
```

Rule 582

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*g^(n - 1)*(g*x)^(m
 - n + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(b*d*(m + n*(p + q + 1)
+ 1)), x] - Dist[g^n/(b*d*(m + n*(p + q + 1) + 1)), Int[(g*x)^(m - n)*(a +
b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m - n + 1) + (a*f*d*(m + n*q + 1) + b*(f
*c*(m + n*p + 1) - e*d*(m + n*(p + q + 1) + 1)))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0] && GtQ[m, n - 1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^4(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{5/2}}\,dx,x,\sin(e+f\right.}{f}$$

$$=\frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan^3(e+fx)}{3f}-\frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx\right.}{}$$

$$=-\frac{(a+2b)\sin^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{f}+\frac{\left(a+b\,\mathrm{s}\right.}{}$$

$$=-\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a+2b}{}$$

$$=-\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a+2b}{}$$

$$=-\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{(a+2b}{}$$

$$=-\frac{(3a+8b)\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{8(a+2}{}$$

Mathematica [A] time = 2.74, size = 211, normalized size = 0.77

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(\left(64a^2+160ab+17b^2\right)\cos(2(e+fx))+32a^2-2b(6a+17b)\cos(4(e+fx))+108ab-b^2\cos(6(e+fx))+18b^2\right)}{4\sqrt{2}}-4a(5a+8b)\sqrt{}}{12f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^4,x]
```

```
[Out] (32*a*(a + 2*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 4*a*(5*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
 + f*x, -(b/a)] - ((32*a^2 + 108*a*b + 18*b^2 + (64*a^2 + 160*a*b + 17*b^2)
*Cos[2*(e + f*x)] - 2*b*(6*a + 17*b)*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)
])*Sec[e + f*x]^2*Tan[e + f*x])/(4*Sqrt[2]))/(12*f*Sqrt[2*a + b - b*Cos[2*(
e + f*x)]])
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

maple [A] time = 2.84, size = 419, normalized size = 1.52

$$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b^2\sin\left(fx+e\right)\left(\cos^6\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\right.}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(3*a+7*b)*cos(f*x+e)^4*sin(f
*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+13*a*b+9*b^2)*cos(f
*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^
2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/
2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a+8*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-8*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a-16*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(-(a+
b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(sin(f*x+e)-1)/(1+sin(
f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^4\,\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**4,x)
```

```
[Out] Timed out
```

3.506 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx$

Optimal. Leaf size=222

$$\frac{\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}}{}$$

```
[Out] 4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f-1/3*(7*a+8*b)*Ellipt
icE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*a*(a+b)*EllipticF(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a
+b*sin(f*x+e)^2)^(1/2)+(a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)/f
```

Rubi [A] time = 0.23, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 467, 528, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - ((7*a +
8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f
*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a
*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e
 + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x])/f
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

]

Rule 467

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^q)/(b*n*(p + 1)), x] - Dist[e^n/(b*n*(p + 1)), Int[(e*x)^(m -
n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 1)*Simp[c*(m - n + 1) + d*(m + n*(q
 - 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0
] && IGtQ[n, 0] && LtQ[p, -1] && GtQ[q, 0] && GtQ[m - n + 1, 0] && IntBinom
ialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\tan^2(e+fx)\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^{3/2}}{\left(1-x^2\right)^{3/2}}dx, x, \sin(e+fx\right.}{f}$$

$$= \frac{\left(a+b\sin^2(e+fx)\right)^{3/2}\tan(e+fx)}{f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} + \frac{\left(a+b\sin^2(e+\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{(7a+8b)\sqrt{\cos^2}}{}$$

Mathematica [A] time = 2.82, size = 174, normalized size = 0.78

$$\frac{\sqrt{2}\tan(e+fx)\left(24a^2-4b(2a+3b)\cos(2(e+fx))+40ab-b^2\cos(4(e+fx))+13b^2\right)+32a(a+b)\sqrt{\frac{2a-b\cos(}{}}}{24f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2)*Tan[e + f*x]^2,x]
```

```
[Out] (-8*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x,
 -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e
+ f*x, -(b/a)] + Sqrt[2]*(24*a^2 + 40*a*b + 13*b^2 - 4*b*(2*a + 3*b)*Cos[2*
(e + f*x)] - b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/(24*f*Sqrt[2*a + b - b*Cos
[2*(e + f*x)]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*
x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\tan\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

maple [B] time = 2.65, size = 515, normalized size = 2.32

$$-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\, b^2 \sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x)
[Out] 1/3*(-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*sin(f*x+e)*cos(f*x+e)^
4-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(a+b)*cos(f*x+e)^2*sin(f*x
+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*x+e)
+4*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)+4*a*(cos
(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1
/a*b)^(1/2))*b*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)-7*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a^2-8*(cos(f*x+e)^2)^(1/2)*(-b
/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*(-b*cos
(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*b)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1
)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}} \tan\left(fx+e\right)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2)*tan(f*x+e)^2,x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*tan(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(e+f\,x\right)^2\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2)*tan(f*x+e)**2,x)
[Out] Timed out
```

3.507 $\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=154

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin\cdots}}{3f\sqrt{\frac{b\cdots}{\cdots}}}$$

```
[Out] -1/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/f+2/3*(2*a+b)*(cos(f*
x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)
^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*(a+b)*(cos(f*x+e)^2)^(1/2)/cos
(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+
b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.17, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.375, Rules used = {3180, 3172, 3178, 3177, 3183, 3182}

$$-\frac{b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{a(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{3f\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin\cdots}}{3f\sqrt{\frac{b\cdots}{\cdots}}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) + (2*(2*a +
 b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) - (a*(a + b)*EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Si
n[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3180

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p - 1))/(2*f*p), x] + Dist[
1/(2*p), Int[(a + b*Sin[e + f*x]^2)^(p - 2)*Simp[a*(b + 2*a*p) + b*(2*a + b
)*(2*p - 1)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a +
 b, 0] && GtQ[p, 1]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] && !GtQ[a, 0]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{1}{3}\int \frac{a(3a+b)+2b(2a+b)\sin}{\sqrt{a+b\sin^2(e+f}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{1}{3}(a(a+b))\int \frac{1}{\sqrt{a+b\sin^2(e+}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{\left(2(2a+b)\sqrt{a+b\sin^2(e+fx)}\right.}{3\sqrt{1+\frac{b\sin^2(}{a}}}$$

$$= -\frac{b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}+\frac{2(2a+b)E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+}}{3f\sqrt{1+\frac{b\sin^2(e+fx}{a}}}$$

Mathematica [A] time = 0.77, size = 156, normalized size = 1.01

$$\frac{b\sin(2(e+fx))(-2a+b\cos(2(e+fx))-b)-2\sqrt{2}\,a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+4\sqrt{2}\,a(2a+b)\sqrt{}}{6\sqrt{2}\,f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*Sqrt[2]*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e +
 f*x, -(b/a)] - 2*Sqrt[2]*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*
EllipticF[e + f*x, -(b/a)] + b*(-2*a - b + b*Cos[2*(e + f*x)])*Sin[2*(e + f
*x)])/(6*Sqrt[2]*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^{\frac{3}{2}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^(3/2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.39, size = 266, normalized size = 1.73

$$\frac{-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)a^2}{3}-\frac{a\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin(fx+e),\sqrt{-\frac{b}{a}}\right)b}{3}+ -\ 4}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^(3/2),x)
[Out] (-1/3*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2-1/3*a*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b+4/3*(cos(f*x+e)^2)^(1/2)*((a+b*si
n(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2/3*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a*b+1/3*b^2*sin(f*x+e)^5+1/3*a*b*sin(f*x+e)^3-1/3*b^2*sin(f*x+e)^3-1/3*a
*b*sin(f*x+e))/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(fx+e\right)^2+a\right)^{\frac{3}{2}} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int((a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Timed out
```

3.508 $\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=223

$$\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\,\sec}{}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(3/2)/f+4/3*b*cos(f*x+e)*sin(f*x+e)*(a+b*sin
(f*x+e)^2)^(1/2)/f-1/3*(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e
)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2
)+4/3*a*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^
(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 473, 528, 524, 426, 424, 421, 419}

$$\frac{4b\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f}+\frac{4a(a+b)\sqrt{\cos^2(e+fx)}\,\sec}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (4*b*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e +
 f*x]*(a + b*Sin[e + f*x]^2)^(3/2))/f - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*Ell
ipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2
])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*a*(a + b)*Sqrt[Cos[e + f*x]^2]
*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f
*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/c]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
```

```
]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff, x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}dx = \frac{\left(\sqrt{\cos^2(e+fx)}\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}\left(a+bx^2\right)^{3/2}}{x^2}dx,x,\sin(e+f\right.}{f}$$

$$= -\frac{\cot(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\sec(e+fx)\right.}{}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\right.}{f}$$

$$= \frac{4b\cos(e+fx)\sin(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f} - \frac{\cot(e+fx)\left(a+b\right.}{f}$$

Mathematica [A] time = 2.23, size = 173, normalized size = 0.78

$$\frac{\sqrt{2}\cot(e+fx)\left(-24a^2+4b(2a-b)\cos(2(e+fx))-8ab+b^2\cos(4(e+fx))+3b^2\right)+32a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx}{a}}}{24f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (Sqrt[2]*(-24*a^2 - 8*a*b + 3*b^2 + 4*(2*a - b)*b*Cos[2*(e + f*x)] + b^2*Co
s[4*(e + f*x)])*Cot[e + f*x] - 8*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f
*x)])/a]*EllipticE[e + f*x, -(b/a)] + 32*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*
(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(24*f*Sqrt[2*a + b - b*Cos[2*(e
+ f*x)]])
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

maple [A] time = 1.65, size = 204, normalized size = 0.91

$$\frac{\sin\left(fx+e\right)\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\left(4\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+4\,\mathrm{EllipticF}\left(\sin\left(\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/3*(sin(f*x+e)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b
)^(1/2))*b)+b^2*cos(f*x+e)^6+(2*a*b-2*b^2)*cos(f*x+e)^4+(-3*a^2-2*a*b+b^2)*
cos(f*x+e)^2)/sin(f*x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} \cot\left(fx+e\right)^{2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^{2}\,\left(b\sin\left(e+f\,x\right)^{2}+a\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^2*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a + b \sin^{2}{\left(e + fx \right)}\right)^{\frac{3}{2}} \cot^{2}{\left(e + fx \right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(3/2)*cot(e + f*x)**2, x)
```

3.509 $\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx$

Optimal. Leaf size=276

$$\frac{(3a-5b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\cos^2(e+fx)}{}$$

```
[Out] -1/3*cot(f*x+e)^3*(a+b*sin(f*x+e)^2)^(3/2)/f+(a-b)*cos(f*x+e)^2*cot(f*x+e)*
(a+b*sin(f*x+e)^2)^(1/2)/f+1/3*(3*a-5*b)*cos(f*x+e)*sin(f*x+e)*(a+b*sin(f*x
+e)^2)^(1/2)/f+8/3*(a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos
(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*
(5*a-3*b)*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2
)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.35, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {3196, 473, 580, 528, 524, 426, 424, 421, 419}

$$\frac{(3a-5b)\sin(e+fx)\cos(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f}-\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f}+\frac{(a-b)\cos^2(e+fx)}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a - b)*Cos[e + f*x]^2*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/f + ((3*a
 - 5*b)*Cos[e + f*x]*Sin[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*f) - (Cot[e
 + f*x]^3*(a + b*Sin[e + f*x]^2)^(3/2))/(3*f) + (8*(a - b)*Sqrt[Cos[e + f*x
]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*(a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
1 + (b*Sin[e + f*x]^2)/a])/(3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 473

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^p*(c + d*x^n)^q)/(e*(m
 + 1)), x] - Dist[n/(e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^(p - 1)*(c
 + d*x^n)^(q - 1)*Simp[b*c*p + a*d*q + b*d*(p + q)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q, 0] && LtQ
[m, -1] && GtQ[p, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 528

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (
f_.)*(x_)^(n_)), x_Symbol] :> Simp[(f*x*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)/
(b*(n*(p + q + 1) + 1)), x] + Dist[1/(b*(n*(p + q + 1) + 1)), Int[(a + b*x^
n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f + b*e*n*(p + q + 1)) + (d*(b*e -
 a*f) + f*n*q*(b*c - a*d) + b*d*e*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{
a, b, c, d, e, f, n, p}, x] && GtQ[q, 0] && NeQ[n*(p + q + 1) + 1, 0]
```

Rule 580

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^q)/(a*g*(m + 1)), x] - Dist[1/(a*g^n*(m + 1)), I
nt[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^(q - 1)*Simp[c*(b*e - a*f)*(m +
1) + e*n*(b*c*(p + 1) + a*d*q) + d*((b*e - a*f)*(m + 1) + b*e*n*(p + q + 1)
)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && IGtQ[n, 0] && Gt
Q[q, 0] && LtQ[m, -1] &&  !(EqQ[q, 1] && SimplerQ[e + f*x^n, c + d*x^n])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}{x^4}\,dx, x, \sin(e+\right.}{f}$$

$$= -\frac{\cot^3(e+fx)\left(a+b\sin^2(e+fx)\right)^{3/2}}{3f} + \frac{\left(2\sqrt{\cos^2(e+fx)}\ \sec(e+fx\right.}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} - \frac{\cot^3(e+fx)\left(\right.}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos}{}$$

$$= \frac{(a-b)\cos^2(e+fx)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f} + \frac{(3a-5b)\cos}{}$$

Mathematica [A] time = 4.79, size = 218, normalized size = 0.79

$$\frac{-4\left(5a^2+2ab-3b^2\right)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)-\frac{\cot(e+fx)\csc^2(e+fx)\left(\left(64a^2-32ab-79b^2\right)\cos(2(e+fx))-32a^2-2b(6a-11b\right.}{4\sqrt{2}}}{12f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4*(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-1/4*((-32*a^2 + 44*a*b + 58*b^2 + (64*a^2 - 32*a*b - 79*b^2)*Cos[2*(e + f
*x)] - 2*(6*a - 11*b)*b*Cos[4*(e + f*x)] - b^2*Cos[6*(e + f*x)])*Cot[e + f*
x]*Csc[e + f*x]^2)/Sqrt[2] + 32*a*(a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)
])/a]*EllipticE[e + f*x, -(b/a)] - 4*(5*a^2 + 2*a*b - 3*b^2)*Sqrt[(2*a + b
- b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(12*f*Sqrt[2*a + b - b
*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.60, size = 0, normalized size = 0.00

$$\text{integral}\left(-\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*
x + e)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

maple [A] time = 1.74, size = 419, normalized size = 1.52

$$-\frac{-b^2\left(\sin^8\left(fx+e\right)\right)+5\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2\left(fx+e\right)\right)}{a}}\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)-}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(-b^2*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+2*b*(cos(f*x+e)^2)^(
1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*si
n(f*x+e)^3-3*b^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+8*
(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/
a*b)^(1/2))*a*b*sin(f*x+e)^3+3*a*b*sin(f*x+e)^6-3*b^2*sin(f*x+e)^6+4*a^2*si
n(f*x+e)^4-8*a*b*sin(f*x+e)^4+4*b^2*sin(f*x+e)^4-5*a^2*sin(f*x+e)^2+5*a*b*s
in(f*x+e)^2+a^2)/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}\cot\left(fx+e\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4*(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(3/2)*cot(f*x + e)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \cot\left(e+f\,x\right)^4\,{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4*(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4*(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Timed out
```

3.510 $\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=134

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}+\frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4f(a+b)}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2}}{8f(a+b)^2}$$

```
[Out] 1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)
^(5/2)/f-1/8*(8*a+5*b)*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f+1/4*
sec(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.17, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 89, 78, 63, 208}

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{5/2}}+\frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4f(a+b)}-\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2}}{8f(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(
8*(a + b)^(5/2)*f) - ((8*a + 5*b)*Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]
)/(8*(a + b)^2*f) + (Sec[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*(a + b)*
f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a+b)+2(a+b)x}{(1-x)^2\sqrt{a+bx}}\,dx,x,\sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f}$$

$$= -\frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8(a+b)^2f} + \frac{\sec^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4(a+b)f}$$

$$= \frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{5/2}f} - \frac{(8a+5b)\sec^2(e+fx)\sqrt{a+b\sin^2(}}{8(a+b)^2f}$$

Mathematica [A] time = 0.47, size = 108, normalized size = 0.81

$$\frac{(8a^2+8ab+3b^2)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right) + \sqrt{a+b}\,\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,(2(a+b)\sec^2(e+fx) -}{8f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]] +
Sqrt[a + b]*Sec[e + f*x]^2*(-8*a - 5*b + 2*(a + b)*Sec[e + f*x]^2)*Sqrt[a +
 b*Sin[e + f*x]^2])/(8*(a + b)^(5/2)*f)
```

fricas [A] time = 0.68, size = 328, normalized size = 2.45

$$\left[\frac{(8a^2+8ab+3b^2)\sqrt{a+b}\cos(fx+e)^4\log\left(\frac{b\cos(fx+e)^2-2\sqrt{-b\cos(fx+e)^2+a+b}\sqrt{a+b}-2a-2b}{\cos(fx+e)^2}\right) - 2\left((8a^2+13ab+\right.}{16(a^3+3a^2b+3ab^2+b^3)f\cos(fx+e)^4}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*((8*a^2 + 8*a*b + 3*b^2)*sqrt(a + b)*cos(f*x + e)^4*log((b*cos(f*x +
e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x +
 e)^2) - 2*((8*a^2 + 13*a*b + 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2
)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f
*x + e)^4), -1/8*((8*a^2 + 8*a*b + 3*b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f
*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))*cos(f*x + e)^4 + ((8*a^2 + 13*a*b
+ 5*b^2)*cos(f*x + e)^2 - 2*a^2 - 4*a*b - 2*b^2)*sqrt(-b*cos(f*x + e)^2 + a
 + b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f*cos(f*x + e)^4)]
```

giac [B] time = 3.14, size = 2580, normalized size = 19.25

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/4*((8*a^2 + 8*a*b + 3*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqrt(-a -
b)) - 2*(8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 + 8*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b + 3*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^6*a^(5/2) - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^6*a^(3/2)*b - 21*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^6*sqrt(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt
(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
1/2*e)^2 + a))^5*a^3 - 408*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2
*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
 a))^5*a^2*b - 269*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a
*b^2 - 44*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*
(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1
/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) + 40*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 493*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 292*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^4 + 1304*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^3*a^3*b + 1345*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^3*a^2*b^2 + 104*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^3*a*b^3 - 176*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))^3*b^4 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
```

```
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a
^(9/2) + 600*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)
*b + 1865*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^
2 + 1880*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3
 + 528*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 -
 232*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 904*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^4*b - 1079*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3*b^2 - 60*(sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))*a^2*b^3 + 560*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))*a*b^4 + 192*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))*b^5 - 104*a^(11/2) - 584*a^(9/2)*b - 1351*a^(7/2)*b^2 - 1588*a^(5
/2)*b^3 - 912*a^(3/2)*b^4 - 192*sqrt(a)*b^5)/(((sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a))*sqrt(a) - 3*a - 4*b)^4*(a^2 + 2*a*b + b^2)))/f
```

maple [B] time = 3.43, size = 644, normalized size = 4.81

$$\left(8\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)a^4+24\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)a^3b+27\ln\left(\frac{2}{-}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/16*((8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(
f*x+e)+a))*a^4+24*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)-b*sin(f*x+e)+a))*a^3*b+27*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+14*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+3*ln(2/(1+sin(f*x+e))*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4+8*ln(2/(sin(f*x+e
)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4+24*ln(2/(
sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*
b+27*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+
e)+a))*a^2*b^2+14*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a*b^3+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4)*cos(f*x+e)^4-2*(a+b-b*cos(f*x+e)^2)^(1/2
)*(a+b)^(5/2)*(8*a+5*b)*cos(f*x+e)^2+4*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)*a+4*(a+b)^(5/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b)/(a+b)^(5/2)/cos(f*x+e)^4/(
a^2+2*a*b+b^2)/f
```

maxima [B] time = 0.42, size = 248, normalized size = 1.85

$$-\frac{\frac{\left(8a^2b^3+8ab^4+3b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\left(a^2+2ab+b^2\right)\sqrt{a+b}}-\frac{2\left(\left(8ab^4+5b^5\right)\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}-\left(8a^2b^4+11ab^5+3b^6\right)\sqrt{b\sin\left(fx+e\right)^2+a}\right)}{a^4+4a^3b+6a^2b^2+4ab^3+b^4+\left(b\sin\left(fx+e\right)^2+a\right)^2\left(a^2+2ab+b^2\right)-2\left(a^3+3a^2b+3ab^2+b^3\right)\left(b\sin\right.}}{16b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2*b^3 + 8*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt
(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*s
qrt(a + b)) - 2*((8*a*b^4 + 5*b^5)*(b*sin(f*x + e)^2 + a)^(3/2) - (8*a^2*b^
4 + 11*a*b^5 + 3*b^6)*sqrt(b*sin(f*x + e)^2 + a))/(a^4 + 4*a^3*b + 6*a^2*b^
2 + 4*a*b^3 + b^4 + (b*sin(f*x + e)^2 + a)^2*(a^2 + 2*a*b + b^2) - 2*(a^3 +
 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{{\tan\left(e+f\,x\right)}^5}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{5}{\left(e + f x \right)}}{\sqrt{a + b \sin^{2}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

3.511 $\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=81

$$\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

```
[Out] -1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f+1/
2*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)/f
```

Rubi [A] time = 0.10, antiderivative size = 81, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3194, 78, 63, 208}

$$\frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2f(a+b)}-\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(3/
2)*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*(a + b)*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{2b(a+b)f}$$

$$= -\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{3/2}f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2(a+b)f}$$

Mathematica [A] time = 0.22, size = 77, normalized size = 0.95

$$-\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a+b}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -1/2*(((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(a + b)^(
3/2) - (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(a + b))/f
```

fricas [A] time = 0.59, size = 220, normalized size = 2.72

$$\left[\frac{(2a+b)\sqrt{a+b}\cos\left(fx+e\right)^2\log\left(\frac{b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}(a\right.}{4\left(a^2+2ab+b^2\right)f\cos\left(fx+e\right)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((2*a + b)*sqrt(a + b)*cos(f*x + e)^2*log((b*cos(f*x + e)^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)
, 1/2*((2*a + b)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-
a - b)/(a + b))*cos(f*x + e)^2 + sqrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/(
(a^2 + 2*a*b + b^2)*f*cos(f*x + e)^2)]
```

giac [B] time = 1.29, size = 793, normalized size = 9.79

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] ((2*a + b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)
- sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)) - 2*(2*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sq
rt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
+ 1/2*e)^2 + a))^3*b + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))^2*a^(3/2) + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqr
t(a)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)
*b^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2
*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2*(a + b)
))/f
```

maple [B] time = 3.03, size = 353, normalized size = 4.36

$$-\left(2\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)a^2+3\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)ab+\ln\left(\frac{2\sqrt{a+}}{}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/4*(-(2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^2+3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a*b+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*b^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(
f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b+ln(2/(1+sin(f*x+e))*((a+b)^(1/
2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^2)*cos(f*x+e)^2+2*(a+b-b*c
os(f*x+e)^2)^(1/2)*(a+b)^(3/2))/(a+b)^(5/2)/cos(f*x+e)^2/f
```

maxima [A] time = 0.54, size = 124, normalized size = 1.53

$$-\frac{\frac{2\sqrt{b\sin\left(fx+e\right)^2+a}\,b^3}{\left(b\sin\left(fx+e\right)^2+a\right)(a+b)-a^2-2\,ab-b^2}-\frac{\left(2\,ab^2+b^3\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{(a+b)^{\frac{3}{2}}}}{4\,b^2 f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(2*sqrt(b*sin(f*x + e)^2 + a)*b^3/((b*sin(f*x + e)^2 + a)*(a + b) - a^
2 - 2*a*b - b^2) - (2*a*b^2 + b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a
 + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/(a + b)^(3/2))/(b^2*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^3}{\sqrt{b\,\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^3\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.512 $\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=36

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/f/(a+b)^(1/2)
```

Rubi [A] time = 0.05, antiderivative size = 36, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{\sqrt{a+b}\,f}$$

Mathematica [A] time = 0.04, size = 38, normalized size = 1.06

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a-b\cos^2(e+fx)+b}}{\sqrt{a+b}}\right)}{f\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] ArcTanh[Sqrt[a + b - b*Cos[e + f*x]^2]/Sqrt[a + b]]/(Sqrt[a + b]*f)
```

fricas [A] time = 0.53, size = 112, normalized size = 3.11

$$\left[\frac{\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2\,a-2\,b}{\cos\left(fx+e\right)^2}\right)}{2\sqrt{a+b}\,f}, -\frac{\sqrt{-a-b}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a-b}}{a+b}\right)}{(a+b)f}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b)
- 2*a - 2*b)/cos(f*x + e)^2)/(sqrt(a + b)*f), -sqrt(-a - b)*arctan(sqrt(-b*
cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b))/((a + b)*f)]
```

giac [B] time = 0.58, size = 98, normalized size = 2.72

$$-\frac{2\,\arctan\left(-\frac{\sqrt{a}\,\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}-\sqrt{a}}{2\sqrt{-a-b}}\right)}{\sqrt{-a-b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] -2*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a
))/sqrt(-a - b))/(sqrt(-a - b)*f)
```

maple [B] time = 2.91, size = 113, normalized size = 3.14

$$\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}+2b\sin\left(fx+e\right)+2a}{\sin\left(fx+e\right)-1}\right)}{2\sqrt{a+b}\,f}+\frac{\ln\left(\frac{2\sqrt{a+b}\,\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}-2b\sin\left(fx+e\right)+2a}{1+\sin\left(fx+e\right)}\right)}{2\sqrt{a+b}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/2/(a+b)^(1/2)/f*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))+1/2/(a+b)^(1/2)/f*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
```

maxima [B] time = 0.42, size = 106, normalized size = 2.94

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)}{\sqrt{a+b}}-\frac{\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)}{\sqrt{a+b}}}{2f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*
(sin(f*x + e) + 1)))/sqrt(a + b) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(sin(
f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/sqrt(a + b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\tan\left(e+f\,x\right)}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.513 $\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=33

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/f/a^(1/2)

Rubi [A] time = 0.06, antiderivative size = 33, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

[In] Int[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]

[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Mathematica [A] time = 0.03, size = 33, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{\sqrt{a}\,f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/Sqrt[a + b*Sin[e + f*x]^2],x]
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(Sqrt[a]*f))
```

fricas [A] time = 0.58, size = 100, normalized size = 3.03

$$\left[\frac{\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^2-1}\right)}{2\sqrt{a}\,f}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{-a}}{a}\right)}{af}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
[Out] [1/2*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) -
2*a - b)/(cos(f*x + e)^2 - 1))/(sqrt(a)*f), sqrt(-a)*arctan(sqrt(-b*cos(f*x
 + e)^2 + a + b)*sqrt(-a)/a)/(a*f)]
```

giac [A] time = 0.14, size = 31, normalized size = 0.94

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*f)
```

maple [A] time = 0.98, size = 42, normalized size = 1.27

$$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x)
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f
```

maxima [A] time = 0.31, size = 25, normalized size = 0.76

$$-\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
[Out] -arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/(sqrt(a)*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot\left(e+f\,x\right)}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2),x)
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(1/2),x)
[Out] Integral(cot(e + f*x)/sqrt(a + b*sin(e + f*x)**2), x)
```

3.514 $\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=75

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

```
[Out] 1/2*(2*a+b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f-1/2*csc(f*x
+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.09, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3194, 78, 63, 208}

$$\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f}-\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] ((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2/Sqrt[a]])/(2*a^(3/2)*f) - (Cs
c[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(2*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= -\frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af} - \frac{(2a+b)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{2abf}$$

$$= \frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{3/2}f} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{2af}$$

Mathematica [A] time = 0.17, size = 71, normalized size = 0.95

$$\frac{\frac{(2a+b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}} - \frac{\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a}}{2f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((2*a + b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/a^(3/2) - (Csc[e + 
 f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/a)/(2*f)
```

fricas [A] time = 0.60, size = 220, normalized size = 2.93

$$\left[\frac{\left((2\,a+b)\cos\left(fx+e\right)^2-2\,a-b\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a}-2\,a-b\right)}{\cos\left(fx+e\right)^2-1}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a}}{4\left(a^2f\cos\left(fx+e\right)^2-a^2f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2
- 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)
) + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f*x + e)^2 - a^2*f), -1
/2*(((2*a + b)*cos(f*x + e)^2 - 2*a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x +
e)^2 + a + b)*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*f*cos(f
*x + e)^2 - a^2*f)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Warning, need to choose a branch for the roo
t of a polynomial with parameters. This might be wrong.The choice was done
assuming [a,b]=[45,77]Warning, need to choose a branch for the root of a po
lynomial with parameters. This might be wrong.The choice was done assuming
[a,b]=[-97,-38]Unable to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Un
able to check sign: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sig
n: (2*pi/t_nostep/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/t_nostep
/2)>(-2*pi/t_nostep/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to
 check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/
x/2)Unable to check sign: (2*pi/x/2)>(-2*pi/x/2)Unable to check sign: (2*pi
/t_nostep/2)>(-2*pi/t_nostep/2)Evaluation time: 0.65Unable to divide, perha
ps due to rounding error%%%{1,[4,0]%%%}+%%%{%%%{-2,[1]%%%},[2,0]%%%}+%%%{%%
%{1,[2]%%%},[0,0]%%%} / %%%{%%%{1,[1]%%%},[4,0]%%%}+%%%{%%%{-2,[2]%%%},[2,0
]%%%}+%%%{%%%{1,[3]%%%},[0,0]%%%} Error: Bad Argument Value
```

maple [A] time = 1.71, size = 114, normalized size = 1.52

$$\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{\sqrt{a}\,f}-\frac{\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{2fa\sin\left(fx+e\right)^2}+\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{2f\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f-1/2/f/a
/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+1/2/f*b/a^(3/2)*ln((2*a+2*a^(1/2)*(a
+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.30, size = 77, normalized size = 1.03

$$\frac{\frac{2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{\sqrt{a}}+\frac{b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{b\sin\left(fx+e\right)^2+a}}{a\sin\left(fx+e\right)^2}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + b*arcsinh(a/(sqrt
(a*b)*abs(sin(f*x + e))))/a^(3/2) - sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x +
 e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^3}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/sqrt(a + b*sin(e + f*x)**2), x)
```

3.515 $\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=126

$$\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

```
[Out] -1/8*(8*a^2+8*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/
f+1/8*(8*a+3*b)*csc(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/4*csc(f*x+e)^
4*(a+b*sin(f*x+e)^2)^(1/2)/a/f
```

Rubi [A] time = 0.13, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 89, 78, 63, 208}

$$-\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}+\frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f}-\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8*a
^(5/2)*f) + ((8*a + 3*b)*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(8*a^2*
f) - (Csc[e + f*x]^4*Sqrt[a + b*Sin[e + f*x]^2])/(4*a*f)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-3b)+2ax}{x^2\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \ldots$$

$$= \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{8a^2f} - \frac{\csc^4(e+fx)\sqrt{a+b\sin^2(e+fx)}}{4af} + \ldots$$

$$= -\frac{\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f} + \frac{(8a+3b)\csc^2(e+fx)\sqrt{a+b\sin^2(e\ldots}}{8a^2f}$$

Mathematica [A] time = 0.36, size = 101, normalized size = 0.80

$$\frac{\sqrt{a}\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\left(-2a\csc^2(e+fx)+8a+3b\right)-\left(8a^2+8ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{5/2}f}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-((8*a^2 + 8*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]) + S
qrt[a]*Csc[e + f*x]^2*(8*a + 3*b - 2*a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f
*x]^2])/(8*a^(5/2)*f)
```

fricas [A] time = 0.61, size = 388, normalized size = 3.08

$$\left[\frac{\left(\left(8a^2+8ab+3b^2\right)\cos\left(fx+e\right)^4-2\left(8a^2+8ab+3b^2\right)\cos\left(fx+e\right)^2+8a^2+8ab+3b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+\ldots\right.\right.}{\ldots}\right.}{16\left(a^3f\cos\left(fx+e\right)^4-2a^3f\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*
cos(f*x + e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2
*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) -
 2*((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2
+ a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^3*f), 1/8*(((8
*a^2 + 8*a*b + 3*b^2)*cos(f*x + e)^4 - 2*(8*a^2 + 8*a*b + 3*b^2)*cos(f*x +
e)^2 + 8*a^2 + 8*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
b)*sqrt(-a)/a) - ((8*a^2 + 3*a*b)*cos(f*x + e)^2 - 6*a^2 - 3*a*b)*sqrt(-b*c
os(f*x + e)^2 + a + b))/(a^3*f*cos(f*x + e)^4 - 2*a^3*f*cos(f*x + e)^2 + a^
3*f)]
```

giac [B] time = 1.03, size = 898, normalized size = 7.13

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] -1/64*(sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a)*(tan(1/2*f*x + 1/2*e)^2/a - (13*a + 6*b)/a^2) - 8*
(8*a^2 + 8*a*b + 3*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))/sqrt(-a))/(sqrt(-a)*a^2) - 4*(8*a^(5/2) + 8*a^(3/2)*b + 3*sqrt(a)
*b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(
3/2) - 2*sqrt(a)*b))/a^3 + 4*(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*ta
n(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e
)^2 + a))^3*a^2 + 16*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3
*a*b + 6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) - 8*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 20*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b - 10*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a*b^2 - 7*a^(7/2) - 4*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^2))/f
```

maple [A] time = 1.97, size = 219, normalized size = 1.74

$$-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{\sqrt{a}\,f}+\frac{\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{fa\sin\left(fx+e\right)^{2}}-\frac{b\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f\,a^{\frac{3}{2}}}-\frac{\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{4fa\sin\left(fx+e\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/a^(1/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))/f+1/f/a/
sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/f*b/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*
sin(f*x+e)^2)^(1/2))/sin(f*x+e))-1/4/f/a/sin(f*x+e)^4*(a+b*sin(f*x+e)^2)^(1
```

```
/2)+3/8/f/a^2*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-3/8/f/a^(5/2)*b^2*ln(
(2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.32, size = 158, normalized size = 1.25

$$-\frac{\frac{8\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{\sqrt{a}}+\frac{8\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{3}{2}}}+\frac{3\,b^2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\,|\sin(fx+e)|}\right)}{a^{\frac{5}{2}}}-\frac{8\,\sqrt{b\sin(fx+e)^2+a}}{a\sin(fx+e)^2}-\frac{3\,\sqrt{b\sin(fx+e)^2+a}\,b}{a^2\sin(fx+e)^2}+\cdots}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/sqrt(a) + 8*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b^2*arcsinh(a/(sqrt(a*b)*abs(sin(f
*x + e))))/a^(5/2) - 8*sqrt(b*sin(f*x + e)^2 + a)/(a*sin(f*x + e)^2) - 3*sq
rt(b*sin(f*x + e)^2 + a)*b/(a^2*sin(f*x + e)^2) + 2*sqrt(b*sin(f*x + e)^2 +
 a)/(a*sin(f*x + e)^4))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^5}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/sqrt(a + b*sin(e + f*x)**2), x)
```

3.516 $\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=246

$$-\frac{2(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{a\sqrt{\cos^2(e+fx)}\ \mathrm{s}}{}$$

```
[Out] 2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1
/2)*(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*a*Ell
ipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*
x+e)^2/a)^(1/2)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a+b)*(a+b*sin(f*x+e
)^2)^(1/2)*tan(f*x+e)/(a+b)^2/f+1/3*sec(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)*t
an(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.24, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{2(2a+b)\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)^2}+\frac{\tan(e+fx)\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3f(a+b)}-\frac{a\sqrt{\cos^2(e+fx)}\ \mathrm{s}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(2*a + b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*S
ec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^2*f*Sqrt[1 + (b*Sin[e +
f*x]^2)/a]) - (a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a
)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)*f*Sqrt[a + b*Sin
[e + f*x]^2]) - (2*(2*a + b)*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)^2*f) + (Sec[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/(3*(a
 + b)*f)
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{(1-x^2)^{5/2}\sqrt{a+bx^2}}\,dx,x,\sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Su}}{3}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f}$$

$$= -\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{3(a+b)^2f} + \frac{\sec^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3(a+b)f}$$

$$= \frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+}}{3(a+b)^2f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 2.14, size = 188, normalized size = 0.76

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(2\left(4a^2+3ab+b^2\right)\cos(2(e+fx))+(2a+b)(2a-b\cos(4(e+fx))-b)\right)}{\sqrt{2}} - 2a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{6f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (4*a*(2*a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 2*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((2*(4*a^2 + 3*a*b + b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a -
b - b*Cos[4*(e + f*x)]))*Sec[e + f*x]^2*Tan[e + f*x])/Sqrt[2])/(6*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b*cos(f*x + e)^2
- a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 2.83, size = 377, normalized size = 1.53

$$-\frac{2\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\,b\left(2a+b\right)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*(2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(2*a+b)*sin(f*x+e)*cos
(f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(4*a^2+7*a*b+3*b^2)*co
s(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b
+b^2)*sin(f*x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^
(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(EllipticF(sin(f*x+e),(-1/a*b)^(1
/2))*a+EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*EllipticE(sin(f*x+e),(-1/a*
b)^(1/2))*a-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(1+sin(
f*x+e))/(sin(f*x+e)-1)/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f
*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\tan\left(e+f\,x\right)}^4}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(e + f x \right)}}{\sqrt{a + b \sin^{2}{\left(e + f x \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.517 $\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=109

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*si
n(f*x+e)^2)^(1/2)/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)+(a+b*sin(f*x+e)^2)^(1/
2)*tan(f*x+e)/(a+b)/f
```

Rubi [A] time = 0.12, antiderivative size = 109, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3196, 471, 426, 424}

$$\frac{\tan(e+fx)\sqrt{a+b\sin^2(e+fx)}}{f(a+b)}-\frac{\sqrt{\cos^2(e+fx)}\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)}{f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])) +
 (Sqrt[a + b*Sin[e + f*x]^2]*Tan[e + f*x])/((a + b)*f)
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
```

```
e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\,d\right.}{(a+b)f}$$

$$= \frac{\sqrt{a+b\sin^2(e+fx)}\,\tan(e+fx)}{(a+b)f} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx}\right.}{(a+b)f\sqrt{1+\frac{b\,s}{}}}$$

$$= -\frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}}{(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}} + \frac{\sqrt{a}}{}$$

Mathematica [A] time = 0.38, size = 100, normalized size = 0.92

$$\frac{\sqrt{2}\,\tan(e+fx)(2a-b\cos(2(e+fx))+b)-2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{2f(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (-2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + S
qrt[2]*(2*a + b - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(2*(a + b)*f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.50, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b*cos(f*x + e)^2
 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [B] time = 2.76, size = 222, normalized size = 2.04

$$\frac{-\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\, b\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right.}}{\left(a+b\right)\sqrt{-\left(a+b\left(\sin^2\left(fx+\right.\right.\right.}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] (-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*sin(f*x+e)*cos(f*x+e)^2+(-b*
cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a+b)*sin(f*x+e)-a*(cos(f*x+e)^2)^(1
/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^
(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))/(a+b)/(-(a+b*sin(f*x+e)^2)*(sin
(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(tan(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^2}{\sqrt{b\sin\left(e+f\,x\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.518 $\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] (cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin
(f*x+e)^2/a)^(1/2)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.03, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3183, 3182}

$$\frac{\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (EllipticF[e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(f*Sqrt[a + b*S
in[e + f*x]^2])
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{1}{\sqrt{a+b\sin^2(e+fx)}}\,dx &= \frac{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}\int \frac{1}{\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}\,dx}{\sqrt{a+b\sin^2(e+fx)}}\\ &= \frac{F\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}{f\sqrt{a+b\sin^2(e+fx)}}\end{aligned}$$

Mathematica [A] time = 0.08, size = 60, normalized size = 1.18

$$\frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(f*Sqrt
[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.43, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b\cos\left(fx+e\right)^2-a-b},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b*cos(f*x + e)^2 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [C] time = 0.37, size = 60, normalized size = 1.18

$$\frac{\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)-a-b}{a}}\,\mathrm{am}^{-1}\left(fx+e\middle|\frac{i\sqrt{b}}{\sqrt{a}}\right)}{f\sqrt{a+b-b\left(\cos^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] 1/f/(a+b-b*cos(f*x+e)^2)^(1/2)*(-(b*cos(f*x+e)^2-a-b)/a)^(1/2)*InverseJacob
iAM(f*x+e,I/a^(1/2)*b^(1/2))
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{1}{\sqrt{b\sin\left(e+fx\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + b\sin^2\left(e + fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + b*sin(e + f*x)**2), x)
```

3.519 $\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=106

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] -cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a/f-EllipticE(sin(f*x+e),(-b/a)^(1/2))
*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a/f/(1+b*sin(f*x+
e)^2/a)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3196, 475, 21, 426, 424}

$$-\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af}-\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{af\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a*f)) - (Sqrt[Cos[e + f*x]^2]*
EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x
]^2])/(a*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)]])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 475

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q)
/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a + b*x^n)^
p*(c + d*x^n)^(q - 1)*Simp[c*b*(m + 1) + n*(b*c*(p + 1) + a*d*q) + d*(b*(m
+ 1) + b*n*(p + q + 1))*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, p}, x] &&
NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[0, q, 1] && LtQ[m, -1] && IntBinomia
```

```
lQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-a-b}{\sqrt{1-x^2}\,\sqrt{}}\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{\sqrt{1-x^2}}\right.}{af}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+f}\right.}{af\sqrt{1+\frac{b\sin^2}{}}}$$

$$= -\frac{\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{af} - \frac{\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec(}{af\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.36, size = 101, normalized size = 0.95

$$-\frac{\cot(e+fx)\sqrt{2a-b\cos(2(e+fx))+b}}{\sqrt{2}\,af} - \frac{\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] -((Sqrt[2*a + b - b*Cos[2*(e + f*x)]]*Cot[e + f*x])/(Sqrt[2]*a*f)) - (Sqrt[
(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)])/(f*Sqrt[2*a +
 b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^2}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b*cos(f*x + e)^2
- a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.58, size = 120, normalized size = 1.13

$$-\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\sin\left(fx+e\right)\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-b\left(\cos^{4}\left(fx+e\right)\right)+(}{a\sin\left(fx+e\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*sin(f*x+e)*Ellip
ticE(sin(f*x+e),(-1/a*b)^(1/2))-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)/a/sin(f*
x+e)/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(f*x + e)^2/sqrt(b*sin(f*x + e)^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^{2}}{\sqrt{b\sin\left(e+f\,x\right)^{2}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{2}\left(e+fx\right)}{\sqrt{a+b\sin^{2}\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/sqrt(a + b*sin(e + f*x)**2), x)
```

3.520 $\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx$

Optimal. Leaf size=240

$$\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}+\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] 2/3*(2*a+b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f-1/3*cot(f*x+e)*csc(f*
x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a/f+2/3*(2*a+b)*EllipticE(sin(f*x+e),(-b/a)
^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b
*sin(f*x+e)^2/a)^(1/2)-1/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x
+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a/f/(a+b*sin(f*x+e)^2)^
(1/2)
```

Rubi [A] time = 0.27, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3196, 474, 583, 524, 426, 424, 421, 419}

$$\frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f}+\frac{2(2a+b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (2*(2*a + b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*f) - (Cot[e +
f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*f) + (2*(2*a + b)*Sqrt
[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[
a + b*Sin[e + f*x]^2])/(3*a^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((a + b)*
Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*S
qrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 474

```
Int[((e_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_)
)^(q_), x_Symbol] :> Simp[(c*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
(q - 1))/(a*e*(m + 1)), x] - Dist[1/(a*e^n*(m + 1)), Int[(e*x)^(m + n)*(a +
 b*x^n)^p*(c + d*x^n)^(q - 2)*Simp[c*(c*b - a*d)*(m + 1) + c*n*(b*c*(p + 1)
 + a*d*(q - 1)) + d*((c*b - a*d)*(m + 1) + c*b*n*(p + q))*x^n, x], x], x] /
; FreeQ[{a, b, c, d, e, p}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && GtQ[q,
 1] && LtQ[m, -1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\sqrt{a+bx^2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= -\frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3af} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Sub}}{}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e}}{3af}$$

$$= \frac{2(2a+b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2f} - \frac{\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e}}{3af}$$

Mathematica [A] time = 3.97, size = 186, normalized size = 0.78

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left((2a+b)(2a+b\cos(4(e+fx))+3b)-2\left(4a^2+5ab+2b^2\right)\cos(2(e+fx))\right)}{\sqrt{2}} - 2a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right)+}{6a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/Sqrt[a + b*Sin[e + f*x]^2],x]
```

```
[Out] (((-2*(4*a^2 + 5*a*b + 2*b^2)*Cos[2*(e + f*x)] + (2*a + b)*(2*a + 3*b + b*C
os[4*(e + f*x)]))*Cot[e + f*x]*Csc[e + f*x]^2)/Sqrt[2] + 4*a*(2*a + b)*Sqrt
[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] - 2*a*(a + b)
*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(6*a^2*
f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4}{b\cos\left(fx+e\right)^2-a-b}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b*cos(f*x + e)^2
 - a - b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\sqrt{b\sin\left(fx+e\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/sqrt(b*sin(f*x + e)^2 + a), x)
```

maple [A] time = 1.96, size = 351, normalized size = 1.46

$$-\frac{\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^2(fx+e)\right)}{a}}\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2\left(\sin^3\left(fx+e\right)\right)+b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-4*(cos(f*x+
e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*a^2*sin(f*x+e)^3-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Ell
ipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+4*a*b*sin(f*x+e)^6+2*b^2
*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-2*b^2*sin(f*x+e)^4-5*a^
2*sin(f*x+e)^2-a*b*sin(f*x+e)^2+a^2)/a^2/sin(f*x+e)^3/cos(f*x+e)/(a+b*sin(f
*x+e)^2)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cot\left(e+f\,x\right)}^4}{\sqrt{b\,{\sin\left(e+f\,x\right)}^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4\left(e+fx\right)}{\sqrt{a+b\sin^2\left(e+fx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(1/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/sqrt(a + b*sin(e + f*x)**2), x)
```

3.521 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=177

$$-\frac{8a^2-8ab-b^2}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{}{8f(}$$

```
[Out] 1/8*(8*a^2-8*a*b-b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(
7/2)/f+1/8*(-8*a^2+8*a*b+b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/8*(8*a+3
*b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/4*sec(f*x+e)^4/(a+b)/
f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {3194, 89, 78, 51, 63, 208}

$$-\frac{8a^2-8ab-b^2}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8f(a+b)^{7/2}}+\frac{\sec^4(e+fx)}{4f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{}{8f(}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((8*a^2 - 8*a*b - b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(8*
(a + b)^(7/2)*f) - (8*a^2 - 8*a*b - b^2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e +
f*x]^2]) - ((8*a + 3*b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2]) + Sec[e + f*x]^4/(4*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-b)+2(a+b)x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^4(e+fx)}{4(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(8a^2-8a\right.}{}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{4(a+b)}$$

$$= -\frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(8a+3b)\sec^2(e+fx)}{8(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{4(a+b)}$$

$$= \frac{\left(8a^2-8ab-b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{7/2}f} - \frac{8a^2-8ab-b^2}{8(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - $$

Mathematica [C] time = 0.47, size = 107, normalized size = 0.60

$$\frac{\left(-8a^2+8ab+b^2\right){}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)-\frac{1}{2}(a+b)\sec^4(e+fx)((8a+3b)\cos(2(e+fx))+4a-b)}{8f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((-8*a^2 + 8*a*b + b^2)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]
^2)/(a + b)] - ((a + b)*(4*a - b + (8*a + 3*b)*Cos[2*(e + f*x)])*Sec[e + f*
x]^4)/2)/(8*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [A] time = 0.78, size = 593, normalized size = 3.35

$$\left[-\frac{\left(\left(8\,a^{2}b-8\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{6}-\left(8\,a^{3}-9\,ab^{2}-b^{3}\right)\cos\left(fx+e\right)^{4}\right)\sqrt{a+b}\,\log\left(\frac{b\cos\left(fx+e\right)^{2}+2\sqrt{-b\cos\left(fx+e\right)^{2}+a}}{\cos\left(fx+e\right)^{2}}\right.}{16\left(\left(a^{4}b+4\,a^{3}b^{2}+6\,a^{2}b^{3}+4\,ab^{4}+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/16*(((8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)
*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)
^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) - 2*((8*a^3 - 9*a*b^2
- b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2 - 2*b^3 + (8*a^3 + 19*a^2
*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*cos(f*x + e)^6 - (a^5 + 5*a
^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f*cos(f*x + e)^4), -1/8*(((
8*a^2*b - 8*a*b^2 - b^3)*cos(f*x + e)^6 - (8*a^3 - 9*a*b^2 - b^3)*cos(f*x +
 e)^4)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a
+ b)) - ((8*a^3 - 9*a*b^2 - b^3)*cos(f*x + e)^4 - 2*a^3 - 6*a^2*b - 6*a*b^2
 - 2*b^3 + (8*a^3 + 19*a^2*b + 14*a*b^2 + 3*b^3)*cos(f*x + e)^2)*sqrt(-b*co
s(f*x + e)^2 + a + b))/((a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f*c
os(f*x + e)^6 - (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*f
*cos(f*x + e)^4)]
```

giac [B] time = 4.03, size = 2776, normalized size = 15.68

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/4*(4*((a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*b^5)*tan(1/2*f*x
+ 1/2*e)^2/(a^7*b + 7*a^6*b^2 + 21*a^5*b^3 + 35*a^4*b^4 + 35*a^3*b^5 + 21*a
^2*b^6 + 7*a*b^7 + b^8) + (a^6*b + 4*a^5*b^2 + 6*a^4*b^3 + 4*a^3*b^4 + a^2*
b^5)/(a^7*b + 7*a^6*b^2 + 21*a^5*b^3 + 35*a^4*b^4 + 35*a^3*b^5 + 21*a^2*b^6
 + 7*a*b^7 + b^8))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + (8*a^2 - 8*a*b - b^2)*arctan(-1/2*(s
qrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2
*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/
((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a - b)) - 2*(8*(sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^6*a^(5/2) - 32*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
6*a^(3/2)*b - 25*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
```

```
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqr
t(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 -
 352*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b - 113*(sq
rt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*
f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 28*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x +
 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*
b*tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) - 64*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 561*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 - 116*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^3*a^4 + 1088*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^3*a^3*b + 597*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3
*a^2*b^2 - 504*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3
 - 112*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqr
t(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f
*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 608*(sqrt(a)*t
an(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1
/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 1565*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 952*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 - 176*(sqrt(a)*tan(1/2*
f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2
 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 736*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))*a^4*b - 483*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*
tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2
*e)^2 + a))*a^3*b^2 + 532*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*a^2*b^3 + 496*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^
4 - 64*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 - 104*a^(11
/2) - 512*a^(9/2)*b - 979*a^(7/2)*b^2 - 836*a^(5/2)*b^3 - 208*a^(3/2)*b^4 +
 64*sqrt(a)*b^5)/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*((sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^4))/f
```

maple [B] time = 13.20, size = 3763, normalized size = 21.26

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/16/(a^4*b^2*cos(f*x+e)^4+4*a^3*b^3*cos(f*x+e)^4+6*a^2*b^4*cos(f*x+e)^4+4
*a*b^5*cos(f*x+e)^4+b^6*cos(f*x+e)^4-2*a^5*b*cos(f*x+e)^2-10*a^4*b^2*cos(f*
x+e)^2-20*a^3*b^3*cos(f*x+e)^2-20*a^2*b^4*cos(f*x+e)^2-10*a*b^5*cos(f*x+e)^
2-2*b^6*cos(f*x+e)^2+a^6+6*a^5*b+15*a^4*b^2+20*a^3*b^3+15*a^2*b^4+6*a*b^5+b
^6)/cos(f*x+e)^4/(a+b)^(3/2)*(-4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a^3
-4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*b^3+ln(2/(1+sin(f*x+e))*((a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^6*cos(f*x+e)^8+ln(2/(sin(
f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*cos(
f*x+e)^8-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*si
n(f*x+e)+a))*b^6*cos(f*x+e)^6-2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^6-8*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^6*cos(f*x+e)^4+ln(
2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b
^6*cos(f*x+e)^4-8*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/
2)+b*sin(f*x+e)+a))*a^6*cos(f*x+e)^4+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^6*cos(f*x+e)^4-2*(a+b)^(3/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)*b^4*cos(f*x+e)^8-2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(
3/2)*b^3*cos(f*x+e)^6+4*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b^4*cos(f*x+
e)^6+8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^3*cos(f*x+e)^4
-12*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a^2*b-12*(a+b)^(3/2)*(a+b-b*cos(
f*x+e)^2)^(3/2)*a*b^2-8*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8-8*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^8+9*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*a^2*b^4*cos(f*x+e)^8+10*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8-8*ln(2/(sin(f*x+e)-1)*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^8-8*ln
(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*
a^3*b^3*cos(f*x+e)^8+9*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^8-2*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^6-38*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
*a^2*b^4*cos(f*x+e)^6+10*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)
^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^8+16*ln(2/(1+sin(f*x+e))*((a+b)
^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^6+32*ln
(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*
a^4*b^2*cos(f*x+e)^6-8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*
a^4*cos(f*x+e)^4+16*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^4*cos(f*x+e)^4
-2*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b^4*cos(f*x+e)^4+16*(a+b)^(3/2)*(
a+b-b*cos(f*x+e)^2)^(3/2)*a^3*cos(f*x+e)^2+6*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^
2)^(3/2)*b^3*cos(f*x+e)^2-22*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^6+16*ln(2/(sin(f*x+e)-1)*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^6+1
2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+
a))*a*b^5*cos(f*x+e)^4-24*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e
)^2)^(1/2)+b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^4-15*ln(2/(sin(f*x+e)-1)*((a+b
)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^4+20
*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a
))*a^3*b^3*cos(f*x+e)^4+30*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^4+30*ln(2/(1+sin(f*x+e))*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^4*cos(f*x+e)^4
+12*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e
)+a))*a*b^5*cos(f*x+e)^4+32*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^4*b^2*cos(f*x+e)^6-2*ln(2/(sin(f*x+e)-1)*((
a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^6
-38*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e
)+a))*a^2*b^4*cos(f*x+e)^6-22*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^5*cos(f*x+e)^6-24*ln(2/(1+sin(f*x+e))*(
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^5*b*cos(f*x+e)^4-
```

```
15*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)
+a))*a^4*b^2*cos(f*x+e)^4+20*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b^3*cos(f*x+e)^4+16*(a+b)^(3/2)*(a+b-b*c
os(f*x+e)^2)^(3/2)*a*b^2*cos(f*x+e)^6+16*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^
2+b^3)/b^2)^(1/2)*a^3*b*cos(f*x+e)^6-64*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^6-80*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^
2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^6-32*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)*a^3*b*cos(f*x+e)^6+36*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a*b^3*cos(
f*x+e)^6-8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a*b^2*cos(f*
x+e)^4-32*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a^2*b*cos(f*x+e)^4-32*(a+b
)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b^2*cos(f*x+e)^4+24*(a+b)^(3/2)*(-b*co
s(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^3*b*cos(f*x+e)^4+72*(a+b)^(3/2)*(-b*cos
(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^4+40*(a+b)^(3/2)*(-b*co
s(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^4+16*(a+b)^(3/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)*a^3*b*cos(f*x+e)^4-18*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^
(1/2)*a^2*b^2*cos(f*x+e)^4-20*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a*b^3*
cos(f*x+e)^4+38*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a^2*b*cos(f*x+e)^2+2
8*(a+b)^(3/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b^2*cos(f*x+e)^2-8*(a+b)^(3/2)*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b^2*cos(f*x+e)^8+40*(a+b)^(3/2)*
(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^3*cos(f*x+e)^8+16*(a+b)^(3/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)*a^2*b^2*cos(f*x+e)^8-16*(a+b)^(3/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)*a*b^3*cos(f*x+e)^8+8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)
/b^2)^(3/2)*a^2*b*cos(f*x+e)^6+8*(a+b)^(3/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(3/2)*a*b^2*cos(f*x+e)^6)/f
```

maxima [**B**] time = 0.79, size = 334, normalized size = 1.89

$$-\frac{\frac{\left(8\,a^2b^3-8\,ab^4-b^5\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^2+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^2+a}+\sqrt{a+b}}\right)}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\sqrt{a+b}}+\frac{2\left(8\,a^4b^3+16\,a^3b^4+8\,a^2b^5+\left(8\,a^2b^3-8\,ab^4-b^5\right)\left(b\sin\left(fx+e\right)^2+a\right)^2-\left(16\,a^3b\right.\right.}{\left(a^3+3\,a^2b+3\,ab^2+b^3\right)\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}-2\left(a^4+4\,a^3b+6\,a^2b^2+4\,ab^3+b^4\right)\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}}{16\,b^3f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/16*((8*a^2*b^3 - 8*a*b^4 - b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a
 + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*sqrt(a + b)) + 2*(8*a^4*b^3 + 16*a^3*b^4 + 8*a^2*b^5 + (8*a^2*b^3
- 8*a*b^4 - b^5)*(b*sin(f*x + e)^2 + a)^2 - (16*a^3*b^3 + 8*a^2*b^4 - 7*a*b
^5 + b^6)*(b*sin(f*x + e)^2 + a))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f
*x + e)^2 + a)^(5/2) - 2*(a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin
(f*x + e)^2 + a)^(3/2) + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4
 + b^5)*sqrt(b*sin(f*x + e)^2 + a)))/(b^3*f)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^5}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e + fx\right)}{\left(a + b\sin^2\left(e + fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.522 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{2a-b}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -1/2*(2*a-b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f+1/
2*(2*a-b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+1/2*sec(f*x+e)^2/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-b}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{5/2}}+\frac{\sec^2(e+fx)}{2f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(5/
2)*f) + (2*a - b)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Sec[e + f*x]
^2/(2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subs}}{}$$

$$= \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)}{2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a-b)\,\text{Subs}}{}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{5/2}f} + \frac{2a-b}{2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{se}}{2(a+b)f}$$

Mathematica [C] time = 0.11, size = 75, normalized size = 0.64

$$\frac{(2a-b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)+(a+b)\sec^2(e+fx)}{2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a - b)*Hypergeometric2F1[-1/2, 1, 1/2, (a + b*Sin[e + f*x]^2)/(a + b)]
+ (a + b)*Sec[e + f*x]^2)/(2*(a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [B] time = 0.62, size = 445, normalized size = 3.77

$$\left[-\frac{\left(\left(2ab-b^2\right)\cos\left(fx+e\right)^4-\left(2a^2+ab-b^2\right)\cos\left(fx+e\right)^2\right)\sqrt{a+b}\,\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-}{\cos\left(fx+e\right)^2}\right.}{4\left(\left(a^3b+3a^2b^2+3ab^3+b^4\right)f\cos\left(fx+e\right)^4-\left(a^4+4a^3b+6\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b - b^2)*cos(f*x + e)^2)*
sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(
a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*((2*a^2 + a*b - b^2)*cos(f*x + e)^2
 + a^2 + 2*a*b + b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2
+ 3*a*b^3 + b^4)*f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 +
b^4)*f*cos(f*x + e)^2), 1/2*(((2*a*b - b^2)*cos(f*x + e)^4 - (2*a^2 + a*b -
 b^2)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*s
qrt(-a - b)/(a + b)) - ((2*a^2 + a*b - b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4)*
f*cos(f*x + e)^4 - (a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*f*cos(f*x +
e)^2)]
```

giac [B] time = 1.61, size = 1011, normalized size = 8.57

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] (((a^4*b + 3*a^3*b^2 + 3*a^2*b^3 + a*b^4)*tan(1/2*f*x + 1/2*e)^2/(a^5*b + 5
*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6) + (a^4*b + 3*a^3*b^2 +
3*a^2*b^3 + a*b^4)/(a^5*b + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 +
 b^6))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan
(1/2*f*x + 1/2*e)^2 + a) + (2*a - b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^2 + 2*a*b + b^2)*sqr
t(-a - b)) - 2*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a
+ (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sqrt(a)*tan
(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2
*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*f*x + 1/2
*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*ta
n(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^2
 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*sqrt(a) - 3*a - 4*b)^2*(a^2 + 2*a*b + b^2)))/f
```

maple [B] time = 10.46, size = 2194, normalized size = 18.59

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/4/(a^3*b^2*cos(f*x+e)^4+3*a^2*b^3*cos(f*x+e)^4+3*a*b^4*cos(f*x+e)^4+b^5*c
os(f*x+e)^4-2*a^4*b*cos(f*x+e)^2-8*a^3*b^2*cos(f*x+e)^2-12*a^2*b^3*cos(f*x+
e)^2-8*a*b^4*cos(f*x+e)^2-2*b^5*cos(f*x+e)^2+a^5+5*a^4*b+10*a^3*b^2+10*a^2*
b^3+5*a*b^4+b^5)/cos(f*x+e)^2/(a+b)^(1/2)*(2*b^3/(a+b)^(1/2)*(a+b-b*cos(f*x
```

```
+e)^2)^(3/2)+2/(a+b)^(1/2)*a^3*(a+b-b*cos(f*x+e)^2)^(3/2)+6*b/(a+b)^(1/2)*a
^2*(a+b-b*cos(f*x+e)^2)^(3/2)+6*b^2/(a+b)^(1/2)*a*(a+b-b*cos(f*x+e)^2)^(3/2
)-cos(f*x+e)^6*(2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a-6*(
a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b-4*(a+b-b*cos(f*x+e)^2)
^(1/2)*(a+b)^(1/2)*a+2*b*(a+b-b*cos(f*x+e)^2)^(1/2)*(a+b)^(1/2)+2*ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2+ln(
2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a
*b-ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)
+a))*b^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*si
n(f*x+e)+a))*a^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a*b-ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*b^2)*b^2+2*cos(f*x+e)^4*((a+b)^(1/2)*(-b*cos(f*x+e
)^2+(a*b^2+b^3)/b^2)^(3/2)*a+(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^
(3/2)*b+(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*b+2*(a+b)^(1/2)*(-b*cos(f*x+
e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2-4*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/
b^2)^(1/2)*a*b-6*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2-4*
(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)*a*b+2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*b^2+2*ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3+3*a^2*b*l
n(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))
-b^3*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+
e)+a))+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(
f*x+e)+a))*a^3+3*a^2*b*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))-b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*
x+e)^2)^(1/2)+b*sin(f*x+e)+a)))*b+cos(f*x+e)^2*(2*(a+b)^(1/2)*(-b*cos(f*x+e
)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2-2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b
^2)^(3/2)*b^2-4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(3/2)*a*b-4*(a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(3/2)*b^2-2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2
)^(1/2)*a^3+2*(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2*b+10*
(a+b)^(1/2)*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^2+6*(a+b)^(1/2)*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^3+4*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)*a^3+6*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2*b-2*(a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)*b^3-2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^4-5*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^3*b-3*ln(2/(1+sin(f*x+e))*((a+b)^
(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^2+ln(2/(1+sin(f*x+e
))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a*b^3+ln(2/(1+s
in(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b^4-2*l
n(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))
*a^4-5*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*
x+e)+a))*a^3*b-3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2
)+b*sin(f*x+e)+a))*a^2*b^2+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+
e)^2)^(1/2)+b*sin(f*x+e)+a))*a*b^3+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*
cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b^4))/f
```

maxima [A] time = 1.07, size = 193, normalized size = 1.64

$$\frac{\frac{\left(2\,ab^{2}-b^{3}\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}+\sqrt{a+b}}\right)}{\left(a^{2}+2\,ab+b^{2}\right)\sqrt{a+b}}-\frac{2\left(2\,a^{2}b^{2}+2\,ab^{3}-\left(2\,ab^{2}-b^{3}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)\right)}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}\left(a^{2}+2\,ab+b^{2}\right)-\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{b\sin\left(fx+e\right)^{2}+a}}}{4\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/4*((2*a*b^2 - b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(sqrt(b
*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^2 + 2*a*b + b^2)*sqrt(a + b)) - 2*
(2*a^2*b^2 + 2*a*b^3 - (2*a*b^2 - b^3)*(b*sin(f*x + e)^2 + a))/((b*sin(f*x
```

```
+ e)^2 + a)^(3/2)*(a^2 + 2*a*b + b^2) - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqr
t(b*sin(f*x + e)^2 + a)))/(b^2*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^3}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.523 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=63

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{1}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(3/2)/f-1/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 63, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{3/2}}-\frac{1}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(3/2)*f) - 1/((a +
 b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}-\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{b(a+b)f}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{3/2}f} - \frac{1}{(a+b)f\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 0.07, size = 54, normalized size = 0.86

$$-\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{f(a+b)\sqrt{a-b\cos^2(e+fx)+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(Hypergeometric2F1[-1/2, 1, 1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a + b)*
f*Sqrt[a + b - b*Cos[e + f*x]^2]))
```

fricas [B] time = 0.56, size = 281, normalized size = 4.46

$$\left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a+b}\,\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}-2a-2b}{\cos\left(fx+e\right)^2}\right)+2\sqrt{-b\cos\left(fx+e\right)^2+a+}}{2\left(\left(a^2b+2ab^2+b^3\right)f\cos\left(fx+e\right)^2-\left(a^3+3a^2b+3ab^2+b^3\right)f\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x +
 e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f), -((b*cos(f*x + e)^2 - a - b)*sq
rt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - s
qrt(-b*cos(f*x + e)^2 + a + b)*(a + b))/((a^2*b + 2*a*b^2 + b^3)*f*cos(f*x
+ e)^2 - (a^3 + 3*a^2*b + 3*a*b^2 + b^3)*f)]
```

giac [B] time = 0.89, size = 250, normalized size = 3.97

$$-\frac{\frac{\frac{\left(a^2b+2\,ab^2+b^3\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^3b+3\,a^2b^2+3\,ab^3+b^4}+\frac{a^2b+2\,ab^2+b^3}{a^3b+3\,a^2b^2+3\,ab^3+b^4}}{\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}}+\frac{2\arctan\left(-\frac{\sqrt{a}\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,b\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+a}-\sqrt{a}}{2\,\sqrt{-a-b}}\right)}{(a+b)\sqrt{-a-b}}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -(((a^2*b + 2*a*b^2 + b^3)*tan(1/2*f*x + 1/2*e)^2/(a^3*b + 3*a^2*b^2 + 3*a*
b^3 + b^4) + (a^2*b + 2*a*b^2 + b^3)/(a^3*b + 3*a^2*b^2 + 3*a*b^3 + b^4))/s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a) + 2*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a + b)*sqrt(-a - b)))/f
```

maple [B] time = 8.49, size = 1317, normalized size = 20.90

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/2/a/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(f*x+e)^4-2*a^3*b*c
os(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2*b^4*cos(f*x+e)^2+
a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)
*b^2-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^3-2*(a+b-b*cos(f*x+e)^2)^(1/
2)*a*b^2-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a^2+(-b*cos(f*x+e)^2+(a*b^
2+b^3)/b^2)^(1/2)*a^3-2*(a+b-b*cos(f*x+e)^2)^(1/2)*a^3+a^2*b*(-b*cos(f*x+e)
^2+(a*b^2+b^3)/b^2)^(1/2)-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a*b^2-4*a
^2*b*(a+b-b*cos(f*x+e)^2)^(1/2)+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b^2+ln(2/(sin(f*x+e)-1)*((a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a^3+ln(2/
(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+
b)^(1/2)*a*b^2+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-
b*sin(f*x+e)+a))*(a+b)^(1/2)*a^3+2*a^2*b*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(
a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)+2*a^2*b*ln(2/(sin(f*
x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2
)+b^2*(ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*
x+e)+a))*(a+b)^(1/2)*a+ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2)*a-2*(a+b-b*cos(f*x+e)^2)^(1/2)*a+(-b*c
os(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a-(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2
)*b)*cos(f*x+e)^4-b*(2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2
)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a^2+2*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)
*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*(a+b)^(1/2)*a*b+2*ln(2/(sin(f*
x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*(a+b)^(1/2
)*a^2+2*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f
*x+e)+a))*(a+b)^(1/2)*a*b+(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*a+(-b*cos
(f*x+e)^2+(a*b^2+b^3)/b^2)^(3/2)*b-4*(a+b-b*cos(f*x+e)^2)^(1/2)*a^2-4*(a+b-
b*cos(f*x+e)^2)^(1/2)*a*b+2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^2-2*(
-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*b^2)*cos(f*x+e)^2)/f
```

maxima [B] time = 0.66, size = 143, normalized size = 2.27

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}\right)}{(a+b)^{\frac{3}{2}}}-\frac{\operatorname{arsinh}\left(-\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}-\frac{a}{\sqrt{ab}\left(\sin\left(fx+e\right)-1\right)}\right)}{(a+b)^{\frac{3}{2}}}+\frac{2}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a+\sqrt{b\sin\left(fx+e\right)^2+a}\,b}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/2*(arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) + 1)) - a/(sqrt(a*b)*
(sin(f*x + e) + 1)))/(a + b)^(3/2) - arcsinh(-b*sin(f*x + e)/(sqrt(a*b)*(si
n(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e) - 1)))/(a + b)^(3/2) + 2/(sqr
t(b*sin(f*x + e)^2 + a)*a + sqrt(b*sin(f*x + e)^2 + a)*b))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan\left(e+f\,x\right)}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.524 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=57

$$\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(3/2)/f+1/a/f/(a+b*sin(f*x+e)^
2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 57, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{af\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(3/2)*f)) + 1/(a*f*Sqrt[a
+ b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^2(e+fx)\right)}{2af}$$
$$= \frac{1}{af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^2(e+fx)}\right)}{abf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{3/2}f} + \frac{1}{af\sqrt{a+b\sin^2(e+fx)}}$$

Mathematica [C] time = 0.05, size = 46, normalized size = 0.81

$$\frac{{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)}{af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a]/(a*f*Sqrt[a + b*S
in[e + f*x]^2])
```

fricas [B] time = 0.57, size = 225, normalized size = 3.95

$$\left[\frac{\left(b\cos\left(fx+e\right)^2-a-b\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-b\right)}{\cos\left(fx+e\right)^2-1}\right)-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,a}{2\left(a^2bf\cos\left(fx+e\right)^2-\left(a^3+a^2b\right)f\right)},\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/2*((b*cos(f*x + e)^2 - a - b)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-
b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*sqrt
(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 - (a^3 + a^2*b)*f),
((b*cos(f*x + e)^2 - a - b)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/a) - sqrt(-b*cos(f*x + e)^2 + a + b)*a)/(a^2*b*f*cos(f*x + e)^2 -
 (a^3 + a^2*b)*f)]
```

giac [A] time = 0.16, size = 57, normalized size = 1.00

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,af}+\frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,af}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a*f) + 1/(sqrt(b*sin(
f*x + e)^2 + a)*a*f)
```

maple [A] time = 1.58, size = 64, normalized size = 1.12

$$\frac{1}{af\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^{2}\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f\,a^{\frac{3}{2}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a/f/(a+b*sin(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.30, size = 46, normalized size = 0.81

$$-\frac{\frac{\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}-\frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a}}{f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -(arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) - 1/(sqrt(b*sin(f*x + e)
^2 + a)*a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\cot\left(e+f\,x\right)}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.525 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=110

$$\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] 1/2*(2*a+3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/2*(-2*a
-3*b)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)^2
)^(1/2)
```

Rubi [A] time = 0.12, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$-\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f}-\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((2*a + 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(5/2)*f) - (
2*a + 3*b)/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc[e + f*x]^2/(2*a*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int \frac{}{x}\right.}{}$$

$$= -\frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e+fx)}{2af\sqrt{a+b\sin^2(e+fx)}} - \frac{(2a+3b)\,\text{Subst}\left(\int -\right.}{}$$

$$= \frac{(2a+3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{5/2}f} - \frac{2a+3b}{2a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^2(e}{2af\sqrt{a+b\sin}}$$

Mathematica [C] time = 0.10, size = 70, normalized size = 0.64

$$\frac{-(2a+3b)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)-a\csc^2(e+fx)}{2a^2f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-(a*Csc[e + f*x]^2) - (2*a + 3*b)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*S
in[e + f*x]^2)/a])/(2*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [B] time = 0.58, size = 406, normalized size = 3.69

$$\left[\frac{\left(\left(2\,ab+3\,b^2\right)\cos\left(fx+e\right)^4-\left(2\,a^2+7\,ab+6\,b^2\right)\cos\left(fx+e\right)^2+2\,a^2+5\,ab+3\,b^2\right)\sqrt{a}\,\log\left(\frac{2\left(b\cos\left(fx+e\right)^2-2\right.}{}\right.}{4\left(a^3bf\cos\left(fx+e\right)^4-\left(a^4+2\,a^3b\right)f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/4*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x + e
)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(-b*co
s(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*((2*a^2
+ 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f*x + e)^2 + (a^4 + a^3*b)*f
), -1/2*(((2*a*b + 3*b^2)*cos(f*x + e)^4 - (2*a^2 + 7*a*b + 6*b^2)*cos(f*x
+ e)^2 + 2*a^2 + 5*a*b + 3*b^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a
+ b)*sqrt(-a)/a) - ((2*a^2 + 3*a*b)*cos(f*x + e)^2 - 3*a^2 - 3*a*b)*sqrt(-b
*cos(f*x + e)^2 + a + b))/(a^3*b*f*cos(f*x + e)^4 - (a^4 + 2*a^3*b)*f*cos(f
*x + e)^2 + (a^4 + a^3*b)*f)]
```

giac [B] time = 1.01, size = 525, normalized size = 4.77

$$-\frac{\frac{\left(\frac{(a^5b+a^4b^2)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^6b+a^5b^2}+\frac{2(5a^5b+11a^4b^2+6a^3b^3)}{a^6b+a^5b^2}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+\frac{9a^5b+17a^4b^2+8a^3b^3}{a^6b+a^5b^2}}{\sqrt{a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^4+2a\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+4b\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2+a}}+\frac{2\left(2a^{\frac{3}{2}}+3\sqrt{a}b\right)\log\left(\left|-\left(\sqrt{a}\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2-\sqrt{a\tan\left(\frac{1}{2}\right.}\right.\right.\right.}{}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/8*((((a^5*b + a^4*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^6*b + a^5*b^2) + 2*(5*a
^5*b + 11*a^4*b^2 + 6*a^3*b^3)/(a^6*b + a^5*b^2))*tan(1/2*f*x + 1/2*e)^2 +
(9*a^5*b + 17*a^4*b^2 + 8*a^3*b^3)/(a^6*b + a^5*b^2))/sqrt(a*tan(1/2*f*x +
1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) + 2
*(2*a^(3/2) + 3*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^3 - 2*((sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^2*sqrt(a) - a^(3/2))*a^2))/f
```

maple [A] time = 1.78, size = 159, normalized size = 1.45

$$-\frac{1}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}+\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f\,a^{\frac{3}{2}}}-\frac{1}{2fa\sin\left(fx+e\right)^2\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}-\frac{}{2f\,a^2\sqrt{a+}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/a/f/(a+b*sin(f*x+e)^2)^(1/2)+1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+
e)^2)^(1/2))/sin(f*x+e))-1/2/f/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)-3/2/
f/a^2*b/(a+b*sin(f*x+e)^2)^(1/2)+3/2/f/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin
(f*x+e)^2)^(1/2))/sin(f*x+e))
```

maxima [A] time = 0.51, size = 117, normalized size = 1.06

$$\frac{\frac{2\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}+\frac{3\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{5}{2}}}-\frac{2}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a}-\frac{3\,b}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{1}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a\sin\left(fx+e\right)^{2}}}{2\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] 1/2*(2*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 3*b*arcsinh(a/(sq
rt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 2/(sqrt(b*sin(f*x + e)^2 + a)*a) - 3*
b/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 1/(sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*
x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{3}{\left(e+fx \right)}}{\left(a+b\sin^{2}{\left(e+fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.526 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=167

$$\frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -1/8*(8*a^2+24*a*b+15*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2
)/f+1/8*(8*a^2+24*a*b+15*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/8*(8*a+5*b)*
csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)-1/4*csc(f*x+e)^4/a/f/(a+b*sin(f
*x+e)^2)^(1/2)
```

Rubi [A] time = 0.16, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f}+\frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] -((8*a^2 + 24*a*b + 15*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(7/2)*f) + (8*a^2 + 24*a*b + 15*b^2)/(8*a^3*f*Sqrt[a + b*Sin[e + f*x]^2]
) + ((8*a + 5*b)*Csc[e + f*x]^2)/(8*a^2*f*Sqrt[a + b*Sin[e + f*x]^2]) - Csc
[e + f*x]^4/(4*a*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-5b)+2ax}{x^2(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(8a^2+24ab+15b^2\right)\text{S}}{}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+}}$$

$$= \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(8a+5b)\csc^2(e+fx)}{8a^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\csc^4(e+fx)}{4af\sqrt{a+b\sin^2(e+}}$$

$$= -\frac{\left(8a^2+24ab+15b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{7/2}f} + \frac{8a^2+24ab+15b^2}{8a^3f\sqrt{a+b\sin^2(e+fx)}} + $$

Mathematica [C] time = 0.33, size = 94, normalized size = 0.56

$$\frac{\left(8a^2+24ab+15b^2\right)\,{}_2F_1\left(-\frac{1}{2},1;\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+a\csc^2(e+fx)\left(-2a\csc^2(e+fx)+8a+5b\right)}{8a^3f\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (a*Csc[e + f*x]^2*(8*a + 5*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 24*a*b + 15*b
^2)*Hypergeometric2F1[-1/2, 1, 1/2, 1 + (b*Sin[e + f*x]^2)/a])/(8*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

fricas [B] time = 0.62, size = 652, normalized size = 3.90

$$\left[\frac{\left(\left(8\,a^2b+24\,ab^2+15\,b^3\right)\cos\left(fx+e\right)^6-\left(8\,a^3+48\,a^2b+87\,ab^2+45\,b^3\right)\cos\left(fx+e\right)^4-8\,a^3-32\,a^2b-39\,ab^2\right.}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] [1/16*(((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b +
87*a*b^2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 +
(16*a^3 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*c
os(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f
*x + e)^2 - 1)) - 2*((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x + e)^4 + 14*a^3
+ 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos(f*x + e)^2)*sqrt
(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^5 + 3*a^4*b)*f*co
s(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 + a^4*b)*f), 1/8*(
((8*a^2*b + 24*a*b^2 + 15*b^3)*cos(f*x + e)^6 - (8*a^3 + 48*a^2*b + 87*a*b^
2 + 45*b^3)*cos(f*x + e)^4 - 8*a^3 - 32*a^2*b - 39*a*b^2 - 15*b^3 + (16*a^3
 + 72*a^2*b + 102*a*b^2 + 45*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*c
os(f*x + e)^2 + a + b)*sqrt(-a)/a) - ((8*a^3 + 24*a^2*b + 15*a*b^2)*cos(f*x
 + e)^4 + 14*a^3 + 29*a^2*b + 15*a*b^2 - (24*a^3 + 53*a^2*b + 30*a*b^2)*cos
(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^4*b*f*cos(f*x + e)^6 - (a^
5 + 3*a^4*b)*f*cos(f*x + e)^4 + (2*a^5 + 3*a^4*b)*f*cos(f*x + e)^2 - (a^5 +
 a^4*b)*f)]
```

giac [B] time = 1.45, size = 1150, normalized size = 6.89

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] -1/64*(((((a^8*b + a^7*b^2)*tan(1/2*f*x + 1/2*e)^2/(a^9*b + a^8*b^2) - (11*
a^8*b + 21*a^7*b^2 + 10*a^6*b^3)/(a^9*b + a^8*b^2))*tan(1/2*f*x + 1/2*e)^2
- (89*a^8*b + 297*a^7*b^2 + 328*a^6*b^3 + 120*a^5*b^4)/(a^9*b + a^8*b^2))*t
an(1/2*f*x + 1/2*e)^2 - (77*a^8*b + 219*a^7*b^2 + 206*a^6*b^3 + 64*a^5*b^4)
/(a^9*b + a^8*b^2))/sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - 8*(8*a^2 + 24*a*b + 15*b^2)*arctan(
-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))/sqrt(-a))/(sqrt(-a)*a
^3) - 4*(8*a^(5/2) + 24*a^(3/2)*b + 15*sqrt(a)*b^2)*log(abs(-(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^4 + 4*
(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*ta
n(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^2 + 20*(sqrt(a)
*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
```

```
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b + 14*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2
 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2
*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^2*a^(3/2)*b - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a^3 - 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2
*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2*b
 - 18*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a
*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^2 - 7*a^(7/2
) - 8*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1
/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 -
 a)^2*a^3))/f
```

maple [A] time = 2.01, size = 288, normalized size = 1.72

$$\frac{1}{af\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}-\frac{\ln\left(\frac{2a+2\sqrt{a}\,\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}{\sin\left(fx+e\right)}\right)}{f\,a^{\frac{3}{2}}}+\frac{1}{fa\sin\left(fx+e\right)^2\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}+\frac{}{f\,a^2\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] 1/a/f/(a+b*sin(f*x+e)^2)^(1/2)-1/f/a^(3/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e
)^2)^(1/2))/sin(f*x+e))+1/f/a/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+3/f/a^2
*b/(a+b*sin(f*x+e)^2)^(1/2)-3/f/a^(5/2)*b*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2))/sin(f*x+e))-1/4/f/a/sin(f*x+e)^4/(a+b*sin(f*x+e)^2)^(1/2)+5/8/f/
a^2*b/sin(f*x+e)^2/(a+b*sin(f*x+e)^2)^(1/2)+15/8/f/a^3*b^2/(a+b*sin(f*x+e)^
2)^(1/2)-15/8/f/a^(7/2)*b^2*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^(1/2))/sin
(f*x+e))
```

maxima [A] time = 0.33, size = 219, normalized size = 1.31

$$-\frac{\frac{8\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{3}{2}}}+\frac{24\,b\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{5}{2}}}+\frac{15\,b^2\,\mathrm{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{7}{2}}}-\frac{8}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a}-\frac{24\,b}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2}-}{8\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] -1/8*(8*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(3/2) + 24*b*arcsinh(a/(
sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b^2*arcsinh(a/(sqrt(a*b)*abs(sin
(f*x + e))))/a^(7/2) - 8/(sqrt(b*sin(f*x + e)^2 + a)*a) - 24*b/(sqrt(b*sin(
f*x + e)^2 + a)*a^2) - 15*b^2/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 8/(sqrt(b*
sin(f*x + e)^2 + a)*a*sin(f*x + e)^2) - 5*b/(sqrt(b*sin(f*x + e)^2 + a)*a^2
*sin(f*x + e)^2) + 2/(sqrt(b*sin(f*x + e)^2 + a)*a*sin(f*x + e)^4))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.01

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.527 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=292

$$-\frac{4a\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{4a\sqrt{\cos}}{}$$

```
[Out] 1/3*(7*a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*
(7*a-b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*
(a+b*sin(f*x+e)^2)^(1/2)/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*a*Ellipti
cF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)
^2/a)^(1/2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-4/3*a*tan(f*x+e)/(a+b)^2/f/(
a+b*sin(f*x+e)^2)^(1/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{4a\tan(e+fx)}{3f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{\tan(e+fx)\sec^2(e+fx)}{3f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{4a\sqrt{\cos}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f
*x]^2]) + ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -
(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^3*f*Sqrt[1 + (b*
Sin[e + f*x]^2)/a]) - (4*a*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*
x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)^2*f*Sq
rt[a + b*Sin[e + f*x]^2]) - (4*a*Tan[e + f*x])/(3*(a + b)^2*f*Sqrt[a + b*Si
n[e + f*x]^2]) + (Sec[e + f*x]^2*Tan[e + f*x])/(3*(a + b)*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^{5/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\mathrm{Subst}\left(\int \frac{a}{\left(1-x^2\right)^{3}}\right.}{3(a+b)f}$$

$$= -\frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(}{3(a+b)}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(}{3(a+b)}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{4a\tan(e+fx)}{3(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sec^2(}{3(a+b)}$$

$$= \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a-b)\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+f\right.}{3(a+b)^3f\sqrt{1}}$$

Mathematica [A] time = 2.28, size = 197, normalized size = 0.67

$$\frac{-\frac{\tan(e+fx)\sec^2(e+fx)\left(4\left(4a^2-3ab+b^2\right)\cos(2(e+fx))+8a^2+b(b-7a)\cos(4(e+fx))-21ab-5b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+f\right.}{6f(a+b)^3\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (2*a*(7*a - b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(
b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*
x, -(b/a)] - ((8*a^2 - 21*a*b - 5*b^2 + 4*(4*a^2 - 3*a*b + b^2)*Cos[2*(e +
f*x)] + b*(-7*a + b)*Cos[4*(e + f*x)])*Sec[e + f*x]^2*Tan[e + f*x])/(2*Sqrt
[2]))/(6*(a + b)^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 3.10, size = 368, normalized size = 1.26

$$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\,b\,(7a-b)\sin\left(fx+e\right)\left(\cos^4\left(fx+e\right)\right)-4\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(7*a-b)*sin(f*x+e)*cos(f
*x+e)^4-4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a+b)*cos(f*x+e)^2*s
in(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(a^2+2*a*b+b^2)*sin(f*
x+e)-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*
cos(f*x+e)^2+(a+b)/a)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)
)*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2)/(1+sin(f*x+e))/(s
in(f*x+e)-1)/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b
)^3/cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^4}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{4}{\left(e+fx\right)}}{\left(a+b\sin^{2}{\left(e+fx\right)}\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.528 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=224

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2b\sin(e+fx)\cos(e+fx)}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

```
[Out] -2*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)-2*EllipticE(s
in(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^
(1/2)/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(f*x+e),(-b/a)^(1/2
))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/(a+b)/f/(a+b*
sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.23, antiderivative size = 224, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 471, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}-\frac{2b\sin(e+fx)\cos(e+fx)}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}\,F\left(\sin\right.}{f(a+b)\sqrt{a+b\sin^2(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*b*Cos[e + f*x]*Sin[e + f*x])/((a + b)^2*f*Sqrt[a + b*Sin[e + f*x]^2]) -
 (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[a + b*Sin[e + f*x]^2])/((a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a])
 + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*
x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) +
 Tan[e + f*x]/((a + b)*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\right)\text{Subst}\left(\int \frac{a-bx^2}{\sqrt{1-x^2}\,(a+bx}\right.}{(a+b)f}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+fx\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos^2(e+f\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\tan(e+fx)}{(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(2\sqrt{\cos^2(e+f\right.}{}$$

$$= -\frac{2b\cos(e+fx)\sin(e+fx)}{(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{2\sqrt{\cos^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))\middle|-\frac{b}{a}\right)\sec}{(a+b)^2f\sqrt{1+\frac{b\sin^2(e}{a}}}$$

Mathematica [A] time = 0.89, size = 145, normalized size = 0.65

$$\frac{2\tan(e+fx)(a-b\cos(2(e+fx))) + \sqrt{2}\,(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\middle|-\frac{b}{a}\right) - 2\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e\right.}{\sqrt{2}\,f(a+b)^2\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*Sqrt[2]*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b
/a)] + Sqrt[2]*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e +
 f*x, -(b/a)] + 2*(a - b*Cos[2*(e + f*x)])*Tan[e + f*x])/(Sqrt[2]*(a + b)^2
*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 2.63, size = 283, normalized size = 1.26

$$-\frac{\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+\left(a+b\right)\left(\cos^2\left(fx+e\right)\right)}\left(2\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\sin\left(f\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
[Out] -(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*(cos(f*x+e)^2)^(1/2)*(-b/a*c
os(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))-a*(cos(f*
x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*
b)^(1/2))-b*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Elliptic
F(sin(f*x+e),(-1/a*b)^(1/2))+2*sin(f*x+e)*cos(f*x+e)^2*b-a*sin(f*x+e)-b*sin
(f*x+e))/(a+b)^2/(-(a+b*sin(f*x+e)^2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/
cos(f*x+e)/(a+b*sin(f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.529 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=101

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)+(cos(f*x+e)^2)^(
1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin(f*x+e)^2)^(1/2)
/a/(a+b)/f/(1+b*sin(f*x+e)^2/a)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.250, Rules used = {3184, 21, 3178, 3177}

$$\frac{b\sin(e+fx)\cos(e+fx)}{af(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{af(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(a*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) + (E
llipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e + f*x]^2])/(a*(a + b)*f*Sqrt[1 +
 (b*Sin[e + f*x]^2)/a])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a-b\sin^2(e+fx)}{\sqrt{a+b\sin^2(e+fx)}}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\int \sqrt{a+b\sin^2(e+fx)}\,dx}{a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\sqrt{a+b\sin^2(e+fx)}\int \sqrt{1+\frac{b\sin^2(e+fx)}{a}}\,dx}{a(a+b)\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{a(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{E\left(e+fx\middle|-\frac{b}{a}\right)\sqrt{a+b\sin^2(e+fx)}}{a(a+b)f\sqrt{1+\frac{b\sin^2(e+fx)}{a}}}$$

Mathematica [A] time = 0.15, size = 90, normalized size = 0.89

$$\frac{2a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e+fx\middle|-\frac{b}{a}\right)+\sqrt{2}\,b\sin(2(e+fx))}{2af(a+b)\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-3/2),x]
```

```
[Out] (2*a*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sq
rt[2]*b*Sin[2*(e + f*x)])/(2*a*(a + b)*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]]
)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)/(b^2*cos(f*x + e)^4 - 2*(a*b + b^2
)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

maple [A] time = 1.82, size = 103, normalized size = 1.02

$$\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,a\,\mathrm{EllipticE}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)+\sin\left(fx+e\right)\left(\cos^2\left(fx+e\right)\right)b}{a\left(a+b\right)\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] ((cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*a*EllipticE(sin(f*x
+e),(-1/a*b)^(1/2))+sin(f*x+e)*cos(f*x+e)^2*b)/a/(a+b)/cos(f*x+e)/(a+b*sin(
f*x+e)^2)^(1/2)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-3/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\left(b\sin\left(e+fx\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-3/2), x)
```

3.530 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=209

$$-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f}-\frac{2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(1/2)-2*cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/
2)/a^2/f-2*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/
2)*(a+b*sin(f*x+e)^2)^(1/2)/a^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)+EllipticF(sin(
f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(
1/2)/a/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.24, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3196, 469, 583, 524, 426, 424, 421, 419}

$$-\frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f}-\frac{2\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin(e+fx))|-\frac{b}{a}\right)}{a^2f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] Cot[e + f*x]/(a*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*Cot[e + f*x]*Sqrt[a + b*
Sin[e + f*x]^2])/(a^2*f) - (2*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e +
 f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(a^2*f*Sqrt[1 + (b
*Sin[e + f*x]^2)/a]) + (Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(a*f*Sqrt[a + b*Sin[
e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
```

```
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && !GtQ[a, 0
]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] && !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] && !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-2+x^2}{x^2\sqrt{1-x^2}\sqrt{a+bx^2}}\,d\right.}{af}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} + \frac{\left(\sqrt{\cos^2(e+fx)}\,\text{se}\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\text{s}\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{\left(2\sqrt{\cos^2(e+fx)}\,\text{s}\right.}{}$$

$$= \frac{\cot(e+fx)}{af\sqrt{a+b\sin^2(e+fx)}} - \frac{2\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{a^2f} - \frac{2\sqrt{\cos^2(e+fx)}\,E}{}$$

Mathematica [A] time = 0.75, size = 142, normalized size = 0.68

$$\frac{-2\cot(e+fx)(a-b\cos(2(e+fx))+b)+\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+fx\left|-\frac{b}{a}\right.\right)-2\sqrt{2}\,a\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,E\left(e\right.}{\sqrt{2}\,a^2f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (-2*(a + b - b*Cos[2*(e + f*x)])*Cot[e + f*x] - 2*Sqrt[2]*a*Sqrt[(2*a + b -
 b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -(b/a)] + Sqrt[2]*a*Sqrt[(2*a +
b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f*x, -(b/a)])/(Sqrt[2]*a^2*f*Sqrt[
2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^2}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.61, size = 141, normalized size = 0.67

$$\frac{\sin\left(fx+e\right)\sqrt{-\frac{b\left(\cos^2\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,a\left(\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)-2\,\mathrm{EllipticE}\left(\sin\left(fx\right.\right.\right.}{\sin\left(fx+e\right)a^2\cos\left(fx+e\right)\sqrt{a+b\left(\sin^2\left(fx+e\right)\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x)
[Out] (sin(f*x+e)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*(Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))-2*EllipticE(sin(f*x+e),(-1/a*b)^(1/2)))+2*b
*cos(f*x+e)^4+(-a-2*b)*cos(f*x+e)^2)/sin(f*x+e)/a^2/cos(f*x+e)/(a+b*sin(f*x
+e)^2)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2),x)
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(3/2),x)
[Out] Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.531 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx$

Optimal. Leaf size=297

$$\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}+\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^3f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] (a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(1/2)+1/3*(7*a+8*b)*
cot(f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/f-1/3*(3*a+4*b)*cot(f*x+e)*csc(f*x+
e)^2*(a+b*sin(f*x+e)^2)^(1/2)/a^2/b/f+1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-
b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/f/
(1+b*sin(f*x+e)^2/a)^(1/2)-4/3*(a+b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec
(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*sin(f*x+
e)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {3196, 468, 583, 524, 426, 424, 421, 419}

$$\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f}-\frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^2bf}-\frac{4(a+b)\sqrt{\cos\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(a*b*f*Sqrt[a + b*Sin[e + f*x]^2]) +
((7*a + 8*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f) - ((3*a + 4
*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^2*b*f) + (
(7*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Se
c[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*f*Sqrt[1 + (b*Sin[e + f*x]^2)
/a]) - (4*(a + b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/
a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e
+ f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] && !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] && !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{3/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-3a-4b+\cdots}{x^4\sqrt{1-x^2}\cdots}\right.}{abf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a+4b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(\cdots}}{3a^2bf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3\cdots}{\cdots}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3\cdots}{\cdots}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3\cdots}{\cdots}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{abf\sqrt{a+b\sin^2(e+fx)}} + \frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f} - \frac{(3\cdots}{\cdots}$$

Mathematica [A] time = 3.82, size = 199, normalized size = 0.67

$$\frac{\frac{\cot(e+fx)\csc^2(e+fx)\left(-4\left(4a^2+11ab+8b^2\right)\cos(2(e+fx))+8a^2+b(7a+8b)\cos(4(e+fx))+37ab+24b^2\right)}{2\sqrt{2}} - 8a(a+b)\sqrt{\frac{2a-b\cos(2(e+fx))+b}{a}}\,F\left(e+f\cdots\right.}{6a^3f\sqrt{2a-b\cos(2(e+fx))+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(3/2),x]
```

```
[Out] (((8*a^2 + 37*a*b + 24*b^2 - 4*(4*a^2 + 11*a*b + 8*b^2)*Cos[2*(e + f*x)] +
b*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x]*Csc[e + f*x]^2)/(2*Sqrt[2]) +
2*a*(7*a + 8*b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticE[e + f*x, -
(b/a)] - 8*a*(a + b)*Sqrt[(2*a + b - b*Cos[2*(e + f*x)])/a]*EllipticF[e + f
*x, -(b/a)])/(6*a^3*f*Sqrt[2*a + b - b*Cos[2*(e + f*x)]])
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4}{b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b^2*cos(f*x + e)^4
 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{4}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(3/2), x)
```

maple [A] time = 1.91, size = 353, normalized size = 1.19

$$-\frac{4\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\ \mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}\left(\sin^{3}\left(fx+e\right)\right)+4b\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x)
```

```
[Out] -1/3*(4*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*sin(f*x+e)^3+4*b*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*sin(f*x+e)^3-7*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)
*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b*sin(f*x+e)^3+7*a*b*sin(f*x+e)^6+8
*b^2*sin(f*x+e)^6+4*a^2*sin(f*x+e)^4-3*a*b*sin(f*x+e)^4-8*b^2*sin(f*x+e)^4-
5*a^2*sin(f*x+e)^2-4*a*b*sin(f*x+e)^2+a^2)/a^3/sin(f*x+e)^3/cos(f*x+e)/(a+b
*sin(f*x+e)^2)^(1/2)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(3/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^4}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(3/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{4}{\left(e+fx \right)}}{\left(a+b\sin^{2}{\left(e+fx \right)}\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(3/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(3/2), x)
```

3.532 $\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=218

$$-\frac{8a^2-24ab+3b^2}{8f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}-\frac{8a^2-24ab+3b^2}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+}}{\sqrt{a+b}}\right.}{8f(a+b)^{9/2}}$$

```
[Out] 1/8*(8*a^2-24*a*b+3*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b
)^(9/2)/f+1/24*(-8*a^2+24*a*b-3*b^2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)-1/8
*(8*a+b)*sec(f*x+e)^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/4*sec(f*x+e)^4/(
a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(-8*a^2+24*a*b-3*b^2)/(a+b)^4/f/(a+b*si
n(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.28, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$-\frac{8a^2-24ab+3b^2}{8f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}-\frac{8a^2-24ab+3b^2}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+}}{\sqrt{a+b}}\right.}{8f(a+b)^{9/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((8*a^2 - 24*a*b + 3*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/
(8*(a + b)^(9/2)*f) - (8*a^2 - 24*a*b + 3*b^2)/(24*(a + b)^3*f*(a + b*Sin[e
 + f*x]^2)^(3/2)) - ((8*a + b)*Sec[e + f*x]^2)/(8*(a + b)^2*f*(a + b*Sin[e
+ f*x]^2)^(3/2)) + Sec[e + f*x]^4/(4*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)
) - (8*a^2 - 24*a*b + 3*b^2)/(8*(a + b)^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
```

```
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x^2}{(1-x)^3(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^4(e+fx)}{4(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\text{Subst}\left(\int \frac{\frac{1}{2}(4a-3b)+2(a+b)x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4(a+b)f}$$

$$= -\frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^4(e+fx)}{4(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(8a^2-2}{}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+}$$

$$= -\frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(8a+b)\sec^2(e+fx)}{8(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{4(a+}$$

$$= \frac{\left(8a^2-24ab+3b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{8(a+b)^{9/2}f} - \frac{8a^2-24ab+3b^2}{24(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Mathematica [C] time = 0.48, size = 107, normalized size = 0.49

$$\frac{\left(-8a^2+24ab-3b^2\right)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)-\frac{3}{2}(a+b)\sec^4(e+fx)((8a+b)\cos(2(e+fx))+4a-3b)}{24f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((-8*a^2 + 24*a*b - 3*b^2)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e +
f*x]^2)/(a + b)] - (3*(a + b)*(4*a - 3*b + (8*a + b)*Cos[2*(e + f*x)])*Sec[
e + f*x]^4)/2)/(24*(a + b)^3*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.85, size = 995, normalized size = 4.56

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 - 24*a*b^3 + 3*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a
^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 -
 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sq
rt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*
(3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8*
a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^3
 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4*
b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
+ 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(f
*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2*
b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4), -1/24*(3*((8*a^2*b^2 - 24*a*b^3 + 3
*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x
+ e)^6 + (8*a^4 - 8*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4)*
sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) -
 (3*(8*a^3*b - 16*a^2*b^2 - 21*a*b^3 + 3*b^4)*cos(f*x + e)^6 - 4*(8*a^4 - 8
*a^3*b - 37*a^2*b^2 - 18*a*b^3 + 3*b^4)*cos(f*x + e)^4 + 6*a^4 + 24*a^3*b +
 36*a^2*b^2 + 24*a*b^3 + 6*b^4 - 3*(8*a^4 + 25*a^3*b + 27*a^2*b^2 + 11*a*b^
3 + b^4)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^5*b^2 + 5*a^4
*b^3 + 10*a^3*b^4 + 10*a^2*b^5 + 5*a*b^6 + b^7)*f*cos(f*x + e)^8 - 2*(a^6*b
 + 6*a^5*b^2 + 15*a^4*b^3 + 20*a^3*b^4 + 15*a^2*b^5 + 6*a*b^6 + b^7)*f*cos(
f*x + e)^6 + (a^7 + 7*a^6*b + 21*a^5*b^2 + 35*a^4*b^3 + 35*a^3*b^4 + 21*a^2
*b^5 + 7*a*b^6 + b^7)*f*cos(f*x + e)^4)]
```

giac [B] time = 5.04, size = 3826, normalized size = 17.55

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/12*(3*(8*a^2 - 24*a*b + 3*b^2)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)
^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1
/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b))/((a^4 + 4*a^3*b + 6*a^2*b^2
 + 4*a*b^3 + b^4)*sqrt(-a - b)) + 4*((((4*a^17*b^2 + 51*a^16*b^3 + 294*a^15
*b^4 + 1001*a^14*b^5 + 2184*a^13*b^6 + 3003*a^12*b^7 + 2002*a^11*b^8 - 1287
*a^10*b^9 - 5148*a^9*b^10 - 7007*a^8*b^11 - 6006*a^7*b^12 - 3549*a^6*b^13 -
 1456*a^5*b^14 - 399*a^4*b^15 - 66*a^3*b^16 - 5*a^2*b^17)*tan(1/2*f*x + 1/2
```

*e)^2/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4 + 816*a^15*b^5 + 3060*a^14*b^6
 + 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^11*b^9 + 43758*a^10*b^10 + 4862
0*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13 + 18564*a^6*b^14 + 8568*a^5*b^
15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b^18 + 18*a*b^19 + b^20) + 3*(4
*a^17*b^2 + 55*a^16*b^3 + 342*a^15*b^4 + 1253*a^14*b^5 + 2912*a^13*b^6 + 40
95*a^12*b^7 + 2002*a^11*b^8 - 5291*a^10*b^9 - 15444*a^9*b^10 - 22451*a^8*b^
11 - 22022*a^7*b^12 - 15561*a^6*b^13 - 8008*a^5*b^14 - 2947*a^4*b^15 - 738*
a^3*b^16 - 113*a^2*b^17 - 8*a*b^18)/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4
+ 816*a^15*b^5 + 3060*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^1
1*b^9 + 43758*a^10*b^10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13
+ 18564*a^6*b^14 + 8568*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b
^18 + 18*a*b^19 + b^20))*tan(1/2*f*x + 1/2*e)^2 + 3*(4*a^17*b^2 + 55*a^16*b
^3 + 342*a^15*b^4 + 1253*a^14*b^5 + 2912*a^13*b^6 + 4095*a^12*b^7 + 2002*a^
11*b^8 - 5291*a^10*b^9 - 15444*a^9*b^10 - 22451*a^8*b^11 - 22022*a^7*b^12 -
 15561*a^6*b^13 - 8008*a^5*b^14 - 2947*a^4*b^15 - 738*a^3*b^16 - 113*a^2*b^
17 - 8*a*b^18)/(a^18*b^2 + 18*a^17*b^3 + 153*a^16*b^4 + 816*a^15*b^5 + 3060
*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8 + 31824*a^11*b^9 + 43758*a^10*b^
10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 31824*a^7*b^13 + 18564*a^6*b^14 + 85
68*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17 + 153*a^2*b^18 + 18*a*b^19 + b^2
0))*tan(1/2*f*x + 1/2*e)^2 + (4*a^17*b^2 + 51*a^16*b^3 + 294*a^15*b^4 + 100
1*a^14*b^5 + 2184*a^13*b^6 + 3003*a^12*b^7 + 2002*a^11*b^8 - 1287*a^10*b^9
 - 5148*a^9*b^10 - 7007*a^8*b^11 - 6006*a^7*b^12 - 3549*a^6*b^13 - 1456*a^5*
b^14 - 399*a^4*b^15 - 66*a^3*b^16 - 5*a^2*b^17)/(a^18*b^2 + 18*a^17*b^3 + 1
53*a^16*b^4 + 816*a^15*b^5 + 3060*a^14*b^6 + 8568*a^13*b^7 + 18564*a^12*b^8
 + 31824*a^11*b^9 + 43758*a^10*b^10 + 48620*a^9*b^11 + 43758*a^8*b^12 + 318
24*a^7*b^13 + 18564*a^6*b^14 + 8568*a^5*b^15 + 3060*a^4*b^16 + 816*a^3*b^17
 + 153*a^2*b^18 + 18*a*b^19 + b^20))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 6*(8*(sqrt(a)*ta
n(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/
2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^7*a^2 - 8*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))^7*a*b - 5*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))^7*b^2 - 56*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))^6*a^(5/2) - 8*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x +
 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6
*a^(3/2)*b - 29*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*
e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^6*sqrt
(a)*b^2 - 120*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)
^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^3 -
296*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a^2*b + 43*(sqrt
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*a*b^2 - 12*(sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^5*b^3 + 136*(sqrt(a)*tan(1/2*f*x + 1
/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*
tan(1/2*f*x + 1/2*e)^2 + a))^4*a^(7/2) - 168*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))^4*a^(5/2)*b - 629*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*
x + 1/2*e)^2 + a))^4*a^(3/2)*b^2 + 60*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqr
t(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x +
 1/2*e)^2 + a))^4*sqrt(a)*b^3 + 344*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))^3*a^4 + 872*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))^3*a^3*b - 151*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/

```
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a^
2*b^2 - 1112*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^
4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*a*b^3 -
 48*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*t
an(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^4 + 24*(sqrt(a
)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
+ 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(9/2) + 616*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(7/2)*b + 1265*(sqrt(a)*tan(1/2
*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^
2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2)*b^2 + 24*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b^3 - 880*(sqrt(a)*tan(1/2*f*x
+ 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4
*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b^4 - 232*(sqrt(a)*tan(1/2*f*x +
1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b
*tan(1/2*f*x + 1/2*e)^2 + a))*a^5 - 568*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a^4*b + 113*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(
1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^
2 + a))*a^3*b^2 + 1124*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
 + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))
*a^2*b^3 + 432*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a*b^4 -
 320*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*
tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*b^5 - 104*a^(11/2
) - 440*a^(9/2)*b - 607*a^(7/2)*b^2 - 84*a^(5/2)*b^3 + 496*a^(3/2)*b^4 + 32
0*sqrt(a)*b^5)/((a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*((sqrt(a)*tan(1
/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e
)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^4))/f
```

maple [B] time = 8.43, size = 2139, normalized size = 9.81

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)+
(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2/f*b^4*a^2/(b+(-a*b)
^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*
cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/2/f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)
^(1/2))^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^
2)/b)^(1/2)+1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(-a*b)^(1/2)/(
sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+7/16/f*b^3/(
b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x
+e)^2)^(1/2)-7/16/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a+b)/(sin
(f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)+1/2/f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+
(-a*b)^(1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*
b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))-1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(
1/2))^4/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*
x+e)+2*a)/(1+sin(f*x+e)))+7/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3
*a/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+
2*a)/(1+sin(f*x+e)))+7/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3*a/(a
+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/
(sin(f*x+e)-1))+1/2/f*b^4*a^2/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^
(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin
(f*x+e)-1))-1/f*b^5*a/(b+(-a*b)^(1/2))^4/(-b+(-a*b)^(1/2))^4/(a+b)^(1/2)*ln
```

```
((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-
1))+1/16/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(sin(f*x+e)-1)^
2*(a+b-b*cos(f*x+e)^2)^(1/2)+1/16/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2)
)^2/(a+b)/(1+sin(f*x+e))^2*(a+b-b*cos(f*x+e)^2)^(1/2)-3/16/f*b^3/(b+(-a*b)^
(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^2/(sin(f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1
/2)-1/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(1+sin(f*x+e))*
(a+b-b*cos(f*x+e)^2)^(1/2)+3/16/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^
2/(a+b)^2/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)-1/12/f*b^2*a/(b+(-a*b)^
(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2
+(a*b+b^2)/b)^(1/2)-1/12/f*b^2*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(
1/2))^3/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/1
2/f*b^2*a/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b
)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/f*b^2*(-a*b)^(1/2)/(b+(-a*b)^(
1/2))^3/(-b+(-a*b)^(1/2))^3/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a
*b+b^2)/b)^(1/2)+1/16/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)/(s
in(f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)-1/16/f*b^5/(b+(-a*b)^(1/2))^3/(-b+(
-a*b)^(1/2))^3/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b
*sin(f*x+e)+2*a)/(sin(f*x+e)-1))-1/16/f*b^5/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(
1/2))^3/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*
x+e)+2*a)/(1+sin(f*x+e)))+3/16/f*b^4/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2
/(a+b)^(5/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*
a)/(1+sin(f*x+e)))-1/16/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^
(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+s
in(f*x+e)))+3/16/f*b^4/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(5/2)*l
n((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)
-1))-1/16/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(3/2)*ln((2*(a
+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))
```

maxima [B] time = 0.66, size = 424, normalized size = 1.94

$$-\frac{\frac{3\left(8\,a^{2}b^{3}-24\,ab^{4}+3\,b^{5}\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}+\sqrt{a+b}}\right)}{\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\sqrt{a+b}}+\frac{2\left(8\,a^{5}b^{3}+24\,a^{4}b^{4}+24\,a^{3}b^{5}+8\,a^{2}b^{6}+3\left(8\,a^{2}b^{3}-24\,ab^{4}+3\,b^{5}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)^{3}-5\left(8\,a^{3}b^{3}-16\cdots\right.\right.}{\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{7}{2}}-2\left(a^{5}+5\,a^{4}b+10\,a^{3}b^{2}+10\,a^{2}b^{3}+5\,ab^{4}+b^{5}\right)\cdots}}{48\,b^{3}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/48*(3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*log((sqrt(b*sin(f*x + e)^2 + a) - s
qrt(a + b))/(sqrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^4 + 4*a^3*b + 6
*a^2*b^2 + 4*a*b^3 + b^4)*sqrt(a + b)) + 2*(8*a^5*b^3 + 24*a^4*b^4 + 24*a^3
*b^5 + 8*a^2*b^6 + 3*(8*a^2*b^3 - 24*a*b^4 + 3*b^5)*(b*sin(f*x + e)^2 + a)^
3 - 5*(8*a^3*b^3 - 16*a^2*b^4 - 21*a*b^5 + 3*b^6)*(b*sin(f*x + e)^2 + a)^2
+ 8*(a^4*b^3 - 4*a^3*b^4 - 11*a^2*b^5 - 6*a*b^6)*(b*sin(f*x + e)^2 + a))/((
a^4 + 4*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(7/2) - 2
*(a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)*(b*sin(f*x + e)^
2 + a)^(5/2) + (a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*
b^5 + b^6)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b^3*f)
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^5\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.533 $\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=153

$$\frac{2a-3b}{2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{2a-3b}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}+\frac{}{2f(a+b}$$

```
[Out] -1/2*(2*a-3*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(7/2)/f+
1/6*(2*a-3*b)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/2*sec(f*x+e)^2/(a+b)/f/(
a+b*sin(f*x+e)^2)^(3/2)+1/2*(2*a-3*b)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.15, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$\frac{2a-3b}{2f(a+b)^3\sqrt{a+b\sin^2(e+fx)}}+\frac{2a-3b}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2f(a+b)^{7/2}}+\frac{}{2f(a+b}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -((2*a - 3*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]])/(2*(a + b)^(
7/2)*f) + (2*a - 3*b)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + Sec[e
+ f*x]^2/(2*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (2*a - 3*b)/(2*(a + b
)^3*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a-3b)\,\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx, x, \sin^2(e$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a-}{}$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{2(a+}$$

$$= \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)}{2(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{2(a+}$$

$$= -\frac{(2a-3b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{2(a+b)^{7/2}f} + \frac{2a-3b}{6(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{}{2(a}$$

Mathematica [C] time = 0.12, size = 76, normalized size = 0.50

$$\frac{(2a-3b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)+a}{a+b}\right)+3(a+b)\sec^2(e+fx)}{6f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a - 3*b)*Hypergeometric2F1[-3/2, 1, -1/2, (a + b*Sin[e + f*x]^2)/(a + b
)] + 3*(a + b)*Sec[e + f*x]^2)/(6*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.68, size = 769, normalized size = 5.03

$$\left[-\frac{3\left(\left(2ab^2-3b^3\right)\cos\left(fx+e\right)^6-2\left(2a^2b-ab^2-3b^3\right)\cos\left(fx+e\right)^4+\left(2a^3+a^2b-4ab^2-3b^3\right)\cos\left(fx+\right.\right.}{12\left(\left(a^4b^2+4a^3b^3+6a^2b^4+4ab^5+b^6\right)f\cos\left(fx+\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/12*(3*((2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*c
os(f*x + e)^4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(a +
b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(a + b) -
2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^4
- 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*c
os(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^4*b^2 + 4*a^3*b^3 + 6*a
^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b + 5*a^4*b^2 + 10*a^3*b^
3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 + (a^6 + 6*a^5*b + 15*a^4*
b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*cos(f*x + e)^2), 1/6*(3*((
2*a*b^2 - 3*b^3)*cos(f*x + e)^6 - 2*(2*a^2*b - a*b^2 - 3*b^3)*cos(f*x + e)^
4 + (2*a^3 + a^2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-a - b)*arctan(s
qrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a - b)/(a + b)) - (3*(2*a^2*b - a*b^2
- 3*b^3)*cos(f*x + e)^4 - 3*a^3 - 9*a^2*b - 9*a*b^2 - 3*b^3 - 4*(2*a^3 + a^
2*b - 4*a*b^2 - 3*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a
^4*b^2 + 4*a^3*b^3 + 6*a^2*b^4 + 4*a*b^5 + b^6)*f*cos(f*x + e)^6 - 2*(a^5*b
 + 5*a^4*b^2 + 10*a^3*b^3 + 10*a^2*b^4 + 5*a*b^5 + b^6)*f*cos(f*x + e)^4 +
(a^6 + 6*a^5*b + 15*a^4*b^2 + 20*a^3*b^3 + 15*a^2*b^4 + 6*a*b^5 + b^6)*f*co
s(f*x + e)^2)]
```

giac [B] time = 2.29, size = 1792, normalized size = 11.71

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*a - 3*b)*arctan(-1/2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan
(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)
^2 + a) - sqrt(a))/sqrt(-a - b))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*sqrt(-a -
 b)) + 2*((((2*a^13*b^2 + 21*a^12*b^3 + 99*a^11*b^4 + 275*a^10*b^5 + 495*a^
9*b^6 + 594*a^8*b^7 + 462*a^7*b^8 + 198*a^6*b^9 - 55*a^4*b^11 - 33*a^3*b^12
 - 9*a^2*b^13 - a*b^14)*tan(1/2*f*x + 1/2*e)^2/(a^14*b^2 + 14*a^13*b^3 + 91
*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 + 2002*a^9*b^7 + 3003*a^8*b^8 + 34
32*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11 + 1001*a^4*b^12 + 364*a^3*b^13 +
 91*a^2*b^14 + 14*a*b^15 + b^16) + 3*(2*a^13*b^2 + 23*a^12*b^3 + 119*a^11*b
^4 + 363*a^10*b^5 + 715*a^9*b^6 + 924*a^8*b^7 + 726*a^7*b^8 + 198*a^6*b^9 -
 264*a^5*b^10 - 385*a^4*b^11 - 253*a^3*b^12 - 97*a^2*b^13 - 21*a*b^14 - 2*b
^15)/(a^14*b^2 + 14*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 +
 2002*a^9*b^7 + 3003*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11
 + 1001*a^4*b^12 + 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))*tan(1/2*
f*x + 1/2*e)^2 + 3*(2*a^13*b^2 + 23*a^12*b^3 + 119*a^11*b^4 + 363*a^10*b^5
+ 715*a^9*b^6 + 924*a^8*b^7 + 726*a^7*b^8 + 198*a^6*b^9 - 264*a^5*b^10 - 38
5*a^4*b^11 - 253*a^3*b^12 - 97*a^2*b^13 - 21*a*b^14 - 2*b^15)/(a^14*b^2 + 1
4*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10*b^6 + 2002*a^9*b^7 + 30
03*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^5*b^11 + 1001*a^4*b^12 +
 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))*tan(1/2*f*x + 1/2*e)^2 + (
2*a^13*b^2 + 21*a^12*b^3 + 99*a^11*b^4 + 275*a^10*b^5 + 495*a^9*b^6 + 594*a
^8*b^7 + 462*a^7*b^8 + 198*a^6*b^9 - 55*a^4*b^11 - 33*a^3*b^12 - 9*a^2*b^13
 - a*b^14)/(a^14*b^2 + 14*a^13*b^3 + 91*a^12*b^4 + 364*a^11*b^5 + 1001*a^10
*b^6 + 2002*a^9*b^7 + 3003*a^8*b^8 + 3432*a^7*b^9 + 3003*a^6*b^10 + 2002*a^
5*b^11 + 1001*a^4*b^12 + 364*a^3*b^13 + 91*a^2*b^14 + 14*a*b^15 + b^16))/(a
*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/
2*e)^2 + a)^(3/2) - 6*(2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f
*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a
))^3*a + (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b + 2*(sqrt
```

```
(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*
x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2) + 5*(sqrt(a)*tan(
1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*
e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*sqrt(a)*b - 2*(sqrt(a)*tan(1/2*f*
x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^2 - (sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a*b + 4*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*
f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 +
a))*b^2 - 2*a^(5/2) - 5*a^(3/2)*b - 4*sqrt(a)*b^2)/((a^3 + 3*a^2*b + 3*a*b^
2 + b^3)*((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 +
 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2 - 2*(sqrt(
a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x
 + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*sqrt(a) - 3*a - 4*b)^2))/f
```

maple [B] time = 6.28, size = 1256, normalized size = 8.21

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/2/f*b^3/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1
/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))*a-1/2/f*
b^4/(b+(-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a
+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin(f*x+e)+2*a)/(sin(f*x+e)-1))-1/2/f*b^3/(b+(
-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)
*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*a+1/2/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a
*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b
+b^2)/b)^(1/2)-1/4/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)/(sin(
f*x+e)-1)*(a+b-b*cos(f*x+e)^2)^(1/2)+1/4/f*b^3/(b+(-a*b)^(1/2))^2/(-b+(-a*b
)^(1/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+2*b*sin
(f*x+e)+2*a)/(sin(f*x+e)-1))-1/12/f*b/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^
2/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/12/f*
b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/a/(sin(f*x+e)+(-a*b)^
(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2/f*b^3/(b+(-a*b)^(1/2))^3/(
-b+(-a*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)
-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))*a-1/2/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a
*b)^(1/2))^3/(a+b)^(1/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-2*b*s
in(f*x+e)+2*a)/(1+sin(f*x+e)))+1/4/f*b^2/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2
))^2/(a+b)/(1+sin(f*x+e))*(a+b-b*cos(f*x+e)^2)^(1/2)+1/4/f*b^3/(b+(-a*b)^(1
/2))^2/(-b+(-a*b)^(1/2))^2/(a+b)^(3/2)*ln((2*(a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-2*b*sin(f*x+e)+2*a)/(1+sin(f*x+e)))-1/12/f*b/(b+(-a*b)^(1/2))^2/(-
b+(-a*b)^(1/2))^2/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/
b)^(1/2)-1/12/f*b*(-a*b)^(1/2)/(b+(-a*b)^(1/2))^2/(-b+(-a*b)^(1/2))^2/a/(si
n(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)+1/2/f*b^3/(b+(
-a*b)^(1/2))^3/(-b+(-a*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)
*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*a-1/2/f*b^4/(b+(-a*b)^(1/2))^3/(-b+(-a
*b)^(1/2))^3/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b
+b^2)/b)^(1/2)
```

maxima [A] time = 0.78, size = 262, normalized size = 1.71

$$\frac{\frac{3\,\left(2\,ab^{2}-3\,b^{3}\right)\log\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}-\sqrt{a+b}}{\sqrt{b\sin\left(fx+e\right)^{2}+a}+\sqrt{a+b}}\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\sqrt{a+b}}-\frac{2\,\left(2\,a^{3}b^{2}+4\,a^{2}b^{3}+2\,ab^{4}-3\,\left(2\,ab^{2}-3\,b^{3}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)^{2}+2\,\left(2\,a^{2}b^{2}-ab^{3}-3\,b^{4}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)\right)}{\left(a^{3}+3\,a^{2}b+3\,ab^{2}+b^{3}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}-\left(a^{4}+4\,a^{3}b+6\,a^{2}b^{2}+4\,ab^{3}+b^{4}\right)\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}}}{12\,b^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/12*(3*(2*a*b^2 - 3*b^3)*log((sqrt(b*sin(f*x + e)^2 + a) - sqrt(a + b))/(s
qrt(b*sin(f*x + e)^2 + a) + sqrt(a + b)))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*
sqrt(a + b)) - 2*(2*a^3*b^2 + 4*a^2*b^3 + 2*a*b^4 - 3*(2*a*b^2 - 3*b^3)*(b*
sin(f*x + e)^2 + a)^2 + 2*(2*a^2*b^2 - a*b^3 - 3*b^4)*(b*sin(f*x + e)^2 + a
))/((a^3 + 3*a^2*b + 3*a*b^2 + b^3)*(b*sin(f*x + e)^2 + a)^(5/2) - (a^4 + 4
*a^3*b + 6*a^2*b^2 + 4*a*b^3 + b^4)*(b*sin(f*x + e)^2 + a)^(3/2)))/(b^2*f)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.534 $\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=91

$$-\frac{1}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{1}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

```
[Out] arctanh((a+b*sin(f*x+e)^2)^(1/2)/(a+b)^(1/2))/(a+b)^(5/2)/f-1/3/(a+b)/f/(a+
b*sin(f*x+e)^2)^(3/2)-1/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$-\frac{1}{f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}-\frac{1}{3f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{f(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a + b]]/((a + b)^(5/2)*f) - 1/(3*(a
 + b)*f*(a + b*Sin[e + f*x]^2)^(3/2)) - 1/((a + b)^2*f*Sqrt[a + b*Sin[e + f
*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\tan(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{5/2}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{(1-x)(a+bx)^{3/2}}\,dx,x,\sin^2(e+fx)\right)}{2(a+b)f}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{(1}\right.}{}$$

$$= -\frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{}{1}\right.}{}$$

$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a+b}}\right)}{(a+b)^{5/2}f} - \frac{1}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{1}{(a+b)^2 f\sqrt{a+b\sin}}$$

Mathematica [C] time = 0.08, size = 56, normalized size = 0.62

$$-\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};1-\frac{b\cos^2(e+fx)}{a+b}\right)}{3f(a+b)\left(a-b\cos^2(e+fx)+b\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/3*Hypergeometric2F1[-3/2, 1, -1/2, 1 - (b*Cos[e + f*x]^2)/(a + b)]/((a +
 b)*f*(a + b - b*Cos[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.60, size = 521, normalized size = 5.73

$$\left[\frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{a+b}\,\log\left(\frac{b\cos\left(fx+e\right)^2-2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\sqrt{a+b}}{\cos\left(fx+e\right)^2}\right.}{6\left(\left(a^3b^2+3\,a^2b^3+3\,ab^4+b^5\right)f\cos\left(fx+e\right)^4-2\left(a^4b+4\,a^3b^2+6\,a^2b^3+4\,ab^4+b^5\right)f\,c\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a + b)*log((b*cos(f*x + e)^2 - 2*sqrt(-b*cos(f*x + e)^2 + a + b)*
sqrt(a + b) - 2*a - 2*b)/cos(f*x + e)^2) + 2*(3*(a*b + b^2)*cos(f*x + e)^2
- 4*a^2 - 8*a*b - 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2
*b^3 + 3*a*b^4 + b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 +
 4*a*b^4 + b^5)*f*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3
 + 5*a*b^4 + b^5)*f), -1/3*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x +
 e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a - b)*arctan(sqrt(-b*cos(f*x + e)^2 + a +
```

```
b)*sqrt(-a - b)/(a + b)) - (3*(a*b + b^2)*cos(f*x + e)^2 - 4*a^2 - 8*a*b -
 4*b^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/((a^3*b^2 + 3*a^2*b^3 + 3*a*b^4 +
b^5)*f*cos(f*x + e)^4 - 2*(a^4*b + 4*a^3*b^2 + 6*a^2*b^3 + 4*a*b^4 + b^5)*f
*cos(f*x + e)^2 + (a^5 + 5*a^4*b + 10*a^3*b^2 + 10*a^2*b^3 + 5*a*b^4 + b^5)
*f)]
```

giac [B] time = 1.22, size = 842, normalized size = 9.25

$$-\frac{\left(\left(\left(\frac{\left(4a^9b^2+33a^8b^3+120a^7b^4+252a^6b^5+336a^5b^6+294a^4b^7+168a^3b^8+60a^2b^9+12ab^{10}+b^{11}\right)\tan\left(\frac{1}{2}fx+\frac{1}{2}e\right)^2}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7+210a^4b^8+120a^3b^9+45a^2b^{10}+10ab^{11}+b^{12}}+\frac{3\left(4a^9b^2+37a^8b^3+152a^7b^4+364a^6b^5+560a^5b^6+574\right.}{a^{10}b^2+10a^9b^3+45a^8b^4+120a^7b^5+210a^6b^6+252a^5b^7}\right.\right.\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/3*(((((4*a^9*b^2 + 33*a^8*b^3 + 120*a^7*b^4 + 252*a^6*b^5 + 336*a^5*b^6
+ 294*a^4*b^7 + 168*a^3*b^8 + 60*a^2*b^9 + 12*a*b^10 + b^11)*tan(1/2*f*x +
1/2*e)^2/(a^10*b^2 + 10*a^9*b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 +
252*a^5*b^7 + 210*a^4*b^8 + 120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12) +
 3*(4*a^9*b^2 + 37*a^8*b^3 + 152*a^7*b^4 + 364*a^6*b^5 + 560*a^5*b^6 + 574*
a^4*b^7 + 392*a^3*b^8 + 172*a^2*b^9 + 44*a*b^10 + 5*b^11)/(a^10*b^2 + 10*a^
9*b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8
+ 120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 + 3
*(4*a^9*b^2 + 37*a^8*b^3 + 152*a^7*b^4 + 364*a^6*b^5 + 560*a^5*b^6 + 574*a^
4*b^7 + 392*a^3*b^8 + 172*a^2*b^9 + 44*a*b^10 + 5*b^11)/(a^10*b^2 + 10*a^9*
b^3 + 45*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8 +
120*a^3*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))*tan(1/2*f*x + 1/2*e)^2 + (4*
a^9*b^2 + 33*a^8*b^3 + 120*a^7*b^4 + 252*a^6*b^5 + 336*a^5*b^6 + 294*a^4*b^
7 + 168*a^3*b^8 + 60*a^2*b^9 + 12*a*b^10 + b^11)/(a^10*b^2 + 10*a^9*b^3 + 4
5*a^8*b^4 + 120*a^7*b^5 + 210*a^6*b^6 + 252*a^5*b^7 + 210*a^4*b^8 + 120*a^3
*b^9 + 45*a^2*b^10 + 10*a*b^11 + b^12))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan
(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) + 6*arctan(-1/2
*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(
1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a) - sqrt(a))/sqrt(-a - b
))/((a^2 + 2*a*b + b^2)*sqrt(-a - b)))/f
```

maple [B] time = 5.35, size = 898, normalized size = 9.87

$$\frac{-8a^3b^3\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab^2+b^3}{b^2}}\,\sqrt{a+b}-8a^2b^4\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab^2+b^3}{b^2}}\,\sqrt{a+b}+3a^4b^3\ln\left(\frac{2\sqrt{a+b}}{}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/b^3/(a+b)^(1/2)/a^2/(a^2*b^2*cos(f*x+e)^4+2*a*b^3*cos(f*x+e)^4+b^4*cos(
f*x+e)^4-2*a^3*b*cos(f*x+e)^2-6*a^2*b^2*cos(f*x+e)^2-6*a*b^3*cos(f*x+e)^2-2
*b^4*cos(f*x+e)^2+a^4+4*a^3*b+6*a^2*b^2+4*a*b^3+b^4)*(-8*a^3*b^3*(-b*cos(f*
x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)-8*a^2*b^4*(-b*cos(f*x+e)^2+(a*b^2
+b^3)/b^2)^(1/2)*(a+b)^(1/2)+3*a^4*b^3*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+3*a^4*b^3*ln(2/(1+sin(f*x+e))*((a+
b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))+3*ln(2/(sin(f*x+e)-1)*
((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a^2*b^5+3*ln(2/(1+
sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a^2*b^
```

```
5+6*a^3*b^4*ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*s
in(f*x+e)+a))+6*a^3*b^4*ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^
2)^(1/2)-b*sin(f*x+e)+a))+3*a^2*b^5*(ln(2/(sin(f*x+e)-1)*((a+b)^(1/2)*(a+b-
b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))+ln(2/(1+sin(f*x+e))*((a+b)^(1/2)*(a+
b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a)))*cos(f*x+e)^4+6*cos(f*x+e)^2*a^2*b
^4*((-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*(a+b)^(1/2)-ln(2/(1+sin(f*x+e))
*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*a-ln(2/(1+sin(f*x
+e))*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)-b*sin(f*x+e)+a))*b-ln(2/(sin(f
*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*a-ln(2/(s
in(f*x+e)-1)*((a+b)^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+b*sin(f*x+e)+a))*b))/f
```

maxima [B] time = 0.77, size = 203, normalized size = 2.23

$$-\frac{\frac{2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}a+\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{3}{2}}b}+\frac{6}{\sqrt{b\sin\left(fx+e\right)^2+a}\,a^2+2\sqrt{b\sin\left(fx+e\right)^2+a}\,ab+\sqrt{b\sin\left(fx+e\right)^2+a}\,b^2}+\frac{3\,\mathrm{arsinh}\left(\frac{b\sin\left(fx+e\right)}{\sqrt{ab}\left(\sin\left(fx+e\right)+1\right)}-\right.}{(a+b)^{\frac{5}{2}}}}{6f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/6*(2/((b*sin(f*x + e)^2 + a)^(3/2)*a + (b*sin(f*x + e)^2 + a)^(3/2)*b) +
 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2 + 2*sqrt(b*sin(f*x + e)^2 + a)*a*b + sqr
t(b*sin(f*x + e)^2 + a)*b^2) + 3*arcsinh(b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x
 + e) + 1)) - a/(sqrt(a*b)*(sin(f*x + e) + 1)))/(a + b)^(5/2) - 3*arcsinh(-
b*sin(f*x + e)/(sqrt(a*b)*(sin(f*x + e) - 1)) - a/(sqrt(a*b)*(sin(f*x + e)
- 1)))/(a + b)^(5/2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(e+f\,x\right)}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan{\left(e + f x \right)}}{\left(a + b \sin^{2}{\left(e + f x \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.535 $\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=83

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

```
[Out] -arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(5/2)/f+1/3/a/f/(a+b*sin(f*x+e
)^2)^(3/2)+1/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.09, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3194, 51, 63, 208}

$$\frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}}-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f}+\frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -(ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]]/(a^(5/2)*f)) + 1/(3*a*f*(a +
b*Sin[e + f*x]^2)^(3/2)) + 1/(a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff, x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$
$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{Subst}\left(\int \frac{1}{x(a+bx)^{3/2}}\,dx, x, \sin^2(e+fx)\right)}{2af}$$
$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \text{s}\right.}{2a^2f}$$
$$= \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{\right.}}{a^2bf}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{a^{5/2}f} + \frac{1}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{1}{a^2f\sqrt{a+b\sin^2(e+fx}}$$

Mathematica [C] time = 0.06, size = 49, normalized size = 0.59

$$\frac{{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2)/a]/(3*a*f*(a + b*Si
n[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.60, size = 382, normalized size = 4.60

$$\left[\frac{3\left(b^2\cos\left(fx+e\right)^4-2\left(ab+b^2\right)\cos\left(fx+e\right)^2+a^2+2\,ab+b^2\right)\sqrt{a}\log\left(\frac{2\left(b\cos\left(fx+e\right)^2+2\sqrt{-b\cos\left(fx+e\right)^2+a+b}\sqrt{a}-2a-\right.}{\cos\left(fx+e\right)^2-1}\right.}{6\left(a^3b^2f\cos\left(fx+e\right)^4-2\left(a^4b+a^3b^2\right)f\cos\left(fx+e\right)^2+\left(a^5-\right.\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/6*(3*(b^2*cos(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b +
b^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e)^2 + a + b)*sq
rt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*a*b*cos(f*x + e)^2 - 4*a^2 -
3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4 - 2*(a^4*
b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f), 1/3*(3*(b^2*c
os(f*x + e)^4 - 2*(a*b + b^2)*cos(f*x + e)^2 + a^2 + 2*a*b + b^2)*sqrt(-a)*
arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*a*b*cos(f*x + e)^2
```

```
- 4*a^2 - 3*a*b)*sqrt(-b*cos(f*x + e)^2 + a + b))/(a^3*b^2*f*cos(f*x + e)^4
 - 2*(a^4*b + a^3*b^2)*f*cos(f*x + e)^2 + (a^5 + 2*a^4*b + a^3*b^2)*f)]
```

giac [A] time = 0.18, size = 74, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{b\sin\left(fx+e\right)^{2}+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,a^{2}f}+\frac{3\,b\sin\left(fx+e\right)^{2}+4\,a}{3\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a^{2}f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] arctan(sqrt(b*sin(f*x + e)^2 + a)/sqrt(-a))/(sqrt(-a)*a^2*f) + 1/3*(3*b*sin
(f*x + e)^2 + 4*a)/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*f)
```

maple [B] time = 3.43, size = 271, normalized size = 3.27

$$\frac{7\sqrt{-b\left(\cos^{2}\left(fx+e\right)\right)+\frac{ab+b^{2}}{b}}}{12f\,a^{2}\sqrt{-ab}\,\left(\sin\left(fx+e\right)-\frac{\sqrt{-ab}}{b}\right)}-\frac{\sqrt{-b\left(\cos^{2}\left(fx+e\right)\right)+\frac{ab+b^{2}}{b}}}{12f\,a^{2}b\left(\sin\left(fx+e\right)+\frac{\sqrt{-ab}}{b}\right)^{2}}-\frac{7\sqrt{-b\left(\cos^{2}\left(fx+e\right)\right)+\frac{ab+b^{2}}{b}}}{12f\,a^{2}\sqrt{-ab}\,\left(\sin\left(fx+e\right)+\frac{\sqrt{-ab}}{b}\right)}-\frac{\ln}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b
^2)/b)^(1/2)-1/12/f/a^2/b/(sin(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a
*b+b^2)/b)^(1/2)-7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*co
s(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/f/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)
^2)^(1/2))/sin(f*x+e))-1/12/f/a^2/b/(sin(f*x+e)-(-a*b)^(1/2)/b)^2*(-b*cos(f
*x+e)^2+(a*b+b^2)/b)^(1/2)
```

maxima [A] time = 0.38, size = 66, normalized size = 0.80

$$-\frac{\frac{3\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{5}{2}}}-\frac{3}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a}}{3\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/3*(3*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) - 3/(sqrt(b*sin(f*
x + e)^2 + a)*a^2) - 1/((b*sin(f*x + e)^2 + a)^(3/2)*a))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\cot\left(e+f\,x\right)}{{\left(b\,{\sin\left(e+f\,x\right)}^{2}+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(e+fx\right)}{\left(a+b\sin^{2}\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.536 $\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=143

$$\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^2(e+f}{2af\left(a+b\sin^2(e\right.}$$

```
[Out] 1/2*(2*a+5*b)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(7/2)/f+1/6*(-2*a
-5*b)/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/2*csc(f*x+e)^2/a/f/(a+b*sin(f*x+e)^2
)^(3/2)+1/2*(-2*a-5*b)/a^3/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.14, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3194, 78, 51, 63, 208}

$$-\frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e+fx)}}-\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f}-\frac{\csc^2(e+}{2af\left(a+b\sin^2(\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((2*a + 5*b)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(2*a^(7/2)*f) - (
2*a + 5*b)/(6*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) - Csc[e + f*x]^2/(2*a*f*(
a + b*Sin[e + f*x]^2)^(3/2)) - (2*a + 5*b)/(2*a^3*f*Sqrt[a + b*Sin[e + f*x]
^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\,\text{Subst}\left(\int \frac{1}{x(a+bx)^{5/2}}\,dx, x, \sin^2(e+fx)\right)}{4af}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(2a+5b)\,\text{Subst}\left(\int\right.}{}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e}}$$

$$= -\frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e+fx)}{2af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2a+5b}{2a^3f\sqrt{a+b\sin^2(e}}$$

$$= \frac{(2a+5b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{2a^{7/2}f} - \frac{2a+5b}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\csc^2(e}{2af\left(a+b\sin\right.}$$

Mathematica [C] time = 0.25, size = 69, normalized size = 0.48

$$-\frac{(2a+5b)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+3a\csc^2(e+fx)}{6a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^3/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -1/6*(3*a*Csc[e + f*x]^2 + (2*a + 5*b)*Hypergeometric2F1[-3/2, 1, -1/2, 1 +
 (b*Sin[e + f*x]^2)/a])/(a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

fricas [B] time = 0.61, size = 666, normalized size = 4.66

$$\left[\frac{3\left(\left(2\,ab^2+5\,b^3\right)\cos\left(fx+e\right)^6-\left(4\,a^2b+16\,ab^2+15\,b^3\right)\cos\left(fx+e\right)^4-2\,a^3-9\,a^2b-12\,ab^2-5\,b^3+\left(2\,a^3+\right.\right.}{12\left(\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/12*(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*
cos(f*x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 2
6*a*b^2 + 15*b^3)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 - 2*sqrt(
-b*cos(f*x + e)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) + 2*(3*
(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b + 15*a*b^2 - 2*(4*a^
3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x + e)^2 + a + b))/(
a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^4 + (a^6 +
4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b + a^4*b^2)*f), -1/6*
(3*((2*a*b^2 + 5*b^3)*cos(f*x + e)^6 - (4*a^2*b + 16*a*b^2 + 15*b^3)*cos(f*
x + e)^4 - 2*a^3 - 9*a^2*b - 12*a*b^2 - 5*b^3 + (2*a^3 + 13*a^2*b + 26*a*b^
2 + 15*b^3)*cos(f*x + e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)
*sqrt(-a)/a) - (3*(2*a^2*b + 5*a*b^2)*cos(f*x + e)^4 + 11*a^3 + 26*a^2*b +
15*a*b^2 - 2*(4*a^3 + 16*a^2*b + 15*a*b^2)*cos(f*x + e)^2)*sqrt(-b*cos(f*x
+ e)^2 + a + b))/(a^4*b^2*f*cos(f*x + e)^6 - (2*a^5*b + 3*a^4*b^2)*f*cos(f*
x + e)^4 + (a^6 + 4*a^5*b + 3*a^4*b^2)*f*cos(f*x + e)^2 - (a^6 + 2*a^5*b +
a^4*b^2)*f)]
```

giac [B] time = 1.36, size = 751, normalized size = 5.25

$$-\frac{\left(\left(\left(\left(\frac{3\left(a^{12}b^2+2\,a^{11}b^3+a^{10}b^4\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2}{a^{13}b^2+2\,a^{12}b^3+a^{11}b^4}+\frac{4\left(11\,a^{12}b^2+42\,a^{11}b^3+51\,a^{10}b^4+20\,a^9b^5\right)}{a^{13}b^2+2\,a^{12}b^3+a^{11}b^4}\right)\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+\frac{6\left(19\,a^{12}b^2+90\,a^{11}b^3+163\,a^{10}b^4+132\,a^9b^5+40\,a^8b^6\right)}{a^{13}b^2+2\,a^{12}b^3+a^{11}b^4}\right)\right.\right.}{\left(a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^4+2\,a\tan\left(\frac{1}{2}\,fx+\frac{1}{2}\,e\right)^2+4\,b\tan\right.}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/24*(((((3*(a^12*b^2 + 2*a^11*b^3 + a^10*b^4)*tan(1/2*f*x + 1/2*e)^2/(a^1
3*b^2 + 2*a^12*b^3 + a^11*b^4) + 4*(11*a^12*b^2 + 42*a^11*b^3 + 51*a^10*b^4
 + 20*a^9*b^5)/(a^13*b^2 + 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 +
 6*(19*a^12*b^2 + 90*a^11*b^3 + 163*a^10*b^4 + 132*a^9*b^5 + 40*a^8*b^6)/(a
^13*b^2 + 2*a^12*b^3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 + 12*(9*a^12*b^2 +
 42*a^11*b^3 + 73*a^10*b^4 + 56*a^9*b^5 + 16*a^8*b^6)/(a^13*b^2 + 2*a^12*b^
3 + a^11*b^4))*tan(1/2*f*x + 1/2*e)^2 + 7*(5*a^12*b^2 + 18*a^11*b^3 + 21*a^
10*b^4 + 8*a^9*b^5)/(a^13*b^2 + 2*a^12*b^3 + a^11*b^4))/(a*tan(1/2*f*x + 1/
2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2)
 + 6*(2*a^(3/2) + 5*sqrt(a)*b)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - s
qrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x
 + 1/2*e)^2 + a))*a - a^(3/2) - 2*sqrt(a)*b))/a^4 - 6*((sqrt(a)*tan(1/2*f*x
 + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 +
4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^(3/2) + 2*(sqrt(a)*tan(1/2*f*x + 1/2*e)^
2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/
2*f*x + 1/2*e)^2 + a))*sqrt(a)*b + a^2)/(((sqrt(a)*tan(1/2*f*x + 1/2*e)^2 -
```

```
 sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f
*x + 1/2*e)^2 + a))^2*sqrt(a) - a^(3/2))*a^3))/f
```

maple [B] time = 4.28, size = 1038, normalized size = 7.26

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/6/a^(13/2)/b^2/(b^2*cos(f*x+e)^6-2*a*b*cos(f*x+e)^4-3*b^2*cos(f*x+e)^4+a^
2*cos(f*x+e)^2+4*cos(f*x+e)^2*a*b+3*b^2*cos(f*x+e)^2-a^2-2*a*b-b^2)*(3*(a+b
-b*cos(f*x+e)^2)^(1/2)*a^(11/2)*b^2-6*a^6*b^2*ln(2/sin(f*x+e)*(a^(1/2)*(a+b
-b*cos(f*x+e)^2)^(1/2)+a))+3*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^4+8*a^(11
/2)*b^2*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)+20*(-b*cos(f*x+e)^2+(a*b^2+
b^3)/b^2)^(1/2)*a^(9/2)*b^3+6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b^3+12*(-b
*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b^4-27*a^5*b^3*ln(2/sin(f*x+e)
*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))-36*a^4*b^4*ln(2/sin(f*x+e)*(a^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x
+e)^2)^(1/2)+a))*a^3*b^5+3*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1
/2)+a))*a^3*b^4*(2*a+5*b)*cos(f*x+e)^6+3*cos(f*x+e)^4*b^3*(2*(-b*cos(f*x+e)
^2+(a*b^2+b^3)/b^2)^(1/2)*a^(9/2)+(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b+4*(-
b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b-4*ln(2/sin(f*x+e)*(a^(1/2)*
(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^5-16*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f
*x+e)^2)^(1/2)+a))*a^4*b-15*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(
1/2)+a))*a^3*b^2)-cos(f*x+e)^2*b^2*(8*(-b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/
2)*a^(11/2)+6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(9/2)*b+26*(-b*cos(f*x+e)^2+(a*b
^2+b^3)/b^2)^(1/2)*a^(9/2)*b+6*(a+b-b*cos(f*x+e)^2)^(1/2)*a^(7/2)*b^2+24*(-
b*cos(f*x+e)^2+(a*b^2+b^3)/b^2)^(1/2)*a^(7/2)*b^2-6*ln(2/sin(f*x+e)*(a^(1/2
)*(a+b-b*cos(f*x+e)^2)^(1/2)+a))*a^6-39*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos
(f*x+e)^2)^(1/2)+a))*a^5*b-78*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)
^(1/2)+a))*a^4*b^2-45*ln(2/sin(f*x+e)*(a^(1/2)*(a+b-b*cos(f*x+e)^2)^(1/2)+a
))*a^3*b^3))/f
```

maxima [A] time = 0.44, size = 156, normalized size = 1.09

$$\frac{\frac{6\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{5}{2}}}+\frac{15\,b\,\operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{7}{2}}}-\frac{6}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{2}}-\frac{2}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}}a}-\frac{15\,b}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\,a^{3}}-\frac{5\,b}{\left(b\sin\left(fx+e\right)^{2}\right.}}{6\,f}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^3/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] 1/6*(6*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 15*b*arcsinh(a/(s
qrt(a*b)*abs(sin(f*x + e))))/a^(7/2) - 6/(sqrt(b*sin(f*x + e)^2 + a)*a^2) -
 2/((b*sin(f*x + e)^2 + a)^(3/2)*a) - 15*b/(sqrt(b*sin(f*x + e)^2 + a)*a^3)
 - 5*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 3/((b*sin(f*x + e)^2 + a)^(3/2)
*a*sin(f*x + e)^2))/f
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot\left(e+f\,x\right)^3}{{\left(b\,\sin\left(e+f\,x\right)^2+a\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^3/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**3/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(cot(e + f*x)**3/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.537 $\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=208

$$\frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8a^2}{24a^3f\left(a\right.}$$

```
[Out] -1/8*(8*a^2+40*a*b+35*b^2)*arctanh((a+b*sin(f*x+e)^2)^(1/2)/a^(1/2))/a^(9/2
)/f+1/24*(8*a^2+40*a*b+35*b^2)/a^3/f/(a+b*sin(f*x+e)^2)^(3/2)+1/8*(8*a+7*b)
*csc(f*x+e)^2/a^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/4*csc(f*x+e)^4/a/f/(a+b*sin(
f*x+e)^2)^(3/2)+1/8*(8*a^2+40*a*b+35*b^2)/a^4/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.20, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3194, 89, 78, 51, 63, 208}

$$\frac{8a^2+40ab+35b^2}{8a^4f\sqrt{a+b\sin^2(e+fx)}}+\frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{(8a+}{8a^2f\left(a\right.}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] -((8*a^2 + 40*a*b + 35*b^2)*ArcTanh[Sqrt[a + b*Sin[e + f*x]^2]/Sqrt[a]])/(8
*a^(9/2)*f) + (8*a^2 + 40*a*b + 35*b^2)/(24*a^3*f*(a + b*Sin[e + f*x]^2)^(3
/2)) + ((8*a + 7*b)*Csc[e + f*x]^2)/(8*a^2*f*(a + b*Sin[e + f*x]^2)^(3/2))
- Csc[e + f*x]^4/(4*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (8*a^2 + 40*a*b + 3
5*b^2)/(8*a^4*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 51

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^(n + 1))/((b*c - a*d)*(m + 1)), x] - Dist[(d*(
m + n + 2))/((b*c - a*d)*(m + 1)), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 89

```
Int[((a_.) + (b_.)*(x_))^2*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^
(p_.), x_Symbol] :> Simp[((b*c - a*d)^2*(c + d*x)^(n + 1)*(e + f*x)^(p + 1)
)/(d^2*(d*e - c*f)*(n + 1)), x] - Dist[1/(d^2*(d*e - c*f)*(n + 1)), Int[(c
+ d*x)^(n + 1)*(e + f*x)^p*Simp[a^2*d^2*f*(n + p + 2) + b^2*c*(d*e*(n + 1)
+ c*f*(p + 1)) - 2*a*b*d*(d*e*(n + 1) + c*f*(p + 1)) - b^2*d*(d*e - c*f)*(n
 + 1)*x, x], x], x] /; FreeQ[{a, b, c, d, e, f, n, p}, x] && (LtQ[n, -1] ||
 (EqQ[n + p + 3, 0] && NeQ[n, -1] && (SumSimplerQ[n, 1] ||  !SumSimplerQ[p,
 1])))
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \frac{\cot^5(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3(a+bx)^{5/2}}\,dx,x,\sin^2(e+fx)\right)}{2f}$$

$$= -\frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\text{Subst}\left(\int \frac{\frac{1}{2}(-8a-7b)+2ax}{x^2(a+bx)^{5/2}}\,dx,x,\sin^2(e+fx)\right)}{4af}$$

$$= \frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^4(e+fx)}{4af\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{\left(8a^2+40ab+35\right.}{}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^4(e+}{4af\left(a+b\sin^2\right.}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^4(e+}{4af\left(a+b\sin^2\right.}$$

$$= \frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{(8a+7b)\csc^2(e+fx)}{8a^2f\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{\csc^4(e+}{4af\left(a+b\sin^2\right.}$$

$$= -\frac{\left(8a^2+40ab+35b^2\right)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^2(e+fx)}}{\sqrt{a}}\right)}{8a^{9/2}f}+\frac{8a^2+40ab+35b^2}{24a^3f\left(a+b\sin^2(e+fx)\right)^{3/2}}$$

Mathematica [C] time = 0.84, size = 117, normalized size = 0.56

$$\frac{\left(8a^2+40ab+35b^2\right)\csc^2(e+fx)\,{}_2F_1\left(-\frac{3}{2},1;-\frac{1}{2};\frac{b\sin^2(e+fx)}{a}+1\right)+3a\csc^4(e+fx)\left(-2a\csc^2(e+fx)+8a+7b\right)}{24a^3f\sqrt{a+b\sin^2(e+fx)}\left(a\csc^2(e+fx)+b\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^5/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (3*a*Csc[e + f*x]^4*(8*a + 7*b - 2*a*Csc[e + f*x]^2) + (8*a^2 + 40*a*b + 35
*b^2)*Csc[e + f*x]^2*Hypergeometric2F1[-3/2, 1, -1/2, 1 + (b*Sin[e + f*x]^2
)/a])/(24*a^3*f*(b + a*Csc[e + f*x]^2)*Sqrt[a + b*Sin[e + f*x]^2])
```

fricas [B] time = 0.67, size = 984, normalized size = 4.73

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] [1/48*(3*((8*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*
a^2*b^2 + 115*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*
b^2 + 450*a*b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2
+ 110*a*b^3 + 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b
^4)*cos(f*x + e)^2)*sqrt(a)*log(2*(b*cos(f*x + e)^2 + 2*sqrt(-b*cos(f*x + e
)^2 + a + b)*sqrt(a) - 2*a - b)/(cos(f*x + e)^2 - 1)) - 2*(3*(8*a^3*b + 40*
a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*a^2*b^2 + 31
5*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 105*a*b^3 + (8
8*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sqrt(-b*cos(f*
x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5*b^2)*f*cos(
f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(a^7 + 3*a^6*
b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f), 1/24*(3*((8
*a^2*b^2 + 40*a*b^3 + 35*b^4)*cos(f*x + e)^8 - 2*(8*a^3*b + 56*a^2*b^2 + 11
5*a*b^3 + 70*b^4)*cos(f*x + e)^6 + (8*a^4 + 88*a^3*b + 323*a^2*b^2 + 450*a*
b^3 + 210*b^4)*cos(f*x + e)^4 + 8*a^4 + 56*a^3*b + 123*a^2*b^2 + 110*a*b^3
+ 35*b^4 - 2*(8*a^4 + 64*a^3*b + 171*a^2*b^2 + 185*a*b^3 + 70*b^4)*cos(f*x
+ e)^2)*sqrt(-a)*arctan(sqrt(-b*cos(f*x + e)^2 + a + b)*sqrt(-a)/a) - (3*(8
*a^3*b + 40*a^2*b^2 + 35*a*b^3)*cos(f*x + e)^6 - (32*a^4 + 232*a^3*b + 500*
a^2*b^2 + 315*a*b^3)*cos(f*x + e)^4 - 50*a^4 - 205*a^3*b - 260*a^2*b^2 - 10
5*a*b^3 + (88*a^4 + 413*a^3*b + 640*a^2*b^2 + 315*a*b^3)*cos(f*x + e)^2)*sq
rt(-b*cos(f*x + e)^2 + a + b))/(a^5*b^2*f*cos(f*x + e)^8 - 2*(a^6*b + 2*a^5
*b^2)*f*cos(f*x + e)^6 + (a^7 + 6*a^6*b + 6*a^5*b^2)*f*cos(f*x + e)^4 - 2*(
a^7 + 3*a^6*b + 2*a^5*b^2)*f*cos(f*x + e)^2 + (a^7 + 2*a^6*b + a^5*b^2)*f)]
```

giac [B] time = 1.99, size = 1411, normalized size = 6.78

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] -1/192*(((((3*((a^17*b^2 + 2*a^16*b^3 + a^15*b^4)*tan(1/2*f*x + 1/2*e)^2/(a
^18*b^2 + 2*a^17*b^3 + a^16*b^4) - (9*a^17*b^2 + 32*a^16*b^3 + 37*a^15*b^4
+ 14*a^14*b^5)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 -
 2*(197*a^17*b^2 + 1106*a^16*b^3 + 2181*a^15*b^4 + 1832*a^14*b^5 + 560*a^13
*b^6)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 - 6*(165*a
^17*b^2 + 1072*a^16*b^3 + 2761*a^15*b^4 + 3526*a^14*b^5 + 2232*a^13*b^6 + 5
60*a^12*b^7)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e)^2 - 3
```

```
*(307*a^17*b^2 + 1958*a^16*b^3 + 4835*a^15*b^4 + 5792*a^14*b^5 + 3376*a^13*
b^6 + 768*a^12*b^7)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))*tan(1/2*f*x + 1/2*e
)^2 - (295*a^17*b^2 + 1552*a^16*b^3 + 2859*a^15*b^4 + 2242*a^14*b^5 + 640*a
^13*b^6)/(a^18*b^2 + 2*a^17*b^3 + a^16*b^4))/(a*tan(1/2*f*x + 1/2*e)^4 + 2*
a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a)^(3/2) - 24*(8*a^
2 + 40*a*b + 35*b^2)*arctan(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1
/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2
 + a))/sqrt(-a))/(sqrt(-a)*a^4) - 12*(8*a^(5/2) + 40*a^(3/2)*b + 35*sqrt(a)
*b^2)*log(abs(-(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e
)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a - a^(
3/2) - 2*sqrt(a)*b))/a^5 + 12*(6*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*t
an(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*
e)^2 + a))^3*a^2 + 24*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x
+ 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^
3*a*b + 22*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4
+ 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^3*b^2 + 5*(
sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/
2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(5/2) + 8*(sqrt(a)*
tan(1/2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x +
1/2*e)^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))^2*a^(3/2)*b - 8*(sqrt(a)*tan(1/
2*f*x + 1/2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)
^2 + 4*b*tan(1/2*f*x + 1/2*e)^2 + a))*a^3 - 28*(sqrt(a)*tan(1/2*f*x + 1/2*e
)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(
1/2*f*x + 1/2*e)^2 + a))*a^2*b - 26*(sqrt(a)*tan(1/2*f*x + 1/2*e)^2 - sqrt(
a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*tan(1/2*f*x + 1
/2*e)^2 + a))*a*b^2 - 7*a^(7/2) - 12*a^(5/2)*b)/(((sqrt(a)*tan(1/2*f*x + 1/
2*e)^2 - sqrt(a*tan(1/2*f*x + 1/2*e)^4 + 2*a*tan(1/2*f*x + 1/2*e)^2 + 4*b*t
an(1/2*f*x + 1/2*e)^2 + a))^2 - a)^2*a^4))/f
```

maple [B] time = 5.60, size = 901, normalized size = 4.33

$$\frac{7\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{12f\,a^2\sqrt{-ab}\,\left(\sin\left(fx+e\right)-\frac{\sqrt{-ab}}{b}\right)}+\frac{13b\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}}{6f\,a^3\sqrt{-ab}\,\left(\sin\left(fx+e\right)-\frac{\sqrt{-ab}}{b}\right)}+\frac{19\sqrt{-b\left(\cos^2\left(fx+e\right)\right)+\frac{ab+b^2}{b}}\;b^2}{12f\,a^4\sqrt{-ab}\,\left(\sin\left(fx+e\right)-\frac{\sqrt{-ab}}{b}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 7/12/f/a^2/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b
^2)/b)^(1/2)+13/6/f/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)-(-a*b)^(1/2)/b)*(-b*cos(
f*x+e)^2+(a*b+b^2)/b)^(1/2)+19/12/f/a^4/(-a*b)^(1/2)/(sin(f*x+e)-(-a*b)^(1/
2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)*b^2-1/12/f/a^2/b/(sin(f*x+e)-(-a*
b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/f/a^3/(sin(f*x+e)-(-a
*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^4*b/(sin(f*x+e)
-(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^2/b/(sin(f*
x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/6/f/a^3/(sin(f
*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-1/12/f/a^4*b/(s
in(f*x+e)+(-a*b)^(1/2)/b)^2*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2)-7/12/f/a^2/
(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a*b+b^2)/b)^(1/2
)-13/6/f/a^3/(-a*b)^(1/2)*b/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*cos(f*x+e)^2+(a
*b+b^2)/b)^(1/2)-19/12/f/a^4/(-a*b)^(1/2)/(sin(f*x+e)+(-a*b)^(1/2)/b)*(-b*c
os(f*x+e)^2+(a*b+b^2)/b)^(1/2)*b^2-1/f/a^(5/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f
*x+e)^2)^(1/2))/sin(f*x+e))-5/f/a^(7/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2
)^(1/2))/sin(f*x+e))*b-35/8/f/a^(9/2)*ln((2*a+2*a^(1/2)*(a+b*sin(f*x+e)^2)^
(1/2))/sin(f*x+e))*b^2+1/f/a^3/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)+11/8/f
/a^4*b/sin(f*x+e)^2*(a+b*sin(f*x+e)^2)^(1/2)-1/4/f/a^3/sin(f*x+e)^4*(a+b*si
n(f*x+e)^2)^(1/2)
```

maxima [A] time = 0.40, size = 280, normalized size = 1.35

$$-\frac{\frac{24 \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{5}{2}}} + \frac{120\, b \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{7}{2}}} + \frac{105\, b^{2} \operatorname{arsinh}\left(\frac{a}{\sqrt{ab}\left|\sin\left(fx+e\right)\right|}\right)}{a^{\frac{9}{2}}} - \frac{24}{\sqrt{b\sin\left(fx+e\right)^{2}+a}\, a^{2}} - \frac{8}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{3}{2}} a} - \cdots}{\cdots}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^5/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] -1/24*(24*arcsinh(a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(5/2) + 120*b*arcsinh(
a/(sqrt(a*b)*abs(sin(f*x + e))))/a^(7/2) + 105*b^2*arcsinh(a/(sqrt(a*b)*abs
(sin(f*x + e))))/a^(9/2) - 24/(sqrt(b*sin(f*x + e)^2 + a)*a^2) - 8/((b*sin(
f*x + e)^2 + a)^(3/2)*a) - 120*b/(sqrt(b*sin(f*x + e)^2 + a)*a^3) - 40*b/((
b*sin(f*x + e)^2 + a)^(3/2)*a^2) - 105*b^2/(sqrt(b*sin(f*x + e)^2 + a)*a^4)
 - 35*b^2/((b*sin(f*x + e)^2 + a)^(3/2)*a^3) - 24/((b*sin(f*x + e)^2 + a)^(
3/2)*a*sin(f*x + e)^2) - 21*b/((b*sin(f*x + e)^2 + a)^(3/2)*a^2*sin(f*x + e
)^2) + 6/((b*sin(f*x + e)^2 + a)^(3/2)*a*sin(f*x + e)^4))/f
```

mupad [F(-1)] time = 0.00, size = -1, normalized size = -0.00

Hanged

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^5/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] \text{Hanged}
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(e + fx\right)}{\left(a + b\sin^{2}\left(e + fx\right)\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**5/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**5/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.538 $\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}} dx$

Optimal. Leaf size=348

$$-\frac{2(2a-b)\tan(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8b(a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}+\frac{b(5a-3b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{}{3}$$

```
[Out] 1/3*(5*a-3*b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(3/2)+8/
3*(a-b)*b*cos(f*x+e)*sin(f*x+e)/(a+b)^4/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a-b
)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*s
in(f*x+e)^2)^(1/2)/(a+b)^4/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(5*a-3*b)*Ellip
ticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+
e)^2/a)^(1/2)/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-2/3*(2*a-b)*tan(f*x+e)/(a+
b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*sec(f*x+e)^2*tan(f*x+e)/(a+b)/f/(a+b*si
n(f*x+e)^2)^(3/2)
```

Rubi [A] time = 0.43, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 470, 527, 524, 426, 424, 421, 419}

$$-\frac{2(2a-b)\tan(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{8b(a-b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^4\sqrt{a+b\sin^2(e+fx)}}+\frac{b(5a-3b)\sin(e+fx)\cos(e+fx)}{3f(a+b)^3\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{}{3}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((5*a - 3*b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^3*f*(a + b*Sin[e + f*x
]^2)^(3/2)) + (8*(a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^4*f*Sqrt[a
 + b*Sin[e + f*x]^2]) + (8*(a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Si
n[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*(a + b)^4*
f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - ((5*a - 3*b)*Sqrt[Cos[e + f*x]^2]*Ellip
ticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)
/a])/(3*(a + b)^3*f*Sqrt[a + b*Sin[e + f*x]^2]) - (2*(2*a - b)*Tan[e + f*x]
)/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2)) + (Sec[e + f*x]^2*Tan[e + f*
x])/(3*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 470

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[(a*e^(2*n - 1)*(e*x)^(m - 2*n + 1)*(a + b*x^n)^
(p + 1)*(c + d*x^n)^(q + 1))/(b*n*(b*c - a*d)*(p + 1)), x] + Dist[e^(2*n)/(
b*n*(b*c - a*d)*(p + 1)), Int[(e*x)^(m - 2*n)*(a + b*x^n)^(p + 1)*(c + d*x^
n)^q*Simp[a*c*(m - 2*n + 1) + (a*d*(m - n + n*q + 1) + b*c*n*(p + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[m - n + 1, n] && IntBinomialQ[a, b, c, d, e, m, n,
p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^4}{\left(1-x^2\right)^{5/2}\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{a}{(1-x}\right.}{3(a+b)f}$$

$$= -\frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\sec^2(e+fx)\tan(e+fx)}{3(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\text{co}\right.}{}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{2(2a-b)\tan(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\text{s}}{3(a}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(a+}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(a+}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} - \frac{}{3(a+}$$

$$= \frac{(5a-3b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^3f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{8(a-b)b\cos(e+fx)\sin(e+fx)}{3(a+b)^4f\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a-}{}$$

Mathematica [A] time = 3.42, size = 235, normalized size = 0.68

$$2ab\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(\left(-5a^2-2ab+3b^2\right)F\left(e+fx\middle|-\frac{b}{a}\right)+8a(a-b)E\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}\,b\left(2ab(a+b)\text{si}\right.$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*a*(a - b)*EllipticE[e +
f*x, -(b/a)] + (-5*a^2 - 2*a*b + 3*b^2)*EllipticF[e + f*x, -(b/a)]) + Sqrt[
2]*b*(2*a*b*(a + b)*Sin[2*(e + f*x)] + 4*(a - b)*b*(2*a + b - b*Cos[2*(e +
f*x)])*Sin[2*(e + f*x)] - 4*(a - b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Tan[e
+ f*x] + (a + b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Sec[e + f*x]^2*Tan[e + f*
x]))/(6*b*(a + b)^4*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^4}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 3.50, size = 667, normalized size = 1.92

$$-\frac{-8\sqrt{-b\left(\cos^4\left(fx+e\right)\right)+(a+b)\left(\cos^2\left(fx+e\right)\right)}\, b^2(a-b)\sin\left(fx+e\right)\left(\cos^6\left(fx+e\right)\right)+\sqrt{-b\left(\cos^4\left(fx+e\right)\right)}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(-8*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(a-b)*sin(f*x+e)*co
s(f*x+e)^6+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(13*a^2+2*a*b-11*b^
2)*cos(f*x+e)^4*sin(f*x+e)-2*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(2*
a^3+3*a^2*b-b^3)*cos(f*x+e)^2*sin(f*x+e)+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^
2)^(1/2)*(a^3+3*a^2*b+3*a*b^2+b^3)*sin(f*x+e)+(cos(f*x+e)^2)^(1/2)*(-b*cos(
f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*b*(5*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+2*EllipticF(sin(f*x+e),(-1/a*b)^(1/
2))*a*b-3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-8*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b)*cos(f*x+e)^4
-(cos(f*x+e)^2)^(1/2)*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(-b/a*cos(
f*x+e)^2+(a+b)/a)^(1/2)*(5*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+7*Ellip
ticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b-EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
a*b^2-3*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^3-8*EllipticE(sin(f*x+e),(-1
/a*b)^(1/2))*a^3+8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2)*cos(f*x+e)^2
)/(a+b*sin(f*x+e)^2)^(3/2)/(1+sin(f*x+e))/(sin(f*x+e)-1)/(-(a+b*sin(f*x+e)^
2)*(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)^4/cos(f*x+e)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^4}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(tan(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^4\left(e+fx\right)}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(tan(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.539 $\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=292

$$\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{4b\sin(e+fx)\cos(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{4\sqrt{\cos^2(e}}{}$$

```
[Out] -4/3*b*cos(f*x+e)*sin(f*x+e)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(3/2)-1/3*(7*a-b)
*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)^3/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a-b)*E
llipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(
f*x+e)^2)^(1/2)/a/(a+b)^3/f/(1+b*sin(f*x+e)^2/a)^(1/2)+4/3*EllipticF(sin(f*
x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/
2)/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+tan(f*x+e)/(a+b)/f/(a+b*sin(f*x+e)^2)
^(3/2)
```

Rubi [A] time = 0.31, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.320, Rules used = {3196, 471, 527, 524, 426, 424, 421, 419}

$$\frac{\tan(e+fx)}{f(a+b)\left(a+b\sin^2(e+fx)\right)^{3/2}}-\frac{b(7a-b)\sin(e+fx)\cos(e+fx)}{3af(a+b)^3\sqrt{a+b\sin^2(e+fx)}}-\frac{4b\sin(e+fx)\cos(e+fx)}{3f(a+b)^2\left(a+b\sin^2(e+fx)\right)^{3/2}}+\frac{4\sqrt{\cos^2(e}}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-4*b*Cos[e + f*x]*Sin[e + f*x])/(3*(a + b)^2*f*(a + b*Sin[e + f*x]^2)^(3/2
)) - ((7*a - b)*b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)^3*f*Sqrt[a + b*Si
n[e + f*x]^2]) - ((7*a - b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f
*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a*(a + b)^3*f*Sqr
t[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin
[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*(a + b)
^2*f*Sqrt[a + b*Sin[e + f*x]^2]) + Tan[e + f*x]/((a + b)*f*(a + b*Sin[e + f
*x]^2)^(3/2))
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 471

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)
*(c + d*x^n)^(q + 1))/(n*(b*c - a*d)*(p + 1)), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 527

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*x*(a + b*x^n)^(p + 1)*(c +
d*x^n)^(q + 1))/(a*n*(b*c - a*d)*(p + 1)), x] + Dist[1/(a*n*(b*c - a*d)*(p
+ 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*c
- a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ
[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\tan^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{x^2}{(1-x^2)^{3/2}(a+bx^2)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{a-3b}{\sqrt{1-x^2}\,(a+}\right.}{(a+b)f}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\tan(e+fx)}{(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{(a+b)}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{(a+b)}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} + \frac{}{(a+b)}$$

$$= -\frac{4b\cos(e+fx)\sin(e+fx)}{3(a+b)^2f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{(7a-b)b\cos(e+fx)\sin(e+fx)}{3a(a+b)^3f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a-b}{}$$

Mathematica [A] time = 2.66, size = 199, normalized size = 0.68

$$\frac{8a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\right) - 2a^2(7a-b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}E\left(e+fx\middle|-\frac{b}{a}\right) + \frac{\tan(e+fx)(24a^3+}{}}{6af(a+b)^3(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-2*a^2*(7*a - b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*
x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Ellipti
cF[e + f*x, -(b/a)] + ((24*a^3 + 4*a^2*b + 5*a*b^2 + b^3 - 4*a*b*(11*a + 3*
b)*Cos[2*(e + f*x)] + (7*a - b)*b^2*Cos[4*(e + f*x)])*Tan[e + f*x])/Sqrt[2]
)/(6*a*(a + b)^3*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\tan\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx-\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*tan(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(fx+e\right)^2}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(tan(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [B] time = 3.01, size = 851, normalized size = 2.91

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*((-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b^2*(7*a-b)*sin(f*x+e)*cos(
f*x+e)^4-(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*b*(11*a^2+10*a*b-b^2)*c
os(f*x+e)^2*sin(f*x+e)+3*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*a*(a^2+
2*a*b+b^2)*sin(f*x+e)-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(-b*cos(f*x+e)^4+(a
+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(4*EllipticF(sin(f*x+e),(-
1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*EllipticE(sin(f*
x+e),(-1/a*b)^(1/2))*a+EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b)*cos(f*x+e)^2
+4*(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*co
s(f*x+e)^2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+8*(-b*co
s(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^
2+(a+b)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+4*(-b*cos(f*x+e
)^4+(a+b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)
/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2-7*(-b*cos(f*x+e)^4+(a+
b)*cos(f*x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3-6*(-b*cos(f*x+e)^4+(a+b)*cos(f*
x+e)^2)^(1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b+(-b*cos(f*x+e)^4+(a+b)*cos(f*x+e)^2)^(
1/2)*(cos(f*x+e)^2)^(1/2)*(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*EllipticE(sin(f
*x+e),(-1/a*b)^(1/2))*a*b^2)/(a+b*sin(f*x+e)^2)^(3/2)/(-(a+b*sin(f*x+e)^2)*
(sin(f*x+e)-1)*(1+sin(f*x+e)))^(1/2)/(a+b)^3/a/cos(f*x+e)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(e+f\,x\right)^2}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
[Out] int(tan(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^2(e + fx)}{\left(a + b\sin^2(e + fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
[Out] Integral(tan(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.540 $\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+f}{3af(a+b)\left(a+b\sin^2(e+\right.}$$

```
[Out] 1/3*b*cos(f*x+e)*sin(f*x+e)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*b*(2*a+b
)*cos(f*x+e)*sin(f*x+e)/a^2/(a+b)^2/f/(a+b*sin(f*x+e)^2)^(1/2)+2/3*(2*a+b)*
(cos(f*x+e)^2)^(1/2)/cos(f*x+e)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*(a+b*sin
(f*x+e)^2)^(1/2)/a^2/(a+b)^2/f/(1+b*sin(f*x+e)^2/a)^(1/2)-1/3*(cos(f*x+e)^2
)^(1/2)/cos(f*x+e)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*(1+b*sin(f*x+e)^2/a)^
(1/2)/a/(a+b)/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.26, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.438, Rules used = {3184, 3173, 3172, 3178, 3177, 3183, 3182}

$$\frac{2b(2a+b)\sin(e+fx)\cos(e+fx)}{3a^2f(a+b)^2\sqrt{a+b\sin^2(e+fx)}}+\frac{2(2a+b)\sqrt{a+b\sin^2(e+fx)}\,E\left(e+fx\middle|-\frac{b}{a}\right)}{3a^2f(a+b)^2\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}+\frac{b\sin(e+fx)\cos(e+f}{3af(a+b)\left(a+b\sin^2(e+\right.}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (b*Cos[e + f*x]*Sin[e + f*x])/(3*a*(a + b)*f*(a + b*Sin[e + f*x]^2)^(3/2))
+ (2*b*(2*a + b)*Cos[e + f*x]*Sin[e + f*x])/(3*a^2*(a + b)^2*f*Sqrt[a + b*S
in[e + f*x]^2]) + (2*(2*a + b)*EllipticE[e + f*x, -(b/a)]*Sqrt[a + b*Sin[e
+ f*x]^2])/(3*a^2*(a + b)^2*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) - (EllipticF[
e + f*x, -(b/a)]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a*(a + b)*f*Sqrt[a + b*
Sin[e + f*x]^2])
```

Rule 3172

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)]^2)/Sqrt[(a_) + (b_.)*sin[(e_.) +
 (f_.)*(x_)]^2], x_Symbol] :> Dist[B/b, Int[Sqrt[a + b*Sin[e + f*x]^2], x],
 x] + Dist[(A*b - a*B)/b, Int[1/Sqrt[a + b*Sin[e + f*x]^2], x], x] /; FreeQ
[{a, b, e, f, A, B}, x]
```

Rule 3173

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]^2), x_Symbol] :> -Simp[((A*b - a*B)*Cos[e + f*x]*Sin[e + f*x]
*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(a + b)*(p + 1)), x] - Dist[1/(2*a*
(a + b)*(p + 1)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)*Simp[a*B - A*(2*a*(p +
 1) + b*(2*p + 3)) + 2*(A*b - a*B)*(p + 2)*Sin[e + f*x]^2, x], x], x] /; Fr
eeQ[{a, b, e, f, A, B}, x] && LtQ[p, -1] && NeQ[a + b, 0]
```

Rule 3177

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(Sqrt[a
]*EllipticE[e + f*x, -(b/a)])/f, x] /; FreeQ[{a, b, e, f}, x] && GtQ[a, 0]
```

Rule 3178

```
Int[Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[a
+ b*Sin[e + f*x]^2]/Sqrt[1 + (b*Sin[e + f*x]^2)/a], Int[Sqrt[1 + (b*Sin[e +
 f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3182

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Simp[(1*El
lipticF[e + f*x, -(b/a)])/(Sqrt[a]*f), x] /; FreeQ[{a, b, e, f}, x] && GtQ[
a, 0]
```

Rule 3183

```
Int[1/Sqrt[(a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2], x_Symbol] :> Dist[Sqrt[
1 + (b*Sin[e + f*x]^2)/a]/Sqrt[a + b*Sin[e + f*x]^2], Int[1/Sqrt[1 + (b*Sin
[e + f*x]^2)/a], x], x] /; FreeQ[{a, b, e, f}, x] &&  !GtQ[a, 0]
```

Rule 3184

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_), x_Symbol] :> -Simp[(b*Co
s[e + f*x]*Sin[e + f*x]*(a + b*Sin[e + f*x]^2)^(p + 1))/(2*a*f*(p + 1)*(a +
 b)), x] + Dist[1/(2*a*(p + 1)*(a + b)), Int[(a + b*Sin[e + f*x]^2)^(p + 1)
*Simp[2*a*(p + 1) + b*(2*p + 3) - 2*b*(p + 2)*Sin[e + f*x]^2, x], x], x] /;
 FreeQ[{a, b, e, f}, x] && NeQ[a + b, 0] && LtQ[p, -1]
```

Rubi steps

$$\int \frac{1}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\int \frac{-3a-2b+b\sin^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{3/2}}\,dx}{3a(a+b)}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{-a(3a\cdots}{\cdots}}{\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} - \frac{\int \frac{\cdots}{\sqrt{a+b\cdots}}}{3a\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(2(2a\cdots\right.}{\cdots}$$

$$= \frac{b\cos(e+fx)\sin(e+fx)}{3a(a+b)f\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2b(2a+b)\cos(e+fx)\sin(e+fx)}{3a^2(a+b)^2f\sqrt{a+b\sin^2(e+fx)}} + \frac{2(2a+\cdots}{\cdots}$$

Mathematica [A] time = 1.35, size = 172, normalized size = 0.77

$$\frac{-\sqrt{2}\,b\sin(2(e+fx))\left(-5a^2+b(2a+b)\cos(2(e+fx))-5ab-b^2\right)-a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}F\left(e+fx\middle|-\frac{b}{a}\cdots\right.}{3a^2f(a+b)^2(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^(-5/2),x]
```

```
[Out] (2*a^2*(2*a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e + f*x
, -(b/a)] - a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticF[
e + f*x, -(b/a)] - Sqrt[2]*b*(-5*a^2 - 5*a*b - b^2 + b*(2*a + b)*Cos[2*(e +
 f*x)])*Sin[2*(e + f*x)])/(3*a^2*(a + b)^2*f*(2*a + b - b*Cos[2*(e + f*x)])
^(3/2))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^{2}+a+b}}{b^{3}\cos\left(fx+e\right)^{6}-3\left(ab^{2}+b^{3}\right)\cos\left(fx+e\right)^{4}-a^{3}-3\,a^{2}b-3\,ab^{2}-b^{3}+3\left(a^{2}b+2\,ab^{2}+b^{3}\right)\cos\left(f\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)/(b^3*cos(f*x + e)^6 - 3*(a*b^2 +
b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2*b + 2*a*b^2 +
b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

maple [B] time = 2.09, size = 547, normalized size = 2.45

$$-\frac{\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b\left(\sin^{2}\left(fx+e\right)\right)}{a}}\,\text{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^{2}b\left(\sin^{2}\left(fx+e\right)\right)+\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,\sqrt{\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*((cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e
),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^2+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^
2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2-4*(cos(
f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^
(1/2))*a^2*b*sin(f*x+e)^2-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/
2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^2+4*a*b^2*sin(f*x+
e)^5+2*b^3*sin(f*x+e)^5+(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*E
llipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3+a^2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(
f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-4*(cos(f*x+e)^2)^
(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3
-2*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(
-1/a*b)^(1/2))*a^2*b+5*a^2*b*sin(f*x+e)^3-a*b^2*sin(f*x+e)^3-2*b^3*sin(f*x+
e)^3-5*a^2*b*sin(f*x+e)-3*a*b^2*sin(f*x+e))/(a+b*sin(f*x+e)^2)^(3/2)/a^2/(a
+b)^2/cos(f*x+e)/f
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^(-5/2), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{1}{\left(b\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(1/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\left(a+b\sin^2\left(e+fx\right)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral((a + b*sin(e + f*x)**2)**(-5/2), x)
```

3.541 $\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)}-\frac{(7a+8b)\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}\right.}{3a^3f(a+b)\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] 1/3*cot(f*x+e)/a/f/(a+b*sin(f*x+e)^2)^(3/2)+1/3*(3*a+4*b)*cot(f*x+e)/a^2/(a
+b)/f/(a+b*sin(f*x+e)^2)^(1/2)-1/3*(7*a+8*b)*cot(f*x+e)*(a+b*sin(f*x+e)^2)^
(1/2)/a^3/(a+b)/f-1/3*(7*a+8*b)*EllipticE(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+
e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^3/(a+b)/f/(1+b*sin(f*x+e
)^2/a)^(1/2)+4/3*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f*x+e)*(cos(f*x+e)^
2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^2/f/(a+b*sin(f*x+e)^2)^(1/2)
```

Rubi [A] time = 0.37, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3196, 469, 579, 583, 524, 426, 424, 421, 419}

$$-\frac{(7a+8b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3f(a+b)}+\frac{(3a+4b)\cot(e+fx)}{3a^2f(a+b)\sqrt{a+b\sin^2(e+fx)}}+\frac{4\sqrt{\cos^2(e+fx)}\ \sec(e+fx)\sqrt{}}{3a^2f\sqrt{}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] Cot[e + f*x]/(3*a*f*(a + b*Sin[e + f*x]^2)^(3/2)) + ((3*a + 4*b)*Cot[e + f*
x])/(3*a^2*(a + b)*f*Sqrt[a + b*Sin[e + f*x]^2]) - ((7*a + 8*b)*Cot[e + f*x
]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^3*(a + b)*f) - ((7*a + 8*b)*Sqrt[Cos[e +
 f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Si
n[e + f*x]^2])/(3*a^3*(a + b)*f*Sqrt[1 + (b*Sin[e + f*x]^2)/a]) + (4*Sqrt[C
os[e + f*x]^2]*EllipticF[ArcSin[Sin[e + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1
+ (b*Sin[e + f*x]^2)/a])/(3*a^2*f*Sqrt[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 469

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((e*x)^(m + 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^
q)/(a*e*n*(p + 1)), x] + Dist[1/(a*n*(p + 1)), Int[(e*x)^m*(a + b*x^n)^(p +
 1)*(c + d*x^n)^(q - 1)*Simp[c*(m + n*(p + 1) + 1) + d*(m + n*(p + q + 1) +
 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] &&
 IGtQ[n, 0] && LtQ[p, -1] && LtQ[0, q, 1] && IntBinomialQ[a, b, c, d, e, m,
 n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\sqrt{1-x^2}}{x^2\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-4+3\ldots}{x^2\sqrt{1-x^2}\,(a\ldots}\ldots\right)}{3af}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e+\ldots}\right.}{\ldots}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot\ldots}{\ldots}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot\ldots}{\ldots}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot\ldots}{\ldots}$$

$$= \frac{\cot(e+fx)}{3af\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{(3a+4b)\cot(e+fx)}{3a^2(a+b)f\sqrt{a+b\sin^2(e+fx)}} - \frac{(7a+8b)\cot\ldots}{\ldots}$$

Mathematica [A] time = 2.66, size = 209, normalized size = 0.73

$$\frac{8a^2(a+b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2} F\left(e+fx\middle|-\frac{b}{a}\right) - 2a^2(7a+8b)\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2} E\left(e+fx\middle|-\frac{b}{a}\right) - \frac{\cot(e+fx)(\ldots}{\ldots}}{6a^3f(a+b)(2a-b\cos(2(e+fx))+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^2/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (-(((24*a^3 + 68*a^2*b + 69*a*b^2 + 24*b^3 - 4*b*(11*a^2 + 19*a*b + 8*b^2)*
Cos[2*(e + f*x)] + b^2*(7*a + 8*b)*Cos[4*(e + f*x)])*Cot[e + f*x])/Sqrt[2])
 - 2*a^2*(7*a + 8*b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*EllipticE[e +
 f*x, -(b/a)] + 8*a^2*(a + b)*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*Elli
pticF[e + f*x, -(b/a)])/(6*a^3*(a + b)*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/
2))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^2}{b^3\cos\left(fx+e\right)^6 - 3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4 - a^3 - 3a^2b - 3ab^2 - b^3 + 3\left(a^2b+2ab^2+b^3\right)\cos\left(f\ldots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^2/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^{2}}{\left(b\sin\left(fx+e\right)^{2}+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^2/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.99, size = 411, normalized size = 1.43

$$-\sqrt{-\frac{b\left(\cos^{2}\left(fx+e\right)\right)}{a}+\frac{a+b}{a}}\,\sqrt{\frac{\cos\left(2fx+2e\right)}{2}+\frac{1}{2}}\,ab\left(4\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a+4\,\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] 1/3*(-(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)^2)^(1/2)*a*b*(4*Ellipti
cF(sin(f*x+e),(-1/a*b)^(1/2))*a+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b-7*
EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2
))*b)*sin(f*x+e)*cos(f*x+e)^2+(-b/a*cos(f*x+e)^2+(a+b)/a)^(1/2)*(cos(f*x+e)
^2)^(1/2)*a*(4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^2+8*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a*b+4*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*b^2-7*Ellipt
icE(sin(f*x+e),(-1/a*b)^(1/2))*a^2-15*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*
a*b-8*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*b^2)*sin(f*x+e)+(-7*a*b^2-8*b^3)
*cos(f*x+e)^6+(11*a^2*b+26*a*b^2+16*b^3)*cos(f*x+e)^4+(-3*a^3-14*a^2*b-19*a
*b^2-8*b^3)*cos(f*x+e)^2)/sin(f*x+e)/a^3/(a+b)/(a+b*sin(f*x+e)^2)^(3/2)/cos
(f*x+e)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^2/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\mathrm{cot}\left(e+f\,x\right)}^2}{{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^2/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**2/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**2/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.542 $\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx$

Optimal. Leaf size=348

$$\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f}+\frac{8(a+2b)\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\sqrt{a+b\sin^2(e+fx)}\,E\left(\sin^{-1}(\sin\right.}{3a^4f\sqrt{\frac{b\sin^2(e+fx)}{a}+1}}$$

```
[Out] 1/3*(a+b)*cot(f*x+e)*csc(f*x+e)^2/a/b/f/(a+b*sin(f*x+e)^2)^(3/2)+2/3*(a+3*b
)*cot(f*x+e)*csc(f*x+e)^2/a^2/b/f/(a+b*sin(f*x+e)^2)^(1/2)+8/3*(a+2*b)*cot(
f*x+e)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f-1/3*(3*a+8*b)*cot(f*x+e)*csc(f*x+e)^2
*(a+b*sin(f*x+e)^2)^(1/2)/a^3/b/f+8/3*(a+2*b)*EllipticE(sin(f*x+e),(-b/a)^(
1/2))*sec(f*x+e)*(cos(f*x+e)^2)^(1/2)*(a+b*sin(f*x+e)^2)^(1/2)/a^4/f/(1+b*s
in(f*x+e)^2/a)^(1/2)-1/3*(5*a+8*b)*EllipticF(sin(f*x+e),(-b/a)^(1/2))*sec(f
*x+e)*(cos(f*x+e)^2)^(1/2)*(1+b*sin(f*x+e)^2/a)^(1/2)/a^3/f/(a+b*sin(f*x+e)
^2)^(1/2)
```

Rubi [A] time = 0.52, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.360, Rules used = {3196, 468, 579, 583, 524, 426, 424, 421, 419}

$$\frac{8(a+2b)\cot(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^4f}-\frac{(3a+8b)\cot(e+fx)\csc^2(e+fx)\sqrt{a+b\sin^2(e+fx)}}{3a^3bf}+\frac{2(a+3b)\cot}{3a^2bf\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] ((a + b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a*b*f*(a + b*Sin[e + f*x]^2)^(3/2)
) + (2*(a + 3*b)*Cot[e + f*x]*Csc[e + f*x]^2)/(3*a^2*b*f*Sqrt[a + b*Sin[e +
 f*x]^2]) + (8*(a + 2*b)*Cot[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f)
 - ((3*a + 8*b)*Cot[e + f*x]*Csc[e + f*x]^2*Sqrt[a + b*Sin[e + f*x]^2])/(3*
a^3*b*f) + (8*(a + 2*b)*Sqrt[Cos[e + f*x]^2]*EllipticE[ArcSin[Sin[e + f*x]]
, -(b/a)]*Sec[e + f*x]*Sqrt[a + b*Sin[e + f*x]^2])/(3*a^4*f*Sqrt[1 + (b*Sin
[e + f*x]^2)/a]) - ((5*a + 8*b)*Sqrt[Cos[e + f*x]^2]*EllipticF[ArcSin[Sin[e
 + f*x]], -(b/a)]*Sec[e + f*x]*Sqrt[1 + (b*Sin[e + f*x]^2)/a])/(3*a^3*f*Sqr
t[a + b*Sin[e + f*x]^2])
```

Rule 419

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> S
imp[(1*EllipticF[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[a]*Sqrt[c]*Rt
[-(d/c), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ
[a, 0] &&  !(NegQ[b/a] && SimplerSqrtQ[-(b/a), -(d/c)])
```

Rule 421

```
Int[1/(Sqrt[(a_) + (b_.)*(x_)^2]*Sqrt[(c_) + (d_.)*(x_)^2]), x_Symbol] :> D
ist[Sqrt[1 + (d*x^2)/c]/Sqrt[c + d*x^2], Int[1/(Sqrt[a + b*x^2]*Sqrt[1 + (d
*x^2)/c]), x], x] /; FreeQ[{a, b, c, d}, x] &&  !GtQ[c, 0]
```

Rule 424

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Simp[
(Sqrt[a]*EllipticE[ArcSin[Rt[-(d/c), 2]*x], (b*c)/(a*d)])/(Sqrt[c]*Rt[-(d/c
```

```
), 2]), x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] && GtQ[a, 0]
```

Rule 426

```
Int[Sqrt[(a_) + (b_.)*(x_)^2]/Sqrt[(c_) + (d_.)*(x_)^2], x_Symbol] :> Dist[
Sqrt[a + b*x^2]/Sqrt[1 + (b*x^2)/a], Int[Sqrt[1 + (b*x^2)/a]/Sqrt[c + d*x^2
], x], x] /; FreeQ[{a, b, c, d}, x] && NegQ[d/c] && GtQ[c, 0] &&  !GtQ[a, 0
]
```

Rule 468

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> -Simp[((c*b - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1)*
(c + d*x^n)^(q - 1))/(a*b*e*n*(p + 1)), x] + Dist[1/(a*b*n*(p + 1)), Int[(e
*x)^m*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q - 2)*Simp[c*(c*b*n*(p + 1) + (c*b
- a*d)*(m + 1)) + d*(c*b*n*(p + 1) + (c*b - a*d)*(m + n*(q - 1) + 1))*x^n,
x], x], x] /; FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n,
0] && LtQ[p, -1] && GtQ[q, 1] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 524

```
Int[((e_) + (f_.)*(x_)^(n_))/(Sqrt[(a_) + (b_.)*(x_)^(n_)]*Sqrt[(c_) + (d_.
)*(x_)^(n_)]), x_Symbol] :> Dist[f/b, Int[Sqrt[a + b*x^n]/Sqrt[c + d*x^n],
x], x] + Dist[(b*e - a*f)/b, Int[1/(Sqrt[a + b*x^n]*Sqrt[c + d*x^n]), x], x
] /; FreeQ[{a, b, c, d, e, f, n}, x] &&  !(EqQ[n, 2] && ((PosQ[b/a] && PosQ
[d/c]) || (NegQ[b/a] && (PosQ[d/c] || (GtQ[a, 0] && ( !GtQ[c, 0] || Simpler
SqrtQ[-(b/a), -(d/c)]))))))
```

Rule 579

```
Int[((g_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> -Simp[((b*e - a*f)*(g*x)^(m
+ 1)*(a + b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*g*n*(b*c - a*d)*(p + 1)),
x] + Dist[1/(a*n*(b*c - a*d)*(p + 1)), Int[(g*x)^m*(a + b*x^n)^(p + 1)*(c +
 d*x^n)^q*Simp[c*(b*e - a*f)*(m + 1) + e*n*(b*c - a*d)*(p + 1) + d*(b*e - a
*f)*(m + n*(p + q + 2) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g,
m, q}, x] && IGtQ[n, 0] && LtQ[p, -1]
```

Rule 583

```
Int[((g_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_
))^(q_.)*((e_) + (f_.)*(x_)^(n_)), x_Symbol] :> Simp[(e*(g*x)^(m + 1)*(a +
b*x^n)^(p + 1)*(c + d*x^n)^(q + 1))/(a*c*g*(m + 1)), x] + Dist[1/(a*c*g^n*(
m + 1)), Int[(g*x)^(m + n)*(a + b*x^n)^p*(c + d*x^n)^q*Simp[a*f*c*(m + 1) -
 e*(b*c + a*d)*(m + n + 1) - e*n*(b*c*p + a*d*q) - b*e*d*(m + n*(p + q + 2)
 + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e, f, g, p, q}, x] && IGtQ[n, 0
] && LtQ[m, -1]
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{5/2}}\,dx = \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}}{x^4\left(a+bx^2\right)^{5/2}}\,dx, x, \sin(e+fx)\right)}{f}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} - \frac{\left(\sqrt{\cos^2(e+fx)}\,\sec(e+fx)\right)\text{Subst}\left(\int \frac{-3(a+2b)\cdot}{x^4\sqrt{1-x^2}}\right.}{3abf}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{\left(\sqrt{\cos^2(e\right.}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} - \frac{(3a+8b)}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)}{}$$

$$= \frac{(a+b)\cot(e+fx)\csc^2(e+fx)}{3abf\left(a+b\sin^2(e+fx)\right)^{3/2}} + \frac{2(a+3b)\cot(e+fx)\csc^2(e+fx)}{3a^2bf\sqrt{a+b\sin^2(e+fx)}} + \frac{8(a+2b)}{}$$

Mathematica [A] time = 3.07, size = 226, normalized size = 0.65

$$\frac{2a^2b\left(\frac{2a-b\cos(2(e+fx))+b}{a}\right)^{3/2}\left(8(a+2b)E\left(e+fx\middle|-\frac{b}{a}\right)-(5a+8b)F\left(e+fx\middle|-\frac{b}{a}\right)\right)+\sqrt{2}\,b\left(2ab(a+b)\sin(2(e+fx))\right.}{}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[e + f*x]^4/(a + b*Sin[e + f*x]^2)^(5/2),x]
```

```
[Out] (2*a^2*b*((2*a + b - b*Cos[2*(e + f*x)])/a)^(3/2)*(8*(a + 2*b)*EllipticE[e
+ f*x, -(b/a)] - (5*a + 8*b)*EllipticF[e + f*x, -(b/a)]) + Sqrt[2]*b*(4*(a
+ 2*b)*(2*a + b - b*Cos[2*(e + f*x)])^2*Cot[e + f*x] - a*(2*a + b - b*Cos[2
*(e + f*x)])^2*Cot[e + f*x]*Csc[e + f*x]^2 + 2*a*b*(a + b)*Sin[2*(e + f*x)]
 + 4*b*(a + 2*b)*(2*a + b - b*Cos[2*(e + f*x)])*Sin[2*(e + f*x)]))/(6*a^4*b
*f*(2*a + b - b*Cos[2*(e + f*x)])^(3/2))
```

fricas [F] time = 0.54, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\sqrt{-b\cos\left(fx+e\right)^2+a+b}\,\cot\left(fx+e\right)^4}{b^3\cos\left(fx+e\right)^6-3\left(ab^2+b^3\right)\cos\left(fx+e\right)^4-a^3-3a^2b-3ab^2-b^3+3\left(a^2b+2ab^2+b^3\right)\cos\left(fx+\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="fricas")
```

```
[Out] integral(-sqrt(-b*cos(f*x + e)^2 + a + b)*cot(f*x + e)^4/(b^3*cos(f*x + e)^
6 - 3*(a*b^2 + b^3)*cos(f*x + e)^4 - a^3 - 3*a^2*b - 3*a*b^2 - b^3 + 3*(a^2
*b + 2*a*b^2 + b^3)*cos(f*x + e)^2), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(fx+e\right)^4}{\left(b\sin\left(fx+e\right)^2+a\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="giac")
```

```
[Out] integrate(cot(f*x + e)^4/(b*sin(f*x + e)^2 + a)^(5/2), x)
```

maple [A] time = 1.93, size = 633, normalized size = 1.82

$$-\frac{5\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}\,\sqrt{\frac{a+b(\sin^2(fx+e))}{a}}\;\mathrm{EllipticF}\left(\sin\left(fx+e\right),\sqrt{-\frac{b}{a}}\right)a^2b\left(\sin^5\left(fx+e\right)\right)+8\sqrt{\frac{\cos(2fx+2e)}{2}+\frac{1}{2}}}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x)
```

```
[Out] -1/3*(5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x
+e),(-1/a*b)^(1/2))*a^2*b*sin(f*x+e)^5+8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x
+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5-8*(
cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a
*b)^(1/2))*a^2*b*sin(f*x+e)^5-16*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a
)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a*b^2*sin(f*x+e)^5+8*a*b^2*sin
(f*x+e)^8+16*b^3*sin(f*x+e)^8+5*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)
^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3+8*(cos(f*x+e)^
2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticF(sin(f*x+e),(-1/a*b)^(1/2))*
a^2*b*sin(f*x+e)^3-8*(cos(f*x+e)^2)^(1/2)*((a+b*sin(f*x+e)^2)/a)^(1/2)*Elli
pticE(sin(f*x+e),(-1/a*b)^(1/2))*a^3*sin(f*x+e)^3-16*(cos(f*x+e)^2)^(1/2)*(
(a+b*sin(f*x+e)^2)/a)^(1/2)*EllipticE(sin(f*x+e),(-1/a*b)^(1/2))*a^2*b*sin(
f*x+e)^3+13*a^2*b*sin(f*x+e)^6+16*a*b^2*sin(f*x+e)^6-16*b^3*sin(f*x+e)^6+4*
a^3*sin(f*x+e)^4-7*a^2*b*sin(f*x+e)^4-24*a*b^2*sin(f*x+e)^4-5*a^3*sin(f*x+e
)^2-6*a^2*b*sin(f*x+e)^2+a^3)/sin(f*x+e)^3/a^4/(a+b*sin(f*x+e)^2)^(3/2)/cos
(f*x+e)/f
```

maxima [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)^4/(a+b*sin(f*x+e)^2)^(5/2),x, algorithm="maxima")
```

```
[Out] Timed out
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot\left(e+f\,x\right)^4}{\left(b\,\sin\left(e+f\,x\right)^2+a\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2),x)
```

```
[Out] int(cot(e + f*x)^4/(a + b*sin(e + f*x)^2)^(5/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^4(e+fx)}{\left(a+b\sin^2(e+fx)\right)^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(f*x+e)**4/(a+b*sin(f*x+e)**2)**(5/2),x)
```

```
[Out] Integral(cot(e + f*x)**4/(a + b*sin(e + f*x)**2)**(5/2), x)
```

3.543 $\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx$

Optimal. Leaf size=120

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx\right.}{df(m+1)}$$

```
[Out] AppellF1(1/2+1/2*m,1/2+1/2*m,-p,3/2+1/2*m,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)*(
cos(f*x+e)^2)^(1/2+1/2*m)*(a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^(1+m)/d/f/(1+
m)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.120, Rules used = {3197, 511, 510}

$$\frac{\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^{m+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(e+fx\right.}{df(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*(d*Tan[e
 + f*x])^(1 + m))/(d*f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3197

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*((d_.)*tan[(e_.) + (f_.)
*(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(
ff*(d*Tan[e + f*x])^(m + 1)*(Cos[e + f*x]^2)^((m + 1)/2))/(d*f*Sin[e + f*x]
^(m + 1)), Subst[Int[((ff*x)^m*(a + b*ff^2*x^2)^p)/(1 - ff^2*x^2)^((m + 1)/
2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, d, e, f, m, p}, x] &&  !In
tegerQ[m]
```

Rubi steps

$$\int \left(a+b\sin^2(e+fx)\right)^p (d\tan(e+fx))^m\,dx = \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)(d\tan(e+fx))^{1+m}\right)\text{Subst}\left(\int x^m\right.}{df}$$

$$= \frac{\left(\cos^2(e+fx)^{\frac{1+m}{2}}\sin^{-1-m}(e+fx)\left(a+b\sin^2(e+fx)\right)^p\left(1+\frac{b\sin^2(e}{a}\right.\right.}{}$$

$$= \frac{F_1\left(\frac{1+m}{2};\frac{1+m}{2},-p;\frac{3+m}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right)\cos^2(e+fx)^{\frac{1+m}{2}}}{df(1+m}$$

Mathematica [A] time = 0.50, size = 121, normalized size = 1.01

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{m+1}{2}}(d\tan(e+fx))^m\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{m+1}{2};\frac{m+1}{2},-p;\frac{m+3}{2};\sin^2(\right.}{f(m+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)^p*(d*Tan[e + f*x])^m,x]
```

```
[Out] (AppellF1[(1 + m)/2, (1 + m)/2, -p, (3 + m)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(Cos[e + f*x]^2)^((1 + m)/2)*(a + b*Sin[e + f*x]^2)^p*Tan[e +
f*x]*(d*Tan[e + f*x])^m)/(f*(1 + m)*(1 + (b*Sin[e + f*x]^2)/a)^p)
```

fricas [F] time = 0.81, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(d\tan\left(fx+e\right)\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(d*tan(f*x + e))^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

maple [F] time = 4.01, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

```
[Out] int((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\left(d\tan\left(fx+e\right)\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)^p*(d*tan(f*x+e))^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(d*tan(f*x + e))^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(d\,\tan\left(e+f\,x\right)\right)}^m\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((d*tan(e + f*x))^m*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)**p*(d*tan(f*x+e))**m,x)
```

```
[Out] Timed out
```

3.544 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=102

$$\frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2d(a+b)}-\frac{(a+bp+b)\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)^2}$$

```
[Out] -1/2*(b*p+a+b)*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin(
d*x+c)^2)^(1+p)/(a+b)^2/d/(1+p)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^2)^(1+p)/(
a+b)/d
```

Rubi [A] time = 0.09, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 78, 68}

$$\frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2d(a+b)}-\frac{(a+bp+b)\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

```
[Out] -((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(
a + b)]*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)^2*d*(1 + p)) + (Sec[c +
d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] ||  !(IntegerQ[n] ||  !(EqQ[e, 0] ||  !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x(a+bx)^p}{(1-x)^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\sec^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{(a+b+bp)\,\text{Subst}\left(\int \frac{(a+bx)^p}{1-x}$$
$$= -\frac{(a+b+bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx)\right)^1}{2(a+b)^2d(1+p)}$$

Mathematica [A] time = 0.25, size = 83, normalized size = 0.81

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\left((p+1)(a+b)\sec^2(c+dx)-(a+bp+b)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)\right)}{2d(p+1)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^3,x]
```

```
[Out] ((-((a + b + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)
/(a + b)]) + (a + b)*(1 + p)*Sec[c + d*x]^2)*(a + b*Sin[c + d*x]^2)^(1 + p)
)/(2*(a + b)^2*d*(1 + p))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\tan(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

maple [F] time = 2.01, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^2(dx+c)\right)\right)^p \left(\tan^3(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^2+a\right)^p \tan(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^3, x)
```

mupad **[F]** time = 0.00, size = -1, normalized size = -0.01

$$\int \tan (c + d x)^3 \left(b \sin (c + d x)^2 + a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.545 $\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx$

Optimal. Leaf size=59

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

```
[Out] 1/2*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^2)/(a+b))*(a+b*sin(d*x+c)^2)^(
1+p)/(a+b)/d/(1+p)
```

Rubi [A] time = 0.04, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.095$, Rules used = {3194, 68}

$$\frac{\left(a+b\sin^2(c+dx)\right)^{p+1} {}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)+a}{a+b}\right)}{2d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^2)/(a + b)]*(a + b*
Sin[c + d*x]^2)^(1 + p))/(2*(a + b)*d*(1 + p))
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \left(a+b\sin^2(c+dx)\right)^p \tan(c+dx)\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{1-x}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^2(c+dx)}{a+b}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2(a+b)d(1+p)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 61, normalized size = 1.03

$$\frac{\left(a-b\cos^2(c+dx)+b\right)^{p+1} {}_2F_1\left(1,p+1;p+2;1-\frac{b\cos^2(c+dx)}{a+b}\right)}{2d(p+1)(a+b)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x],x]
[Out] ((a + b - b*Cos[c + d*x]^2)^(1 + p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1 -
(b*Cos[c + d*x]^2)/(a + b)])/(2*(a + b)*d*(1 + p))
```

fricas [F] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^{2}+a+b\right)^{p}\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

maple [F] time = 2.29, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^{2}\left(dx+c\right)\right)\right)^{p}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x)
[Out] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^{2}+a\right)^{p}\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c),x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\tan\left(c+d\,x\right)\,{\left(b\,{\sin\left(c+d\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c),x)
[Out] Timed out
```

3.546 $\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

```
[Out] -1/2*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(1+p)/
a/d/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.095, Rules used = {3194, 65}

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c
 + d*x]^2)^(1 + p))/(2*a*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^2(c+dx)}{a}\right)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.06, size = 54, normalized size = 1.00

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*S
in[c + d*x]^2)^(1 + p))/(a*d*(1 + p))
```

fricas [F] time = 0.44, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

maple [F] time = 2.25, size = 0, normalized size = 0.00

$$\int\cot\left(dx+c\right)\left(a+b\left(\sin^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cot\left(c+d\,x\right)\,{\left(b\,{\sin\left(c+d\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^2\left(c+dx\right)\right)^p\cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Integral((a + b*sin(c + d*x)**2)**p*cot(c + d*x), x)
```

3.547 $\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=95

$$\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2ad}$$

```
[Out] -1/2*csc(d*x+c)^2*(a+b*sin(d*x+c)^2)^(1+p)/a/d+1/2*(-b*p+a)*hypergeom([1, 1
+p],[2+p],1+b*sin(d*x+c)^2/a)*(a+b*sin(d*x+c)^2)^(1+p)/a^2/d/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3194, 78, 65}

$$\frac{(a-bp)\left(a+b\sin^2(c+dx)\right)^{p+1}{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{2a^2d(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{p+1}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(Csc[c + d*x]^2*(a + b*Sin[c + d*x]^2)^(1 + p))/(2*a*d) + ((a - b*p)*Hyper
geometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^2)/a]*(a + b*Sin[c + d*x]
^2)^(1 + p))/(2*a^2*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 78

```
Int[((a_.) + (b_.)*(x_))*((c_.) + (d_.)*(x_))^(n_.)*((e_.) + (f_.)*(x_))^(p
_.), x_Symbol] :> -Simp[((b*e - a*f)*(c + d*x)^(n + 1)*(e + f*x)^(p + 1))/(
f*(p + 1)*(c*f - d*e)), x] - Dist[(a*d*f*(n + p + 2) - b*(d*e*(n + 1) + c*f
*(p + 1)))/(f*(p + 1)*(c*f - d*e)), Int[(c + d*x)^n*(e + f*x)^(p + 1), x],
x] /; FreeQ[{a, b, c, d, e, f, n}, x] && LtQ[p, -1] && ( !LtQ[n, -1] || Int
egerQ[p] || !(IntegerQ[n] || !(EqQ[e, 0] || !(EqQ[c, 0] || LtQ[p, n]))))
```

Rule 3194

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^((m
 + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff*x)^p)/(1 - ff*x)^((m + 1
)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && Intege
rQ[(m - 1)/2]
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)(a+bx)^p}{x^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} - \frac{(a-bp)\,\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x,\right.}{2ad}$$
$$= -\frac{\csc^2(c+dx)\left(a+b\sin^2(c+dx)\right)^{1+p}}{2ad} + \frac{(a-bp)\,{}_2F_1\left(1,1+p;2+p;1\right.}{2a}$$

Mathematica [A] time = 0.43, size = 73, normalized size = 0.77

$$-\frac{\left(a+b\sin^2(c+dx)\right)^{p+1}\left(\frac{(bp-a)\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^2(c+dx)}{a}+1\right)}{p+1}+a\csc^2(c+dx)\right)}{2a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -1/2*((a*Csc[c + d*x]^2 + ((-a + b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, 1
+ (b*Sin[c + d*x]^2)/a])/(1 + p))*(a + b*Sin[c + d*x]^2)^(1 + p))/(a^2*d)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos(dx+c)^2+a+b\right)^p\cot(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
```

```
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^2+a\right)^p\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

maple [F] time = 1.85, size = 0, normalized size = 0.00

$$\int \left(\cot^3(dx+c)\right)\left(a+b\left(\sin^2(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^2+a\right)^p\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^3\,{\left(b\,{\sin\left(c+d\,x\right)}^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.548 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

```
[Out] 1/5*AppellF1(5/2,5/2,-p,7/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^4*(a
+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a)^
p)
```

Rubi [A] time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3196, 511, 510}

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
```

```
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p\tan^4(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\ \sec(c+dx)\right)\mathrm{Subst}\left(\int\frac{x^4\left(a+bx^2\right)^p}{\left(1-x^2\right)^{5/2}}dx,x,\sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\ \sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right.}{d}$$

$$= \frac{F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\ \sin^4(c+dx}{5d}$$

Mathematica [**A**] time = 4.50, size = 102, normalized size = 1.01

$$\frac{\sin^4(c+dx)\sqrt{\cos^2(c+dx)}\ \tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(\frac{5}{2};\frac{5}{2},-p;\frac{7}{2};\sin^2(c+dx),-\frac{b\sin}{}\right.}{5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^4,x]
[Out] (AppellF1[5/2, 5/2, -p, 7/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(5*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

fricas [**F**] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4, x)
```

giac [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

maple [**F**] time = 1.09, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^2\left(dx+c\right)\right)\right)^p\left(\tan^4\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x)
[Out] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x)
```

maxima [**F**] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan\left(c+d\,x\right)^4\,{\left(b\,\sin\left(c+d\,x\right)^2+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.549 $\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=101

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b}{}\right.}{3d}$$

```
[Out] 1/3*AppellF1(3/2,3/2,-p,5/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*sin(d*x+c)^2*(a
+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)*tan(d*x+c)/d/((1+b*sin(d*x+c)^2/a)^
p)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3196, 511, 510}

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b}{}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
```

```
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \left(a+b\sin^2(c+dx)\right)^p \tan^2(c+dx)\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\text{Subst}\left(\int \frac{x^2\left(a+bx^2\right)^p}{\left(1-x^2\right)^{3/2}}\,dx, x, \sin(c+dx)\right)}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\text{S}}{d}$$

$$= \frac{F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\sin^2(c+dx)\,(}{3d}$$

Mathematica [A] time = 0.31, size = 102, normalized size = 1.01

$$\frac{\sin^2(c+dx)\sqrt{\cos^2(c+dx)}\,\tan(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(\frac{3}{2};\frac{3}{2},-p;\frac{5}{2};\sin^2(c+dx),-\frac{b\sin^2(c}{a}\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x]^2,x]
[Out] (AppellF1[3/2, 3/2, -p, 5/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*Sqrt[
Cos[c + d*x]^2]*Sin[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p*Tan[c + d*x])/(3*d*
((a + b*Sin[c + d*x]^2)/a)^p)
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^2, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

maple [F] time = 1.46, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^2\left(dx+c\right)\right)\right)^p\left(\tan^2\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x)
[Out] int((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\tan\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^2)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*tan(d*x + c)^2, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan(c + d\,x)^2\,\left(b\sin(c + d\,x)^2 + a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**2)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.550 $\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=97

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\right.}{d}$$

```
[Out] -AppellF1(-1/2,-1/2,-p,1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)*sec(d
*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)^2/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = $\{3196, 511, 510\}$

$$-\frac{\sqrt{\cos^2(c+dx)}\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
(1 + (b*Sin[c + d*x]^2)/a)^p))
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx=\frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\text{Subst}\left(\int\frac{\sqrt{1-x^2}\left(a+bx^2\right)^p}{x^2}dx,x,\sin(c+d\right.}{d}$$

$$=\frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right.}{d}$$

$$=-\frac{F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\csc(c\right.}{d}$$

Mathematica [A] time = 0.20, size = 98, normalized size = 1.01

$$-\frac{\sqrt{\cos^2(c+dx)}\,\csc(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(-\frac{1}{2};-\frac{1}{2},-p;\frac{1}{2};\sin^2(c+dx),-\right.}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^2)^p,x]
[Out] -((AppellF1[-1/2, -1/2, -p, 1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(d*
((a + b*Sin[c + d*x]^2)/a)^p))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int\left(\cot^2\left(dx+c\right)\right)\left(a+b\left(\sin^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x)
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^2, x)
```

mupad [**F**] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^2\,\left(b\sin(c+d\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [**F(-1)**] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.551 $\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx$

Optimal. Leaf size=101

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+\right.}{3d}$$

```
[Out] -1/3*AppellF1(-3/2,-3/2,-p,-1/2,sin(d*x+c)^2,-b*sin(d*x+c)^2/a)*csc(d*x+c)^
3*sec(d*x+c)*(a+b*sin(d*x+c)^2)^p*(cos(d*x+c)^2)^(1/2)/d/((1+b*sin(d*x+c)^2
/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3196, 511, 510}

$$-\frac{\sqrt{\cos^2(c+dx)}\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{b\sin^2(c+dx)}{a}+1\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
```

```
[Out] -(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a)]*S
qrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^p)/(
3*d*(1 + (b*Sin[c + d*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3196

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.)*tan[(e_.) + (f_.)*(x_)]^
(m_), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Dist[(ff^(m +
1)*Sqrt[Cos[e + f*x]^2])/(f*Cos[e + f*x]), Subst[Int[(x^m*(a + b*ff^2*x^2)^
p)/(1 - ff^2*x^2)^((m + 1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b,
 e, f, p}, x] && IntegerQ[m/2] &&  !IntegerQ[p]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^2(c+dx)\right)^p\,dx = \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\right)\mathrm{Subst}\left(\int \frac{\left(1-x^2\right)^{3/2}\left(a+bx^2\right)^p}{x^4}\,dx, x, \sin(c+dx\right.}{d}$$

$$= \frac{\left(\sqrt{\cos^2(c+dx)}\,\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(1+\frac{b\sin^2(c+dx)}{a}\right)^{-p}\right)\mathrm{S}}{d}$$

$$= -\frac{F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\frac{b\sin^2(c+dx)}{a}\right)\sqrt{\cos^2(c+dx)}\,\csc^3(c+}{3d}$$

Mathematica [A] time = 3.36, size = 102, normalized size = 1.01

$$-\frac{\sqrt{\cos^2(c+dx)}\,\csc^3(c+dx)\sec(c+dx)\left(a+b\sin^2(c+dx)\right)^p\left(\frac{a+b\sin^2(c+dx)}{a}\right)^{-p}F_1\left(-\frac{3}{2};-\frac{3}{2},-p;-\frac{1}{2};\sin^2(c+dx),-\right.}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^2)^p,x]
[Out] -1/3*(AppellF1[-3/2, -3/2, -p, -1/2, Sin[c + d*x]^2, -((b*Sin[c + d*x]^2)/a
)]*Sqrt[Cos[c + d*x]^2]*Csc[c + d*x]^3*Sec[c + d*x]*(a + b*Sin[c + d*x]^2)^
p)/(d*((a + b*Sin[c + d*x]^2)/a)^p)
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(-b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^4, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="fricas")
[Out] integral((-b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

maple [F] time = 1.11, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(dx+c\right)\right)\left(a+b\left(\sin^2\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x)
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^2+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^2)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^2 + a)^p*cot(d*x + c)^4, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot(c+d\,x)^4\,\left(b\sin(c+d\,x)^2+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p,x)
```

```
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**2)**p,x)
```

```
[Out] Timed out
```

3.552 $\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=153

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log\left(a+b\,s\right.}{3a}$$

```
[Out] -1/2*csc(x)^2/a-ln(sin(x))/a-1/3*b^(2/3)*ln(a^(1/3)+b^(1/3)*sin(x))/a^(5/3)
+1/6*b^(2/3)*ln(a^(2/3)-a^(1/3)*b^(1/3)*sin(x)+b^(2/3)*sin(x)^2)/a^(5/3)+1/
3*ln(a+b*sin(x)^3)/a+1/3*b^(2/3)*arctan(1/3*(a^(1/3)-2*b^(1/3)*sin(x))/a^(1
/3)*3^(1/2))/a^(5/3)*3^(1/2)
```

Rubi [A] time = 0.19, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.667, Rules used = {3230, 1834, 1871, 200, 31, 634, 617, 204, 628, 260}

$$\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\sin^2(x)\right)}{6a^{5/3}}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\tan^{-1}\left(\frac{\sqrt[3]{a}-2\sqrt[3]{b}\sin(x)}{\sqrt{3}\sqrt[3]{a}}\right)}{\sqrt{3}\,a^{5/3}}+\frac{\log\left(a+b\,s\right.}{3a}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (b^(2/3)*ArcTan[(a^(1/3) - 2*b^(1/3)*Sin[x])/(Sqrt[3]*a^(1/3))])/(Sqrt[3]*a
^(5/3)) - Csc[x]^2/(2*a) - Log[Sin[x]]/a - (b^(2/3)*Log[a^(1/3) + b^(1/3)*S
in[x]])/(3*a^(5/3)) + (b^(2/3)*Log[a^(2/3) - a^(1/3)*b^(1/3)*Sin[x] + b^(2/
3)*Sin[x]^2])/(6*a^(5/3)) + Log[a + b*Sin[x]^3]/(3*a)
```

Rule 31

```
Int[((a_) + (b_.)*(x_))^(-1), x_Symbol] :> Simp[Log[RemoveContent[a + b*x,
x]]/b, x] /; FreeQ[{a, b}, x]
```

Rule 200

```
Int[((a_) + (b_.)*(x_)^3)^(-1), x_Symbol] :> Dist[1/(3*Rt[a, 3]^2), Int[1/(
Rt[a, 3] + Rt[b, 3]*x), x], x] + Dist[1/(3*Rt[a, 3]^2), Int[(2*Rt[a, 3] - R
t[b, 3]*x)/(Rt[a, 3]^2 - Rt[a, 3]*Rt[b, 3]*x + Rt[b, 3]^2*x^2), x], x] /; F
reeQ[{a, b}, x]
```

Rule 204

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> -Simp[ArcTan[(Rt[-b, 2]*x)/Rt[
-a, 2]]/(Rt[-a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (LtQ[
a, 0] || LtQ[b, 0])
```

Rule 260

```
Int[(x_)^(m_.)/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Simp[Log[RemoveConten
t[a + b*x^n, x]]/(b*n), x] /; FreeQ[{a, b, m, n}, x] && EqQ[m, n - 1]
```

Rule 617

```
Int[((a_) + (b_.)*(x_) + (c_.)*(x_)^2)^(-1), x_Symbol] :> With[{q = 1 - 4*S
implify[(a*c)/b^2]}, Dist[-2/b, Subst[Int[1/(q - x^2), x], x, 1 + (2*c*x)/b
], x] /; RationalQ[q] && (EqQ[q^2, 1] ||  !RationalQ[b^2 - 4*a*c])] /; Free
Q[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0]
```

Rule 628

```
Int[((d_) + (e_.)*(x_))/((a_.) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> S
imp[(d*Log[RemoveContent[a + b*x + c*x^2, x]])/b, x] /; FreeQ[{a, b, c, d,
e}, x] && EqQ[2*c*d - b*e, 0]
```

Rule 634

```
Int[((d_.) + (e_.)*(x_))/((a_) + (b_.)*(x_) + (c_.)*(x_)^2), x_Symbol] :> D
ist[(2*c*d - b*e)/(2*c), Int[1/(a + b*x + c*x^2), x], x] + Dist[e/(2*c), In
t[(b + 2*c*x)/(a + b*x + c*x^2), x], x] /; FreeQ[{a, b, c, d, e}, x] && NeQ
[2*c*d - b*e, 0] && NeQ[b^2 - 4*a*c, 0] &&  !NiceSqrtQ[b^2 - 4*a*c]
```

Rule 1834

```
Int[((Pq_)*((c_.)*(x_))^(m_.))/((a_) + (b_.)*(x_)^(n_)), x_Symbol] :> Int[E
xpandIntegrand[((c*x)^m*Pq)/(a + b*x^n), x], x] /; FreeQ[{a, b, c, m}, x] &
& PolyQ[Pq, x] && IntegerQ[n] &&  !IGtQ[m, 0]
```

Rule 1871

```
Int[(P2_)/((a_) + (b_.)*(x_)^3), x_Symbol] :> With[{A = Coeff[P2, x, 0], B
= Coeff[P2, x, 1], C = Coeff[P2, x, 2]}, Int[(A + B*x)/(a + b*x^3), x] + Di
st[C, Int[x^2/(a + b*x^3), x], x] /; EqQ[a*B^3 - b*A^3, 0] ||  !RationalQ[a
/b]] /; FreeQ[{a, b}, x] && PolyQ[P2, x, 2]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cot^3(x)}{a+b\sin^3(x)}\,dx = \text{Subst}\left(\int \frac{1-x^2}{x^3\left(a+bx^3\right)}\,dx, x, \sin(x)\right)$$
$$= \text{Subst}\left(\int \left(\frac{1}{ax^3}-\frac{1}{ax}+\frac{b\left(-1+x^2\right)}{a\left(a+bx^3\right)}\right)dx, x, \sin(x)\right)$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{b\,\text{Subst}\left(\int \frac{-1+x^2}{a+bx^3}\,dx, x, \sin(x)\right)}{a}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b\,\text{Subst}\left(\int \frac{1}{a+bx^3}\,dx, x, \sin(x)\right)}{a}+\frac{b\,\text{Subst}\left(\int \frac{x^2}{a+bx^3}\,dx, x, \sin(x)\right)}{a}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}-\frac{b\,\text{Subst}\left(\int \frac{1}{\sqrt[3]{a}+\sqrt[3]{b}\,x}\,dx, x, \sin(x)\right)}{3a^{5/3}}-\frac{b\,\text{Subs}}{}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{\log\left(a+b\sin^3(x)\right)}{3a}+\frac{b^{2/3}\,\text{Subst}\left(\int\right.}{}$$
$$= -\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2/3}-\sqrt[3]{a}\sqrt[3]{b}\sin(x)+b^{2/3}\right.}{6a^{5/3}}$$
$$= \frac{b^{2/3}\tan^{-1}\left(\frac{1-\frac{2\sqrt[3]{b}\sin(x)}{\sqrt[3]{a}}}{\sqrt{3}}\right)}{\sqrt{3}\,a^{5/3}}-\frac{\csc^2(x)}{2a}-\frac{\log(\sin(x))}{a}-\frac{b^{2/3}\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)}{3a^{5/3}}+\frac{b^{2/3}\log\left(a^{2}\right.}{}$$

Mathematica [A] time = 0.31, size = 143, normalized size = 0.93

$$\frac{2\left(a^{2/3}-(-1)^{2/3}b^{2/3}\right)\log\left(-(-1)^{2/3}\sqrt[3]{a}-\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}-b^{2/3}\right)\log\left(\sqrt[3]{a}+\sqrt[3]{b}\sin(x)\right)+2\left(a^{2/3}+\sqrt[3]{-1}\,b^{2/3}\right)\mathrm{l}}{6a^{5/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]^3/(a + b*Sin[x]^3),x]
```

```
[Out] (-3*a^(2/3)*Csc[x]^2 - 6*a^(2/3)*Log[Sin[x]] + 2*(a^(2/3) - (-1)^(2/3)*b^(2
/3))*Log[-((-1)^(2/3)*a^(1/3)) - b^(1/3)*Sin[x]] + 2*(a^(2/3) - b^(2/3))*Lo
g[a^(1/3) + b^(1/3)*Sin[x]] + 2*(a^(2/3) + (-1)^(1/3)*b^(2/3))*Log[a^(1/3)
+ (-1)^(2/3)*b^(1/3)*Sin[x]])/(6*a^(5/3))
```

fricas [C] time = 68.27, size = 1764, normalized size = 11.53

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="fricas")
```

```
[Out] -1/12*(6*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1
/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) +
1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^
2)*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a
^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4 - 4*b^2*cos(x)^2 - 4*a*b*sin(
x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5
+ 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 + 4*b^2)*((3*(I*sqrt(3) + 1)*(
-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a^3 + 2*a^2)*
sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^
```

```
(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*
(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2) + sqrt(1/3)*((3*(I*sqrt(3) + 1)*(
-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^5 - 8*a^2
*b*sin(x) + 4*a^3 - 4*(a^3*b*sin(x) - a^4)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 +
1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a))*sqrt(((3*(I*sqrt(3) + 1)
*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)*a + 4)/a^2))/b^2) - 6*sqrt(1/3)*(a*cos(x)^2 - a)*sqrt(((3*(I*sqrt(3)
+ 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 +
 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/
3) - 2/a)*a + 4)/a^2)*arctan(-1/8*(2*sqrt(1/3)*sqrt((3*(I*sqrt(3) + 1)*(-1/
54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4 - 4*b^2*co
s(x)^2 - 4*a*b*sin(x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) + 1)*(-1/54/
a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 + 4*b^2)*((
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)*a^3 + 2*a^2)*sqrt(((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/5
4*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1
/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2) - sqrt(1/3)*((
3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) -
 2/a)^2*a^5 - 8*a^2*b*sin(x) + 4*a^3 - 4*(a^3*b*sin(x) - a^4)*(3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a))*sqrt(
((3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3)
 - 2/a)^2*a^2 + 4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2
- b^2)/a^5)^(1/3) - 2/a)*a + 4)/a^2))/b^2) + (a*cos(x)^2 - a)*(3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)*log(1/
4*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3
) - 2/a)^2*a^4 - b^2*cos(x)^2 + 2*a*b*sin(x) + (a^2*b*sin(x) + a^3)*(3*(I*s
qrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)
+ a^2 + b^2) - ((a*cos(x)^2 - a)*(3*(I*sqrt(3) + 1)*(-1/54/a^3 + 1/54*b^2/a
^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 6*cos(x)^2 - 6)*log((3*(I*sqrt(3)
 + 1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a)^2*a^4
- 4*b^2*cos(x)^2 - 4*a*b*sin(x) - 2*(a^2*b*sin(x) - 2*a^3)*(3*(I*sqrt(3) +
1)*(-1/54/a^3 + 1/54*b^2/a^5 + 1/54*(a^2 - b^2)/a^5)^(1/3) - 2/a) + 4*a^2 +
 4*b^2) + 12*(cos(x)^2 - 1)*log(-1/2*sin(x)) - 6)/(a*cos(x)^2 - a)
```

giac [A] time = 0.17, size = 144, normalized size = 0.94

$$\frac{b\left(-\frac{a}{b}\right)^{\frac{1}{3}}\log\left(\left|-\left(-\frac{a}{b}\right)^{\frac{1}{3}}+\sin(x)\right|\right)}{3\,a^2}+\frac{\log\left(\left|b\sin(x)^3+a\right|\right)}{3\,a}-\frac{\log\left(\left|\sin(x)\right|\right)}{a}-\frac{\sqrt{3}\,\left(-ab^2\right)^{\frac{1}{3}}\arctan\left(\frac{\sqrt{3}\left(\left(-\frac{a}{b}\right)^{\frac{1}{3}}+2\,\sin(x)\right)}{3\left(-\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{3\,a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="giac")
```

```
[Out] 1/3*b*(-a/b)^(1/3)*log(abs(-(-a/b)^(1/3) + sin(x)))/a^2 + 1/3*log(abs(b*sin
(x)^3 + a))/a - log(abs(sin(x)))/a - 1/3*sqrt(3)*(-a*b^2)^(1/3)*arctan(1/3*
sqrt(3)*((-a/b)^(1/3) + 2*sin(x))/(-a/b)^(1/3))/a^2 - 1/6*(-a*b^2)^(1/3)*lo
g(sin(x)^2 + (-a/b)^(1/3)*sin(x) + (-a/b)^(2/3))/a^2 - 1/2/(a*sin(x)^2)
```

maple [A] time = 0.31, size = 126, normalized size = 0.82

$$-\frac{1}{2a\sin(x)^2}-\frac{\ln(\sin(x))}{a}-\frac{\ln\left(\sin(x)+\left(\frac{a}{b}\right)^{\frac{1}{3}}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\ln\left(\sin^2(x)-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}-\frac{\sqrt{3}\,\arctan\left(\frac{\sqrt{3}\left(\frac{2\sin(x)}{\left(\frac{a}{b}\right)^{\frac{1}{3}}}-1\right)}{3}\right)}{3a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a+b*sin(x)^3),x)
```

```
[Out] -1/2/a/sin(x)^2-ln(sin(x))/a-1/3/a/(a/b)^(2/3)*ln(sin(x)+(a/b)^(1/3))+1/6/a
/(a/b)^(2/3)*ln(sin(x)^2-(a/b)^(1/3)*sin(x)+(a/b)^(2/3))-1/3/a/(a/b)^(2/3)*
3^(1/2)*arctan(1/3*3^(1/2)*(2/(a/b)^(1/3)*sin(x)-1))+1/3*ln(a+b*sin(x)^3)/a
```

maxima [A] time = 1.00, size = 152, normalized size = 0.99

$$-\frac{\sqrt{3}\left(b\left(3\left(\frac{a}{b}\right)^{\frac{1}{3}}-\frac{2a}{b}\right)+2a\right)\arctan\left(-\frac{\sqrt{3}\left(\left(\frac{a}{b}\right)^{\frac{1}{3}}-2\sin(x)\right)}{3\left(\frac{a}{b}\right)^{\frac{1}{3}}}\right)}{9a^2}+\frac{\left(2\left(\frac{a}{b}\right)^{\frac{2}{3}}+1\right)\log\left(\sin(x)^2-\left(\frac{a}{b}\right)^{\frac{1}{3}}\sin(x)+\left(\frac{a}{b}\right)^{\frac{2}{3}}\right)}{6a\left(\frac{a}{b}\right)^{\frac{2}{3}}}+\frac{\left(\left(\frac{a}{b}\right)^{\frac{2}{3}}\right.}{}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)^3/(a+b*sin(x)^3),x, algorithm="maxima")
```

```
[Out] -1/9*sqrt(3)*(b*(3*(a/b)^(1/3) - 2*a/b) + 2*a)*arctan(-1/3*sqrt(3)*((a/b)^(
1/3) - 2*sin(x))/(a/b)^(1/3))/a^2 + 1/6*(2*(a/b)^(2/3) + 1)*log(sin(x)^2 -
(a/b)^(1/3)*sin(x) + (a/b)^(2/3))/(a*(a/b)^(2/3)) + 1/3*((a/b)^(2/3) - 1)*l
og((a/b)^(1/3) + sin(x))/(a*(a/b)^(2/3)) - log(sin(x))/a - 1/2/(a*sin(x)^2)
```

mupad [B] time = 17.54, size = 2003, normalized size = 13.09

result too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)^3/(a + b*sin(x)^3),x)
```

```
[Out] symsum(log(-(256*(64*b^7*tan(x/2) + 32*a*b^6 - 44*a^3*b^4 + 15*a^5*b^2 - 10
24*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*b^8*tan(x/2)^2
 - 84*a^2*b^5*tan(x/2) + 26*a^4*b^3*tan(x/2) + 48*a*b^6*tan(x/2)^2 - 16*roo
t(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^2*b^6 + 328*root(2
7*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^4*b^4 - 165*root(27*a
^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^6*b^2 - 70*a^3*b^4*tan(x
/2)^2 + 25*a^5*b^2*tan(x/2)^2 - 48*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e -
 a^2 + b^2, e, k)^2*a^3*b^6 - 915*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e -
a^2 + b^2, e, k)^2*a^5*b^4 + 630*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a
^2 + b^2, e, k)^2*a^7*b^2 + 873*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^
2 + b^2, e, k)^3*a^6*b^4 - 810*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
 + b^2, e, k)^3*a^8*b^2 + 864*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
+ b^2, e, k)^4*a^7*b^4 - 405*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
 b^2, e, k)^4*a^9*b^2 - 1296*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
 b^2, e, k)^5*a^8*b^4 + 1215*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
 b^2, e, k)^5*a^10*b^2 - 608*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
 b^2, e, k)*a*b^7*tan(x/2) - 8880*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e -
a^2 + b^2, e, k)^2*a^3*b^6*tan(x/2)^2 + 5067*root(27*a^5*e^3 - 27*a^4*e^2 +
 9*a^3*e - a^2 + b^2, e, k)^2*a^5*b^4*tan(x/2)^2 + 1050*root(27*a^5*e^3 - 2
7*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^7*b^2*tan(x/2)^2 + 27648*root(27
*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^4*b^6*tan(x/2)^2 - 1
5543*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^6*b^4*ta
n(x/2)^2 - 1350*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3
*a^8*b^2*tan(x/2)^2 - 27648*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 +
b^2, e, k)^4*a^5*b^6*tan(x/2)^2 + 10800*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^
3*e - a^2 + b^2, e, k)^4*a^7*b^4*tan(x/2)^2 - 675*root(27*a^5*e^3 - 27*a^4*
e^2 + 9*a^3*e - a^2 + b^2, e, k)^4*a^9*b^2*tan(x/2)^2 + 9072*root(27*a^5*e^
3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^8*b^4*tan(x/2)^2 + 2025*roo
```

```
t(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^10*b^2*tan(x/2)^
2 + 1566*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^3*b^5*
tan(x/2) - 610*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)*a^
5*b^3*tan(x/2) + 1760*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e
, k)*a^2*b^6*tan(x/2)^2 - 260*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
+ b^2, e, k)*a^4*b^4*tan(x/2)^2 - 275*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*
e - a^2 + b^2, e, k)*a^6*b^2*tan(x/2)^2 + 1536*root(27*a^5*e^3 - 27*a^4*e^2
 + 9*a^3*e - a^2 + b^2, e, k)^2*a^2*b^7*tan(x/2) - 9870*root(27*a^5*e^3 - 2
7*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^4*b^5*tan(x/2) + 5238*root(27*a^
5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^2*a^6*b^3*tan(x/2) + 31968*
root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^5*b^5*tan(x/2
) - 21150*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k)^3*a^7*b
^3*tan(x/2) - 57888*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e,
k)^4*a^6*b^5*tan(x/2) + 40824*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^3*e - a^2
+ b^2, e, k)^4*a^8*b^3*tan(x/2) + 41472*root(27*a^5*e^3 - 27*a^4*e^2 + 9*a^
3*e - a^2 + b^2, e, k)^5*a^7*b^5*tan(x/2) - 30456*root(27*a^5*e^3 - 27*a^4*
e^2 + 9*a^3*e - a^2 + b^2, e, k)^5*a^9*b^3*tan(x/2)))/a^3)*root(27*a^5*e^3
- 27*a^4*e^2 + 9*a^3*e - a^2 + b^2, e, k), k, 1, 3) - 1/(8*a*tan(x/2)^2) -
tan(x/2)^2/(8*a) - log(tan(x/2))/a
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(x)}{a + b\sin^3(x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)**3/(a+b*sin(x)**3),x)
```

```
[Out] Integral(cot(x)**3/(a + b*sin(x)**3), x)
```

3.553 $\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx$

Optimal. Leaf size=45

$$\frac{2}{3}\sqrt{a+b\sin^3(x)}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2)
```

Rubi [A] time = 0.07, antiderivative size = 45, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2}{3}\sqrt{a+b\sin^3(x)}-\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(x)\sqrt{a+b\sin^3(x)}\,dx &= \text{Subst}\left(\int \frac{\sqrt{a+bx^3}}{x}\,dx, x, \sin(x)\right)\\ &= \frac{1}{3}\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{1}{3}a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^3(x)\right)\\ &= \frac{2}{3}\sqrt{a+b\sin^3(x)} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^3(x)}\right)}{3b}\\ &= -\frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right) + \frac{2}{3}\sqrt{a+b\sin^3(x)}\end{aligned}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 1.00

$$\frac{2}{3}\sqrt{a+b\sin^3(x)} - \frac{2}{3}\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^3],x]
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/3 + (2*Sqrt[a + b*Sin[x]
^3])/3
```

fricas [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
[Out] Timed out
```

giac [A] time = 0.15, size = 38, normalized size = 0.84

$$\frac{2\,a\arctan\left(\frac{\sqrt{b\sin(x)^3+a}}{\sqrt{-a}}\right)}{3\sqrt{-a}} + \frac{2}{3}\sqrt{b\sin(x)^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
[Out] 2/3*a*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a) + 2/3*sqrt(b*sin(x)^3
+ a)
```

maple [A] time = 1.93, size = 34, normalized size = 0.76

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^3(x)\right)}}{\sqrt{a}}\right)\sqrt{a}}{3} + \frac{2\sqrt{a+b\left(\sin^3(x)\right)}}{3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))*a^(1/2)+2/3*(a+b*sin(x)^3)^(1/2)
```

maxima [A] time = 0.56, size = 52, normalized size = 1.16

$$\frac{1}{3}\sqrt{a}\log\left(\frac{\sqrt{b\sin(x)^3+a}-\sqrt{a}}{\sqrt{b\sin(x)^3+a}+\sqrt{a}}\right)+\frac{2}{3}\sqrt{b\sin(x)^3+a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*sqrt(a)*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + sq
rt(a))) + 2/3*sqrt(b*sin(x)^3 + a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(x)\sqrt{b\sin(x)^3+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*sin(x)^3)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*sin(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^3(x)}\cot(x)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**3)*cot(x), x)
```

3.554 $\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$

Optimal. Leaf size=28

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)

Rubi [A] time = 0.07, antiderivative size = 28, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^3],x]

[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^3}}\,dx, x, \sin(x)\right)$$
$$= \frac{1}{3}\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^3(x)\right)$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^3(x)}\right)}{3b}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Mathematica [**A**] time = 0.02, size = 28, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^3(x)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^3],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^3]/Sqrt[a]])/(3*Sqrt[a])
```

fricas [**F(-2)**] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="fricas")
```

```
[Out] Exception raised: TypeError >>  Error detected within library code:   faile
d of mode Union(SparseUnivariatePolynomial(SimpleAlgebraicExtension(InnerPr
imeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^2+3*?+1)),fail
ed) cannot be coerced to mode SparseUnivariatePolynomial(SimpleAlgebraicExt
ension(InnerPrimeField(7),SparseUnivariatePolynomial(InnerPrimeField(7)),?^
2+3*?+1))
```

giac [**A**] time = 0.12, size = 24, normalized size = 0.86

$$\frac{2\,\arctan\left(\frac{\sqrt{b\sin(x)^3+a}}{\sqrt{-a}}\right)}{3\,\sqrt{-a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="giac")
```

```
[Out] 2/3*arctan(sqrt(b*sin(x)^3 + a)/sqrt(-a))/sqrt(-a)
```

maple [**A**] time = 0.28, size = 21, normalized size = 0.75

$$-\frac{2\,\text{arctanh}\left(\frac{\sqrt{a+b\left(\sin^3(x)\right)}}{\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^3)^(1/2),x)
```

```
[Out] -2/3*arctanh((a+b*sin(x)^3)^(1/2)/a^(1/2))/a^(1/2)
```

maxima [A] time = 1.07, size = 39, normalized size = 1.39

$$\frac{\log\left(\frac{\sqrt{b\sin(x)^3+a}-\sqrt{a}}{\sqrt{b\sin(x)^3+a}+\sqrt{a}}\right)}{3\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^3)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/3*log((sqrt(b*sin(x)^3 + a) - sqrt(a))/(sqrt(b*sin(x)^3 + a) + sqrt(a)))/
sqrt(a)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\cot(x)}{\sqrt{b\sin(x)^3+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*sin(x)^3)^(1/2),x)
```

```
[Out] int(cot(x)/(a + b*sin(x)^3)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^3(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**3)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**3), x)
```

3.555 $\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx$

Optimal. Leaf size=59

$$\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}$$

```
[Out] -1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))*a^(1/2)/d+1/2*(a+b*sin(d*x+c
)^4)^(1/2)/d
```

Rubi [A] time = 0.09, antiderivative size = 59, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = $\{3229, 266, 50, 63, 208\}$

$$\frac{\sqrt{a+b\sin^4(c+dx)}}{2d}-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(2*d) + Sqrt[a + b*S
in[c + d*x]^4]/(2*d)
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
```

```
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot(c+dx)\sqrt{a+b\sin^4(c+dx)}\,dx = \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx^2}}{x}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= \frac{\sqrt{a+b\sin^4(c+dx)}}{2d} + \frac{a\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}$$
$$= -\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2d} + \frac{\sqrt{a+b\sin^4(c+dx)}}{2d}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 0.93

$$-\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - \sqrt{a+b\sin^4(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Sqrt[a + b*Sin[
c + d*x]^4])/d
```

fricas [A] time = 0.65, size = 195, normalized size = 3.31

$$\left[\frac{\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\,\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\,\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+\right.}}{4\,d}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d
*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4
- 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b))/d, 1/2*(sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 +
 a + b)*sqrt(-a)/a) + sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/
d]
```

giac [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Timed out
```

maple [F] time = 1.73, size = 0, normalized size = 0.00

$$\int \cot \left(dx+c\right) \sqrt{a+b \left(\sin^{4}\left(dx+c\right)\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [A] time = 0.61, size = 68, normalized size = 1.15

$$\frac{\sqrt{a} \log\left(\frac{\sqrt{b \sin(dx+c)^{4}+a}-\sqrt{a}}{\sqrt{b \sin(dx+c)^{4}+a}+\sqrt{a}}\right)+2 \sqrt{b \sin\left(dx+c\right)^{4}+a}}{4\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] 1/4*(sqrt(a)*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c
)^4 + a) + sqrt(a))) + 2*sqrt(b*sin(d*x + c)^4 + a))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot\left(c+d\,x\right)\,\sqrt{b\,{\sin\left(c+d\,x\right)}^4+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{4}\left(c+dx\right)} \cot\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(c + d*x)**4)*cot(c + d*x), x)
```

3.556 $\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2d(a+b)}-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d(a+b)^{3/2}}$$

```
[Out] -1/2*a*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/(a+
b)^(3/2)/d+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(1/2)/(a+b)/d
```

Rubi [A] time = 0.12, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3229, 807, 725, 206}

$$\frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2d(a+b)}-\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2d(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -(a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])
])/(2*(a + b)^(3/2)*d) + (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(2*(a
+ b)*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{x}{(1-x)^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} - \frac{a\,\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2(a+b)d}$$
$$= \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d} + \frac{a\,\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)d}$$
$$= -\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2(a+b)^{3/2}d} + \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2(a+b)d}$$

Mathematica [A] time = 0.20, size = 85, normalized size = 0.96

$$-\frac{\frac{a\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{(a+b)^{3/2}} - \frac{\sec^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{a+b}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*((a*ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x
]^4])])/(a + b)^(3/2) - (Sec[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a + b)
)/d
```

fricas [B] time = 0.64, size = 361, normalized size = 4.06

$$\left[\frac{\sqrt{a+b}\,a\cos(dx+c)^2\log\left(\frac{\left(ab+2b^2\right)\cos(dx+c)^4-4\left(ab+b^2\right)\cos(dx+c)^2+2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\left(b\cos(dx+c)^2-a-b\right)\sqrt{a+b}+}{\cos(dx+c)^4}\right.}{4\left(a^2+2ab+b^2\right)d\cos(dx+c)^2}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*(sqrt(a + b)*a*cos(d*x + c)^2*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a
*b + b^2)*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a
 + b)*(b*cos(d*x + c)^2 - a - b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d
*x + c)^4) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(a + b))
/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2), -1/2*(a*sqrt(-a - b)*arctan(sqrt(b
*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a - b)*sq
rt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b + b^2)*cos(d*x + c)^2 + a^2
 + 2*a*b + b^2))*cos(d*x + c)^2 - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^
2 + a + b)*(a + b))/((a^2 + 2*a*b + b^2)*d*cos(d*x + c)^2)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^3}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.52, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(dx+c\right)}{\sqrt{a+b\left(\sin^{4}\left(dx+c\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{3}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^3/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\tan\left(c+d\,x\right)^{3}}{\sqrt{b\sin\left(c+d\,x\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(tan(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{3}\left(c+dx\right)}{\sqrt{a+b\sin^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(tan(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.557 $\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=51

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\,\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

```
[Out] 1/2*arctanh((a+b*sin(d*x+c)^2)/(a+b)^(1/2)/(a+b*sin(d*x+c)^4)^(1/2))/d/(a+b
)^(1/2)
```

Rubi [A] time = 0.06, antiderivative size = 51, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.130, Rules used = {3229, 725, 206}

$$\frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\,\sqrt{a+b\sin^4(c+dx)}}\right)}{2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[(a + b*Sin[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b*Sin[c + d*x]^4])]/(2
*Sqrt[a + b]*d)
```

Rule 206

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1*ArcTanh[(Rt[-b, 2]*x)/
Rt[a, 2]])/(Rt[a, 2]*Rt[-b, 2]), x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 725

```
Int[1/(((d_) + (e_.)*(x_))*Sqrt[(a_) + (c_.)*(x_)^2]), x_Symbol] :> -Subst[
Int[1/(c*d^2 + a*e^2 - x^2), x], x, (a*e - c*d*x)/Sqrt[a + c*x^2]] /; FreeQ
[{a, c, d, e}, x]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\tan(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1-x)\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{1}{a+b-x^2}\,dx, x, \frac{-a-b\sin^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\right)}{2d}$$
$$= \frac{\tanh^{-1}\left(\frac{a+b\sin^2(c+dx)}{\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}}\right)}{2\sqrt{a+b}\,d}$$

Mathematica [A] time = 0.09, size = 65, normalized size = 1.27

$$\frac{\tanh^{-1}\left(\frac{a-b\cos^2(c+dx)+b}{\sqrt{a+b}\sqrt{a+b\cos^4(c+dx)-2b\cos^2(c+dx)+b}}\right)}{2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[(a + b - b*Cos[c + d*x]^2)/(Sqrt[a + b]*Sqrt[a + b - 2*b*Cos[c + d*
x]^2 + b*Cos[c + d*x]^4])]/(2*Sqrt[a + b]*d)
```

fricas [B] time = 0.60, size = 240, normalized size = 4.71

$$\left[\frac{\log\left(\frac{(ab+2b^2)\cos(dx+c)^4-4(ab+b^2)\cos(dx+c)^2-2\sqrt{b\cos(dx+c)^4-2b\cos(dx+c)^2+a+b}\,(b\cos(dx+c)^2-a-b)\sqrt{a+b}+2a^2+4ab+2b^2}{\cos(dx+c)^4}\right)}{4\sqrt{a+b}\,d}, \frac{\sqrt{-a-}}{}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(((a*b + 2*b^2)*cos(d*x + c)^4 - 4*(a*b + b^2)*cos(d*x + c)^2 - 2*s
qrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(b*cos(d*x + c)^2 - a -
b)*sqrt(a + b) + 2*a^2 + 4*a*b + 2*b^2)/cos(d*x + c)^4)/(sqrt(a + b)*d), 1/
2*sqrt(-a - b)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(
b*cos(d*x + c)^2 - a - b)*sqrt(-a - b)/((a*b + b^2)*cos(d*x + c)^4 - 2*(a*b
 + b^2)*cos(d*x + c)^2 + a^2 + 2*a*b + b^2))/((a + b)*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [A] time = 0.51, size = 72, normalized size = 1.41

$$\frac{\ln\left(\frac{2a+2b-2b\left(\cos^2(dx+c)\right)+2\sqrt{a+b}\sqrt{a+b-2b\left(\cos^2(dx+c)\right)+b\left(\cos^4(dx+c)\right)}}{\cos(dx+c)^2}\right)}{2d\sqrt{a+b}}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)

[Out] 1/2/d/(a+b)^(1/2)*ln((2*a+2*b-2*b*cos(d*x+c)^2+2*(a+b)^(1/2)*(a+b-2*b*cos(d*x+c)^2+b*cos(d*x+c)^4)^(1/2))/cos(d*x+c)^2)

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)}{\sqrt{b\sin(dx+c)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")

[Out] integrate(tan(d*x + c)/sqrt(b*sin(d*x + c)^4 + a), x)

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{\tan(c+d\,x)}{\sqrt{b\sin(c+d\,x)^4 + a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)

[Out] int(tan(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(c+dx)}{\sqrt{a + b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(tan(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)

[Out] Integral(tan(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)

3.558 $\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=35

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

[Out] -1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)

Rubi [A] time = 0.07, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.174, Rules used = {3229, 266, 63, 208}

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

[In] Int[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]

[Out] -ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d)

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}$$
$$= -\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Mathematica [A] time = 0.02, size = 35, normalized size = 1.00

$$-\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(Sqrt[a]*d)
```

fricas [A] time = 0.59, size = 140, normalized size = 4.00

$$\left[\frac{\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2-2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\,\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)}{4\,\sqrt{a}\,d}, \frac{\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}}{a}\right.}{2\,ad}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 - 2*sqrt(b*cos(d*x + c)^4
 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(d
*x + c)^2 + 1))/(sqrt(a)*d), 1/2*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*
b*cos(d*x + c)^2 + a + b)*sqrt(-a)/a)/(a*d)]
```

giac [A] time = 0.62, size = 31, normalized size = 0.89

$$\frac{\arctan\left(\frac{\sqrt{b\sin(dx+c)^4+a}}{\sqrt{-a}}\right)}{2\,\sqrt{-a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] 1/2*arctan(sqrt(b*sin(d*x + c)^4 + a)/sqrt(-a))/(sqrt(-a)*d)
```

maple [F] time = 1.55, size = 0, normalized size = 0.00

$$\int \frac{\cot(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [A] time = 0.68, size = 50, normalized size = 1.43

$$\frac{\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{4\sqrt{a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] 1/4*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a)
+ sqrt(a)))/(sqrt(a)*d)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(c+d\,x)}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(cot(c + d*x)/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(cot(c + d*x)/sqrt(a + b*sin(c + d*x)**4), x)
```

3.559 $\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=70

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

```
[Out] 1/2*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/d/a^(1/2)-1/2*csc(d*x+c)^2*(a
+b*sin(d*x+c)^4)^(1/2)/a/d
```

Rubi [A] time = 0.10, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3229, 807, 266, 63, 208}

$$\frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d}-\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]]/(2*Sqrt[a]*d) - (Csc[c + d*x]^2
*Sqrt[a + b*Sin[c + d*x]^4])/(2*a*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
```

```
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{1-x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^4(c+dx)\right)}{4d}$$
$$= -\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad} - \frac{\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^4(c+dx)}\right)}{2bd}$$
$$= \frac{\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{2\sqrt{a}\,d} - \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Mathematica [A] time = 0.08, size = 66, normalized size = 0.94

$$\frac{\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - \csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - Csc[c + d*x]^2*Sqrt[
a + b*Sin[c + d*x]^4])/(2*a*d)
```

fricas [A] time = 0.60, size = 247, normalized size = 3.53

$$\left[\frac{\left(\cos(dx+c)^2-1\right)\sqrt{a}\log\left(\frac{8\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+2\,\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}\,\sqrt{a}+2\,a+b\right)}{\cos(dx+c)^4-2\,\cos(dx+c)^2+1}\right)+2\sqrt{b\cos(dx+}}{4\left(ad\cos(dx+c)^2-ad\right)}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [1/4*((cos(d*x + c)^2 - 1)*sqrt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x +
c)^2 + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a
+ b)/(cos(d*x + c)^4 - 2*cos(d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2
*b*cos(d*x + c)^2 + a + b))/(a*d*cos(d*x + c)^2 - a*d), -1/2*((cos(d*x + c)
^2 - 1)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
*sqrt(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b))/(a*d*co
s(d*x + c)^2 - a*d)]
```

giac [F(-2)] time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,x):;OUTPUT:Evaluation time: 3.09Not invertible Error: B
ad Argument Value
```

maple [F] time = 1.81, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [A] time = 1.96, size = 79, normalized size = 1.13

$$-\frac{\frac{\log\left(\frac{\sqrt{b\sin(dx+c)^4+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^4+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{2\sqrt{b\sin(dx+c)^4+a}}{a\sin(dx+c)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/4*(log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a
) + sqrt(a)))/sqrt(a) + 2*sqrt(b*sin(d*x + c)^4 + a)/(a*sin(d*x + c)^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+d\,x)^3}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^3/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^3(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**3/sqrt(a + b*sin(c + d*x)**4), x)
```

3.560 $\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=108

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}$$

```
[Out] -1/4*(2*a-b)*arctanh((a+b*sin(d*x+c)^4)^(1/2)/a^(1/2))/a^(3/2)/d+csc(d*x+c)
^2*(a+b*sin(d*x+c)^4)^(1/2)/a/d-1/4*csc(d*x+c)^4*(a+b*sin(d*x+c)^4)^(1/2)/a
/d
```

Rubi [A] time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3229, 1807, 807, 266, 63, 208}

$$-\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d}-\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad}+\frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((2*a - b)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]])/(4*a^(3/2)*d) + (C
sc[c + d*x]^2*Sqrt[a + b*Sin[c + d*x]^4])/(a*d) - (Csc[c + d*x]^4*Sqrt[a +
b*Sin[c + d*x]^4])/(4*a*d)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 807

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> -Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))
/(2*(p + 1)*(c*d^2 + a*e^2)), x] + Dist[(c*d*f + a*e*g)/(c*d^2 + a*e^2), In
t[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d, e, f, g, m, p}
, x] && NeQ[c*d^2 + a*e^2, 0] && EqQ[Simplify[m + 2*p + 3], 0]
```

Rule 1807

```
Int[(Pq_)*((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2)^(p_), x_Symbol] :> With[{
Q = PolynomialQuotient[Pq, c*x, x], R = PolynomialRemainder[Pq, c*x, x]}, S
```

```
imp[(R*(c*x)^(m + 1)*(a + b*x^2)^(p + 1))/(a*c*(m + 1)), x] + Dist[1/(a*c*(
m + 1)), Int[(c*x)^(m + 1)*(a + b*x^2)^p*ExpandToSum[a*c*(m + 1)*Q - b*R*(m
 + 2*p + 3)*x, x], x], x]] /; FreeQ[{a, b, c, p}, x] && PolyQ[Pq, x] && LtQ
[m, -1] && (IntegerQ[2*p] || NeQ[Expon[Pq, x], 1])
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\text{Subst}\left(\int \frac{(1-x)^2}{x^3\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= -\frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} - \frac{\text{Subst}\left(\int \frac{4a-(2a-b)x}{x^2\sqrt{a+bx^2}}\,dx, x, \sin^2(c+dx)\right)}{4ad}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Sub}}{}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Sub}}{}$$

$$= \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \frac{\csc^4(c+dx)\sqrt{a+b\sin^4(c+dx)}}{4ad} + \frac{(2a-b)\,\text{Sub}}{}$$

$$= -\frac{(2a-b)\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right)}{4a^{3/2}d} + \frac{\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)}}{ad} - \csc^4(c+dx$$

Mathematica [A] time = 2.90, size = 141, normalized size = 1.31

$$-\frac{2a^{3/2}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^4(c+dx)}}{\sqrt{a}}\right) - 4a\csc^2(c+dx)\sqrt{a+b\sin^4(c+dx)} + b\sqrt{a+b\sin^4(c+dx)}\left(\frac{a\csc^4(c+dx)}{b} - \frac{\tanh^{-1}(}{\sqrt{b}}\right.}{4a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^5/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/4*(2*a^(3/2)*ArcTanh[Sqrt[a + b*Sin[c + d*x]^4]/Sqrt[a]] - 4*a*Csc[c + d
*x]^2*Sqrt[a + b*Sin[c + d*x]^4] + b*Sqrt[a + b*Sin[c + d*x]^4]*((a*Csc[c +
 d*x]^4)/b - ArcTanh[Sqrt[1 + (b*Sin[c + d*x]^4)/a]]/Sqrt[1 + (b*Sin[c + d*
x]^4)/a]))/(a^2*d)
```

fricas [A] time = 0.65, size = 371, normalized size = 3.44

$$\left[-\frac{\left((2\,a-b)\cos\left(dx+c\right)^{4}-2\,(2\,a-b)\cos\left(dx+c\right)^{2}+2\,a-b\right)\sqrt{a}\,\log\left(\frac{8\left(b\cos(dx+c)^{4}-2\,b\cos(dx+c)^{2}+2\,\sqrt{b\cos(dx+c)^{4}-2}\cdots\right.}{\cos(dx+c)^{4}-2\,\cos(dx+c)^{2}\cdots}\right.}{8\left(a^{2}d\cos\left(dx+c\right)^{4}-2\,a^{2}d\cos\cdots\right.}\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a - b)*sq
rt(a)*log(8*(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + 2*sqrt(b*cos(d*x + c)^
4 - 2*b*cos(d*x + c)^2 + a + b)*sqrt(a) + 2*a + b)/(cos(d*x + c)^4 - 2*cos(
d*x + c)^2 + 1)) + 2*sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4
*a*cos(d*x + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 +
a^2*d), 1/4*(((2*a - b)*cos(d*x + c)^4 - 2*(2*a - b)*cos(d*x + c)^2 + 2*a -
 b)*sqrt(-a)*arctan(sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*sqr
t(-a)/a) - sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)*(4*a*cos(d*x
 + c)^2 - 3*a))/(a^2*d*cos(d*x + c)^4 - 2*a^2*d*cos(d*x + c)^2 + a^2*d)]
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot\left(dx+c\right)^{5}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^5/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.88, size = 0, normalized size = 0.00

$$\int \frac{\cot^{5}\left(dx+c\right)}{\sqrt{a+b\left(\sin^{4}\left(dx+c\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [A] time = 0.69, size = 166, normalized size = 1.54

$$-\frac{\frac{2\,\sqrt{b\sin(dx+c)^{4}+a}\,b}{\left(b\sin(dx+c)^{4}+a\right)a-a^{2}}-\frac{2\,\log\left(\frac{\sqrt{b\sin(dx+c)^{4}+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^{4}+a}+\sqrt{a}}\right)}{\sqrt{a}}+\frac{b\log\left(\frac{\sqrt{b\sin(dx+c)^{4}+a}-\sqrt{a}}{\sqrt{b\sin(dx+c)^{4}+a}+\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{8\,\sqrt{b\sin(dx+c)^{4}+a}}{a\sin(dx+c)^{2}}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^5/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] -1/8*(2*sqrt(b*sin(d*x + c)^4 + a)*b/((b*sin(d*x + c)^4 + a)*a - a^2) - 2*l
og((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d*x + c)^4 + a) + sqr
t(a)))/sqrt(a) + b*log((sqrt(b*sin(d*x + c)^4 + a) - sqrt(a))/(sqrt(b*sin(d
```

```
*x + c)^4 + a) + sqrt(a)))/a^(3/2) - 8*sqrt(b*sin(d*x + c)^4 + a)/(a*sin(d*
x + c)^2))/d
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cot(c+dx)^5}{\sqrt{b\sin(c+dx)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^5/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^5(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**5/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**5/sqrt(a + b*sin(c + d*x)**4), x)
```

3.561 $\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=411

$$\frac{\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{d\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}+\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\right.}{}$$

```
[Out] -a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/
2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan((a+
b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*
tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2
)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*a^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/(a+b)^(3/4)/d/(a+b*sin(d*x+c)^4)^(
1/2)+cos(d*x+c)*sin(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/d/(a+b)^
(1/2)/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
```

Rubi [A] time = 0.38, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.160, Rules used = {3232, 1139, 1103, 1195}

$$\frac{\sin(c+dx)\cos(c+dx)\left((a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a\right)}{d\sqrt{a+b}\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)}+\frac{\sqrt[4]{a}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]*Sin[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4
))/(Sqrt[a + b]*d*Sqrt[a + b*Sin[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c +
 d*x]^2)) - (a^(1/4)*Cos[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c
 + d*x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[
c + d*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a]
 + Sqrt[a + b]*Tan[c + d*x]^2)^2])/((a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]
^4]) + (a^(1/4)*Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*
x])/a^(1/4)], (1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)*Sqrt[(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sq
rt[a + b]*Tan[c + d*x]^2)^2])/(2*(a + b)^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]
)
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
```

```
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 3232

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[((d*ff*x)^m*ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\tan^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{x^2}{\sqrt{a+2ax^2+(a+b)x^4}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\left(\sqrt{a}\,\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a\cdots}}\right.}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos(c+dx)\sin(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{\sqrt{a+b}\,d\sqrt{a+b\sin^4(c+dx)}\left(\sqrt{a}+\sqrt{a+b}\tan^2(c+dx)\right)} - \frac{\sqrt[4]{a}\cos^2(c+\cdots}{\cdots}$$

Mathematica [C] time = 6.15, size = 291, normalized size = 0.71

$$-\frac{2i\sqrt{2}\sqrt{a}\cos^2(c+dx)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\sqrt{1+\left(1+\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\left(E\left(i\sinh^{-1}\left(\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\tan(c\cdots\right.\right.\right.}{d\left(\sqrt{a}+i\sqrt{b}\right)\sqrt{1-\frac{i\sqrt{b}}{\sqrt{a}}}\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(\cdots}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Tan[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] ((-2*I)*Sqrt[2]*Sqrt[a]*Cos[c + d*x]^2*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqr
t[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]
- EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a]
 + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sqrt[1 + (1 - (I*Sqrt[b])/Sqrt[a])*Ta
n[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x]^2])/((Sqrt[a]
 + I*Sqrt[b])*Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*d*Sqrt[8*a + 3*b - 4*b*Cos[2*(c
 + d*x)] + b*Cos[4*(c + d*x)]])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan\left(dx+c\right)^{2}}{\sqrt{b\cos\left(dx+c\right)^{4}-2\,b\cos\left(dx+c\right)^{2}+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{2}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.54, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(dx+c\right)}{\sqrt{a+b\left(\sin^{4}\left(dx+c\right)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
[Out] int(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{2}}{\sqrt{b\sin\left(dx+c\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\tan\left(c+d\,x\right)^{2}}{\sqrt{b\sin\left(c+d\,x\right)^{4}+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(tan(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{2}\left(c+dx\right)}{\sqrt{a+b\sin^{4}\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(tan(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.562 $\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\,\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] 1/2*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)^(1/2)/co
s(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*arctan((a+b)^(1
/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+2*a*tan(d
*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^(1/2)*(a^
(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(1/4)/(a+b)^(1/4)/d/(a+b*sin(d*x+c)^4)^(1
/2)
```

Rubi [A] time = 0.08, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 16, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.125, Rules used = {3210, 1103}

$$\frac{\cos^2(c+dx)\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\,\tan^2(c+dx)+\sqrt{a}\right)^2}}\,F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)}{2\sqrt[4]{a}\,d\sqrt[4]{a+b}\,\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (Cos[c + d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (
1 - Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a
+ 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c
 + d*x]^2)^2])/(2*a^(1/4)*(a + b)^(1/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 3210

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_), x_Symbol] :> With[{ff =
FreeFactors[Tan[e + f*x], x]}, Dist[(ff*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f
*x]^2)^(2*p))/(f*(a + 2*a*Tan[e + f*x]^2 + (a + b)*Tan[e + f*x]^4)^p), Subs
t[Int[(a + 2*a*ff^2*x^2 + (a + b)*ff^4*x^4)^p/(1 + ff^2*x^2)^(2*p + 1), x],
 x, Tan[e + f*x]/ff], x]] /; FreeQ[{a, b, e, f, p}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{1}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{\sqrt{a+2ax^2+(a+}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= \frac{\cos^2(c+dx)F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\,\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\frac{\sqrt{a}}{\sqrt{a+b}}\right)\right)\left(\sqrt{a}+\sqrt{a+b}\,\tan^2(c+dx\right.}{2\sqrt[4]{a}\,\sqrt[4]{a+b}\,d\sqrt{a+b\sin^4(c+dx)}}$$

Mathematica [C] time = 2.74, size = 304, normalized size = 1.88

$$\frac{2\sqrt{2}\left(\sqrt{b}+i\sqrt{a}\right)\sin^2(c+dx)\tan(c+dx)\left(2\sqrt{a}+i\sqrt{b}\cos(2(c+dx))-i\sqrt{b}\right)\left(2i\sqrt{a}+\sqrt{b}\cos(2(c+dx))-\sqrt{b}\right)}{\sqrt{a}\,d(8a-4b\cos(2(c+dx}$$

Antiderivative was successfully verified.

```
[In] Integrate[1/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] (2*Sqrt[2]*(I*Sqrt[a] + Sqrt[b])*(2*Sqrt[a] - I*Sqrt[b] + I*Sqrt[b]*Cos[2*(
c + d*x)])*((2*I)*Sqrt[a] - Sqrt[b] + Sqrt[b]*Cos[2*(c + d*x)])*Sqrt[(1 - (
(2*I)*Sqrt[a])/Sqrt[b] - Cos[2*(c + d*x)])*Csc[c + d*x]^2]*Sqrt[(Cot[c + d*
x]^2*(I*Sqrt[a]*Sqrt[b] - a*Csc[c + d*x]^2))/(Sqrt[a] - I*Sqrt[b])^2]*Ellip
ticF[ArcSin[Sqrt[((-I)*Sqrt[b] + Sqrt[a]*Csc[c + d*x]^2)/(Sqrt[a] - I*Sqrt[
b])]], 1/2 + ((I/2)*Sqrt[a])/Sqrt[b]]*Sin[c + d*x]^2*Tan[c + d*x])/(Sqrt[a]
*d*(8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)])^(3/2))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{1}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [B] time = 3.39, size = 396, normalized size = 2.44

$$-\frac{\sqrt{\left(4a+\left(\cos^2\left(2dx+2c\right)\right)b+b-2b\cos\left(2dx+2c\right)\right)\left(\sin^2\left(2dx+2c\right)\right)}\,\sqrt{-ab}\,\sqrt{\frac{\left(-b+\sqrt{-ab}\right)\left(-1+\cos(2dx+2c)\right)}{\sqrt{-ab}\,\left(\cos(2dx+2c)+1\right)}}\,\left(\cos(2\right.}{\left(-b+\sqrt{-ab}\right)\sqrt{\frac{\left(-1+\cos(2dx+2c)\right)\left(\cos(2dx+2c)+1\right)\left(-b\cos(2dx+2c)+2\sqrt{-ab}+b\right)\left(b\cos(2dx+2c\right.}{b}}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] -((4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))*sin(2*d*x+2*c)^2)^(1/2)*(-a
*b)^(1/2)*((-b+(-a*b)^(1/2))*(-1+cos(2*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*
c)+1))^(1/2)*(cos(2*d*x+2*c)+1)^2*((-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)/(-a
*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*((b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)-b)/(-
a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2)*EllipticF(((-b+(-a*b)^(1/2))*(-1+cos(2
*d*x+2*c))/(-a*b)^(1/2)/(cos(2*d*x+2*c)+1))^(1/2),((b+(-a*b)^(1/2))/(-b+(-a
*b)^(1/2)))^(1/2))/(-b+(-a*b)^(1/2))/(1/b*(-1+cos(2*d*x+2*c))*(cos(2*d*x+2*
c)+1)*(-b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)+b)*(b*cos(2*d*x+2*c)+2*(-a*b)^(1/2)
```

```
-b))^(1/2)/sin(2*d*x+2*c)/(4*a+cos(2*d*x+2*c)^2*b+b-2*b*cos(2*d*x+2*c))^(1/
2)/d
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{b\sin\left(dx+c\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
[Out] integrate(1/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{b\sin\left(c+d\,x\right)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + b*sin(c + d*x)^4)^(1/2),x)
[Out] int(1/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a+b\sin^4\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+b*sin(d*x+c)**4)**(1/2),x)
[Out] Integral(1/sqrt(a + b*sin(c + d*x)**4), x)
```

3.563 $\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$

Optimal. Leaf size=477

$$\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\right.\right.}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

```
[Out] -(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))^2)
^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticE(sin(2*arctan
((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*((a+
2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)^2)^
(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(1/2)
+1/2*(a+b)^(1/4)*cos(d*x+c)^2*(cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))
)^2)^(1/2)/cos(2*arctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4)))*EllipticF(sin(2*ar
ctan((a+b)^(1/4)*tan(d*x+c)/a^(1/4))),1/2*(2-2*a^(1/2)/(a+b)^(1/2))^(1/2))*
((a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)
^2)^(1/2)*(a^(1/2)+(a+b)^(1/2)*tan(d*x+c)^2)/a^(3/4)/d/(a+b*sin(d*x+c)^4)^(
1/2)-cos(d*x+c)^2*cot(d*x+c)*(a+2*a*tan(d*x+c)^2+(a+b)*tan(d*x+c)^4)/a/d/(a
+b*sin(d*x+c)^4)^(1/2)+cos(d*x+c)*sin(d*x+c)*(a+b)^(1/2)*(a+2*a*tan(d*x+c)^
2+(a+b)*tan(d*x+c)^4)/a/d/(a+b*sin(d*x+c)^4)^(1/2)/(a^(1/2)+(a+b)^(1/2)*tan
(d*x+c)^2)
```

Rubi [A] time = 0.37, antiderivative size = 477, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.240, Rules used = {3232, 1123, 12, 1139, 1103, 1195}

$$\frac{\sqrt[4]{a+b}\cos^2(c+dx)\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)\sqrt{\frac{(a+b)\tan^4(c+dx)+2a\tan^2(c+dx)+a}{\left(\sqrt{a+b}\tan^2(c+dx)+\sqrt{a}\right)^2}}F\left(2\tan^{-1}\left(\frac{\sqrt[4]{a+b}\tan(c+dx)}{\sqrt[4]{a}}\right)|\frac{1}{2}\left(1-\right.\right.}{2a^{3/4}d\sqrt{a+b\sin^4(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -((Cos[c + d*x]^2*Cot[c + d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*
x]^4))/(a*d*Sqrt[a + b*Sin[c + d*x]^4])) + (Sqrt[a + b]*Cos[c + d*x]*Sin[c
+ d*x]*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))/(a*d*Sqrt[a + b*S
in[c + d*x]^4]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)) - ((a + b)^(1/4)*Cos
[c + d*x]^2*EllipticE[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 -
Sqrt[a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*
a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d
*x]^2)^2])/(a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4]) + ((a + b)^(1/4)*Cos[c +
d*x]^2*EllipticF[2*ArcTan[((a + b)^(1/4)*Tan[c + d*x])/a^(1/4)], (1 - Sqrt[
a]/Sqrt[a + b])/2]*(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2)*Sqrt[(a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)/(Sqrt[a] + Sqrt[a + b]*Tan[c + d*x]^2
)^2])/(2*a^(3/4)*d*Sqrt[a + b*Sin[c + d*x]^4])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 1103

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q = Rt[c
/a, 4]}, Simp[((1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^2)^2)]*
```

```
EllipticF[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(2*q*Sqrt[a + b*x^2 + c*x^4]
), x]] /; FreeQ[{a, b, c}, x] && NeQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1123

```
Int[((d_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4)^(p_), x_Symbol]
:> Simp[((d*x)^(m + 1)*(a + b*x^2 + c*x^4)^(p + 1))/(a*d*(m + 1)), x] - Dis
t[1/(a*d^2*(m + 1)), Int[(d*x)^(m + 2)*(b*(m + 2*p + 3) + c*(m + 4*p + 5)*x
^2)*(a + b*x^2 + c*x^4)^p, x], x] /; FreeQ[{a, b, c, d, p}, x] && NeQ[b^2 -
 4*a*c, 0] && LtQ[m, -1] && IntegerQ[2*p] && (IntegerQ[p] || IntegerQ[m])
```

Rule 1139

```
Int[(x_)^2/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbol] :> With[{q =
 Rt[c/a, 2]}, Dist[1/q, Int[1/Sqrt[a + b*x^2 + c*x^4], x], x] - Dist[1/q, I
nt[(1 - q*x^2)/Sqrt[a + b*x^2 + c*x^4], x], x]] /; FreeQ[{a, b, c}, x] && N
eQ[b^2 - 4*a*c, 0] && PosQ[c/a]
```

Rule 1195

```
Int[((d_) + (e_.)*(x_)^2)/Sqrt[(a_) + (b_.)*(x_)^2 + (c_.)*(x_)^4], x_Symbo
l] :> With[{q = Rt[c/a, 4]}, -Simp[(d*x*Sqrt[a + b*x^2 + c*x^4])/(a*(1 + q^
2*x^2)), x] + Simp[(d*(1 + q^2*x^2)*Sqrt[(a + b*x^2 + c*x^4)/(a*(1 + q^2*x^
2)^2)]*EllipticE[2*ArcTan[q*x], 1/2 - (b*q^2)/(4*c)])/(q*Sqrt[a + b*x^2 + c
*x^4]), x] /; EqQ[e + d*q^2, 0]] /; FreeQ[{a, b, c, d, e}, x] && NeQ[b^2 -
4*a*c, 0] && PosQ[c/a]
```

Rule 3232

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^4)^(p_)*((d_.)*tan[(e_.) + (f_.)*
(x_)])^(m_), x_Symbol] :> With[{ff = FreeFactors[Tan[e + f*x], x]}, Dist[(f
f*(a + b*Sin[e + f*x]^4)^p*(Sec[e + f*x]^2)^(2*p))/(f*Apart[a*(1 + Tan[e +
f*x]^2)^2 + b*Tan[e + f*x]^4]^p), Subst[Int[((d*ff*x)^m*ExpandToSum[a*(1 +
ff^2*x^2)^2 + b*ff^4*x^4, x]^p)/(1 + ff^2*x^2)^(2*p + 1), x], x, Tan[e + f*
x]/ff], x]] /; FreeQ[{a, b, d, e, f, m}, x] && IntegerQ[p - 1/2]
```

Rubi steps

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx = \frac{\left(\cos^2(c+dx)\sqrt{a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)}\right)\text{Subst}\left(\int \frac{1}{x^2\sqrt{a+2ax^2+(\ldots}}\right.}{d\sqrt{a+b\sin^4(c+dx)}}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\cos^2(\ldots\right.}{\ldots}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left((a+b\ldots\right.}{\ldots}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\left(\sqrt{a+\ldots}\right.}{\ldots}$$

$$= -\frac{\cos^2(c+dx)\cot(c+dx)\left(a+2a\tan^2(c+dx)+(a+b)\tan^4(c+dx)\right)}{ad\sqrt{a+b\sin^4(c+dx)}} + \frac{\sqrt{a+b\ldots}}{\ldots}$$

Mathematica [C] time = 11.30, size = 378, normalized size = 0.79

$$-\frac{\cot(c+dx)\sqrt{8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b}}{2\sqrt{2}ad}-\frac{\cos^4(c+dx)\left(\frac{\left(\sqrt{a}\sqrt{b}+ia\right)\sec^2(c+dx)\sqrt{1+\left(1-\frac{i\sqrt{b}}{\sqrt{a}}\right)\tan^2(c+dx)}\cdots}{\cdots}\right)}{\cdots}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^2/Sqrt[a + b*Sin[c + d*x]^4],x]
```

```
[Out] -1/2*(Sqrt[8*a + 3*b - 4*b*Cos[2*(c + d*x)] + b*Cos[4*(c + d*x)]]*Cot[c + d
*x])/(Sqrt[2]*a*d) - (Cos[c + d*x]^4*(a*Sec[c + d*x]^4*Tan[c + d*x] + b*Tan
[c + d*x]^5 + ((I*a + Sqrt[a]*Sqrt[b])*(EllipticE[I*ArcSinh[Sqrt[1 - (I*Sqr
t[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a] + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])]
- EllipticF[I*ArcSinh[Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]*Tan[c + d*x]], (Sqrt[a]
 + I*Sqrt[b])/(Sqrt[a] - I*Sqrt[b])])*Sec[c + d*x]^2*Sqrt[1 + (1 - (I*Sqrt[
b])/Sqrt[a])*Tan[c + d*x]^2]*Sqrt[1 + (1 + (I*Sqrt[b])/Sqrt[a])*Tan[c + d*x
]^2])/Sqrt[1 - (I*Sqrt[b])/Sqrt[a]]))/(a*d*Sqrt[Cos[c + d*x]^4*(a + 2*a*Tan
[c + d*x]^2 + (a + b)*Tan[c + d*x]^4)])
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\cot(dx+c)^2}{\sqrt{b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(cot(d*x + c)^2/sqrt(b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)
, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

maple [F] time = 1.71, size = 0, normalized size = 0.00

$$\int\frac{\cot^2(dx+c)}{\sqrt{a+b\left(\sin^4(dx+c)\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

```
[Out] int(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\cot(dx+c)^2}{\sqrt{b\sin(dx+c)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2/(a+b*sin(d*x+c)^4)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(cot(d*x + c)^2/sqrt(b*sin(d*x + c)^4 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cot(c+d\,x)^2}{\sqrt{b\sin(c+d\,x)^4+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2),x)
```

```
[Out] int(cot(c + d*x)^2/(a + b*sin(c + d*x)^4)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot^2(c+dx)}{\sqrt{a+b\sin^4(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2/(a+b*sin(d*x+c)**4)**(1/2),x)
```

```
[Out] Integral(cot(c + d*x)**2/sqrt(a + b*sin(c + d*x)**4), x)
```

3.564 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^m(c+dx)\left(a+b\sin^4(c+dx)\right)^p, x\right)$$

[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]

[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx = \int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Mathematica [A] time = 5.75, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m,x]

[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^m, x]

fricas [A] time = 2.70, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4 - 2\,b\cos(dx+c)^2 + a + b\right)^p \tan(dx+c)^m, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="fricas")

[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^m, x)

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4 + a\right)^p \tan(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="giac")

[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)

maple [A] time = 5.07, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}\left(\tan^{m}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c+d\,x\right)^{m}\,\left(b\sin\left(c+d\,x\right)^{4}+a\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**m,x)
[Out] Timed out
```

3.565 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=279

$$-\frac{(a+2bp+b)\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}-\frac{(a\cdots}{}$$

```
[Out] -1/4*(2*b*p+a+b)*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*si
n(d*x+c)^4)^(1+p)/(a+b)^2/d/(1+p)+1/2*sec(d*x+c)^2*(a+b*sin(d*x+c)^4)^(1+p)
/(a+b)/d-1/2*(2*b*p+a+b)*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c)^4
/a)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)+1/2*
b*(1+2*p)*hypergeom([1/2, -p],[3/2],-b*sin(d*x+c)^4/a)*sin(d*x+c)^2*(a+b*si
n(d*x+c)^4)^p/(a+b)/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A] time = 0.29, antiderivative size = 279, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.435, Rules used = {3229, 835, 844, 246, 245, 757, 430, 429, 444, 68}

$$-\frac{(a+2bp+b)\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d(a+b)}+\frac{b(2\cdots}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] -((a + b + 2*b*p)*Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)
/(a + b)]*(a + b*Sin[c + d*x]^4)^(1 + p))/(4*(a + b)^2*d*(1 + p)) + (Sec[c
+ d*x]^2*(a + b*Sin[c + d*x]^4)^(1 + p))/(2*(a + b)*d) - ((a + b + 2*b*p)*A
ppellF1[1/2, 1, -p, 3/2, Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d
*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
+ (b*(1 + 2*p)*Hypergeometric2F1[1/2, -p, 3/2, -((b*Sin[c + d*x]^4)/a)]*Sin
[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p)/(2*(a + b)*d*(1 + (b*Sin[c + d*x]^4)/
a)^p)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] && !IntegerQ[m] && IntegerQ[n]
```

Rule 245

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[a^p*x*Hypergeometric2F
1[-p, 1/n, 1/n + 1, -((b*x^n)/a)], x] /; FreeQ[{a, b, n, p}, x] && !IGtQ[p
, 0] && !IntegerQ[1/n] && !ILtQ[Simplify[1/n + p], 0] && (IntegerQ[p] ||
GtQ[a, 0])
```

Rule 246

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x
^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(1 + (b*x^n)/a)^p, x], x]
 /; FreeQ[{a, b, n, p}, x] && !IGtQ[p, 0] && !IntegerQ[1/n] && !ILtQ[Sim
plify[1/n + p], 0] && !(IntegerQ[p] || GtQ[a, 0])
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
```

```
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 835

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Simp[((e*f - d*g)*(d + e*x)^(m + 1)*(a + c*x^2)^(p + 1))/
((m + 1)*(c*d^2 + a*e^2)), x] + Dist[1/((m + 1)*(c*d^2 + a*e^2)), Int[(d +
e*x)^(m + 1)*(a + c*x^2)^p*Simp[(c*d*f + a*e*g)*(m + 1) - c*(e*f - d*g)*(m
+ 2*p + 3)*x, x], x], x] /; FreeQ[{a, c, d, e, f, g, p}, x] && NeQ[c*d^2 +
a*e^2, 0] && LtQ[m, -1] && (IntegerQ[m] || IntegerQ[p] || IntegersQ[2*m, 2*
p])
```

Rule 844

```
Int[((d_.) + (e_.)*(x_))^(m_)*((f_.) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p
_.), x_Symbol] :> Dist[g/e, Int[(d + e*x)^(m + 1)*(a + c*x^2)^p, x], x] + D
ist[(e*f - d*g)/e, Int[(d + e*x)^m*(a + c*x^2)^p, x], x] /; FreeQ[{a, c, d,
 e, f, g, m, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !IGtQ[m, 0]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^3(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^p}{(1-x)^2}\,dx, x, \sin^2(c+dx)\right)}{2d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{\text{Subst}\left(\int \frac{(a+b(1+2p)x)\left(a+bx^2\right)^p}{1-x}\,dx,\right.}{2(a+b)d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{(b(1+2p))\,\text{Subst}\left(\int \left(a+bx^2\right)^p\right.}{2(a+b)d}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} - \frac{(a+b+2bp)\,\text{Subst}\left(\int \left(\frac{\left(a+bx^2\right)^p}{1-x^2}\right.\right.}{2(a}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^4(c}{a}\right.}{}$$

$$= \frac{\sec^2(c+dx)\left(a+b\sin^4(c+dx)\right)^{1+p}}{2(a+b)d} + \frac{b(1+2p)\,{}_2F_1\left(\frac{1}{2},-p;\frac{3}{2};-\frac{b\sin^4(c}{a}\right.}{}$$

$$-\frac{(a+b+2bp)\,{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+dx)\right)^{1+}}{4(a+b)^2d(1+p)}$$

Mathematica [B] time = 18.56, size = 922, normalized size = 3.30

$$-\frac{2\left(\sqrt{-ab}-b\right)\left(b+\sqrt{-ab}\right)(2p-1)F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+}{\sqrt{-ab}-b}\right.}{(a+b)^2dp\left(b(2p-1)F_1\left(-2p;-p,-p;1-2p;-\frac{(a+b)\sec^2(c+dx)}{\sqrt{-ab}-b},\frac{(a+b)\sec^2(c+dx)}{b+\sqrt{-ab}}\right)\cos^2(c+dx)+\left(b+\sqrt{-ab}\right)pF_1\left(1-\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^3,x]
```

```
[Out] (-2*(-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*(-1 + 2*p)*AppellF1[-2*p, -p, -p
, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c
+ d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2*(a + b + (a - Sqrt[-(a*b)])*Co
t[c + d*x]^2)*(a + b + (a + Sqrt[-(a*b)])*Cot[c + d*x]^2)*Sin[c + d*x]^4*(a
 + b*Sin[c + d*x]^4)^p)/((a + b)^2*d*p*((b + Sqrt[-(a*b)])*p*AppellF1[1 - 2
*p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((
a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] - (-b + Sqrt[-(a*b)])*p*AppellF1
[1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)]
)), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + b*(-1 + 2*p)*AppellF1[-2
*p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a +
 b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2)*(8*a + 3*b - 4*b*Co
s[2*(c + d*x)] + b*Cos[4*(c + d*x)])) - ((-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*
b)])*(-1 + p)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)
/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Sec[c +
 d*x]^2*(a + b*Sin[c + d*x]^4)^p*(-a + Sqrt[-(a*b)] - (a + b)*Tan[c + d*x]^
2)*(a + Sqrt[-(a*b)] + (a + b)*Tan[c + d*x]^2))/((a + b)^2*d*(-1 + 2*p)*(2*
b*(-1 + p)*AppellF1[1 - 2*p, -p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-
b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + p*((b +
Sqrt[-(a*b)])*AppellF1[2 - 2*p, 1 - p, -p, 3 - 2*p, -(((a + b)*Sec[c + d*x]
^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + (b
```

```
- Sqrt[-(a*b)])*AppellF1[2 - 2*p, -p, 1 - p, 3 - 2*p, -(((a + b)*Sec[c + d
*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])])*
Sec[c + d*x]^2)*(a + 2*a*Tan[c + d*x]^2 + (a + b)*Tan[c + d*x]^4))
```

fricas [F] time = 0.57, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

maple [F] time = 2.62, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\left(\tan^3\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^3,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int\tan\left(c+d\,x\right)^3\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**3,x)
```

```
[Out] Timed out
```

3.566 $\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx$

Optimal. Leaf size=141

$$\frac{\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d}+\frac{\left(a+b\sin^4(c+dx\right.}{}$$

```
[Out] 1/4*hypergeom([1, 1+p],[2+p],(a+b*sin(d*x+c)^4)/(a+b))*(a+b*sin(d*x+c)^4)^(
1+p)/(a+b)/d/(1+p)+1/2*AppellF1(1/2,1,-p,3/2,sin(d*x+c)^4,-b*sin(d*x+c)^4/a
)*sin(d*x+c)^2*(a+b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A] time = 0.12, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3229, 757, 430, 429, 444, 68}

$$\frac{\sin^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}F_1\left(\frac{1}{2};1,-p;\frac{3}{2};\sin^4(c+dx),-\frac{b\sin^4(c+dx)}{a}\right)}{2d}+\frac{\left(a+b\sin^4(c+dx\right.}{}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, (a + b*Sin[c + d*x]^4)/(a + b)]*(a + b*
Sin[c + d*x]^4)^(1 + p))/(4*(a + b)*d*(1 + p)) + (AppellF1[1/2, 1, -p, 3/2,
 Sin[c + d*x]^4, -((b*Sin[c + d*x]^4)/a)]*Sin[c + d*x]^2*(a + b*Sin[c + d*x
]^4)^p)/(2*d*(1 + (b*Sin[c + d*x]^4)/a)^p)
```

Rule 68

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((
b*c - a*d)^n*(a + b*x)^(m + 1)*Hypergeometric2F1[-n, m + 1, m + 2, -((d*(a
+ b*x))/(b*c - a*d))])/(b^(n + 1)*(m + 1)), x] /; FreeQ[{a, b, c, d, m}, x]
 && NeQ[b*c - a*d, 0] &&  !IntegerQ[m] && IntegerQ[n]
```

Rule 429

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Simp[a^p*c^q*x*AppellF1[1/n, -p, -q, 1 + 1/n, -((b*x^n)/a), -((d*x^n)/c)
], x] /; FreeQ[{a, b, c, d, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1]
 && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 430

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_), x_Symbol]
:> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p],
 Int[(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /; FreeQ[{a, b, c, d, n, p, q}
, x] && NeQ[b*c - a*d, 0] && NeQ[n, -1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 444

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n_))^(q_.
), x_Symbol] :> Dist[1/n, Subst[Int[(a + b*x)^p*(c + d*x)^q, x], x, x^n], x
] /; FreeQ[{a, b, c, d, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && EqQ[m - n +
 1, 0]
```

Rule 757

```
Int[((d_) + (e_.)*(x_))^(m_)*((a_) + (c_.)*(x_)^2)^(p_), x_Symbol] :> Int[E
xpandIntegrand[(a + c*x^2)^p, (d/(d^2 - e^2*x^2) - (e*x)/(d^2 - e^2*x^2))^(
```

```
-m), x], x] /; FreeQ[{a, c, d, e, p}, x] && NeQ[c*d^2 + a*e^2, 0] &&  !Inte
gerQ[p] && ILtQ[m, 0]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan(c+dx)\,dx = \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{1-x}\,dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{\left(a+bx^2\right)^p}{1-x^2}-\frac{x\left(a+bx^2\right)^p}{-1+x^2}\right)dx, x, \sin^2(c+dx)\right)}{2d}$$
$$= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{1-x^2}\,dx, x, \sin^2(c+dx)\right)}{2d} - \frac{\text{Subst}\left(\int \frac{x\left(a+bx^2\right)^p}{-1+x^2}\,dx, x, \sin^2\right.}{2d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{-1+x}\,dx, x, \sin^4(c+dx)\right)}{4d} + \frac{\left(\left(a+b\sin^4(c+dx)\right)^p\left(1+\right.\right.}{}$$
$$= \frac{{}_2F_1\left(1,1+p;2+p;\frac{a+b\sin^4(c+dx)}{a+b}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4(a+b)d(1+p)} + \frac{F_1\left(\frac{1}{2};1,\right.}{}$$

Mathematica [B] time = 9.90, size = 463, normalized size = 3.28

$$\frac{2(2p-1)\left(\sqrt{-ab}-b\right)\left(\sqrt{-ab}+b\right)\sin^4(c+dx)\cos^2(c+dx)\left(\left(a-\sqrt{-}\right.\right.}{dp(a+b)^2(8a-4b\cos(2(c+dx))+b\cos(4(c+dx))+3b)\left(p\left(\sqrt{-ab}+b\right)F_1\left(1-2p;1-p,-p;2-2p;-\frac{(a+b)\,\mathrm{se}}{\sqrt{-}}\right.\right.}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x],x]
```

```
[Out] (2*(-b + Sqrt[-(a*b)])*(b + Sqrt[-(a*b)])*(-1 + 2*p)*AppellF1[-2*p, -p, -p,
 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a + b)*Sec[c +
 d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2*(a + b + (a - Sqrt[-(a*b)])*Cot
[c + d*x]^2)*(a + b + (a + Sqrt[-(a*b)])*Cot[c + d*x]^2)*Sin[c + d*x]^4*(a
+ b*Sin[c + d*x]^4)^p)/((a + b)^2*d*p*((b + Sqrt[-(a*b)])*p*AppellF1[1 - 2*
p, 1 - p, -p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a
 + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] - (-b + Sqrt[-(a*b)])*p*AppellF1[
1 - 2*p, -p, 1 - p, 2 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])
), ((a + b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])] + b*(-1 + 2*p)*AppellF1[-2*
p, -p, -p, 1 - 2*p, -(((a + b)*Sec[c + d*x]^2)/(-b + Sqrt[-(a*b)])), ((a +
b)*Sec[c + d*x]^2)/(b + Sqrt[-(a*b)])]*Cos[c + d*x]^2)*(8*a + 3*b - 4*b*Cos
[2*(c + d*x)] + b*Cos[4*(c + d*x)]))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b\right)^p\tan(dx+c), x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(dx+c\right)^{4}+a\right)^{p} \tan \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

maple [F] time = 4.72, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p} \tan \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(dx+c\right)^{4}+a\right)^{p} \tan \left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \tan \left(c+d\,x\right)\,{\left(b\,{\sin \left(c+d\,x\right)}^4+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

3.567 $\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=54

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

```
[Out] -1/4*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(1+p)/
a/d/(1+p)
```

Rubi [A] time = 0.06, antiderivative size = 54, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3229, 266, 65}

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] -(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
 + d*x]^4)^(1 + p))/(4*a*d*(1 + p))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx &= \frac{\text{Subst}\left(\int \frac{\left(a+bx^2\right)^p}{x}\,dx,x,\sin^2(c+dx)\right)}{2d}\\ &= \frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sin^4(c+dx)\right)}{4d}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}\end{aligned}$$

Mathematica [A] time = 0.09, size = 54, normalized size = 1.00

$$-\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] -1/4*(Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*S
in[c + d*x]^4)^(1 + p))/(a*d*(1 + p))
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

maple [F] time = 4.47, size = 0, normalized size = 0.00

$$\int\cot\left(dx+c\right)\left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cot\left(c+d\,x\right)\,{\left(b\,{\sin\left(c+d\,x\right)}^4+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^4)^p, x)
```

sympy **[F(-1)]** time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

3.568 $\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=127

$$\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}}{2d}$$

```
[Out] 1/4*hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^4/a)*(a+b*sin(d*x+c)^4)^(1+p)/a
/d/(1+p)-1/2*csc(d*x+c)^2*hypergeom([-1/2, -p],[1/2],-b*sin(d*x+c)^4/a)*(a+
b*sin(d*x+c)^4)^p/d/((1+b*sin(d*x+c)^4/a)^p)
```

Rubi [A] time = 0.10, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {3229, 764, 365, 364, 266, 65}

$$\frac{\left(a+b\sin^4(c+dx)\right)^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{4ad(p+1)}-\frac{\csc^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^4)/a]*(a + b*Sin[c
+ d*x]^4)^(1 + p))/(4*a*d*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-1/2
, -p, 1/2, -((b*Sin[c + d*x]^4)/a)]*(a + b*Sin[c + d*x]^4)^p)/(2*d*(1 + (b*
Sin[c + d*x]^4)/a)^p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] && !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] && !IGtQ[p, 0]
 && !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 764

```
Int[(x_)^(m_.)*((f_) + (g_.)*(x_))*((a_) + (c_.)*(x_)^2)^(p_.), x_Symbol] :
> Dist[f, Int[x^m*(a + c*x^2)^p, x], x] + Dist[g, Int[x^(m + 1)*(a + c*x^2)
```

```
^p, x], x] /; FreeQ[{a, c, f, g, p}, x] && IntegerQ[m] &&  !IntegerQ[2*p]
```

Rule 3229

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^(n_))^(p_.)*tan[(e_.) + (f_.)*(x_
)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x]^2, x]}, Dist[ff^
((m + 1)/2)/(2*f), Subst[Int[(x^((m - 1)/2)*(a + b*ff^(n/2)*x^(n/2))^p)/(1
- ff*x)^((m + 1)/2), x], x, Sin[e + f*x]^2/ff], x]] /; FreeQ[{a, b, e, f, p
}, x] && IntegerQ[(m - 1)/2] && IntegerQ[n/2]
```

Rubi steps

$$\int \cot^3(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\frac{\text{Subst}\left(\int\frac{(1-x)\left(a+bx^2\right)^p}{x^2}\,dx,x,\sin^2(c+dx)\right)}{2d}$$

$$=\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{x^2}\,dx,x,\sin^2(c+dx)\right)}{2d}-\frac{\text{Subst}\left(\int\frac{\left(a+bx^2\right)^p}{x}\,dx,x,\sin^2\right.}{2d}$$

$$=-\frac{\text{Subst}\left(\int\frac{(a+bx)^p}{x}\,dx,x,\sin^4(c+dx)\right)}{4d}+\frac{\left(\left(a+b\sin^4(c+dx)\right)^p\left(1+\right.\right.}{}$$

$$=\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^4(c+dx)}{a}\right)\left(a+b\sin^4(c+dx)\right)^{1+p}}{4ad(1+p)}-\frac{\csc^2(c}{}$$

Mathematica [A] time = 0.62, size = 119, normalized size = 0.94

$$\frac{\left(a+b\sin^4(c+dx)\right)^p\left(\frac{\left(a+b\sin^4(c+dx)\right){}_2F_1\left(1,p+1;p+2;\frac{b\sin^4(c+dx)}{a}+1\right)}{a(p+1)}-2\csc^2(c+dx)\left(\frac{b\sin^4(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{1}{2},-p;\frac{1}{2};\right.\right.}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^4)^p*((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c
 + d*x]^4)/a]*(a + b*Sin[c + d*x]^4))/(a*(1 + p)) - (2*Csc[c + d*x]^2*Hyper
geometric2F1[-1/2, -p, 1/2, -((b*Sin[c + d*x]^4)/a)])/(1 + (b*Sin[c + d*x]^
4)/a)^p))/(4*d)
```

fricas [F] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^3,
x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

maple [F] time = 2.49, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right) \left(a+b\left(\sin^{4}\left(dx+c\right)\right)\right)^{p}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{4}+a\right)^{p}\cot\left(dx+c\right)^{3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\mathrm{cot}\left(c+d\,x\right)}^3\,{\left(b\,{\sin\left(c+d\,x\right)}^4+a\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

3.569 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx=\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Mathematica [A] time = 41.65, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4,x]
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^4, x]
```

fricas [A] time = 0.58, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\tan\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^4,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\tan\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

maple [A] time = 1.66, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4\left(dx+c\right)\right)\right)^p \left(\tan^4\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p \tan\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c+d\,x\right)^4\,\left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.570 $\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx=\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Mathematica [A] time = 1.83, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(c+dx)\right)^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2,x]
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p*Tan[c + d*x]^2, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b\right)^p\tan(dx+c)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*tan(d*x + c)^2,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p \tan(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

maple [A] time = 2.30, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4(dx+c)\right)\right)^p \left(\tan^2(dx+c)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
[Out] int((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p \tan(dx+c)^2\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p*tan(d*x+c)^2,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*tan(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan(c+d\,x)^2 \left(b\sin(c+d\,x)^4+a\right)^p dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p*tan(d*x+c)**2,x)
[Out] Timed out
```

3.571 $\int \left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=17

$$\text{Int}\left(\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx=\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A] time = 0.20, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[(a + b*Sin[c + d*x]^4)^p, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

maple [A] time = 2.50, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^4(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^4)^p,x)
```

```
[Out] int((a+b*sin(d*x+c)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(b\sin\left(c+d\,x\right)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^4)^p,x)
```

```
[Out] int((a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**4)**p,x)
```

```
[Out] Timed out
```

3.572 $\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A] time = 1.50, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^4)^p, x]
```

fricas [A] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos\left(dx+c\right)^4-2\,b\cos\left(dx+c\right)^2+a+b\right)^p\cot\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^2,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^4+a\right)^p\cot\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

maple [A] time = 2.31, size = 0, normalized size = 0.00

$$\int \left(\cot^2(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p\cot(dx+c)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^2\left(b\sin(c+d\,x)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**4)**p,x)
[Out] Timed out
```

3.573 $\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

Rubi [A] time = 0.04, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Mathematica [A] time = 35.15, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)\left(a+b\sin^4(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p,x]
```

```
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^4)^p, x]
```

fricas [A] time = 0.56, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\cos(dx+c)^4-2\,b\cos(dx+c)^2+a+b\right)^p\cot(dx+c)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="fricas")
```

```
[Out] integral((b*cos(d*x + c)^4 - 2*b*cos(d*x + c)^2 + a + b)^p*cot(d*x + c)^4,
x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

maple [A] time = 1.81, size = 0, normalized size = 0.00

$$\int \left(\cot^4(dx+c)\right)\left(a+b\left(\sin^4(dx+c)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^4+a\right)^p\cot(dx+c)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^4)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^4 + a)^p*cot(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot(c+d\,x)^4\left(b\sin(c+d\,x)^4+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p,x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^4)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**4)**p,x)
[Out] Timed out
```

3.574 $\int (a+b\sin^n(c+dx))^3 \tan^m(c+dx)\,dx$

Optimal. Leaf size=306

$$\frac{a^3\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2}\right.}{d(m+n+1)}$$

```
[Out] a^3*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/
(1+m)+3*a^2*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/
2*n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n
)+3*a*b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n],[3/
2+1/2*m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)+b^3*
(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+3/2*n],[3/2+1/2*
m+3/2*n],sin(d*x+c)^2)*sin(d*x+c)^(3*n)*tan(d*x+c)^(1+m)/d/(1+m+3*n)
```

Rubi [A] time = 0.43, antiderivative size = 306, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{3a^2b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}(m+n+1);\frac{1}{2}(m+n+3);\sin^2(c+dx)\right)}{d(m+n+1)}+\frac{a^3\tan\ldots}{\ldots}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

```
[Out] (a^3*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (3*a^2*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeomet
ric2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x
]^n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (3*a*b^2*(Cos[c + d*x]^2)^((1 +
 m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c
 + d*x]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n)) + (b^
3*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(1 + m)/2, (1 + m + 3*n)/2
, (3 + m + 3*n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^(3*n)*Tan[c + d*x]^(1 + m))
/(d*(1 + m + 3*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^3\tan^m(c+dx)\,dx = \int \left(a^3\tan^m(c+dx)+3a^2b\sin^n(c+dx)\tan^m(c+dx)+3ab^2\sin^{2n}(c+$$

$$= a^3\int \tan^m(c+dx)\,dx+\left(3a^2b\right)\int \sin^n(c+dx)\tan^m(c+dx)\,dx+\left(3ab\right.$$

$$= \frac{a^3\operatorname{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(3a^2b\cos^{1+m}(c+dx)\sin^{-1-m}(c\right.$$

$$= \frac{a^3\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{3a^2b\cos^2(c+d}{}$$

Mathematica [C] time = 19.60, size = 3544, normalized size = 11.58

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^3*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m +
n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d
*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan
[(c + d*x)/2]*Tan[c + d*x]^m*(a^3*Tan[c + d*x]^m + 3*a^2*b*Sin[c + d*x]^n*T
an[c + d*x]^m + 3*a*b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m + b^3*Sin[c + d*x
]^(3*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c
+ d*x]^2*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^
2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3
*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*
n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*Tan[c + d*x]^(-1
 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^3*Appell
F1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a^2*AppellF1[(1 + m + n
)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1
+ m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2
 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, T
```

```
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]
^n)/(1 + m + 3*n))))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(-1 + m)*((a^3*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a
^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((
3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3
*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x
)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n))))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x
)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Ta
n[c + d*x]^m + 2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*
Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((3*a^2*AppellF1[
(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1
[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m +
 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Si
n[c + d*x]^n)/(1 + m + 3*n))) + b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(
(3*a^2*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])/(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^
n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*AppellF1[(1 + m + 3*n)/2, m, 1
 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c +
 d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)))*Tan[(c + d*x)/2] + (a^3*(-(((
1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)) + (m*(1 + m
)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m)))/(1 + m) + b
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(b*n*Cos[c + d*x]*(Sec[(c + d*x)/2]^
2)^n*Sin[c + d*x]^(-1 + n)*((3*a*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 +
m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n) + (b*Appel
lF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[
(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 3*n)) + b*n
*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((3*a*AppellF1[1/2 + m/2 + n, m, 1 +
 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + 2*n
) + (b*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m +
 3*n))*Tan[(c + d*x)/2] + (3*a^2*(-(((1 + n)*(1 + m + n)*AppellF1[1 + (1 +
m + n)/2, m, 2 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)) + (m*(1 + m + n)*App
ellF1[1 + (1 + m + n)/2, 1 + m, 1 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)))/
(1 + m + n) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((b*n*AppellF1[(1 + m
 + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n))/(1 + m + 3
*n) + (b*n*AppellF1[(1 + m + 3*n)/2, m, 1 + 3*n, (3 + m + 3*n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*Tan[(
c + d*x)/2])/(1 + m + 3*n) + (3*a*(-(((1/2 + m/2 + n)*(1 + 2*n)*AppellF1[3/
2 + m/2 + n, m, 2 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2 + n)) + (m*(1/2 + m/2
 + n)*AppellF1[3/2 + m/2 + n, 1 + m, 1 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2
+ n)))/(1 + m + 2*n) + (b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(-(((1 + 3*
n)*(1 + m + 3*n)*AppellF1[1 + (1 + m + 3*n)/2, m, 2 + 3*n, 1 + (3 + m + 3*n
)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d
*x)/2])/(3 + m + 3*n)) + (m*(1 + m + 3*n)*AppellF1[1 + (1 + m + 3*n)/2, 1 +
 m, 1 + 3*n, 1 + (3 + m + 3*n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + 3*n)))/(1 + m + 3*n))))*Tan[c
```

```
+ d*x]^m))
```

fricas [F] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^3\sin\left(dx+c\right)^{3n}+3\,ab^2\sin\left(dx+c\right)^{2n}+3\,a^2b\sin\left(dx+c\right)^n+a^3\right)\tan\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b^3*sin(d*x + c)^(3*n) + 3*a*b^2*sin(d*x + c)^(2*n) + 3*a^2*b*sin
(d*x + c)^n + a^3)*tan(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^3\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^3\left(\tan^m\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^3\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^3*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^3*tan(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int{\tan\left(c+d\,x\right)}^m\,{\left(a+b\,{\sin\left(c+d\,x\right)}^n\right)}^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3,x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^3, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**3*tan(d*x+c)**m,x)
[Out] Timed out
```

3.575 $\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx$

Optimal. Leaf size=215

$$\frac{a^2\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+}{2}\right.}{d(m+n+1)}$$

```
[Out] a^2*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/
(1+m)+2*a*b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/2*
n],[3/2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n)+
b^2*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+n],[3/2+1/2*
m+n],sin(d*x+c)^2)*sin(d*x+c)^(2*n)*tan(d*x+c)^(1+m)/d/(1+m+2*n)
```

Rubi [A] time = 0.30, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.217, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{a^2\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{2ab\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+}{2}\right.}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (a^2*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*
x]^(1 + m))/(d*(1 + m)) + (2*a*b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometri
c2F1[(1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^
n*Tan[c + d*x]^(1 + m))/(d*(1 + m + n)) + (b^2*(Cos[c + d*x]^2)^((1 + m)/2)
*Hypergeometric2F1[(1 + m)/2, (1 + m + 2*n)/2, (3 + m + 2*n)/2, Sin[c + d*x
]^2]*Sin[c + d*x]^(2*n)*Tan[c + d*x]^(1 + m))/(d*(1 + m + 2*n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
```

```
IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] && !
IntegerQ[n]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^2\tan^m(c+dx)\,dx = \int \left(a^2\tan^m(c+dx)+2ab\sin^n(c+dx)\tan^m(c+dx)+b^2\sin^{2n}(c+dx\right.$$
$$= a^2\int \tan^m(c+dx)\,dx+(2ab)\int \sin^n(c+dx)\tan^m(c+dx)\,dx+b^2\int \text{s}$$
$$= \frac{a^2\,\text{Subst}\left(\int \frac{x^m}{1+x^2}\,dx,x,\tan(c+dx)\right)}{d}+\left(2ab\cos^{1+m}(c+dx)\sin^{-1-m}(c\right.$$
$$= \frac{a^2\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)}+\frac{2ab\cos^2(c+dx}{}$$

Mathematica [C] time = 15.37, size = 2368, normalized size = 11.01

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^2*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*AppellF1[(1 + m)/2, m, 1, (3 +
 m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)
/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2
 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2
]*Tan[c + d*x]^m*(a^2*Tan[c + d*x]^m + 2*a*b*Sin[c + d*x]^n*Tan[c + d*x]^m
+ b^2*Sin[c + d*x]^(2*n)*Tan[c + d*x]^m))/(d*(2*m*(Cos[c + d*x]*Sec[(c + d*
x)/2]^2)^m*Sec[c + d*x]^2*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[
c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m
, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c
+ d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*Tan[c + d*x
]^(-1 + m) + Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*((a^2*A
ppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m
+ n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/
(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1
 + m + 2*n)))*Tan[c + d*x]^m + 2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 +
m)*((a^2*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*Appell
F1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2])/(1 + m + n) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 +
n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c +
d*x]^n)/(1 + m + 2*n)))*Tan[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]
) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m + 2*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*n*Cos[c + d*x]*(Sec[(
```

```
c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n)*((2*a*AppellF1[(1 + m + n)/2, m, 1 +
 n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n) +
(b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n)) +
 b*n*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((2*a*AppellF1[(1 + m + n)/2, m,
 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])/(1 + m + n
) + (b*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)/(1 + m + 2*n
))*Tan[(c + d*x)/2] + (a^2*(-(((1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3
 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
 + d*x)/2])/(3 + m)) + (m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3
+ m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/(3 + m)))/(1 + m) + b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*((b*
n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n
))/(1 + m + 2*n) + (b*n*AppellF1[1/2 + m/2 + n, m, 1 + 2*n, 3/2 + m/2 + n,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*Tan[(c + d*x)/2])/(1 + m + 2*n) + (b*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x
]^n*(-(((1/2 + m/2 + n)*(1 + 2*n)*AppellF1[3/2 + m/2 + n, m, 2 + 2*n, 5/2 +
 m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2])/(3/2 + m/2 + n)) + (m*(1/2 + m/2 + n)*AppellF1[3/2 + m/2 + n,
1 + m, 1 + 2*n, 5/2 + m/2 + n, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec
[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3/2 + m/2 + n)))/(1 + m + 2*n) + (2*a*(-
(((1 + n)*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, m, 2 + n, 1 + (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/(3 + m + n)) + (m*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, 1 + m, 1 +
 n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*
x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n)))/(1 + m + n)))*Tan[c + d*x]^m))
```

fricas [F] time = 0.53, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b^2\sin\left(dx+c\right)^{2\,n}+2\,ab\sin\left(dx+c\right)^n+a^2\right)\tan\left(dx+c\right)^m,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="fricas")
```

```
[Out] integral((b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^2)*tan(d*x + c)
^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^2\tan\left(dx+c\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^2\left(\tan^m\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{2}\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^2*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^2*tan(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.00

$$\int \tan\left(c+d\,x\right)^{m}\,{\left(a+b\,\sin\left(c+d\,x\right)^{n}\right)}^{2} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2,x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^2, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a+b\sin^{n}\left(c+dx\right)\right)^{2}\tan^{m}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**2*tan(d*x+c)**m,x)
[Out] Integral((a + b*sin(c + d*x)**n)**2*tan(c + d*x)**m, x)
```

3.576 $\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx$

Optimal. Leaf size=124

$$\frac{a\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}\right.}{d(m+n+1)}$$

```
[Out] a*hypergeom([1, 1/2+1/2*m],[3/2+1/2*m],-tan(d*x+c)^2)*tan(d*x+c)^(1+m)/d/(1
+m)+b*(cos(d*x+c)^2)^(1/2+1/2*m)*hypergeom([1/2+1/2*m, 1/2+1/2*m+1/2*n],[3/
2+1/2*m+1/2*n],sin(d*x+c)^2)*sin(d*x+c)^n*tan(d*x+c)^(1+m)/d/(1+m+n)
```

Rubi [A] time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.238, Rules used = {3234, 3476, 364, 2602, 2577}

$$\frac{a\tan^{m+1}(c+dx)\,{}_2F_1\left(1,\frac{m+1}{2};\frac{m+3}{2};-\tan^2(c+dx)\right)}{d(m+1)}+\frac{b\cos^2(c+dx)^{\frac{m+1}{2}}\tan^{m+1}(c+dx)\sin^n(c+dx)\,{}_2F_1\left(\frac{m+1}{2},\frac{1}{2}\right.}{d(m+n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (a*Hypergeometric2F1[1, (1 + m)/2, (3 + m)/2, -Tan[c + d*x]^2]*Tan[c + d*x]
^(1 + m))/(d*(1 + m)) + (b*(Cos[c + d*x]^2)^((1 + m)/2)*Hypergeometric2F1[(
1 + m)/2, (1 + m + n)/2, (3 + m + n)/2, Sin[c + d*x]^2]*Sin[c + d*x]^n*Tan[
c + d*x]^(1 + m))/(d*(1 + m + n))
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 2577

```
Int[(cos[(e_.) + (f_.)*(x_)]*(b_.))^(n_)*((a_.)*sin[(e_.) + (f_.)*(x_)])^(m
_), x_Symbol] :> Simp[(b^(2*IntPart[(n - 1)/2] + 1)*(b*Cos[e + f*x])^(2*Fra
cPart[(n - 1)/2])*(a*Sin[e + f*x])^(m + 1)*Hypergeometric2F1[(1 + m)/2, (1
- n)/2, (3 + m)/2, Sin[e + f*x]^2])/(a*f*(m + 1)*(Cos[e + f*x]^2)^FracPart[
(n - 1)/2]), x] /; FreeQ[{a, b, e, f, m, n}, x]
```

Rule 2602

```
Int[((a_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((b_.)*tan[(e_.) + (f_.)*(x_)])^(
n_), x_Symbol] :> Dist[(a*Cos[e + f*x]^(n + 1)*(b*Tan[e + f*x])^(n + 1))/(b
*(a*Sin[e + f*x])^(n + 1)), Int[(a*Sin[e + f*x])^(m + n)/Cos[e + f*x]^n, x]
, x] /; FreeQ[{a, b, e, f, m, n}, x] &&  !IntegerQ[n]
```

Rule 3234

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*((d_.)*tan[(e
_.) + (f_.)*(x_)])^(m_.), x_Symbol] :> Int[ExpandTrig[(d*tan[e + f*x])^m*(a
 + b*(c*sin[e + f*x])^n)^p, x], x] /; FreeQ[{a, b, c, d, e, f, m, n}, x] &&
 IGtQ[p, 0]
```

Rule 3476

```
Int[((b_.)*tan[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Dist[b/d, Subst[Int[
x^n/(b^2 + x^2), x], x, b*Tan[c + d*x]], x] /; FreeQ[{b, c, d, n}, x] &&  !
```

IntegerQ[n]

Rubi steps

$$\int (a+b\sin^n(c+dx))\tan^m(c+dx)\,dx = \int \left(a\tan^m(c+dx)+b\sin^n(c+dx)\tan^m(c+dx)\right)\,dx$$
$$= a\int \tan^m(c+dx)\,dx + b\int \sin^n(c+dx)\tan^m(c+dx)\,dx$$
$$= \frac{a\,\mathrm{Subst}\left(\int \frac{x^m}{1+x^2}\,dx, x, \tan(c+dx)\right)}{d} + \left(b\cos^{1+m}(c+dx)\sin^{-1-m}(c+dx\right.$$
$$= \frac{a\,{}_2F_1\left(1,\frac{1+m}{2};\frac{3+m}{2};-\tan^2(c+dx)\right)\tan^{1+m}(c+dx)}{d(1+m)} + \frac{b\cos^2(c+dx)^{\frac{1+m}{2}}}{}$$

Mathematica [C] time = 13.50, size = 1395, normalized size = 11.25

result too large to display

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)*Tan[c + d*x]^m,x]
```

```
[Out] (2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*AppellF1[(1 + m)/2, m
, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*Appell
F1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[(c + d*x)/2]*Tan[c + d*
x]^m*(a*Tan[c + d*x]^m + b*Sin[c + d*x]^n*Tan[c + d*x]^m))/(d*(1 + m)*(1 +
m + n)*((2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*Sec[c + d*x]^2*(a*(1 + m +
 n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Ta
n[(c + d*x)/2]*Tan[c + d*x]^(-1 + m))/((1 + m)*(1 + m + n)) + (Sec[(c + d*x
)/2]^2*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^m*(a*(1 + m + n)*AppellF1[(1 + m)/
2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + b*(1 + m)*Ap
pellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c
+ d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*Tan[c + d*x]^m)/((1 + m
)*(1 + m + n)) + (2*m*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(-1 + m)*(a*(1 + m
+ n)*AppellF1[(1 + m)/2, m, 1, (3 + m)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2] + b*(1 + m)*AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n)*T
an[(c + d*x)/2]*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])*Tan[c + d*x]^m)/((1 + m)*(1 + m + n)) + (2*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^m*Tan[(c + d*x)/2]*(b*(1 + m)*n*AppellF1[(1
+ m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Cos[c + d*x]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^(-1 + n) + b*(1 + m)*n*
AppellF1[(1 + m + n)/2, m, 1 + n, (3 + m + n)/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*Tan[(c + d*x)/2] + a*(
1 + m + n)*(-(((1 + m)*AppellF1[1 + (1 + m)/2, m, 2, 1 + (3 + m)/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 +
 m)) + (m*(1 + m)*AppellF1[1 + (1 + m)/2, 1 + m, 1, 1 + (3 + m)/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 +
m)) + b*(1 + m)*(Sec[(c + d*x)/2]^2)^n*Sin[c + d*x]^n*(-(((1 + n)*(1 + m +
n)*AppellF1[1 + (1 + m + n)/2, m, 2 + n, 1 + (3 + m + n)/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/(3 + m + n))
 + (m*(1 + m + n)*AppellF1[1 + (1 + m + n)/2, 1 + m, 1 + n, 1 + (3 + m + n)
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2])/(3 + m + n)))*Tan[c + d*x]^m)/((1 + m)*(1 + m + n))))
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int\left(a+b\left(\sin^{n}\left(dx+c\right)\right)\right)\left(\tan^{m}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
[Out] int((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^{n}+a\right)\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)*tan(d*x+c)^m,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)*tan(d*x + c)^m, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\tan\left(c+d\,x\right)^{m}\,\left(a+b\sin\left(c+d\,x\right)^{n}\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n),x)
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a+b\sin^{n}\left(c+dx\right)\right)\tan^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)*tan(d*x+c)**m,x)
[Out] Integral((a + b*sin(c + d*x)**n)*tan(c + d*x)**m, x)
```

3.577 $\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)},x\right)$$

```
[Out] Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx = \int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Mathematica [A] time = 2.45, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{a+b\sin^n(c+dx)}\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n),x]
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n), x]
```

fricas [A] time = 0.46, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\frac{\tan\left(dx+c\right)^m}{b\sin\left(dx+c\right)^n+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="fricas")
[Out] integral(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^m}{b\sin\left(dx+c\right)^n+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="giac")
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

maple [A] time = 1.57, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(dx+c\right)}{a+b\left(\sin^{n}\left(dx+c\right)\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan\left(dx+c\right)^{m}}{b\sin\left(dx+c\right)^{n}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n),x, algorithm="maxima")
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan\left(c+d\,x\right)^{m}}{a+b\sin\left(c+d\,x\right)^{n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n),x)
[Out] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n), x)
```

sympy [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}\left(c+dx\right)}{a+b\sin^{n}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n),x)
[Out] Integral(tan(c + d*x)**m/(a + b*sin(c + d*x)**n), x)
```

3.578 $\int \frac{\tan^m(c+dx)}{\left(a+b\sin^n(c+dx)\right)^2} dx$

Optimal. Leaf size=26

$$\text{Int}\left(\frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2}, x\right)$$

```
[Out] Unintegrable(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

Rubi [A] time = 0.06, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Int[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
[Out] Defer[Int][Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

Rubi steps

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx = \int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx$$

Mathematica [A] time = 24.11, size = 0, normalized size = 0.00

$$\int \frac{\tan^m(c+dx)}{(a+b\sin^n(c+dx))^2} dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2,x]
[Out] Integrate[Tan[c + d*x]^m/(a + b*Sin[c + d*x]^n)^2, x]
```

fricas [A] time = 0.45, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{\tan(dx+c)^m}{b^2\sin(dx+c)^{2n} + 2ab\sin(dx+c)^n + a^2}, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="fricas")
[Out] integral(tan(d*x + c)^m/(b^2*sin(d*x + c)^(2*n) + 2*a*b*sin(d*x + c)^n + a^
2), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^m}{\left(b\sin(dx+c)^n + a\right)^2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="giac")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

maple [F(-1)] time = 180.00, size = 0, normalized size = 0.00

$$\int \frac{\tan^{m}(dx+c)}{(a+b(\sin^{n}(dx+c)))^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

```
[Out] int(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\tan(dx+c)^{m}}{\left(b\sin(dx+c)^{n}+a\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)^m/(a+b*sin(d*x+c)^n)^2,x, algorithm="maxima")
```

```
[Out] integrate(tan(d*x + c)^m/(b*sin(d*x + c)^n + a)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \frac{\tan(c+d\,x)^{m}}{\left(a+b\sin(c+d\,x)^{n}\right)^{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2,x)
```

```
[Out] int(tan(c + d*x)^m/(a + b*sin(c + d*x)^n)^2, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(tan(d*x+c)**m/(a+b*sin(d*x+c)**n)**2,x)
```

```
[Out] Timed out
```

3.579 $\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx$

Optimal. Leaf size=47

$$\frac{2\sqrt{a+b\sin^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))*a^(1/2)/n+2*(a+b*sin(x)^n)^(1/2)/n
```

Rubi [A] time = 0.08, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.333, Rules used = {3230, 266, 50, 63, 208}

$$\frac{2\sqrt{a+b\sin^n(x)}}{n}-\frac{2\sqrt{a}\,\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/n + (2*Sqrt[a + b*Sin[x]
^n])/n
```

Rule 50

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
((a + b*x)^(m + 1)*(c + d*x)^n)/(b*(m + n + 1)), x] + Dist[(n*(b*c - a*d))/
(b*(m + n + 1)), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \cot(x)\sqrt{a+b\sin^n(x)}\,dx = \text{Subst}\left(\int \frac{\sqrt{a+bx^n}}{x}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{\sqrt{a+bx}}{x}\,dx, x, \sin^n(x)\right)}{n}$$
$$= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{a\,\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}$$
$$= \frac{2\sqrt{a+b\sin^n(x)}}{n} + \frac{(2a)\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}$$
$$= -\frac{2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n} + \frac{2\sqrt{a+b\sin^n(x)}}{n}$$

Mathematica [A] time = 0.03, size = 45, normalized size = 0.96

$$\frac{2\sqrt{a+b\sin^n(x)} - 2\sqrt{a}\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]*Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*Sqrt[a]*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]] + 2*Sqrt[a + b*Sin[x]^n])
/n
```

fricas [A] time = 0.46, size = 97, normalized size = 2.06

$$\left[\frac{\sqrt{a}\log\left(\frac{b\sin(x)^n - 2\sqrt{b\sin(x)^n+a}\sqrt{a}+2a}{\sin(x)^n}\right) + 2\sqrt{b\sin(x)^n+a}}{n}, \frac{2\left(\sqrt{-a}\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\sqrt{-a}}{a}\right) + \sqrt{b\sin(x)^n+a}\right)}{n}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
```

```
[Out] [(sqrt(a)*log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)
 + 2*sqrt(b*sin(x)^n + a))/n, 2*(sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(
-a)/a) + sqrt(b*sin(x)^n + a))/n]
```

giac [A] time = 0.15, size = 46, normalized size = 0.98

$$\frac{2\left(\frac{ab\arctan\left(\frac{\sqrt{b\sin(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}} + \sqrt{b\sin(x)^n+a}\,b\right)}{bn}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
```

```
[Out] 2*(a*b*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/sqrt(-a) + sqrt(b*sin(x)^n + a
)*b)/(b*n)
```

maple [A] time = 0.11, size = 38, normalized size = 0.81

$$\frac{2\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}-2\sqrt{a}\ \operatorname{arctanh}\left(\frac{\sqrt{a+b\left(\sin^{n}\left(x\right)\right)}}{\sqrt{a}}\right)}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a+b*sin(x)^n)^(1/2),x)
```

```
[Out] 1/n*(2*(a+b*sin(x)^n)^(1/2)-2*a^(1/2)*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))
)
```

maxima [A] time = 0.89, size = 57, normalized size = 1.21

$$\frac{\sqrt{a}\log\left(\frac{\sqrt{b\sin(x)^{n}+a}-\sqrt{a}}{\sqrt{b\sin(x)^{n}+a}+\sqrt{a}}\right)}{n}+\frac{2\sqrt{b\sin(x)^{n}+a}}{n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] sqrt(a)*log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(a
)))/n + 2*sqrt(b*sin(x)^n + a)/n
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int \cot(x)\sqrt{a+b\sin(x)^{n}}\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)*(a + b*sin(x)^n)^(1/2),x)
```

```
[Out] int(cot(x)*(a + b*sin(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a+b\sin^{n}\left(x\right)}\cot\left(x\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)*(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(sqrt(a + b*sin(x)**n)*cot(x), x)
```

3.580 $\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$

Optimal. Leaf size=29

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

Rubi [A] time = 0.08, antiderivative size = 29, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 15, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.267, Rules used = {3230, 266, 63, 208}

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
```

```
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

Rule 63

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - (a*d)/b +
(d*x^p)/b)^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 208

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-(a/b), 2]*ArcTanh[x/
Rt[-(a/b), 2]])/a, x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx = \text{Subst}\left(\int \frac{1}{x\sqrt{a+bx^n}}\,dx, x, \sin(x)\right)$$
$$= \frac{\text{Subst}\left(\int \frac{1}{x\sqrt{a+bx}}\,dx, x, \sin^n(x)\right)}{n}$$
$$= \frac{2\,\text{Subst}\left(\int \frac{1}{-\frac{a}{b}+\frac{x^2}{b}}\,dx, x, \sqrt{a+b\sin^n(x)}\right)}{bn}$$
$$= -\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Mathematica [A] time = 0.01, size = 29, normalized size = 1.00

$$-\frac{2\tanh^{-1}\left(\frac{\sqrt{a+b\sin^n(x)}}{\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[x]/Sqrt[a + b*Sin[x]^n],x]
[Out] (-2*ArcTanh[Sqrt[a + b*Sin[x]^n]/Sqrt[a]])/(Sqrt[a]*n)
```

fricas [A] time = 0.48, size = 74, normalized size = 2.55

$$\left[\frac{\log\left(\frac{b\sin(x)^n-2\sqrt{b\sin(x)^n+a}\,\sqrt{a}+2\,a}{\sin(x)^n}\right)}{\sqrt{a}\,n}, \frac{2\sqrt{-a}\,\arctan\left(\frac{\sqrt{b\sin(x)^n+a}\,\sqrt{-a}}{a}\right)}{an}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="fricas")
[Out] [log((b*sin(x)^n - 2*sqrt(b*sin(x)^n + a)*sqrt(a) + 2*a)/sin(x)^n)/(sqrt(a)
*n), 2*sqrt(-a)*arctan(sqrt(b*sin(x)^n + a)*sqrt(-a)/a)/(a*n)]
```

giac [A] time = 0.13, size = 27, normalized size = 0.93

$$\frac{2\arctan\left(\frac{\sqrt{b\sin(x)^n+a}}{\sqrt{-a}}\right)}{\sqrt{-a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="giac")
[Out] 2*arctan(sqrt(b*sin(x)^n + a)/sqrt(-a))/(sqrt(-a)*n)
```

maple [A] time = 0.16, size = 24, normalized size = 0.83

$$-\frac{2\,\mathrm{arctanh}\left(\frac{\sqrt{a+b\left(\sin^n(x)\right)}}{\sqrt{a}}\right)}{n\sqrt{a}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a+b*sin(x)^n)^(1/2),x)
```

```
[Out] -2*arctanh((a+b*sin(x)^n)^(1/2)/a^(1/2))/n/a^(1/2)
```

maxima [A] time = 0.81, size = 41, normalized size = 1.41

$$\frac{\log\left(\frac{\sqrt{b\sin(x)^n+a}-\sqrt{a}}{\sqrt{b\sin(x)^n+a}+\sqrt{a}}\right)}{\sqrt{a}\,n}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)^n)^(1/2),x, algorithm="maxima")
```

```
[Out] log((sqrt(b*sin(x)^n + a) - sqrt(a))/(sqrt(b*sin(x)^n + a) + sqrt(a)))/(sqr
t(a)*n)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.03

$$\int \frac{\cot(x)}{\sqrt{a+b\sin(x)^n}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(x)/(a + b*sin(x)^n)^(1/2),x)
```

```
[Out] int(cot(x)/(a + b*sin(x)^n)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\cot(x)}{\sqrt{a+b\sin^n(x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(x)/(a+b*sin(x)**n)**(1/2),x)
```

```
[Out] Integral(cot(x)/sqrt(a + b*sin(x)**n), x)
```

3.581 $\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^m(c+dx)\,(a+b\sin^n(c+dx))^p\,,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$$

Mathematica [A] time = 4.10, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan^m(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^m, x]
```

fricas [A] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin(dx+c)^n+a\right)^p\tan(dx+c)^m\,,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^n+a\right)^p\tan(dx+c)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

maple [A] time = 3.74, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^p\,(\tan^m(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{m}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^m,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^m, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c+d\,x\right)^{m}\,\left(a+b\sin\left(c+d\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)^m*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**m,x)
```

```
[Out] Timed out
```

3.582 $\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\tan^3(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$$

Mathematica [A] time = 22.87, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan^3(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^3, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^3,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

maple [A] time = 1.09, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\left(\tan^3\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^3,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^3, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c+d\,x\right)^{3}\,\left(a+b\,\sin\left(c+d\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(tan(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**3,x)
[Out] Timed out
```

3.583 $\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$

Optimal. Leaf size=24

$$\mathrm{Int}\left(\tan(c+dx)\,(a+b\sin^n(c+dx))^p\,,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

Rubi [A] time = 0.03, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Mathematica [A] time = 0.96, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x],x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x], x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

maple [A] time = 1.03, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\tan\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(dx + c\right)^{n} + a\right)^{p} \tan\left(dx + c\right) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c),x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c), x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c + d\,x\right)\,\left(a + b\sin\left(c + d\,x\right)^{n}\right)^{p} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c),x)
```

```
[Out] Timed out
```

3.584 $\int \cot(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$

Optimal. Leaf size=55

$$-\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

```
[Out] -hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(1+p)/a/d/
n/(1+p)
```

Rubi [A] time = 0.08, antiderivative size = 55, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3230, 266, 65}

$$-\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\begin{aligned}\int \cot(c+dx)\,(a+b\sin^n(c+dx))^p\,dx &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx,x,\sin(c+dx)\right)}{d}\\ &= \frac{\operatorname{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx,x,\sin^n(c+dx)\right)}{dn}\\ &= -\frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)}\end{aligned}$$

Mathematica [A] time = 0.05, size = 55, normalized size = 1.00

$$-\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] -((Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[
c + d*x]^n)^(1 + p))/(a*d*n*(1 + p)))
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right),x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

maple [F] time = 0.96, size = 0, normalized size = 0.00

$$\int\cot\left(dx+c\right)\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.02

$$\int\cot\left(c+d\,x\right)\,{\left(a+b\,{\sin\left(c+d\,x\right)}^n\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(cot(c + d*x)*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int (a + b\sin^n(c + dx))^p \cot(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)*(a+b*sin(d*x+c)**n)**p,x)
[Out] Integral((a + b*sin(c + d*x)**n)**p*cot(c + d*x), x)
```

3.585 $\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx$

Optimal. Leaf size=136

$$\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}-\frac{\csc^2(c+dx)\,(a+b\sin^n(c+dx))^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)}{2d}$$

```
[Out] hypergeom([1, 1+p],[2+p],1+b*sin(d*x+c)^n/a)*(a+b*sin(d*x+c)^n)^(1+p)/a/d/n
/(1+p)-1/2*csc(d*x+c)^2*hypergeom([-p, -2/n],[(-2+n)/n],-b*sin(d*x+c)^n/a)*
(a+b*sin(d*x+c)^n)^p/d/((1+b*sin(d*x+c)^n/a)^p)
```

Rubi [A] time = 0.15, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.261, Rules used = {3230, 1844, 365, 364, 266, 65}

$$\frac{(a+b\sin^n(c+dx))^{p+1}\,{}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{adn(p+1)}-\frac{\csc^2(c+dx)\,(a+b\sin^n(c+dx))^p\left(\frac{b\sin^n(c+dx)}{a}+1\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] (Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin[c + d*x]^n)/a]*(a + b*Sin[c
+ d*x]^n)^(1 + p))/(a*d*n*(1 + p)) - (Csc[c + d*x]^2*Hypergeometric2F1[-2/n
, -p, -((2 - n)/n), -((b*Sin[c + d*x]^n)/a)]*(a + b*Sin[c + d*x]^n)^p)/(2*d
*(1 + (b*Sin[c + d*x]^n)/a)^p)
```

Rule 65

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[((c + d*x
)^(n + 1)*Hypergeometric2F1[-m, n + 1, n + 2, 1 + (d*x)/c])/(d*(n + 1)*(-(d
/(b*c)))^m), x] /; FreeQ[{b, c, d, m, n}, x] &&  !IntegerQ[n] && (IntegerQ[
m] || GtQ[-(d/(b*c)), 0])
```

Rule 266

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[1/n, Subst[
Int[x^(Simplify[(m + 1)/n] - 1)*(a + b*x)^p, x], x, x^n], x] /; FreeQ[{a, b
, m, n, p}, x] && IntegerQ[Simplify[(m + 1)/n]]
```

Rule 364

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Simp[(a^
p*(c*x)^(m + 1)*Hypergeometric2F1[-p, (m + 1)/n, (m + 1)/n + 1, -((b*x^n)/a
)])/(c*(m + 1)), x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0] && (ILt
Q[p, 0] || GtQ[a, 0])
```

Rule 365

```
Int[((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> Dist[(a^
IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x^n)/a)^FracPart[p], Int[(c*x)^
m*(1 + (b*x^n)/a)^p, x], x] /; FreeQ[{a, b, c, m, n, p}, x] &&  !IGtQ[p, 0]
 &&  !(ILtQ[p, 0] || GtQ[a, 0])
```

Rule 1844

```
Int[(Pq_)*((c_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.), x_Symbol] :> I
nt[ExpandIntegrand[(c*x)^m*Pq*(a + b*x^n)^p, x], x] /; FreeQ[{a, b, c, m, n
```

```
, p}, x] && (PolyQ[Pq, x] || PolyQ[Pq, x^n]) && !IGtQ[m, 0]
```

Rule 3230

```
Int[((a_) + (b_.)*((c_.)*sin[(e_.) + (f_.)*(x_)])^(n_))^(p_.)*tan[(e_.) + (
f_.)*(x_)]^(m_.), x_Symbol] :> With[{ff = FreeFactors[Sin[e + f*x], x]}, Di
st[ff^(m + 1)/f, Subst[Int[(x^m*(a + b*(c*ff*x)^n)^p)/(1 - ff^2*x^2)^((m +
1)/2), x], x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, e, f, n, p}, x] && I
LtQ[(m - 1)/2, 0]
```

Rubi steps

$$\int \cot^3(c+dx)\,(a+b\sin^n(c+dx))^p\,dx = \frac{\text{Subst}\left(\int \frac{\left(1-x^2\right)(a+bx^n)^p}{x^3}\,dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \left(\frac{(a+bx^n)^p}{x^3} - \frac{(a+bx^n)^p}{x}\right)dx, x, \sin(c+dx)\right)}{d}$$
$$= \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x^3}\,dx, x, \sin(c+dx)\right)}{d} - \frac{\text{Subst}\left(\int \frac{(a+bx^n)^p}{x}\,dx, x, \sin(c+\ldots\right.}{d}$$
$$= -\frac{\text{Subst}\left(\int \frac{(a+bx)^p}{x}\,dx, x, \sin^n(c+dx)\right)}{dn} + \frac{\left((a+b\sin^n(c+dx))^p\left(1+\frac{b\sin\ldots}{\ldots}\right.\right.}{\ldots}$$
$$= \frac{{}_2F_1\left(1,1+p;2+p;1+\frac{b\sin^n(c+dx)}{a}\right)(a+b\sin^n(c+dx))^{1+p}}{adn(1+p)} - \frac{\csc^2(c+\ldots}{\ldots}$$

Mathematica [A] time = 1.03, size = 129, normalized size = 0.95

$$\frac{(a+b\sin^n(c+dx))^p\left(\frac{2\left(a+b\sin^n(c+dx)\right){}_2F_1\left(1,p+1;p+2;\frac{b\sin^n(c+dx)}{a}+1\right)}{an(p+1)} - \csc^2(c+dx)\left(\frac{b\sin^n(c+dx)}{a}+1\right)^{-p}{}_2F_1\left(-\frac{2}{n},-p;\frac{n-2}{n};-\ldots\right.\right.}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cot[c + d*x]^3*(a + b*Sin[c + d*x]^n)^p,x]
```

```
[Out] ((a + b*Sin[c + d*x]^n)^p*((2*Hypergeometric2F1[1, 1 + p, 2 + p, 1 + (b*Sin
[c + d*x]^n)/a]*(a + b*Sin[c + d*x]^n))/(a*n*(1 + p)) - (Csc[c + d*x]^2*Hyp
ergeometric2F1[-2/n, -p, (-2 + n)/n, -((b*Sin[c + d*x]^n)/a)])/(1 + (b*Sin[
c + d*x]^n)/a)^p))/(2*d)
```

fricas [F] time = 0.46, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin(dx+c)^n+a\right)^p\cot(dx+c)^3, x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
```

```
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin(dx+c)^n+a\right)^p\cot(dx+c)^3\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

maple [F] time = 1.06, size = 0, normalized size = 0.00

$$\int \left(\cot^{3}\left(dx+c\right)\right)\left(a+b\left(\sin^{n}\left(dx+c\right)\right)\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
[Out] int(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\cot\left(dx+c\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^3*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^3, x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \cot\left(c+d\,x\right)^{3}\,\left(a+b\,\sin\left(c+d\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)^3*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**3*(a+b*sin(d*x+c)**n)**p,x)
[Out] Timed out
```

3.586 $\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$

Optimal. Leaf size=26

$$\mathrm{Int}\left(\tan^4(c+dx)\,(a+b\sin^n(c+dx))^p\,,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx=\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Mathematica [A] time = 25.25, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan^4(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^4, x]
```

fricas [A] time = 0.50, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{4},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

maple [A] time = 0.96, size = 0, normalized size = 0.00

$$\int \left(a+b\left(\sin^{n}\left(dx+c\right)\right)\right)^{p}\left(\tan^{4}\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^{n}+a\right)^{p}\tan\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^4,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c+d\,x\right)^{4}\,{\left(a+b\,{\sin\left(c+d\,x\right)}^{n}\right)}^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**4,x)
```

```
[Out] Timed out
```

3.587 $\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$

Optimal. Leaf size=26

$$\operatorname{Int}\left(\tan^2(c+dx)\,(a+b\sin^n(c+dx))^p\,,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx = \int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$$

Mathematica [A] time = 2.75, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p \tan^2(c+dx)\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p*Tan[c + d*x]^2, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\tan\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

maple [A] time = 0.91, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^p\left(\tan^2(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(dx + c\right)^{n} + a\right)^{p} \tan\left(dx + c\right)^{2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p*tan(d*x+c)^2,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*tan(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \tan\left(c + d\,x\right)^2\,\left(a + b\sin\left(c + d\,x\right)^n\right)^p\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(tan(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p*tan(d*x+c)**2,x)
```

```
[Out] Timed out
```

3.588 $\int (a+b\sin^n(c+dx))^p\,dx$

Optimal. Leaf size=17

$$\mathrm{Int}\left((a+b\sin^n(c+dx))^p\,,x\right)$$

```
[Out] Unintegrable((a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A] time = 0.01, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int (a+b\sin^n(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int (a+b\sin^n(c+dx))^p\,dx=\int (a+b\sin^n(c+dx))^p\,dx$$

Mathematica [A] time = 0.66, size = 0, normalized size = 0.00

$$\int (a+b\sin^n(c+dx))^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[(a + b*Sin[c + d*x]^n)^p, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\mathrm{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p, x)
```

maple [A] time = 0.68, size = 0, normalized size = 0.00

$$\int (a+b(\sin^n(dx+c)))^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int((a+b*sin(d*x+c)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(dx + c\right)^{n} + a\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.06

$$\int \left(a + b \sin \left(c + d\,x\right)^{n}\right)^{p} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int((a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

3.589 $\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A] time = 2.09, size = 0, normalized size = 0.00

$$\int \cot^2(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[Cot[c + d*x]^2*(a + b*Sin[c + d*x]^n)^p, x]
```

fricas [A] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^2,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

maple [A] time = 0.97, size = 0, normalized size = 0.00

$$\int \left(\cot^2\left(dx+c\right)\right)\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

```
[Out] int(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin \left(dx+c\right)^{n}+a\right)^{p} \cot \left(dx+c\right)^{2} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^2*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
```

```
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^2, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot \left(c+d\,x\right)^2\,{\left(a+b\,{\sin \left(c+d\,x\right)}^n\right)}^p \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p,x)
```

```
[Out] int(cot(c + d*x)^2*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**2*(a+b*sin(d*x+c)**n)**p,x)
```

```
[Out] Timed out
```

3.590 $\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$

Optimal. Leaf size=26

$$\text{Int}\left(\cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p,x\right)$$

```
[Out] Unintegrable(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

Rubi [A] time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Defer[Int][Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

Rubi steps

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx=\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Mathematica [A] time = 36.68, size = 0, normalized size = 0.00

$$\int \cot^4(c+dx)\left(a+b\sin^n(c+dx)\right)^p\,dx$$

Verification is Not applicable to the result.

```
[In] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p,x]
[Out] Integrate[Cot[c + d*x]^4*(a + b*Sin[c + d*x]^n)^p, x]
```

fricas [A] time = 0.49, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^4,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="fricas")
[Out] integral((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

giac [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(dx+c\right)^n+a\right)^p\cot\left(dx+c\right)^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="giac")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

maple [A] time = 1.08, size = 0, normalized size = 0.00

$$\int \left(\cot^4\left(dx+c\right)\right)\left(a+b\left(\sin^n\left(dx+c\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
[Out] int(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x)
```

maxima [A] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b \sin\left(dx+c\right)^{n}+a\right)^{p} \cot\left(dx+c\right)^{4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)^4*(a+b*sin(d*x+c)^n)^p,x, algorithm="maxima")
[Out] integrate((b*sin(d*x + c)^n + a)^p*cot(d*x + c)^4, x)
```

mupad [A] time = 0.00, size = -1, normalized size = -0.04

$$\int \cot\left(c+d\,x\right)^{4}\,\left(a+b\sin\left(c+d\,x\right)^{n}\right)^{p}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p,x)
[Out] int(cot(c + d*x)^4*(a + b*sin(c + d*x)^n)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cot(d*x+c)**4*(a+b*sin(d*x+c)**n)**p,x)
[Out] Timed out
```

3.591 $\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{(2a-b)\sqrt{\sin(2e+2fx)}\,F\left(e+fx-\frac{\pi}{4}\middle|2\right)}{3fg^2\sqrt{d\sin(e+fx)}\,\sqrt{g\cos(e+fx)}}+\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}$$

```
[Out] 2/3*(a+b)*(d*sin(f*x+e))^(1/2)/d/f/g/(g*cos(f*x+e))^(3/2)-1/3*(2*a-b)*(sin(
e+1/4*Pi+f*x)^2)^(1/2)/sin(e+1/4*Pi+f*x)*EllipticF(cos(e+1/4*Pi+f*x),2^(1/2
))*sin(2*f*x+2*e)^(1/2)/f/g^2/(g*cos(f*x+e))^(1/2)/(d*sin(f*x+e))^(1/2)
```

Rubi [A] time = 0.18, antiderivative size = 114, normalized size of antiderivative = 1.07, number of steps used = 7, number of rules used = 7, integrand size = 37, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.189, Rules used = {3202, 457, 329, 237, 335, 275, 232}

$$\frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}}-\frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+fx))^{3/2}F\left(\frac{1}{2}\csc^{-1}(\sin(e+fx))\middle|2\right)}{3d^2fg(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x]]),x]
```

```
[Out] (2*(a + b)*Sqrt[d*Sin[e + f*x]])/(3*d*f*g*(g*Cos[e + f*x])^(3/2)) - (2*(2*a
 - b)*(1 - Csc[e + f*x]^2)^(3/4)*EllipticF[ArcCsc[Sin[e + f*x]]/2, 2]*(d*Si
n[e + f*x])^(3/2))/(3*d^2*f*g*(g*Cos[e + f*x])^(3/2))
```

Rule 232

```
Int[((a_) + (b_.)*(x_)^2)^(-3/4), x_Symbol] :> Simp[(2*EllipticF[(1*ArcSin[
Rt[-(b/a), 2]*x])/2, 2])/(a^(3/4)*Rt[-(b/a), 2]), x] /; FreeQ[{a, b}, x] &&
 GtQ[a, 0] && NegQ[b/a]
```

Rule 237

```
Int[((a_) + (b_.)*(x_)^4)^(-3/4), x_Symbol] :> Dist[(x^3*(1 + a/(b*x^4))^(3
/4))/(a + b*x^4)^(3/4), Int[1/(x^3*(1 + a/(b*x^4))^(3/4)), x], x] /; FreeQ[
{a, b}, x]
```

Rule 275

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k = GCD[m
+ 1, n]}, Dist[1/k, Subst[Int[x^((m + 1)/k - 1)*(a + b*x^(n/k))^p, x], x, x
^k], x] /; k != 1] /; FreeQ[{a, b, p}, x] && IGtQ[n, 0] && IntegerQ[m]
```

Rule 329

```
Int[((c_.)*(x_))^(m_)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> With[{k =
 Denominator[m]}, Dist[k/c, Subst[Int[x^(k*(m + 1) - 1)*(a + (b*x^(k*n))/c^
n)^p, x], x, (c*x)^(1/k)], x]] /; FreeQ[{a, b, c, p}, x] && IGtQ[n, 0] && F
ractionQ[m] && IntBinomialQ[a, b, c, n, m, p, x]
```

Rule 335

```
Int[(x_)^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_), x_Symbol] :> -Subst[Int[(a +
b/x^n)^p/x^(m + 2), x], x, 1/x] /; FreeQ[{a, b, p}, x] && ILtQ[n, 0] && Int
egerQ[m]
```

Rule 457

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_.)*((c_) + (d_.)*(x_)^(n
_)), x_Symbol] :> -Simp[((b*c - a*d)*(e*x)^(m + 1)*(a + b*x^n)^(p + 1))/(a*
b*e*n*(p + 1)), x] - Dist[(a*d*(m + 1) - b*c*(m + n*(p + 1) + 1))/(a*b*n*(p
 + 1)), Int[(e*x)^m*(a + b*x^n)^(p + 1), x], x] /; FreeQ[{a, b, c, d, e, m,
 n}, x] && NeQ[b*c - a*d, 0] && LtQ[p, -1] && (( !IntegerQ[p + 1/2] && NeQ[
p, -5/4]) ||  !RationalQ[m] || (IGtQ[n, 0] && ILtQ[p + 1/2, 0] && LeQ[-1, m
, -(n*(p + 1))]))
```

Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rubi steps

$$\int \frac{a+b\sin^2(e+fx)}{(g\cos(e+fx))^{5/2}\sqrt{d\sin(e+fx)}}\,dx = \frac{\cos^2(e+fx)^{3/4}\,\text{Subst}\left(\int \frac{a+bx^2}{\sqrt{dx}\left(1-x^2\right)^{7/4}}\,dx, x, \sin(e+fx)\right)}{fg(g\cos(e+fx))^{3/2}}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left((-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\ldots}{\sqrt{dx}\ldots}\right.}{3fg(g\cos(e+fx)\ldots}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\cos^2(e+fx)^{3/4}\right)\text{Subst}\left(\int \frac{\ldots}{(1-\ldots}\right.}{3dfg(g\cos(e+f\ldots}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e\ldots\right.}{3dfg\ldots}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left(2(-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e\ldots\right.}{3dfg(g\ldots}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} + \frac{\left((-2a+b)\left(1-\csc^2(e+fx)\right)^{3/4}(d\sin(e+\ldots\right.}{3dfg(g\cos\ldots}$$

$$= \frac{2(a+b)\sqrt{d\sin(e+fx)}}{3dfg(g\cos(e+fx))^{3/2}} - \frac{2(2a-b)\left(1-\csc^2(e+fx)\right)^{3/4}F\left(\frac{1}{2}\sin^{-1}\ldots\right.}{3d^2fg(g\cos(e+\ldots}$$

Mathematica [C] time = 0.18, size = 102, normalized size = 0.95

$$\frac{2\cos^2(e+fx)^{3/4}\left(5a\sin(e+fx)\,{}_2F_1\left(\frac{1}{4},\frac{7}{4};\frac{5}{4};\sin^2(e+fx)\right)+b\sin^3(e+fx)\,{}_2F_1\left(\frac{5}{4},\frac{7}{4};\frac{9}{4};\sin^2(e+fx)\right)\right)}{5fg\sqrt{d\sin(e+fx)}\,(g\cos(e+fx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sin[e + f*x]^2)/((g*Cos[e + f*x])^(5/2)*Sqrt[d*Sin[e + f*x
]]),x]
```

```
[Out] (2*(Cos[e + f*x]^2)^(3/4)*(5*a*Hypergeometric2F1[1/4, 7/4, 5/4, Sin[e + f*x
]^2]*Sin[e + f*x] + b*Hypergeometric2F1[5/4, 7/4, 9/4, Sin[e + f*x]^2]*Sin[
e + f*x]^3))/(5*f*g*(g*Cos[e + f*x])^(3/2)*Sqrt[d*Sin[e + f*x]])
```

fricas [F] time = 0.47, size = 0, normalized size = 0.00

$$\text{integral}\left(-\frac{\left(b\cos\left(fx+e\right)^{2}-a-b\right)\sqrt{g\cos\left(fx+e\right)}\sqrt{d\sin\left(fx+e\right)}}{dg^{3}\cos\left(fx+e\right)^{3}\sin\left(fx+e\right)},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="fricas")
```

```
[Out] integral(-(b*cos(f*x + e)^2 - a - b)*sqrt(g*cos(f*x + e))*sqrt(d*sin(f*x +
e))/(d*g^3*cos(f*x + e)^3*sin(f*x + e)), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)^{2}+a}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

maple [B] time = 1.03, size = 327, normalized size = 3.06

$$\left(-2\cos\left(fx+e\right)\sin\left(fx+e\right)\sqrt{-\frac{-1+\cos\left(fx+e\right)-\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)+\sin\left(fx+e\right)}{\sin\left(fx+e\right)}}\sqrt{\frac{-1+\cos\left(fx+e\right)}{\sin\left(fx+e\right)}}\,\text{EllipticF}\left(\sqrt{-\frac{-1+\text{c}}{}}\right.\right.$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x)
```

```
[Out] 1/3/f*(-2*cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1
/2)*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+
e))^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1
/2))*a+cos(f*x+e)*sin(f*x+e)*(-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2)
*((-1+cos(f*x+e)+sin(f*x+e))/sin(f*x+e))^(1/2)*((-1+cos(f*x+e))/sin(f*x+e))
^(1/2)*EllipticF((-(-1+cos(f*x+e)-sin(f*x+e))/sin(f*x+e))^(1/2),1/2*2^(1/2)
)*b+2^(1/2)*cos(f*x+e)*a+2^(1/2)*cos(f*x+e)*b-2^(1/2)*a-2^(1/2)*b)*cos(f*x+
e)*sin(f*x+e)/(-1+cos(f*x+e))/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2)*2^(
1/2)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{b\sin\left(fx+e\right)^{2}+a}{\left(g\cos\left(fx+e\right)\right)^{\frac{5}{2}}\sqrt{d\sin\left(fx+e\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)^2)/(g*cos(f*x+e))^(5/2)/(d*sin(f*x+e))^(1/2),x, a
lgorithm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)/((g*cos(f*x + e))^(5/2)*sqrt(d*sin(f*x + e
))), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{b\sin\left(e+f\,x\right)^2+a}{\left(g\,\cos\left(e+f\,x\right)\right)^{5/2}\,\sqrt{d\,\sin\left(e+f\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
x)
```

```
[Out] int((a + b*sin(e + f*x)^2)/((g*cos(e + f*x))^(5/2)*(d*sin(e + f*x))^(1/2)),
 x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sin(f*x+e)**2)/(g*cos(f*x+e))**(5/2)/(d*sin(f*x+e))**(1/2),x
)
```

```
[Out] Timed out
```

3.592 $\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\, d$

Optimal. Leaf size=137

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\ldots\right.}{df(n+1)}$$

```
[Out] c*AppellF1(1/2+1/2*n,1/2-1/2*m,-p,3/2+1/2*n,sin(f*x+e)^2,-b*sin(f*x+e)^2/a)
*(c*cos(f*x+e))^(-1+m)*(cos(f*x+e)^2)^(1/2-1/2*m)*(d*sin(f*x+e))^(1+n)*(a+b
*sin(f*x+e)^2)^p/d/f/(1+n)/((1+b*sin(f*x+e)^2/a)^p)
```

Rubi [A] time = 0.21, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3202, 511, 510}

$$\frac{c\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^{m-1}(d\sin(e+fx))^{n+1}\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\frac{1-m}{2},-p;\ldots\right.}{df(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (c*AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e
 + f*x]^2)/a)]*(c*Cos[e + f*x])^(-1 + m)*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Si
n[e + f*x])^(1 + n)*(a + b*Sin[e + f*x]^2)^p)/(d*f*(1 + n)*(1 + (b*Sin[e +
f*x]^2)/a)^p)
```

Rule 510

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[(a^p*c^q*(e*x)^(m + 1)*AppellF1[(m + 1)/n, -p, -
q, 1 + (m + 1)/n, -((b*x^n)/a), -((d*x^n)/c)])/(e*(m + 1)), x] /; FreeQ[{a,
 b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] && NeQ[m, n
 - 1] && (IntegerQ[p] || GtQ[a, 0]) && (IntegerQ[q] || GtQ[c, 0])
```

Rule 511

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Dist[(a^IntPart[p]*(a + b*x^n)^FracPart[p])/(1 + (b*x
^n)/a)^FracPart[p], Int[(e*x)^m*(1 + (b*x^n)/a)^p*(c + d*x^n)^q, x], x] /;
FreeQ[{a, b, c, d, e, m, n, p, q}, x] && NeQ[b*c - a*d, 0] && NeQ[m, -1] &&
 NeQ[m, n - 1] &&  !(IntegerQ[p] || GtQ[a, 0])
```

Rule 3202

```
Int[(cos[(e_.) + (f_.)*(x_)]*(c_.))^(m_)*((d_.)*sin[(e_.) + (f_.)*(x_)])^(n
_.)*((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)]^2)^(p_.), x_Symbol] :> With[{ff =
 FreeFactors[Sin[e + f*x], x]}, Dist[(ff*c^(2*IntPart[(m - 1)/2] + 1)*(c*Co
s[e + f*x])^(2*FracPart[(m - 1)/2]))/(f*(Cos[e + f*x]^2)^FracPart[(m - 1)/2
]), Subst[Int[(d*ff*x)^n*(1 - ff^2*x^2)^((m - 1)/2)*(a + b*ff^2*x^2)^p, x],
 x, Sin[e + f*x]/ff], x]] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] &&  !Int
egerQ[m]
```

Rubi steps

$$\int (c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\,dx=\frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\right)\text{Sub}}{}$$

$$=\frac{\left(c(c\cos(e+fx))^{2\left(-\frac{1}{2}+\frac{m}{2}\right)}\cos^2(e+fx)^{\frac{1}{2}-\frac{m}{2}}\left(a+\right.\right.}{}$$

$$=\frac{cF_1\left(\frac{1+n}{2};\frac{1-m}{2},-p;\frac{3+n}{2};\sin^2(e+fx),-\frac{b\sin^2(e+fx)}{a}\right.}{}$$

Mathematica [A] time = 0.84, size = 135, normalized size = 0.99

$$\frac{\tan(e+fx)\cos^2(e+fx)^{\frac{1-m}{2}}(c\cos(e+fx))^m(d\sin(e+fx))^n\left(a+b\sin^2(e+fx)\right)^p\left(\frac{b\sin^2(e+fx)}{a}+1\right)^{-p}F_1\left(\frac{n+1}{2};\right.}{f(n+1)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(c*Cos[e + f*x])^m*(d*Sin[e + f*x])^n*(a + b*Sin[e + f*x]^2)^p,x]
```

```
[Out] (AppellF1[(1 + n)/2, (1 - m)/2, -p, (3 + n)/2, Sin[e + f*x]^2, -((b*Sin[e +
 f*x]^2)/a)]*(c*Cos[e + f*x])^m*(Cos[e + f*x]^2)^((1 - m)/2)*(d*Sin[e + f*x
])^n*(a + b*Sin[e + f*x]^2)^p*Tan[e + f*x])/(f*(1 + n)*(1 + (b*Sin[e + f*x]
^2)/a)^p)
```

fricas [F] time = 1.84, size = 0, normalized size = 0.00

$$\text{integral}\left(\left(-b\cos\left(fx+e\right)^2+a+b\right)^p\left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n,x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="fricas")
```

```
[Out] integral((-b*cos(f*x + e)^2 + a + b)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^
n, x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\left(b\sin\left(fx+e\right)^2+a\right)^p\left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="giac")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

maple [F] time = 8.13, size = 0, normalized size = 0.00

$$\int\left(c\cos\left(fx+e\right)\right)^m\left(d\sin\left(fx+e\right)\right)^n\left(a+b\left(\sin^2\left(fx+e\right)\right)\right)^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

```
[Out] int((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x)
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sin\left(fx+e\right)^{2}+a\right)^{p}\left(c\cos\left(fx+e\right)\right)^{m}\left(d\sin\left(fx+e\right)\right)^{n}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))^m*(d*sin(f*x+e))^n*(a+b*sin(f*x+e)^2)^p,x, algorit
hm="maxima")
```

```
[Out] integrate((b*sin(f*x + e)^2 + a)^p*(c*cos(f*x + e))^m*(d*sin(f*x + e))^n, x
)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int {\left(c\,\cos\left(e+f\,x\right)\right)}^m\,{\left(d\,\sin\left(e+f\,x\right)\right)}^n\,{\left(b\,{\sin\left(e+f\,x\right)}^2+a\right)}^p\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p,x)
```

```
[Out] int((c*cos(e + f*x))^m*(d*sin(e + f*x))^n*(a + b*sin(e + f*x)^2)^p, x)
```

sympy [F(-1)] time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*cos(f*x+e))**m*(d*sin(f*x+e))**n*(a+b*sin(f*x+e)**2)**p,x)
```

```
[Out] Timed out
```

3.593 $\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}\,E\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}}$$

```
[Out] (cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticE(sin(e+f*x
+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2)
/f/(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(1/2)
```

Rubi [F] time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] ((I/2)*Defer[Subst][Defer[Int][Sqrt[a + (c + b*x)^2/(1 + x^2)]/(I - x), x],
 x, Tan[e + f*x]])/f + ((I/2)*Defer[Subst][Defer[Int][Sqrt[a + (c + b*x)^2/
(1 + x^2)]/(I + x), x], x, Tan[e + f*x]])/f
```

Rubi steps

$$\begin{aligned}\int \sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}\,dx &= \frac{\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{1+x^2}\,dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{\text{Subst}\left(\int \left(\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i-x)}+\frac{i\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{2(i+x)}\right)dx,x,\tan(e+fx)\right)}{f}\\ &= \frac{i\,\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{i-x}\,dx,x,\tan(e+fx)\right)}{2f}+\frac{i\,\text{Subst}\left(\int \frac{\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}{i+x}\right.}{2}\end{aligned}$$

Mathematica [B] time = 1.67, size = 325, normalized size = 4.11

$$-\frac{\left(\left(b^2-c^2\right)\sin(2(e+fx))+2bc\cos(2(e+fx))\right)\sqrt{2a+\left(c^2-b^2\right)\cos(2(e+fx))+b^2+2bc\sin(2(e+fx))+c^2}}{\sqrt{2}\,f\sqrt{\left(b^2+c^2\right)^2}\sqrt{\frac{\left(\left(b^2-c^2\right)\sin(2(e+fx))+2bc\cos(2(e+fx))\right)^2}{\left(b^2+c^2\right)^2}}\sqrt{\frac{2a+\left(c^2-b^2\right)\cos}{2a}}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] -((EllipticE[ArcSin[Sqrt[(Sqrt[(b^2 + c^2)^2] + (b^2 - c^2)*Cos[2*(e + f*x)
] - 2*b*c*Sin[2*(e + f*x)])/Sqrt[(b^2 + c^2)^2]]/Sqrt[2]], (2*Sqrt[(b^2 + c
^2)^2])/(2*a + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[2*a + b^2 + c^2 + (-b
^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)]]*(2*b*c*Cos[2*(e + f*x)
] + (b^2 - c^2)*Sin[2*(e + f*x)]))/(Sqrt[2]*Sqrt[(b^2 + c^2)^2]*f*Sqrt[(2*a
 + b^2 + c^2 + (-b^2 + c^2)*Cos[2*(e + f*x)] + 2*b*c*Sin[2*(e + f*x)])/(2*a
 + b^2 + c^2 + Sqrt[(b^2 + c^2)^2])]*Sqrt[(2*b*c*Cos[2*(e + f*x)] + (b^2 -
c^2)*Sin[2*(e + f*x)])^2/(b^2 + c^2)^2]))
```

fricas [F] time = 0.51, size = 0, normalized size = 0.00

$$\operatorname{integral}\left(\sqrt{2\,bc\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(b^{2}-c^{2}\right)\cos\left(fx+e\right)^{2}+b^{2}+a},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^2
+ b^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

maple [B] time = 4.68, size = 4067972, normalized size = 51493.32

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^{2}+a}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int\sqrt{a+{\left(c\,\cos\left(e+f\,x\right)+b\,\sin\left(e+f\,x\right)\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
```

```
[Out] int((a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

sympy [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a + \left(b \sin \left(e + fx\right) + c \cos \left(e + fx\right)\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
[Out] Integral(sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

3.594 $\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$

Optimal. Leaf size=79

$$\frac{\sqrt{\frac{(b\sin(e+fx)+c\cos(e+fx))^2}{a}+1}\,F\left(e+fx+\tan^{-1}(b,c)|-\frac{b^2+c^2}{a}\right)}{f\sqrt{a+(b\sin(e+fx)+c\cos(e+fx))^2}}$$

```
[Out] (cos(e+f*x+arctan(b,c))^2)^(1/2)/cos(e+f*x+arctan(b,c))*EllipticF(sin(e+f*x
+arctan(b,c)),((-b^2-c^2)/a)^(1/2))*(1+(c*cos(f*x+e)+b*sin(f*x+e))^2/a)^(1/
2)/f/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2)
```

Rubi [F] time = 0.70, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.000, Rules used = {}

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx$$

Verification is Not applicable to the result.

```
[In] Int[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] ((I/2)*Defer[Subst][Defer[Int][1/((I - x)*Sqrt[a + (c + b*x)^2/(1 + x^2)]),
 x], x, Tan[e + f*x]])/f + ((I/2)*Defer[Subst][Defer[Int][1/((I + x)*Sqrt[a
 + (c + b*x)^2/(1 + x^2)]), x], x, Tan[e + f*x]])/f
```

Rubi steps

$$\int \frac{1}{\sqrt{a+(c\cos(e+fx)+b\sin(e+fx))^2}}\,dx = \frac{\text{Subst}\left(\int \frac{1}{(1+x^2)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{\text{Subst}\left(\int \left(\frac{i}{2(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}+\frac{i}{2(i+x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\right)dx, x, \tan(e+fx)\right)}{f}$$

$$= \frac{i\,\text{Subst}\left(\int \frac{1}{(i-x)\sqrt{a+\frac{(c+bx)^2}{1+x^2}}}\,dx, x, \tan(e+fx)\right)}{2f} + \frac{i\,\text{Subst}\left(\int \frac{1}{(i+x)\sqrt{a+}}\right.}{}$$

Mathematica [C] time = 1.59, size = 529, normalized size = 6.70

$$\frac{\sqrt{2}\sec\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(\sin\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)-1\right)}{2a+bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}+b^2+c^2}}\sqrt{-\frac{bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}\left(\sin\left(\tan^{-1}\left(\frac{c^2-b^2}{2bc}\right)+2(e+fx)\right)\right.}{2a-bc\sqrt{\frac{(b^2+c^2)^2}{b^2c^2}}+b^2+c^2}}}{}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[1/Sqrt[a + (c*Cos[e + f*x] + b*Sin[e + f*x])^2],x]
```

```
[Out] (Sqrt[2]*AppellF1[1/2, 1/2, 1/2, 3/2, (2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^
2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*a + b^2
 + c^2 - b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]), (2*a + b^2 + c^2 + b*c*Sqrt[(b
^2 + c^2)^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]])/(2*
a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)])]*Sec[2*(e + f*x) + ArcTa
n[(-b^2 + c^2)/(2*b*c)]]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(-1 + Si
n[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 + b*c*Sqrt
[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[-((b*c*Sqrt[(b^2 + c^2)^2/(b^2*c^2)]*(1 +
 Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]))/(2*a + b^2 + c^2 - b*c*S
qrt[(b^2 + c^2)^2/(b^2*c^2)]))]*Sqrt[2*a + b^2 + c^2 + b*c*Sqrt[(b^2 + c^2)
^2/(b^2*c^2)]*Sin[2*(e + f*x) + ArcTan[(-b^2 + c^2)/(2*b*c)]]])/(b*c*Sqrt[(
b^2 + c^2)^2/(b^2*c^2)]*f)
```

fricas [F] time = 0.48, size = 0, normalized size = 0.00

$$\text{integral}\left(\frac{1}{\sqrt{2\,bc\cos\left(fx+e\right)\sin\left(fx+e\right)-\left(b^2-c^2\right)\cos\left(fx+e\right)^2+b^2+a}},x\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="fricas")
```

```
[Out] integral(1/sqrt(2*b*c*cos(f*x + e)*sin(f*x + e) - (b^2 - c^2)*cos(f*x + e)^
2 + b^2 + a), x)
```

giac [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="giac")
```

```
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

maple [C] time = 3.76, size = 258179, normalized size = 3268.09

output too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x)
```

```
[Out] result too large to display
```

maxima [F] time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{\left(c\cos\left(fx+e\right)+b\sin\left(fx+e\right)\right)^2+a}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))^2)^(1/2),x, algorithm="maxima")
```

```
[Out] integrate(1/sqrt((c*cos(f*x + e) + b*sin(f*x + e))^2 + a), x)
```

mupad [F] time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{1}{\sqrt{a+\left(c\,\cos\left(e+f\,x\right)+b\,\sin\left(e+f\,x\right)\right)^2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2),x)
```

```
[Out] int(1/(a + (c*cos(e + f*x) + b*sin(e + f*x))^2)^(1/2), x)
```

sympy **[F]** time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{1}{\sqrt{a + \left(b \sin\left(e + fx\right) + c \cos\left(e + fx\right)\right)^2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(1/(a+(c*cos(f*x+e)+b*sin(f*x+e))**2)**(1/2),x)
```

```
[Out] Integral(1/sqrt(a + (b*sin(e + f*x) + c*cos(e + f*x))**2), x)
```

Chapter 4

Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.0.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)


GradeAntiderivative[result_,optimal_] :=
  If[ExpnType[result]<=ExpnType[optimal],
    If[FreeQ[result,Complex] || Not[FreeQ[optimal,Complex]],
      If[LeafCount[result]<=2*LeafCount[optimal],
        "A",
      "B"],
    "C"],
  If[FreeQ[result,Integrate] && FreeQ[result,Int],
    "C",
  "F"]]


(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
```

```
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
        Exp,Log,
        Sin,Cos,Tan,Cot,Sec,Csc,
        ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
        Sinh,Cosh,Tanh,Coth,Sech,Csch,
        ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
        Erf, Erfc, Erfi,
        FresnelS, FresnelC,
        ExpIntegralE, ExpIntegralEi, LogIntegral,
        SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
        Gamma, LogGamma, PolyGamma,
        Zeta, PolyLog, ProductLog,
        EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]
```

```
AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.0.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems

GradeAntiderivative := proc(result,optimal)
local leaf_count_result, leaf_count_optimal,ExpnType_result,ExpnType_optimal,
   debug:=false;

      leaf_count_result:=leafcount(result);
      #do NOT call ExpnType() if leaf size is too large. Recursion problem
      if leaf_count_result > 500000 then
          return "B";
      fi;

      leaf_count_optimal:=leafcount(optimal);

      ExpnType_result:=ExpnType(result);
      ExpnType_optimal:=ExpnType(optimal);

      if debug then
         print("ExpnType_result",ExpnType_result," ExpnType_optimal=",
   ExpnType_optimal);
      fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

  #This check below actually is not needed, since I only
  #call this grading only for passed integrals. i.e. I check
  #for "F" before calling this. But no harm of keeping it here.
  #just in case.


  if not type(result,freeof('int')) then
     return "F";
  end if;


  if ExpnType_result<=ExpnType_optimal then
      if debug then
          print("ExpnType_result<=ExpnType_optimal");
      fi;
```

```
      if is_contains_complex(result) then
         if is_contains_complex(optimal) then
            if debug then
               print("both result and optimal complex");
            fi;
            #both result and optimal complex
            if leaf_count_result<=2*leaf_count_optimal then
               return "A";
            else
               return "B";
            end if
         else #result contains complex but optimal is not
            if debug then
               print("result contains complex but optimal is not");
            fi;
            return "C";
         end if
      else # result do not contain complex
           # this assumes optimal do not as well
           if debug then
               print("result do not contain complex, this assumes optimal do
   not as well");
           fi;
           if leaf_count_result<=2*leaf_count_optimal then
              if debug then
                 print("leaf_count_result<=2*leaf_count_optimal");
              fi;
              return "A";
           else
              if debug then
                 print("leaf_count_result>2*leaf_count_optimal");
              fi;
              return "B";
           end if
      end if
  else #ExpnType(result) > ExpnType(optimal)
      if debug then
         print("ExpnType(result) > ExpnType(optimal)");
      fi;
      return "C";
  end if

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
# 8 = integrate function
```

```
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:
```

```
AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.0.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#              Port of original Maple grading function by
#              Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#              added 'exp_polar'
from sympy import *

def leaf_count(expr):
    #sympy do not have leaf count function. This is approximation
    return round(1.7*count_ops(expr))

def is_sqrt(expr):
    if isinstance(expr,Pow):
        if expr.args[1] == Rational(1,2):
            return True
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
            asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
            asinh,acosh,atanh,acoth,asech,acsch
        ]

def is_special_function(func):
    return func in [ erf,erfc,erfi,
             fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
             gamma,loggamma,digamma,zeta,polylog,LambertW,
             elliptic_f,elliptic_e,elliptic_pi,exp_polar
         ]

def is_hypergeometric_function(func):
    return func in [hyper]

def is_appell_function(func):
    return func in [appellf1]
```

```
def is_atom(expn):
    try:
        if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
           return True
        else:
           return False

    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):   #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):  #type(op(2,expn),'integer')
            return expnType(expn.args[0])   #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):  #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))  #max(2,ExpnType(op(1,expn
   )))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,
   ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or
   type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):  #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))  #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)   #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,
   expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType
   ,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9
```

```
#main function
def grade_antiderivative(result,optimal):

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        return "F"

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
   well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```

4.0.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
```

```
        else:
            return False
    else:
        return False

def is_elementary_function(func):
    debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    debug=False
    if debug: print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
           'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','
    sinh_integral'
           'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
           'polylog','lambert_w','elliptic_f','elliptic_e',
           'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','
    hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']   #[appellf1] can't find this in
    sagemath

def is_atom(expn):

    debug=False
    if debug: print ("Enter is_atom")


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-
    sagemath-equivalent-to-atomic-type-in-maple/
    try:
        if expn.parent() is SR:
```

```
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens")
    :
            return expn in expn.parent().base_ring() or expn in expn.parent().
    gens()
        return False

    except AttributeError as error:
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))   #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational: #type(isinstance(expn.args[0],
    Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.
    args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer
    )
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],
    Rational)
            if type(expn.operands()[0])==Rational: #isinstance(expn.args[0],
    Rational)
                return 1
            else:
                return max(2,expnType(expn.operands()[0]))  #max(2,expnType(
    expn.args[0]))
        else:
            return max(3,expnType(expn.operands()[0]),expnType(expn.operands()
    [1])) #max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1]))
    elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #
    isinstance(expn,Add) or isinstance(expn,Mul)
        m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
        m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
        return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.operator()):  #is_elementary_function(expn
    .func)
        return max(3,expnType(expn.operands()[0]))
    elif is_special_function(expn.operator()): #is_special_function(expn.func)
        m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(
    expn.args)))
        return max(4,m1)   #max(4,m1)
    elif is_hypergeometric_function(expn.operator()): #
    is_hypergeometric_function(expn.func)
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(5,m1)   #max(5,m1)
    elif is_appell_function(expn.operator()):
```

```
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(6,m1)   #max(6,m1)
    elif str(expn).find("Integral") != -1: #this will never happen, since it
                #is checked before calling the grading function that is passed.
                #but kept it here.
        m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(
    expn.args)))
        return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    if debug: print ("Enter grade_antiderivative for sagemath")

    leaf_count_result  = tree_size(result) #leaf_count(result)
    leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

    if debug: print ("leaf_count_result=", leaf_count_result, "
    leaf_count_optimal=",leaf_count_optimal)


    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",
    expnType_optimal)

    if expnType_result <= expnType_optimal:
        if result.has(I):
            if optimal.has(I): #both result and optimal complex
                if leaf_count_result <= 2*leaf_count_optimal:
                    return "A"
                else:
                    return "B"
            else: #result contains complex but optimal is not
                return "C"
        else: # result do not contain complex, this assumes optimal do not as
    well
            if leaf_count_result <= 2*leaf_count_optimal:
                return "A"
            else:
                return "B"
    else:
        return "C"
```